Phase diagram of dense two-color QCD with $N_f = 2$ flavors of staggered quarks

V.V. Braguta[1,2,3,4], E.-M. Ilgenfritz[5], A.Yu. Kotov[2,6], A.V. Molochkov[3] and A.A. Nikolaev[2,3]

[1]Institute for High Energy Physics, Protvino, Russia,
[2]Institute of Theoretical and Experimental Physics, Moscow, Russia,
[3]Far Eastern Federal University, Vladivostok, Russia,
[4]Moscow Institute of Physics and Technology, Dolgoprudny, Russia,
[5]Joint Institute for Nuclear Research, BLTP, Dubna, Russia,
[6]National Research Nuclear University (MEPhI), Moscow, Russia

Institut für Theoretische Physik
Justus-Liebig-Universität
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Outline

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1. Introduction: Why should we consider two-color QCD ?
2. Our lattice set-up
3. Preliminary results at $0 < T < T_\chi$, from $16^3 \times 6$ lattices
4. More detailed results close to $T = 0$, from $16^3 \times 32$ lattices
5. Qualitative summary and comparison with similar work
6. Outlook

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Circumnavigating the sign problem

Sign problem - an obstacle for SU(3) lattice QCD at finite density

So far, the most effective way to deal with fermions in lattice QCD: fermion determinant $=$ a closed expression encoding the fermionic part of the path integral.

Logarithmic derivative of the fermion determinant $\rightarrow$ contribution to driving force in the Hamilton equations of motion for the gauge field (within HMC)

For $\mu \neq 0$ the determinant takes complex values $\rightarrow$ breakdown of importance sampling is unavoidable

Circumnavigating the sign problem

What can a lattice theorist do ?

- Invest heavy efforts to overcome the sign problem for SU(3) QCD or lattice-based effective models:
  1. complex Langevin simulation
  2. density of states method (generic for complex action)
  3. dualization (difficult for non-Abelian theories)
  4. changing the order of integration (strong coupling)
  5. complexification (Lefshetz thimbles)

Circumnavigating the sign problem

What can a lattice theorist do ?

- Alternatively, turn to the few gauge theories with dynamical quarks which are free of the sign problem: G_2, “adjoint” SU(3), SU(2).

  In which respect can this detour be finally helpful for SU(3) QCD ?

  1. Test case to estimate the correctness of other lattice approaches (imaginary chemical potential)
  2. Test case to estimate the correctness of non-perturbative continuum methods of extending $\mu = 0$ to $\mu \neq 0$ ...
  3. Discussion of the role of N_f and of the type of fermion discretization

Finally, QC_2D is interesting of its own !

The Phase Diagram of Four Flavor SU(2) Lattice Gauge Theory at Nonzero Chemical Potential and Temperature
J.B. Kogut, D. Toublan, D.K. Sinclair, Nucl. Phys. B 642 (2002) 181-209

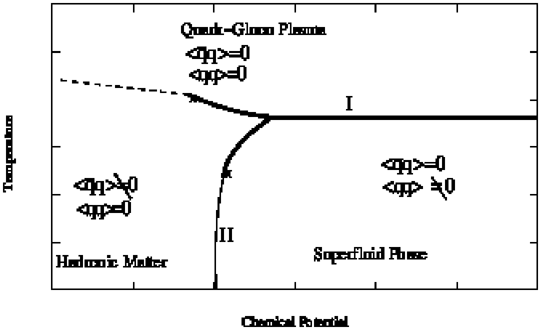

Figure: Schematic phase diagram of QC_2D in the T-μ plane. The thin(thick) line represents second(first) order transitions. The dashed line denotes a crossover.

Are there possibly general features to be seen ?

QCD - like theories at finite baryon density
J.B. Kogut, M.A. Stephanov, D. Toublan, J.J.M. Verbaarschot,
A. Zhitnitsky, Nucl. Phys. B 582 (2000) 477-513
e-Print: hep-ph/0001171

No deeper similarities ? Quarkyonic phase for very large N_c ?

Phases of cold, dense quarks at large N_c
L. McLerran, R.D. Pisarski,
Nucl. Phys. A 796 (2007) 83-100
e-Print: arXiv:0706.2191

Quarkyonic phase

L. McLerran, R.D. Pisarski, Phases of cold, dense quarks at large N_c

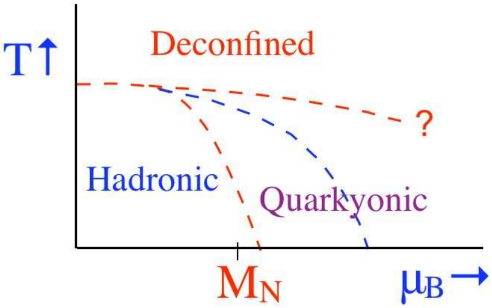

Figure: Schematic phase diagram of large-N_c QCD in the T-μ plane with a separate quarkyonic phase.

