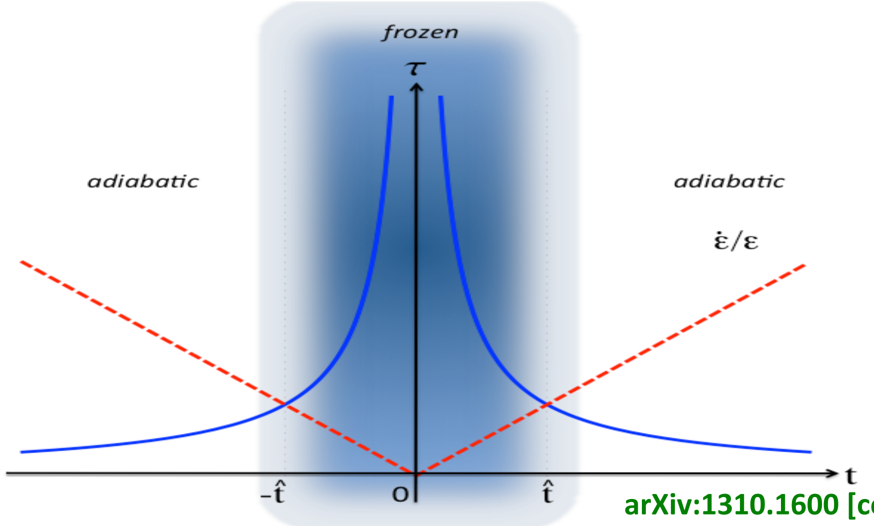
Kibble-Zurek dynamics and off-equilibrium scaling of critical cumulants in the QCD phase diagram

Raju Venugopalan

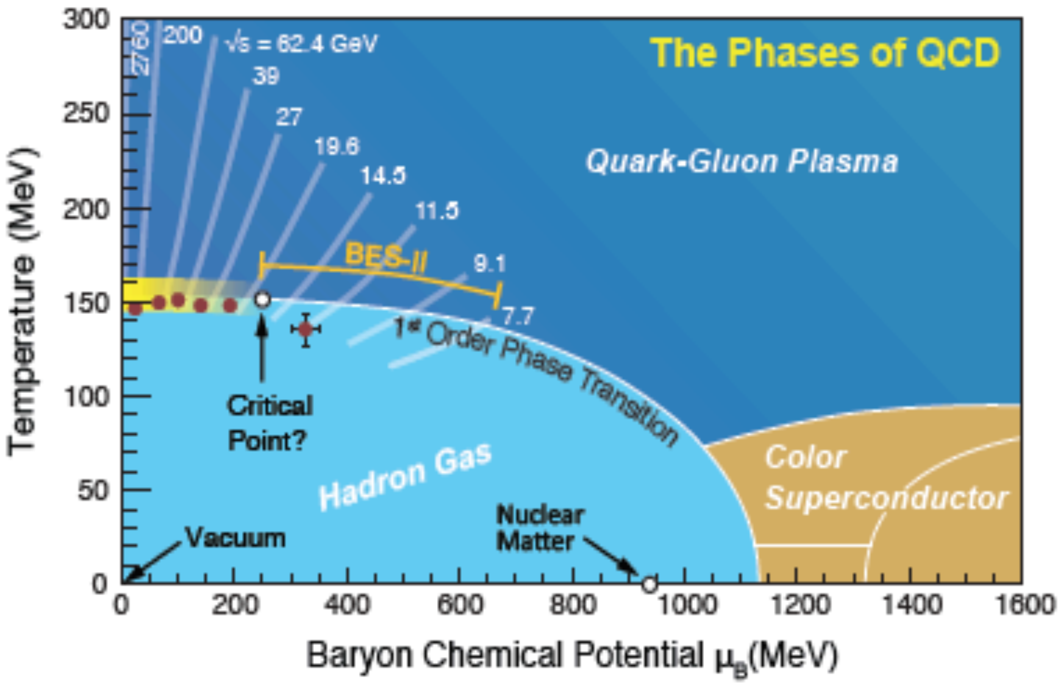
BNL/Heidelberg



arXiv:1310.1600 [cond-mat.stat-mech]

Giessen Seminar, June 28th, 2016

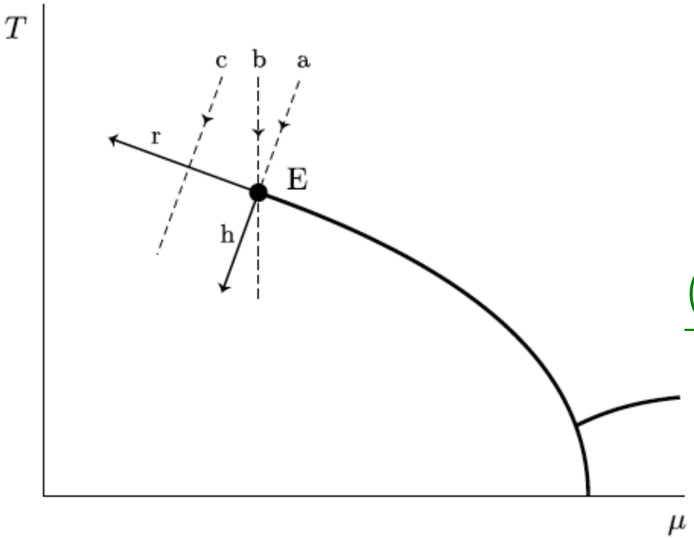
Critical point in the QCD phase diagram?



In HI collisions, trajectories close to a critical point experience large fluctuations in moments of the order parameter

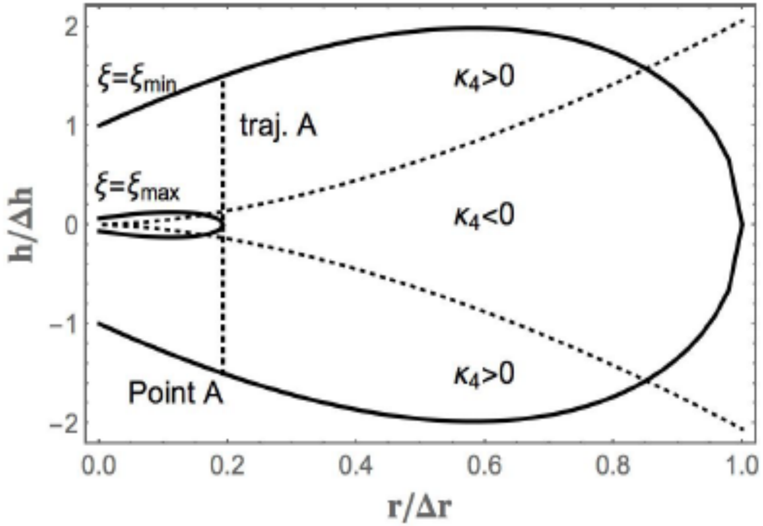
Depending on the static universality class, these moments (and conserved charges they couple to) can display non-monotonic behavior across the CP

QCD critical point and the 3-D Ising model



$$\frac{(T - T_c)}{\Delta T} = \frac{h}{\Delta h}$$

$$\frac{(\mu - \mu_c)}{\Delta \mu} = -\frac{r}{\Delta r}$$



The QCD critical point lies in the static universality class of the 3-D Ising model

Berges, Rajagopal, arXiv:hep-ph/9804233

Very valuable for quantitative studies of critical dynamics:
however, map between the two phase diagrams is non-universal

Critical cumulants in equilibrium

Critical fluctuations in equilibrium described by

$$P_0(\sigma) = \exp\left(-V_4\,\Omega_0(\sigma)\right) \qquad V_4 = V/T$$

with the 3-D Ising effective action

$$\Omega_0(\sigma) = \frac{1}{2}m_\sigma^2(\sigma-\sigma_0)^2 + \frac{\lambda_3}{3}(\sigma-\sigma_0)^3 + \frac{\lambda_4}{4}(\sigma-\sigma_0)^4$$

where σ is the zero mode of the critical field, with $m_\sigma = \xi^{-1}_{eq}$

Work in regime with expansion parameter $\epsilon = \sqrt{\frac{\xi_{\rm eq}^3}{V}} << 1$

We drop the kinetic term σ^2/L^2 << σ^2/ξ^2_{eq} because $L_{\rm micro} << \xi << L$

Critical cumulants in equilibrium

Compute critical cumulants:

$$< \cdots >= \frac{\int_{-\infty}^{\infty} d\sigma\,(\cdots)P(\sigma;\tau)}{\int_{-\infty}^{\infty} P(\sigma;\tau)}$$

