Der Wissenschaftsfonds.

Glueballs: Theoretical Status and Experimental Search

Denis Parganlija

Thanks to:
F. Brünner and A. Rebhan (Vienna)
F. Giacosa (Kielce)
S. Janowski and D. H. Rischke (Frankfurt)
D. Bugg (London)

What does a particle physicist do?

I‘m a physicist so I was thinking about stuff all day.

I even wrote some of it down.

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Dr. Leonard Hofstadter,
The second most famous physicist of all time

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Note: the most famous physicist of all time is of course →

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Particle Production

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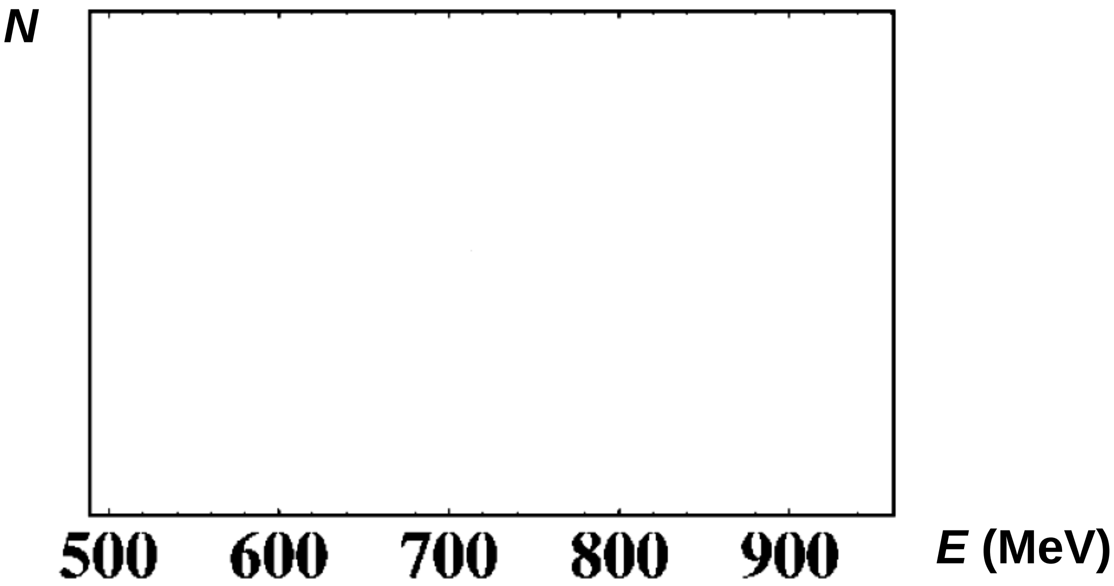

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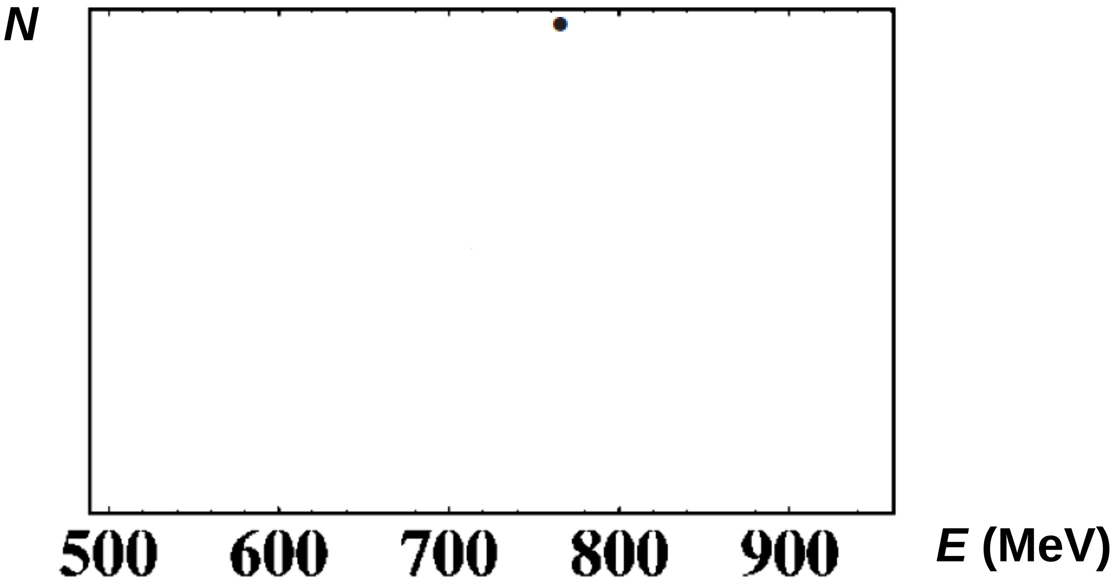

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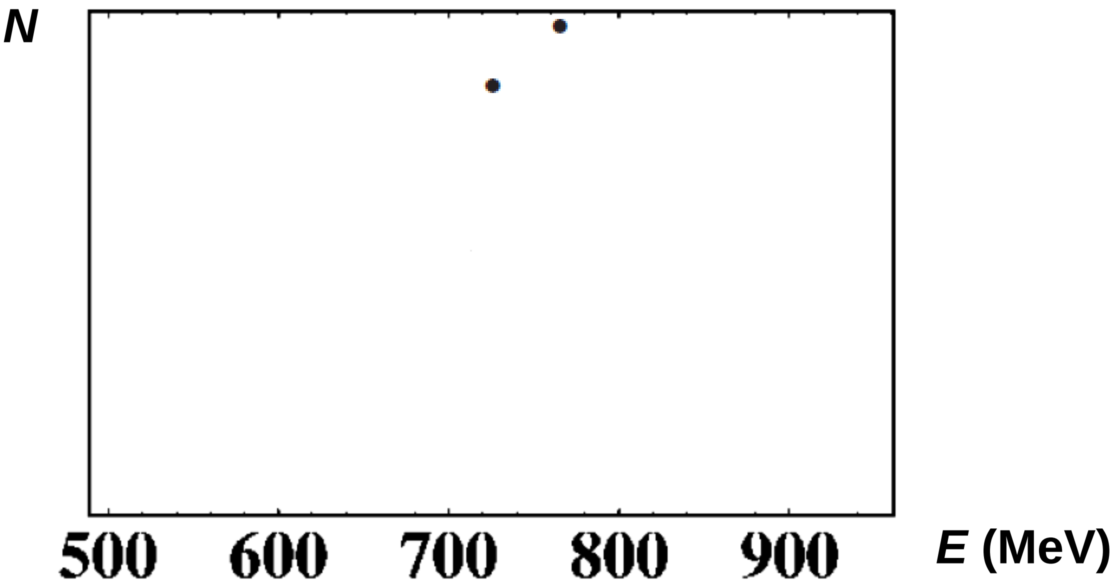

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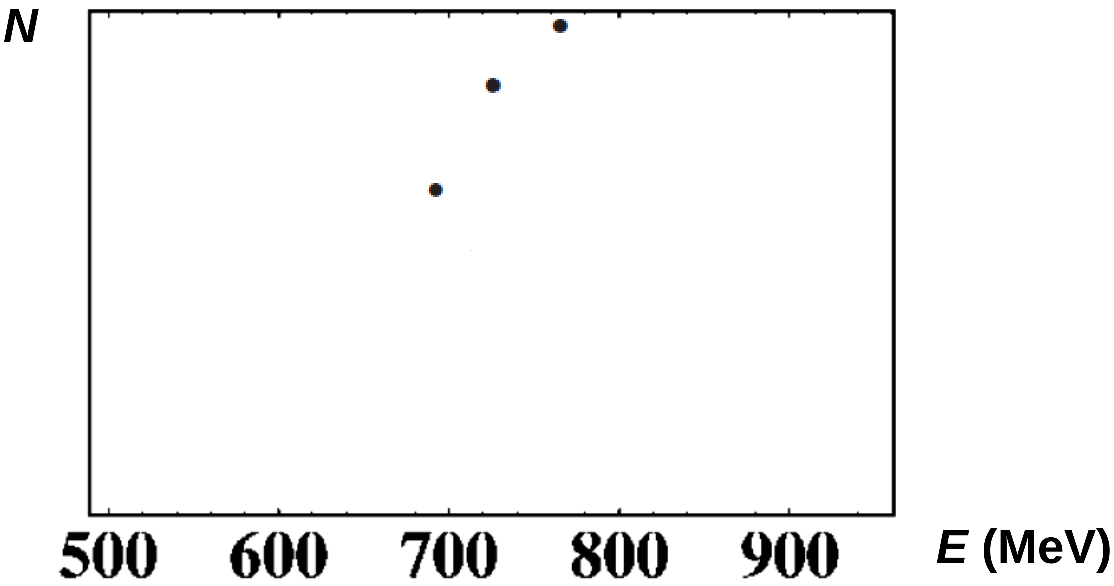