Predictions of Chiral Perturbation Theory (Kogut et al.)

J.B. Kogut, M.A. Stephanov, D. Toublan, J.J.M. Verbaarschot, A. Zhitnitsky, Nucl. Phys. B 582 (2000) 477-513

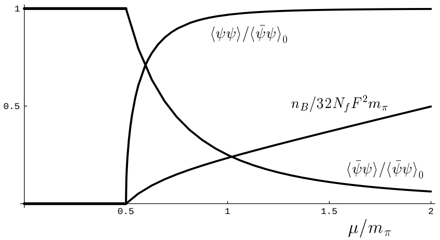

Figure: The magnitudes of the chiral $\langle\bar\psi\psi\rangle$ and the diquark $\langle\psi\psi\rangle$ condensates in units of $\langle\bar\psi\psi\rangle_0 = 2N_fG$ as a function of μ/m_π for zero diquark source. Also the density of the baryon charge in units of $32N_fF^2m_\pi$ is shown.

Predictions of Chiral Perturbation Theory (Kogut et al.)

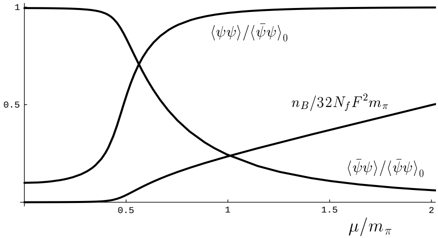

Figure: The magnitudes of the chiral $\langle\bar\psi\psi\rangle$ and the diquark $\langle\psi\psi\rangle$ condensates in units of $2N_f\ \langle\bar\psi\psi\rangle_0$ as a function of μ/m_π at small non-zero diquark source $j = 0.1m$. Also the density of the baryon charge in units of $32N_fF^2m_\pi$ is shown.

Previous lattice studies of QC_2D at $\mu \neq 0$

- $N_f = 8$ staggered fermions, no rooting
  S. Hands, J.B. Kogut, M.P. Lombardo, and S.E. Morrison
  Nucl. Phys. B 558 (1999) 327-346
- $N_f = 4$ staggered fermions, with roooting
  - J.B. Kogut, D. Toublan and D.K. Sinclair
    Phys. Lett. B 514 (2001) 77-87;
  - J.B. Kogut, D. Toublan and D.K. Sinclair
    Nucl. Phys. B 642 (2002) 181-209
- $N_f = 2$ Wilson fermions
  - S. Cotter, P. Giudice, S. Hands, and J.I. Skullerud
    Phys. Rev. D 87 (2013) 034507;
  - T. Makiyama et al. (with A. Nakamura)
    Phys.Rev. D93 (2016) 014505
    (Phase structure of QC_2D at both real and imaginary chemical potential)

Outline

The partition function

For the SU(2) gauge fields Wilson action:

$$S_G = \beta \sum_x \sum_{\mu<\nu=1}^{4} \left(1 - \frac{1}{2}\mathrm{Tr}\, U_{x,\mu\nu}\right)$$

For the fermionic degrees of freedom staggered action:

$$S_F = \sum_{x,y} \overline{\psi}_x M(\mu, m)_{x,y}\psi_y + \frac{\lambda}{2}\sum_x \left(\psi_x^T \tau_2 \psi_x + \overline{\psi}_x \tau_2 \overline{\psi}_x^T\right)$$

with a diquark source term included ($\propto \lambda$, quadratic in $\bar{\psi}$ and in ψ, violating $U_V(1)$ symmetry) and with a staggered hopping term (bilinear in $\bar{\psi}$, ψ)

$$M_{xy} = ma\delta_{xy} + \frac{1}{2}\sum_{\mu=1}^{4} \eta_\mu(x)\left[U_{x,\mu}\delta_{x+\hat{\mu},y}e^{\mu a \delta_{\mu,4}} - U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}e^{-\mu a \delta_{\mu,4}}\right]$$

The partition function

Integrating out the fermions with such an action , one is left with a bosonic path integral (Pf = Pfaffian):

$$Z = \int DU\, e^{-S_G} \cdot Pf \begin{pmatrix} \lambda\tau_2 & M \\ -M^T & \lambda\tau_2 \end{pmatrix} = \int DU\, e^{-S_G} \cdot \left(\det(M^\dagger M + \lambda^2)\right)^{\frac{1}{2}}$$

suitable or $N_f = 4$. Here, however, we simulate for $N_f = 2$

$$Z = \int DU\, e^{-S_G} \cdot \left(\det(M^\dagger M + \lambda^2)\right)^{\frac{1}{4}}$$

Absence of the sign problem for QC_2D

Generally, for lattice Dirac operators holds:

$$[\det M(\mu)]^* = \det M(-\mu^*)$$

For strictly imaginary μ this leads to $\det M(\mu) = \text{ real valued}$

In case of SU(2), a special relation holds:

$$\det M(\mu) = \det\left[(\tau_2 C\gamma_5)^{-1} M(\mu)(\tau_2 C\gamma_5)\right] = [\det M(\mu^*)]^*$$

with $C = \gamma_2\gamma_4$.