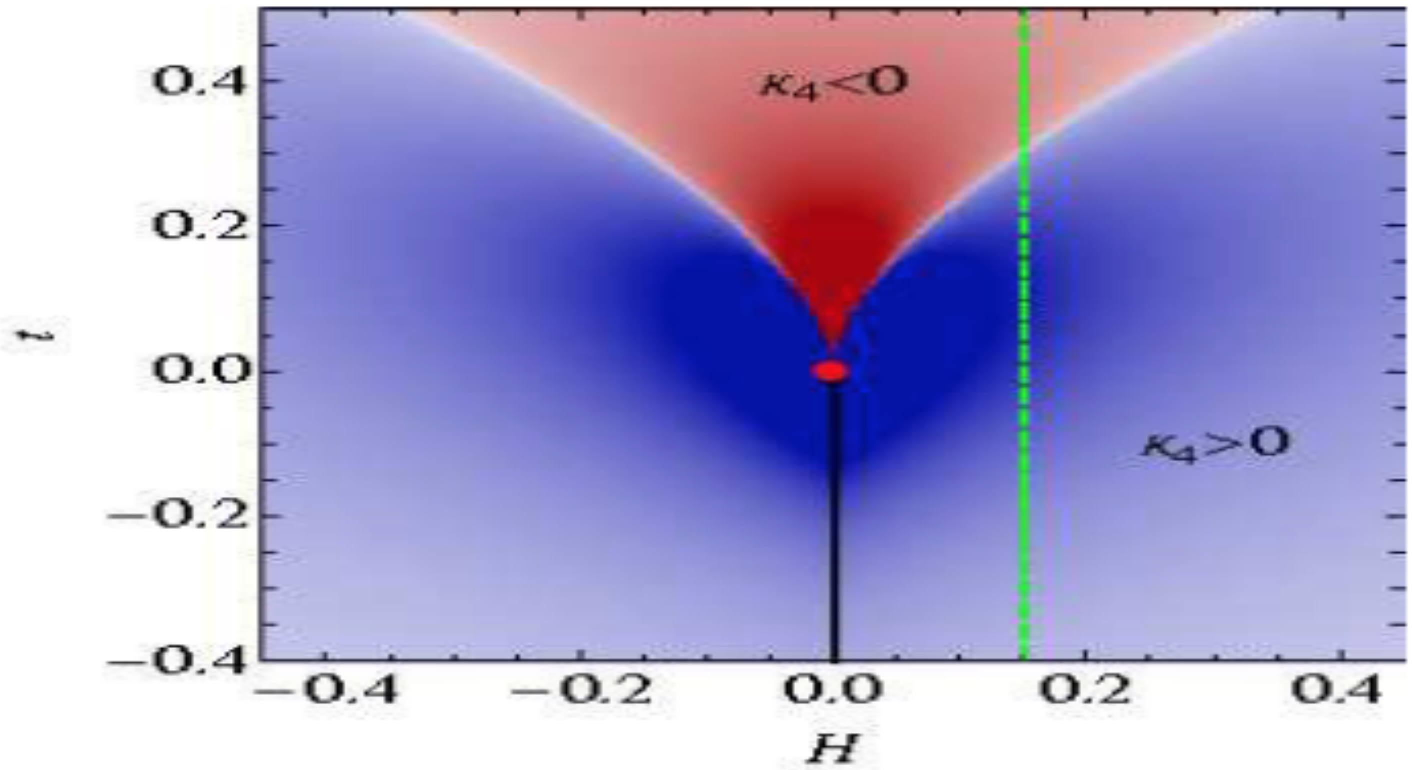
Magnetization: $\mathbf{M_{eq} = \sigma_0 \sim \xi^{-1/2}_{eq}}$

Variance: $\mathbf{\kappa_2 \sim \xi^{2}_{eq}}$

Non-Gaussian cubic (Skewness) and quartic (Kurtosis) cumulants: $\mathbf{\kappa_3 \sim \xi^{9/2}_{eq}}$, $\mathbf{\kappa_4 \sim \xi^{7}_{eq}}$

Stephanov, hep-ph/0809.3450

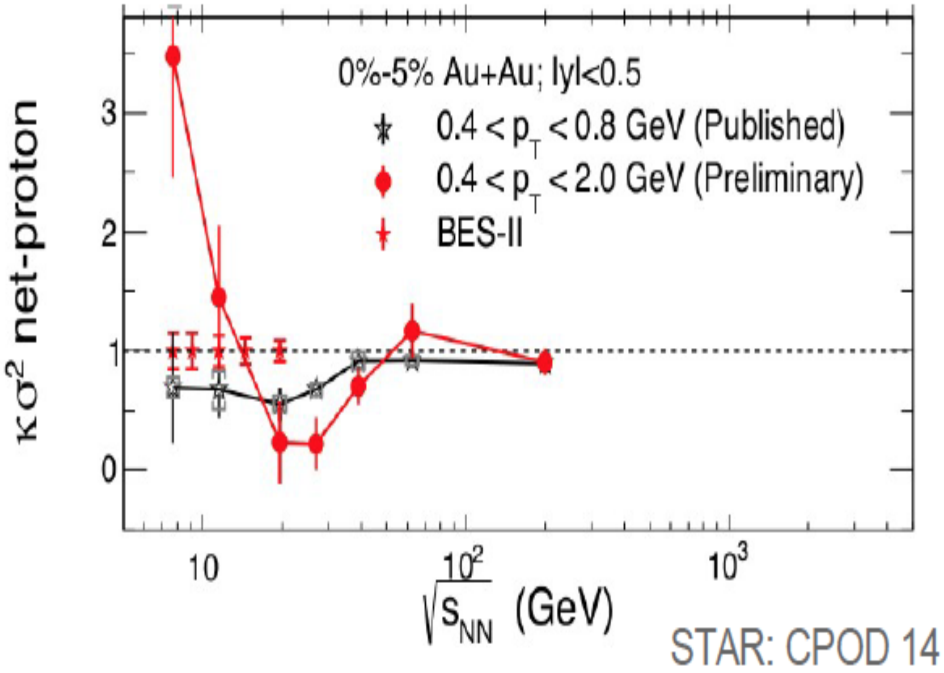
Equilibrium critical dynamics near a critical point



Example: sign of kurtosis (fourth cumulant of critical order parameter) in the universality class of the 3-D Ising model in the crossover regime

Stephanov,arXiv:1104.1627

Equilibrium dynamics in vicinity of critical point



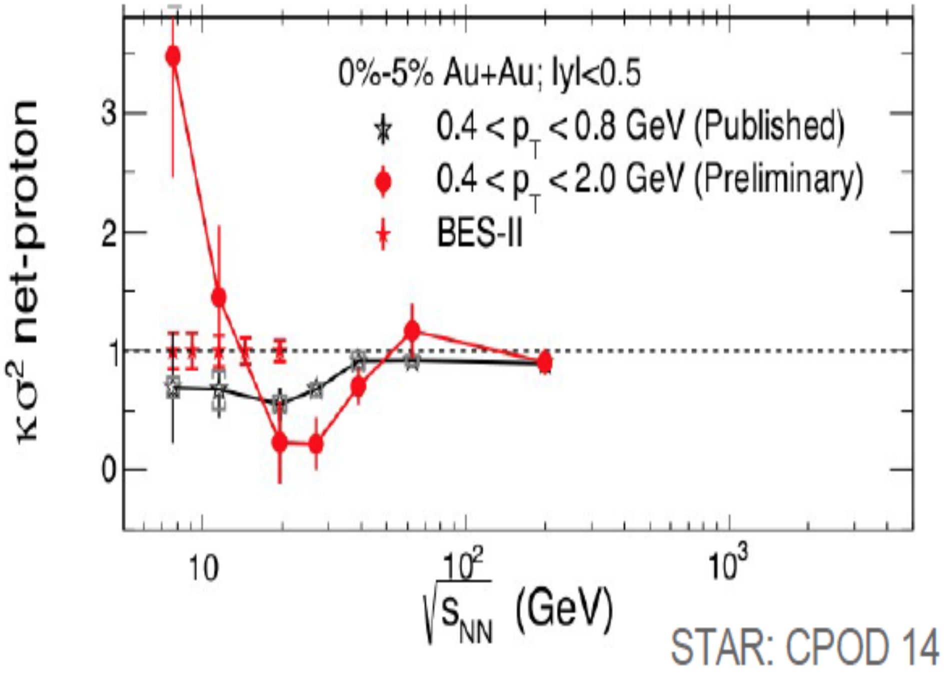
Hints of non-monotonicity seen in heavy-ion experiments
- strong motivation for Beam Energy Scan (BES) II at RHIC

