QCD Thermodynamics at finite T and μ in HTLpt.

NAJMUL HAQUE

Institut für Theoretische Physik,
Universität Giessen

Lunch Club Seminar

About me

Name: Najmul Haque

PhD: 2008-2014

Institute: Saha Institute of Nuclear Physics, Kolkata, India.

Advisor: Prof. Munshigolam Mustafa.

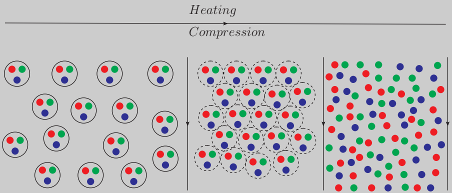
Thesis: Some Applications of Hard Thermal Loop Perturbation Theory in Quark Gluon Plasma

Post Doc:

1. Sept 2014 - April 2015
   Saha Inst.
2. April 2015 - April 2016
   Kent State University, USA with Prof. Michael Strickland.
3. Aug 2016 -
   JLU with Prof. Christian Fischer as a Humboldt Post Doctoral Fellow.

Nuclear matter at extreme condition

- At low energy Quarks and Gluons are confined within hadrons.
- At high temperature and/or at high baryonic density $\Longrightarrow$ Deconfinement $\Longrightarrow$ Produce a new state of matter known as Quark Gluon Plasma (QGP).



QGP $\equiv$ A thermalized state of matter in which (quasi) free quarks and gluons are deconfined from hadrons, so that color degrees of freedom become manifest over larger volume, rather than merely hadronic volume.

Where can we find QGP?

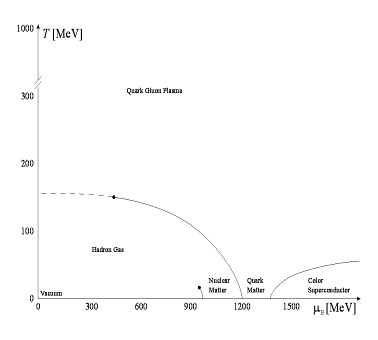
1. In the early Universe, about 10^{-5}s after cosmic ‘Big Bang’. Temperature $(T) \sim 10^{12}K \sim 200$ MeV and net baryon density $\mu_B \rightarrow 0$.
2. In the interior of Astro-physical objects like Neutron star; where mass densities are likely to exceed 10^{15} gm/cm^3 which is about four times the central density of nuclei while the surface temperatures are as low as 10^5 K or less
3. In heavy-ion-collisions experiments.

QCD Phase Diagram & EoS

• At High temperature and/or density Quarks and Gluons become deconfined and produce QGP.

• In ongoing RHIC experiments and and also future FAIR experiments the chemical potential of deconfined Nuclear matter is finite.

• Determination of EoS of hot and dense Nuclear matter is essential to QGP phenomenology.



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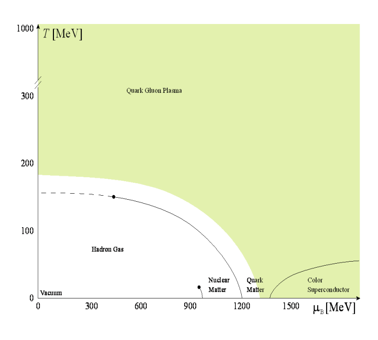

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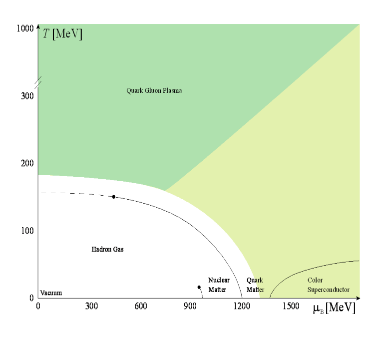


Thermodynamics using Lattice QCD

- The currently most reliable method for determining the equation of state at finite temperature is lattice QCD.
- Due to the sign problem, lattice QCD can not compute EoS at finite baryon chemical potential straightforwardly.
- It can compute thermodynamic functions at small chemical potential by making a Taylor expansion of the partition function around $\mu = 0$ and extrapolating the result as

$$P(T,\mu) = P(T,\mu=0) + \frac{\mu^2}{2}\left.\frac{\partial^2 P}{\partial \mu^2}\right|_{\mu=0} + \frac{\mu^4}{4!}\left.\frac{\partial^4 P}{\partial \mu^4}\right|_{\mu=0} + \cdots$$

- The extrapolations can only be trusted at small chemical potential, it would be nice to have an alternative framework for calculating QCD thermodynamical quantities at finte T and μ.

Thermodynamics using perturbation theory

- At sufficiently high temperature, the value of the strong coupling constant is small ⇒ It works well at high T.
- Unfortunately, it turns out that a strict expansion in the coupling constant does not converges at the temperature those are relevant for heavy-ion collision experiments.
- The source of the poor convergence comes from contributions from soft momenta, $p \sim gT$.
- One needs a way of reorganizing the perturbative series which treats the soft sector more carefully.