Therefore, for strictly real μ:

$$\det M(\mu) = \text{ real} \to \det\left[M^\dagger(\mu)M(\mu)\right] > 0 \qquad (1)$$

Observables for deconfinement and chiral symmery restoration

Polyakov loop:

$$\langle L \rangle = \frac{1}{N_s^3} \sum_{x_1,x_2,x_3=0}^{N_s-1} \frac{1}{2} \left\langle \mathrm{Tr} \prod_{x_4=0}^{N_\tau-1} U_{x,4} \right\rangle$$

Time-like Wilson loop around a rectangular contour $C = R \times T$:

$$W(R,T) = \left\langle \mathrm{Tr} \left[\prod_C U_{x,\mu} \right] \right\rangle$$

Chiral condensate:

$$a^3 \langle \bar{q} q \rangle = a^3 \langle \bar{q}_{i\alpha} q_{i\alpha} \rangle = - \frac{1}{N_s^3 N_\tau} \frac{\partial (log\, Z)}{\partial (ma)}$$

has been obtained by stochastic estimation.
Susceptibilities are still beyond our computational resources !

Observables describing baryon onset and diquark condensation

Baryon density:

$$a^3 n_B = \frac{1}{2}\frac{1}{N_s^3 N_\tau}\frac{\partial(\log Z)}{\partial(\mu a)}$$

Diquark condensate: ($\hat{C}$ = charge conjugation)

$$\begin{aligned} a^3 \langle qq \rangle &= a^3 \left\langle q^T_{i\alpha} \hat{C} \gamma_5 (\tau_2)_{ij} (\sigma_2)_{\alpha\beta} q_{j\beta} \right\rangle \\ &= -\frac{1}{N_s^3 N_\tau}\frac{\partial(\log Z)}{\partial\lambda} \\ &= \frac{2\lambda}{N_s^3 N_\tau}\left\langle \mathrm{Tr}\left[M^\dagger M + \lambda^2\right]^{-1}\right\rangle \end{aligned}$$

Susceptibilities are still beyond our computational resources !

Outline

T- dependence of the Polyakov loop for various μ

Curvature of the crossover line at high temperature separating confinement from deconfinement, beginning at $\mu = 0$

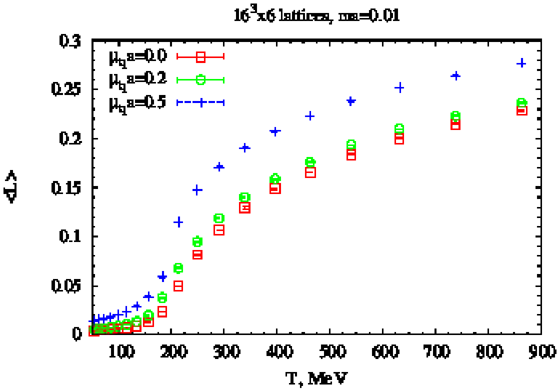

Figure: Polyakov loop as a function of T for three values of the baryon chemical potential μ.

T- dependence of the chiral condensate for various μ

Does there exist a common crossover line at high temperature for deconfinement and chiral symmetry restoration ?

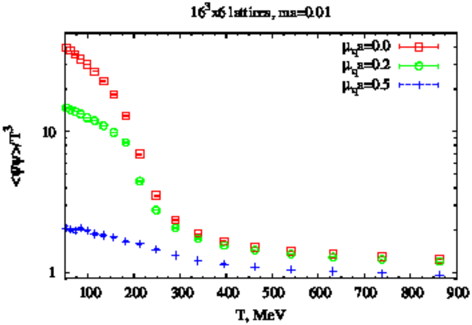

Figure: Chiral condensate as a function of T for three values of the baryon chemical potential μ. The ordinate axis is logarithmic.

A first μ- scan of the baryon density at medium T

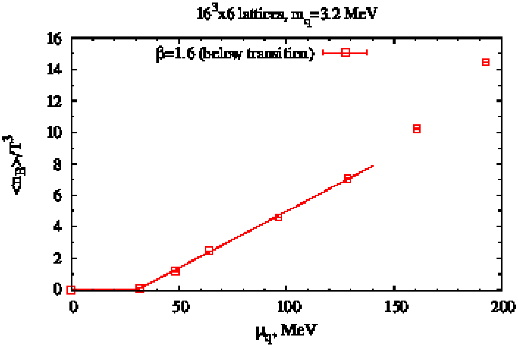

Figure: Baryon number density as a function of μ at a temperature below $T_\chi(\mu = 0)$ (with $N_\tau = 6$ and $\beta = 1.6$), described by a linear fit beyond $\mu^c \approx 30$ MeV.

