Chiral symmetry breaking in continuum QCD

Mario Mitter

Ruprecht-Karls-Universität Heidelberg

Gießen, February 10, 2016

Der Wissenschaftsfonds.

European Research Council
Established by the European Commission

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

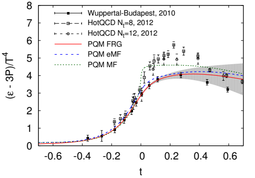
fQCD collaboration - QCD (phase diagram) with FRG:

J. Braun, A. K. Cyrol, L. Fister, W. J. Fu, T. K. Herbst, MM
N. Müller, J. M. Pawlowski, S. Rechenberger, F. Rennecke, N. Strodthoff

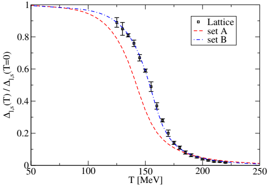
Table of Contents

QCD phase diagram with functional methods

- works well at $\mu = 0$: agreement with lattice



[Herbst, MM, Pawlowski, Schaefer, Stiele, '13]

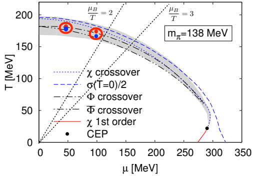


[Luecker, Fischer, Welzbacher, 2014]

[Luecker, Fischer, Fister, Pawlowski, '13]

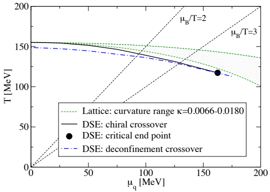
QCD phase diagram with functional methods

- works well at $\mu = 0$: agreement with lattice
- different results at large μ
  (possibly already at small μ)



[Herbst, Pawlowski, Schaefer, 2013]

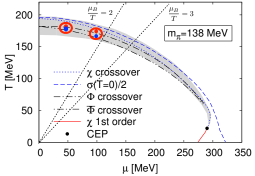
[Braun, Haas, Pawlowski, unpublished]



[Luecker, Fischer, Fister, Pawlowski, '13]

QCD phase diagram with functional methods

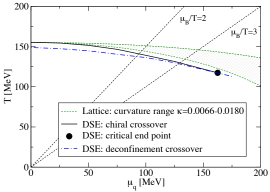
- works well at $\mu = 0$: agreement with lattice
- different results at large μ (possibly already at small μ)



[Herbst, Pawlowski, Schaefer, 2013]

[Braun, Haas, Pawlowski, unpublished]

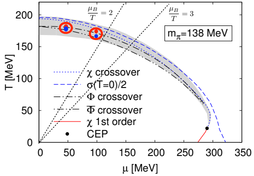
- calculations need model input:
  - Polyakov-quark-meson model with FRG:
    - inital values at $\Lambda \approx \mathcal{O}(\Lambda_{\mathrm{QCD}})$
    - input for Polyakov loop potential
  - quark propagator DSE:
    - IR quark-gluon vertex



[Luecker, Fischer, Fister, Pawlowski, '13]

QCD phase diagram with functional methods

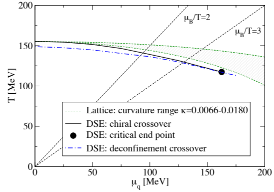
- works well at $\mu = 0$: agreement with lattice
- different results at large μ
  (possibly already at small μ)



[Herbst, Pawlowski, Schaefer, 2013]

[Braun, Haas, Pawlowski, unpublished]

- calculations need model input:
  - Polyakov-quark-meson model with FRG:
    - ★ inital values at $\Lambda \approx \mathcal{O}(\Lambda_{\text{QCD}})$
    - ★ input for Polyakov loop potential
  - quark propagator DSE:
    - ★ IR quark-gluon vertex



possible explanation for disagreement:

- $\mu \neq 0$: relative importance of diagrams changes
  $\Rightarrow$ summed contributions vs. individual contributions

[Luecker, Fischer, Fister, Pawlowski, '13]

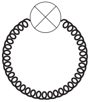
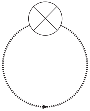
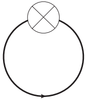
Back to QCD in the vacuum (Wetterich equation)

- use only perturbative QCD input
  - $\alpha_S(\Lambda = \mathcal{O}(10)\ \mathrm{GeV})$
  - $m_q(\Lambda = \mathcal{O}(10)\ \mathrm{GeV})$

Back to QCD in the vacuum (Wetterich equation)

- use only perturbative QCD input
  - $\alpha_S(\Lambda = \mathcal{O}(10)\text{ GeV})$
  - $m_q(\Lambda = \mathcal{O}(10)\text{ GeV})$

- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$

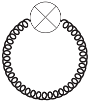
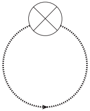
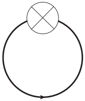
$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2}$  $-$  $-$ 

$\Rightarrow$ effective action $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$

Back to QCD in the vacuum (Wetterich equation)

- use only perturbative QCD input
  - $\alpha_S(\Lambda = \mathcal{O}(10)\ \text{GeV})$
  - $m_q(\Lambda = \mathcal{O}(10)\ \text{GeV})$

- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$

$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2}$  $-$  $-$ 

$\Rightarrow$ effective action $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$

- ∂_k: integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge): ghosts appear

