Axial anomaly’s role on conventional mesons, baryons, and pseudoscalar glueball

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arXiv: 1709.07454

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arXiv: 1708.01061, to appear in PRD

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arXiv:1208.6474 , Phys.Rev. D87 (2013) no.5, 054036

Outline

Brief recall of QCD and the axial anomaly

Axial anomaly and strange-nonstrange mixing

Axial anomaly in the baryonic sector: N(1535) -> Nη

Axial anomaly and the pseudoscalar glueball

Summary

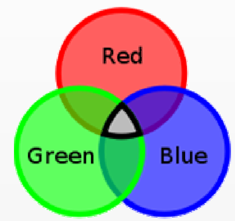
QCD Lagrangian: symmetries and anomalies

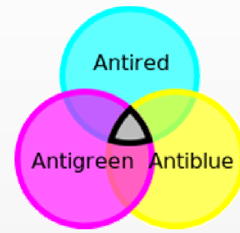
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



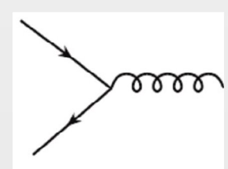


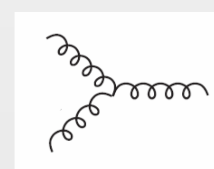
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \ i = u,d,s,...$$

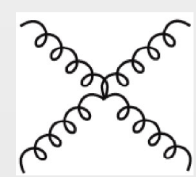
8 type of gluons ($R\overline{G}$, $B\overline{G}$,...)

$$A_\mu^a \ ; \ a = 1,...,8$$

$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}}$$







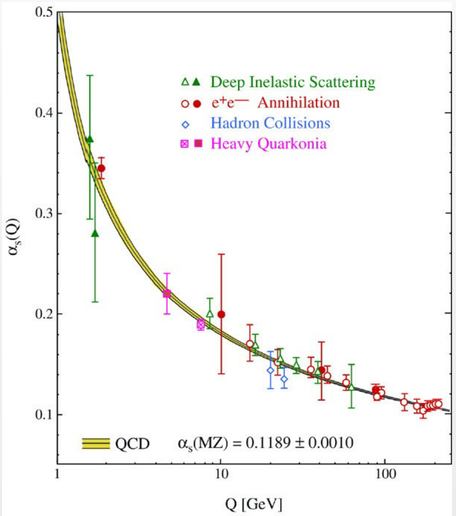
Trace anomaly: the emergence of a dimension

Chiral limit: $m_i = 0$

$x^\mu \to x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250\ \mathrm{MeV}$

$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \to m^*_{gluon} \approx 500 - 800\ \mathrm{MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

The trace anomaly/2

Emergence of a low-energy scale: $\Lambda_{YM} \approx 250\text{ MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

„Effective“ gluon mass

Analytic Structure of the Landau-Gauge Gluon Propagator
Stefan Strauss, Christian S. Fischer, and Christian Kellermann
Phys. Rev. Lett. **109**, 252001 – Published 19 December 2012

Moreover, the gluon is certainly not a massive particle in the usual sense. [...] Within the present accuracy we find 600 MeV < mg < 700 MeV.

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

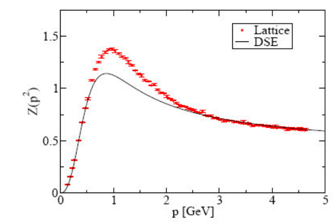
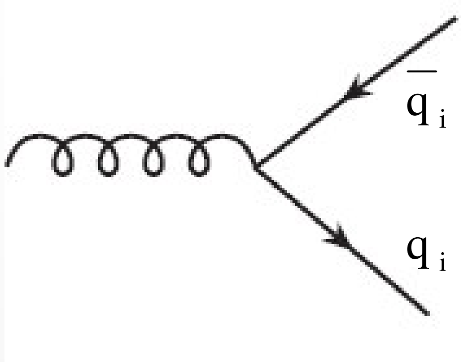


FIG. 2: Results for the gluon dressing function $Z(p^2)$ from lattice calculations [12] compared to the result from DSEs [9].

Flavor symmetry



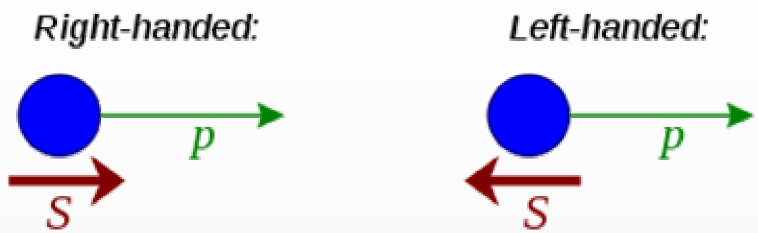
Gluon-quark-antiquark vertex.

