Effective Polyakov loop models for QCD-like theories at finite baryon density

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Motivation

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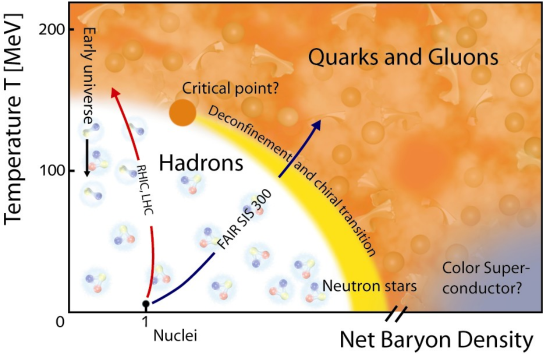

- Exploration of the QCD phase diagram at finite density

Motivation

Sign Problem prohibits Exploration of QCD phase diagram at finite μ

$$\det(D(\mu))^* = \det(D(-\mu))$$

Many proposed solutions to the sign Problem:

- Complex Langevin
- Integration on Lefshetz Thimbles
- Density of states method
- Dualization and flux simulations
- **Effective Polyakov loop theories**

Polyakov loop models for QCD-like theories

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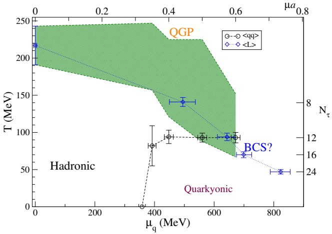
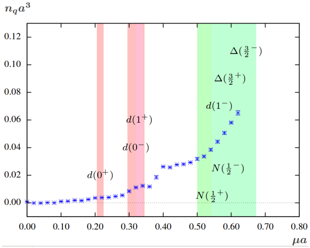
- QCD-like theories, e.g. two-color QCD, G_2 QCD:

  no sign Problem

- Qualitative and quantitative Comparison to results from full theory
- Validity test of eff. Polyakov loop theories
- Better understanding of phase diagram of QCD-like theories



Boz, et al. EPJ A 49 (2013)



Wellegehausen, Maas, von Smekal, PRD 89 (2014)

Effective Polyakov Loop Theories

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- d-1 dim. spin models sharing universal behavior at deconfinement transition with underlying gauge theory

  B.Svetitsky and L.G.Yaffe, Nucl. Phys. B210 (1982) 423

$$L = \mathrm{Tr} \prod_{t=1}^{N_\tau} U_o(\vec{x}, t)$$

- Less computational cost, especially with dynamical fermions and at low $T \Leftrightarrow$ large N_t
- Sign problem at finite μ: Complex Langevin

Effective Action

most general form:

$$S_{\text{eff}} = \sum_{ij} L_i\, K^{(2)}(i,j)\, L_j + \sum_{ijkl} L_i L_j\, K^{(4)}(i,j,k,l)\, L_k L_l + ... + \sum_i h^{(1)}\, L_i + ...$$

can also contain loops in adjoint or higher representations.

B. Svetitsky, Phys. Rept. 132 (1986), Heinzl et al., Phys. Rev. D. 72 (2005)

Find a way to calculate Kernels $K^{(2n)}$ and effective fermions couplings $h^{(n)}$:

$$\exp(-S_{\text{eff}}[L]) = \int \mathcal{D}U\ \delta(L - L[U])\ \exp(-S[U])$$

Different Aproaches

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- Find effective action by inverse Monte-Carlo methods

  Heinzl et al., Phys. Rev. D. 72 (2005)

- Find effective action by relative weights method

  Greensite, Langfeld, Phys. Rev. D. 87 (2013)

- **Use combined strong coupling and hopping expansion**

  Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012)

Effective theory that can be improved order by order

Effective Action

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Integrate out spatial links

$$Z = \int \mathcal{D}U_0\, \mathcal{D}U_i\ \det[D]\, e^{-S_g[U]} = \int \mathcal{D}U_0\, e^{-S_{\text{eff}}[U_0]} = \int \mathcal{D}L\, e^{-S_{\text{eff}}[L]}$$