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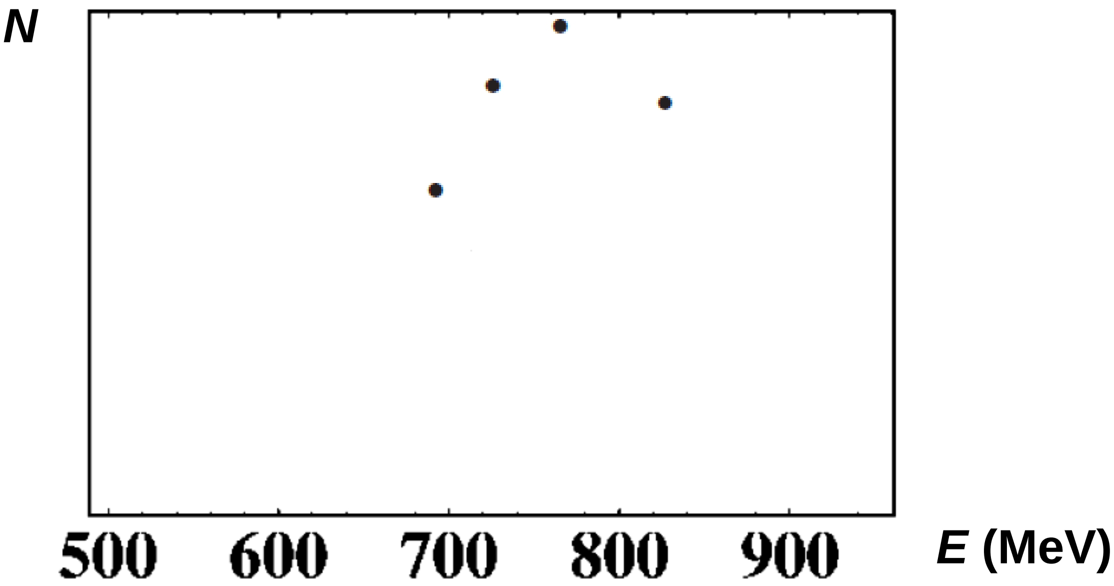

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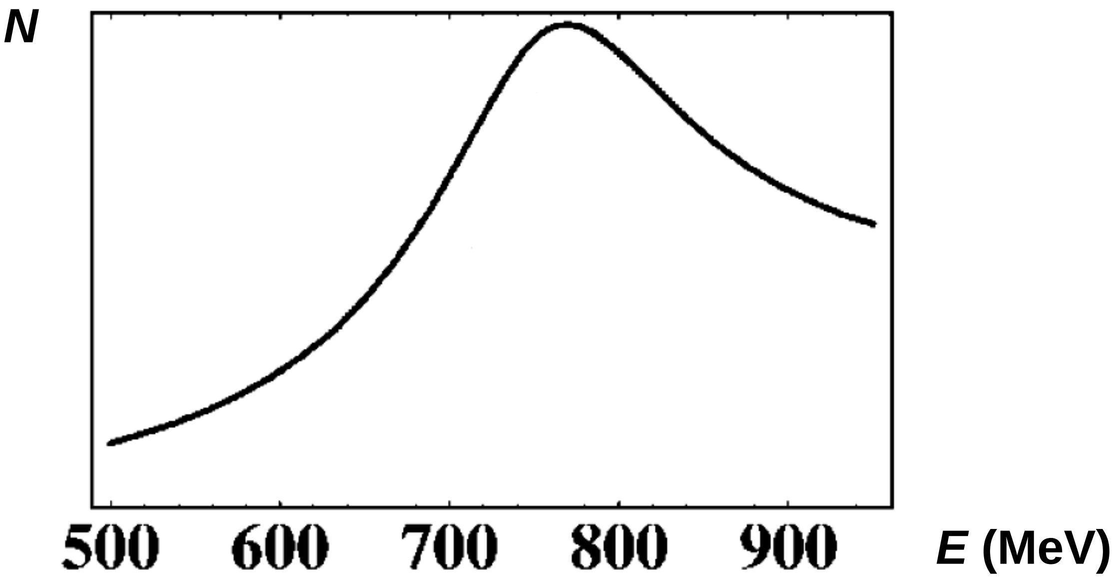

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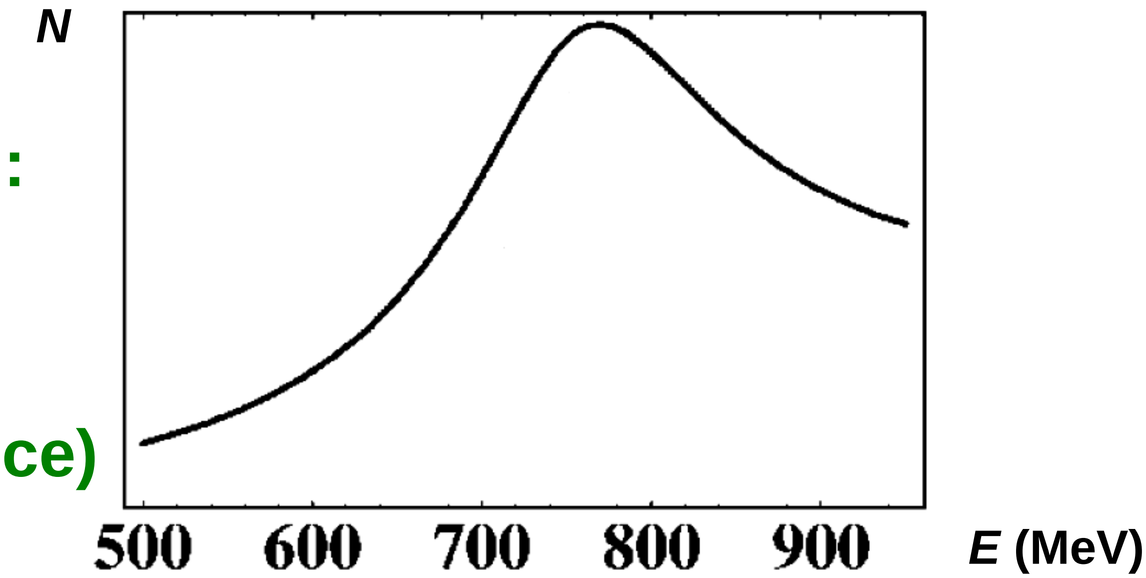

May represent a particle (resonance)