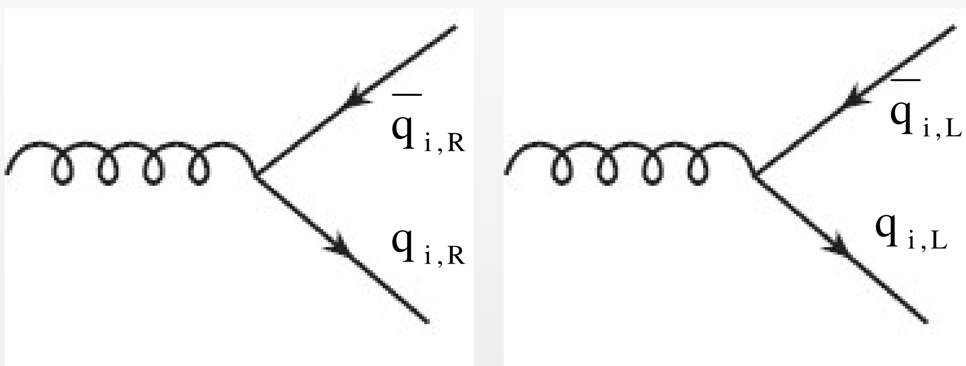
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$U \in U(3)_V \rightarrow U^+ U = 1$$

Chiral symmetry





$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = \underbrace{U(1)_{R+L}}_{\text{baryon number}} \times \underbrace{U(1)_{R-L}}_{\text{anomaly } U(1)_A} \times \underbrace{SU(3)_R \times SU(3)_L}_{\text{SSB into } SU(3)_V}$$

In the chiral limit ($m_i=0$) chiral symmetry is exact

Chiral transformations and axial anomaly

$$G_{\text{fl}} \times U(1)_A = SU(3)_L \times SU(3)_R \times U(1)_A$$

$$\boxed{q_{L,R} \longrightarrow e^{\mp i\alpha/2}\, U_{L,R}\, q_{L,R}}$$

U(1)A chiral

Axial anomaly:

$$\boxed{\partial^{\mu}(\bar{q}^{i}\gamma_{\mu}\gamma_{5}q^{i}) = \frac{3g^2}{16\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})}$$

Spontaneous breaking of chiral symmetry

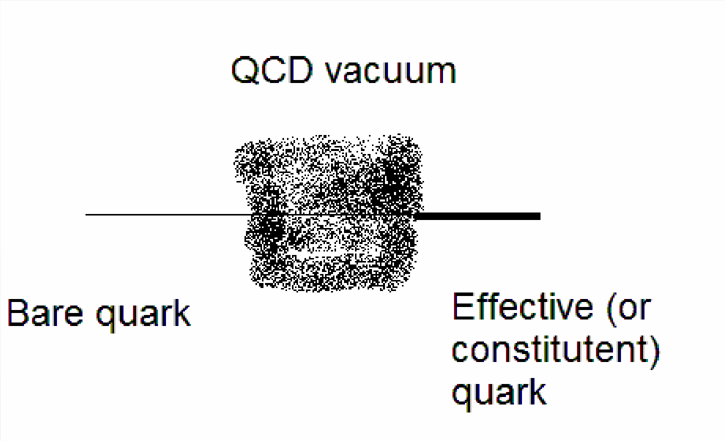
$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$\mathrm{SSB:}\ \mathrm{SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L}}$$

Chiral symmetry → Flavor symmetry

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

$$\mathrm{m \approx m_u \approx m_d \approx 5\ MeV \rightarrow m^* \approx 300\ MeV}$$



$$\mathrm{m_{\rho-meson} \approx 2m^*}$$

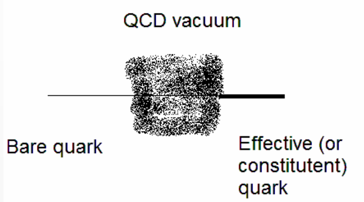
$$\mathrm{m_{proton} \approx 3m^*}$$

Spontnaeous breaking of chiral symmetry/2

$$\mathrm{SSB:}\ \mathrm{SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L}}$$

Chiral symmetry → Flavor symmetry

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

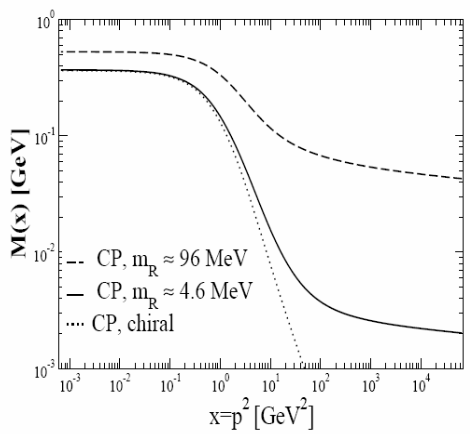


$$\mathrm{m \approx m_u \approx m_d \approx 5\ MeV \rightarrow m^* \approx 300\ MeV}$$