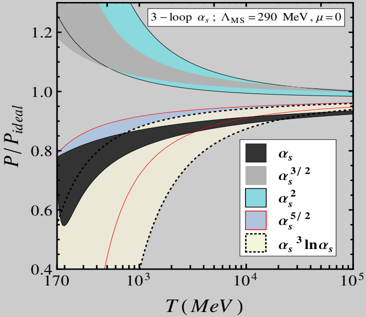
- QCD free energy at finite T known up to three loops (g^5) since 1994

  - Coefficient at $\mathcal{O}(g^6 \log g)$ also known
    (Kajantie, Laine, Rummukainen, Schroder, 2003)

  - Extension to finite chemical potential
    (Vuorinen, 2003)

  - Very poorly convergent: need $T \sim 10^5$ GeV for convergence.

  - Our goal: Find a more convergent gauge-invariant scheme for $T > 2T_c$



- Resulting framework should also be applicable to describe dynamical properties of the QGP

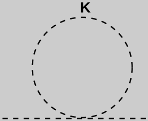
Hard Thermal Loop perturbation theory(HTLpt)

- Hard Thermal Loop resummation originally proposed by Braaten and Pisarski in 1990 and initially applied to calculate quark and gluon damping rate, photon and dilepton production rate etc.
- In 1999, HTLpt was developed by Andersen, Braaten and Strickland using the concept of HTL resummation.
- HTLpt is a gauge invariant reorganization of usual perturbation at finite temperature and finite chemical potential and higher order diagrams contribute to lower order.
- In HTL approximation we define *Two Scales of Momentum*
  1. Hard momentum: $p_0, p \sim T$.
  2. Soft momentum: $p_0, p \sim gT$.
- In HTL approximation we are interested in high temperature limits, so one can take Loop Momentum $\gg$ External Momentum

HTL in scalar theory

$$\text{Lagrangian:} \qquad \mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - g^2\phi^4.$$

The one loop self energy:



After subtraction the vacuum contribution it gives

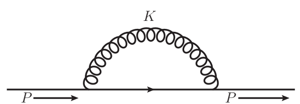
$$\Pi_1 = -12g^2\int\frac{d^4K}{(2\pi)^4}\frac{1}{K^2} = 12g^2T\sum_{n=-\infty}^{n=\infty}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_n^2+k^2} = g^2T^2.$$

Effective propagator

$$D^\star = \frac{1}{P^2-\Pi_1}$$

HTL in Gauge theory: Electron/Quark Propagator

- Two point electron(quark) self energy in HTL



$$\Sigma^{e(q)}_{HTL}(P) = \frac{m^2_{e(q)}}{p}\gamma_0 Q_0\left(\frac{p_0}{p}\right) + \frac{m^2_{e(q)}}{p}\left\{1 - \frac{p_0}{p}Q_0\left(\frac{p_0}{p}\right)\right\}\vec{\gamma}\cdot\hat{p}, \qquad . \quad (1)$$

where

$$m^2_{e(q)} = \frac{1}{8}(C_F)g^2T^2$$

and

$$Q_0\left(\frac{p_0}{p}\right) = \frac{1}{2}\ln\frac{p_0+p}{p_0-p}$$

is Legendre function of second kind.

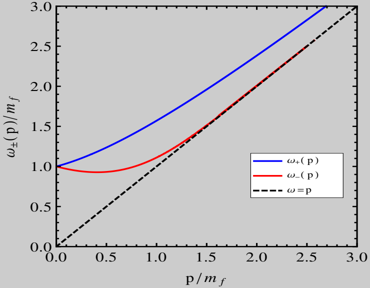
HTL in Gauge theory: Electron(Quark) Propagator

- The full propagator

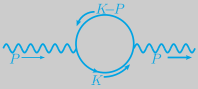
$$iS^\star(P) = \frac{1}{\not{P} - \Sigma(P)} = \frac{1}{2}\left[\frac{\gamma^0 - \vec{\gamma}\cdot\hat{p}}{D_+(P)} + \frac{\gamma^0 + \vec{\gamma}\cdot\hat{p}}{D_-(P)}\right].$$

$$D_\pm(p_0,p) = -p_0 \pm p + \frac{m_q^2}{p}\left[\pm 1 + \frac{1}{2}\left(1 \mp \frac{p_0}{p}\right)\ln\frac{p_0+p}{p_0-p}\right]$$

Dispersion relation: $D_\pm(p_0,p) = 0$



- Photon Self Energy:



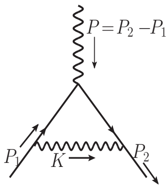
$$\Pi_{\mu\nu} = e^2 \int \frac{d^4K}{(2\pi)^4} \left[\gamma_\mu \not{K} \gamma_\nu (\not{K} - \not{P})\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P).$$

In HTL approximation

$$\Pi_{\mu\nu} = -2m_\gamma^2 \int \frac{d\Omega}{2\pi} \left(\frac{p_0 \hat{K}_\mu \hat{K}_\nu}{\hat{K}\cdot P} - g_{\mu 0} g_{\nu 0} \right) \ , \quad m_\gamma^2 = \frac{e^2T^2}{6}.$$

