Landscape of φ^4 theories

Nicolò Defenu

Outline

- Motivations
- Introduction to Functional Renormalization Group (FRG)
- Application to Spin Systems
  - ND, et al. JHEP 1505(141) October 2014.
  - A. Codello, ND and G. D’Odorico Phys. Rev. D 91, 105003 (2015).
- Application to the sine-Gordon model
  - V. Basco, ND, A. Trombettoni and I. Nandori JHEP Nucl. Phys. B 901 444-460 (2015).
- Future perspectives

Motivations

Phase Transitions

↓

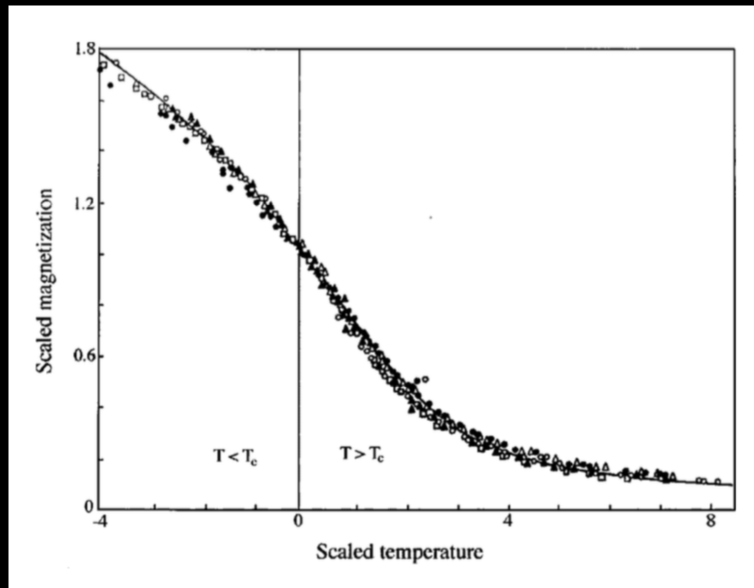
Universality

↓

Renormalization Group

↙ ↘

Perturbative | Non Systematic



↓

Functional Renormalization Group

- Partition Function: generator of the correlation functions

$$\langle \varphi_1^{m_1} \cdots \varphi_N^{m_N} \rangle = \frac{\partial^n Z}{\partial^{m_1} J_1 \cdots J^{m_N} \varphi_N} \qquad \sum_i m_1 = n$$

- Free energy: generator of the connected correlation functions

$$F[J] = -\log\left(Z[J]\right)$$

$$\langle \varphi_1^{m_1} \cdots \varphi_N^{m_N} \rangle_c = \frac{\partial^n F}{\partial^{m_1} J_1 \cdots J^{m_N} \varphi_N}$$

- Effective action: generator of 1PI correlation functions

$$\Gamma[\tilde{\varphi}] = \int d^d x J(x) \tilde{\varphi}(x) - F[J] \quad \tilde{\varphi}(x) = \langle \varphi(x) \rangle$$

Functional RG

Exact flow equation for the effective action

$$\partial_t \Gamma_k[\tilde{\varphi}] = \frac{1}{2}\mathrm{Tr}\left(\frac{\partial_t R_k}{\Gamma^{(2)} + R_k}\right)$$

$k \sim L^{-1} \sim N^{-\frac{1}{d}}$: scale $\qquad$ $k_0 \sim a^{-1} >> 1$: ultraviolet scale

$$\Gamma_{k_0}[\tilde{\varphi}] = S[\tilde{\varphi}] \underset{k_0 > k > 0}{\Longrightarrow} \Gamma_k[\tilde{\varphi}] \underset{k \equiv 0}{\Longrightarrow} \Gamma[\tilde{\varphi}]$$

$$t = \log\left(\frac{k}{k_0}\right)$$

Local Potential Approximation

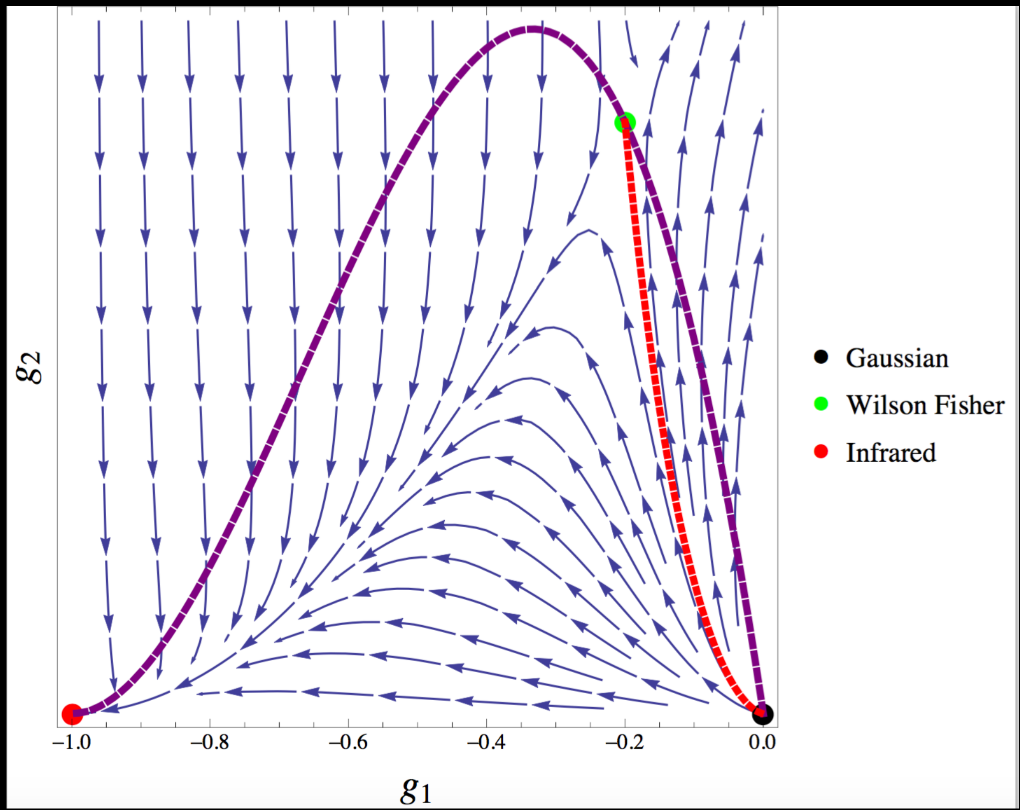
$$\Gamma_k = \int d^d x \left\{ \frac{1}{2}\partial_\mu \varphi_i(x) \partial_\mu \varphi_i(x) + U_k(\rho) \right\}$$

$$\rho = \sum_i \frac{\varphi_i^2}{2} \quad i \in \{1, N\}$$

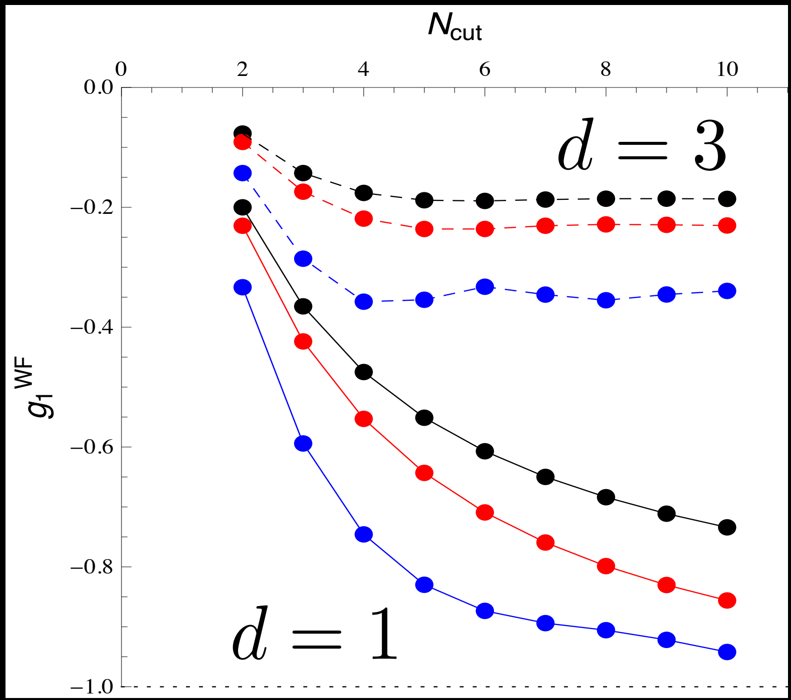
Expansions

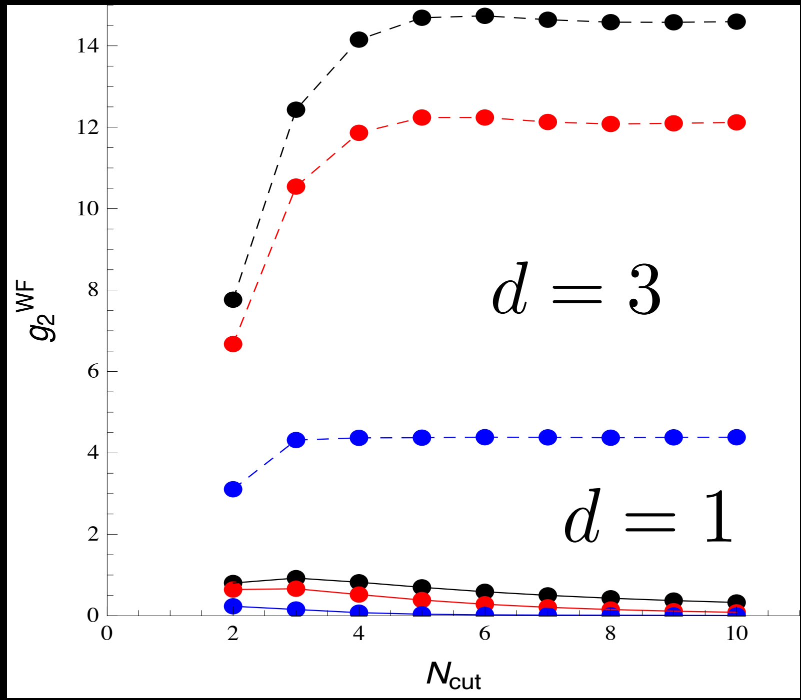
$$U_k(\rho) = \sum_n g_n \frac{\rho^n}{n!}$$