Particle Revolution of 1950-1970

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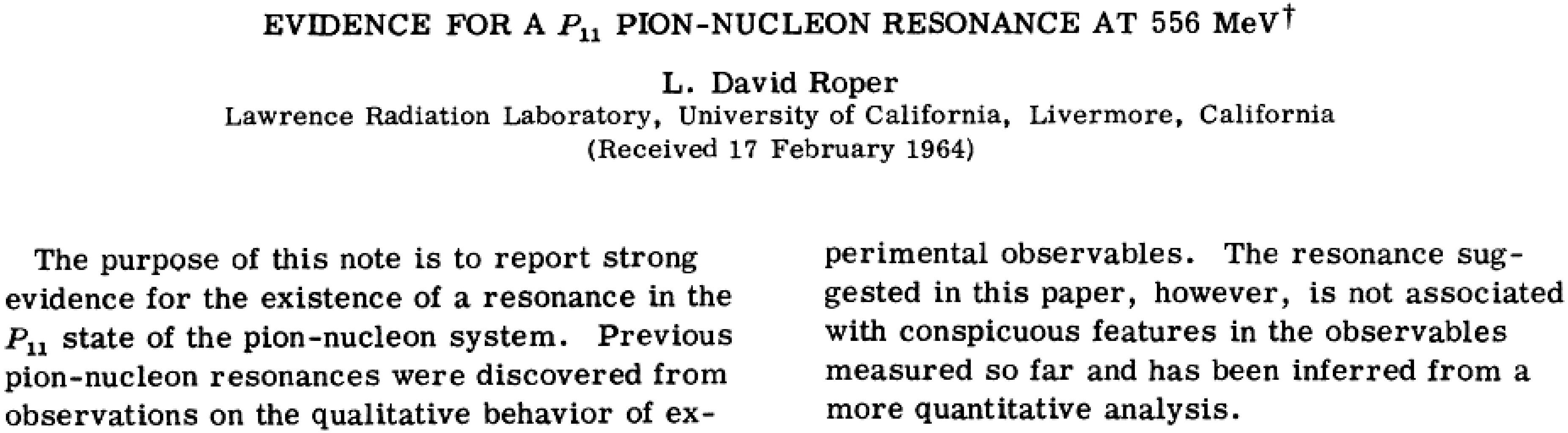
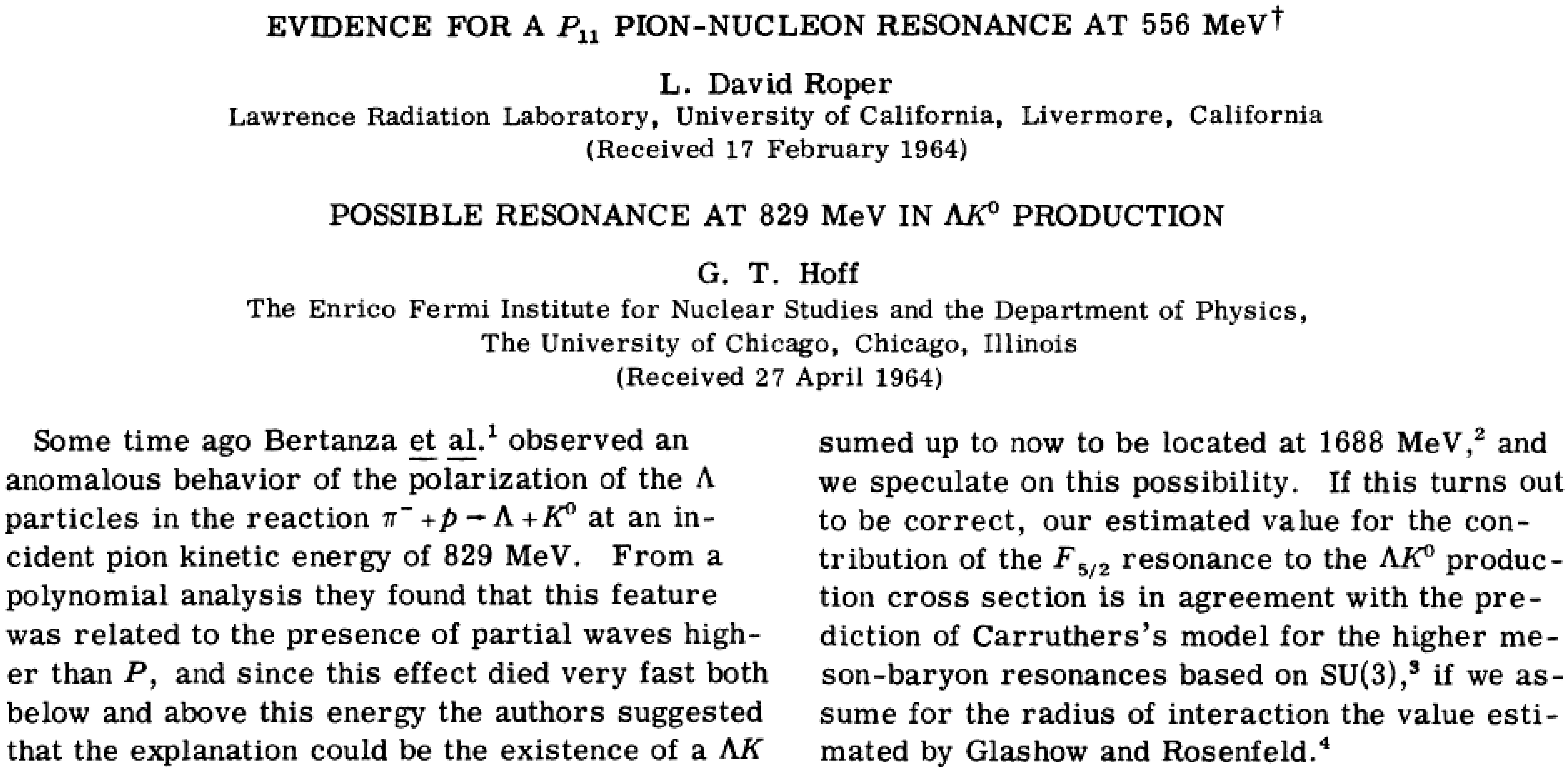


EVIDENCE FOR A P_{11} PION-NUCLEON RESONANCE AT 556 MeV†

L. David Roper
Lawrence Radiation Laboratory, University of California, Livermore, California
(Received 17 February 1964)

The purpose of this note is to report strong evidence for the existence of a resonance in the P_{11} state of the pion-nucleon system. Previous pion-nucleon resonances were discovered from observations on the qualitative behavior of experimental observables. The resonance suggested in this paper, however, is not associated with conspicuous features in the observables measured so far and has been inferred from a more quantitative analysis.

Particle Revolution of 1950-1970



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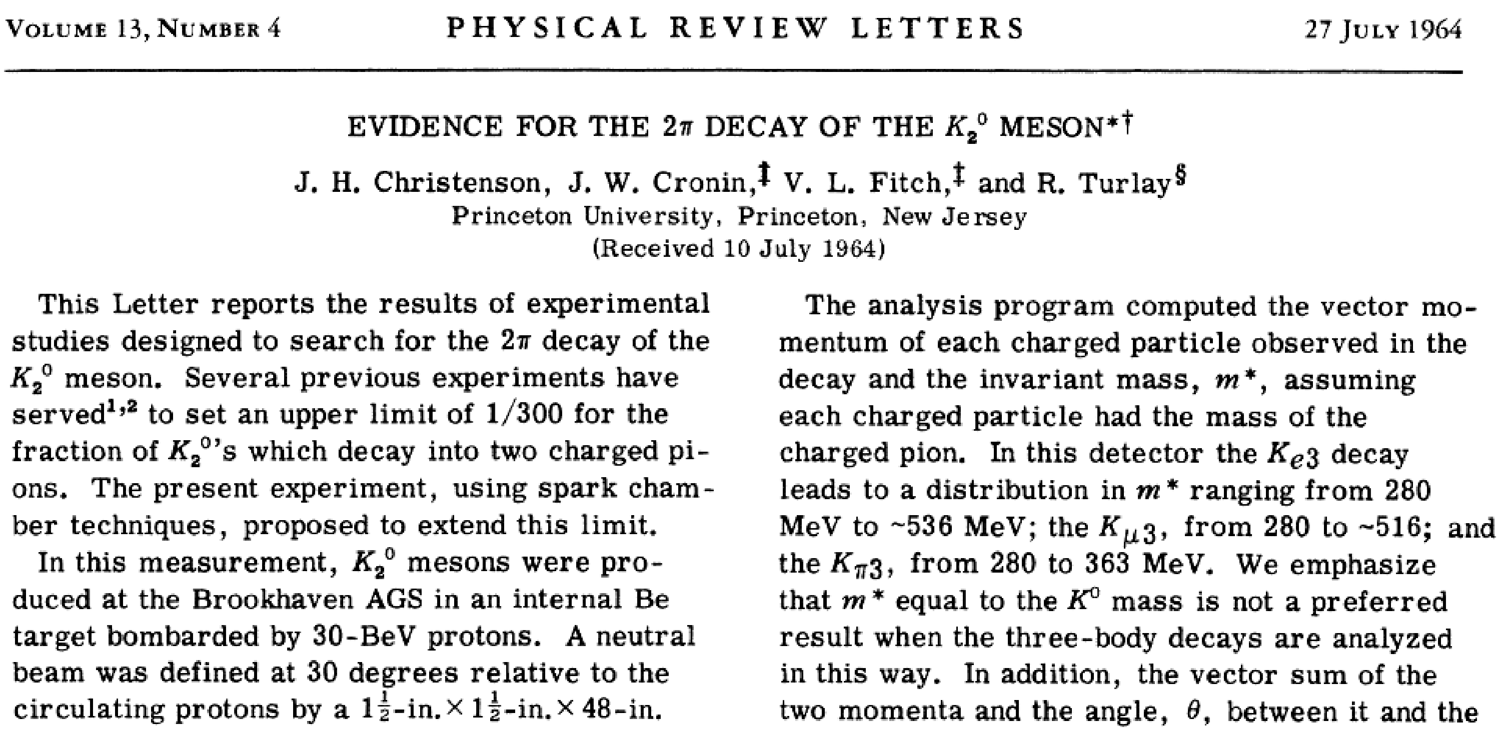
POSSIBLE RESONANCE AT 829 MeV IN ΛK^0 PRODUCTION