Vertex Expansion

- approximation necessary - vertex expansion

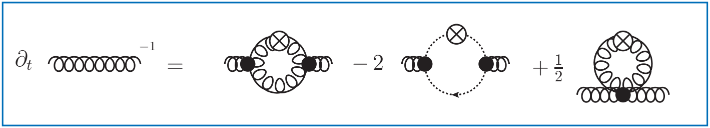
$$\Gamma[\Phi] = \sum_n \int_{p_1,\ldots,p_{n-1}} \Gamma^{(n)}_{\Phi_1\cdots\Phi_n}(p_1,\ldots,p_{n-1})\Phi^1(p_1)\cdots\Phi^n(-p_1-\cdots-p_{n-1})$$

Vertex Expansion

- approximation necessary - vertex expansion

$$\Gamma[\Phi] = \sum_n \int_{p_1,\ldots,p_{n-1}} \Gamma^{(n)}_{\Phi_1\cdots\Phi_n}(p_1,\ldots,p_{n-1})\Phi^1(p_1)\cdots\Phi^n(-p_1-\cdots-p_{n-1})$$

- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
  $\Rightarrow$ equations for 1PI n-point functions, e.g. gluon propagator:

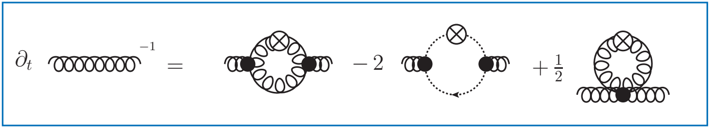


Vertex Expansion

- approximation necessary - vertex expansion

  $$\Gamma[\Phi] = \sum_n \int_{p_1,\dots,p_{n-1}} \Gamma^{(n)}_{\Phi_1\cdots\Phi_n}(p_1,\dots,p_{n-1})\Phi^1(p_1)\cdots\Phi^n(-p_1-\cdots-p_{n-1})$$

- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
  $\Rightarrow$ equations for 1PI n-point functions, e.g. gluon propagator:



- want "apparent convergence" of $\Gamma[\Phi] = \lim_{k\to 0} \Gamma_k[\Phi]$

“Quenched” Landau gauge QCD

- two crucial phenomena: SχSB and confinement
- similar scales - hard to disentangle see e.g. [Williams, Fischer, Heupel, 2015]
- quenched QCD: allows separate investigation:

“Quenched” Landau gauge QCD

- two crucial phenomena: SχSB and confinement
- similar scales - hard to disentangle see e.g. [Williams, Fischer, Heupel, 2015]
- quenched QCD: allows separate investigation:
- matter part [MM, Strodthoff, Pawlowski, 2014]
  (with FRG-YM propagators from [Fischer, Maas, Pawlowski, 2009], [Fister, Pawlowski, unpublished])
- recent results for YM propagators [Cyrol, Fister, MM, Pawlowski, Strodthoff, to be published]

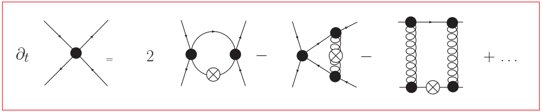
Chiral symmetry breaking

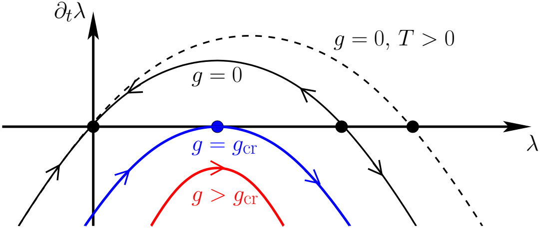
- χSB $\Leftrightarrow$ resonance in 4-Fermi interaction λ (pion pole):

Chiral symmetry breaking

- χSB $\Leftrightarrow$ resonance in 4-Fermi interaction λ (pion pole):
- resonance $\Rightarrow$ singularity without momentum dependency

$$\partial_t \lambda = a\,\lambda^2 + b\,\lambda\,\alpha + c\,\alpha^2, \quad b > 0,\ a, c \leq 0$$

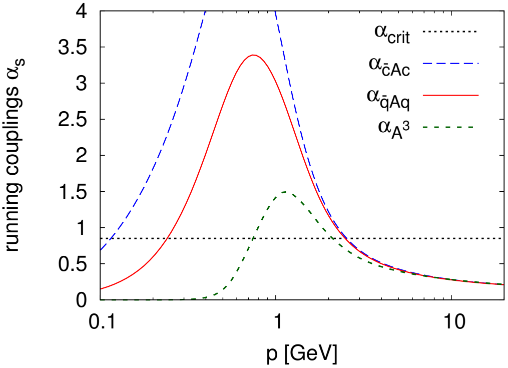




[Braun, 2011]

Effective running couplings

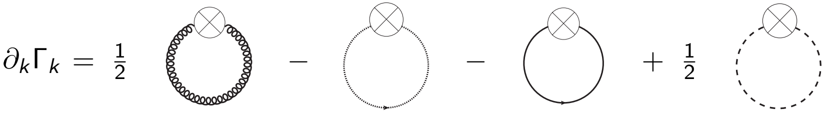
[MM, Pawlowski, Strodthoff, 2014]

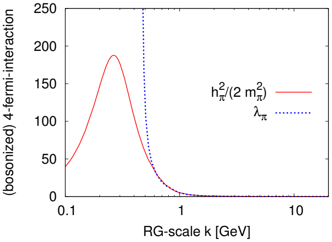


- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

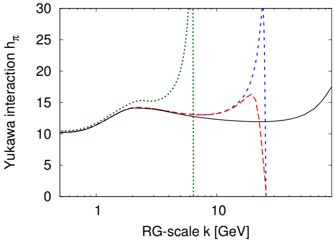
4-Fermi vertex via dynamical hadronization [Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels → meson exchange
- efficient inclusion of momentum dependence ⇒ no singularities
- identifies relevant effective low-energy dofs from QCD