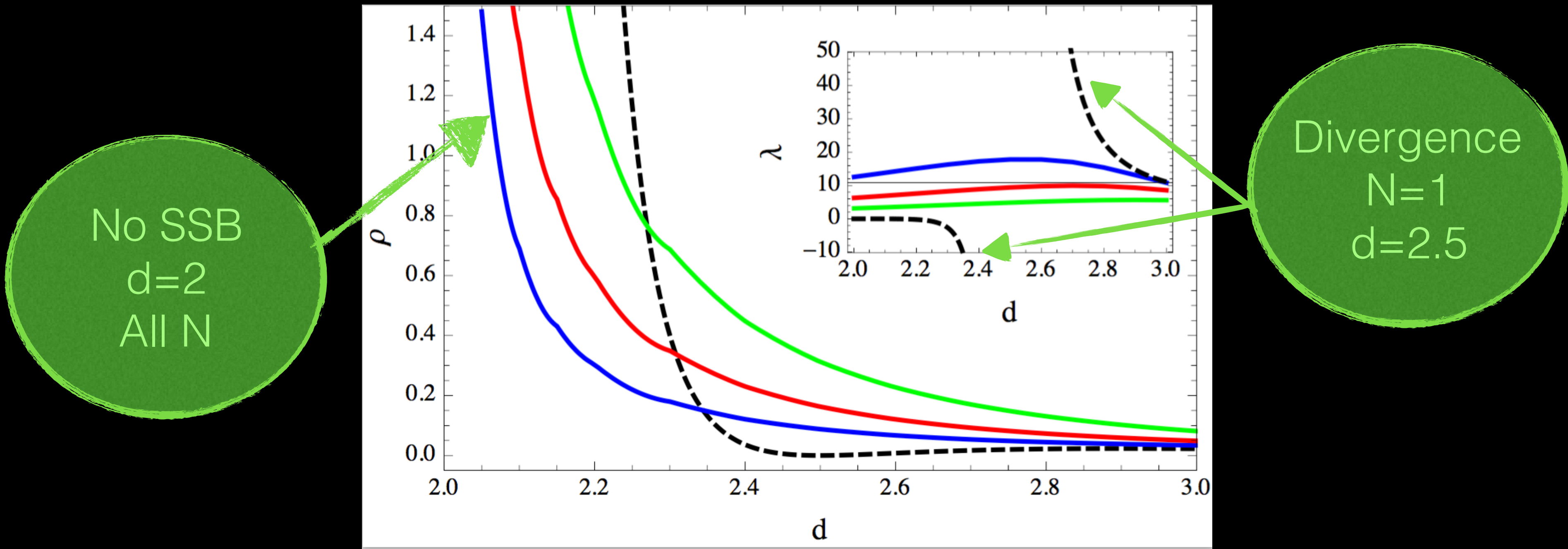
$$U_k(\rho) = \sum_{n=2} \lambda_n \frac{(\rho - \rho_0)^n}{n!}$$



Truncations Picture

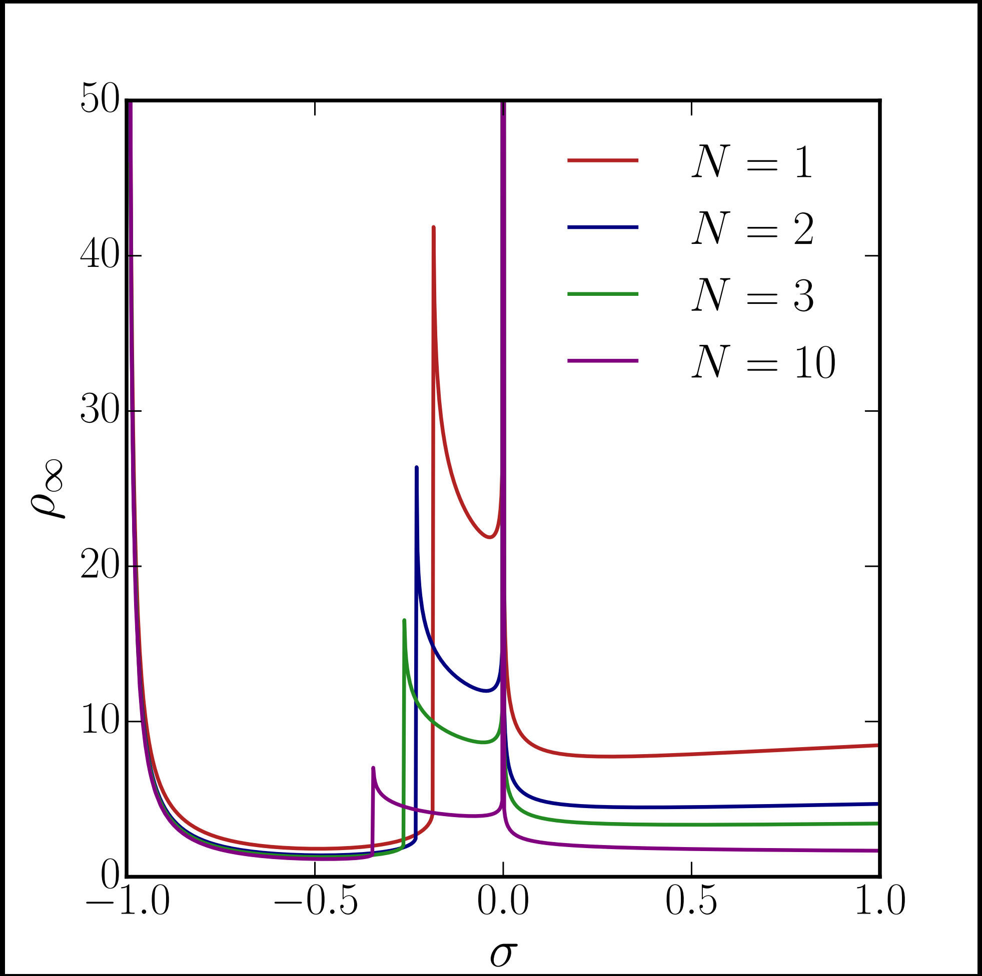


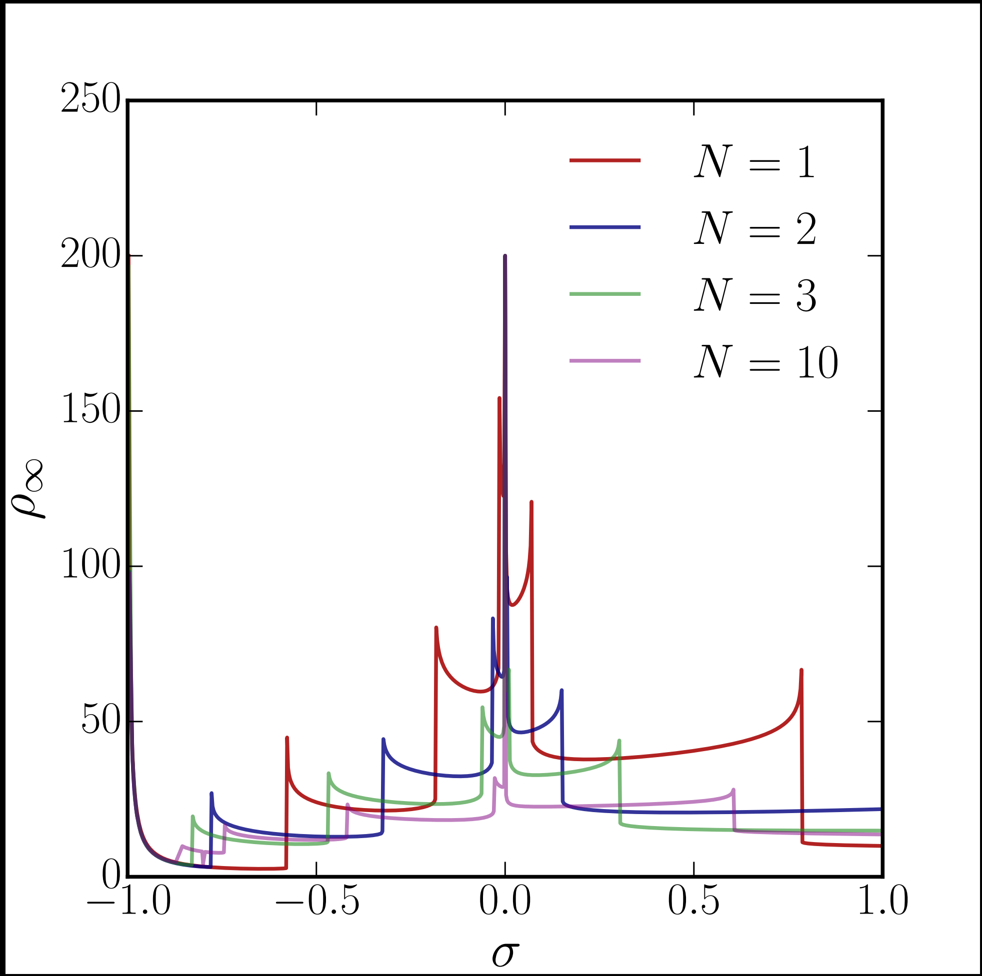




Functional solutions

$$\partial_t U_k(\rho) = d\,U(\rho) - (d-2)\rho\, U^{(1)}(\rho) - \frac{1}{1+U^{(1)}(\rho)+2\,\rho\, U^{(2)}(\rho)} - \frac{N-1}{1+U^{(1)}(\rho)}$$