Notice that the finite-T mass $m_\pi < 100$ MeV at $\beta = 1.6$!

Preliminary summary of results at high and medium T

presented at Lattice 2015 by A. Nikolaev (arxiv:1511.0484)

- Increasing the baryonic chemical potential $\rightarrow$ decreasing $\langle \bar{\psi}\psi \rangle$
- T_c decreases with increasing baryonic chemical potential
- In the confinement phase, n_B rises linearly with μ if $\mu > m_\pi/2$

Outline

Details of the new simulation: a low-temperature scan

just appeared in arXiv: 1605.04090

lattice size $16^3 \times 32$ (representing "near zero temperature")

unimproved Wilson action for the gauge field

inverse gauge coupling fixed at $\beta = 2.15$

$N_f = 2$ flavors of dynamical staggered quarks

lattice spacing $a = 0.112$ fm

physical lattice size $m_\pi L_s a \approx 3.24$

actual temperature $T = 55$ MeV

Other parameters

quark mass:

in physical units $m = 362(4)\ \mathrm{MeV}$ (in lattice units $ma = 0.005$)

μ range under investigation, sequence of diquark source values:

total μ range: $\mu \in [0; 1759]\ \mathrm{MeV}$, $\mu a \in [0.0; 1.0]$

special μ range: $\mu \in [0; 1055]\ \mathrm{MeV}$, $\mu a \in [0.0; 0.6]$, where dependence on λ has been studied at $\lambda = 0.001, 0.00075, 0.0005$

special μ focus on the vicinity of the first phase transition (hadron phase $\rightarrow$ BEC) $\mu = 176, 211, 246\ \mathrm{MeV}$ $\rightarrow$ the detailed dependence on λ was studied at $\lambda = 0.001, 0.000875, 0.00075, 0.000625, 0.0005$

for larger $\mu > 1055\ \mathrm{MeV}$ we have simulated at $\lambda = 0.0005$ only

Algorithm and resources

RHMC (rational hybrid Monte Carlo)
M.A. Clark, Lattice 2006, arXiv:hep-lat/0610048
our code rewritten in CUDA C

number of trajectories
1000 . . . 1500 trajectories per (μ, λ)

Computing resources :

- ITEP supercomputer (“Graphyn” and “Stakan”)
- NRC “Kurchatov Institute” supercomputer (new resource)
- IHEP (Protvino) GPU cluster

all working with Nvidia GPU

Supported by grants from
RFBR, Dynasty foundation, FAIR-Russia Research Center Moscow

Goals of this investigation

- Study the eventual condensed phases (BEC, BCS ..?) of SU(2) QCD, with increasing μ near to $T = 0$
- Are there really more than one condensed phases (both BEC and BCS) for $N_f = 2$ staggered quarks ?
- Is there a quantitative connection with SU(3) QCD ? Baryon density $n_B \approx 1 \ \mathrm{fm}^{-3}$ marks the transition to “quarkyonic matter”.
- Find support for the quarkyonic matter picture of cold and dense QCD

Scale setting and pion mass I

special measurements for $\mu = 0$ and $\lambda = 0$ for calibration performed on the lattice $16^3 \times 32$; fixing $ma = 0.01$ for various $\beta \in [1.9; 2.2]$ for $\beta = 2.15$ also $ma = 0.005$, in order to check (in)dependence on mass

- beta-function : heavy quark potential measured with smearing (1 HYP smearing for temporal links plus 20 APE smearing steps for spatial links).
  Assuming Sommer scale $r_0 = 0.468(4)\ \mathrm{fm}$, one gets $\beta(a)$ which is well described by the two-loop beta function

$$a(\beta) = \frac{1}{\Lambda_L}\left(\frac{4\beta_0}{\beta}\right)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{\beta}{8\beta_0}\right)$$

with $\beta_0 = \frac{3}{8\pi^2}$ and $\beta_1 = \frac{29}{256\pi^4}$ corresponding to $N_c = 2$ and $N_f = 2$

Scale setting and pion mass II

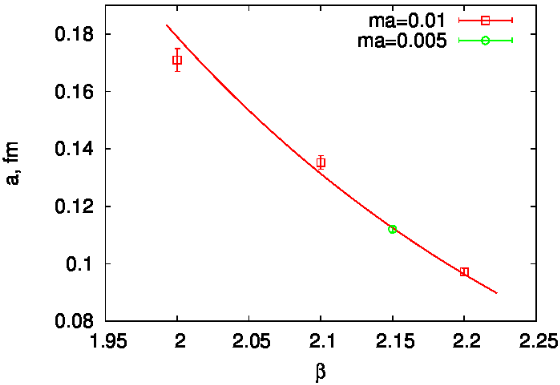

Figure: The dependence of the lattice spacing on the inverse coupling $\beta = 4/g^2$.