- In QCD there are three other diagrams but the form of gluon self energy will be same with $e^2 \to g^2 \left(N_c + \frac{N_f}{2}\right)$

- The three point function



$$\Gamma_\mu(P_1, P_2) = e^2 \int \frac{d^4K}{(2\pi)^4} \left[\gamma_\alpha(\not{K} - \not{P}_2)\gamma_\mu(\not{K} - \not{P}_1)\gamma^\alpha\right] \times \Delta(K)\tilde{\Delta}(P_2 - K)\tilde{\Delta}(P_1 - K)$$

$$\Gamma^\mu_{HTL}(P_1, P_2; P) = \left(\gamma^\mu + m_f^2 \left\langle \frac{\hat{\not{K}}\hat{K}^\mu}{(\hat{K}\cdot P_1)(\hat{K}\cdot P_2)} \right\rangle\right) = (\gamma^\mu + \delta\Gamma^\mu),$$

- HTL two electron-two photon function

$$\Gamma^{\mu\nu}(P_1, P_2; Q_1, Q_2) = -m_f^2 \int \frac{d\Omega}{4\pi} \frac{\hat{\not{K}}\hat{K}^\mu\hat{K}^\nu}{\left[(P_1 + Q_1)\cdot\hat{K}\right]\left(P_2 - Q_1)\cdot\hat{K}\right]} \times \left[\frac{1}{P_1.\hat{K}} + \frac{1}{P_2.\hat{K}}\right].$$

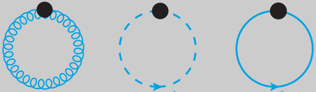
- Total Lagrangian density:

$$\mathcal{L} = (\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL})\big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL},$$
$$\mathcal{L}_{\rm HTL} = (1-\delta) i m_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{Y_\mu}{Y\cdot D} \right\rangle_{\hat{\mathbf{y}}} \psi$$
$$-\frac{1}{2}(1-\delta) m_D^2 \,\mathrm{Tr}\left(F_{\mu\alpha} \left\langle \frac{Y^\alpha Y_\beta}{(Y\cdot D)^2} \right\rangle_{\hat{\mathbf{y}}} F^{\mu\beta} \right),$$

- The HTLpt Lagrangian reduces to the QCD Lagrangian if we set $\delta = 1$.
- Physical observables are calculated in HTLpt by expanding in powers of δ, truncating at some specified order, and then setting $\delta = 1$.
- m_D and m_q are two parameters will be treated as Debye mass and thermal quark mass respectively.

One loop HTL thermodynamics

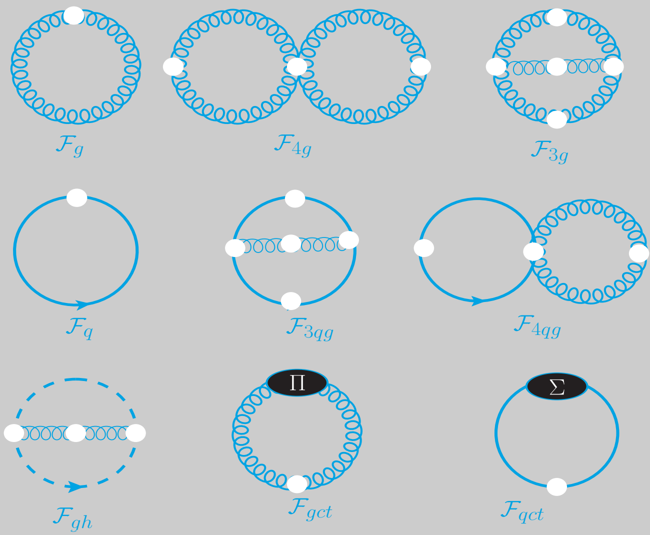
The Feynman diagrams that will contribute to the thermodynamic potential in one loop:



$$
\begin{aligned}
\mathcal{P}(T,\mu) \;=\; & 2N_f N_c T \int \frac{d^3k}{(2\pi)^3} \Bigg[\ln\left(1+e^{-\beta(\omega_+-\mu)}\right) + \ln\left(\frac{1+e^{-\beta(\omega_--\mu)}}{1+e^{-\beta(k-\mu)}}\right) \\
+ & \ln\left(1+e^{-\beta(\omega_++\mu)}\right) + \ln\left(\frac{1+e^{-\beta(\omega_-+\mu)}}{1+e^{-\beta(k+\mu)}}\right) + \beta\omega_+ + \beta(\omega_- - k) \\
+ & \int_{-k}^{k} d\omega \left(\frac{2m_q^2}{\omega^2-k^2}\right) \beta_+(\omega,k) \left[\ln\left(1+e^{-\beta(\omega-\mu)}\right) + \ln\left(1+e^{-\beta(\omega+\mu)}\right) + \beta\omega\right] \Bigg] \\
+ & \quad \text{Gluonic contibution}
\end{aligned}
$$

$$
\rho = \frac{\partial \mathcal{P}}{\partial \mu}; \qquad S = \frac{\partial \mathcal{P}}{\partial T}; \qquad \chi = \frac{\partial^2 \mathcal{P}}{\partial \mu^2}
$$