Nonperturbative propagators, running coupling, and the dynamical quark mass of Landau gauge QCD

C. S. Fischer and R. Alkofer

Phys. Rev. D **67**, 094020 – Published 27 May 2003



Symmetries of QCD: summary

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (trace anomaly) and by small quark masses

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is spontaneously broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (chiral anomaly)

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Axial anomaly and strange-nonstrange mixing

based on

F.G., A . Koenigstein, R.D. Pisarski

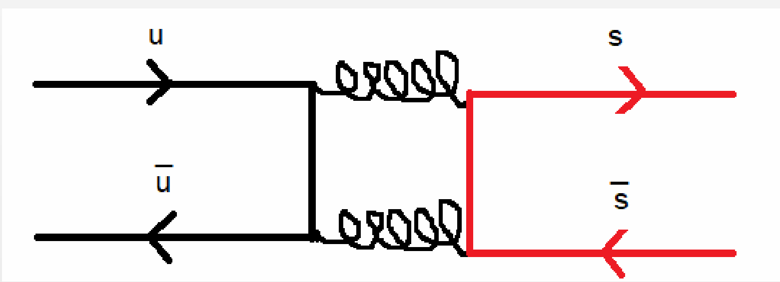
How the axial anomaly controls flavor mixing among mesons

ArXiv: 1709.07454

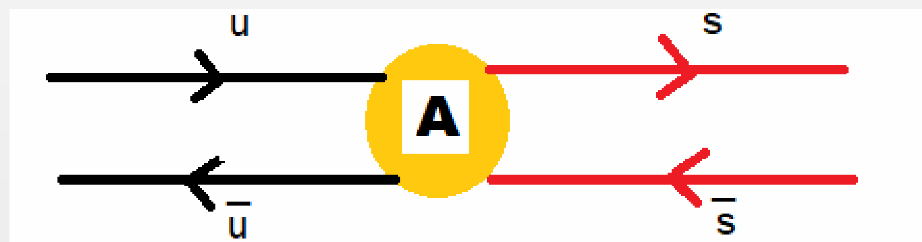
What physical processes we look at:
mixing in the isoscalar sector in a certain multiplet

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{mix} & \sin\theta_{mix} \\ -\sin\theta_{mix} & \cos\theta_{mix} \end{pmatrix} \begin{pmatrix} M_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ M_S = \bar{s}s \end{pmatrix}$$

Such a mixing is suppressed...



But this can be large



(Pseudo)scalar mesons: heterochiral scalars

Uniwersytet Jana Kochanowskiego w Kielcach

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp\mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

$J^{PC},\ ^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d,\, \bar{d}u,\, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s,\, \bar{s}u,\, \bar{d}s,\, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}},\, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
$0^{-+},\ ^1S_0$	$\begin{cases} \pi \\ K \\ \eta, \eta'(958) \end{cases}$	$P^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$	$\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_{\mathrm{R}} q^i_{\mathrm{L}})$	$\Phi \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger$
$0^{++},\ ^3P_0$	$\begin{cases} a_0(1450) \\ K_0^\star(1430) \\ f_0(1370), f_0(1710)^\star \end{cases}$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		

$$\boxed{\Phi \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger}$$

We call the transformation of the matrix Φ **heterochiral**!
We thus have heterochiral scalars.

$\mathrm{tr}(\Phi^\dagger\Phi),\ \mathrm{tr}(\Phi^\dagger\Phi)^2$ are clearly invariant; typical terms for a chiral model.

$\det(\Phi)$ is interesting, since it breaks only U(1)A axial anomaly $\quad \det\Phi \rightarrow e^{-i6\alpha}\det\Phi$

Mixing in the pseudoscalar sector

$$\Phi = S + iP = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & K_0^{*0} & \sigma_S \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42^\circ$$

How to describe the mixing: Anomaly Lagrangian for heterochiral scalars

$$\mathcal{L}_{\Phi}^{\text{anomaly}} = - a_{\text{A}}^{(1)} [\det(\Phi) + \text{c.c.}] - a_{\text{A}}^{(2)} [\det(\Phi) + \text{c.c.}]^2 - a_{\text{A}}^{(3)} [\det(\Phi) - \text{c.c.}]^2 - \ldots$$

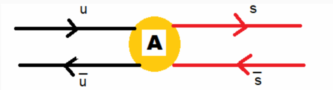
- invariant under $SU(3)_R \times SU(3)_L$, but breaks $U(1)_A$ $\quad \det\Phi \to e^{-i6\alpha} \det\Phi$
- first term: affects the mixing of η and η' and also scalar mixing
- second term: higher order but quite similar to the first
- third term: affects **only** η and η'

Recall the condensation:
$\langle\Phi\rangle \sim \sqrt{3/2}\, f_\pi\, \mathbb{1}$

Pseudoscalar mixing

From the third term only:

$$\mathcal{L}_{\Phi}^{\text{anomaly}} = -\alpha_A \eta_0^2 + \ldots = -\alpha_A \left(\sqrt{2}\eta_N + \eta_S\right)^2 + \ldots$$



$$\theta_P \simeq -\frac{1}{2}\arctan\left[\frac{2\sqrt{2}\alpha_A}{m_K^2 - m_\pi^2 - \alpha_A}\right]$$

The numerical value can be correctly described, see e.g.