Wilson Gauge action:

strong coupling expansion

Wilson quarks:

hopping expansion

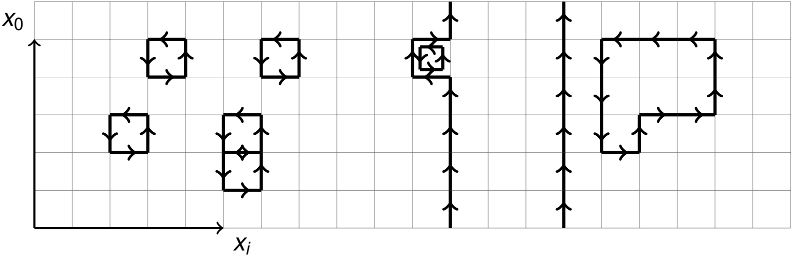
Truncation is valid for heavy quarks on reasonably fine lattices: $a \sim 0.1$ fm

Effective Action

Integrate out spatial links

$$Z = \int \mathcal{D}U_0\, \mathcal{D}U_i\ \det[D]\, e^{-S_g[U]} = \int \mathcal{D}U_0\, e^{-S_{\text{eff}}[U_0]} = \int \mathcal{D}L\, e^{-S_{\text{eff}}[L]}$$

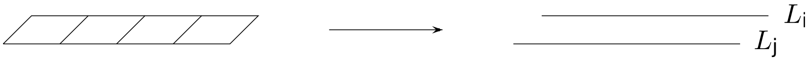
Wilson Gauge action: Wilson quarks:



Effective Action

Contributions to the effective action are graphs winding around the lattice in time direction

Gauge Action:



$$\Rightarrow S_{\text{eff}}^{g} = u^{N_t} \sum_{\langle ij \rangle} L_i L_j + \dots$$

$$u = \frac{\beta}{4} + \mathcal{O}(\beta^2) < 1 \qquad \text{for SU(2)}$$

Effective Action

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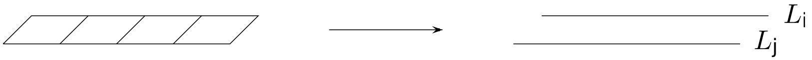
Contributions to the effective action are graphs winding around the lattice in time direction

Gauge Action:



$$\Rightarrow S^g_{\text{eff}} = u^{N_t} \sum_{\langle ij \rangle} L_i L_j + \ldots$$

Low T and strong coupling: $u^{N_t} < 10^{-12}$, omit gauge action

Heavy Fermions and Hopping Expansion

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$$\det[D] = \det[1 - \kappa H] = \exp(-\sum_{i=1}^{\infty} \frac{1}{i} \kappa^i \,\mathrm{Tr}[H^i])$$

Leading order:

$$-S_{\mathrm{LO}} = 2N_f \sum_{\vec{x}} \left[(hL(\vec{x}) + \bar{h}L^\dagger(\vec{x}) \right]$$

$$h(u, \kappa, N_t) = (2\kappa e^{a\mu})^{N_\tau} \exp\left[6N_\tau \kappa^2 \frac{1 - u^{N_\tau}}{1 - u} + \mathcal{O}(u\kappa^2) \right]$$

Heavy Fermions and Hopping Expansion

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- Sum up generalized Polyakov loops

$$\exp\left[-2N_f \sum_{\vec{x}} \sum_{n=1}^{\infty} \left(\frac{(-1)^n}{n} h \,\mathrm{Tr}(W_{\vec{x}}^n) \right)\right] = \prod_{\vec{x}} \det\left[1 + hW_{\vec{x}}\right]^{2N_f}$$

Pietri, Feo, Seiler, Stamatescu, Phys. Rev. D. 76 (2007)

$\longrightarrow$ resumation leads to lattice saturation for large μ

- Also include and expand in spatial hoppings

$$\det[D] = \det[1 - \kappa T]\det[1 - (1 - \kappa T)^{-1} S]$$

Heavy Fermions and Hopping Expansion

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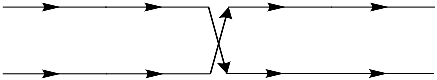
Order κ^2:



$$-S_{\text{eff}} = N_f \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}}\mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