However: according to the dynamical theory of critical phenomena,
the relaxation time of critical modes $\tau_{eff} \sim \xi^{z}$

Hohenberg,Halperin, Rev.Mod.Phys. 49, 435 (1977)

Close to the critical point, critical modes will fall out of equilibrium

Equilibrium dynamics in vicinity of critical point



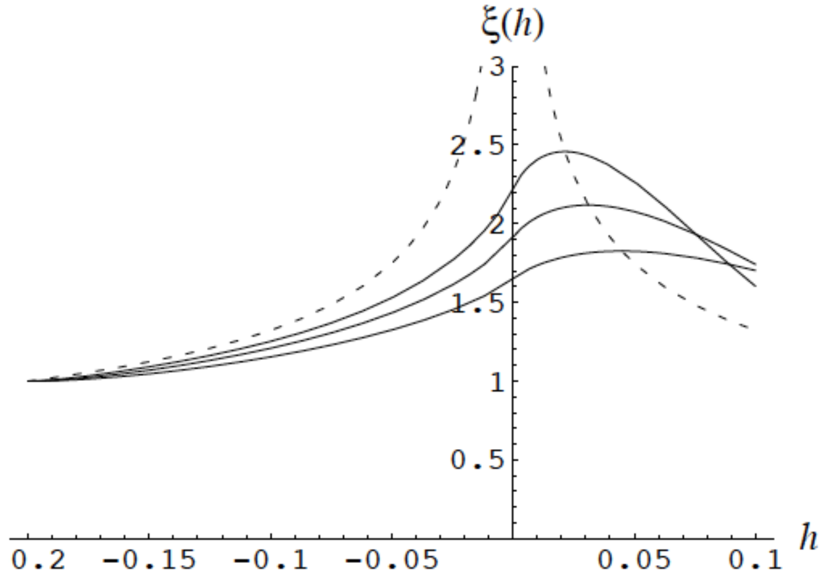
For QCD, ``model H" in H-H classification scheme, z =3 – strong dependence on ξ
-- critical mode is linear combination of chiral condensate and baryon current

Son,Stephanov, hep-ph/0401052
Fujii, hep-ph/0302167

Moments of critical order parameter in trajectories close to the CP will deviate significantly from equilibrium expectations

-- such trajectories will likely retain strong memory effects at freeze-out

Non-equilibrium dynamics in vicinity of critical point



Previous study:

Berdnikov,Rajagopal,hep-ph/9912274

$$\partial_\tau \xi^{-1} = -\tau_{\rm eff}^{-1}(\xi^{-1} - \xi_{\rm eq.}^{-1})$$

This result is reproduced in a Gaussian model for the effective action for the critical field -- equilibrium values of higher cumulants are zero

We will consider here a more general formulation that extracts non-trivial non-equilibrium dynamics of higher cumulants of interest

Mukherjee,Venugopalan,Yin, arXiv:1506.00645, arXiv:1512.08022

Evolution equations for cumulants

Diffusive properties of critical mode described by the Fokker-Planck equation

$$\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{\rm eff}} \left(\partial_\sigma \left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1}\partial_\sigma\right] P(\sigma;\tau)\right)$$

Critical modes receive random kicks from bulk modes--which are in thermal equilibrium

For ε < 1 can re-express as closed form expressions for cumulants

Mukherjee,Venugopalan,Yin, arXiv:1506.00645

$$\partial_\tau M = -\tau_{\rm eff}^{-1}\, p_1(M)\left[1+O(\epsilon)\right]$$

$$\partial_\tau \kappa_n = -n\,\tau_{\rm eff}^{-1}\, p_n(M,\kappa_2,\cdots,\kappa_n)\left[1+O(\epsilon)\right]$$

P_n – polynomials of the magnetization and lower (n-1) moments

Evolution equations for cumulants

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P_n – polynomials of the magnetization and lower (n-1) moments

- **For Gaussian fluctuations, reproduce Berdnikov-Rajagopal results**

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P_n – polynomials of the magnetization and lower (n-1) moments

- **For Gaussian fluctuations, reproduce Berdnikov-Rajagopal results**
- **Near equilibrium: $\delta\kappa_n = \kappa_n - \kappa_n^{eq}$**

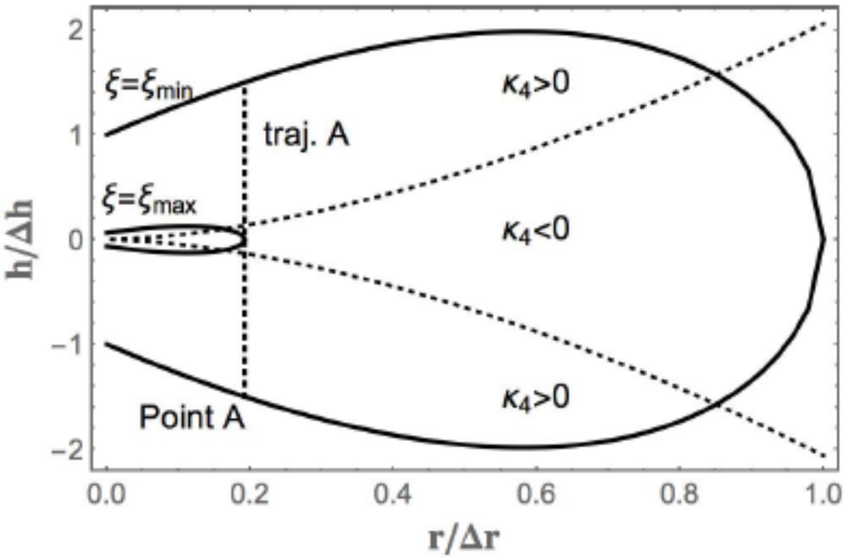
$$\partial_\tau \kappa_2 = -2\,\tau_{\rm eff}^{-1}\, a_2\,\delta\kappa_2$$

$$\partial_\tau \kappa_3 = -3\,\tau_{\rm eff}^{-1}\left[a_2\,\delta\kappa_2 + a_3\,\delta\kappa_3\right]$$

$$\partial_\tau \kappa_4 = -4\,\tau_{\rm eff}^{-1}\left[a_2\,\delta\kappa_2 + a_3\,\delta\kappa_3 + a_4\,\delta\kappa_4\right]$$

Higher cumulants relax more slowly to equilibrium…

Initial condition for evolution of critical modes in heavy-ion collisions: parameters from universality



Fix equilibrium parameters from universality with 3-D Ising model

$$\sigma_0(r,h),\ m_\sigma(r,h),\lambda_3(r,h),\lambda_4(r,h)$$

Zinn-Justin, hep-th/0002136

r & h are reduced temperature and magnetic field respectively

$$\tau_{\rm eff} = \tau_{\rm rel}\left(\frac{\xi_{\rm eq}}{\xi_{\rm min}}\right)^z$$

τ_{rel} is relaxation time at outside edge of critical region

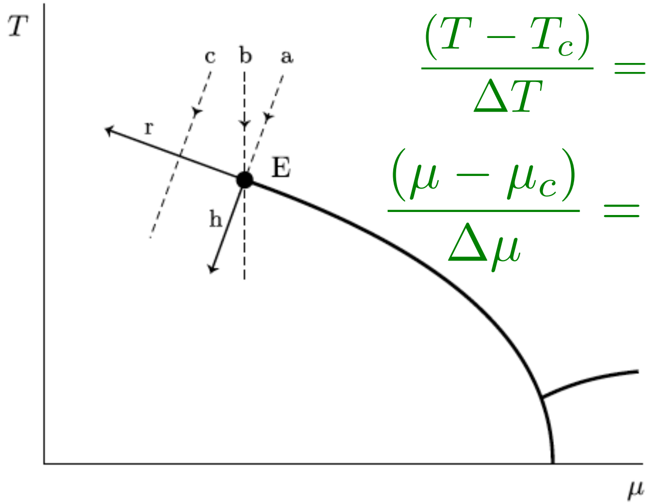
Choose ξ_{max} / ξ_{min} = 3

$$\xi_{\rm eq} \sim |h|^{-2/5}$$

$$=> \xi_{\rm eq} \sim |\frac{(\tau-\tau_c)}{\tau_Q}|^{-2/5} \text{ with } \tau_Q = \tau_c\left(\frac{\Delta T}{T_c(3c_s^2)}\right)$$