G. T. Hoff
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics,
The University of Chicago, Chicago, Illinois
(Received 27 April 1964)

Some time ago Bertanza *et al.*[1] observed an anomalous behavior of the polarization of the Λ particles in the reaction $\pi^- + p \to \Lambda + K^0$ at an incident pion kinetic energy of 829 MeV. From a polynomial analysis they found that this feature was related to the presence of partial waves higher than P, and since this effect died very fast both below and above this energy the authors suggested that the explanation could be the existence of a ΛK

sumed up to now to be located at 1688 MeV,[2] and we speculate on this possibility. If this turns out to be correct, our estimated value for the contribution of the $F_{5/2}$ resonance to the ΛK^0 production cross section is in agreement with the prediction of Carruthers's model for the higher meson-baryon resonances based on SU(3),[3] if we assume for the radius of interaction the value estimated by Glashow and Rosenfeld.[4]

Particle Revolution of 1950-1970



VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

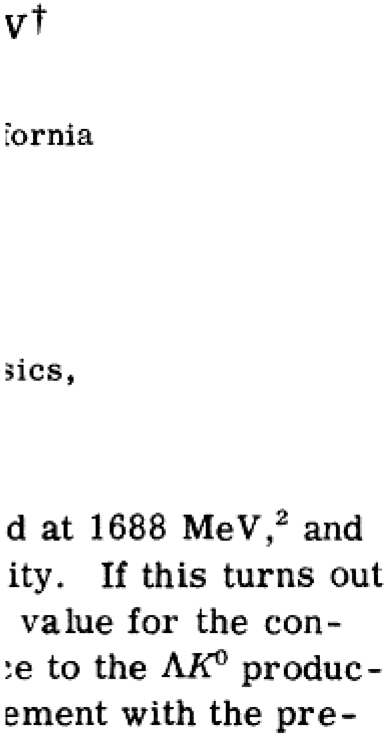
EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have served[1,2] to set an upper limit of 1/300 for the fraction of K_2^0's which decay into two charged pions. The present experiment, using spark chamber techniques, proposed to extend this limit.

In this measurement, K_2^0 mesons were produced at the Brookhaven AGS in an internal Be target bombarded by 30-BeV protons. A neutral beam was defined at 30 degrees relative to the circulating protons by a $1\frac{1}{2}$-in.×$1\frac{1}{2}$-in.×48-in.

The analysis program computed the vector momentum of each charged particle observed in the decay and the invariant mass, m^*, assuming each charged particle had the mass of the charged pion. In this detector the K_{e3} decay leads to a distribution in m^* ranging from 280 MeV to ~536 MeV; the $K_{\mu 3}$, from 280 to ~516; and the $K_{\pi 3}$, from 280 to 363 MeV. We emphasize that m^* equal to the K^0 mass is not a preferred result when the three-body decays are analyzed in this way. In addition, the vector sum of the two momenta and the angle, θ, between it and the



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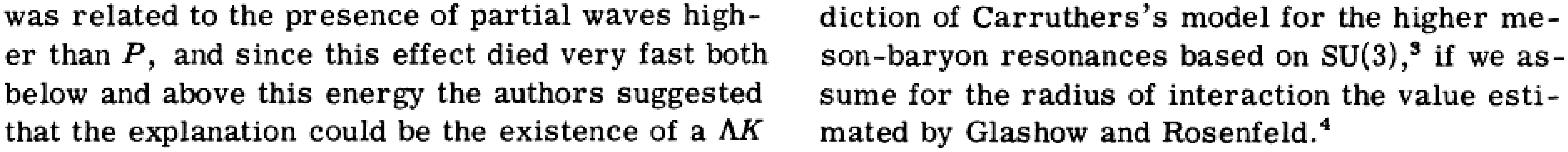

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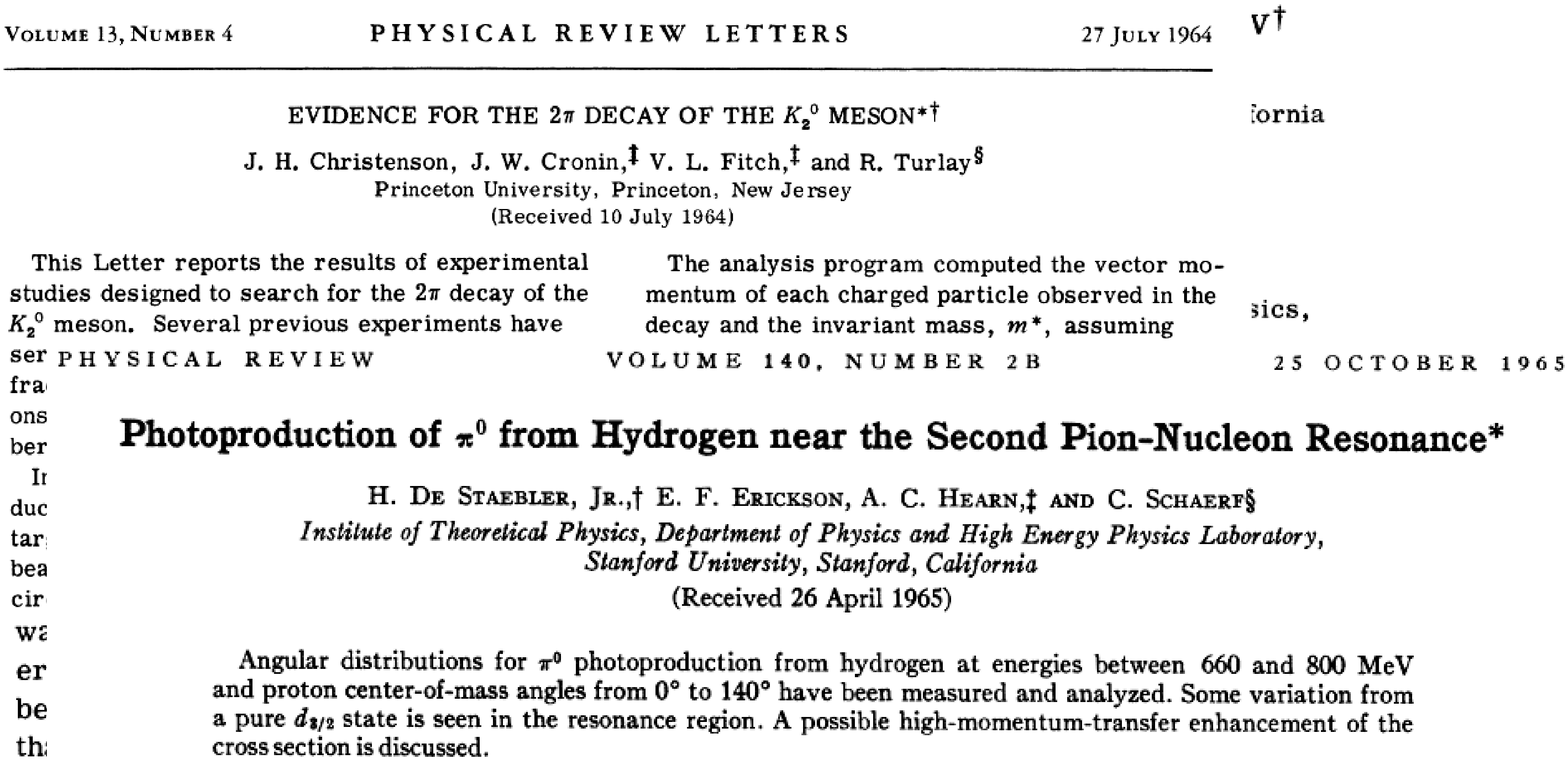

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PHYSICAL REVIEW VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Photoproduction of π^0 from Hydrogen near the Second Pion-Nucleon Resonance*

H. DE STAEBLER, JR.,† E. F. ERICKSON, A. C. HEARN,‡ AND C. SCHAERF§
Institute of Theoretical Physics, Department of Physics and High Energy Physics Laboratory, Stanford University, Stanford, California
(Received 26 April 1965)

Angular distributions for π^0 photoproduction from hydrogen at energies between 660 and 800 MeV and proton center-of-mass angles from 0° to 140° have been measured and analyzed. Some variation from a pure $d_{3/2}$ state is seen in the resonance region. A possible high-momentum-transfer enhancement of the cross section is discussed.