S. D. Bass and A. W. Thomas, Phys. Lett. B **634** (2006) 368 doi:10.1016/j.physletb.2006.01.071 [hep-ph/0507024].

(Axial-)vector mesons: homochiral vectors

J^{PC}, $^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_\mathrm{L}\times SU(3)_\mathrm{R}\times$ $\times U(1)_\mathrm{A}$
1^{--}, $^1S_1$	$\begin{cases} \rho(770) \\ K^*(892) \\ \omega(782), \phi(1020) \end{cases}$	$V_\mu^{ij} = \frac{1}{2}\bar{q}^j\gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L_\mu^{ij} = \bar{q}_\mathrm{L}^j\gamma_\mu q_\mathrm{L}^i)$	$L_\mu \longrightarrow U_\mathrm{L} L_\mu U_\mathrm{L}^\dagger$
1^{++}, $^3P_1$	$\begin{cases} a_1(1260) \\ K_{1,A} \\ f_1(1285), f_1(1420) \end{cases}$	$A_\mu^{ij} = \frac{1}{2}\bar{q}^j\gamma^5\gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R_\mu^{ij} = \bar{q}_\mathrm{R}^j\gamma_\mu q_\mathrm{R}^i)$	$R_\mu \longrightarrow U_\mathrm{R} R_\mu U_\mathrm{R}^\dagger$

$$L_\mu \longrightarrow U_\mathrm{L} L_\mu U_\mathrm{L}^\dagger$$

$$R_\mu \longrightarrow U_\mathrm{R} R_\mu U_\mathrm{R}^\dagger$$

We have here a **homochiral** multiplet.
We call these states as homochiral vectors.

Mixing among vector mesons

$$\begin{pmatrix} \omega(782) \\ \phi(1020) \end{pmatrix} = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix} \begin{pmatrix} \omega_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_V \simeq -3.2°$$

The mixing is very small.
This is understandable: there is no term analogous to the determinant.
Namely, anomlay-driven terms are more complicated, involve derivatives and do not affect isoscalar mixing, e.g. Wess-Zumino like terms:

$$\varepsilon^{\mu\nu\alpha\beta}\mathrm{tr}[L_\mu\Phi(\partial_\nu\Phi^\dagger)\Phi(\partial_\alpha\Phi^\dagger)\Phi(\partial_\beta\Phi^\dagger) + R_\mu\Phi^\dagger(\partial_\nu\Phi)\Phi^\dagger(\partial_\alpha\Phi)\Phi^\dagger(\partial_\beta\Phi)]$$

Mixing among axial-vector mesons

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_{AV} & \sin\theta_{AV} \\ -\sin\theta_{AV} & \cos\theta_{AV} \end{pmatrix} \begin{pmatrix} f_{1,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ f_{1,S} = \bar{s}s \end{pmatrix}$$

θ_{AV} should be small!

Small mixing angle found in the following phenomenological studies:
L. Olbrich, F. Divotgey, F.G., Eur.Phys.J. A**49** (2013) 135 arXiv:1306.1193
Parganlija et al, Phys.Rev. D**87** (2013) no.1, 014011

Ground-state tensors (and their chiral partners): Homochiral tensors

$J^{PC},\ ^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
$2^{++},\ ^3P_2$	$\begin{cases} a_2(1320) \\ K_2^*(1430) \\ f_2(1270), f_2'(1525) \end{cases}$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}_{\mathrm{L}}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q_{\mathrm{L}}^i)$	$L_{\mu\nu} \longrightarrow U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^\dagger$
$2^{--},\ ^3D_2$	$\begin{cases} \rho_2(?) \\ K_2(1820) \\ \omega_2(?), \phi_2(?) \end{cases}$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}_{\mathrm{R}}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q_{\mathrm{R}}^i)$	$R_{\mu\nu} \longrightarrow U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^\dagger$

$$L_{\mu\nu} \longrightarrow U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^\dagger$$

$$R_{\mu\nu} \longrightarrow U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^\dagger$$

Thus, we have **homochiral** tensors. We do not expect large mixing.

Tensor mixing

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos\theta_T & \sin\theta_T \\ -\sin\theta_T & \cos\theta_T \end{pmatrix} \begin{pmatrix} f_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ f_{2,S} = \bar{s}s \end{pmatrix}$$

$$\theta_T \simeq 3.2^\circ$$

As expected, the mixing is very small.

A small mixing is also expected for the (yet unknown) chiral partners of tensor mesons.