Two loop HTL thermodynamics



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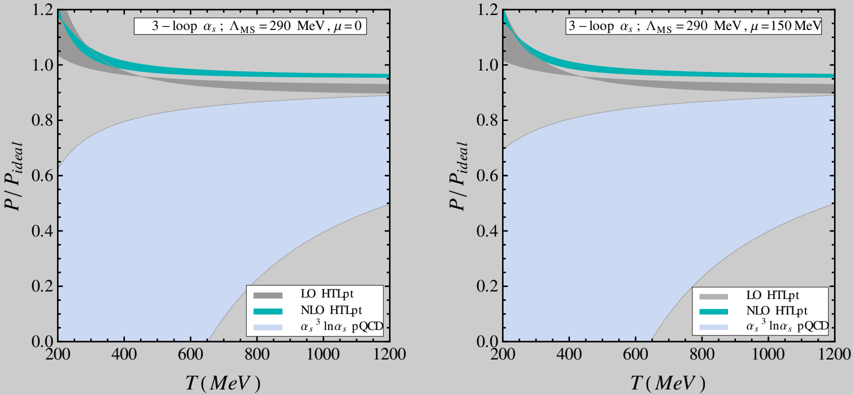
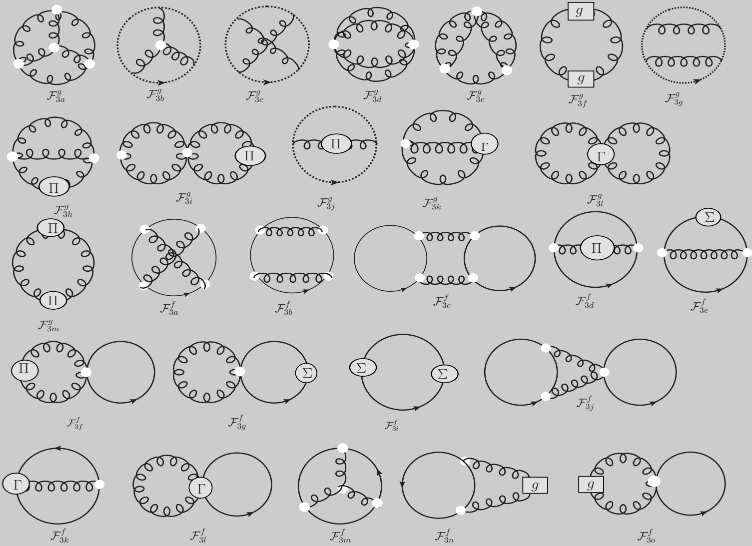
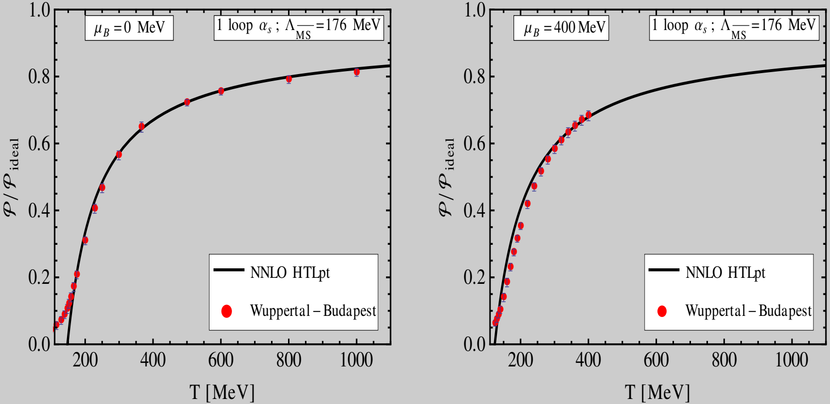