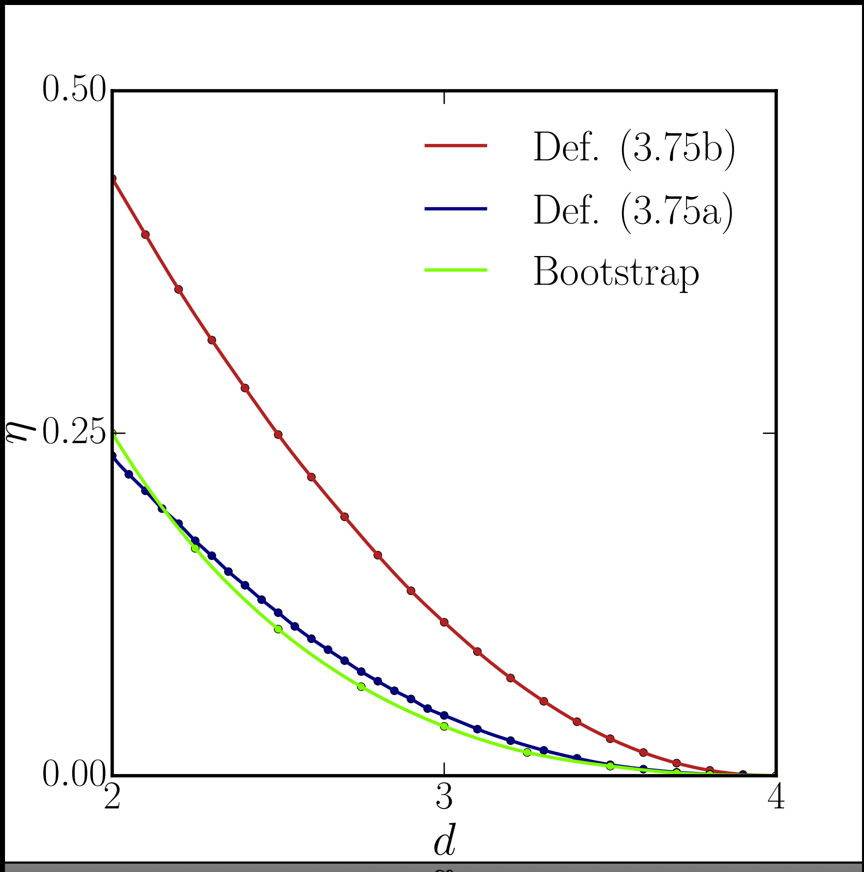




Anomalous Dimension

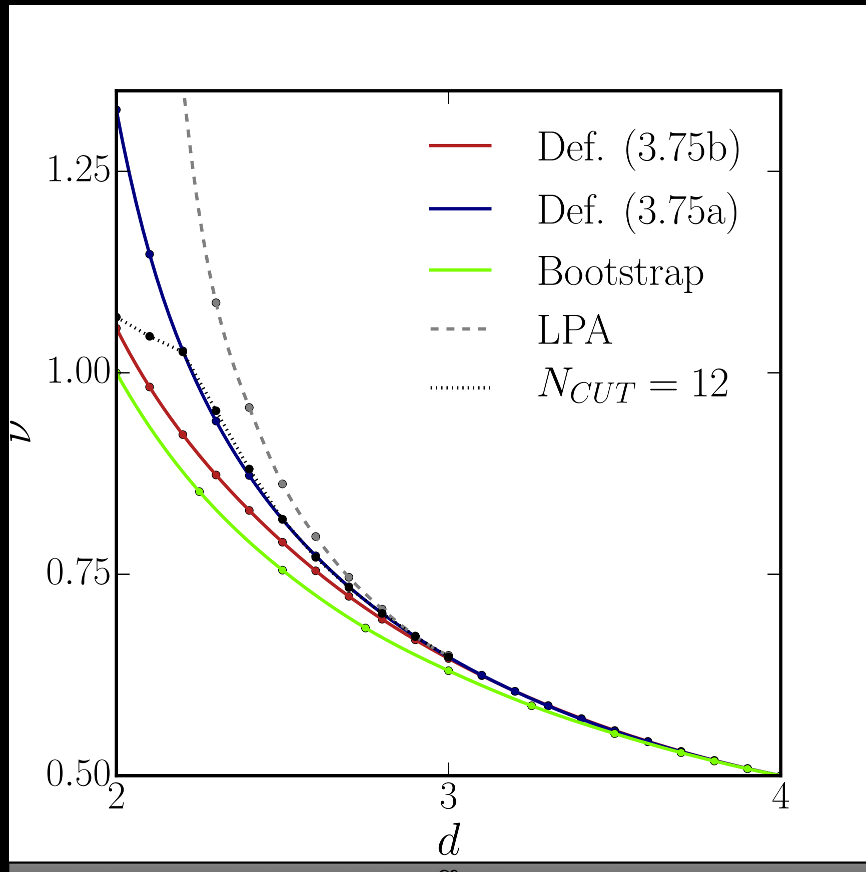
- Massive mode ✗

$$\eta = \frac{4\rho_0 \left(2\,\rho_0\,U^{(3)}(\rho_0) + 3\,U^{(2)}(\rho_0)\right)^2}{(1+2\,\rho_0\,U^{(2)}(\rho_0))^4}$$