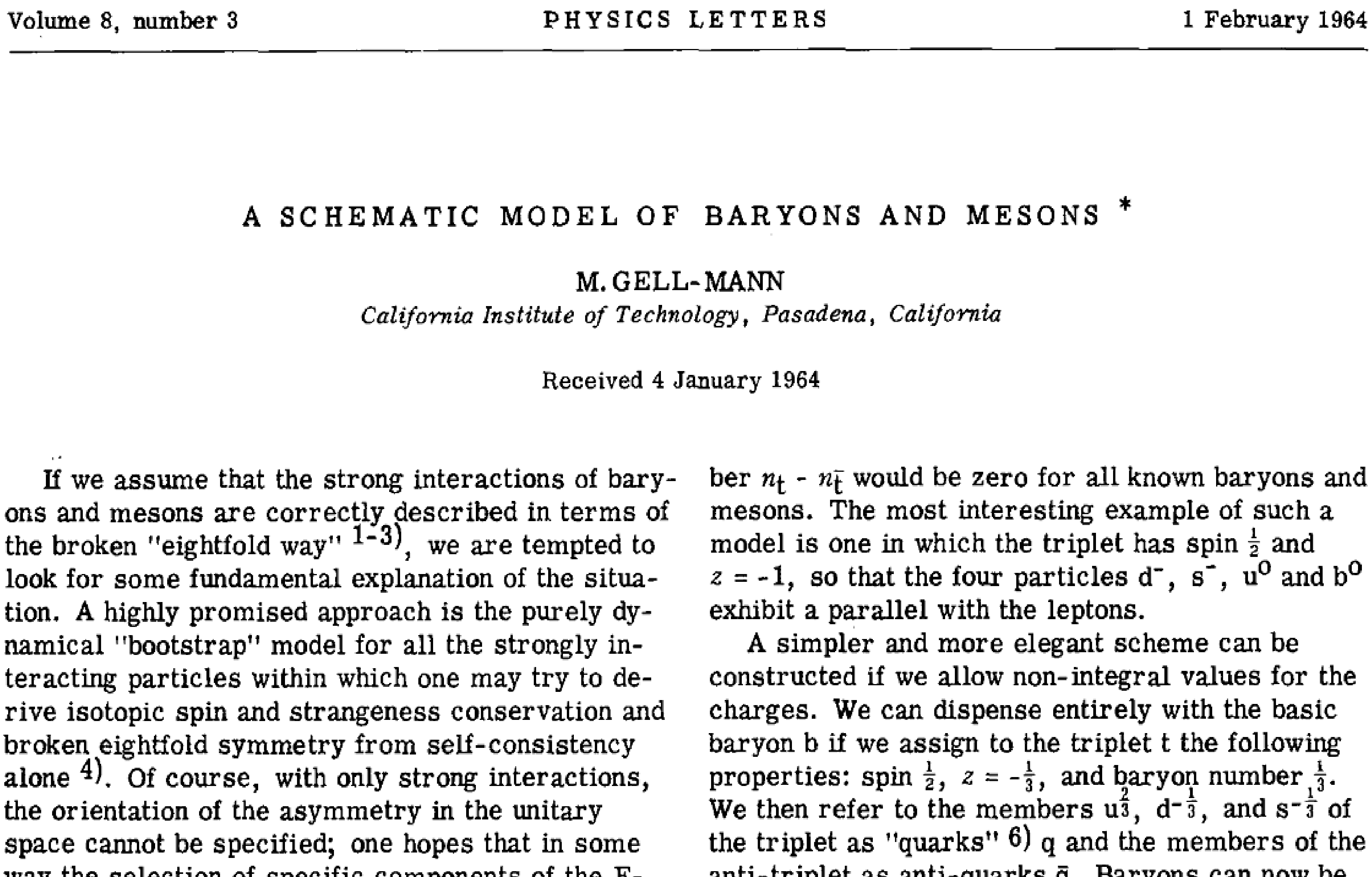
Explanation:

Strong Interaction and its Theory, QCD

Introduction of Quarks

- **Questions:**
  1) Why are there no decays of new particles into all other ones, provided sufficient phase space?
  2) Is there a classification scheme for particles?

Introduction of Quarks



Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" 1-3), we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone 4). Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-

ber n_t - $n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and z = -1, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be

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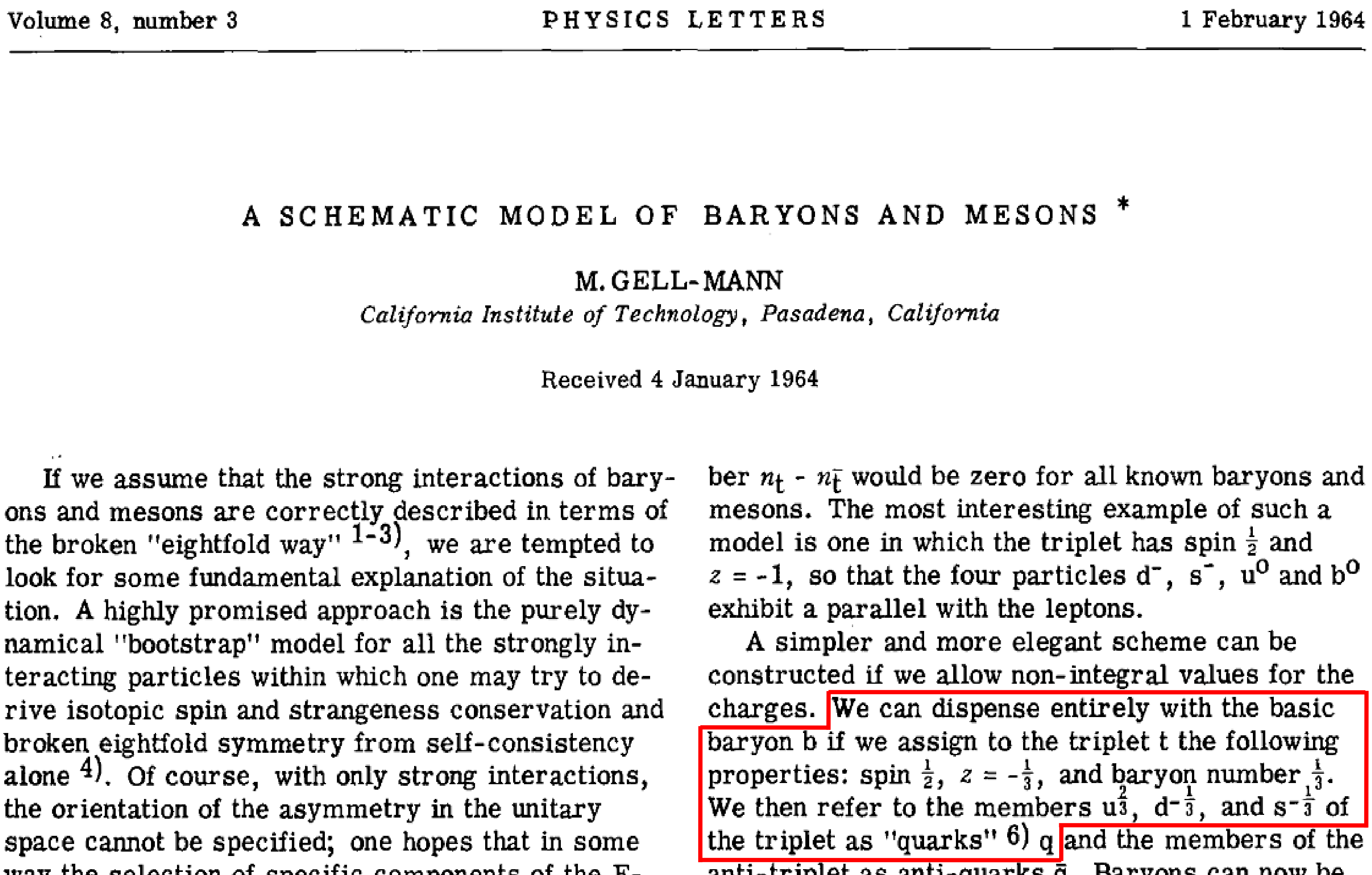