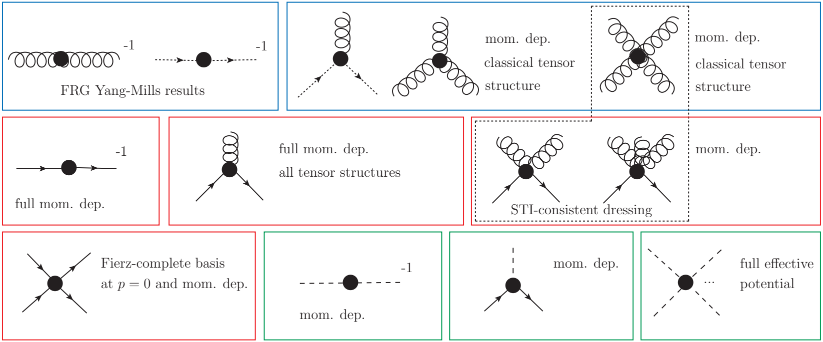
[MM, Strodthoff, Pawlowski, 2014]



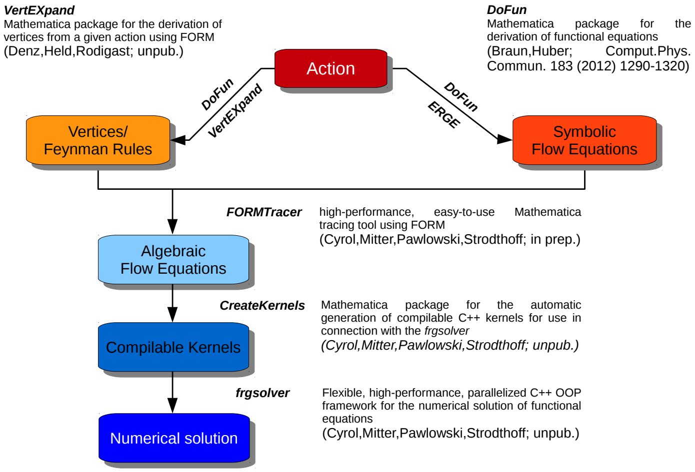
[Braun, Fister, Haas, Pawlowski, Rennecke, 2014]

[MM, Strodthoff, Pawlowski, 2014]

Vertex Expansion in the matter system



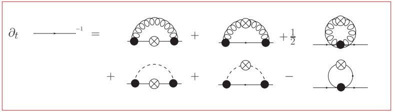
Derivation of equations

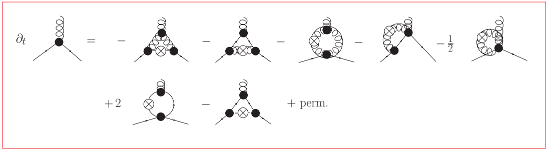


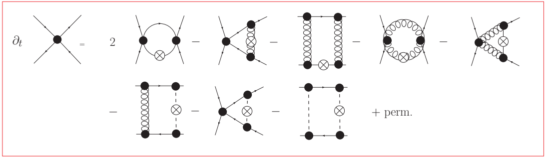
[Cyrol, MM, Pawlowski, Strodthoff, 2013-2016]

Equations in the matter system

[MM, Strodthoff, Pawlowski, 2014]

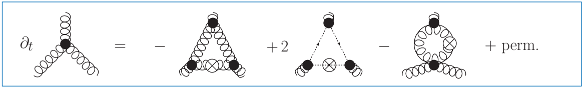


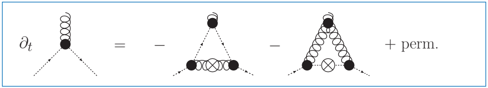


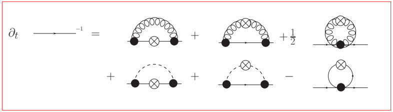


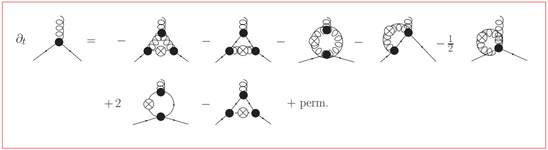
Equations in the matter system

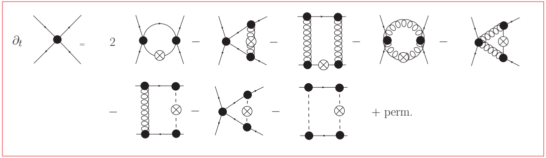
[MM, Strodthoff, Pawlowski, 2014]





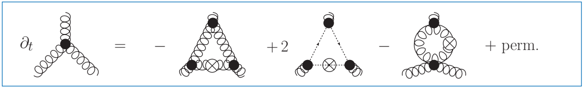


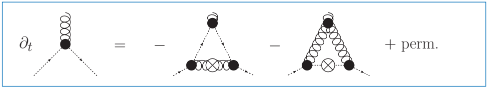


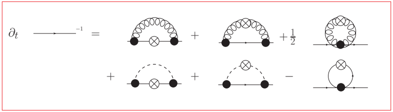


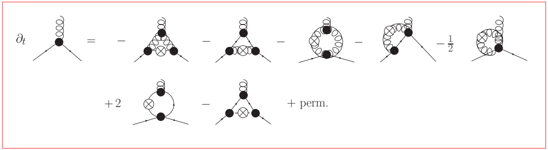
Equations in the matter system

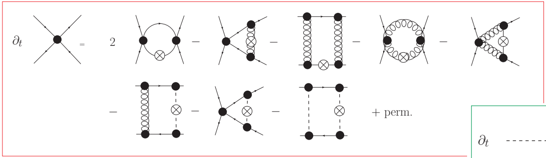
[MM, Strodthoff, Pawlowski, 2014]

