JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Bound States from the nPI effective action

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Overview

1. Motivation and Introduction
2. DSEs and BSEs
3. nPI effective action
4. 3PI DSE results
5. 3PI meson results
6. Outlook and conclusion

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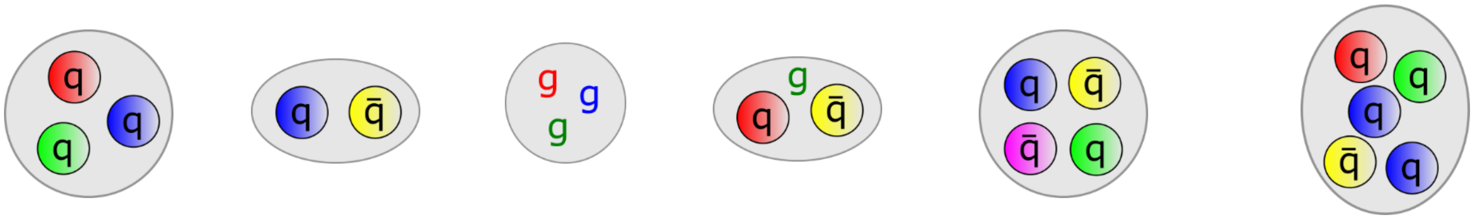
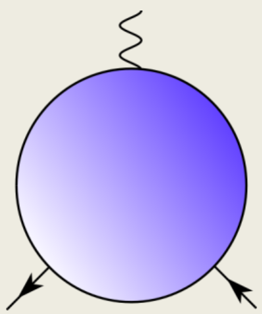
Describe bound-states:



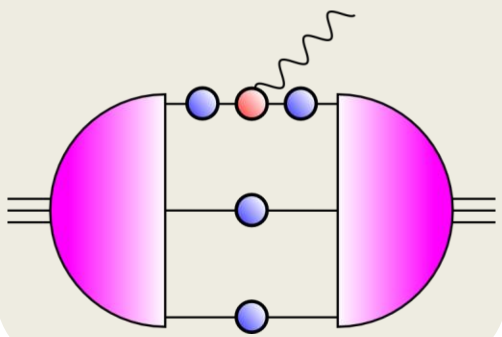
baryons mesons glueballs hybrids tetraquarks pentaquarks

and EM processes:

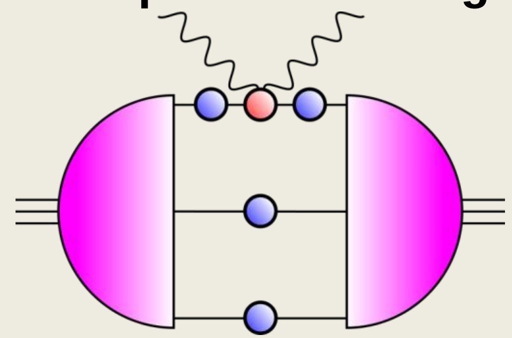
Muon g-2



EM form-factors



Compton Scattering



in terms of QCD's Green's functions.

Functional methods - provide access to Green's functions: **lattice**, **FRG**, **DSE**, **nPI**

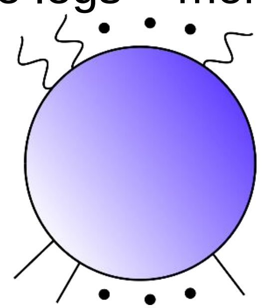
- Covariant, multi-scale (no separation), renormalizable, non-perturbative

Good differences

- (no) sign problem, continuum

There's always a "but"

- Infinite tower of coupled non-linear equations – **truncations** are necessary
- **Computationally and algebraically involved**
- Euclidean - access to time-like properties requires analytic continuation
- More legs = more problems **(larger phase space and number of covariants)**



$$\Gamma^{\mu\nu\ldots}_{ij\ldots}(p_1, p_2, \ldots) = \sum_a F_a(p_1^2, p_2^2, \ldots)\,\tau^{\mu\nu\ldots}_{a,ij\ldots}(p_1, p_2, \ldots)$$

We need various ingredients

Propagators:

- Quark
- Gluon

Vertices:

- Quark-gluon vertex
- Three-gluon vertex

Interaction Kernel:

- Quark-(anti)quark binding

Bound State Amplitude

- Specify quantum numbers
- Describe appropriate state

Symmetries:

- Vector Ward Identity
- Axial-vector Ward Identity

Connection to Gauge sector

Control of "modelling"

Systematic improvements?

DSEs and BSEs

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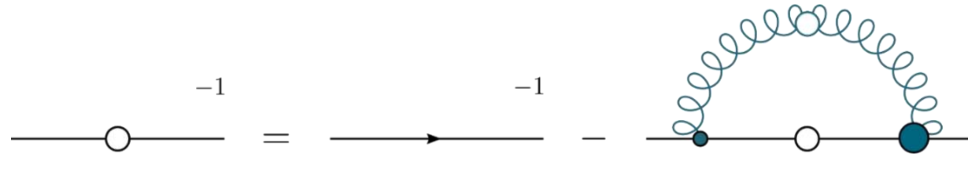
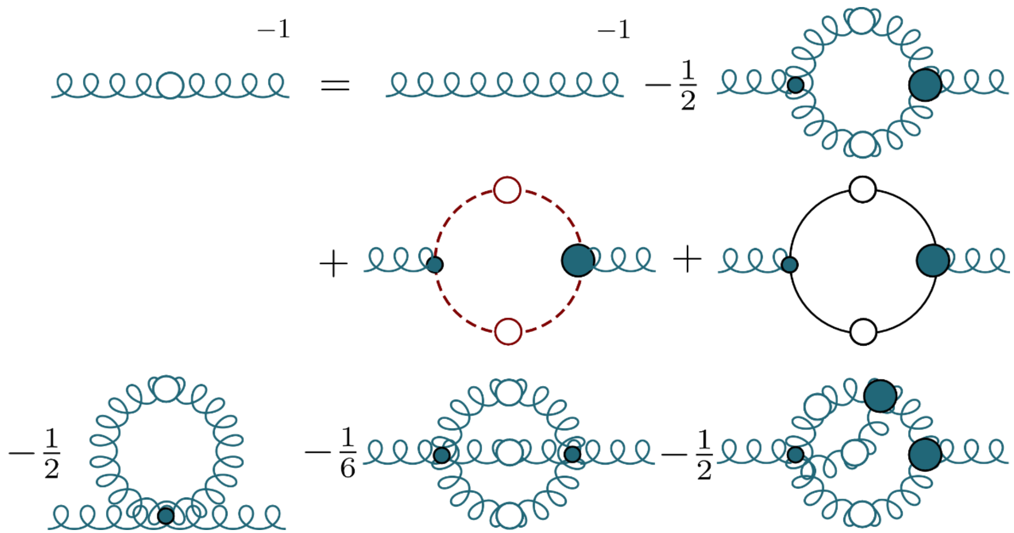
DSEs derived from functional identity:

$$\frac{\delta \Gamma[\phi]}{\delta \phi_i} - \frac{\delta S}{\delta \phi_i}\left[\phi + \frac{\delta^2 W[J]}{\delta J \delta J_k}\frac{\delta}{\delta \phi_k}\right] = 0$$

QFT analogue to EOM



$$S^{-1}(p) = A(p^2)(-i\,\gamma \cdot p + M(p^2))$$



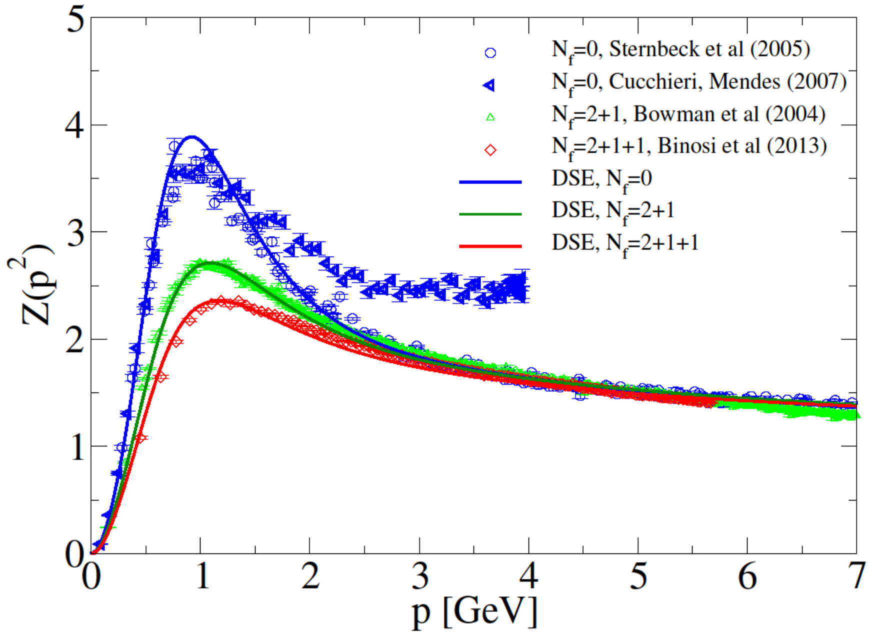
$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