- Goldstone mode ✓

$$\eta = \frac{4\rho_0 U^{(2)}(\rho_0)^2}{(1+2\,\rho_0\,U^{(2)}(\rho_0))^2}$$

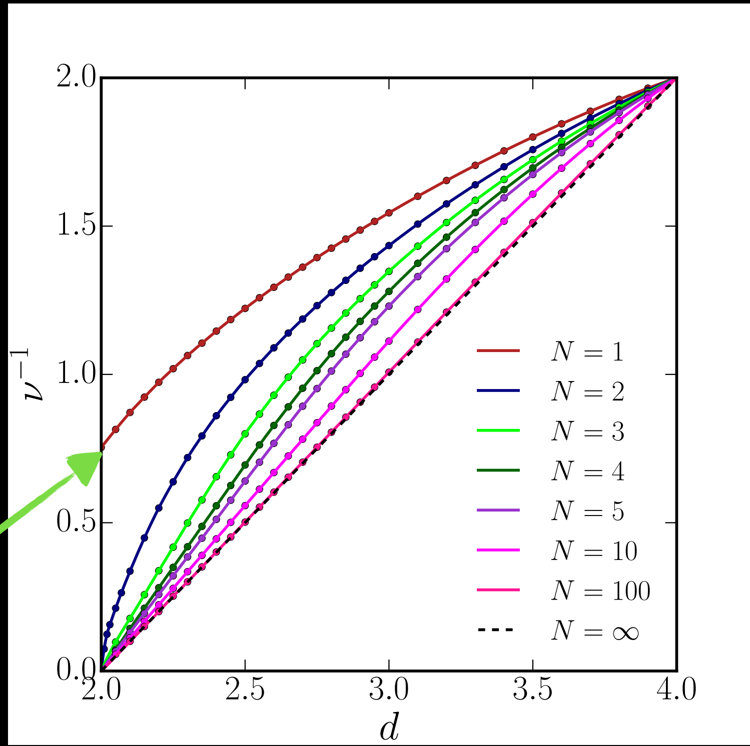


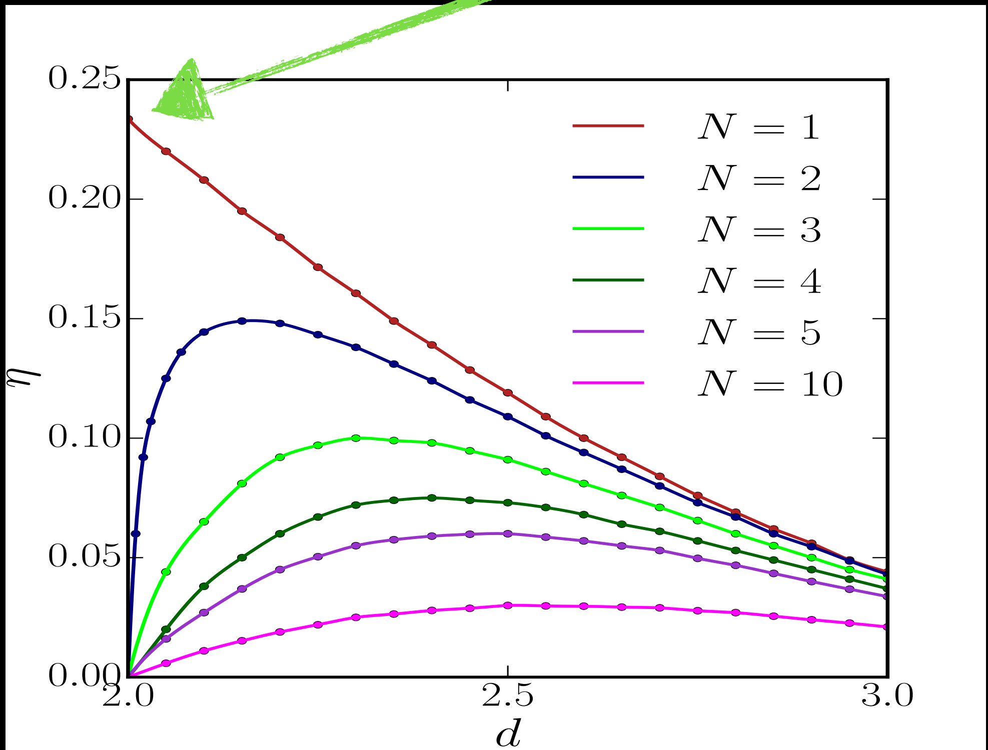
Mermin Wagner Theorem

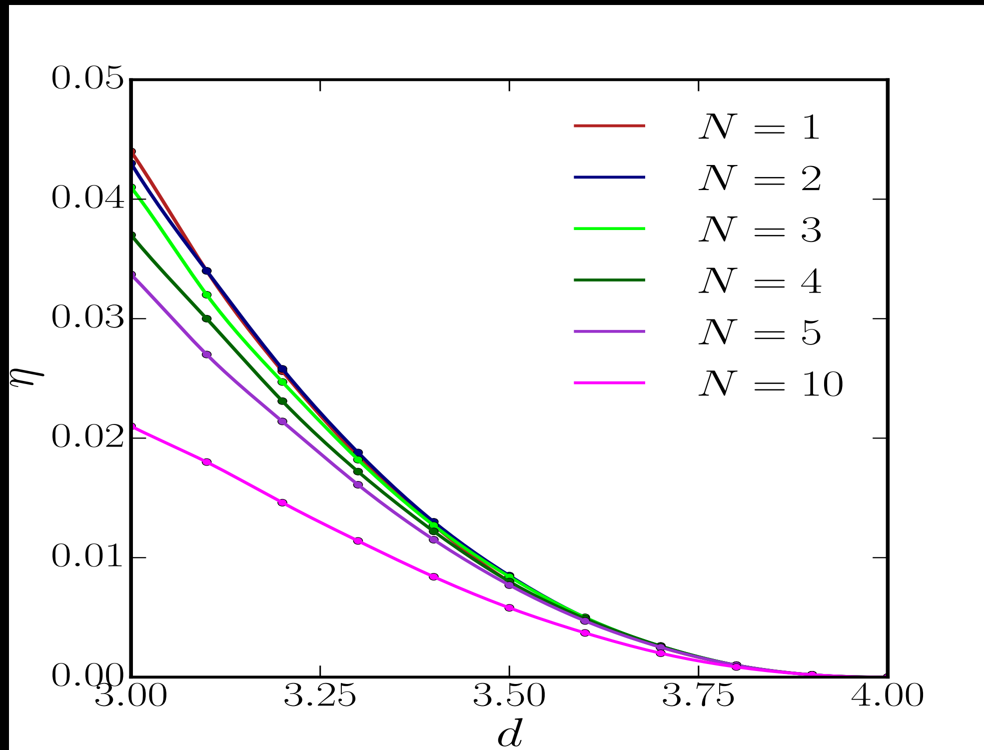
$$\lim_{d \to 2} \eta = 0$$

$$\lim_{d \to 2} \nu = \infty$$

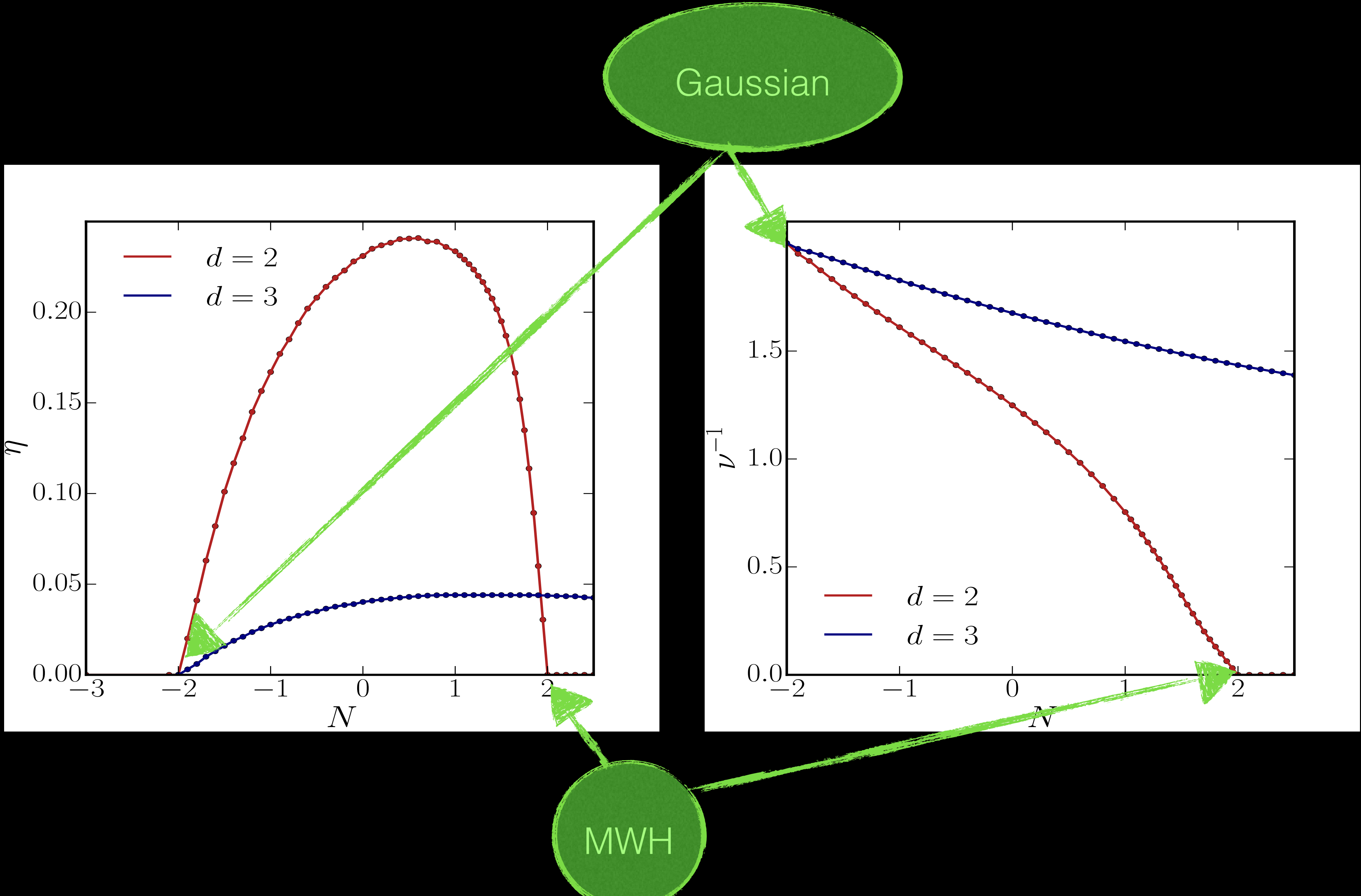
Discrete Symmetry







Phase Transition of Phase Transitions