Heavy Fermions and Hopping Expansion

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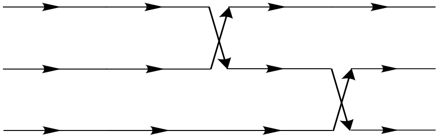
Order κ^4:



Heavy Fermions and Hopping Expansion

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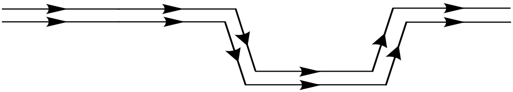
$$
\begin{aligned}
-S_{\text{eff}} = {} & N_f \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\,W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
& + 2N_f^2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2} \\
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
& + 2N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \ .
\end{aligned}
$$

Heavy Fermions and Hopping Expansion

Order κ^4:



in QCD-like theories: fermion lines interchanging "diquarks"

Heavy Fermions and Hopping Expansion

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$$
\begin{aligned}
-S_{\text{eff}} = {} & N_f \sum_{\vec{x}} \log(1 + h\,\mathrm{Tr}\,W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
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& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
& + 2N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + N_f(2N_f - 1)\kappa^4 N_\tau^2 \sum_{x,i} \frac{h^2}{(1 + h\,\mathrm{Tr}\,W_{\vec{x}} + h^2)(1 + h\,\mathrm{Tr}\,W_{\vec{x}+i} + h^2)} \;.
\end{aligned}
$$

Cold and Dense QC_2D with Heavy Quarks

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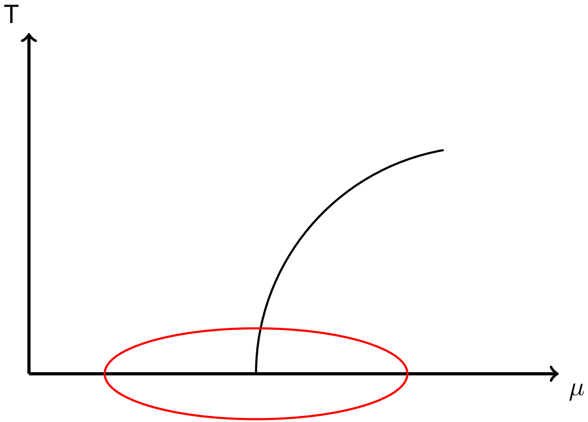

Cold and Dense QC_2D with Heavy Quarks

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- Very heavy quarks:

  m_q = 10.0014 GeV

- Diquark mass fixed to m_d = 20 GeV

  $\rightarrow$ small binding energy

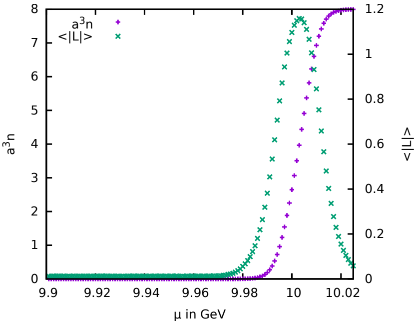
- Silverblaze property

- Lattice saturation, due to Pauli principle

- Deconfinement / Screening ?

m_d = 20 Gev, T = 5 MeV, a = 0.081 fm

$\beta = 2.5$, $\kappa = 0.00802$, $N_t = 484$



PS, von Smekal PRD 92 094504 (2015)

Cold and Dense QC_2D with Heavy Quarks

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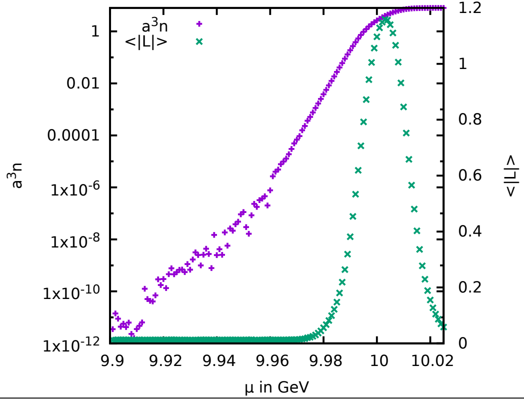
- Two exponential regions
- Thermal excitations of one-quark and two-quark states