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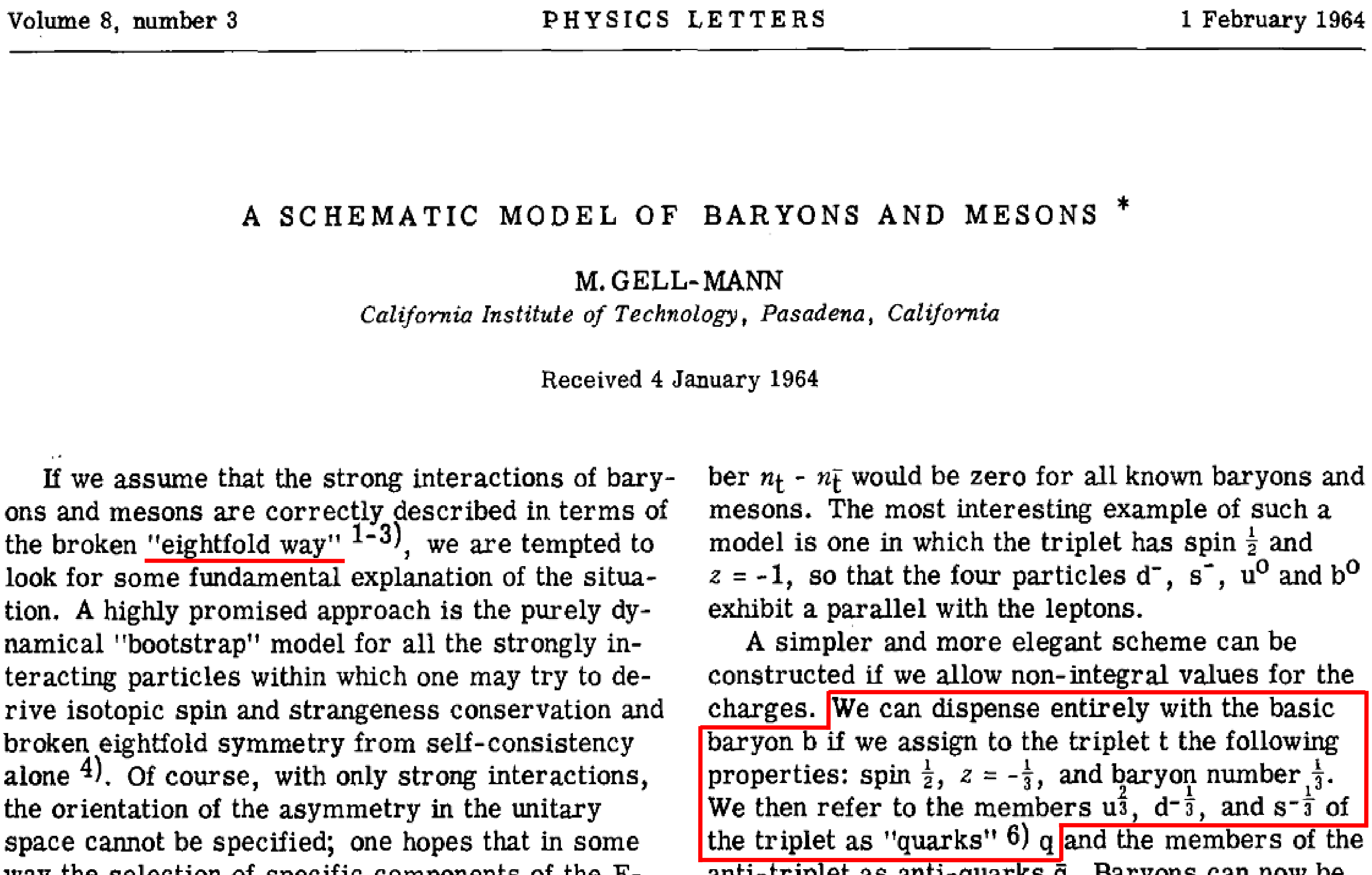

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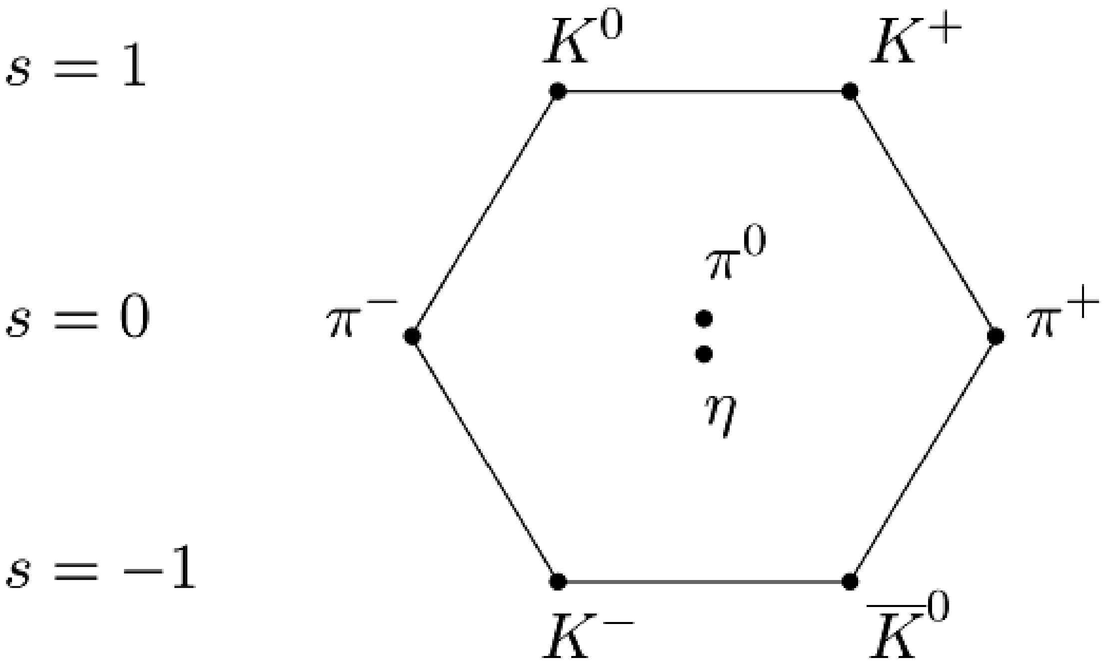

Quantum Chromodynamics

Quantum Chromodynamics

- **QCD Lagrangian:**

$$\mathcal{L} = \bar{q}_f\left(i\gamma^{\mu}D_{\mu} - m_f\right)q_f - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}$$

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Strong coupling energy-dependent

↔ Asymptotic freedom

Hadrons emerge at small energies

↔ Confinement

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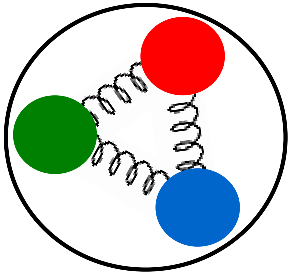

Half-Integer Spin

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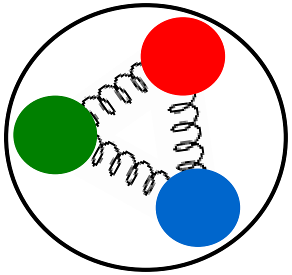

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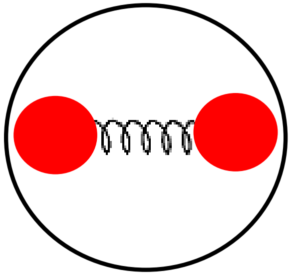

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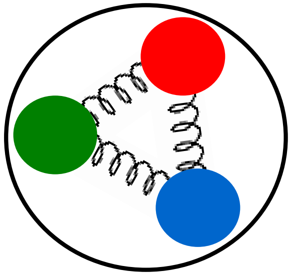

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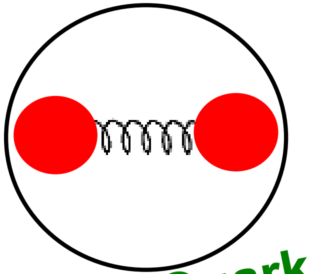

(Quarkonium)

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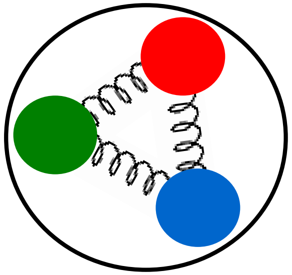

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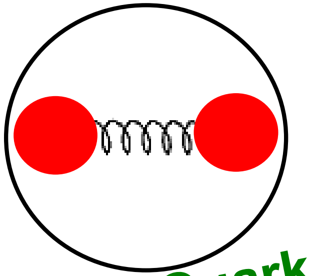