Initial condition: map to QCD critical region

Map to QCD phase diagram: (r,h) ➔ (T,μ)



$$\frac{(T-T_c)}{\Delta T} = \frac{h}{\Delta h}$$

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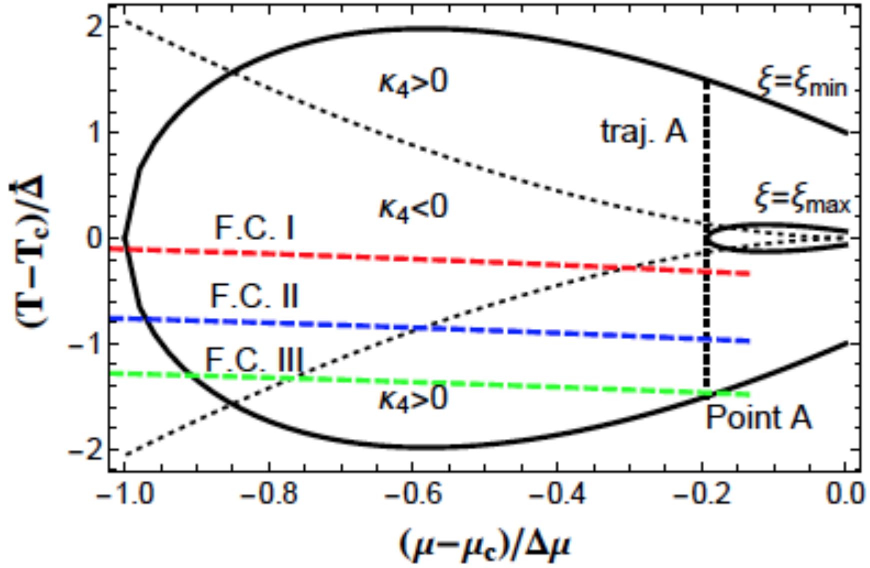


- **Space-time evolution of heavy-ion fireball:**

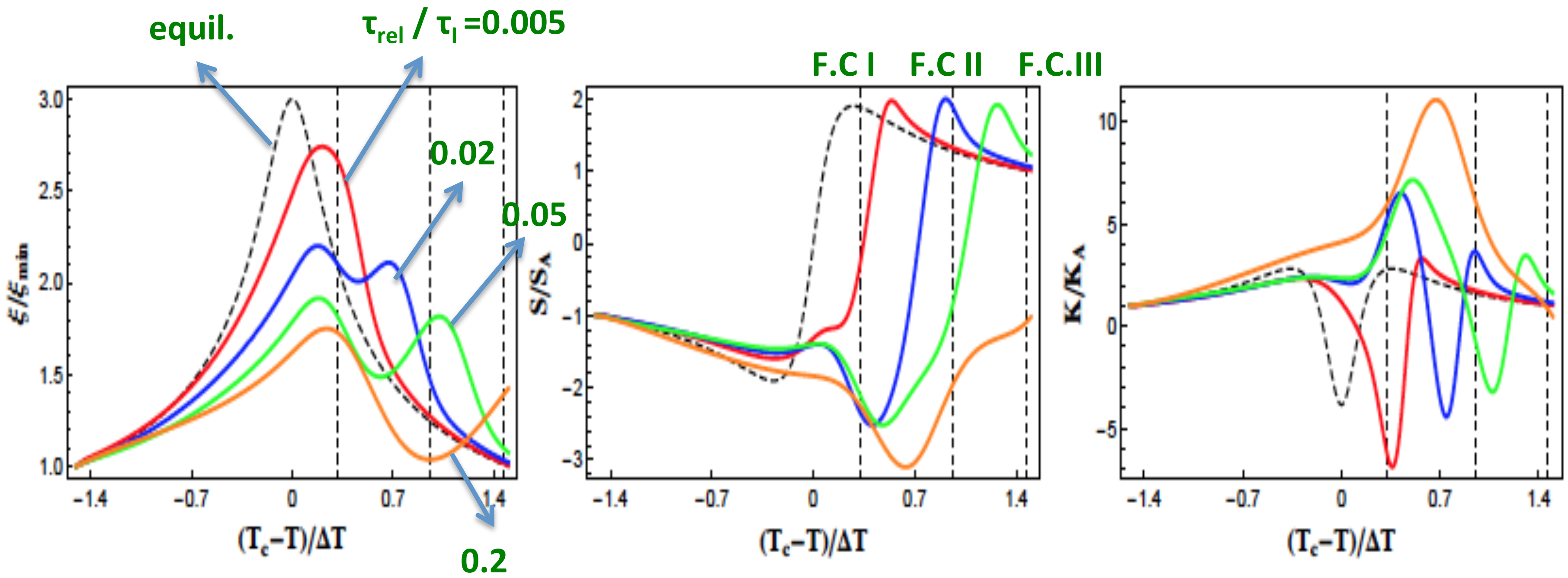
$$\frac{V(\tau)}{V_I} = \left(\frac{\tau}{\tau_I}\right)^{n_V}, \qquad \frac{T(\tau)}{T_I} = \left(\frac{\tau}{\tau_I}\right)^{-n_V c_s^2}$$