Exact equations *in principle*

- Think of as a constraint equation
- Additional constraints provided by WI and STI (difficult to reconcile in practice)

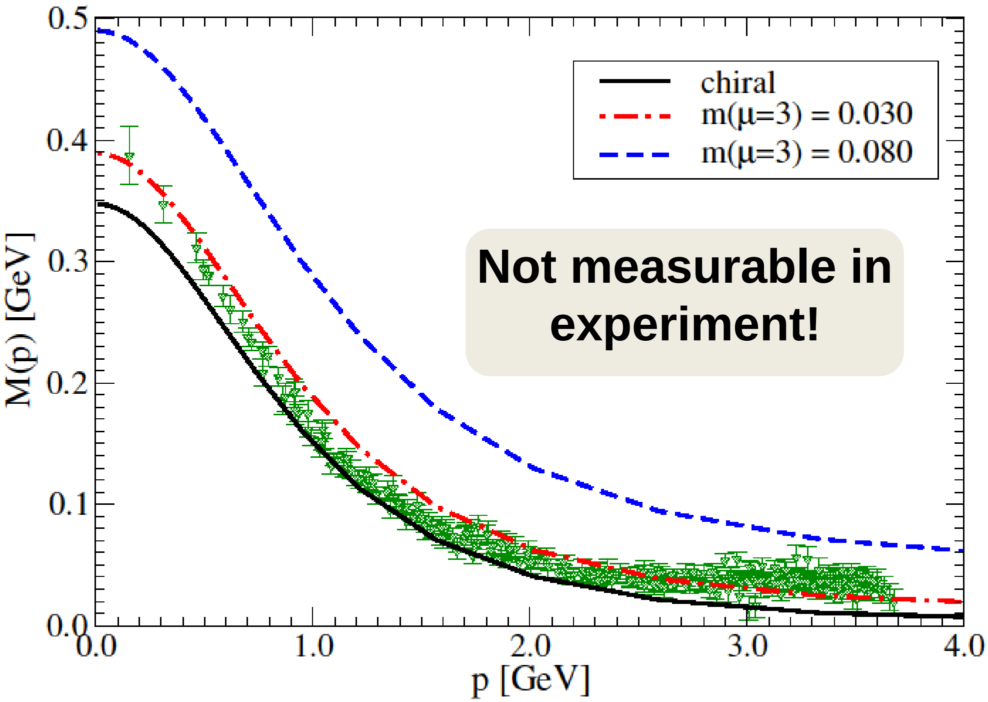
Not closed ☹ so truncation needed

Propagators – typical results



By now, truncations can be quite robust

- Qualitative and quantitative agreement
- Scalar dressings are momentum dependent and encode PT and NP information



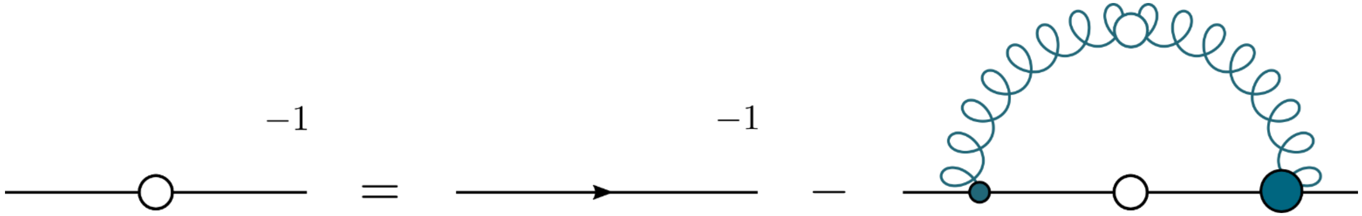
(Data points: Bowman et al)

Same green's functions, can compare with:

- Lattice, FRG, nPI

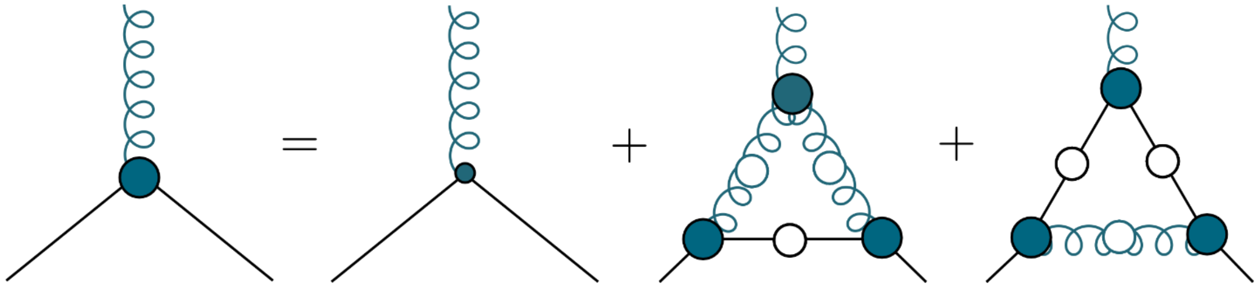
More about truncations later …

By itself, a gluon **will not** generate dynamical chiral symmetry breaking

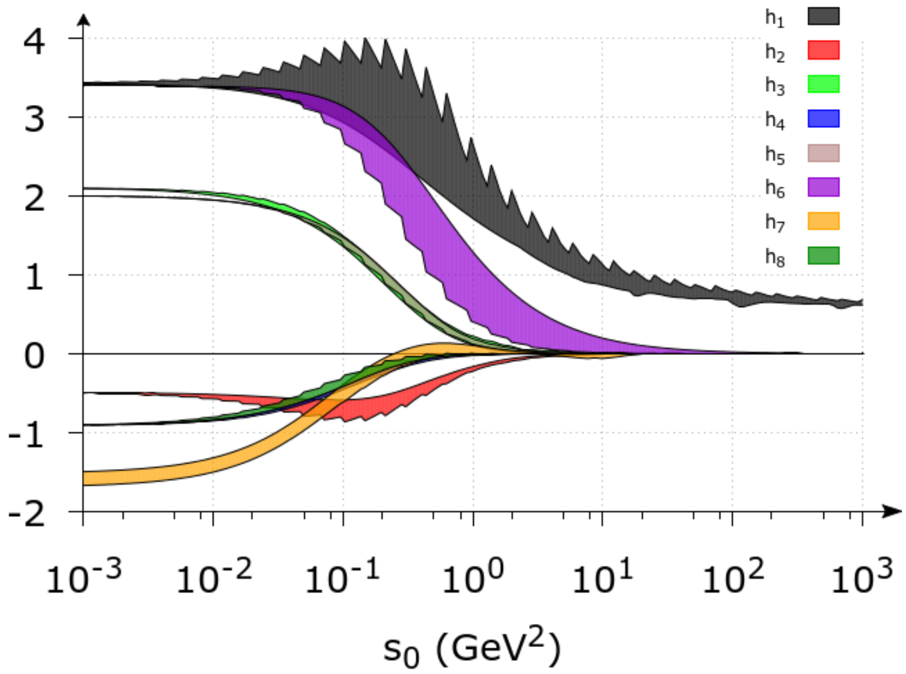


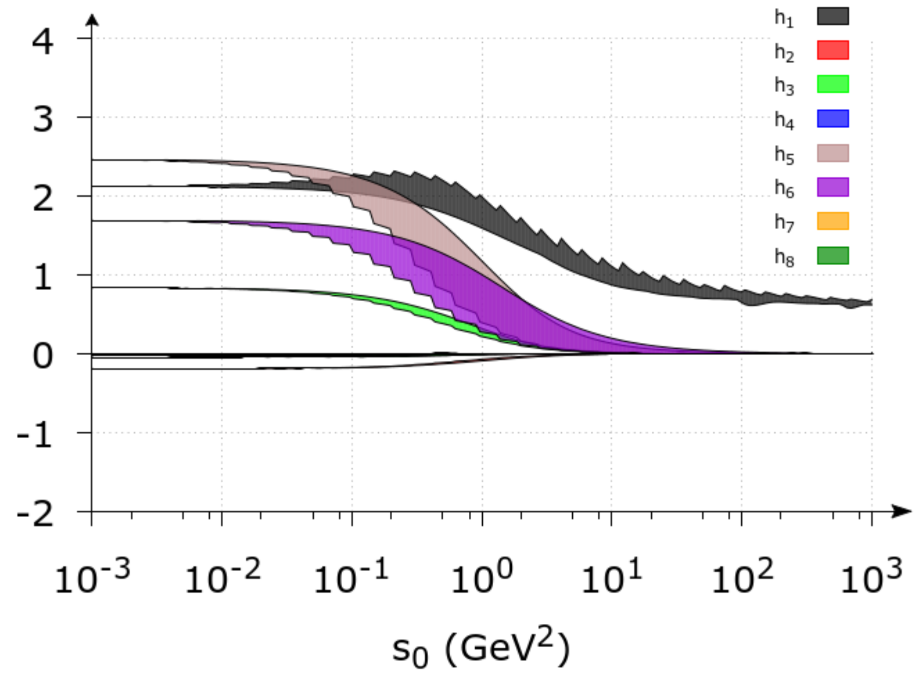
- Non-perturbative enhancements in the **quark-gluon** vertex required.
- **Rainbow-ladder explicitly ignores vertex corrections. These are included implicitly by using an effective interaction to replace the gluon**

e.g.