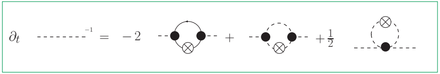


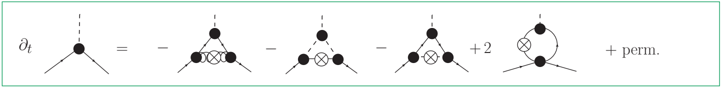








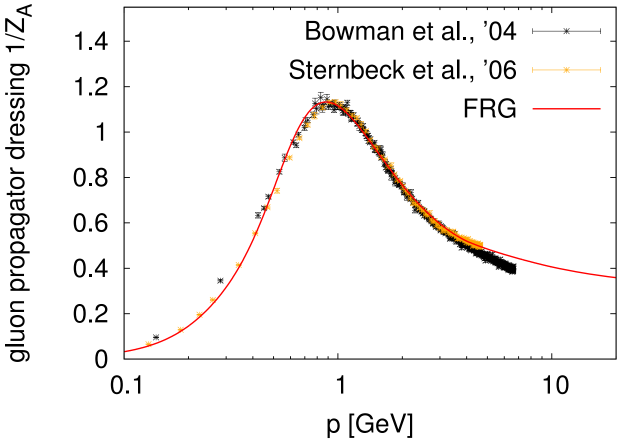




Gluon FRG input

[Fischer, Maas, Pawlowski, 2009], [Fister, Pawlowski, unpublished]

- $\Gamma^{(2)}_{AA}(p) \propto Z_A(p)\, p^2\, \left(\delta^{\mu\nu} - p^\mu p^\nu / p^2\right)$

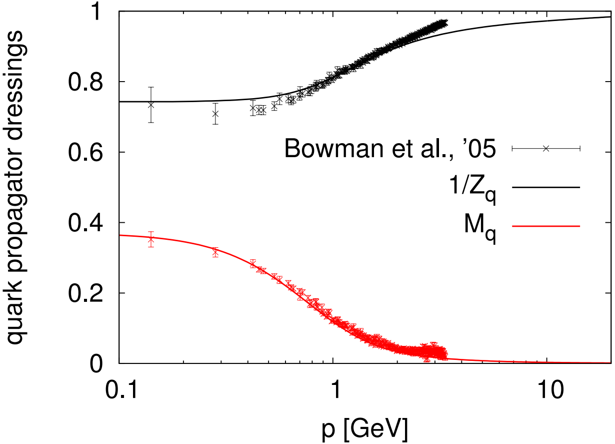


- FRG result ⇒ self-consistent calculation within FRG approach
- sets the scale in comparison to lattice QCD

Quark propagator

[MM, Pawlowski, Strodthoff, 2014]

- $\Gamma^{(2)}_{\bar{q}q}(p) \propto Z_q(p)\, \not{p} + M(p)$

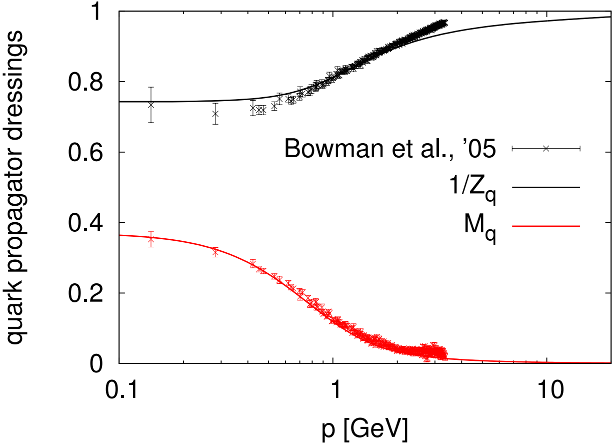


- FRG vs. lattice: bare mass, quenched, scale

Quark propagator

[MM, Pawlowski, Strodthoff, 2014]

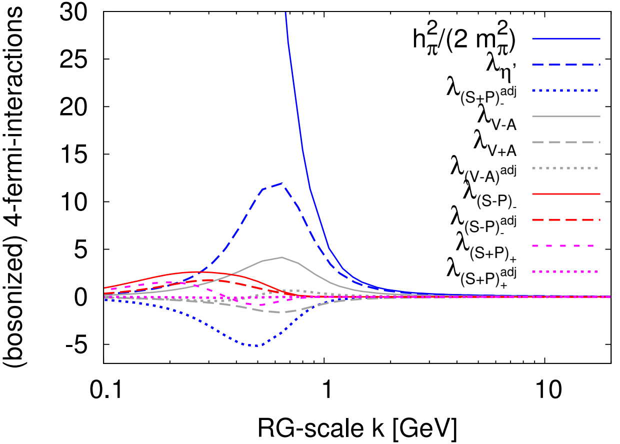
- $\Gamma^{(2)}_{\bar{q}q}(p) \propto Z_q(p)\, \not{p} + M(p)$



- FRG vs. lattice: bare mass, quenched, scale
- agreement not sufficient: need apparent convergence at $\mu \neq 0$

other 4-Fermi channels (mesons)