n_V =3 for Hubble expansion; also choose c_s^2 = 0.15

- **Free parameter τ_{rel} / τ_I**

Reasonable guess: τ_{rel} = 1 fm, τ_I = 10 fm

Results: memory effects on cumulants

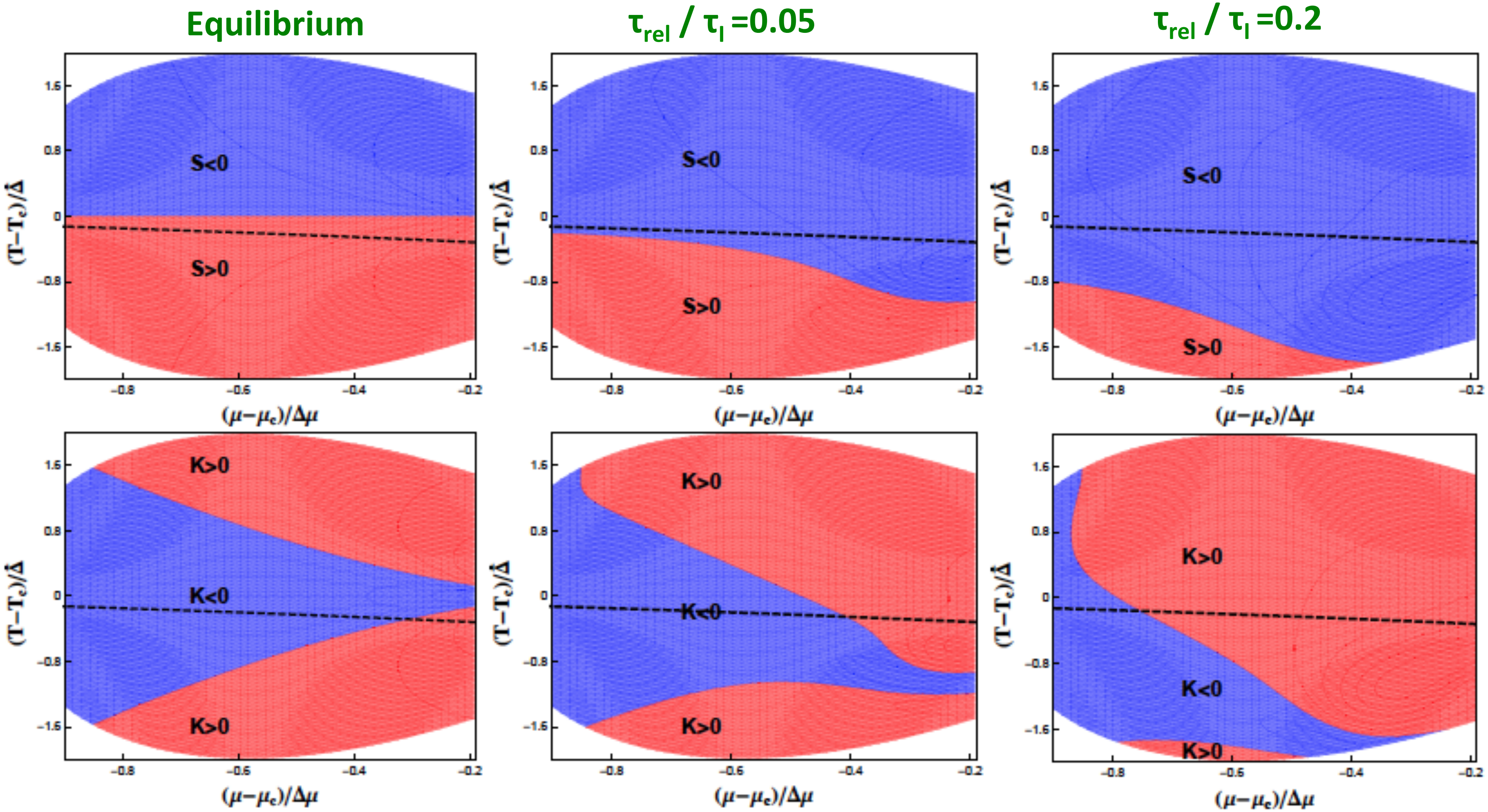


Increasing deviation and even change in sign with increasing τ_{rel} / τ_l

Freeze-out curves from parametrization of particle ratios

Cleymans,Oeschler,Redlich,Wheaton, hep-ph/0511094

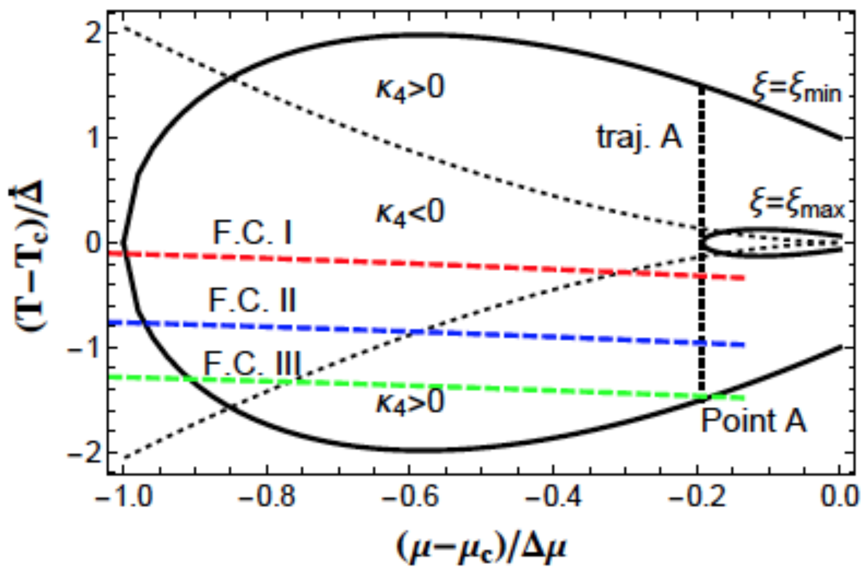
Results: memory effects on cumulants

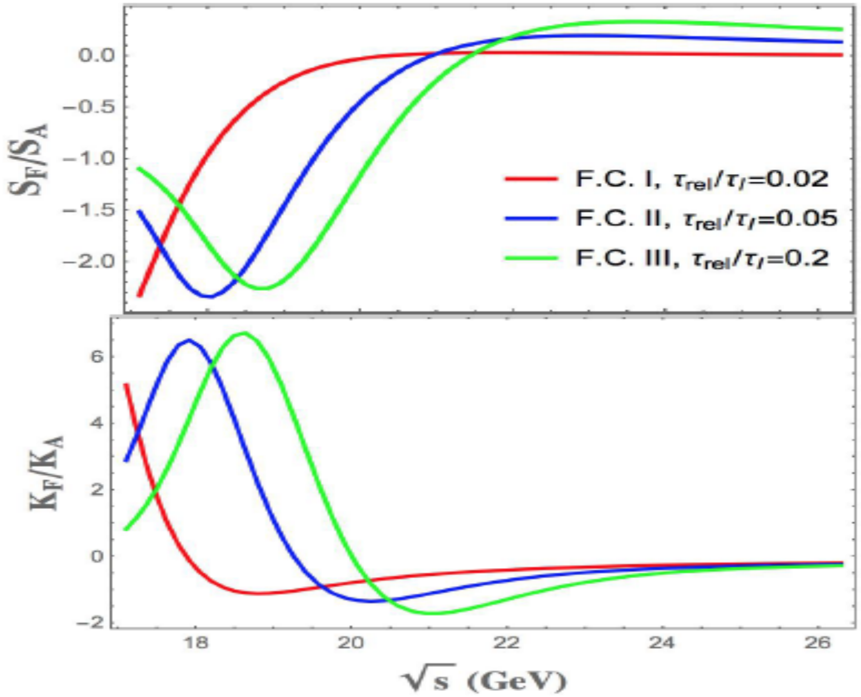


Skewness (top) and Kurtosis (bottom) can change sign for a given μ depending on value of τ_{rel} / τ_l

Cumulants at fireball freeze-out

Mukherjee,Venugopalan,Yin,1512.08022





Different combination of freeze-out curves and relaxation times
… can give similar results

μ (√s) ~ 1/√s   again from particle ratios at freeze-out

Cleymans,Oeschler,Redlich,Wheaton, hep-ph/0511094

Non-universality in heavy-ion collisions

Results are very sensitive to non-universal input:

- **The map from Ising to thermodynamic variables**
- **The details of the trajectory in the (μ_B,T) plane**
- **Relaxation time τ_{eff} of the critical mode**
- **Location of freeze-out in the (μ_B,T) plane**

Is universal information about the critical point overwhelmed by non-universal input ?

Non-equilibrium universality: the Kibble-Zurek mechanism

Originally introduced by Kibble to study the formation and evolution of topological defects in cosmological phase transitions

T.W. Kibble, Physics Reports 67, 183 (1980)

Sir Tom W. B. Kibble
(Born 1932 in Madras, India, Died June 2nd, 2016)

Non-equilibrium universality: the Kibble-Zurek mechanism

Originally introduced by Kibble to study the formation and evolution of topological defects in cosmological phase transitions

T.W. Kibble, Physics Reports 67, 183 (1980)

Generalized by Zurek to discuss a wide range of

Non-equilibrium phenomena; fruitful applications in Quantum phase transitions

W. H. Zurek, Nature 317, 505 (1985);
A. Polkovnikov et al., Rev. Mod. Phys. 83, 863 (2011)

Basic idea present in the old "Landau-Zener" description of excitations in a two-level quantum system

L. Landau, Physics of the Sov. Union 2, 46 (1932);
C. Zener, Proc. R. Soc. A137, 696 (1932);
E. C. G. Stueckelberg, Helvetica Physica Acta 5, 369 (1932);
E. Majorana, Nuovo Cimento 9, 43 (1932)

The Kibble-Zurek Problem: Universality and the Scaling Limit

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Steven S. Gubser and S. L. Sondhi
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(Dated: September 20, 2012)

PUPT-2405

PRL **109,** 015701 (2012) PHYSICAL REVIEW LETTERS week ending 6 JULY 2012

Nonequilibrium Dynamic Critical Scaling of the Quantum Ising Chain

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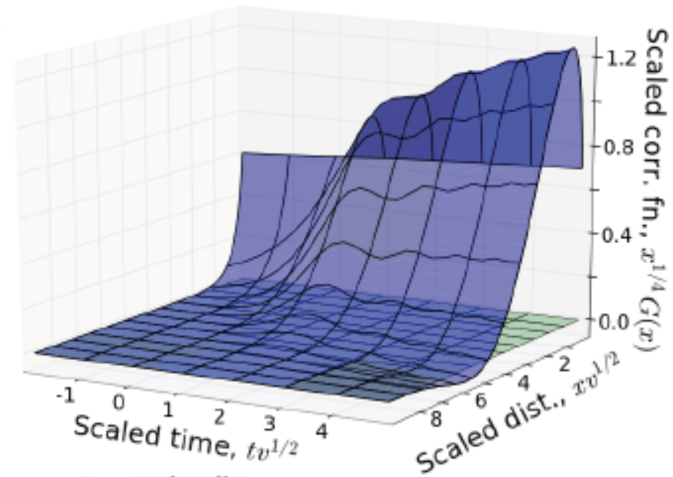
[2]*Princeton Center for Theoretical Science, Princeton University, Princeton, New Jersey 08544, USA*
(Received 2 February 2012; published 2 July 2012)

We solve for the time-dependent finite-size scaling functions of the one-dimensional transverse-field Ising chain during a linear-in-time ramp of the field through the quantum critical point. We then simulate Mott-insulating bosons in a tilted potential, an experimentally studied system in the same equilibrium universality class, and demonstrate that universality holds for the dynamics as well. We find qualitatively