More to life than propagators – quark-gluon vertex



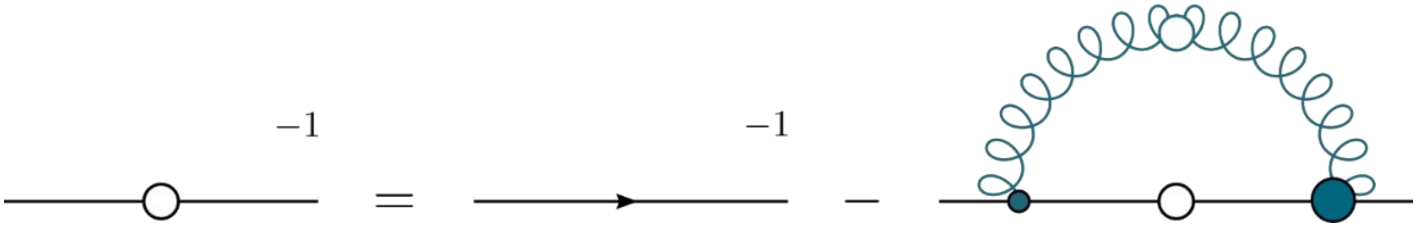


Dynamical chiral symmetry breaking *in the vertex*. **Flavour dependent**

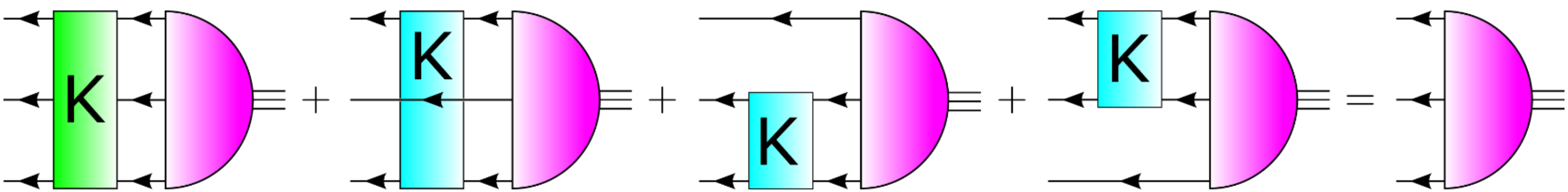
h_1 is the "vector", h_5, h_6 the scalar/vector **anomalous chromomagnetic moments.**

$$\Gamma^a_\mu(l,k) = h_1\gamma^\mu_T + h_2 l^\mu_T \gamma\cdot l + h_3 i l^\mu_T + h_4 (l\cdot \mathrm{k})\frac{i}{2}\left[\gamma^\mu_T,\gamma\cdot l\right] + h_5\frac{i}{2}[\gamma^\mu,\gamma\cdot k]$$
$$+h_6\frac{1}{6}[\gamma^\mu,\gamma\cdot l,\gamma\cdot k] + h_7 t^{\mu\nu}_{(kl)}(l\cdot k)\gamma^\nu + h_8 t^{\mu\nu}_{(kl)}[\gamma^\nu,\gamma\cdot l]$$

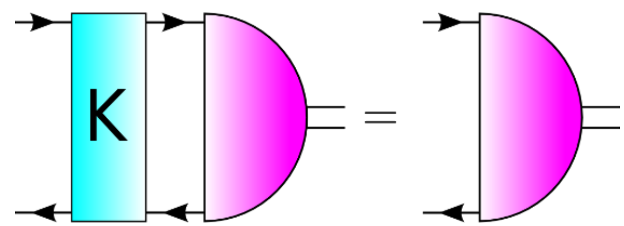
How to connect propagators and vertices…



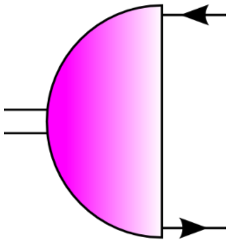
… to bound states of quarks and gluons …



…without breaking everything?



And anyway. What is a bound-state amplitude



Rest frame: $(P^2 = -M^2)$

two independent variables

Meson:

$$\Gamma^{\mu_1\ldots\mu_J} = \begin{pmatrix} Q^{\mu_1\ldots\mu_J} \\ T^{\mu_1\ldots\mu_J} \end{pmatrix} \begin{pmatrix} 1 \\ \gamma_5 \end{pmatrix} D_i(k)\Lambda_\pm$$

$$D_i = \{1, \gamma\cdot k\} \qquad \Lambda_\pm = (1 \pm \widehat{\gamma\cdot P})/2$$

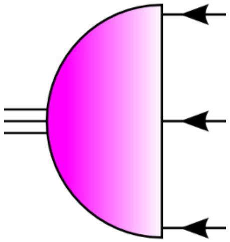
Q and **T:** Angular momentum tensors

Constructed from the traceless part of J-fold tensor products

$$\tilde{Q}^{\mu_1\ldots\mu_J} = k_T^{\{\mu_1}\cdots k_T^{\mu_J\}} \qquad \tilde{T}^{\mu_1\ldots\mu_J} = \gamma_t^{\{\mu_1} k_T^{\mu_2}\cdots k_T^{\mu_J\}}$$

Saturates: no more than four or eight Dirac covariants

Bound-state amplitude: Baryon



five independent variables

Rarita-Schwinger Projector

$$\mathbb{P}^{\mu\nu} = T_P^{\mu\nu} - \frac{1}{3}\gamma_T^\mu \gamma_T^\nu$$

Nucleon/Delta: 64/128 covariants $\quad D_i = \{1, \gamma\cdot k, \gamma\cdot q, (\gamma\cdot k)(\gamma\cdot q)\}$

$$\Gamma^\nu = \begin{pmatrix} 1 & \otimes & 1 \\ \gamma_5 & \otimes & \gamma_5 \\ \gamma_T^\mu & \otimes & \gamma_T^\mu \\ \gamma_T^\mu\gamma_5 & \otimes & \gamma_T^\mu\gamma_5 \end{pmatrix} (D_i\Lambda_\pm\gamma_5 C \quad \otimes \quad D_j\Lambda_+) \begin{pmatrix} \gamma_T^\mu\gamma_5 & \otimes & \mathbb{P}^{\mu\nu} \\ \hat{k}_T^\mu\gamma_5 & \otimes & \mathbb{P}^{\mu\nu} \\ q_t^\mu\gamma_5 & \otimes & \mathbb{P}^{\mu\nu} \end{pmatrix}$$

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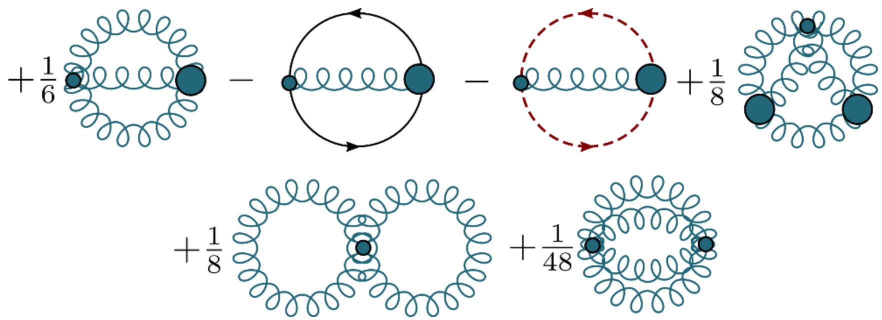
Effective Action: nPI