Scale setting and pion mass III

β	*ma*	*a*, *fm*	M_π, *MeV*
1.9	0.01	0.20(1)	216(6)
2.0	0.01	0.171(4)	311(6)
2.1	0.01	0.135(2)	431(8)
2.2	0.01	0.097(1)	558(11)
2.15	0.005	0.112(1)	362(4)

Table: The lattice spacing a and the pion mass m_π for various values of the inverse coupling β and of the bare quark mass ma.

Scale setting and pion mass IV

- pion mass m_π : from measured pion propagator fitted to

$$C_\pi(t, \vec{q} = 0) = C \cosh\left[m_\pi(t - T/2)\right]$$

The diquark condensate as function of μ

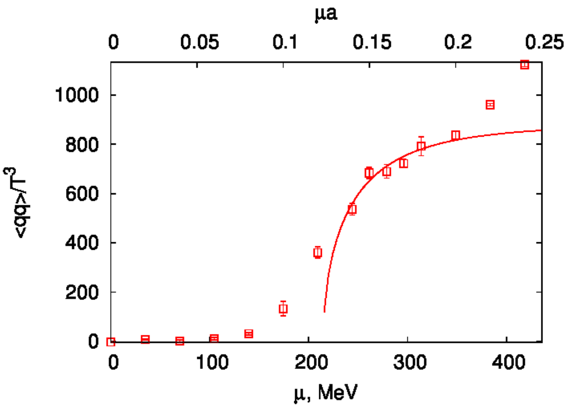

Figure: The diquark condensate $\langle qq \rangle / T^3$ as a function of μ. The data points are compared with the prediction of Chiral Perturbation Theory.

The diquark condensate as function of μ: discussion

According to ChPT

$$\langle qq \rangle = \langle \bar{q}q \rangle_0 \times \sqrt{1 - \left(\frac{\mu^c}{\mu}\right)^4}$$

Applying this formula far from the transition for fitting (see figure) gives $\mu^c = 215(10)\ \mathrm{MeV}$.
Fit by another function: since $\langle \bar{q}q \rangle \propto 1/\mu^2$ is not satisfied, one should replace the fitting formula by

$$\langle qq \rangle = \langle \bar{q}q \rangle_0 \times \sqrt{1 - \left(\frac{\mu^c}{\mu}\right)^{2\alpha}}$$

with $\alpha = 0.78$.
This fitting function gives $\mu^c = 193(10)\ \mathrm{MeV}$.
This gives $m_\pi = 387(20)\ \mathrm{MeV}$, in better agreement to $m_\pi = 362(4)\ \mathrm{MeV}$ (see scale setting) !

The diquark condensate as function of λ

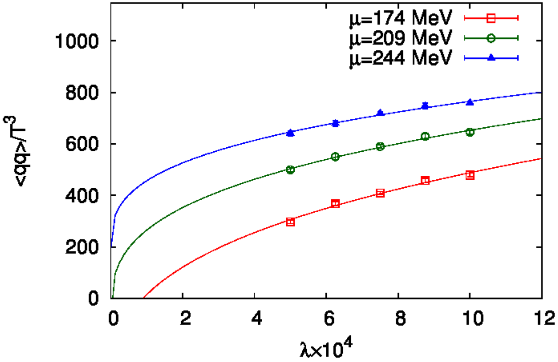

Figure: The diquark condensate $\langle qq \rangle / T^3$ as a function of λ in the vicinity of the first phase transition.

The diquark condensate as function of λ: discussion

According to ChPT, in the limit $\lambda \to 0$ the diquark condensate behaves like $\langle qq \rangle \propto \lambda^{1/3}$.

A fit with $\langle qq \rangle = A + B\lambda^{1/3}$ gives the first non-zero diquark condensate (identified as A) appearing at $\mu = 211\ \mathrm{MeV}$ (closest to μ^c).

For $\mu > 350\ \mathrm{MeV}$ stronger deviations from ChPT: a new regime ?

The diquark condensate: scaling with μ^2

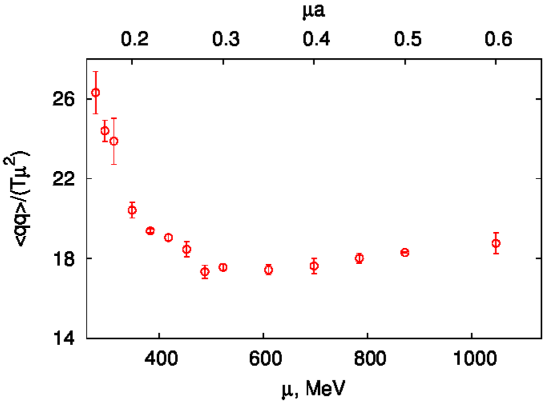

Figure: The ratio $\langle qq \rangle / (T\mu^2)$ as a function of μ around the second phase transition (BEC $\rightarrow$ BCS).