Tri-critical Universality

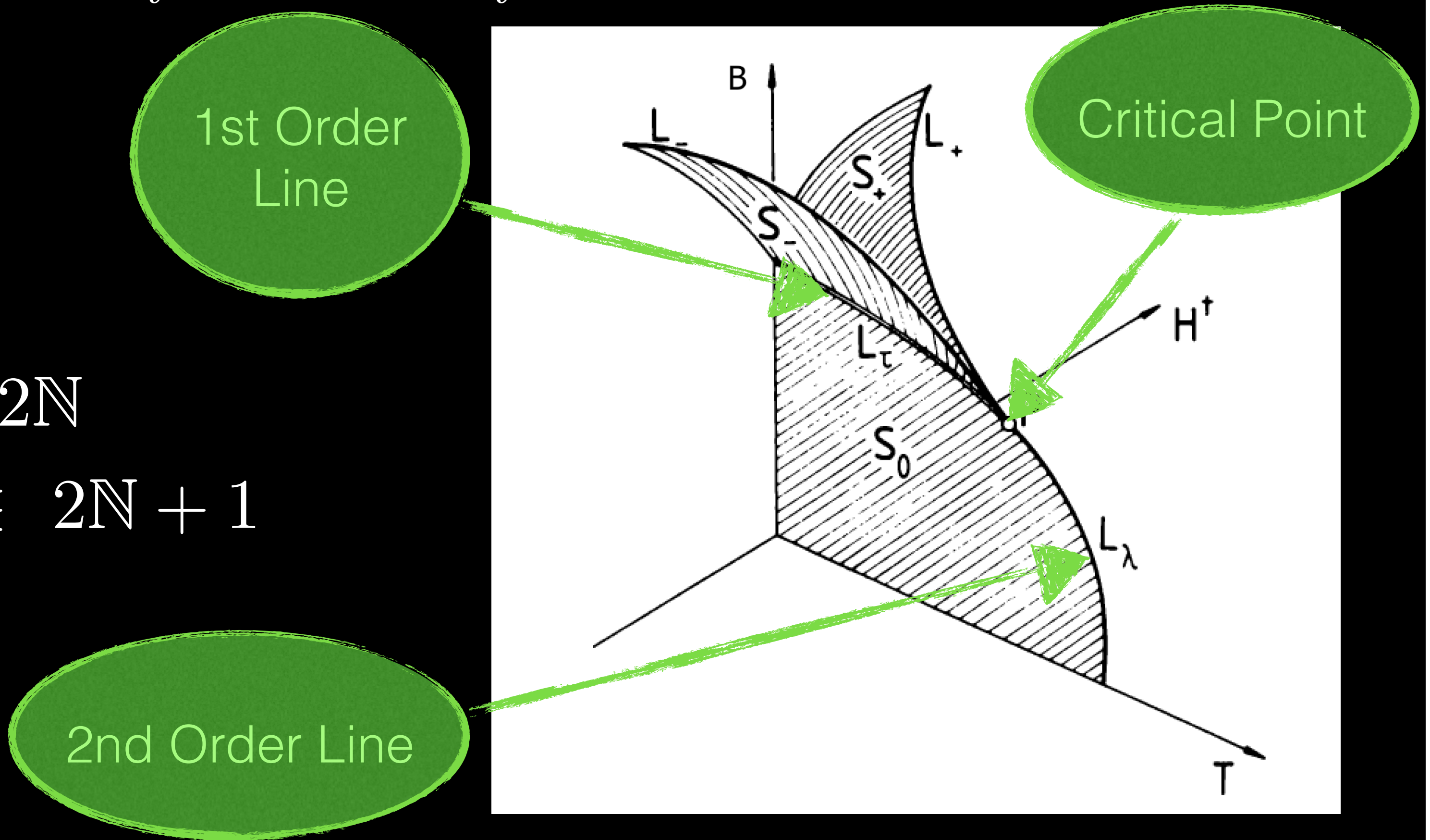
Antiferromagnetic Ising model in a staggered magnetic field

$$H = -\frac{J}{2}\sum_{\langle ij\rangle} S_i S_j - \sum_i B_i S_i - \sum_i H_i^\dagger S_i$$

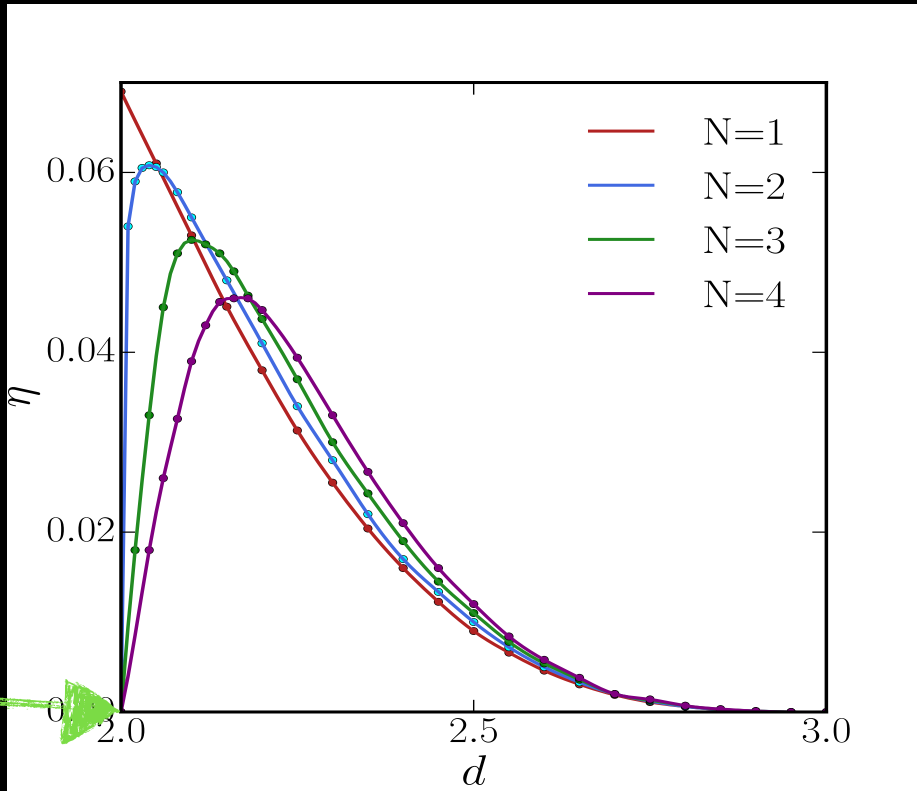
$$J > 0$$

$$B_i \equiv B > 0 \ \forall \ i$$

$$H_i^\dagger = \begin{cases} H \text{ if } i \ \in \ 2\mathbb{N} \\ -H \text{ if } i \ \in \ 2\mathbb{N}+1 \end{cases}$$