Pseudovectors and orbitally excited vectors: Heterochiral vectors

$J^{PC}, {}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
$1^{+-}, {}^1P_1$	$\begin{cases} b_1(1235) \\ K_{1,B} \\ h_1(1170), h_1(1380) \end{cases}$	$P_\mu^{ij} = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi_\mu^{ij} = \bar{q}_{\mathrm{R}}^j \mathrm{i}\overleftrightarrow{D}_\mu q_{\mathrm{L}}^i)$	$\Phi_\mu \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_\mu U_{\mathrm{R}}^\dagger$
$1^{--}, {}^3D_1$	$\begin{cases} \rho(1700) \\ K^*(1680) \\ \omega(1650), \phi(?) \end{cases}$	$S_\mu^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$		

$$\boxed{\Phi_\mu \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_{\mathrm{L}}\, \Phi_\mu\, U_{\mathrm{R}}^\dagger}$$

The pseudovector mesons and the excited vector mesons form a **heterochiral** multiplet. We thus call them heterochiral vectors.
The chiral transformation is just as the (pseudo)scalar mesons (which is also hetero).
Hence, an anomalous Lagrangian is possible for heterochiral vectors.

Excited vector mesons: phi(1930) predicted to be the missing state, see
M. Piotrowska, C. Reisinger and FG.,
``Strong and radiative decays of excited vector mesons and predictions for a new phi(1930)$ resonance,'' Phys. Rev. D **96** (2017) no.5, 054033, arXiv:1708.02593 [hep-ph].

Anomalous Lagrangian for heterochiral vectors

$$\mathcal{L}_{\Phi_\mu}^{\text{anomaly}} = -\, b_{\text{A}}^{(1)}[\text{tr}(\Phi \times \Phi_\mu \cdot \Phi^\mu) + \text{c.c.}] - b_{\text{A}}^{(2)}[\text{tr}(\Phi \times \partial_\mu \Phi \cdot \Phi^\mu) + \text{c.c.}] - b_{\text{A}}^{(3)}[\text{tr}(\Phi \times \Phi \cdot \Phi_\mu) - \text{c.c.}]^2 + \ldots$$

$$(A \times B)^{ii'} = \frac{1}{3!}\epsilon^{ijk}\epsilon^{i'j'k'}A^{jj'}B^{kk'}$$

The first term contains objects as: $\varepsilon^{ijk}\varepsilon^{i'j'k'}\Phi^{ii'}\Phi_\mu^{jj'}\Phi_\mu^{kk'}$

So for the other terms. Such objects are SU(3)RxSU(3)L invariant but break U(1)A.

The first term generates mixing among both nonets (pseudovector and excited vector).

The second term generates decay into (pseudo)scalar states (interesting for future works).

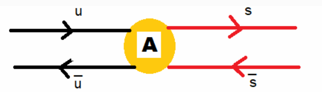
The third terms generates mixing for pseudovectors only.

Pseudovector mixing

Third term only (for simplicity)

$$\mathcal{L}_{\Phi_\mu}^{\text{anomaly}} = -\beta_A \left(\sqrt{2}h_{1,N} + h_{1,SS}\right)^2 + \ldots$$

$$\theta_{PV} \simeq -\frac{1}{2}\arctan\left[\frac{2\sqrt{2}\beta_A}{m_{K_{1,B}}^2 - m_{b_1(1235)}^2 - \beta_A}\right]$$



$$\begin{pmatrix} h_1(1170) \\ h_1(1380) \end{pmatrix} = \begin{pmatrix} \cos\theta_{PV} & \sin\theta_{PV} \\ -\sin\theta_{PV} & \cos\theta_{PV} \end{pmatrix} \begin{pmatrix} h_{1,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ h_{1,S} = \bar{s}s \end{pmatrix}$$

θ_{PV} can be large! (and negative...)

This is a prediction.
Experimental knowledge poor; it does not allow for a phenonemonological study yet.

The mixing of the chiral partners cannot be studies since phi(1930) state was not discovered yet.
Predictions more difficult: If the third term dominates, the mixing angle is small.
If the first term is sizable, a large mixing is also possible.

Pseudotensor mesons (and their chiral partners): heterochiral tensors

J^{PC}, ${}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar u d, \bar d u, \frac{\bar d d - \bar u u}{\sqrt 2}) \\ I=1 & (-\bar u s, \bar s u, \bar d s, \bar s d) \\ I=0 & (\frac{\bar u u + \bar d d}{\sqrt 2}, \bar s s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
2^{-+}, ${}^1D_2$	$\begin{cases} \pi_2(1670) \\ K_2(1770) \\ \eta_2(1645), \eta_2(1870) \end{cases}$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar q^j (\mathrm{i}\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \ldots) q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar q^j_{\mathrm{R}} (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \ldots) q^i_{\mathrm{L}})$	$\Phi_{\mu\nu} \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_{\mu\nu} U_{\mathrm{R}}^\dagger$
2^{++}, ${}^3F_2$	$\begin{cases} a_2(?) \\ K_2^*(?) \\ f_2(?), f_2'(?) \end{cases}$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar q^j (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \ldots) q^i$		

$$\boxed{\Phi_{\mu\nu} \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_{\mathrm{L}}\, \Phi_{\mu\nu}\, U_{\mathrm{R}}^\dagger}$$

Thus, we have heterochiral tensor states.
Transformation just as heterochiral scalars.
Mixing between strange-nonstrange possible.