Cold and Dense QC_2D in the strong coupling limit

$T = 0$, $\mu > 0$ anti-fermions decouple: $h = (2\kappa e^{a\mu})^{N_t}$, $\bar{h} = 0$, $N_f = 1$

$$Z(\beta = 0) = \left[\int dW (1 + hL + h^2)^2\right]^{N_s^3} = [1 + 3h^2 + h^4]^{N_s^3}$$

$$\Rightarrow n = \frac{T}{V}\frac{\partial}{\partial\mu}\log Z = \frac{1}{a^3}\frac{6h^2 + 4h^4}{1 + 3h^2 + h^4}$$

Small but finite Temperature: Mean field Polyakov loop

$$Z = (1 + h\langle L\rangle + h^2)^{2N_f N_s^3}$$

$$\Rightarrow a^3 n = 4N_f \frac{1 + \langle L\rangle e^{\frac{m_q - \mu}{T}}}{1 + 2\langle L\rangle e^{\frac{m_q-\mu}{T}} + \langle L\rangle e^{2\frac{m_q-\mu}{T}}}$$

Cold and Dense QC_2D with Heavy Quarks

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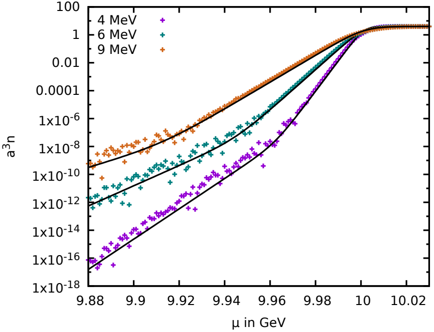
- Behavior of $a^3 n$ is well described by a LO mean field model
- No Fit! All values taken from Simulations or are input
- $\langle L \rangle \approx 5 \cdot 10^{-5} < \langle |L| \rangle = 0.006$
  $\langle L \rangle$ determined by histograms



PS, von Smekal PRD 92 094504 (2015)

Cold and Dense QC_2D with Heavy Quarks

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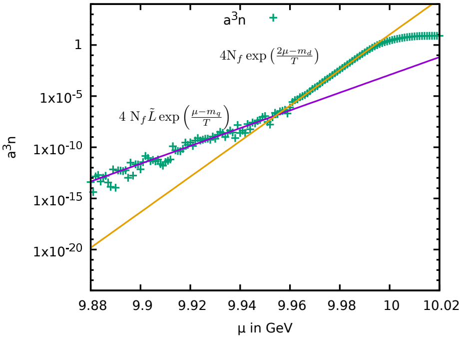
- Most precise description of the second exponential by

$$a^3 n = 4N_f \exp\left(\frac{2\mu - m_d}{T}\right)$$

$$m_d = 19.9986(10)\ \text{GeV}$$

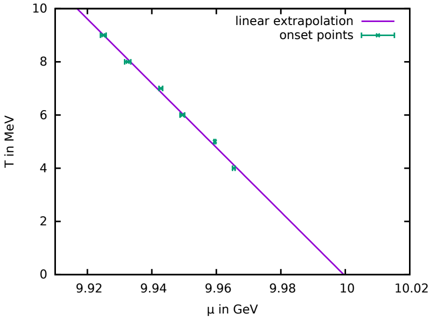
- Bound State
- Model includes Confinement



PS, von Smekal PRD 92 094504 (2015)

Phase Diagram

- Diquark onset but no Bose-Einstein condensate $T > E_{Bind}$.
- Line terminates at $\mu = \frac{m_d}{2}$ according to Silverblaze property
- Deconfinement transition terminates at $\mu > \frac{m_d}{2}$
- Look for BEC at larger κ and β, where diquarks are bound more tightly