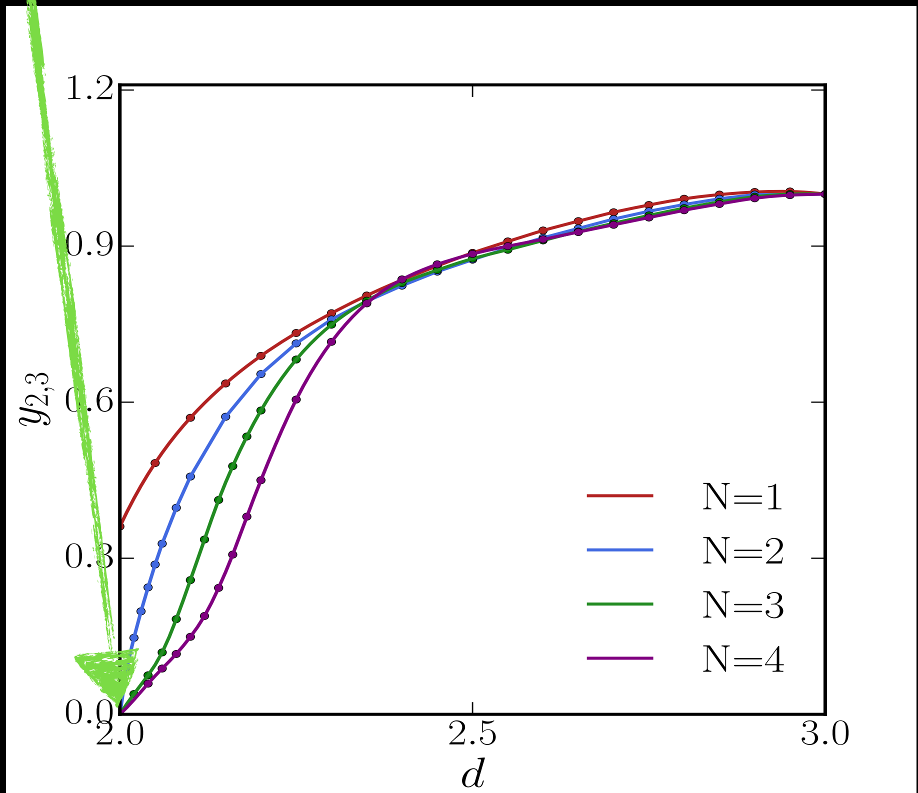


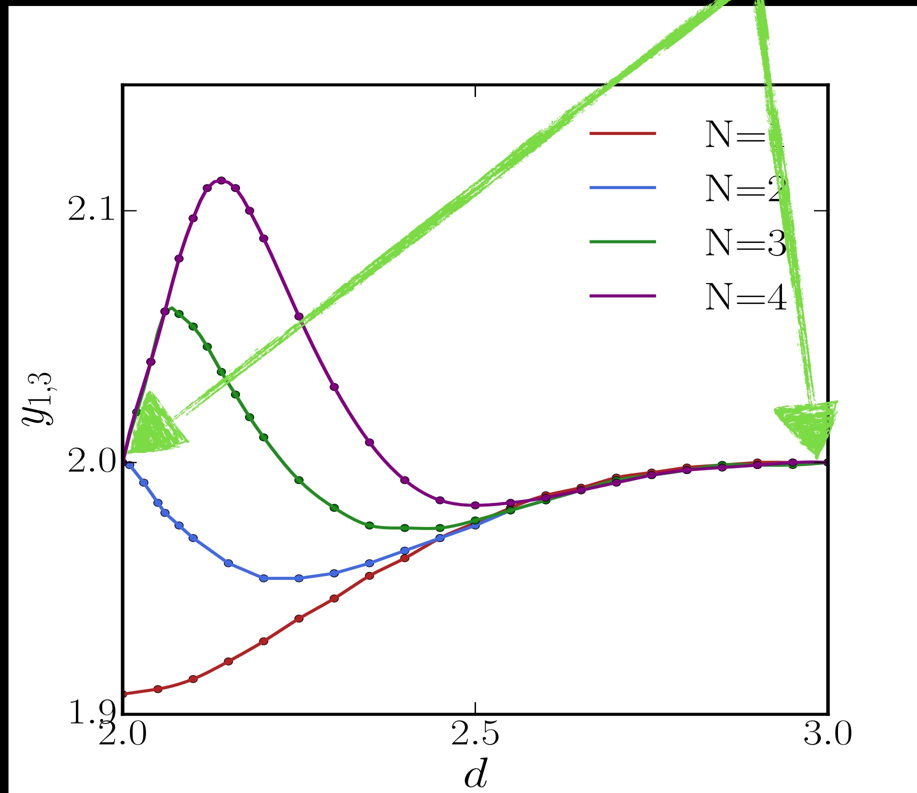
Critical Exponents



MWH

Mean Field Value





Landscape of Field Theories

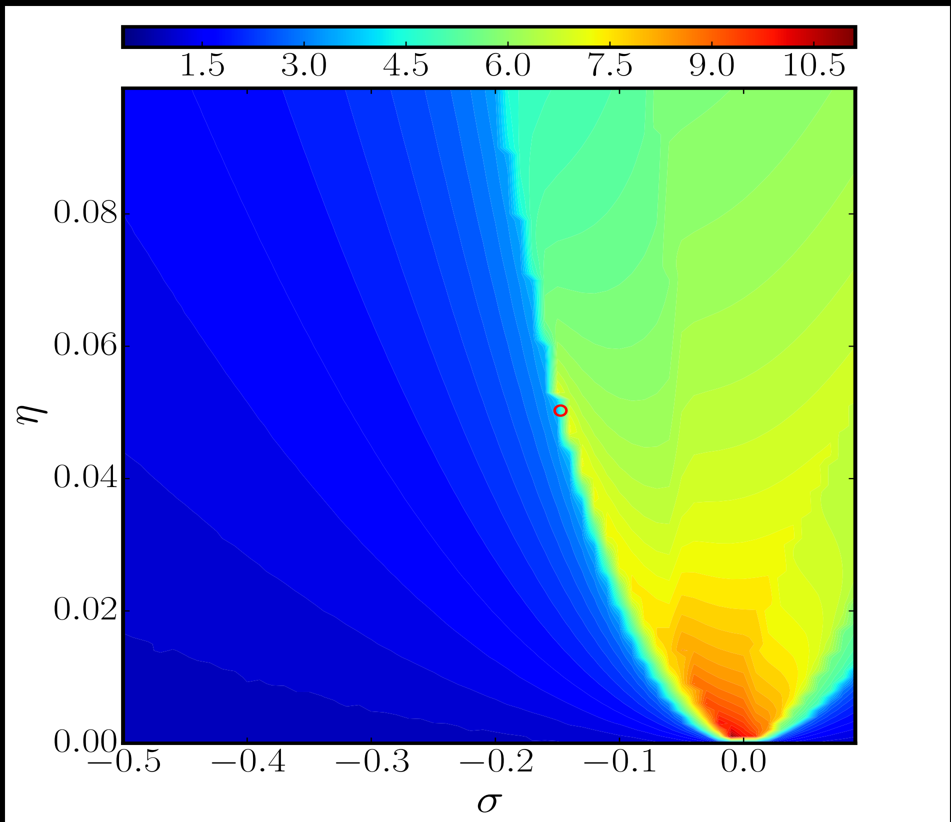
$$N = 1 \quad S[\varphi] = \int d^d x \, \{Z(\varphi)\partial_\mu \varphi(x) \partial_\mu \varphi(x) + V(\varphi)\}$$