Anomalous Lagrangian for heterochiral tensors

$$\boxed{\begin{aligned}\mathcal{L}^{\text{anomaly}}_{\Phi_{\mu\nu}} = &- c_{\text{A}}^{(1)}[\text{tr}(\Phi \times \Phi_{\mu\nu} \cdot \Phi^{\mu\nu}) + \text{c.c.}] \\ &- c_{\text{A}}^{(2)}[\text{tr}(\partial_\mu \Phi \times \partial_\nu \Phi \cdot \Phi^{\mu\nu}) + \text{c.c.}] \\ &- c_{\text{A}}^{(3)}[\text{tr}(\Phi \times \Phi \cdot \Phi_{\mu\nu}) - \text{c.c.}]^2 + \ldots\end{aligned}}$$

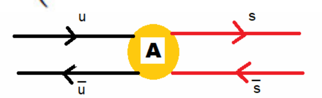
Again, the various terms are $SU(3)_R \times SU(3)_L$ invariant but break $U(1)_A$.

First term generates mixing for pseudotensors and also for their chiral partners.
Second term generates decays of pseudotensor (and partners) into (pseudo)scalars.
Third term generates mixing for pseudotensors only.

Pseudotensor mixing

Third term only

$$\mathcal{L}^{\text{anomaly}}_{\Phi_{\mu\nu}} = -\beta_A \left(\sqrt{2}h_{1,N} + h_{1,SS}\right)^2 + \dots$$



$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\theta_{PT} & \sin\theta_{PT} \\ -\sin\theta_{PT} & \cos\theta_{PT} \end{pmatrix} \begin{pmatrix} \eta_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} = \bar{s}s \end{pmatrix}$$

$$\theta_{PV} \simeq -\frac{1}{2}\arctan\left[\frac{2\sqrt{2}\beta_A}{m^2_{K_{1,B}} - m^2_{b_1(1235)} - \beta_A}\right]$$

According to the phenomenological study in
A. Koenigstein, F.G., Eur.Phys.J. A**52** (2016) no.12, 356, arXiv: 1608.8777:

$$\theta_{PT} \approx -40^0$$

Pseudotensor meson: suprising large mixing?

A. Koenigstein, F.G., Eur.Phys.J. A**52** (2016) no.12, 356, arXiv: 1608.8777

Phenomenology of pseudotensor mesons and the pseudotensor glueball

Abstract. We study the decays of the pseudotensor mesons ($\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1870)$) interpreted as the ground-state nonet of 1^1D_2 $\bar{q}q$ states using interaction Lagrangians which couple them to pseudoscalar, vector, and tensor mesons. While the decays of $\pi_2(1670)$ and $K_2(1770)$ can be well described, the decays of the isoscalar states $\eta_2(1645)$ and $\eta_2(1870)$ can be brought in agreement with the present experimental data only if the mixing angle between nonstrange and strange states is surprisingly large (about $-42°$, similar to the mixing in the pseudoscalar sector, in which the chiral anomaly is active). Such a large mixing angle is however at odd with all other conventional quark-antiquark nonets:

Anomaly in the baryonic sector

based on

L. Olbrich, M. Zetenyi, F.G., D.H. Rischke

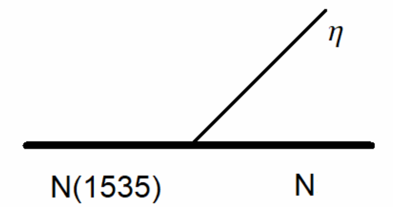
Influence of the axial anomaly on the decay $N(1535) \to N\eta$

ArXiv: 1708.01061, to appear in PRD

Violation of flavour symmetry in N(1535) decays?

Flavour symmetry predicts:

$$\frac{\Gamma_{N(1535)\to N\eta}}{\Gamma_{N(1535)\to N\pi}} \approx \frac{1}{3}\cos^2\theta_P \approx 0.17$$



This is in evident conflict with the experiment (see below).