Three loop HTL thermodynamics



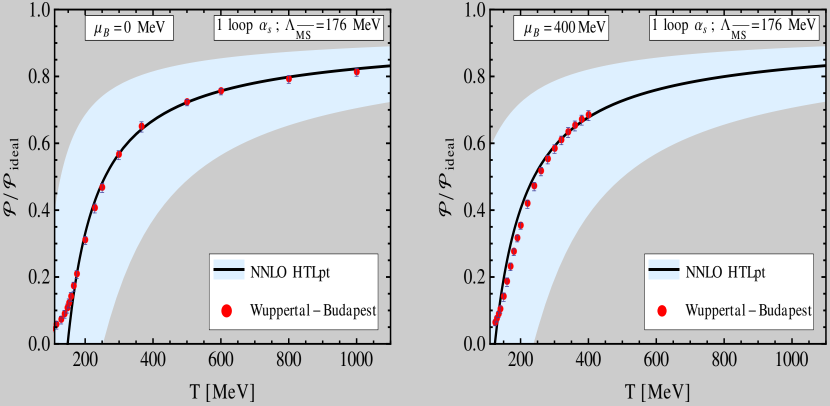
$$
\begin{aligned}
\mathcal{P}_{\rm NNLO} = {}& \frac{d_A\pi^2T^4}{45}\Bigg[\!\!\Bigg[1+\frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat\mu^2+\frac{240}{7}\hat\mu^4\right)-\frac{15}{4}\hat m_D^3-\frac{s_F\alpha_s}{\pi}\Bigg[\frac{5}{8}\left(5+72\hat\mu^2+144\hat\mu^4\right) \\
&+90\hat m_q^2\hat m_D-\frac{15}{2}\left(1+12\hat\mu^2\right)\hat m_D-\frac{15}{2}\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\right)\hat m_D^3\Bigg]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\Bigg[-\frac{45}{2}\hat m_D\left(1+12\hat\mu^2\right) \\
&+\frac{15}{64}\Bigg\{35-32\left(1-12\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}+472\hat\mu^2+1328\hat\mu^4+64\Big(6(1+8\hat\mu^2)\aleph(1,z)+3i\hat\mu(1+4\hat\mu^2)\aleph(0,z) \\
&-36i\hat\mu\aleph(2,z)\Big)\Bigg\}\Bigg]+\left(\frac{s_F\alpha_s}{\pi}\right)^2\Bigg[\frac{5}{4\hat m_D}\left(1+12\hat\mu^2\right)^2+30\left(1+12\hat\mu^2\right)\frac{\hat m_q^2}{\hat m_D}+\frac{25}{12}\Bigg\{\frac{1}{20}\left(1+168\hat\mu^2+2064\hat\mu^4\right) \\
&+\left(1+\frac{72}{5}\hat\mu^2+\frac{144}{5}\hat\mu^4\right)\ln\frac{\hat\Lambda}{2}+\frac{3\gamma_E}{5}\left(1+12\hat\mu^2\right)^2-\frac{8}{5}(1+12\hat\mu^2)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{34}{25}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{72}{5}\Big[3\aleph(3,2z) \\
&+8\aleph(3,z)-12\hat\mu^2\aleph(1,2z)-2(1+8\hat\mu^2)\aleph(1,z)+12i\hat\mu\left(\aleph(2,z)+\aleph(2,2z)\right)-i\hat\mu(1+12\hat\mu^2)\,\aleph(0,z)\Big]\Bigg\} \\
&-\frac{15}{2}\left(1+12\hat\mu^2\right)\left(2\ln\frac{\hat\Lambda}{2}-1-\aleph(z)\right)\hat m_D\Bigg]+\frac{c_A\alpha_s}{3\pi}\frac{s_F\alpha_s}{\pi}\Bigg[\frac{15}{2\hat m_D}\left(1+12\hat\mu^2\right)+90\frac{\hat m_q^2}{\hat m_D} \\
&-\frac{235}{16}\Bigg\{\left(1+\frac{792}{47}\hat\mu^2+\frac{1584}{47}\hat\mu^4\right)\ln\frac{\hat\Lambda}{2}-\frac{24\gamma_E}{47}\left(1+12\mu^2\right)+\frac{319}{940}\left(1+\frac{2040}{319}\hat\mu^2+\frac{38640}{319}\hat\mu^4\right)-\frac{268}{235}\frac{\zeta'(-3)}{\zeta(-3)} \\
&-\frac{144}{47}\left(1+12\hat\mu^2\right)\ln\hat m_D-\frac{44}{47}\left(1+\frac{156}{11}\hat\mu^2\right)\frac{\zeta'(-1)}{\zeta(-1)}-\frac{72}{47}\Big[4i\hat\mu\aleph(0,z)+\left(5-92\hat\mu^2\right)\aleph(1,z)+144i\hat\mu\aleph(2,z) \\
&+52\aleph(3,z)\Big]\Bigg\}+\frac{315}{4}\Bigg\{\left(1+\frac{132}{7}\hat\mu^2\right)\ln\frac{\hat\Lambda}{2}+\frac{11}{7}\left(1+12\hat\mu^2\right)\gamma_E+\frac{9}{14}\left(1+\frac{132}{9}\hat\mu^2\right)+\frac{2}{7}\aleph(z)\Bigg\}\hat m_D\Bigg] \\
&+\frac{c_A\alpha_s}{3\pi}\Bigg[-\frac{15}{4}+\frac{45}{2}\hat m_D-\frac{135}{2}\hat m_D^2-\frac{495}{4}\left(\ln\frac{\hat\Lambda_g}{2}+\frac{5}{22}+\gamma_E\right)\Bigg]+\left(\frac{c_A\alpha_s}{3\pi}\right)^2\Bigg[\frac{45}{4\hat m_D}-\frac{165}{8}\Bigg(\ln\frac{\hat\Lambda_g}{2} \\
&-\frac{72}{11}\ln\hat m_D-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\Bigg)+\frac{1485}{4}\left(\ln\frac{\hat\Lambda_g}{2}-\frac{79}{44}+\gamma_E-\ln 2-\frac{\pi^2}{11}\right)\hat m_D\Bigg]\Bigg]\!\!\Bigg]
\end{aligned}
$$