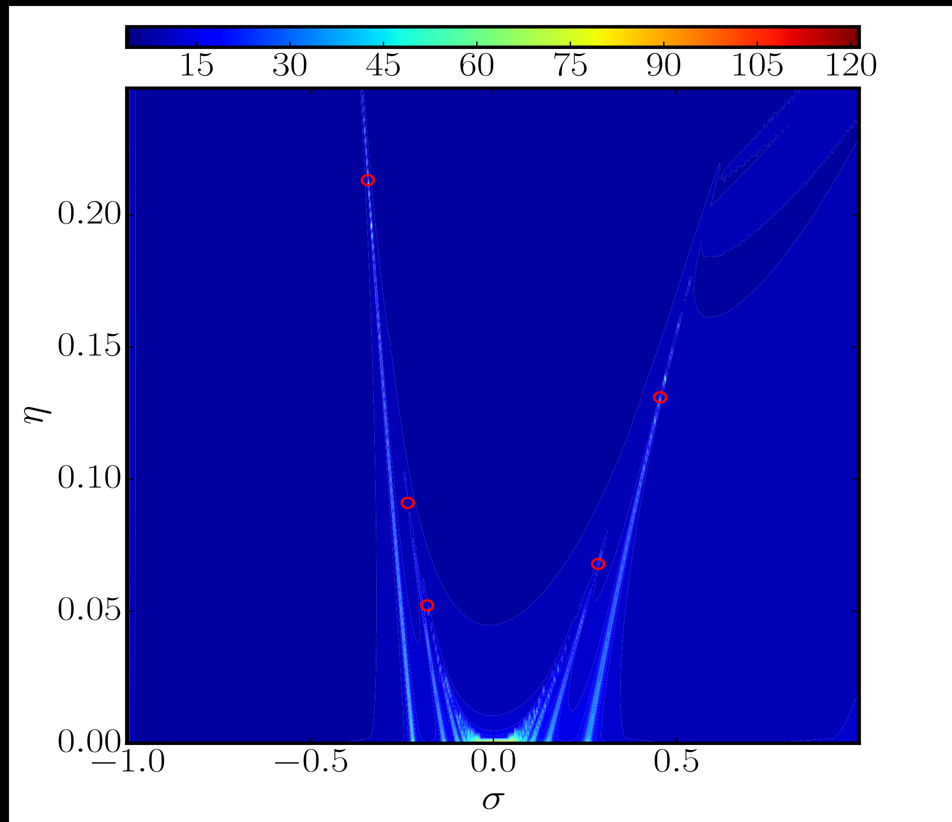
$$V'(0) = 0 \quad V''(0) = \sigma$$

$$Z(0) = 1 \quad Z'(0) = 0$$

$$\sigma = ?$$

$$\eta = ?$$



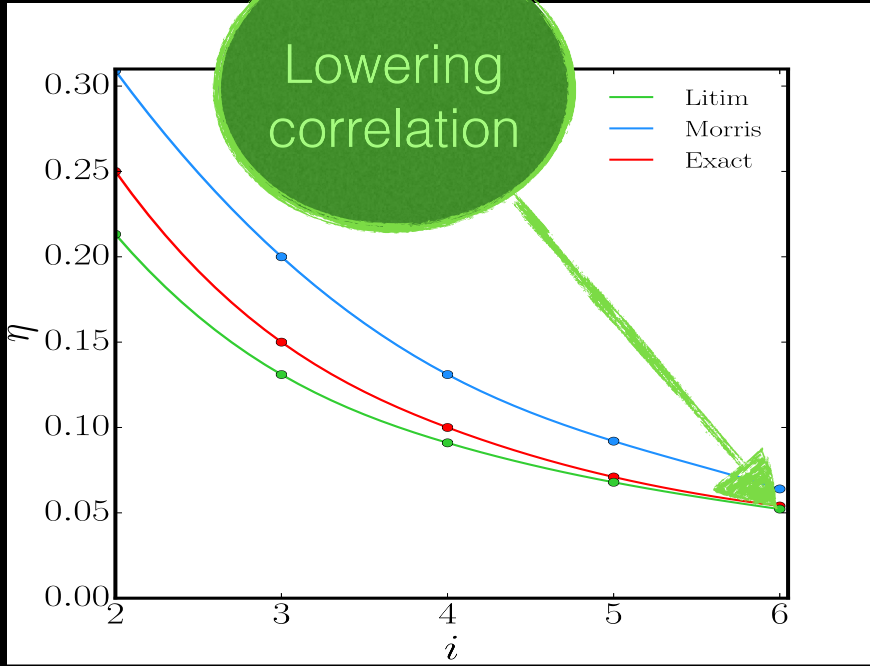


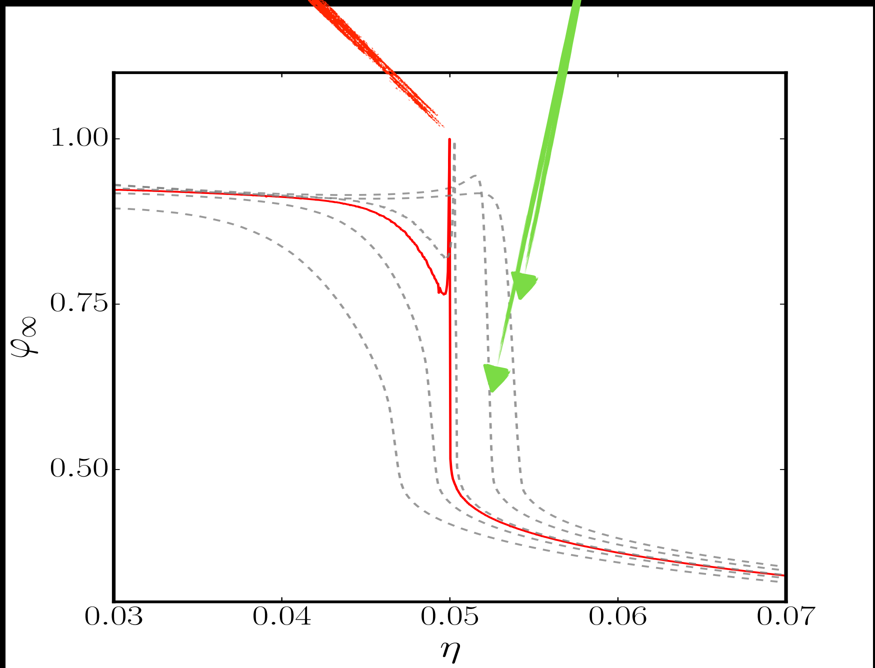
Complete phase diagram

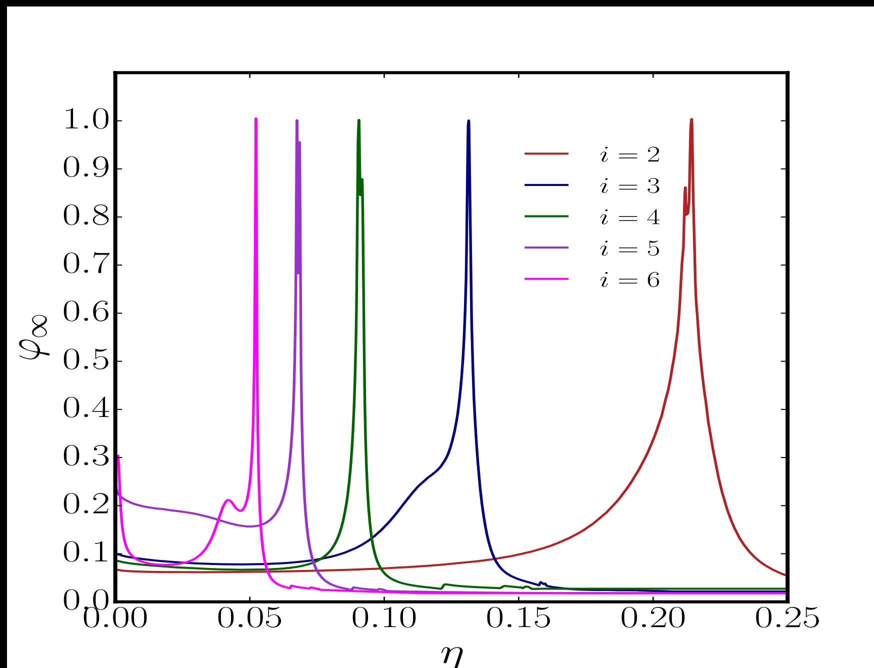
$$\eta = 0.0496$$

Converged value

Intermediate steps







Sine-Gordon Model

XY Model $d = 2$

$$H = -J \sum_{\langle ij \rangle} \cos\left(\theta_i - \theta_j\right)$$

Coulomb Gas

$$H = -\sum_{i \neq j} q_i q_j \log\left|\frac{r_j - r_i}{a}\right|$$

Kosterlitz-Thouless

sine-Gordon

$$S = \int d^d x \left\{\partial_\mu \varphi \partial_\mu \varphi + u(1 - \cos(\beta\varphi))\right\}$$

U(1) field theory $d = 2$

$$S = \int d^d x \left\{\partial_\mu \varphi^* \partial_\mu \varphi + U(\varphi^* \varphi)\right\}$$

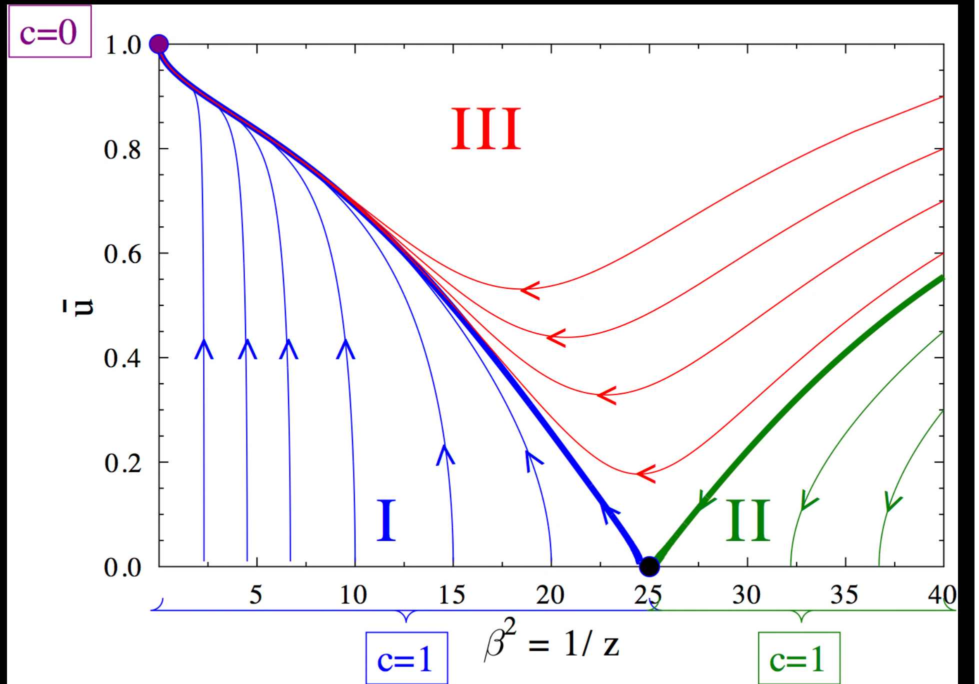
Phase Diagram

Effective Action Ansatz

$$\Gamma_k = \int d^d x \, \{ z_k \partial_\mu \tilde{\varphi} \partial_\mu \tilde{\varphi} + u_k \cos(\tilde{\varphi}) \}$$

$$(2 + \partial_t)\tilde{u}_k = \frac{1}{2\pi \, z_k \tilde{u}_k} \left(1 - \sqrt{1 - \tilde{u}_k^2} \right)$$

$$\partial_t z_k = -\frac{1}{24\pi} \frac{\tilde{u}_k^2}{\sqrt{1 - \tilde{u}_k^2}^{\,3}}$$

