Quarks and pions at finite chemical potential

A study of the QCD phase diagram with Dyson-Schwinger equations

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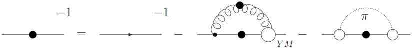
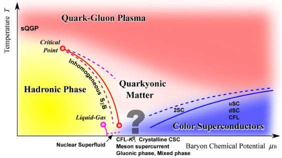
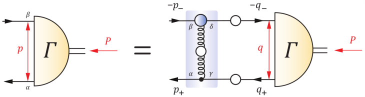
Lunch-club, June 7, 2017

Outline

1. Introduction and Motivation
2. QCD phase diagram
3. Quarks in cold dense matter
4. Pions in cold dense matter
5. Summary and Outlook







QCD phase diagram

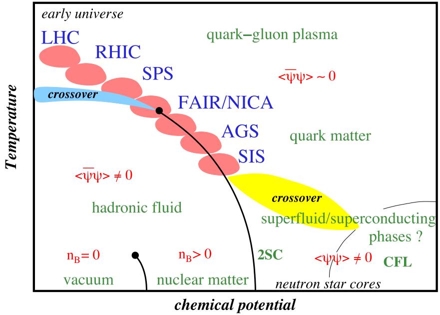


Figure: Schaefer and Wagner, Prog.Part.Nucl.Phys. 62 (2009) 381

Abbreviations:

QGP	Quark-Gluon-Plasma
CEP	Critical end point
SC	Super conducting (phase)
CFL	Color-flavor-locking (phase)
CDM	Cold dense matter
DS	Dyson-Schwinger

Previous Studies

Inclusion of quark flavor effects in the QCD phase diagram:

Luecker and Fischer, Prog.Part.Nucl.Phys. 67(2), 200–205 (2012)
Fischer, Luecker and Welzbacher, Phys.Rev. D 90(3), 34022 (2014)

Influence of baryonic effects in two-color-version of lattice QCD:

Strodthoff, Schaefer and von Smekal, Phys.Rev. D 85, 074007 (2012)

Baryon effects on the location of QCD's CEP in DS approach:

Eichmann, Fischer and Welzbacher, Phys.Rev. D 93, 034013 (2016)

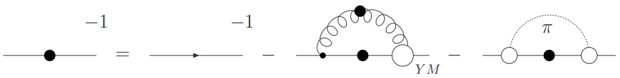
Mesonic back-coupling effects in vacuum and finite T in DS approach:

Fischer, Nickel and Wambach Phys.Rev. D 76, 094009 (2007)
Fischer and Williams Phys.Rev. D 78, 074006 (2008)
Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

Motivation

Open question: Influence of mesonic back-coupling effects onto QCD phase diagram and CEP

- Extend previous calculation to finite chemical potential



Goal of thesis: Reproduction of QCD phase diagram
Enable quark calculation in CDM
Investigate pion in CDM

Origin of Dyson-Schwinger equations

Generating functional:

$$Z[J] = \mathcal{N} \int \mathcal{D}\varphi e^{i(S[\varphi] - \int_x \varphi(x) J(x))}$$

Local translational invariance: $\varphi(x) \to \varphi'(x) = \varphi(x) + \epsilon(x)$

Master-DSE for 1PI Green-functions:

$$\Gamma'_x(\tilde{\varphi}) = \frac{\delta S}{\delta \varphi(x)} \left(\tilde{\varphi} + \int_y \Delta_{.y}[\tilde{\varphi}] \frac{i\delta}{\delta \tilde{\varphi}(y)} \right)$$

Meaning of DSE: Quantum equations of motion for n-point function
Infinite tower of coupled integral equations
Ab initio, if solved completely and self-consistently

Connection to thermodynamics

Grand canonical partition function:

$$Z_{GC}(\beta,\mu_q) = tr\left(e^{-\beta(\hat{H}-\mu_q\hat{N})}\right) = \int_x \langle x|\, e^{-\beta(\hat{H}-\mu_q\hat{N})}\,|x\rangle$$

Particle number operator: $\hat{N} = \int d^4x \hat{\bar{\Psi}}\gamma_0\hat{\Psi}$

Generating functional for finite space-time:

$$Z(x',\tau';x,\tau) = \langle x'|\, e^{-\hat{H}(\tau'-\tau)}\,|x\rangle = \mathcal{N}\int_{x(\tau)}^{x'(\tau')} \mathscr{D}x(\tau'')e^{-S_E(\tau,\tau')}$$

In-medium generating functional:

$$Z_{GC}(\beta,\mu_q) = \mathcal{N}\int_{x(0)=x(\beta)} \mathscr{D}x(\tau)e^{-S_E(0,\beta)+\mu_q\int_0^\beta d\tau\int d^3x\hat{\bar{\Psi}}\gamma_0\hat{\Psi}}$$

Connection to thermodynamics

Finite integration interval and different periodicity conditions

$$\begin{aligned} \Psi(x,\tau) &= -\Psi(x,\tau+\beta) \qquad \text{fermions} \\ \Phi(x,\tau) &= +\Phi(x,\tau+\beta) \qquad \text{bosons} \end{aligned}$$

yield discreet four momentum components

$$p \rightarrow (\vec{p}, \tilde{\omega}_p = \pi T(2n_p + \eta) + i\mu_q), \qquad \eta = \begin{cases} 1 & \text{fermions} \\ 0 & \text{bosons} \end{cases}$$

and a sum over the so called Matsubara-frequencies

$$\int \frac{d^4q}{(2\pi)^4} \rightarrow T \sum_{n_q} \int \frac{d^3q}{(2\pi)^3}$$