NNLO pressure for QCD HTL perturbation theory



- Thick Black Line: Renormalization Scale $\Lambda = 2\pi\sqrt{T^2 + \mu^2/\pi^2}$
-

NNLO pressure for QCD HTL perturbation theory



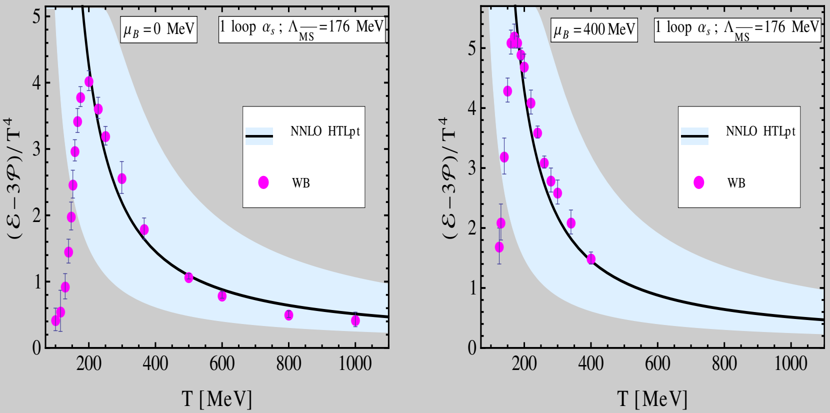
- Thick Black Line: Renormalization Scale $\Lambda = 2\pi\sqrt{T^2 + \mu^2/\pi^2}$
- Band : Varying center value by factor of 2.

Other thermodynamic quantities

1. Energy density: $\mathcal{E} = T\frac{\partial \mathcal{P}}{\partial T} + \mu\frac{\partial \mathcal{P}}{\partial \mu} - \mathcal{P}$
2. Trace anomaly $\mathcal{I} = \mathcal{E} - 3\mathcal{P}$
3. Speed of sound $c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}}$
4. Entropy density $\mathcal{S} = \frac{\partial \mathcal{P}}{\partial T}$
5. Quark number density $\rho = \frac{\partial \mathcal{P}}{\partial \mu}$
6. Quark number susceptibilities $\chi^n(T) = \left.\frac{\partial^n \mathcal{P}}{\partial \mu^n}\right|_{\mu\to 0}$
7. Baryon number susceptibilities $\chi_B^n(T) = \left.\frac{\partial^n \mathcal{P}(\mu_{\mathcal{B}},\mathcal{T})}{\partial \mu_B^n}\right|_{\mu\to 0}$ by changing the chemical potential basis from (μ_u, μ_d, μ_s) to (μ_B, μ_I, μ_S) at $\mu_I = \mu_S = 0$.
8. Later in PRD93(2015) 054045, we have extended thermodynamic calculations in finite isospin chemical potential also.

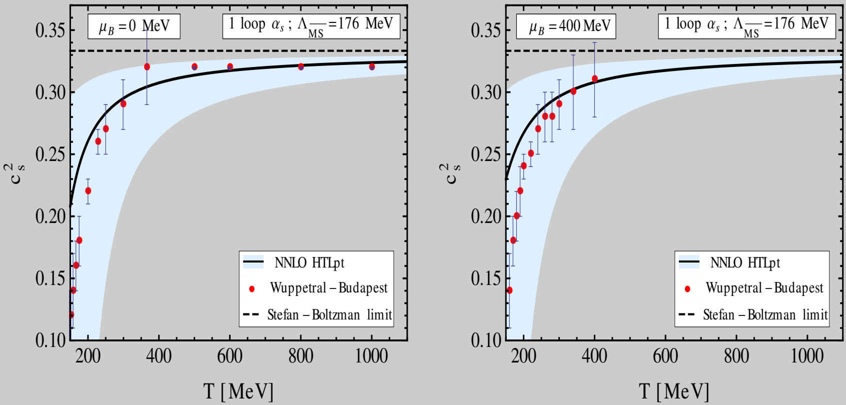
Trace Anomaly

Trace Anomaly=$\mathcal{E} - 3\mathcal{P}$



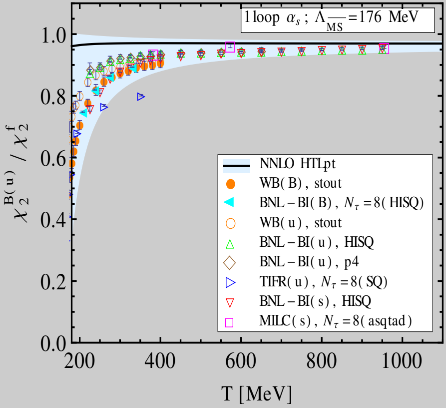
Speed of Sound

$$c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}}$$



Second order Quark Number Susceptibility

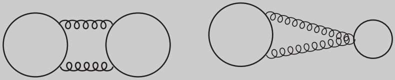
$$\chi_2^u = \frac{\partial^2 \mathcal{P}}{\partial \mu^2}\bigg|_{\mu\to 0}$$