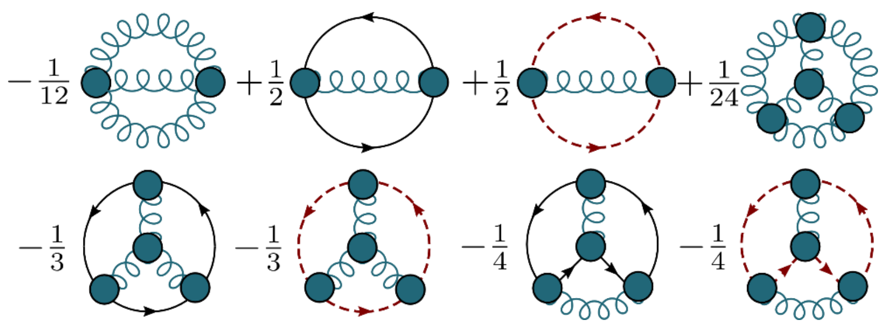
Constructing from $\Gamma[\phi]$ by taking Legendre transf. wrt propagators and vertices.

$$\Gamma[\phi,D,U] = S_{cl}[\phi] + \frac{i}{2} TrLnD^{-1} + \frac{i}{2} Tr\left[D_{(0)}^{-1}D\right] - i\Phi^{0}[\phi,D,U] - i\Phi^{int}[\phi,D,U] + const.$$

Φ^0: non-interacting part



Φ^{int}: interacting part



Construction not known exactly

- Loop expansion in hbar
- Functional derivatives yield DSEs for propagators and vertices
- **Further functional derivatives yield Bethe-Salpeter kernels!**

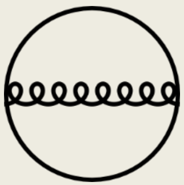
More or less guaranteed to be consistent with symmetries of the action

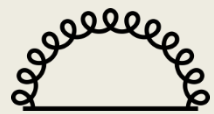
Effective Action: 2PI at two-loop

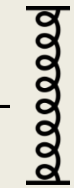
Two-particle irreducible effective action (Legendre transform with respect to fields, propagators):

$$\Gamma[\Psi, G] = S[\Psi] + i\,\mathrm{Tr}\ln G - i\,\mathrm{Tr}\,G_0^{-1} + \Gamma_2[\Psi, G]$$

(See Berges, hep-ph/0409233)

$\Gamma_2 = -\frac{i}{2}$ 

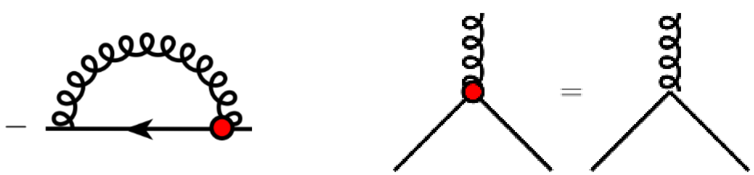
$-\Sigma = \frac{\delta \Gamma_2}{\delta G} = -$ 

$-K = \frac{\delta^2 \Gamma_2}{\delta G \delta G} = -$ 

Quark self-energy, quark-antiquark kernel obtained by functional derivatives

Comparison with exact quark self-energy

implies quark-gluon vertex



Vertices are perturbative

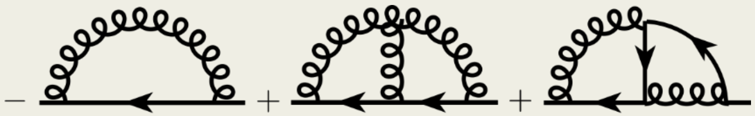
Propagators are resummed

Complement bare vertex (tree-level, no loop corrections) with an RG improvement

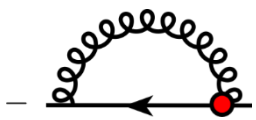
Effective Action: 2PI at three-loop

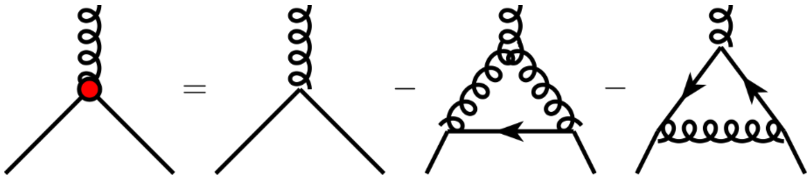
$\Gamma_2 =$

$-\Sigma = \frac{\delta \Gamma_2}{\delta G} =$

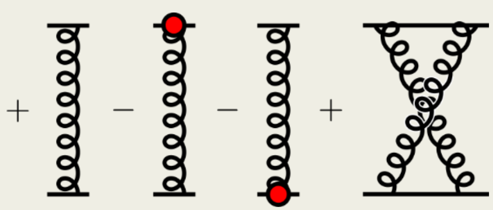


Compare with
quark self-energy:



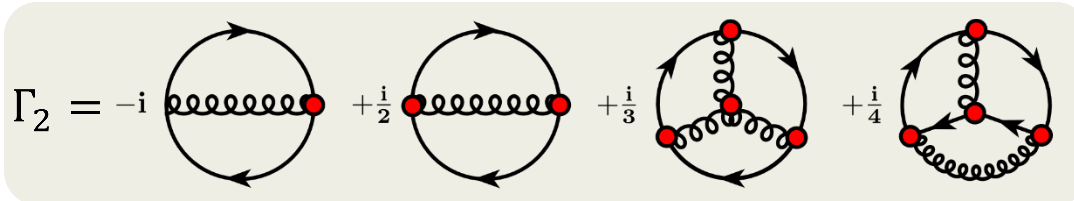


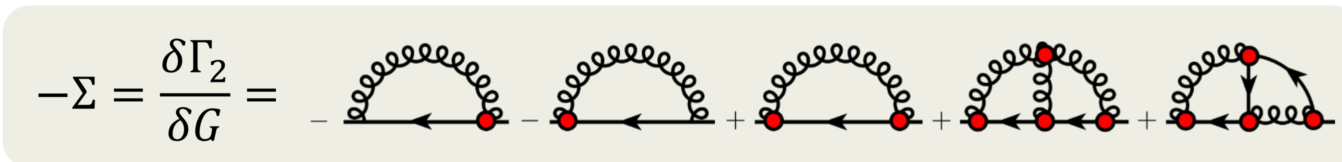
$-K = \frac{\delta^2 \Gamma_2}{\delta G \delta G} =$

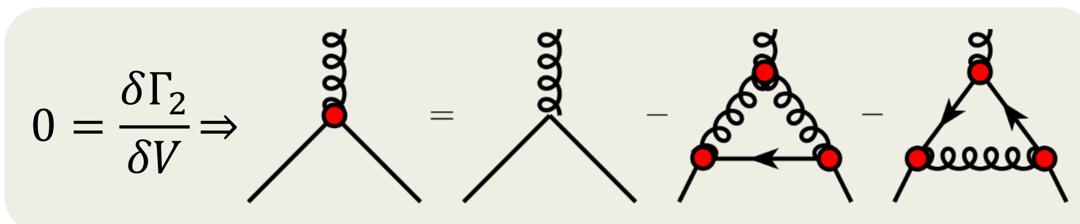


- **Complement** bare vertices with a (constrained) RG improvement
- Select contributions **leading in N_c**

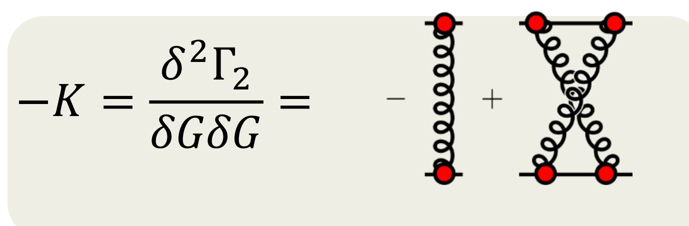
Effective Action: 3PI at three-loop



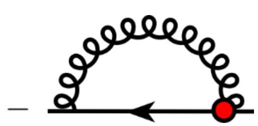




- **BSE kernel is structurally simple**



Reduces to quark self-energy:



Sanchis-Alepuz, RW (arXiv:1503.0596)