PRL **116,** 080601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Universality in the Dynamics of Second-Order Phase Transitions

G. Nikoghosyan,[1,2] R. Nigmatullin,[3] and M. B. Plenio[1]
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(Received 18 November 2013; revised manuscript received 10 February 2015; published 26 February 2016)

Non-equilibrium universality: the Kibble-Zurek mechanism

Mukherjee,Venugopalan,Yin,arXiv:1605.09341

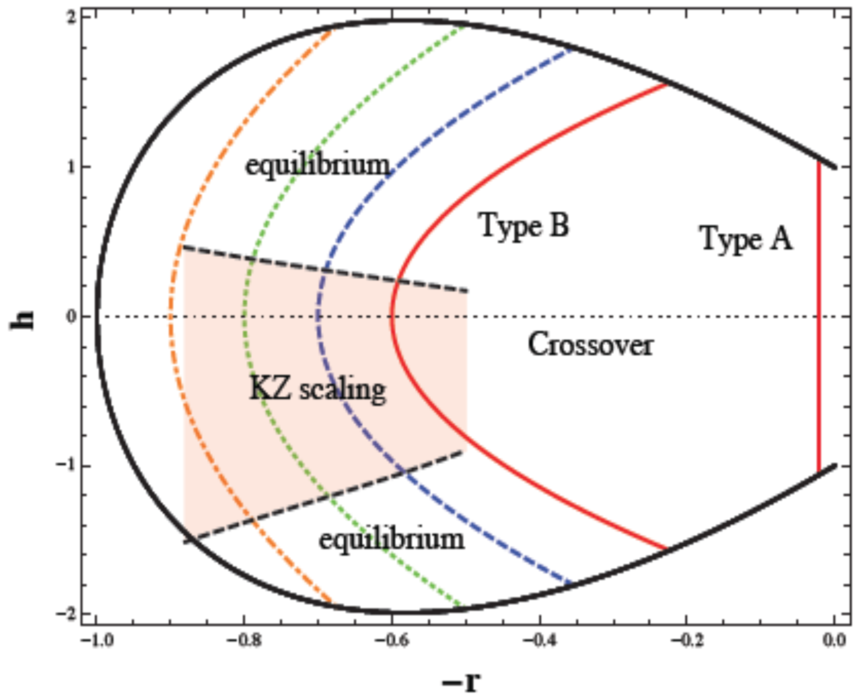
Two competing time scales:

$$\tau_{\text{eff}}(\tau) \qquad \tau_{\text{quench}}(\tau)$$

$$\tau_{\text{quench}} = \min\left(\tau^{\xi}_{\text{quench}},\ \tau^{\theta}_{\text{quench}}\right)$$

$$\tau^{\xi}_{\text{quench}} = \left|\frac{\xi_{\text{eq}}(\tau)}{\partial_\tau \xi_{\text{eq}}(\tau)}\right|$$

$$\tau^{\theta}_{\text{quench}} = \left|\frac{\theta(\tau)}{\partial_\tau \theta(\tau)}\right|$$



Can make an internal map of the Ising variables (r,h) ←→ (ξ,θ)

For example,

$$\kappa_n^{\text{eq.}} \sim \xi_{\text{eq.}}^{-\frac{1}{2}+\frac{5}{2}(n-1)} f_n^{\text{eq.}}(\theta)$$

Non-equilibrium universality: the Kibble-Zurek mechanism

Mukherjee,Venugopalan,Yin,arXiv:1605.09341

Emergent scales:

$$\tau_{KZ} = \tau_{\text{eff}}(\tau^*) = \tau_{\text{quench}}(\tau^*)$$

$$l_{KZ} = \xi_{\text{eq}}(\tau^*) \quad \theta_{KZ} = \theta_{\text{eq}}(\tau^*)$$

$$\tau_{\text{eff}}(\tau) >> \tau_{\text{quench}}(\tau)$$

Frozen critical fluctuations and magnetization angle

Emergent scaling:

$$\kappa_n(\tau;\Gamma) \sim l_{\text{KZ}}^{-\frac{1}{2}+\frac{5}{2}(n-1)}\, \bar{f}_n^I(t;\theta_{\text{KZ}})$$

I represents class of trajectories, Γ non-universal inputs; emergent scales are functionals of these

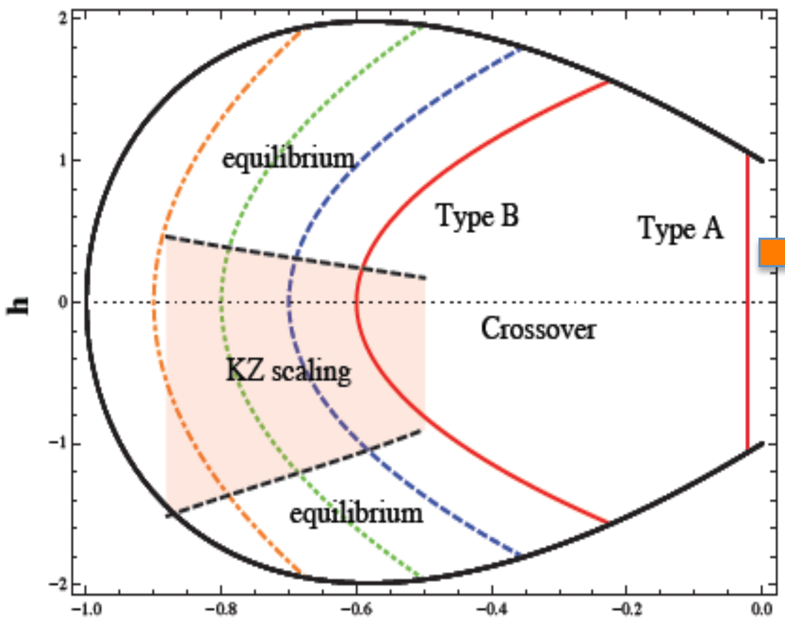
$$\tau_{\text{KZ}}(\Gamma) \quad \theta_{\text{KZ}}(\Gamma) \quad l_{\text{KZ}}(\Gamma)$$