[MM, Pawlowski, Strodthoff, 2014]



- bosonized only σ-π-channel $\Rightarrow$ sufficient
  diquark momentum configuration more important
- other channels: quantitatively not important in loops

Quark-gluon interactions I

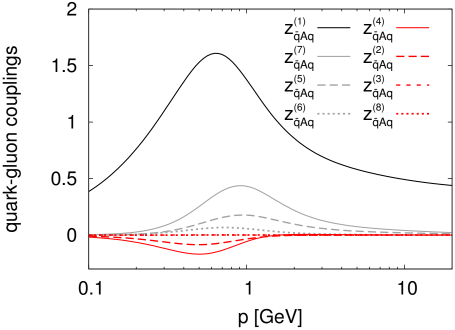
[MM, Pawlowski, Strodthoff, 2014]

- quark-gluon interaction most crucial for chiral symmetry breaking
- full tensor basis ⇒ sufficient chiral symmetry breaking strength?

Quark-gluon interactions I

[MM, Pawlowski, Strodthoff, 2014]

- quark-gluon interaction most crucial for chiral symmetry breaking
- full tensor basis ⇒ sufficient chiral symmetry breaking strength?

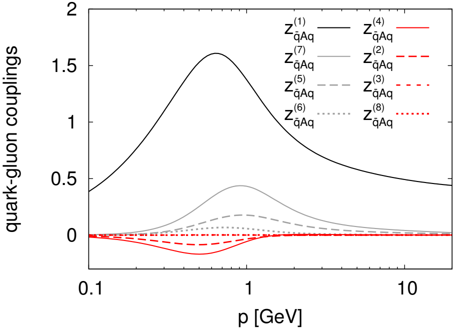


- vertex strength: reflects gluon gap
- 8 tensors (transversally projected):
  - classical tensor
  - chirally symmetric
  - break chiral symmetry

Quark-gluon interactions I

[MM, Pawlowski, Strodthoff, 2014]

- quark-gluon interaction most crucial for chiral symmetry breaking
- full tensor basis ⇒ sufficient chiral symmetry breaking strength?



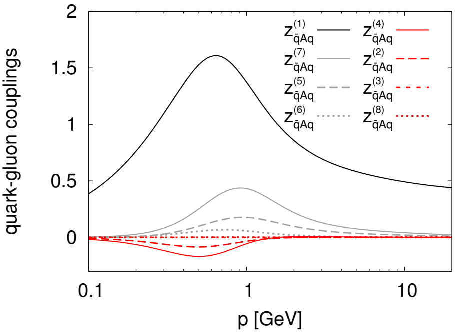
- vertex strength: reflects gluon gap
- 8 tensors (transversally projected):
  - classical tensor
  - chirally symmetric
  - break chiral symmetry

- important non-classical tensors: c.f., [Hopfer et al., 2012], [Williams, 2014], [Aguilar et al., 2014]
  - $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$ ($\frac{1}{2}\mathcal{T}^{(5)}_{\bar{q}Aq} + \mathcal{T}^{(7)}_{\bar{q}Aq}$): increases Z_q/decreases M_q considerably
  - anom. chromomagn. momentum ($\mathcal{T}^{(4)}_{\bar{q}Aq}$) increases M_q moderately

Quark-gluon interactions I

[MM, Pawlowski, Strodthoff, 2014]

- quark-gluon interaction most crucial for chiral symmetry breaking
- full tensor basis ⇒ sufficient chiral symmetry breaking strength?



- vertex strength: reflects gluon gap
- 8 tensors (transversally projected):
  - classical tensor
  - chirally symmetric
  - break chiral symmetry

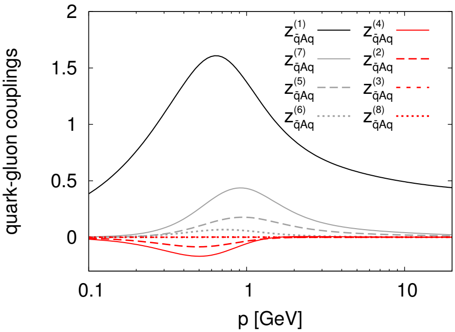
- important non-classical tensors: c.f., [Hopfer et al., 2012], [Williams, 2014], [Aguilar et al., 2014]
  - $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$ $(\frac{1}{2}\mathcal{T}^{(5)}_{\bar{q}Aq} + \mathcal{T}^{(7)}_{\bar{q}Aq})$: increases Z_q/decreases M_q considerably
  - anom. chromomagn. momentum $(\mathcal{T}^{(4)}_{\bar{q}Aq})$ increases M_q moderately

⇒ considerably less chiral symmetry breaking with full tensor basis

Quark-gluon interactions I

[MM, Pawlowski, Strodthoff, 2014]

- quark-gluon interaction most crucial for chiral symmetry breaking
- full tensor basis ⇒ sufficient chiral symmetry breaking strength?



- vertex strength: reflects gluon gap
- 8 tensors (transversally projected):
  - classical tensor
  - chirally symmetric
  - break chiral symmetry

- important non-classical tensors: c.f., [Hopfer et al., 2012], [Williams, 2014], [Aguilar et al., 2014]
  - $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$ $(\frac{1}{2}\mathcal{T}^{(5)}_{\bar{q}Aq} + \mathcal{T}^{(7)}_{\bar{q}Aq})$: increases Z_q/decreases M_q considerably
  - anom. chromomagn. momentum $(\mathcal{T}^{(4)}_{\bar{q}Aq})$ increases M_q moderately

⇒ considerably less chiral symmetry breaking with full tensor basis

- also important ingredient for bound-state equations

Quark-gluon interactions II

- missing strength?