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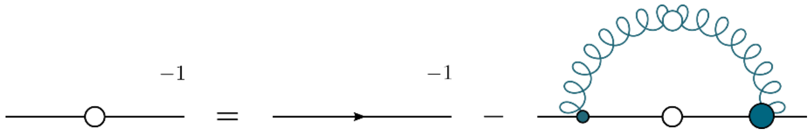
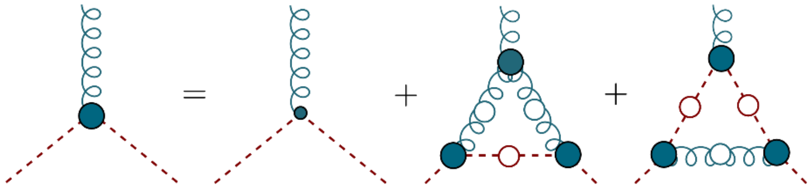
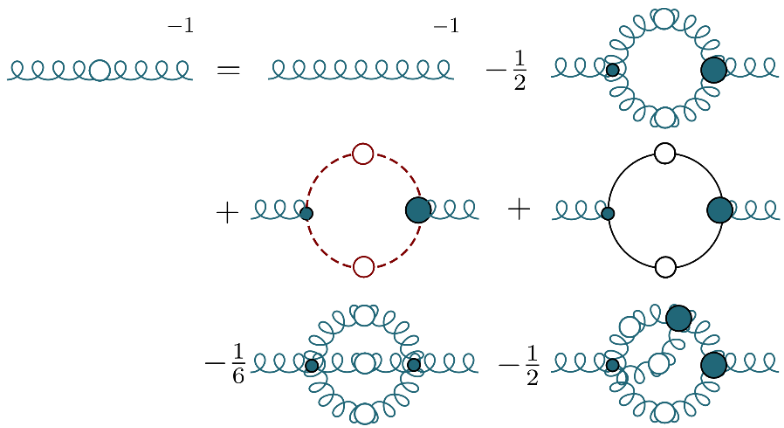
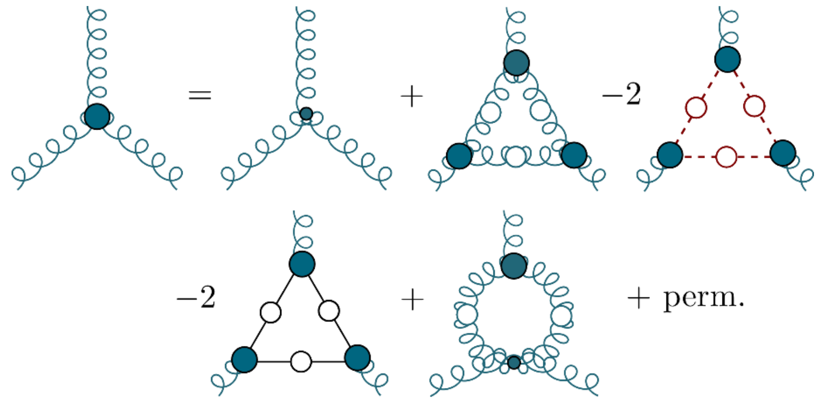
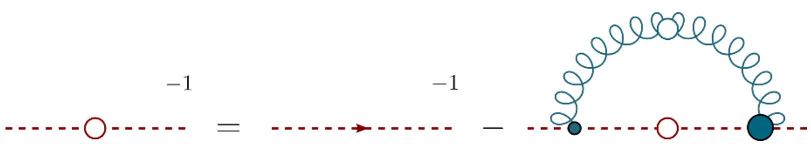