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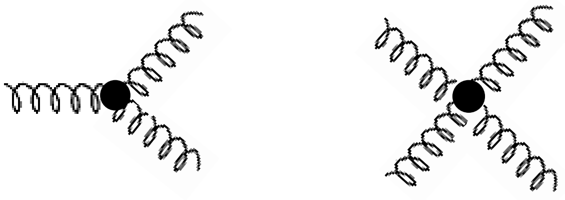

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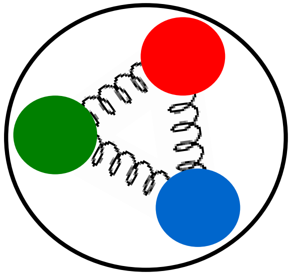

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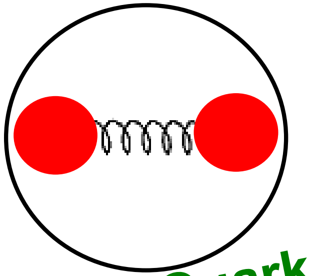

(Quarkonium)

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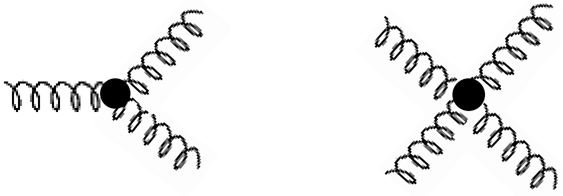
Hadrons emerge at small energies
↔ Confinement

- **Gluons are self-interacting**
- **Gluon bound states: Glueballs!**



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- Note: there are (probably) other ‘exotic’ (non-quarkonium) particles → see, e.g., 1) A. Ali, J. S. Lange and S. Stone, arXiv: 1706.00610 and 2) G. Eichmann, C. S. Fischer and W. Heupel, arXiv:1508.07178

Glueballs and Theory

Two approaches to Low-Energy QCD

Two approaches to Low-Energy QCD

QCD

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First principles:

Effective theories and models:

First principles

↔ **quarks/gluons**

Effective approaches

↔ **hadrons**

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[Wilson; Dürr, Fodor, Gregory, Irving, Katz, Lang, Mohler, Morningstar, Peardon, Prelovsek, …]

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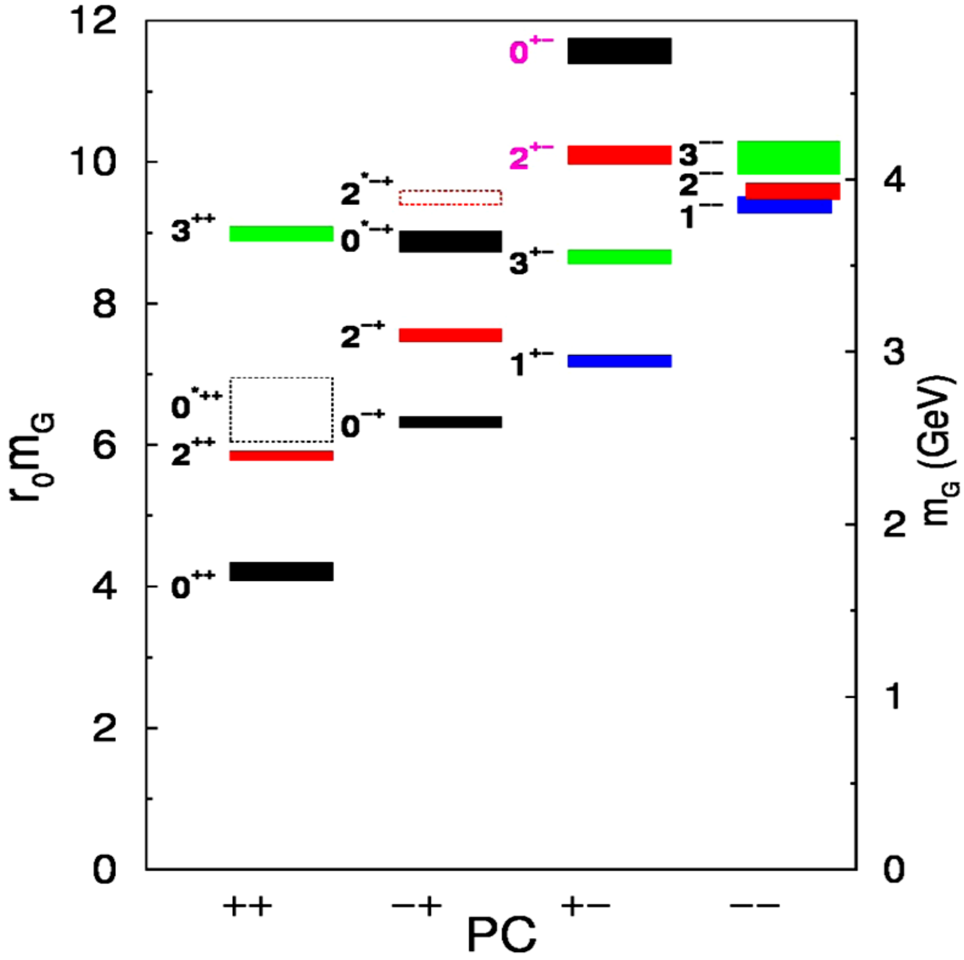
Glueballs should have distinct mass/decay properties

Spectrum of Glueballs

Morningstar & Peardon hep-lat/9901004:



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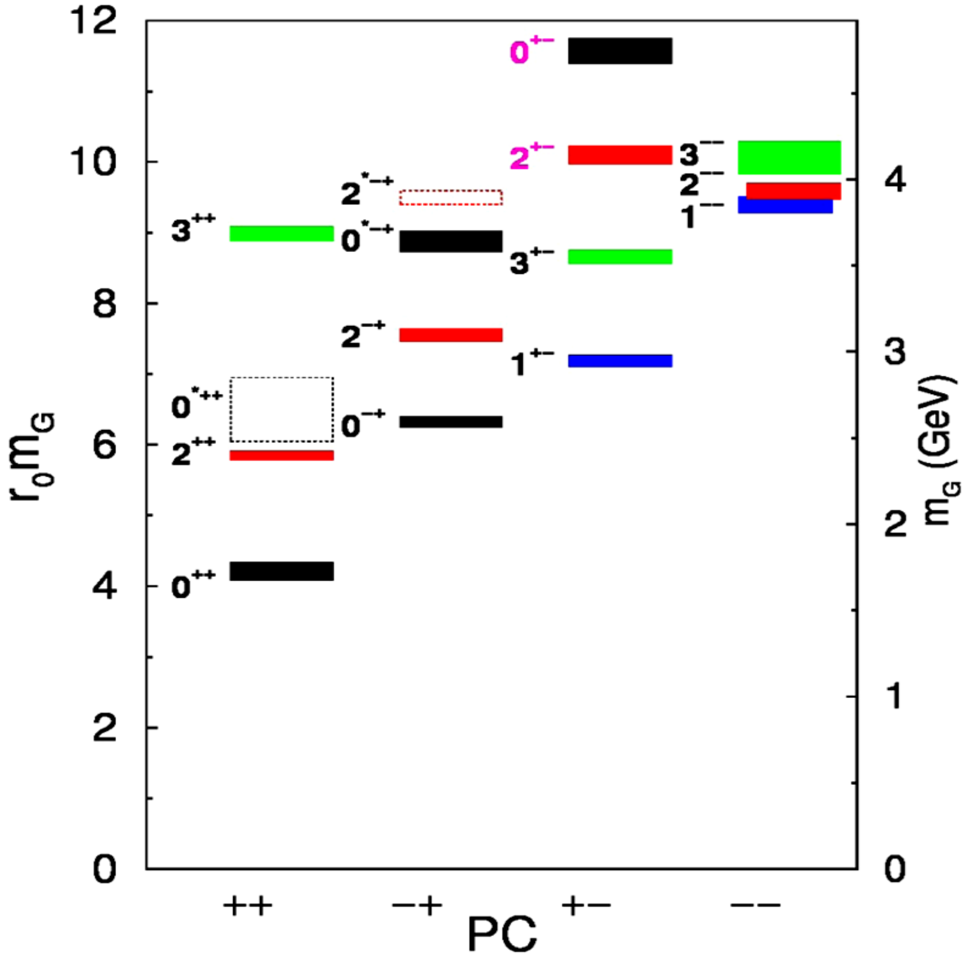