Quark-gluon interactions II

- missing strength?
- expansion in tensor structures $\rightarrow$ expansion in operators $\bar{\psi}\not{D}^n\psi$

  [MM, Pawlowski, Strodthoff, 2014]

Quark-gluon interactions II

- missing strength?
- expansion in tensor structures $\to$ expansion in operators $\bar{\psi}\not{D}^n\psi$

  [MM, Pawlowski, Strodthoff, 2014]

  in particular $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$:
  - contributes to $\bar{q}Aq$, $\bar{q}A^2q$ and $\bar{q}A^3q$
  - contains important non-classical tensors ($\bar{q}Aq$)
  - considerable contribution to quark-gluon vertex ($\bar{q}A^2q$)
  - contribution to $\bar{q}A^3q$ seems unimportant

Quark-gluon interactions II

- missing strength?
- expansion in tensor structures $\rightarrow$ expansion in operators $\bar{\psi}\not{D}^n\psi$

  [MM, Pawlowski, Strodthoff, 2014]

  in particular $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$:
  - contributes to $\bar{q}Aq$, $\bar{q}A^2q$ and $\bar{q}A^3q$
  - contains important non-classical tensors ($\bar{q}Aq$)
  - considerable contribution to quark-gluon vertex ($\bar{q}A^2q$)
  - contribution to $\bar{q}A^3q$ seems unimportant

Quark-gluon interactions II

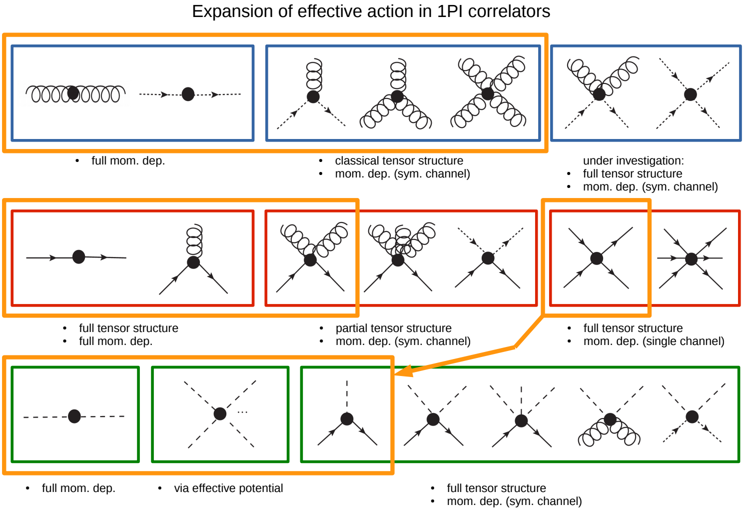
- missing strength?
- expansion in tensor structures → expansion in operators $\bar{\psi}\not{D}^n\psi$

  [MM, Pawlowski, Strodthoff, 2014]

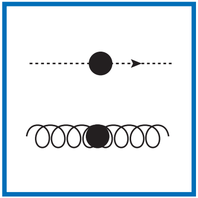
  in particular $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$:
  - contributes to $\bar{q}Aq$, $\bar{q}A^2q$ and $\bar{q}A^3q$
  - contains important non-classical tensors ($\bar{q}Aq$)
  - considerable contribution to quark-gluon vertex ($\bar{q}A^2q$)
  - contribution to $\bar{q}A^3q$ seems unimportant

- explicit calculations of $AA\bar{q}q$-vertex: [MM, Pawlowski, Strodthoff, in prep.]
  - full basis: 63 chirally symmetric tensor elements
  - 15 chirally symmetric tensor elements ($\bar{\psi}\not{D}^3\psi$):
    - ⋆ all seem important
    - ⋆ order of effect similar to $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$
    - ⋆ why? underlying principle?

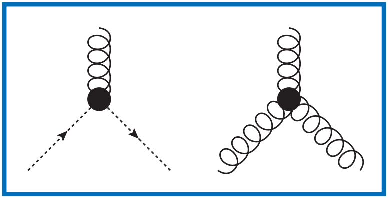
Stability of truncation (apparent convergence)



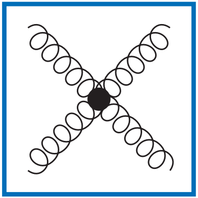
Vertex Expansion in YM theory [Cyrol, Fister, MM, Strodthoff, Pawlowski, to be published]



full. mom. dep.



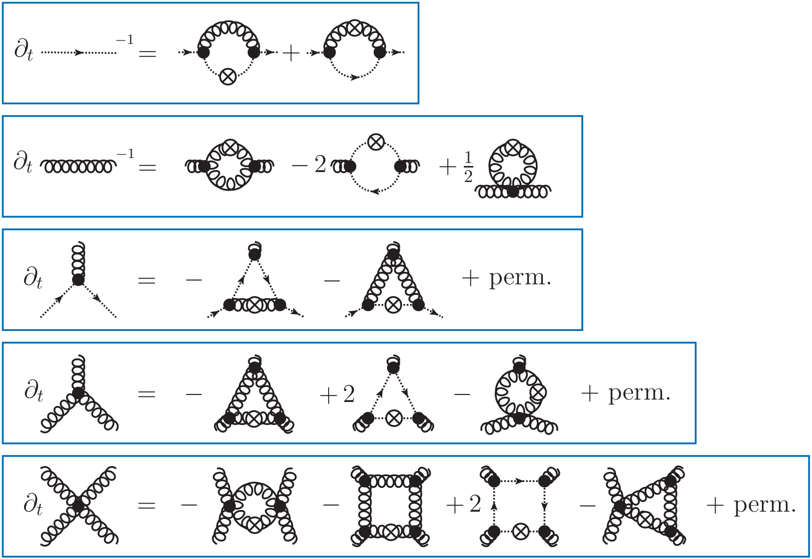
full. mom. dep.



sym. point and tadpole config.