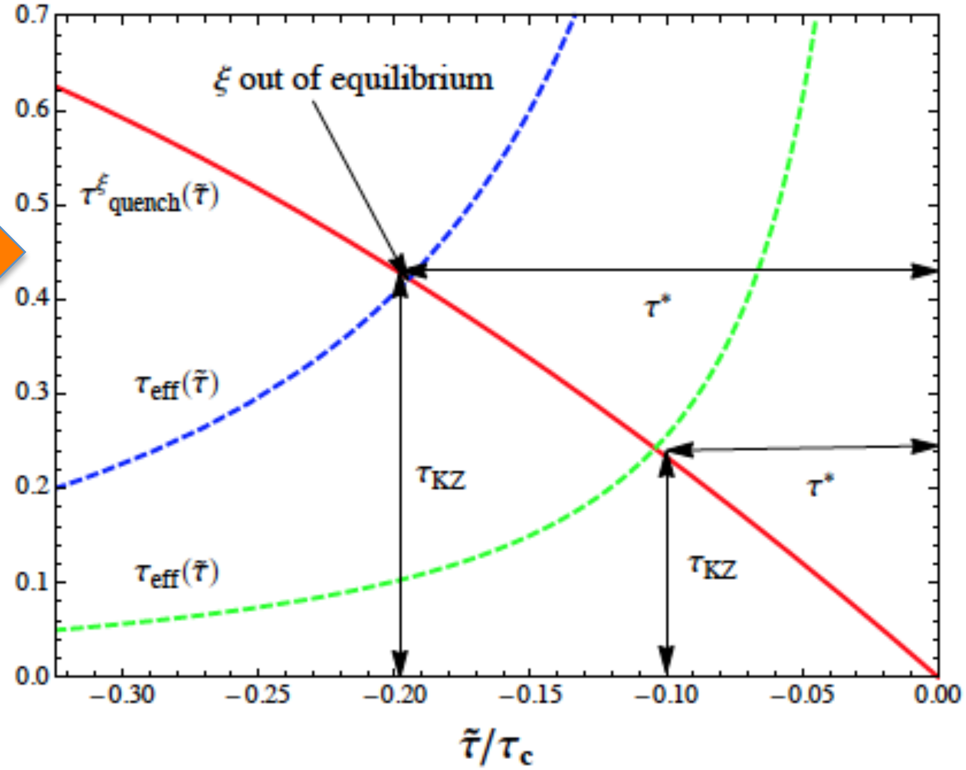
$$\tilde{\tau} = \tau - \tau_c$$

$$t = \tilde{\tau}/\tau_{\text{KZ}}$$

KZ dynamics: example of protocol A

Mukherjee,Venugopalan,Yin,arXiv:1605.09341



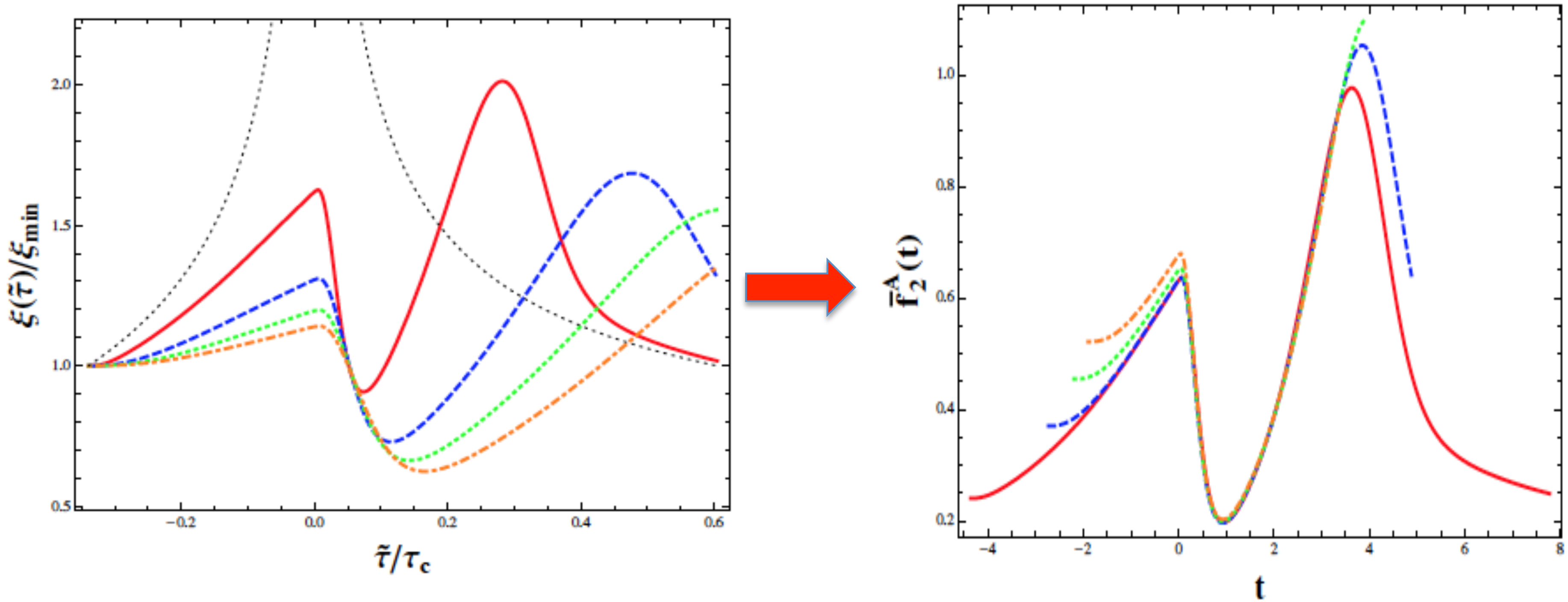


$$\tau_{\text{quench}} = \tau^{\xi}_{\text{quench}}$$

$$\tau^{\xi}_{\text{quench}} = \left| \frac{\xi_{\text{eq}}(\tau)}{\partial_{\tau} \xi_{\text{eq}}(\tau)} \right|$$

KZ dynamics: universality regained

$$\kappa_n(\tau;\Gamma) \sim l_{\mathrm{KZ}}^{-\frac{1}{2}+\frac{5}{2}(n-1)}\,\bar{f}_n^{I}(t;\theta_{\mathrm{KZ}})$$



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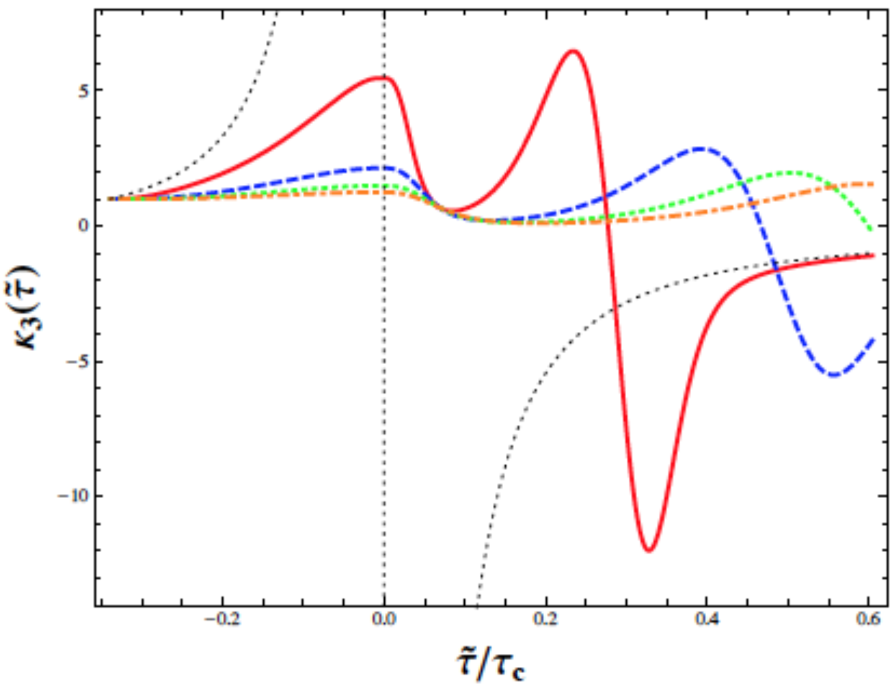
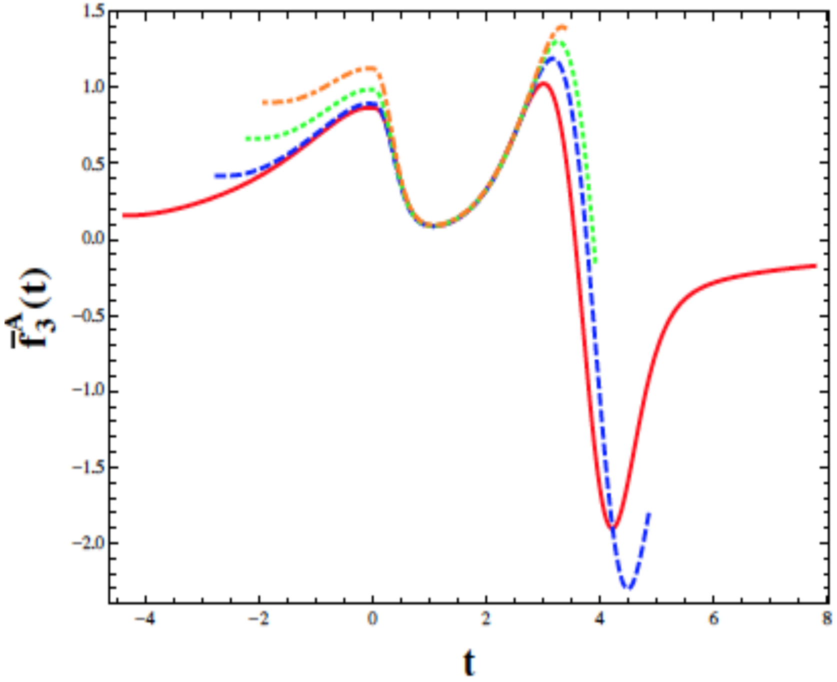
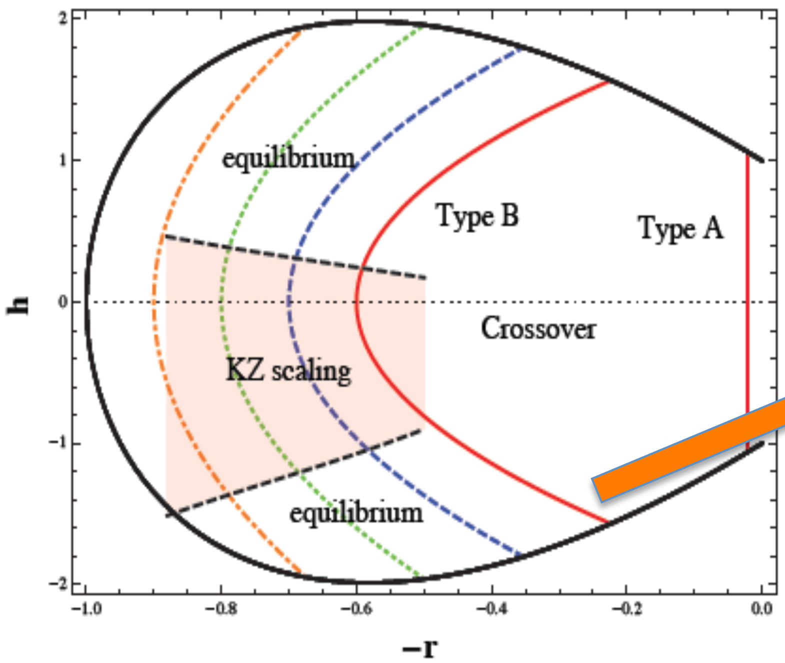
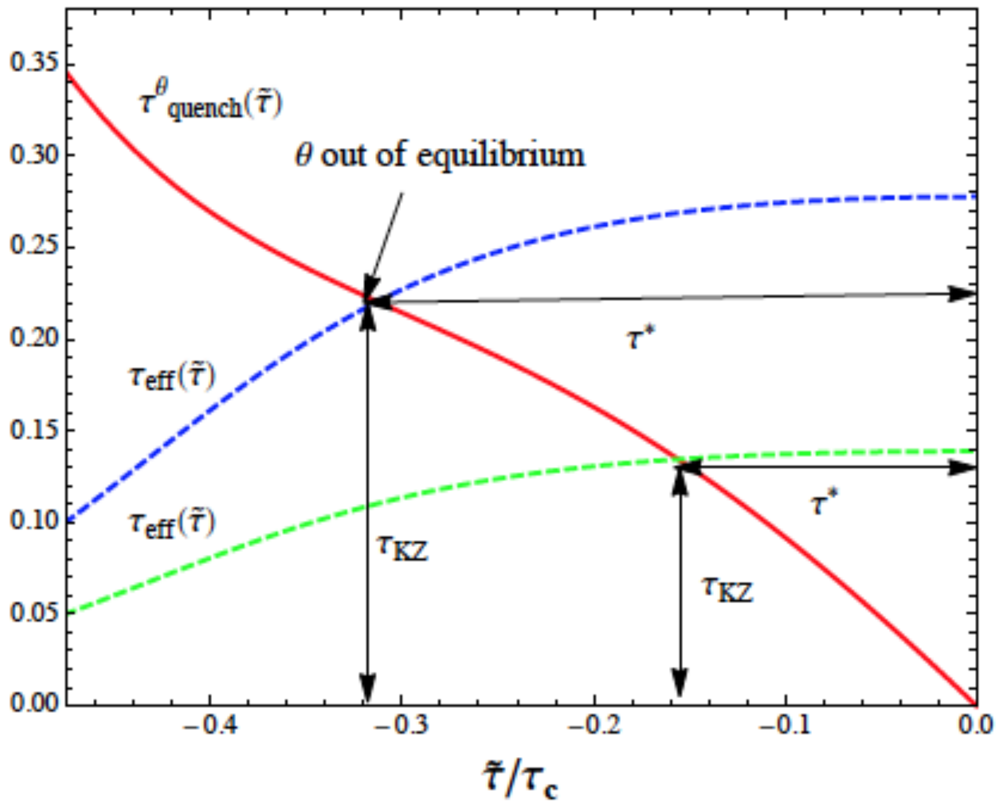