Quark DSE

Graphically:

Mathematically:

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F Z_{1F} \int_q \gamma_\epsilon S(q) \Gamma_\nu(p,q,k) D_{\epsilon\nu}(k)$$

Missing components: Bare S_0 and dressed S quark propagator
Dressed Γ_ν quark-gluon vertex
Dressed gluon propagator $D_{\epsilon\nu}$

Momentum routing: $k = q - p$

Quark tensor structure

Vacuum:

Bare propagator

$S_0^{-1}(p) = Z_2(i\not{p} + \mathbb{1} Z_m m_r)$

Dressed propagator

$S^{-1}(p) = i\not{p}A(p) + \mathbb{1}B(p)$

Medium:

Bare propagator

$S_0^{-1}(p) = Z_2(i\vec{p}\vec{\gamma} + i\tilde{\omega}_p\gamma_4 + \mathbb{1} Z_m m_r)$

Dressed propagator

$S^{-1}(p) =$
$i\vec{p}\vec{\gamma}A(\omega_p, \vec{p}) + i\tilde{\omega}_p\gamma_4 C(\omega_p, \vec{p}) + \mathbb{1}B(\omega_p, \vec{p})$

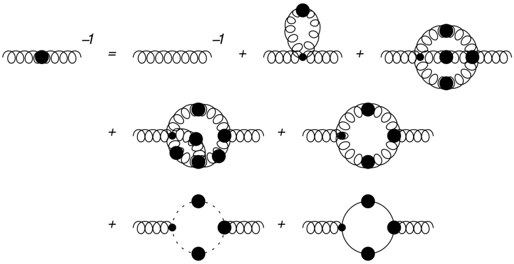
Scalar dressing function B

Vector dressing function A splits up in spatial A and heat bath C part

Comment: $\exists$ D term belonging to tensor structure $\vec{p}\vec{\gamma}\tilde{\omega}_p\gamma_4$ without counterpart in bare propagator $\longrightarrow$ small influence

Gluon DSE

Graphically:



Mathematically:

Momentum routing: $p = q - k$

$$D^{-1}_{\epsilon\nu}(k) = D^{-1}_{0,\epsilon\nu}(k) + \Pi^{YM}_{\epsilon\nu}(k) + \Pi^{QL}_{\epsilon\nu}(k)$$

$$\Pi^{QL}_{\epsilon\nu}(k) = -\frac{g^2 Z_{1F}}{2} \sum_f^{N_f} \int_q tr_D \left[\gamma_\epsilon S(q) \Gamma_\nu(p, q, k) S(p)\right]$$

Gluon tensor structure

Dressed Propagators:

Vacuum

$$D_{\epsilon\nu}(k) = P^{\mathscr{T}}_{\epsilon\nu}(k)\frac{Z(k)}{k^2}$$

Medium

$$D_{\epsilon\nu}(k;T) = \left(P^{T}_{\epsilon\nu}(k)\frac{Z_T(k;T)}{k^2} + P^{L}_{\epsilon\nu}(k)\frac{Z_L(k;T)}{k^2}\right)$$

Projectors

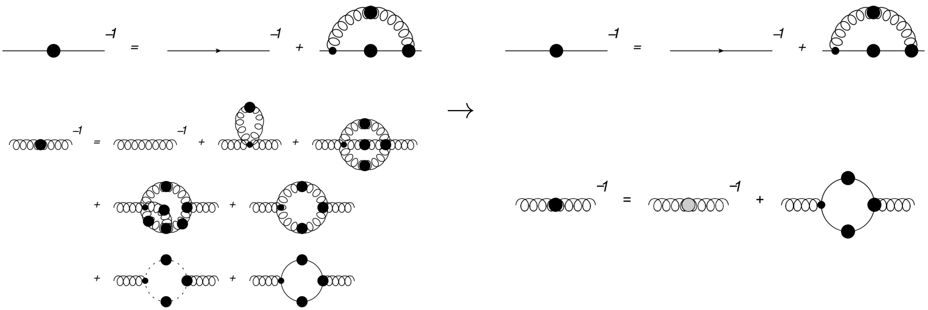
$$P^{T}_{\epsilon\nu}(k) = \left(\delta_{\epsilon\nu} - \frac{k_\epsilon k_\nu}{\vec{k}^2}\right)(1-\delta_{\epsilon 4})(1-\delta_{\nu 4})$$
$$P^{L}_{\epsilon\nu}(k) = P^{\mathscr{T}}_{\epsilon\nu}(k) - P^{T}_{\epsilon\nu}(k)$$
$$P^{\mathscr{T}}_{\epsilon\nu}(k) = \left(\delta_{\epsilon\nu} - \frac{k_\epsilon k_\nu}{k^2}\right)$$

Gluon splits up into a part transversal and a part longitudinal to heat bath

Heat bath: Fourth momentum/ Matsubara component

Truncation scheme one

Gluon truncation:

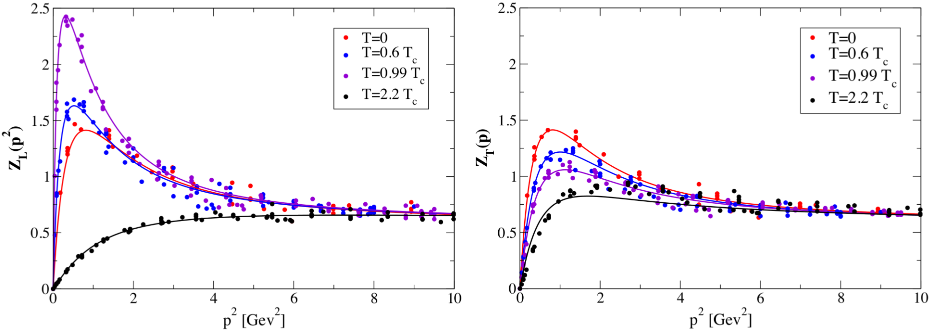


$$D^{-1}_{\epsilon\nu}(k) = \left[D^{\text{lat. que}}_{\epsilon\nu}(k)\right]^{-1} + \Pi^{QL}_{\epsilon\nu}(k)$$

Fischer, Maas and Müller, Eur.Phys.J. C (2010) 68: 165

Truncation scheme one

T-dependent gluon propagator from quenched lattice simulations:



Crucial difference between transversal and longitudinal gluon

Cucchieri, Maas, Mendes, PRD 75 (2007)
CF, Maas, Mueller, EPJC 68 (2010)
Aouane et al., PRD 85 (2012) 034501
Cucchieri, Mendes, PoS FACESQCD 007 (2010)
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503
FRG: Fister, Pawlowski, arXiv:1112.5440

Truncation scheme one

Vertex truncation: STI and perturbative behavior at large momenta constrain vertex

$$\Gamma^f_\nu(p,q,k) = \gamma_\nu \Gamma(k^2)(\delta_{\nu,s}\Sigma_A + \delta_{\nu,4}\Sigma_C)$$
$$\Gamma(k^2) = \frac{d_1}{d_2 + k^2} + \frac{k^2}{\Lambda^2 + k^2}\left(\frac{\beta_0 \alpha(\mu'') \ln[k^2/\Lambda^2 + 1]}{4\pi}\right)^{2\delta}$$

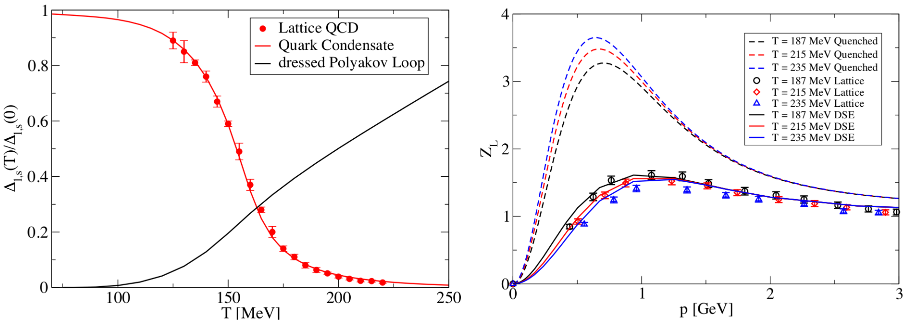
Considers first Ball-Chiu structure: $\Sigma_X = \frac{X(\vec{p}^2,\omega_p)+X(\vec{q}^2,\omega_q)}{2}$

Abelian WTI: from approximated STI
Perturbation theory
Infrared ansatz: d_2 fixed to match gluon input, d_1 fixed via quark condensate

Fischer and Mueller Phys.Rev. D 80, 074029 (2009)

Truncation scheme one

Determination of d_1 and prediction for unquenched gluon:

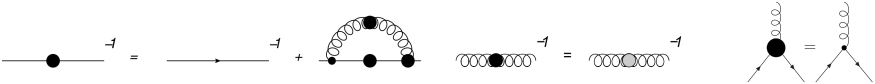


Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

Quantitative agreement: DSE prediction verified by lattice

Truncation scheme two



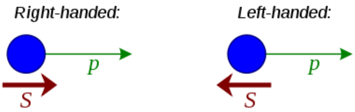
Combining vertex Γ and gluon Z to renormalization-group invariant effective coupling

$$\alpha(\mu) D_{\epsilon\nu}(k) \Gamma^{f}_{\nu}(p, q, k) \propto \alpha(k^2) \frac{P^{\mathcal{T}}_{\epsilon\nu}(k)}{k^2} \gamma_\nu$$

Maris-Tandy ansatz: Simple ansatz, quark flavor decouple

Maris and Tandy, Phys.Rev. C 60, 055214 (1999)

Order parameter



Chirality: Separate transformation of left- and right-handed quarks under chiral sym. group $SU_R(N_f) \times SU_L(N_f)$

Transition property for exchange between left and right handed particles:

$$\left\langle \bar{\Psi}\Psi \right\rangle^f = \left\langle \Psi_L^\dagger \Psi_R \right\rangle^f + \left\langle \Psi_R^\dagger \Psi_L \right\rangle^f$$