Equations in YM theory

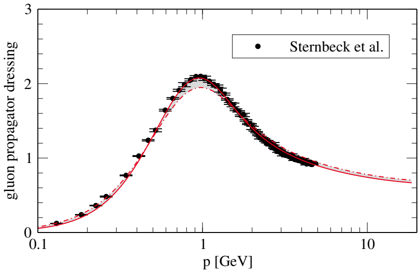
[Cyrol, Fister, MM, Strodthoff, Pawlowski, to be published]

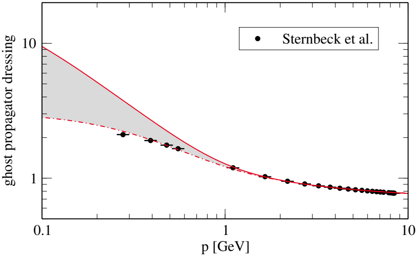


YM propagators

[Cyrol, Fister, MM, Pawlowski, Strodthoff, to be published]

- $\Gamma^{(2)}_{AA}(p) \propto Z_A(p)\, p^2 \, \left(\delta^{\mu\nu} - p^\mu p^\nu / p^2\right)$
- $\Gamma^{(2)}_{\bar{c}c}(p) \propto Z_c(p)\, p^2$



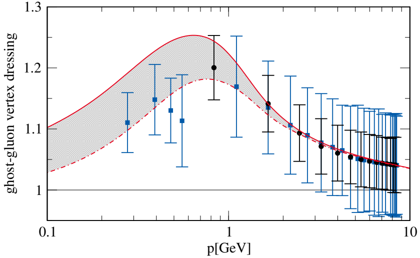


- band: family of decoupling solutions bounded by scaling solution

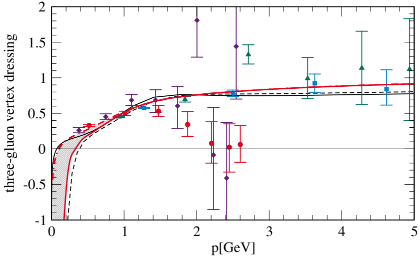
YM vertices I

[Cyrol, Fister, MM, Pawlowski, Strodthoff, to be published]

- comparison to Sternbeck '06



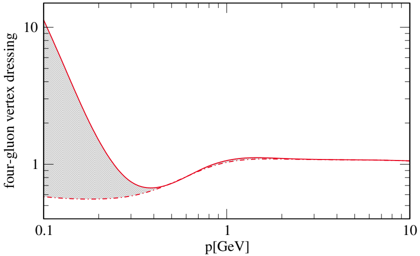
- comparison to
  Cucchieri, Maas, Mendes, '08
  Blum, Huber, MM, von Smekal '14

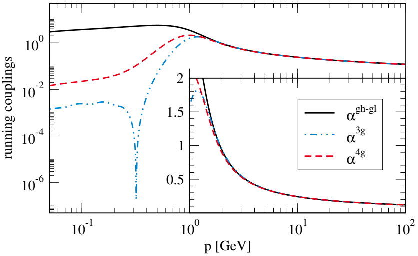


- band: family of decoupling solutions bounded by scaling solution

YM vertices II

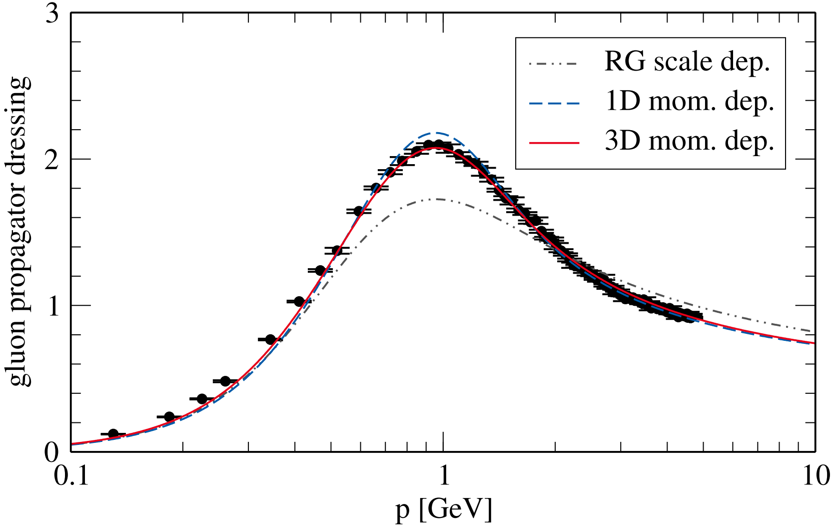
[Cyrol, Fister, MM, Pawlowski, Strodthoff, to be published]