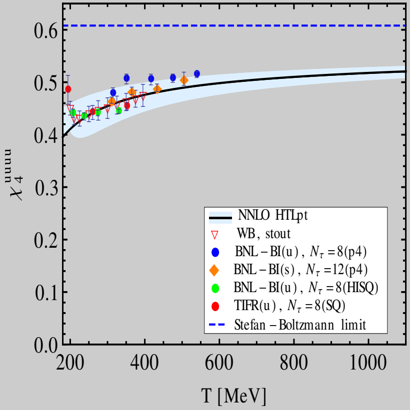
Fourth order Quark Number Susceptibility

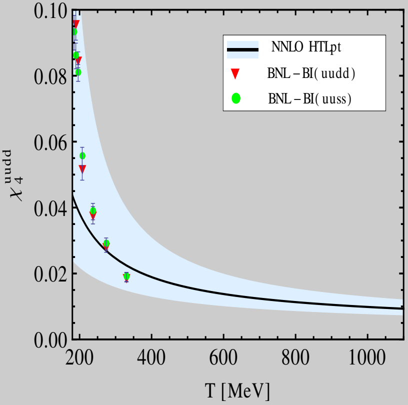
Diagonal Susceptibility $\chi_4^u = \frac{\partial^4 \mathcal{P}}{\partial \mu^4}$
Off-diagonal Susceptibility $\chi_4^{uudd} = \frac{\partial^4 \mathcal{P}}{\partial \mu_u^2 \partial \mu_d^2}$.

The following two diagrams will contribute to only off-diagonal Susceptibility χ_4^{uudd}.



Fourth order Quark Number Susceptibility





Baryon number susceptibilities

$$\chi_B^n(T) \equiv \left.\frac{\partial^n \mathcal{P}}{\partial \mu_B^n}\right|_{\mu_B=0} .$$

$$\chi_2^B = \frac{1}{9}\left[\chi_2^{uu} + \chi_2^{dd} + \chi_2^{ss} + 2\chi_2^{ud} + 2\chi_2^{ds} + 2\chi_2^{us}\right] ,$$

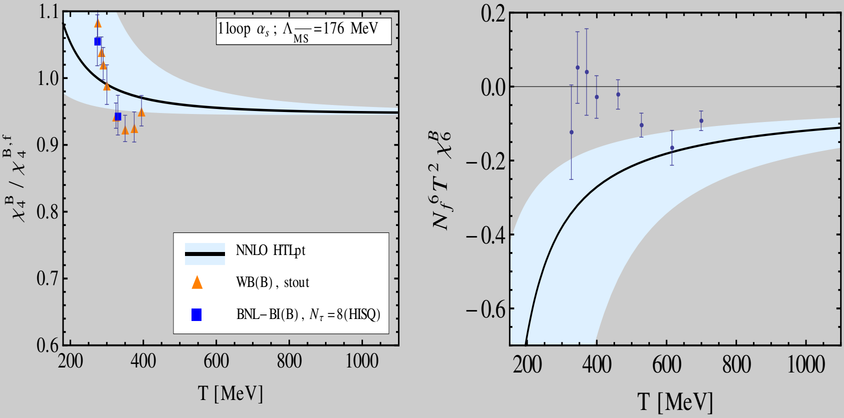
and

$$\begin{aligned}\chi_4^B = \frac{1}{81}\Big[&\chi_4^{uuuu} + \chi_4^{dddd} + \chi_4^{ssss} + 4\chi_4^{uuud} + 4\chi_4^{uuus} \\ &+ 4\chi_4^{dddu} + 4\chi_4^{ddds} + 4\chi_4^{sssu} + 4\chi_4^{sssd} + 6\chi_4^{uudd} \\ &+ 6\chi_4^{ddss} + 6\chi_4^{uuss} + 12\chi_4^{uuds} + 12\chi_4^{ddus} + 12\chi_4^{ssud}\Big].\end{aligned}$$

For $\mu_u = \mu_d = \mu_s = \mu_B/3$,

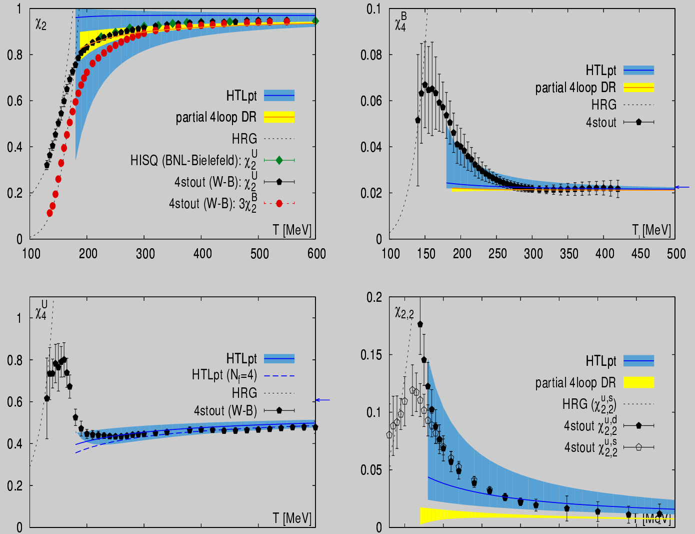
$$\chi_2^B = \frac{1}{3}\chi_2^{uu} \qquad \chi_4^B = \frac{1}{27}\left[\chi_4^{uuuu} + 6\chi_4^{uudd}\right]$$