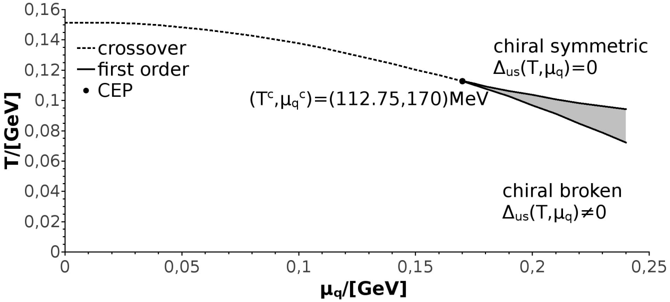
Quark condensate

$$\left\langle \bar{\Psi}\Psi \right\rangle^f = -Z_m Z_2 \int_q tr_{DC} \left[S^f(p) \right]$$

Regularized quark condensate

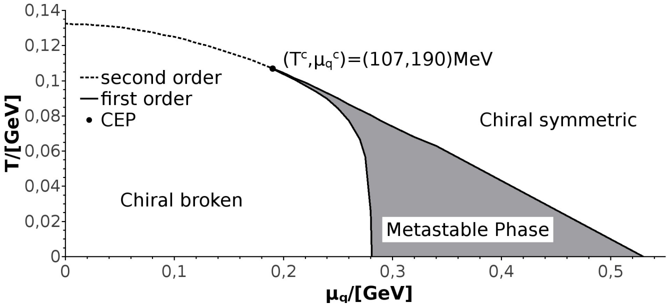
$$\Delta_{f'f} = \left\langle \bar{\Psi}\Psi \right\rangle^{f'} - \frac{m_B^{f'}}{m_B^f} \left\langle \bar{\Psi}\Psi \right\rangle^f$$

Phase diagram truncation one



- Result for unquenched calculation with $N_f = 2 + 1$ quark flavors and lattice fits for the quenched gluon
- Reproduction of results from: Fischer, Luecker and Welzbacher, Phys.Rev. D 90(3) 34022 (2014)
- Small deviations due to new regularization scheme in calculation

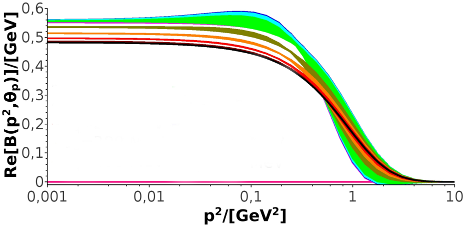
Phase diagram truncation two



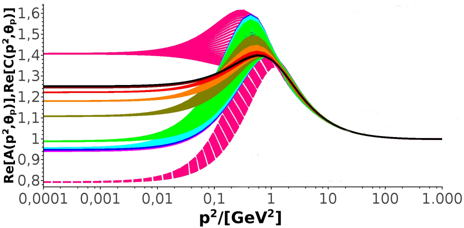
- Quenched calculation, effective model for interaction, chiral limit
- Reproduction of results from: Qin, Chang, Chen, Liu and Roberts, Phys.Rev.Lett. 106, 172301 (2011)
- different CEP due to different order parameter, good agreement for coexistence curves

Results for quark propagator in CDM

Scalar dressing function:



Vector dressing functions:



μ_q=0 MeV	μ_q=500 MeV
μ_q=100 MeV	μ_q=520 MeV
μ_q=200 MeV	μ_q=525 MeV
μ_q=300 MeV	μ_q=529 MeV
μ_q=400 MeV	μ_q=530 MeV

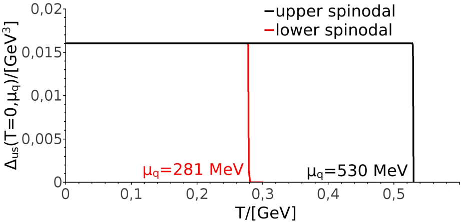
$$S^{-1}(p) = i\vec{p}\vec{\gamma}A(p^2,\theta) + i\tilde{p}_4\gamma_4 C(p^2,\theta) + \mathbb{1}B(p^2,\theta)$$

$A(p^2,\theta)$: solid lines $\qquad$ $C(p^2,\theta)$: dashed lines

degeneration of vector dressing function only in chiral limit

Silverblaze property

Reg. quark condensate in CDM:



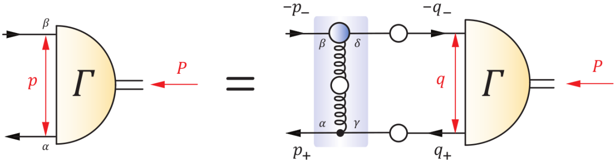
Silverblaze property: In CDM the partition function and observables are independent from chemical potential μ_q, if $\mu_q <$ mass gap of the system Δ.

$$\Delta = \begin{cases} \frac{m_B}{3} & m_B = \text{lightest baryon} \\ \frac{m_M}{2} & m_M = \text{lightest meson} \end{cases}$$

$$\langle \bar{\Psi}\Psi \rangle \sim \int_q S(\vec{q}, q_4 + i\mu_q) \overset{q_4 \to q_4 + i\mu_q}{=} \int_q S(\vec{q}, q_4) \sim \langle \bar{\Psi}\Psi \rangle_{vac}$$

Substitution possible $\Leftrightarrow$ no singularity between 0 and $i\mu_q$ in complex-p_4-plane