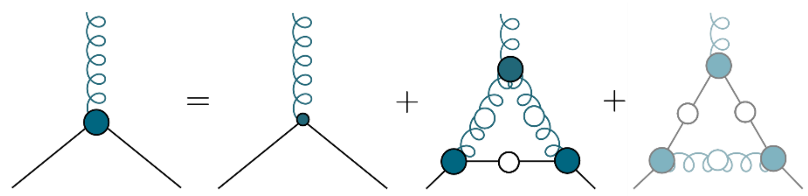


Ghost/gluon solved independently of 3PI

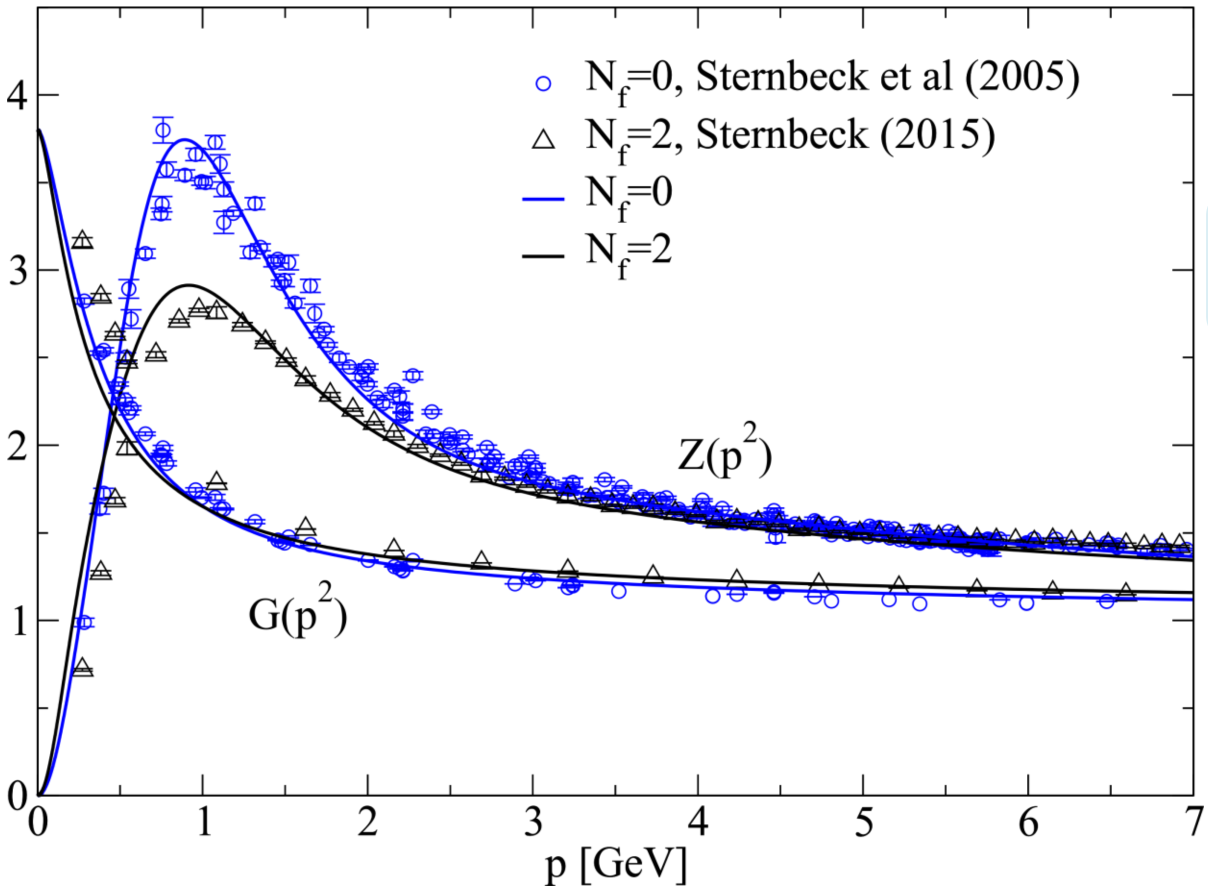








Investigating unquenching effects (quark-loops) on the system



$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

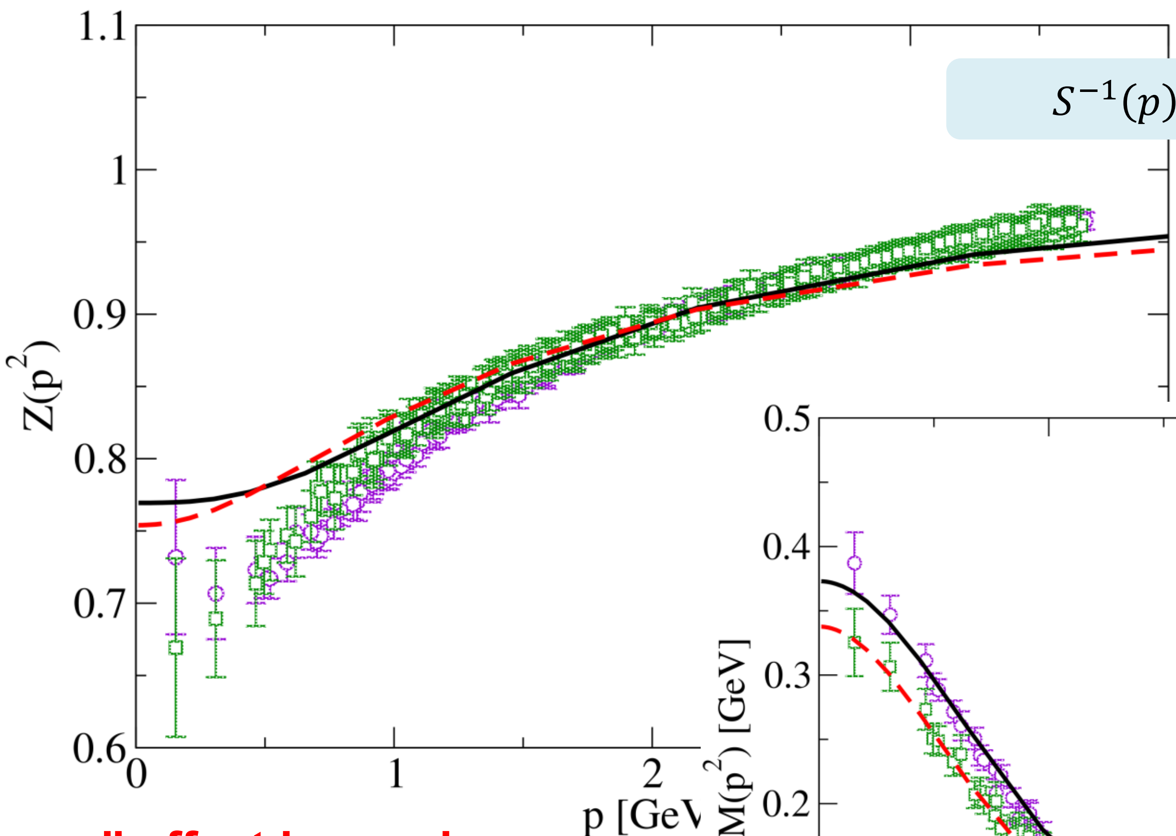
$$D_G(p) = -\frac{G(p^2)}{p^2}$$

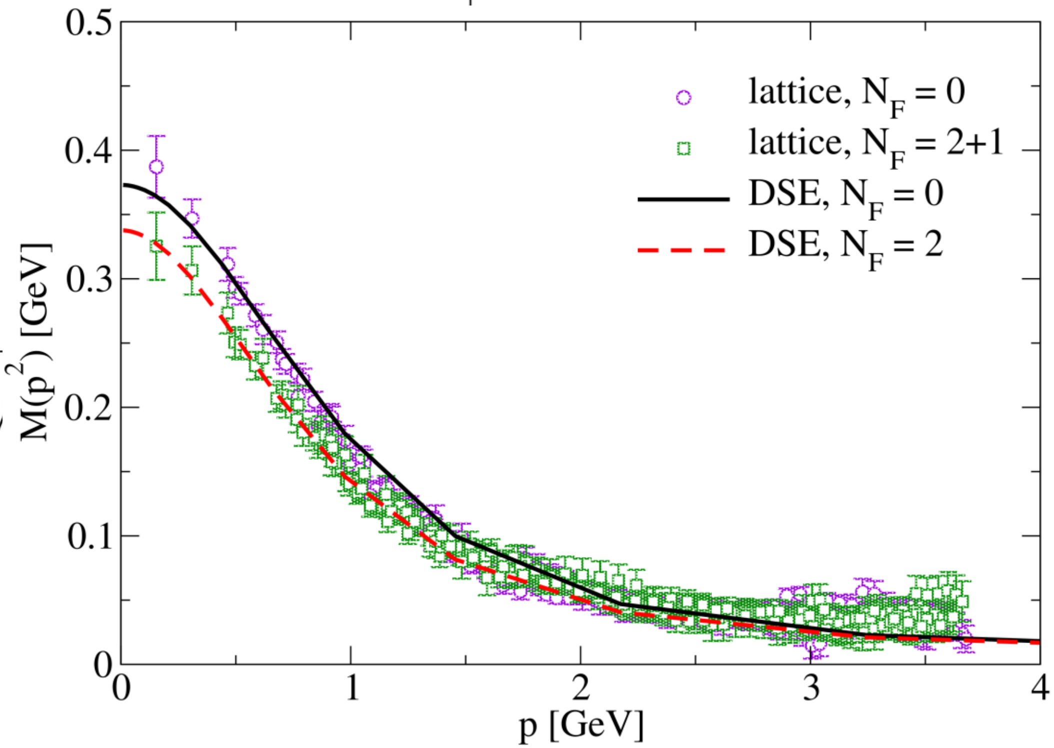
Ghost/Gluon solved such that agreement with quenched/unquenched lattice is obtained

Provides input parameters of model ($g_s, Z_3, \tilde{Z}_3$)

3PI results: DSEs (quark propagator)

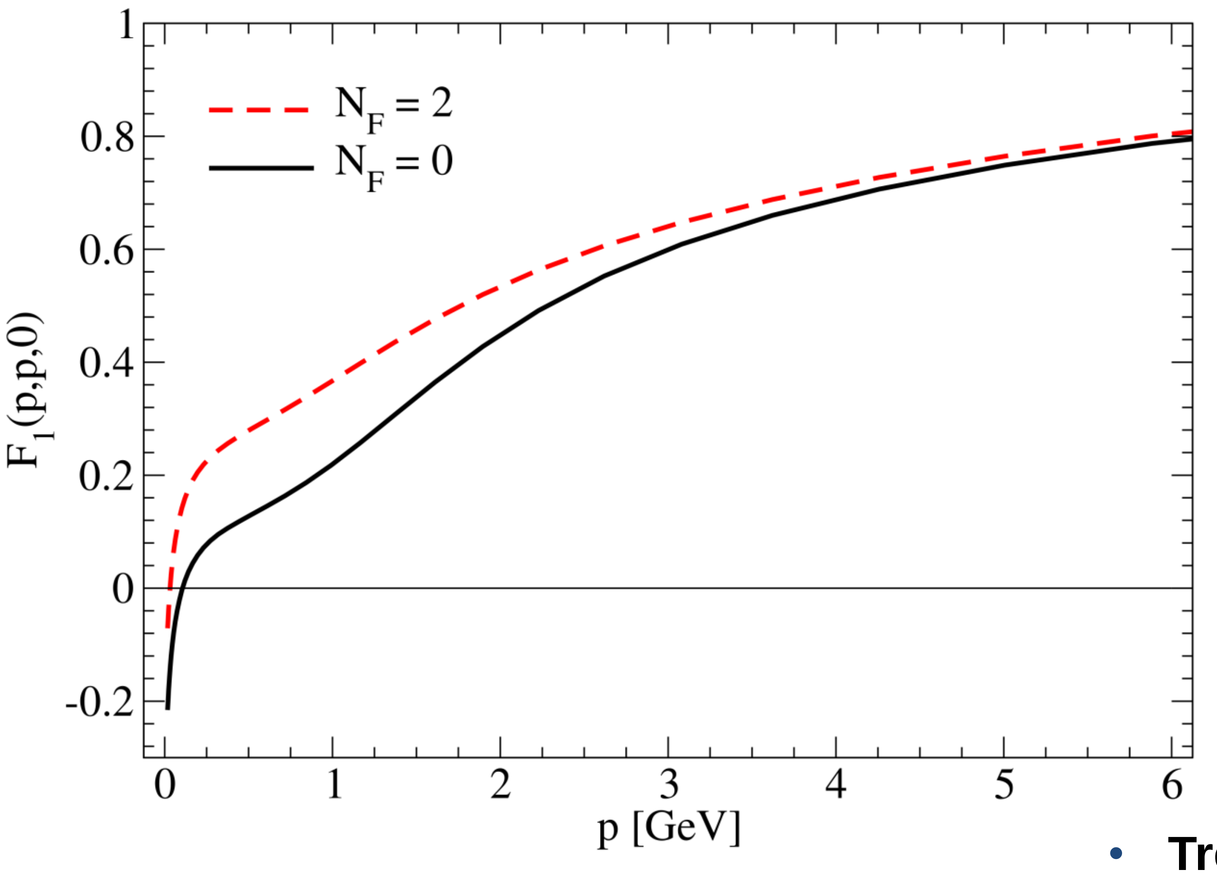
$$S^{-1}(p) = A(p^2)(-i\,\gamma \cdot p + M(p^2))$$





"cusp" effect in quark wavefunction seen for first time

3PI results: DSEs (three-gluon vertex)