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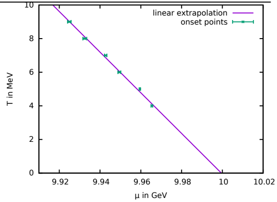
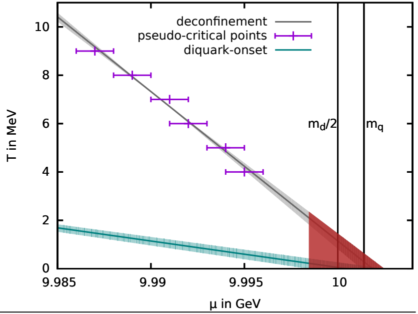
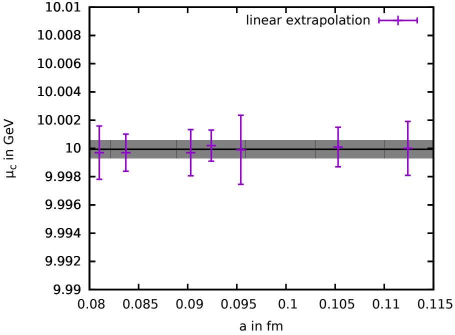




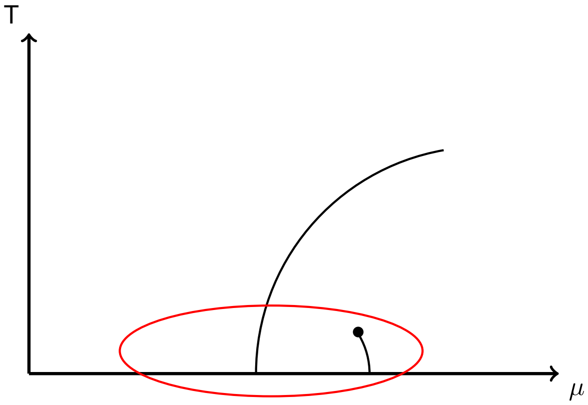
Extrapolation to $T = 0$

- $T = 0$ endpoint is in perfect agreement with $\mu = \frac{m_d}{2}$
- Endpoint is independent of lattice spacing



PS, von Smekal PRD 92 094504 (2015)

Results for heavy and dense G2 QCD



G2 QCD

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- Smallest exceptional Lie Group
- All representation are real
- No sign problem: real and positive for $N_f = 1$
- YM theory has 1st order phase transition
- Spectrum: bosonic and fermionic baryons

$$\{7\} \otimes \{7\} = \{1\} \oplus \{7\} \oplus \{14\} \oplus \{27\}$$
$$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus 4 \cdot \{7\} \oplus 2 \cdot \{14\} \oplus \dots$$
$$\{14\} \otimes \{14\} = \{1\} \oplus \{14\} \oplus \{27\} \oplus \dots,$$
$$\{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \{7\} \oplus 5 \cdot \{14\} \oplus \dots,$$
$$\{7\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \dots$$

diquarks (baryon number 2)

Name	$\mathcal{O}$	T	J	P	C
$d(0^{++})$	$\bar{u}^C \gamma_5 u + c.c.$	SASS	0	+	+
$d(0^{+-})$	$\bar{u}^C \gamma_5 u - c.c.$	SASS	0	+	-
$d(0^{-+})$	$\bar{u}^C u + c.c.$	SASS	0	-	+
$d(0^{--})$	$\bar{u}^C u - c.c.$	SASS	0	-	-
$d(1^{++})$	$\bar{u}^C \gamma_\mu d - \bar{d}^C \gamma_\mu u + c.c.$	SSSA	1	+	+
$d(1^{+-})$	$\bar{u}^C \gamma_\mu d - \bar{d}^C \gamma_\mu u - c.c.$	SSSA	1	+	-
$d(1^{-+})$	$\bar{u}^C \gamma_5 \gamma_\mu d - \bar{d}^C \gamma_5 \gamma_\mu u + c.c.$	SSSA	1	-	+
$d(1^{--})$	$\bar{u}^C \gamma_5 \gamma_\mu d - \bar{d}^C \gamma_5 \gamma_\mu u - c.c.$	SSSA	1	-	-

mesons (baryon number 0)