The diquark condensate scaling with μ^2: discussion

Above $\mu \simeq 520\ \mathrm{MeV}$ one sees a plateau, suggesting proportionality to the surface of the Fermi sphere ....

This is considered as a characteristic feature of the BCS mechanism for the creation of the condensate.

Global view of the diquark condensate

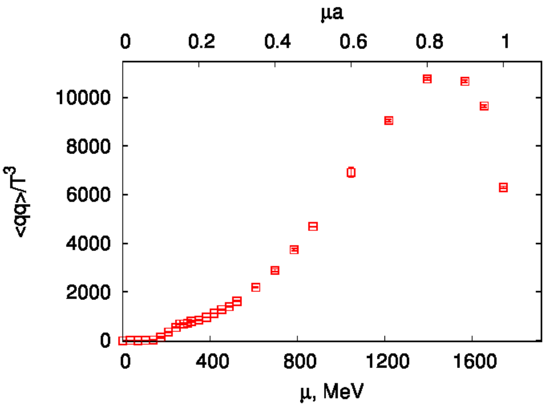

Figure: The diquark condensate $\langle qq \rangle / T^3$ as a function of μ across the two phase transitions.

Global view of the diquark condensate: drop beyond $\mu > 1400$ MeV

saturation effect, interpreted as lattice artefact

The chiral condensate around the first transition

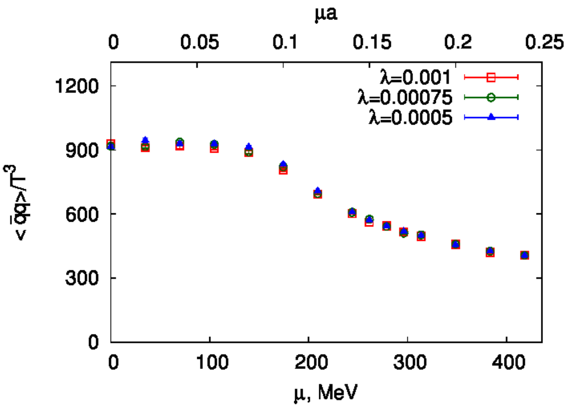

Figure: The chiral condensate $\langle \bar{q}q \rangle / T^3$ as a function of μ for the values $\lambda = 0.001, 0.00075$ and 0.0005 of the diquark source. The chiral condensate turns out independent of λ.

The chiral condensate around the first transition: few comments

The chiral condensate is practically independent of λ.

It starts dropping immediatey above $\mu \simeq 176\ \mathrm{MeV}$, the region of the transition hadron phase $\to$ BEC phase begins.

According to the “circle law”, this marks the beginning of the rise of $\langle qq \rangle$, which is complementary to falling $\langle \bar{q}q \rangle$.

Next: test of the “circle law”. Clear deviation above the transition. For small diquark source λ, a dip is seen marking the transition.

The “circle law” around the first transition

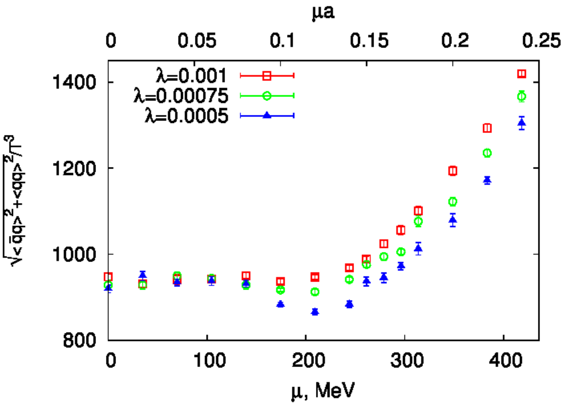

Figure: The combination $\sqrt{\langle qq\rangle^2 + \langle\bar{q}q\rangle^2}/T^3$ of diquark and chiral condensates is almost constant as a function of μ below μ^c. In the limit $\lambda \to 0$, a dip becomes visible at the first phase transition (hadron phase $\to$ BEC).