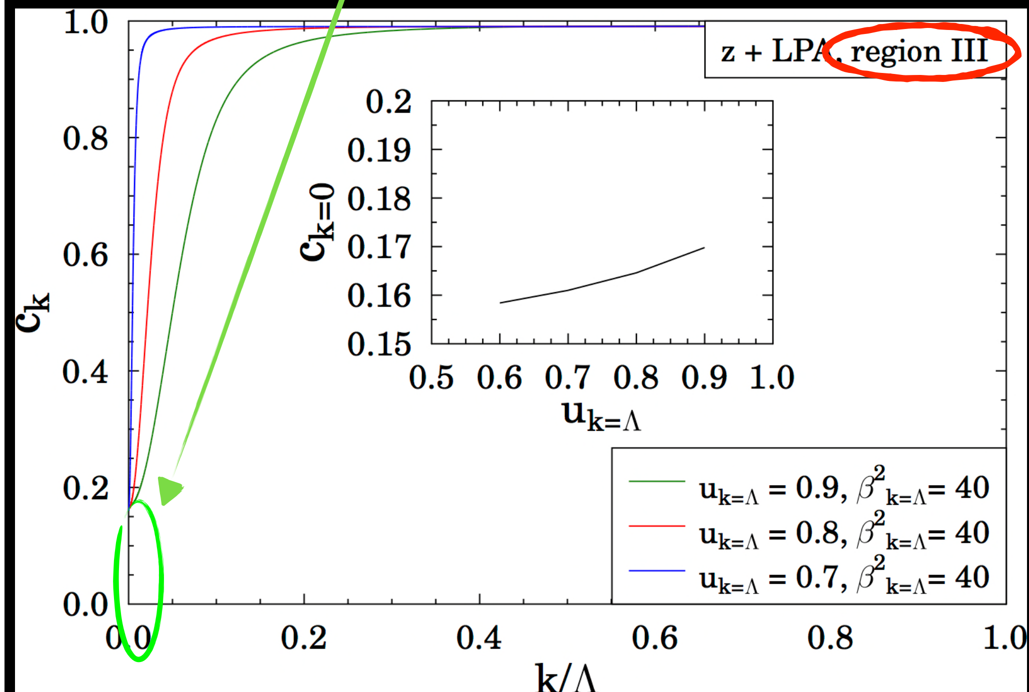


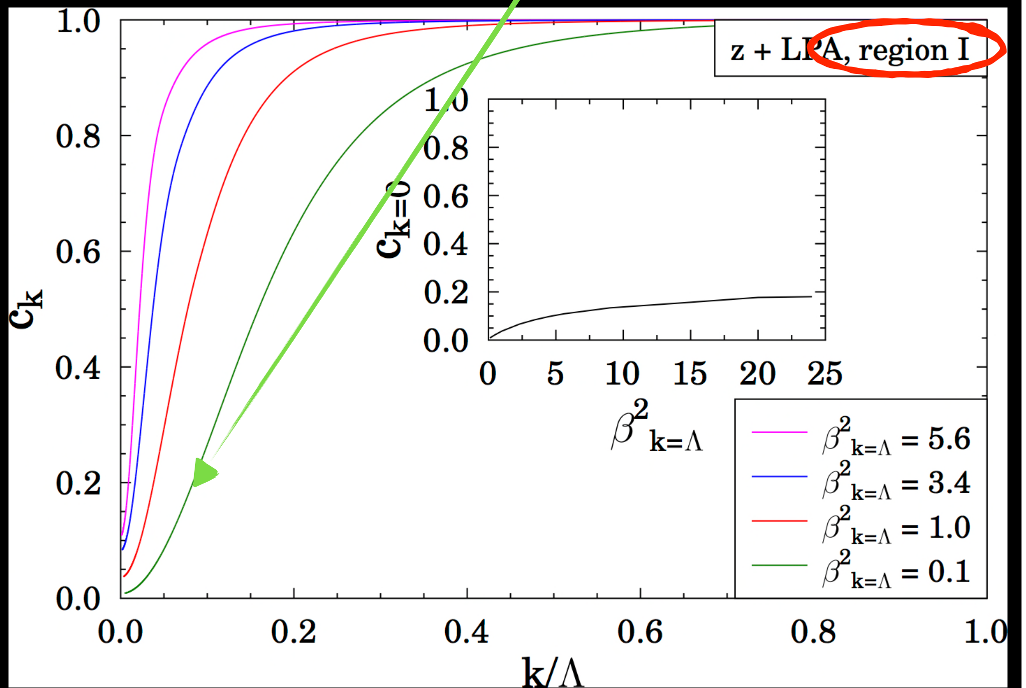
C-function

High frequency contribution?

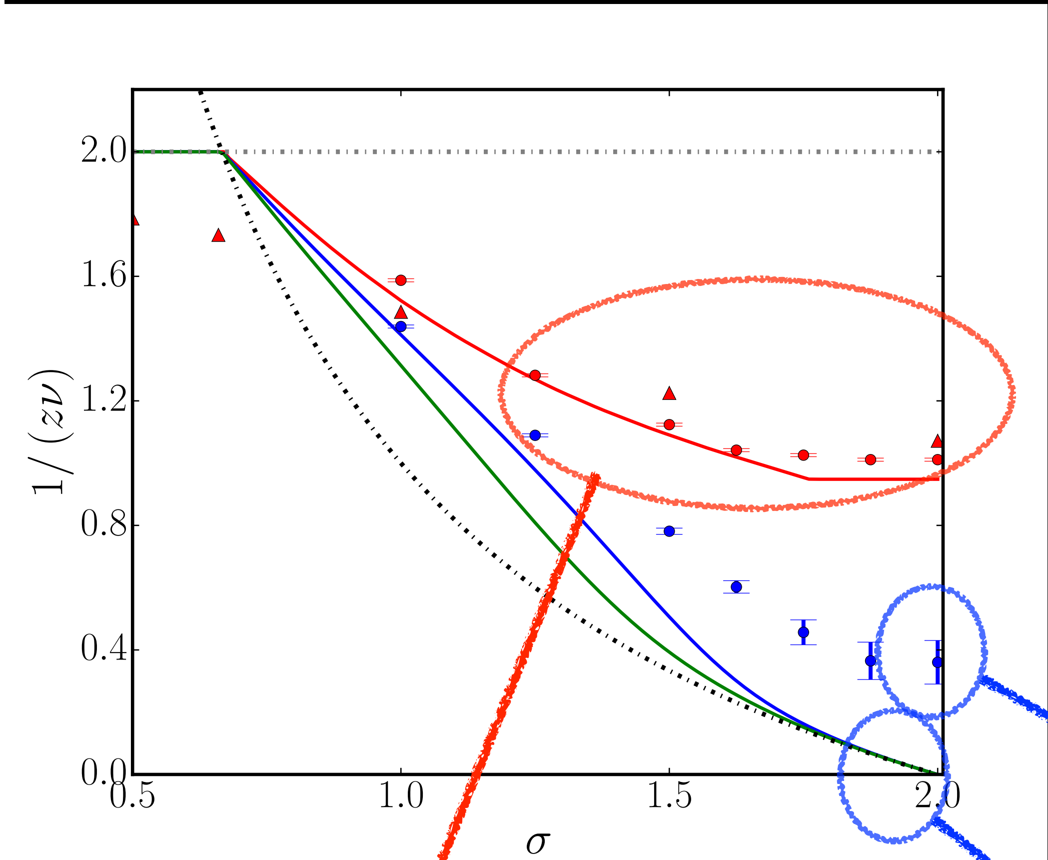
$$\partial_t c_k = \frac{\partial_t \tilde{u}_k}{(1+\tilde{u}_k)^3}$$

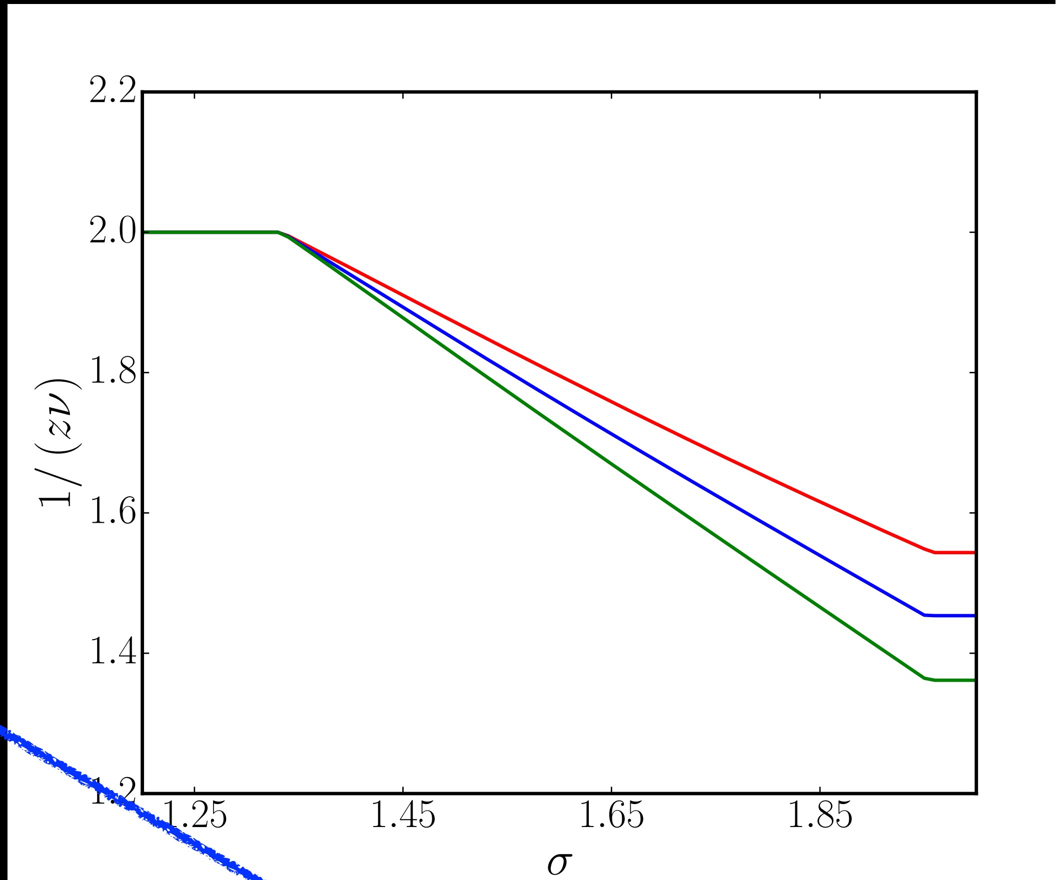
Perturbative Limit





Quantum Long Range





Good (but not perfect) agreement

Finite size effects

Exact Behavior

4. arXiv:1507.04920 [pdf, other]

c-function and central charge of the sine-Gordon model from the non-perturbative renormalization group flow

V. Bacsó, N. Defenu, A. Trombettoni, I. Nándori

Comments: 9 pages, 4 figures, v1, to be published on Nucl. Phys. B

Subjects: **Statistical Mechanics (cond-mat.stat-mech)**; High Energy Physics - Theory (hep-th)

5. arXiv:1410.7024 [pdf, other]

Truncation Effects in the Functional Renormalization Group Study of Spontaneous Symmetry Breaking

N. Defenu, P. Mati, I. G. Marian, I. Nandori, A. Trombettoni

Comments: v2: 11 pages, 7 figures, submitted

Journal-ref: JHEP 1505(141) October 2014

Subjects: **High Energy Physics - Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech)

6. arXiv:1410.3308 [pdf, other]

Critical exponents of O(N) models in fractional dimensions

A. Codello, N. Defenu, G. D'Odorico

Comments: 6 pages, 5 figures, reference added

Journal-ref: Phys. Rev. D 91, 105003 (2015)

Subjects: **High Energy Physics - Theory (hep-th)**; Statistical Mechanics (cond-mat.stat-mech)

Thank You!