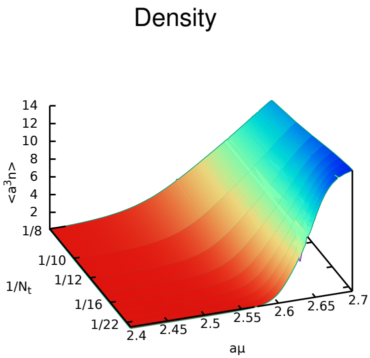
Name	$\mathcal{O}$	T	J	P	C
π	$\bar{u}\gamma_5 d$	SASS	0	-	+
η	$\bar{u}\gamma_5 u$	SASS	0	-	+
a	$\bar{u}d$	SASS	0	+	+
f	$\bar{u}u$	SASS	0	+	+
ρ	$\bar{u}\gamma_\mu d$	SSSA	1	-	+
ω	$\bar{u}\gamma_\mu u$	SSSA	1	-	+
b	$\bar{u}\gamma_5\gamma_\mu d$	SSSA	1	+	+
h	$\bar{u}\gamma_5\gamma_\mu u$	SSSA	1	+	+

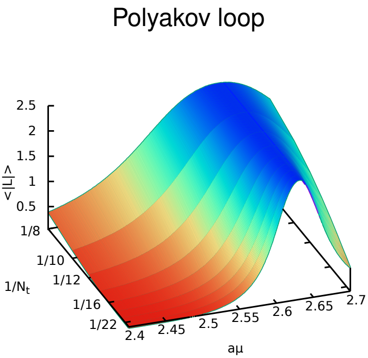
baryons (baryon number 3)

Name	$\mathcal{O}$	T	J	P	C
N	$T^{abc}(\bar{u}_a^C \gamma_5 d_b) u_c$	SAAA	1/2	$\pm$	$\pm$
Δ	$T^{abc}(\bar{u}_a^C \gamma_\mu u_b) u_c$	SSAS	3/2	$\pm$	$\pm$

T: (x, s, C, F)

Phase diagram





- Cold and dens region of the G2 effective theory phase diagram
- $\beta/N_c = 1.4$, $\kappa = 0.0357$, $N_t = 8, ...24$

Cold and Dens Region

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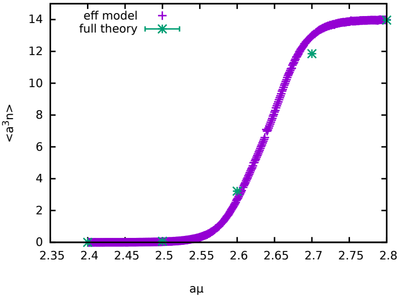
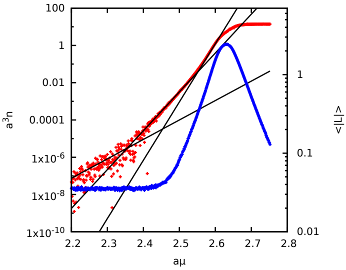
- Deconfinement and lattice saturation
- three regions with different exponential growth



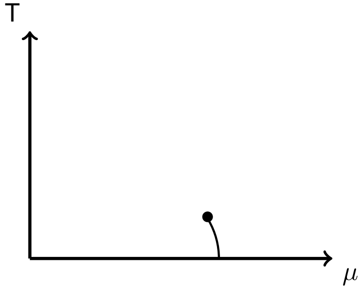


- Comparison of full 4d G2 QCD simulations to effective theory

4d Data by Björn Wellegehausen

Effective Polyakov loop theory for 2 dim G2 QCD

- Less computational cost for full theory
- Simpler phase diagram: no BEC, $U(1)_B$ can not break spontaneously (Mermin-Wagner theorem)



- Effective theory is 1d, can it have a phase transition? (van Hove theorem)

Effective Polyakov loop theory for 2 dim G2 QCD

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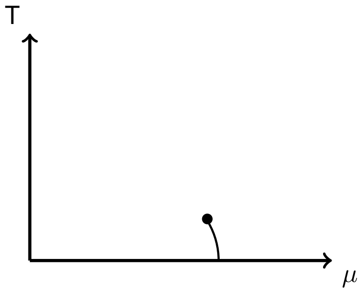

- Effective theory is 1d, can it have a phase transition? (van Hove theorem)

Yes, since there is an external field (dynamical fermions)!