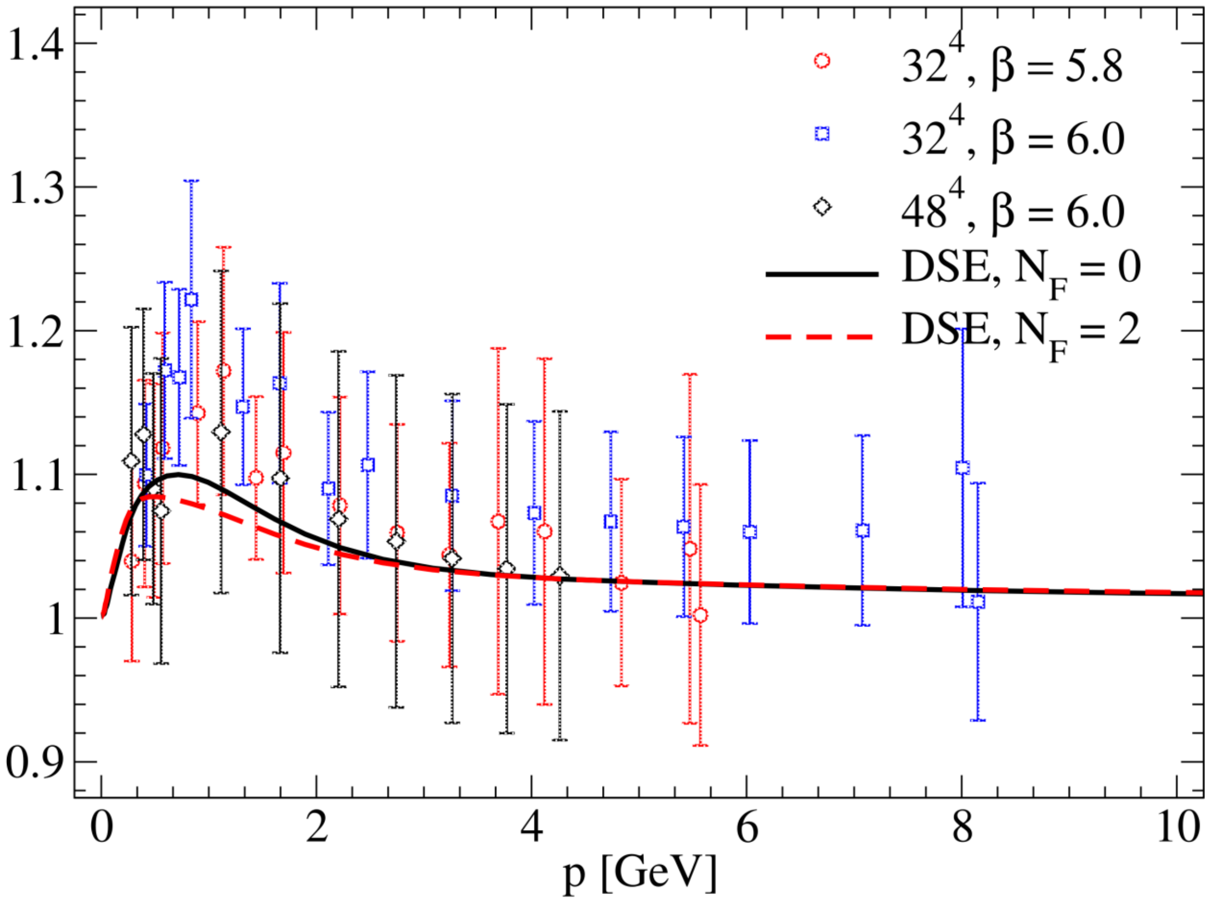
- **Quark-loop enhances three-gluon vertex**
- **Zero crosses pushed to deep IR**

$$\Gamma_{3g}^{\mu\nu\rho}(p,k,q) = F_1(p,k,q)\Gamma_{3g,0}^{\mu\nu\rho}(p,k,q)$$

- **Tree-level structure dominant**
- **Single phase-space slice sufficient (e.g. soft-gluon)**

[Eichmann, RW, Alkofer, Vujinovic]

3PI results: DSEs (ghost-gluon vertex)