Homogeneous BSE



In Rainbow-ladder approximation with effective interaction $\alpha(k^2)$:

$$\Gamma_\pi(P,p) = -4\pi Z_2^2 C_F \int_q \frac{\alpha(k^2)}{k^2} P^{\mathscr{T}}_{\mu\nu}(k)\gamma^\mu S(q_+)\Gamma_\pi(P,q)S(-q_-)\gamma^\nu$$

Eigenvalue equation: $\lambda(P^2)\,|\Gamma(P,p)\rangle = K(P,p)\,|\Gamma(P,p)\rangle$

On-shell condition $P^2 = -M_j^2 \quad \longrightarrow \quad \lambda(P^2) = 1$

Pion amplitude properties

Pion amplitude

$$\Gamma_\pi(P,p) = \gamma_5 \left[-iE(P,p) + \not{P} F(P,p) + \not{p}(Pp) G(P,p) + \left[\not{P}, \not{p}\right] H(P,p) \right]$$

Charge-conjugated pion amplitude:

$$\bar{\Gamma}_\pi(P,p) = \left[C\Gamma_\pi(P,-p)C^{-1} \right]^T$$

C-Parity: $\bar{\Gamma}_\pi(P,p) = \Gamma_\pi(P,p)$

In following: Only consider first tensor-structure

Pion decay constant

Pion decay constant

$$f_\pi P^\mu = Z_2 N_c \int_q tr_D \left[\gamma_5 \gamma^\mu S(q_+) \Gamma_\pi(q,P) S(-q_-)\right]$$

From vacuum to CDM:

$$f_\pi P^\mu \xrightarrow{\mu_q > 0} \left(f_\pi^t P_{\mu\nu}^{\mathscr{L}}(v) + f_\pi^s P_{\mu\nu}^{\mathscr{T}}(v) \right) P^\nu$$

Longitudinal projector $P_{\mu\nu}^{\mathscr{L}}(v) = \frac{v_\mu v_\nu}{v^2}$ with $v = (\vec{0}, 1)$

Present parametrization $P = (\ \vec{0}\ ,\ im_\pi\ ) \rightarrow f_\pi^s = 0$

Pion propagator

Pion velocity:

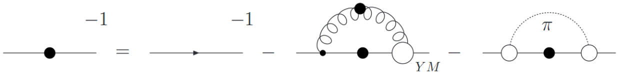
$$u^2 = \left(\frac{f_\pi^s}{f_\pi^t}\right)^2$$

Pion dispersion relation:

$$\omega^2 = u^2\left(\vec{P}^2 + m_\pi^2\right)$$

Pion propagator:

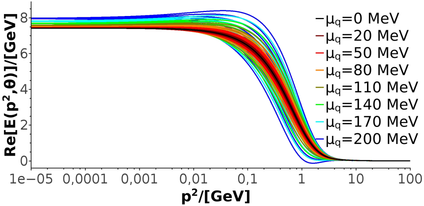
$$D_\pi(P) = \frac{1}{P_4^2 + u^2\left(\vec{P}^2 + m_\pi^2\right)}$$

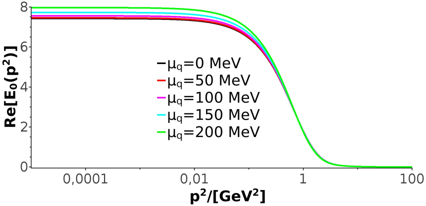


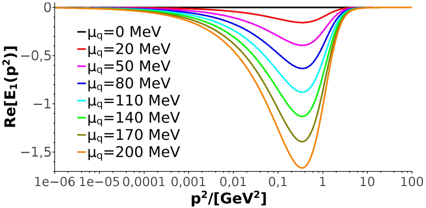
Vacuum: $f_\pi^t \overset{\mu_q \to 0}{=} f_\pi^s \Rightarrow u \to 1$

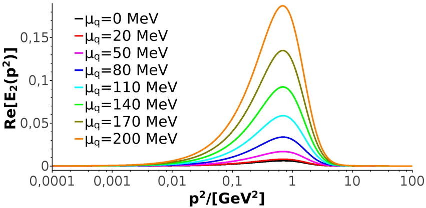
Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

Pion amplitude



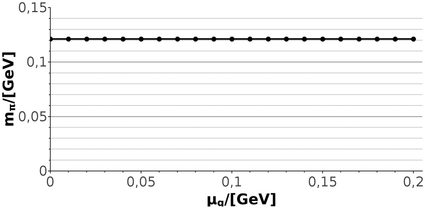


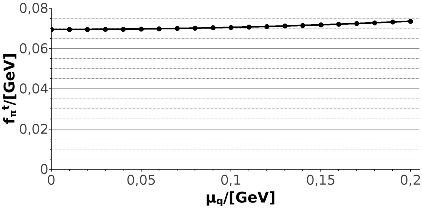




Pion properties

For one tensor structure with quark mass $m_R = 3.7$ MeV at $\mu = 19\,\text{GeV}$:



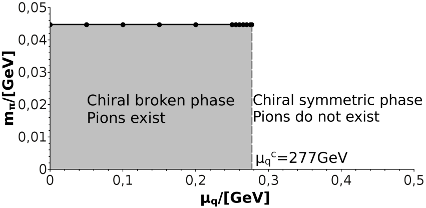


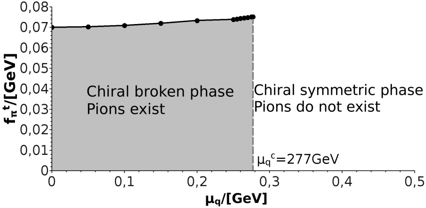
Full tensor structure and quark mass $m_R(\mu = 19\,\text{GeV}) = 3.7$ MeV:
$m_\pi(\mu_q = 0) = 137.4$ MeV and $f_\pi(\mu_q = 0) = 92$ MeV
Source: E.D. Weil, Master thesis (2016)

Silverblaze property fulfilled: mass gap $= \frac{m_M}{2} = 60.5$ MeV

Parametrization $\rightarrow$ only f_π^t

Pion properties





$\mu_q^c = 277$ MeV connected with confinement
But: Corresponds also to phase transition point of chiral condensate

Accordance with previous calculations using different truncations:

Roberts, Phys.Part.Nucl. 30:223-257 (1999)
Roberts and Schmidt, Prog.Part.Nucl.Phys. 45(1) 1-103 (2000)
Liu, Chao, Chang and Wei, Chinese Physics Letters, Volume 22, Number 1

Summary

- Reproduction of QCD phase diagram for (a) $N_f = 2 + 1$ quark flavors with lattice fits and (b) quenched case in chiral limit with effective interaction
  (a) Small variation due to new regularization scheme
  (b) Different CEP due to different order parameter, good agreement of coexistence curves

- Enable quark calculation for CDM for truncation with effective interaction
  - Vacuum and medium limit fulfilled
  - Quark condensate fulfills Silverblaze property

Summary

- Enable pion calculation for CDM for one tensor structure and truncation with effective interaction
  - Vacuum limit fulfilled
  - Pion mass and decay constant fulfill Silverblaze property and match with previous works
  - Pion phase transition at $\mu_q^c = 277$ MeV (corresponds to phase transition point of chiral symmetry)

Outlook

- Extend to full pion tensor structure in vacuum and CDM
- Enable quarks and pions in CDM for unquenched theory with lattice fits
    - Necessary: Quarks in complex plane during iteration
- Test another parametrization for pion calculation
- Enable pion for finite temperature
- Enable pion back-coupling onto the quark
- Investigate pionic back-coupling effects on QCD phase diagram

Any questions?

Thank you for your attention!

Lattice fit functions

Gluon:

$$Z_{T,L}^{\text{lat que}}(k^2) = \frac{x}{(x+1)^2}\left[\left(\frac{\hat{c}}{x + a_{T,L}(T)}\right)^{b_{T,L}(T)} + x\left(\frac{\beta_0 \alpha}{4\pi}\ln(1+x)\right)^{\gamma}\right]$$

Parameters: $x = \frac{k^2}{\Lambda^2}$, $\beta_0 = \frac{11N_c - 2N_f}{3}$, $\hat{c} = 5.87$, $\Lambda = 1.4\,\text{GeV}$, $\gamma = -\frac{13}{22}$, $\alpha(\mu'') = \frac{g^2}{4\pi} = 0.3$

$$a_L(t) = \begin{cases} 0.595 - 0.9025 \cdot t + 0.4005 \cdot t^2 & \text{if } t < 1 \\ 3.6199 \cdot t - 3.4835 & \text{if } t > 1 \end{cases},$$

$$a_T(t) = \begin{cases} 0.595 + 1.1010 \cdot t^2 & \text{if } t < 1 \\ 0.8505 \cdot t - 0.2965 & \text{if } t > 1 \end{cases}$$

$t = \frac{T}{T_c}$, $T_c = 277\,\text{MeV}$

Lattice fit functions

Gluon:

$$Z_{T,L}^{\text{lat que}}(k^2) = \frac{x}{(x+1)^2}\left[\left(\frac{\hat{c}}{x+a_{T,L}(T)}\right)^{b_{T,L}(T)} + x\left(\frac{\beta_0\alpha}{4\pi}\ln(1+x)\right)^{\gamma}\right]$$

Parameters: $x = \frac{k^2}{\Lambda^2}$, $\beta_0 = \frac{11N_c - 2N_f}{3}$, $\hat{c} = 5.87$, $\Lambda = 1.4\,\text{GeV}$, $\gamma = -\frac{13}{22}$, $\alpha(\mu'') = \frac{g^2}{4\pi} = 0.3$

$$b_L(t) = \begin{cases} 1.355 - 0.5741 \cdot t + 0.3287 \cdot t^2 & \text{if } t < 1 \\ 0.1131 \cdot t + 0.9319 & \text{if } t > 1 \end{cases},$$

$$b_T(t) = \begin{cases} 1.355 + 0.5548 \cdot t^2 & \text{if } t < 1 \\ 0.4296 \cdot t + 0.7103 & \text{if } t > 1 \end{cases}$$