Results for a simple flavour invariant approach

	Flavour symmetry [MeV]	Experiment [MeV]
$\Gamma_{N(1535)\to N\pi}$	67.5 ± 19	67.5 ± 19
$\Gamma_{N(1535)\to N\eta}$	$\mathbf{4.3 \pm {}^{1.3}_{1.1}}$	$\mathbf{40 - 91}$
$\Gamma_{\Lambda(1670)\to N\bar{K}}$	$6.0 \pm {}^{1.8}_{1.6}$	$5 - 15$
$\Gamma_{\Lambda(1670)\to \Sigma\pi}$	$21.3 \pm {}^{6.4}_{5.6}$	$6.25 - 27.5$
$\Gamma_{\Lambda(1670)\to \Lambda\eta}$	$\mathbf{0.6 \pm {}^{1.8}_{1.6}}$	$\mathbf{2.5 - 12.5}$

Decays of N(1650) and flavor symmetry: here, it is fine...

Results for a simple flavour invariant approach

	Flavour symmetry [MeV]	Experiment [MeV]
$\Gamma_{N(1650)\to N\pi}$	84 ± 23	84 ± 23
$\Gamma_{N(1650)\to N\eta}$	$8.7 \pm ^{2.6}_{2.2}$	$15.4 - 37.5$
$\Gamma_{N(1650)\to \Lambda K}$	$13.2 \pm ^{3.9}_{3.4}$	$5.5 - 25.5$
$\Gamma_{\Lambda(1800)\to \Lambda\eta}$	$3.09 \pm ^{0.91}_{0.79}$	$2 - 44$

Proposed explanations for N(1535)

Dynamical generation through KΛ and KΣ channels

N. Kaiser, P. B. Siegel and W. Weise, Phys. Lett. B **362** (1995) 23 [nucl-th/9507036].
T. Inoue, E. Oset and M. J. Vicente Vacas, Phys. Rev. C **65** (2002) 035204 [hep-ph/0110333].
P. C. Bruns, M. Mai and U. G. Meissner, Phys. Lett. B **697** (2011) 254 [arXiv:1012.2233 [nucl-th]].
E. J. Garzon and E. Oset, Phys. Rev. C **91** (2015) no.2, 025201 [arXiv:1411.3547 [hep-ph]].
C. S. An and B. S. Zou, Sci. Sin. G **52** (2009) 1452 [arXiv:0910.4452 [nucl-th]].
B. C. Liu and B. S. Zou, Phys. Rev. Lett. **96**, 042002 (2006) [nucl-th/0503069].
X. Cao, J. J. Xie, B. S. Zou and H. S. Xu, Phys. Rev. C **80** (2009) 025203 [arXiv:0905.0260 [nucl-th]].

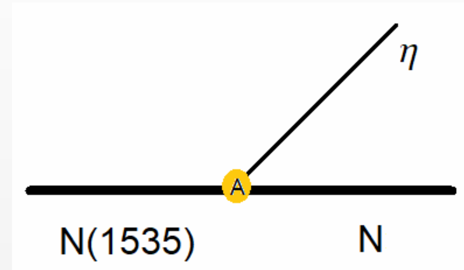
Basically, an sbar-s component is present in N(1535) and explains the coupling to η

New idea: Axial anomaly and N(1535)

There is a simple explanation for an enhanced copling of N(1535) to Nη: the anomaly. Namely, one can write (in the mirror assignment) an anomalous term which couples the nucleon and its chiral partner to the η.

$$\boxed{\mathcal{L}_A^{N_f=2} = \lambda_A^{N_f=2}(\det\Phi - \det\Phi^\dagger)(\bar{\Psi}_2\Psi_1 - \bar{\Psi}_1\Psi_2)}$$

$$\det\Phi - \det\Phi^\dagger = -i\left[(\sigma_N + \phi_N)\eta_N - \boldsymbol{a_0}\cdot\boldsymbol{\pi}\right]$$



In turn, the enhanced N(1535) decay into Nη favors its interpretation as chiral partner of N

Recall that:

$$\begin{pmatrix} N \\ N_* \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}}\begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix}\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\cosh\delta = \frac{m_N + m_{N_*}}{2m_0}$$

$$m_0 = (459 \pm 117)\ \mathrm{MeV}$$

Extension to $N_f = 3$

This is a lot of technical work, see the paper. Yet, the basic idea is the same. A term involving the axial anomaly is possible.

$$\boxed{\mathcal{L}_A^{N_f=3} = \lambda_A^{N_f=3}(\det\Phi - \det\Phi^\dagger)\mathrm{Tr}(\bar{B}_{M_*}B_N - \bar{B}_N B_{M_*} - \bar{B}_{N_*}B_M + \bar{B}_M B_{N_*})}$$

Consequences:

One can understand the enhanced decay $N(1535) \to N\eta$

Further predictions possible, e.g.:
(Experimentally between 2.5 and 12.5 MeV)

$$\Gamma_{\Lambda(1670)\to\Lambda\eta} = (5.1 \pm {}^{2.7}_{2.1})\ \mathrm{MeV}$$