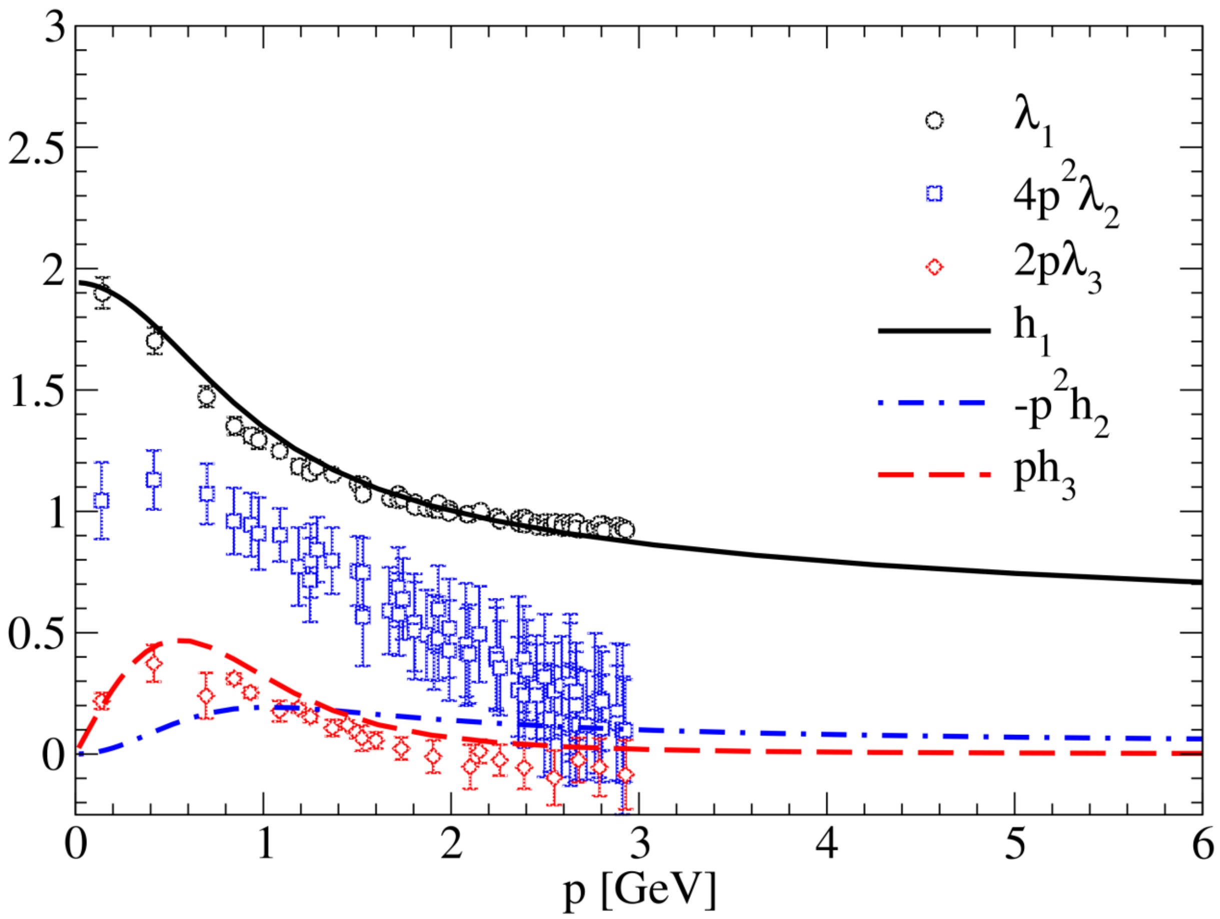


One tensor structure

$$\Gamma^{\mu}_{gh}(l,q) = f(l,q) T^{\mu\nu}_{(q)} l^{\nu}$$

- **Unquenching effects negligible.**
- **Lattice data needs improvement**

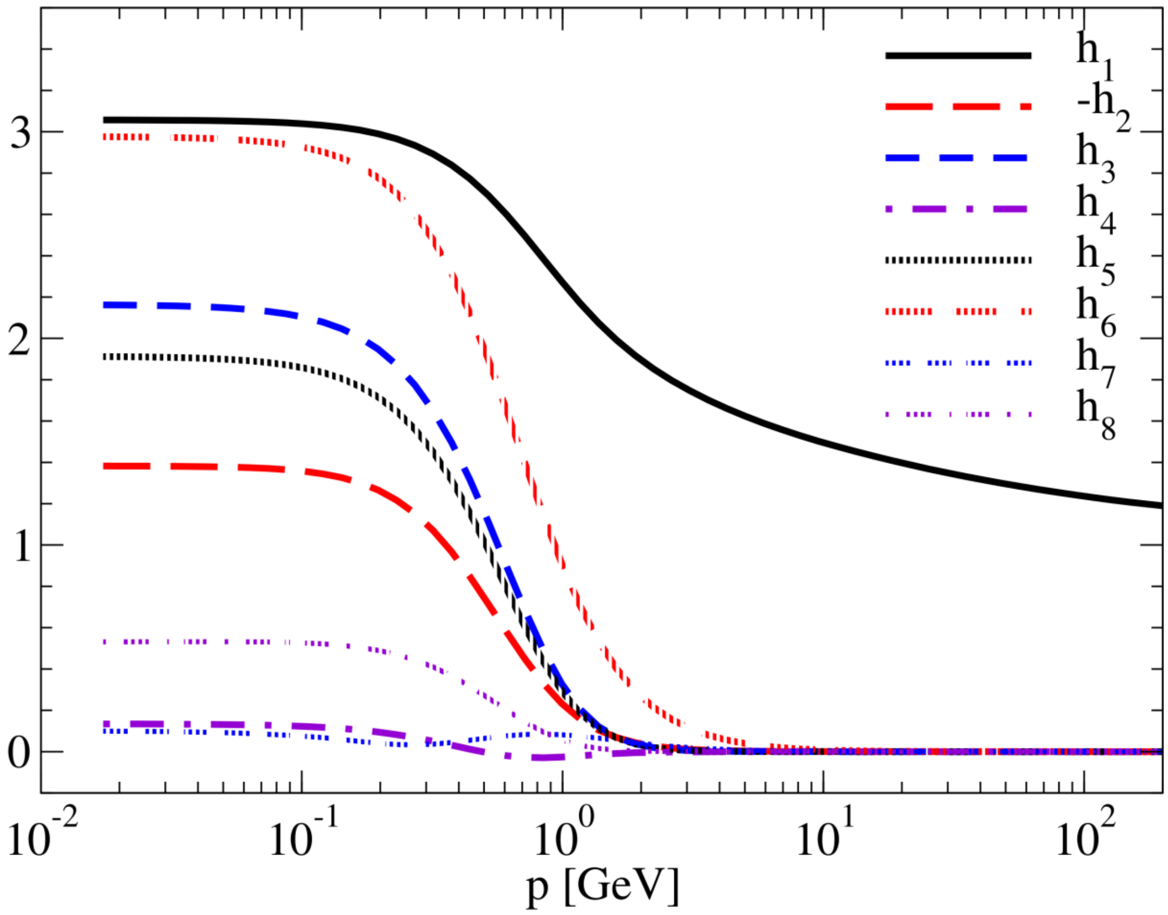
3PI results: DSEs (quenched quark-gluon vertex)



Lattice renormalization procedure is suspect

1st and 3rd structures comparable. Difficult systematics (lattice) in 2nd

3PI results: DSEs (unquenched quark-gluon vertex)



Comparable to quenched (DSE) data. Large corrections beyond tree-level.

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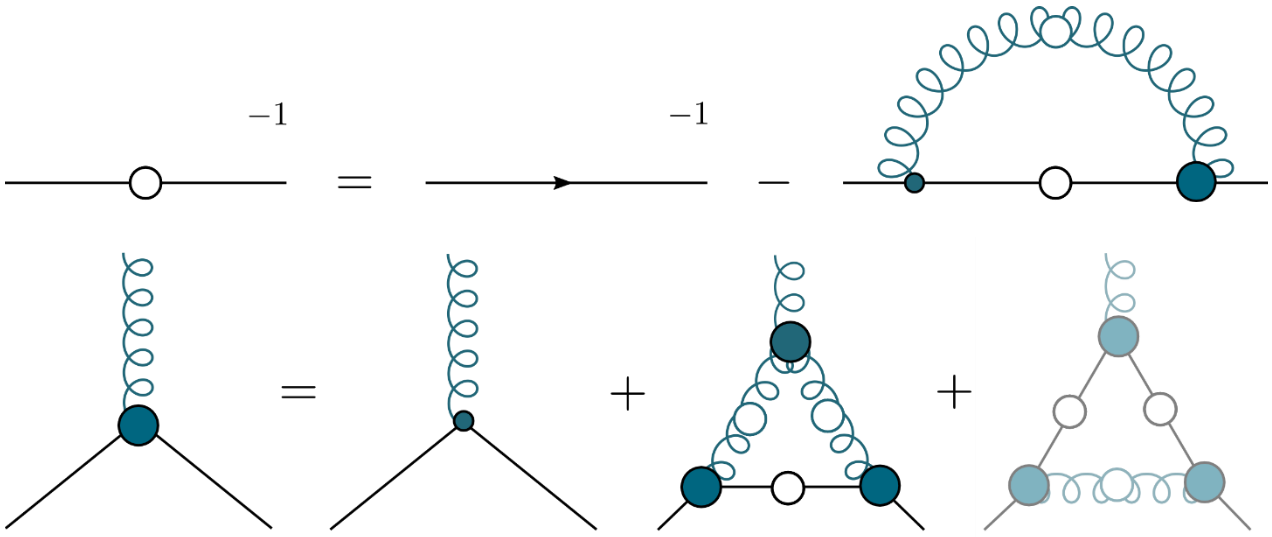
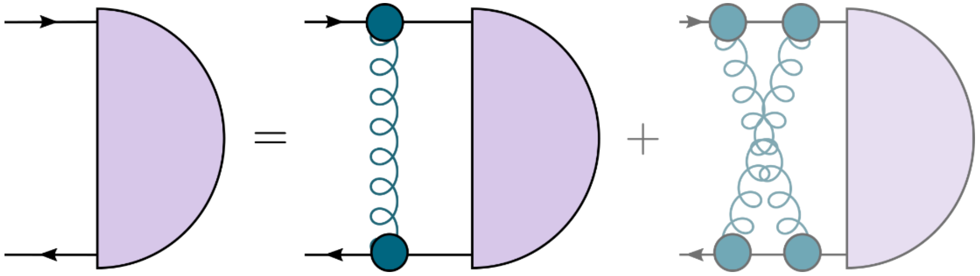


Above are calculated for complex quark momenta.



Homogeneous BSE solved without Pade or fit functions

3PI results: mesons

	RL	2PI-3L		PDG
0^{-+} (π)	$0.14^{\dagger}$	$0.14^{\dagger}$		0.14
0^{++} (σ)	0.64	0.52		0.48(8)
1^{--} (ρ)	0.74	0.77		0.78
1^{++} (a_1)	0.97	0.96		1.23(4)
1^{+-} (b_1)	0.85	1.1		1.23
f_π	$0.092^{\dagger}$	0.103		0.092

Scalar: 2PI-2L (RL) and 2PI-3L it is too light: 500 – 600 MeV

$\rho - a_1$ splitting: 2PI-2L (RL) and 2PI-3L it is too small: 200 MeV

$a_1 - b_1$ splitting: 2PI-2L (RL) and 2PI-3L non-degenerate states

Phenomenology restored by 3PI-3L !!!

Overview

1.Motivation and Introduction

2. DSEs and BSEs

3. nPI effective action

4. 3PI DSE results

5. 3PI meson results

6. Outlook and conclusion

Mesons $q\bar{q}$

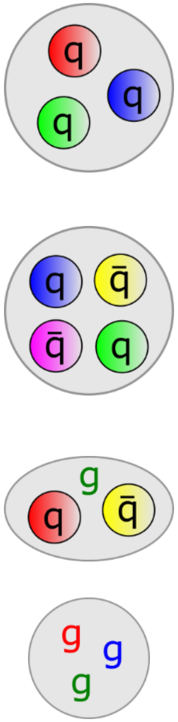
- Only now exploring details of **quark-gluon interaction** on spectrum
- No longer disconnected from gauge sector. Implicit flavor dependence.

Developing framework

- Unified description of mesons and baryons consistent with symmetries
- Calculation of higher spin and/or excited **mesons** and **baryons**

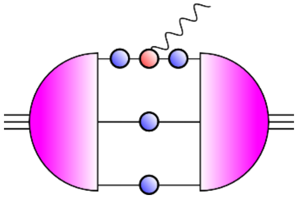
Extensible to other bound-states via nPI

- **Baryons**
- Tetraquarks
- Glueballs and Hybrid mesons

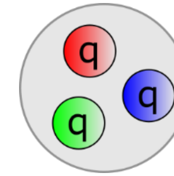


A functional derivative (or two) away …

- Calculation of form-factors, EM transitions and decays



Mesons $q\bar{q}$



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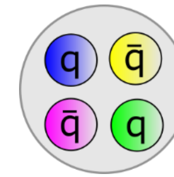

Developing framework

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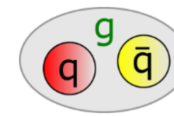
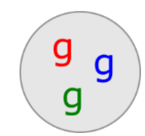


Thank you

Extensible to other bound states via nPI



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