KZ dynamics: more general example (protocol B)

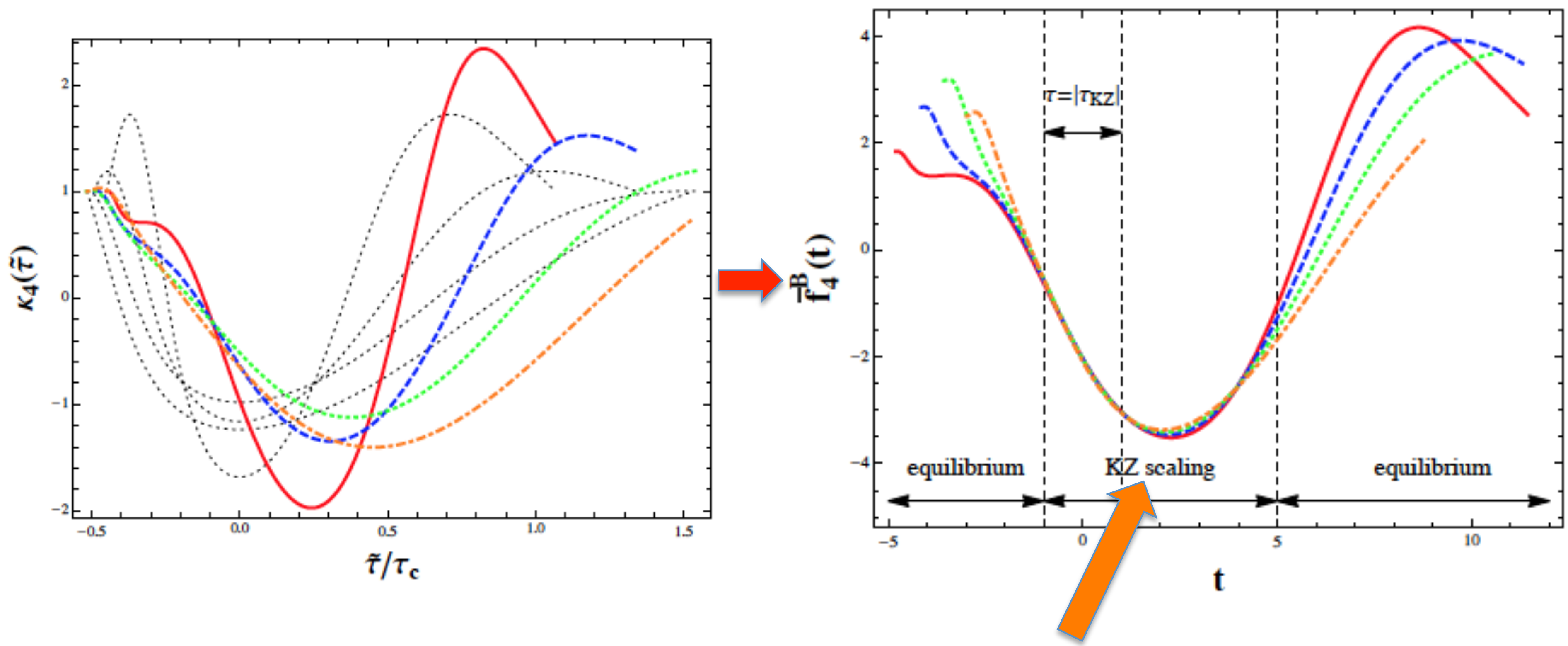




$$\tau_{quench} = \tau^{\theta}_{quench}$$

$$\tau^{\theta}_{quench} = \left|\frac{\theta(\tau)}{\partial_{\tau}\theta(\tau)}\right|$$

KZ dynamics: universality for protocol B



Freeze-out ?

Implications for RHIC's beam energy scan

- RHIC can measure cumulants of the net baryon density with varying energy and centrality – these will couple to critical fluctuations if a CP exists

- If hydro modeling of the bulk dynamics is accurate, for each protocol, can define a corresponding τ_{KZ} , l_{KZ} and θ_{KZ} - these also depend on non-universal inputs from critical properties of QCD matter.

  Read τ_f from position of the freeze-out curve in simulations.

- Compute rescaled cumulant data as $\bar{f}_n^{\mathrm{data}} \equiv \kappa_n^{\mathrm{data}} / l_{\mathrm{KZ}}^{(-\frac{1}{2}+\frac{5(n-1)}{2})}$

  Establish a map : $\kappa_n^{\mathrm{data}} \leftrightarrow \left(\bar{f}_n^{\mathrm{data}}, t_f, \theta_{\mathrm{KZ}}\right)$

  Data points should line up on a single curve by adjusting the non-universal critical inputs

- In parallel, solve cumulant equations along representative protocols and compare theoretically obtained f with rescaled data

Successful theory-data comparison for all cumulants will provide unambiguous evidence for the QCD critical point !