$t = \frac{T}{T_c}$, $T_c = 277\,\text{MeV}$

Lattice fit functions

Vertex:

$$
\begin{aligned}
\Gamma^f_\nu(p,q,k) =& \gamma_\nu \Gamma(x) \left(\delta_{\nu,s}\Sigma_A + \delta_{\nu,4}\Sigma_C\right) \\
\Gamma(x) =& \frac{d1}{d2 + k^2} + \frac{x}{1+x}\left(\frac{\beta_0 \alpha(\mu'') \ln[x+1]}{4\pi}\right)^{2\delta} \\
\Sigma_X =& \frac{X(\vec{p}^2,\omega_p) + X(\vec{q}^2,\omega_q)}{2}
\end{aligned}
$$

Parameters: $x = \frac{k^2}{\Lambda^2}$, $\beta_0 = \frac{11N_c - 2N_f}{3}$, $\alpha(\mu'') = \frac{g^2}{4\pi} = 0.3$, $\delta = \frac{-9N_c}{44N_c - 8N_f}$,
$\Lambda = 1.4\,\text{GeV}$, $d_2 = 0.5\,\text{GeV}^2$,

$$
d_1 = \begin{cases} 4.6 & \text{quenched theory} \\ 8.05 & \text{unquenched theory with } N_f = 2+1 \text{ quark flavors} \end{cases}
$$

Thermal mass

Regularized quark loop:

$$\Pi^{reg}_{\epsilon\nu}(k) = \left[\delta_{\epsilon\alpha}\delta_{\nu\beta} - \delta_{\epsilon\nu}P^{\mathscr{L}}_{\alpha\beta}(k)\right]\Pi^{QL}_{\alpha\beta}(k)$$

$$\Pi^{reg}_{T/L}(k) = \frac{1}{2/1k^2}\Pi^{reg}_{\epsilon\nu}(k)P^{T/L}_{\epsilon\nu}(k)$$

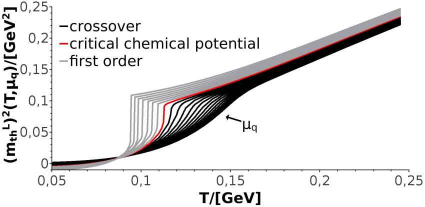
Separation of regular part and thermal mass:

$$\Pi^{reg}_{T/L}(k) = \Pi^{regular}_{T/L}(k) + \frac{2\left[m^{th}_{T/L}(T,\mu_q)\right]^2}{k^2}$$

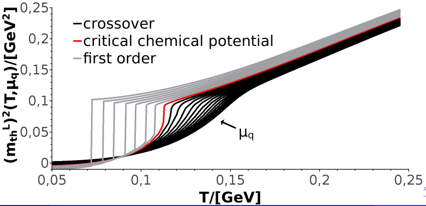
$$\left[m^{th}_{T/L}(T,\mu_q)\right]^2 := \frac{1}{2}\Pi^{reg}_{T/L}(k)\vec{k}^2\Big|_{\omega_k=0,\vec{k}^2\longrightarrow 0}$$

Thermal mass

Upper spinodal:



Lower spinodal:

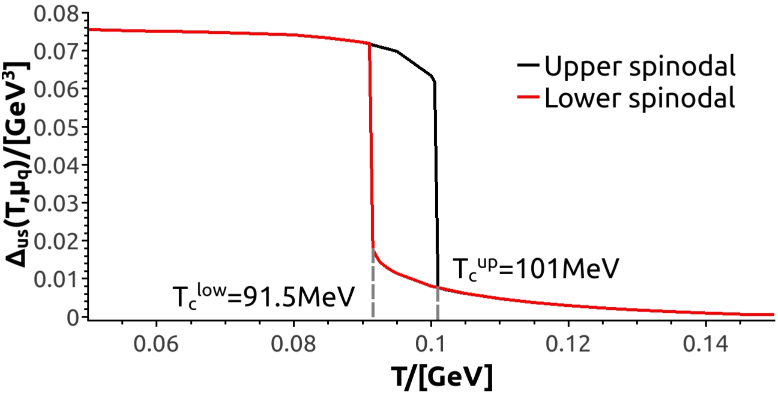


Maris-Tandy interaction

$$\alpha(k^2) = \pi \frac{\eta^7}{\Lambda^4} k^4 e^{-\eta^2 \frac{k^2}{\Lambda^2}} + \frac{2\pi\gamma_m \left(1 - e^{-k^2/\Lambda_t^2}\right)}{\ln\left[e^2 - 1 + \left(1 + k^2/\Lambda_{QCD}^2\right)^2\right]}$$

Parameters: $\gamma_m = \frac{12}{11N_c - 2N_f}$, $\Lambda_t = 1\,\text{GeV}$, $\Lambda_{QCD} = 0.234\,\text{GeV}$

Spinodals


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
$\Delta_{us}(T,\mu_q)/[GeV^3]$
0.06
0.08
0.1
0.12
0.14
T/[GeV]
Upper spinodal
Lower spinodal
T_c^{up}=101MeV
T_c^{low}=91.5MeV

Calculation method

Until now only truncation scheme two (effective interaction)

Changes from medium to CDM:

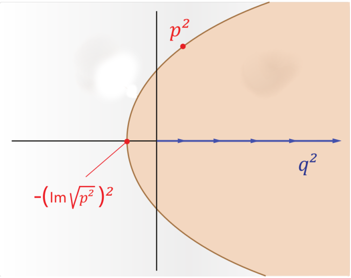
- $(\vec{p}, \tilde{\omega}_p = \omega_p + i\mu_q) \longrightarrow (\vec{p}, \tilde{p}_4 = p_4 + i\mu_q)$
- $\frac{T}{(2\pi)^3} \sum_{\omega_q} \int d\vec{q}^2 \int d\Omega_{3D} \rightarrow \begin{cases} \frac{1}{(2\pi)^4} \int dq_4 \int d\vec{q}^2 \int d\Omega_{3D} & \text{Method A} \\ \frac{1}{(2\pi)^4} \int dq^2 \int d\Omega_{4D} & \text{Method B} \end{cases}$

Method A: Separate integration for spacial and temporal part
Vacuum and medium limit does not work
Method B: Uses hyperspherical coordinates instead
Vacuum and medium limit work perfectly

Complex quark

$$S^{-1}(p \pm i\frac{m_\pi}{2}) = S_0^{-1}(p \pm i\frac{m_\pi}{2}) + Z_2^2 C_F \int_q \gamma_\epsilon S(q) \gamma_\nu P_{\epsilon\nu}^{\mathscr{T}}(\tilde{k}) \frac{\alpha(\tilde{k}^2)}{\tilde{k}^2}$$

Complex gluon momentum: $\tilde{k} = k \mp i\frac{m_\pi}{2}$



$$p_\pm^2 = \left(p \pm \frac{P}{2}\right)^2 \quad \text{and} \quad P = \left(\vec{0}, im_\pi\right)$$

$$\longrightarrow \quad p_\pm^2 = p^2 - \frac{m_\pi^2}{4} \pm im_\pi\sqrt{p^2}$$

$$f(p_0) = \frac{\oint_\gamma \frac{f(p)}{p-p_0} dp}{\oint_\gamma \frac{1}{p-p_0} dp}$$

Parametrization of the pion

Momentum parametrization:

$$P = (0, 0, im_\pi, 0)$$
$$p = (\ |\vec{p}|(\ 0\ ,\ 0\ ,\ 1\ )\ ,\ p_4\ )$$
$$q = (\ |\vec{q}|(\ 0\ ,\ \sin(\Psi_q)\ ,\ \cos(\Psi_q)\ )\ ,\ q_4\ )$$

$$|\vec{p}| = |p|\sin(\theta_p)$$
$$p_4 = |p|\cos(\theta_p)$$

Integral parametrization:

$$\int \frac{d^4q}{(2\pi)^4} = \frac{1}{16\pi^3}\int_{\epsilon^2}^{\Lambda^2} dq^2 q^2 \int_0^\pi d\Psi_q \sin(\Psi_q) \int_0^\pi d\theta_q \sin^2(\theta_q)$$

Chebyshev expansion

Pion amplitude: $\Gamma_\pi(P,p) = \sum_k^4 f_k(P,p)\tau_k(P,p)$

Chebyshev expansion of dressing function $f_k(P^2,p^2,z_p)$**:**

$$f_k(P^2,p^2,z_p) \approx \sum_{j=0}^{\tilde{N}}{}' f_k^j(P^2,p^2)\,T_j(z_p)(i)^j$$

Chebyshev polynomials: $T_n(z_p) = \cos(n\theta_p) = \cos(n\arccos(z_p))$ with $z_p \in [-1,1]$

Recursive formula: $T_j(z_p) = 2z_p T_{j-1}(z_p) - T_{j-2}(z_p)$ with first polynomials $T_0(z_p) = 1$ and $T_1(z_p) = z_p$

C-Parity

Pion amplitude: $\Gamma_\pi(P,p) = \sum_k^4 f_k(P,p)\tau_k(P,p)$

Charge-conjugated pion amplitude: $\bar{\Gamma}_\pi(P,p) = \left[C\Gamma_\pi(P,-p)C^{-1}\right]^T$

C-Parity: $\bar{\Gamma}_\pi(P,p) = \eta_c\Gamma_\pi(P,p)$ with eigenvalue $\eta_c = \pm 1$

$$\begin{aligned} \bar{\tau}_k(P,p) &= \left[C\tau_k(P,-p)C^{-1}\right]^T = \xi_k\tau_k(P,p) \qquad \longrightarrow \qquad \eta_c = \xi_k\tilde{\xi}_k \quad \forall k \\ \bar{f}_k(P,p) &= \left[Cf_k(P,-p)C^{-1}\right]^T = \tilde{\xi}_k f_k(P,p) \end{aligned}$$

Pion: $\eta_c = +1$

Cheby. exp. $\longrightarrow$ $\bar{f}_k(P^2,p^2,z_p) = f_k(P^2,p^2,-z_p) \overset{!}{=} f_k(P^2,p^2,z_p)$

Power-iteration

BSE:

$$\hat{K}(P^2)\,|\Gamma_n(P,p)\rangle = \lambda_n(P^2)\,|\Gamma_{n+1}(P,p)\rangle$$

Iteration number: n

Eigenvalue:

$$\lambda_n(P^2) = \frac{\langle\Gamma_n(P,p)|\,\Gamma_{n+1}(P,p)\rangle}{\langle\Gamma_n(P,p)|\,\Gamma_n(P,p)\rangle}$$

$\lambda_n(P^2) \xrightarrow{n\to\infty} \lambda(P^2)$