Enhanced coupling to η' follows: $g_{\eta' N N_*} \simeq 3.7$

Study of $pn \to pn\eta'$
delivered $g_{\eta' N N_*} \simeq 4.1$

X. Cao and X. G. Lee, Phys. Rev. C **78** (2008) 035207 [arXiv:0804.0656 [nucl-th]].

Pseudoscalar glueball

based on

L. Olbrich, M. Zetenyi, F.G., D.H. Rischke
ArXiv: 1708.01061

W. Eshraim, S. Janowski, F.G., D.H. Rischke

Phys.Rev. D**87** (2013) no.5, 054036
ArXiv: 1208.6474

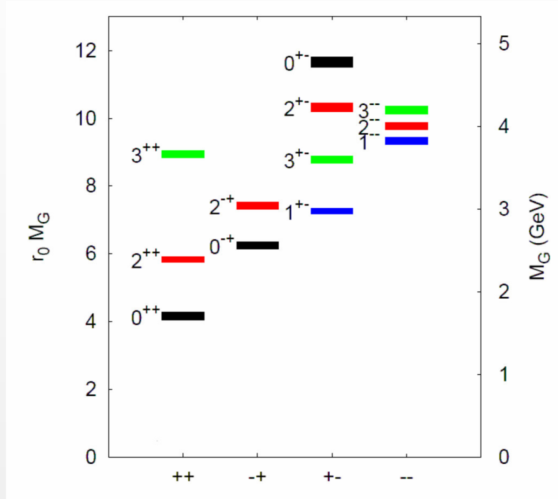
The pseudoscalar glueball and the anomaly

$$\mathcal{L}^{int}_{\tilde{G}\text{-mesons}} = ic_{\tilde{G}\Phi}\tilde{G}\left(\det\Phi - \det\Phi^{\dagger}\right)$$

Quantity	Value
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to\eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to\eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to\eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094

$$\boxed{\Gamma_{\tilde{G}\to\pi\pi\pi} = 0}$$

MG = 2.6 GeV from lattice as been used as an input.

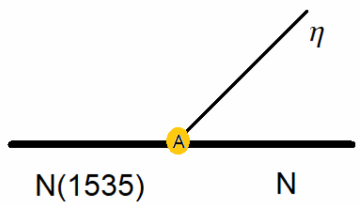


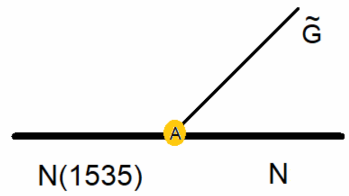
X(2370) found at BESIII is a possible candidate.

Future experimental search, e.g. at BESIII, GlueX, CLAS12, and PANDA.

W. Eshraim, S. Janowski, F.G., D.H. Rischke, Phys.Rev. D**87** (2013) no.5, 054036, ArXiv: 1208.6474
Excited pseudoscalar glueball: W. I. Eshraim and S. Schramm, Phys. Rev. D**95** (2017) no.1, 014028 ArXiv:1606.02207.

Coupling of the pseudoscalar glueball to baryons





$$\mathcal{L}_A^{N_f=2} = \lambda_A^{N_f=2}(\det\Phi - \det\Phi^\dagger)(\bar{\Psi}_2\Psi_1 - \bar{\Psi}_1\Psi_2) \Rightarrow \boxed{\mathcal{L}_{\tilde{G}\Phi}^{N_f=2} = i g_{\tilde{G}\Phi}^{N_f=2}\tilde{G}(\bar{\Psi}_2\Psi_1 - \bar{\Psi}_1\Psi_2)}$$

Strong coupling of the pseudoscalar glueball to NN and also N(1535)N

$$\frac{\Gamma_{\tilde{G}\to\bar{N}N}}{\Gamma_{\tilde{G}\to\bar{N}_*N+\text{ h.c.}}} \simeq 1.96$$

A promising reaction to search the pseudoscalar glueball is:

$$p + \bar{p} \to p + \bar{p}(1535) + \text{ h.c.}$$

See also:

L Olbrich, M. Zetenyi, F.G., D. H. Rischke, arXiv: 1708.01061

W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., Acta Phys. Pol. B, Prc. Suppl. 5/4, arXiv: 1209.3976

Concluding remarks

- Concept of homochirality and heterochirality.
- For heterochiral multiplets an axial-anomalous strange-nonstrange mixing is possible.

  (η-η', but also $\eta_2(1645)$-$\eta_2(1870)$ and evt h1 states)
- For homochiral multiplets no anomalous mixing.

  (ω-phi(1020), $f_2(1270)$-$f_2'(1525)$,..., are nonstrange and strange, resp.)
- Baryons: anomalous coupling of N(1535) to Nη

  Consequences: decay Λ(1670) into Λη and N(1535)Nη' coupling
- Pseudoscalar glueball: anomalous coupling to mesons and baryons.
- Outlook: anomalous decays, detailed study of mixing, anomaly and nucleon-nucleon interation,...

  (Looking forward for exp. results BESIII, Compass, GlueX, CLAS12, PANDA, ...)

Thanks