Global view of the chiral condensate

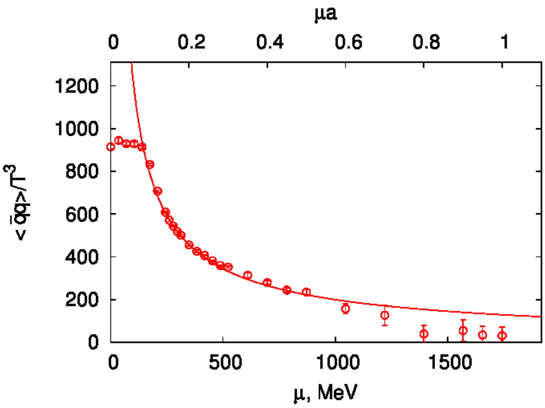

Figure: The chiral condensate $\langle \bar{q}q \rangle / T^3$ as a function of μ. The ChPT prediction does not hold throughout the BEC and BCS phase.

Global view of the chiral condensate: a remarkable deviation from ChPT

A single power describes the drop of the chiral condensate, with an exponent clearly deviating from ChPT:

$$\langle \bar{q}q \rangle \propto \left(\frac{\mu^c}{\mu} \right)^{\alpha}$$

$\alpha = 0.78$ instead of $\alpha = 2$

The chiral limit of the chiral condensate for various μ

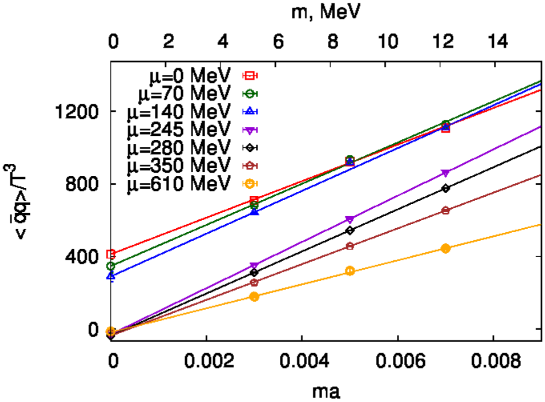

Figure: The linear chiral limit of the chiral condensate $\langle \bar{q}q \rangle / T^3$, taken for different values of the chemical potential.

The chiral limit of the chiral condensate for various μ

The chiral limit of $\langle \bar{q}q \rangle$ vanishes for all $\mu > \mu^c$.

The baryon density

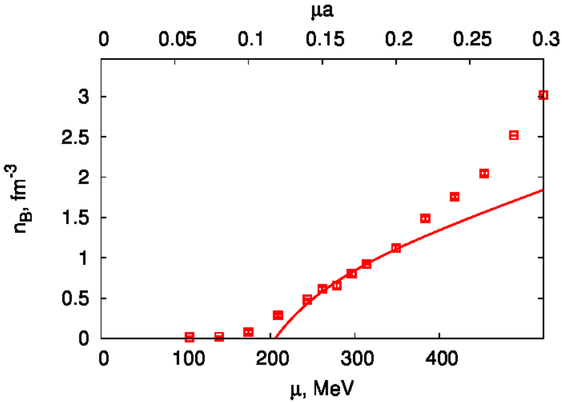

Figure: The baryon density n_B in physical units as a function of μ, compared with the ChPT prediction.

The baryon density: comments

In ChPT it is predicted that $n_B(\lambda) = A + B\lambda^2$.

For $\mu < 176\ \mathrm{MeV}$, the extrapolated baryon density (A) vanishes.

For $\mu > \mu^c$ the μ-dependence is predicted as $n_B = \mu - \frac{(\mu^c)^4}{\mu^3}$

In the interval $\mu \in [263; 350]\ \mathrm{MeV}$, this formula results in a good fit (see figure).

The fit predicts $\mu^c = 207(7)\ \mathrm{MeV}$, close to $193(10)\ \mathrm{MeV}$ from the diquark condensate (with α adapted from μ-dependence of the chiral condensate).

Above this interval, the behavior deviates from ChPT.

The baryon density: scaling with μ^3 (free density)

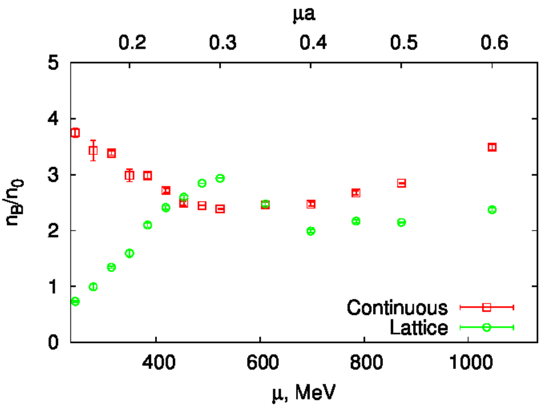

Figure: The ratio n_B/n_0 as a function of the chemical potential μ. For the red symbols , the reference density n_0 denotes the baryon density for free continuum fermions, $n_0 = (4\mu^3)/(3\pi^2)$, whereas for the green symbols the reference density n_0 denotes the baryon density for free lattice fermions.

The gluonic observables

No sign of deconfinement at $T \simeq 0$ throughout all μ !