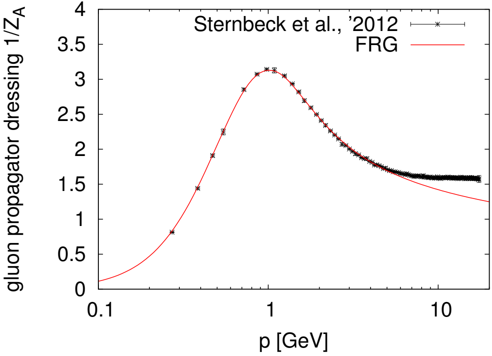


Apparent Convergence

[Cyrol, Fister, MM, Pawlowski, Strodthoff, to be published]



Outlook: unquenched gluon propagator



[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

- self-consistent solution of classical propagators and vertices (1D)
- massless quarks

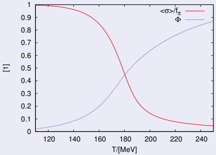
η'-meson (screening) mass at chiral crossover

- small η'-meson mass above chiral crossover? [Kapusta, Kharzeev, McLerran, 1998]
- drop in η' mass at chiral crossover? [Csörgo et al., 2010]

η'-meson (screening) mass at chiral crossover

- small η'-meson mass above chiral crossover? [Kapusta, Kharzeev, McLerran, 1998]
- drop in η' mass at chiral crossover? [Csörgo et al., 2010]

chiral crossover: Polyakov-Quark-Meson model (extended mean-field)

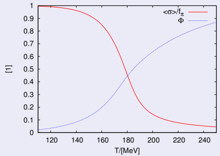


- $N_f = 2$ quark and meson degrees of freedom
- describes chiral crossover
- (de-)confinement via Polyakov loop potential
- $U(1)_A$-anomaly: mesonic 't Hooft determinant

η'-meson (screening) mass at chiral crossover

- small η'-meson mass above chiral crossover? [Kapusta, Kharzeev, McLerran, 1998]
- drop in η' mass at chiral crossover? [Csörgo et al., 2010]

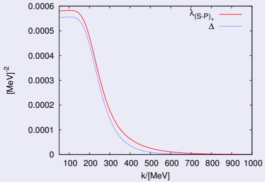
chiral crossover: Polyakov-Quark-Meson model (extended mean-field)



- $N_f = 2$ quark and meson degrees of freedom
- describes chiral crossover
- (de-)confinement via Polyakov loop potential
- $U(1)_A$-anomaly: mesonic 't Hooft determinant

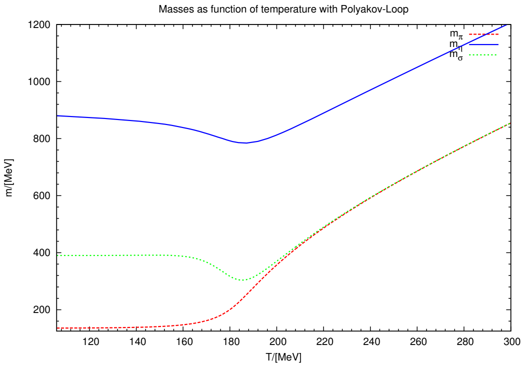
't Hooft determinant

[Heller, MM, 2015]



- RG-scale dependence from fQCD
- temperature dependence $k(T)$:
  - $\lambda_{(S-P)_+,fQCD}(k) \equiv \lambda_{(S-P)_+,PQM}(T)$

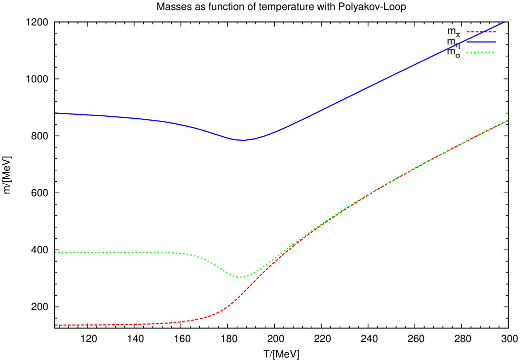
η'-meson (screening) mass at chiral crossover: result



[Heller, MM, 2015]

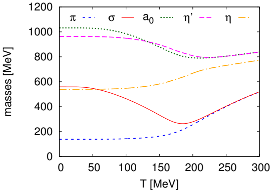
- screening masses!

η'-meson (screening) mass at chiral crossover: result



[Heller, MM, 2015]

- screening masses!
- QM-Model $N_f = 2 + 1$:



[MM, Schaefer, 2013]

- chiral symmetry restoration: $\Rightarrow$ drop in $m_{\eta'}$

Summary and Outlook

(quenched) QCD with functional RG

- QCD phase diagram: need for quantitative precision
- vacuum:
  - sole input $\alpha_S(\Lambda = \mathcal{O}(10)\text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10)\text{ GeV})$
  - good agreement with lattice simulations (sufficient?)
  - (non-perturbative) results:
    - quark-propagator
    - quark-gluon vertex
    - 4-Fermi interaction channels
    - YM-system
  - phenomenology: η'-meson and pion mass splitting

Summary and Outlook

(quenched) QCD with functional RG

- QCD phase diagram: need for quantitative precision
- vacuum:
  - sole input $\alpha_S(\Lambda = \mathcal{O}(10)\text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10)\text{ GeV})$
  - good agreement with lattice simulations (sufficient?)
  - (non-perturbative) results:
    - quark-propagator
    - quark-gluon vertex
    - 4-Fermi interaction channels
    - YM-system
  - phenomenology: η'-meson and pion mass splitting

- unquenching (first results)
- finite temperature/chemical potential
- more checks on convergence of vertex expansion
- bound-state properties (form factor,PDA. . .)