Baryon number susceptibilities

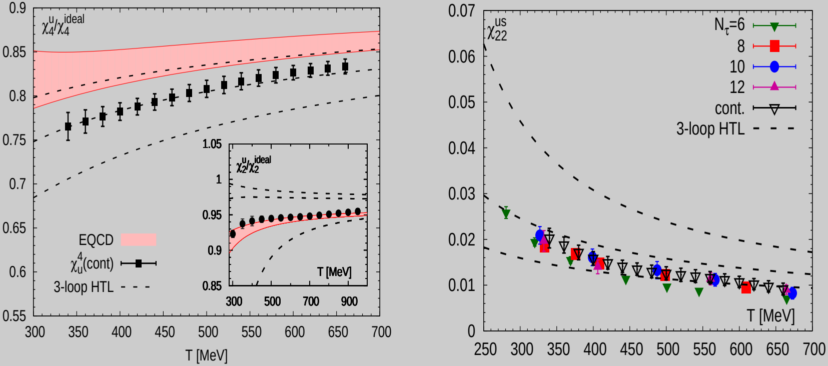


New lattice results

R. Bellwied et. al., PRD92(2015),114505

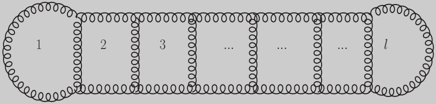


H.T. Ding et. al., PRD92(2015),074043



Further improvements?

1. we can go to higher order (g^6 and $g^6 \log g$) in coupling constant g.



$$Z_l \sim g^{2(l-1)} \left(T \int d^3k \right)^l k^{2(l-1)} (k^2 + m^2)^{-3(l-1)}$$

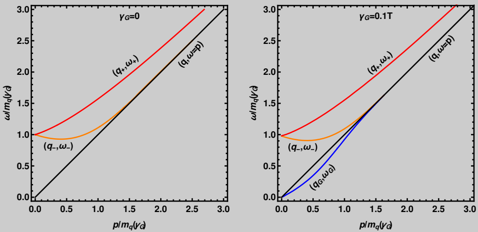
- $l < 4$: Z_l is IR regular.
- $l = 4$: $Z_l \sim g^6 T^4 \log\left(\frac{T}{m}\right)$
- $l > 4$: $Z_l \sim g^6 T^4 \left(\frac{g^2 T}{m}\right)^{l-4}$

- For longitudinal gluons, $m_{el} \sim gT$. So for $l > 4$, $Z_l \sim g^{l+4} T^4$.
- For transverse gluons, $m_{mag} \sim g^2 T$, So for $l > 4$, $Z_l \sim g^6 T^4$.

Conclusion $\longrightarrow$ Complete higher order calculation is not possible.

2. We can take the resummation of magnetic scale($\sim g^2T$)

- In PRL**114**, 161601 (2015), N. Su and K. Tywoniuk calculated quark self energy by taking into account both electric and magnetic scale resummation.



- Later we used that results in PRD93 (2016),065004 to calculate QNS and dilepton rate and we found that it makes the existing results worse.

Conclusion $\longrightarrow$ Resummation of magnetic scales fails to improve existing results at one loop level.

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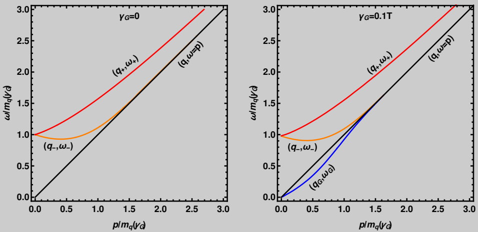

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Conclusion $\longrightarrow$ Resummation of magnetic scales fails to improve existing results at one loop level.

$\Rightarrow$ Alternatively, we can try non-perturbative approach like *Dyson Swinger Equation* to compute thermodynamics in whole $\mu - T$ plane.

Conclusions

- I have discussed about thermodynamic quantities in leading as well as beyond leading order using HTLpt.
- Thermodynamical potential produce correct perturbative order upto g, g^3 and g^5 if one expands for small g in case of one loop, two loop and three loop respectively.
- NNLO pressure is completely analytic and does not depend on any free parameter except the choice of the renormalization scale.
- For three loop case, we found good agreement between our results and LQCD results down to temperature ~ 250 MeV.
- Non-perturbative method like DSE could be interesting to calculate thermodynamics of the deconfined nuclear matter.

Thank you for your attention.