Effective Polyakov loop theory for 2 dim G2 QCD

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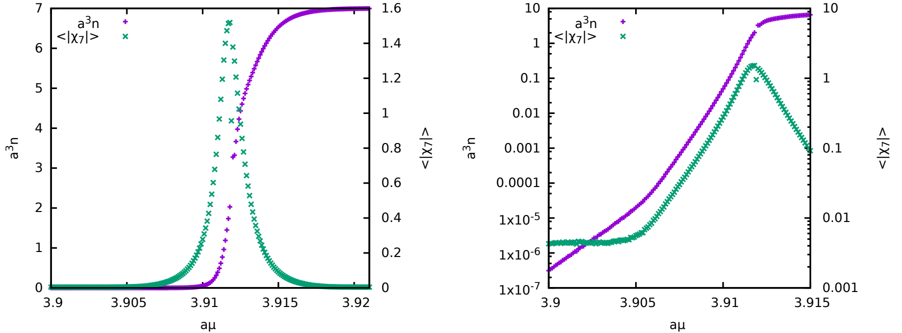
- Strong coupling limit: No diquarks!

$$Z(\beta = 0) = \left[\int dW (1 + (h + h^6)\chi_7 + (h^2 + h^5)(\chi_7 + \chi_{14}) + (h^3 + h^4)(\chi_7^2 - \chi_{14}) + h^7) \right]^{N_s^3}$$

$$= [1 + h^3 + h^4 + h^7]^{N_s^3}$$

- Full theory: diquarks with unusual mass ordering $m_d(0^+) > m_d(1^+)$

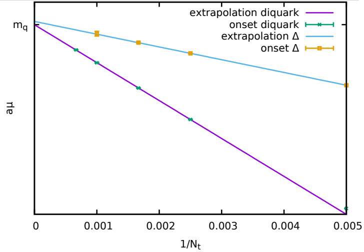
Results

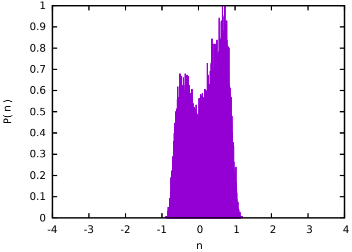


- Cold and dens region of the effective theory for the 2d G2 QCD phase diagram
- $\beta/N_c = 1.39$, $\kappa = 0.01$, $N_t = 800$

Results

- Extrapolation: No bound diquarks
- negative Polyakov loop → negative local particle numbers
- Coexistence of baryon and anti-baryon "phases"
  Also in effective theory for 4d G2 QCD
- Negative Polyakov loops lead to numerical instability





Summary & Conclusion

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Effective Polyakov loop theory for QC_2D

- Thermal diquark excitation onset at $T > 0$
- Extrapolation $T \to 0$ gives $m_d/2$ according to Silverblaze Property
- However no signal for BEC: $T > E_{\text{Bind.}}$

Effective Polyalov loop theory for G2

- Two- and three-quark excitations in agreement with possible spectrum
- Good Agreement between full theory and effective model

Analytic relations for Cold and Dense regime

$$h = \exp\left[N_\tau\left(a\mu + \ln 2\kappa + 6\kappa^2\frac{u - u^{N_\tau}}{1-u}\right)\right] \ ,$$

$$am_d = \text{ArCosh}\left(1 + \frac{((\frac{1}{2\kappa})^2 - 4)((\frac{1}{2\kappa})^2 - 1)}{(2(\frac{1}{2\kappa})^2 - 3)}\right) - 24\kappa^2\frac{u}{1-u} + \mathcal{O}(\kappa^4 u^2, \kappa^2 u^5) \ .$$