Polyakov loop throughout compatible with zero.

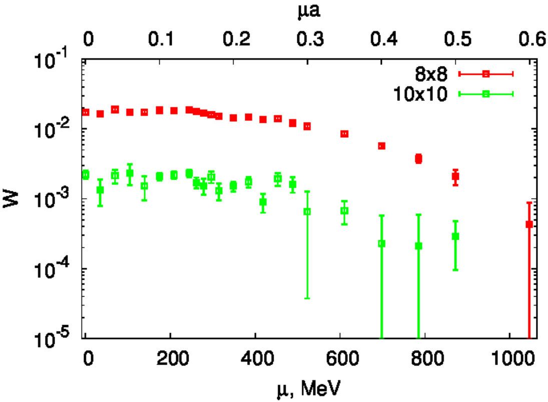

Figure: The time-like Wilson loops for the contours 8×8 and 10×10 as functions of the chemical potential μ.

Outline

Qualitative summary I

- low μ : $0 < \mu < \mu^c = \frac{m_\pi}{2} \simeq 200\ \mathrm{MeV}$:
  hadron phase
  - confinement
  - chiral symmetry broken, chiral condensate $\langle \bar{q}q \rangle \neq 0$
  - diquark condensate $\langle qq \rangle = 0$
  - baryon density n_B vanishing
  - this phase ends with a second order phase transition
  - relevant degrees of freedom : Goldstone bosons

Qualitative summary II

- baryon onset in μ : $\mu^c < \mu < \mu^d \simeq 350\ \mathrm{MeV}$:
  BEC phase, Bose condensation of scalar diquarks
  - rough agreement with ChPT, exception is the chiral condensate $\langle \bar{q}q \rangle$
  - confinement persists
  - chiral symmetry gradually restored
  - complementarily to that, the diquark condensate $\langle qq \rangle$ grows
  - in the chiral limit, chiral condensate $\langle \bar{q}q \rangle \to 0$
  - baryon density n_B starts growing linearly
  - relevant degrees of freedom : Goldstone bosons
  - dilute baryon gas, $n_B < 1\ \mathrm{fm}^{-3}$

Qualitative summary III

- medium μ : $\mu^d < \mu < 500\ \mathrm{MeV}$:
  crossover from BEC to BCS phase
  - increasing deviations from ChPT
  - baryon density reaches a dense regime, $n_B > 1\ \mathrm{fm}^{-3}$
  - confinement persists
  - chiral symmetry almost restored
  - in the chiral limit, chiral condensate $\langle \bar{q}q \rangle \to 0$
  - the diquark condensate $\langle qq \rangle$ grows further

We have $\mu^d = 1.76\mu^c$. A previous NJL analysis gives $\mu^d = [1.65 - 2]\mu^c$.
communicated by Lianyi He, Tsinghua University Beijing
Phys. Rev. D 82, 096003 (2010), also Gao-feng Sun, Lianyi He, Pengfei Zhuang, Phys. Rev. D 75, 096004 (2007)

Qualitative summary IV

- large μ : $500\ \mathrm{MeV} < \mu < 1000\ \mathrm{MeV}$:
  BCS phase
  - different scaling of diquark condensate, $\langle qq \rangle \propto \mu^2$
  - different scaling of baryon density: $n_B \propto \mu^3$
  - relevant degrees of freedom : quarks inside the Fermi sphere
  - condensate of Cooper pairs $\propto$ surface of the Fermi sphere
  - chiral symmetry restored, $\langle \bar{q}q \rangle = 0$
  - probable interpretation : BCS phase

- $\mu > 1000\ \mathrm{MeV}$:
  lattice artefacts, saturation effects ?

Comparison with similar work I

- J. Kogut et al., staggered fermions with rooting for $N_f = 4$:
  partial similarity
  - succession of hadron phase and BEC phase, both well described by ChPT
  - no BCS phase
- S. Hands et al., Wilson fermions for $N_f = 2$:
  no similarity
  - succession of hadron phase and BCS phase (with deconfinement at higher μ)
  - BEC phase missed (due to the absence of chiral symmetry for Wilson fermions)

Comparison with similar work II

- Large N_c scenario (quarkyonic phase) :
  high similarity
  - succession of hadronic, dilute nuclear gas phase (similar to BEC), quarkyonic phase (still confining, chiral symmetry restored)
  - suggestion: quarkyonic phase $\simeq$ BCS phase of QC_2D !

Outline

Next projects

- phase transitions supported by susceptibility measurements (higher statistics urgently needed)
- measure gluon propagator in media with chemical potential, relation to propagators at $\mu = 0$
- $U_A(1)$ violation/restoration in dense matter, such measurements are presently performed (Wilson flow)
- learn more about the second transition (likely a crossover) BEC $\rightarrow$ BCS