Also calculable from Bethe-Salpeter equation:

	Masses (GeV)			
J^{PC}	Lattice	Hamiltonian/ Regge theory	Constituent models	This work
0^{++}	1.71 (5)(8)	1.98	1.71	1.64
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[H. Sanchis-Alepuz, C. S. Fischer, C. Kellermann and L. von Smekal, Phys. Rev. D 92, 034001 (2015) arXiv: 1503.06051 [hep-ph]]

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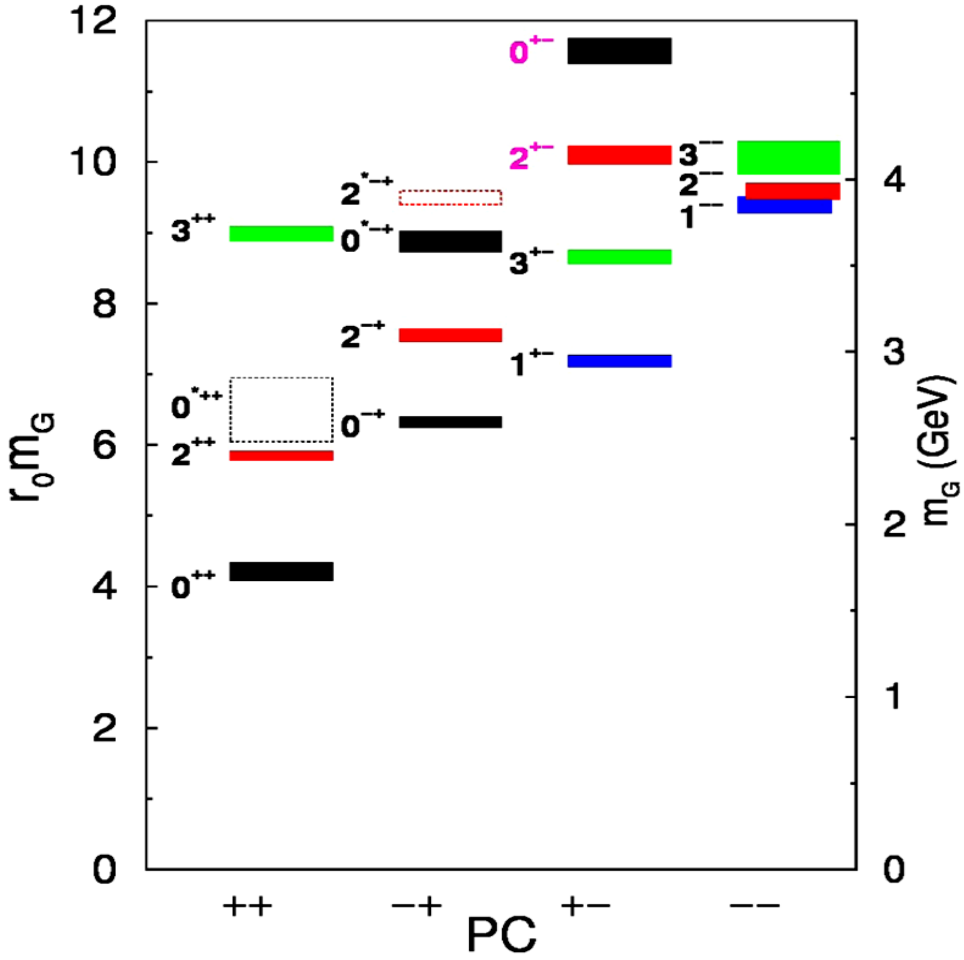

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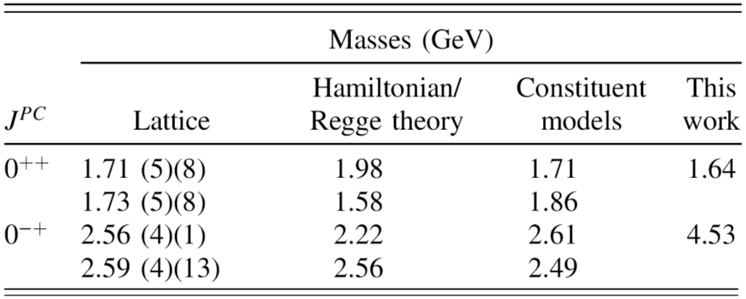

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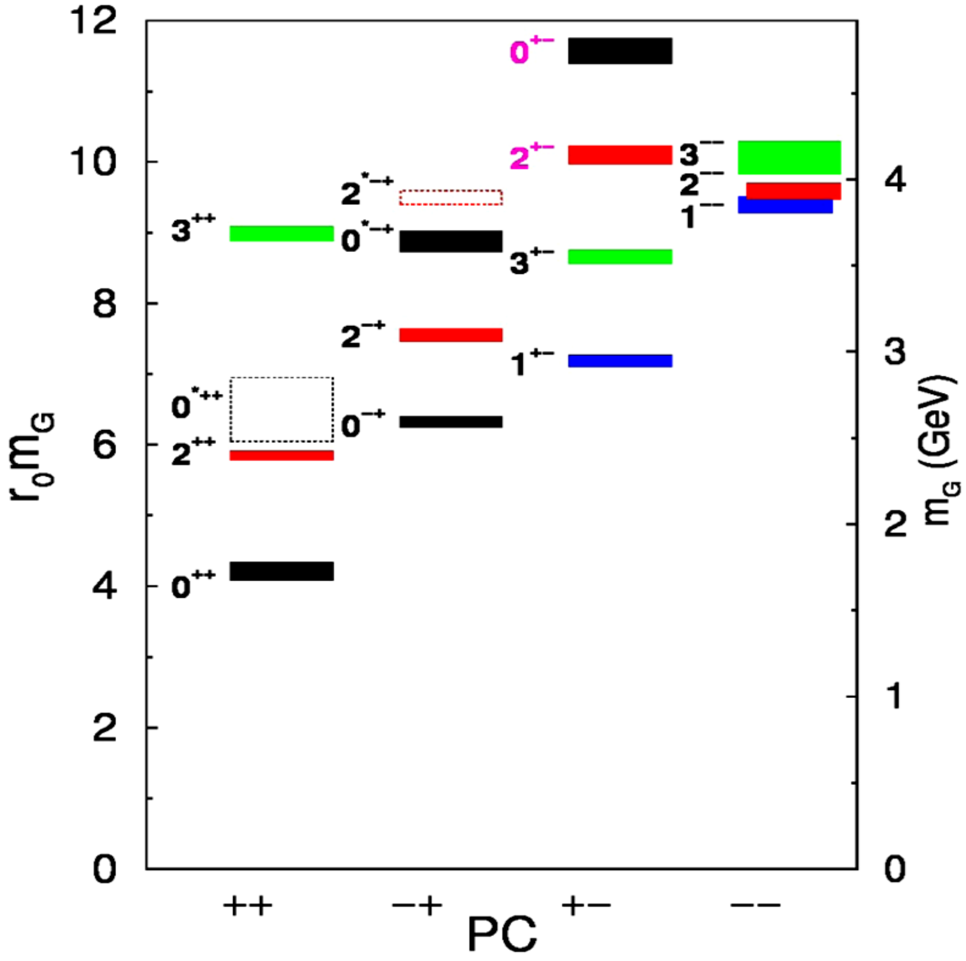

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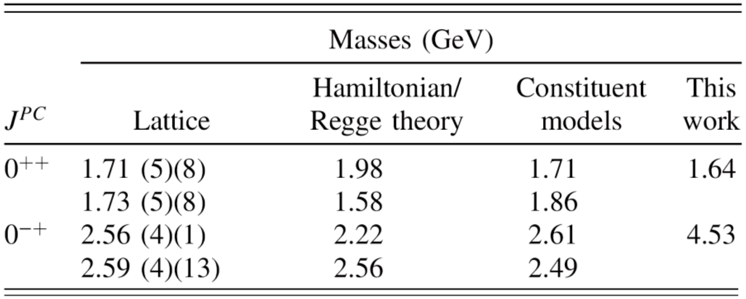

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Model glueball-quarkonia dynamics

Can a Model Ever Describe QCD?

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- **Symmetries of the QCD Lagrangian:** **Poincare; Local $SU(3)_c$ Colour; Global Chiral $U(N_f)$x $U(N_f)$; Dilatational; CPT; Z_n (n = 0, …, $N_c - 1$)**

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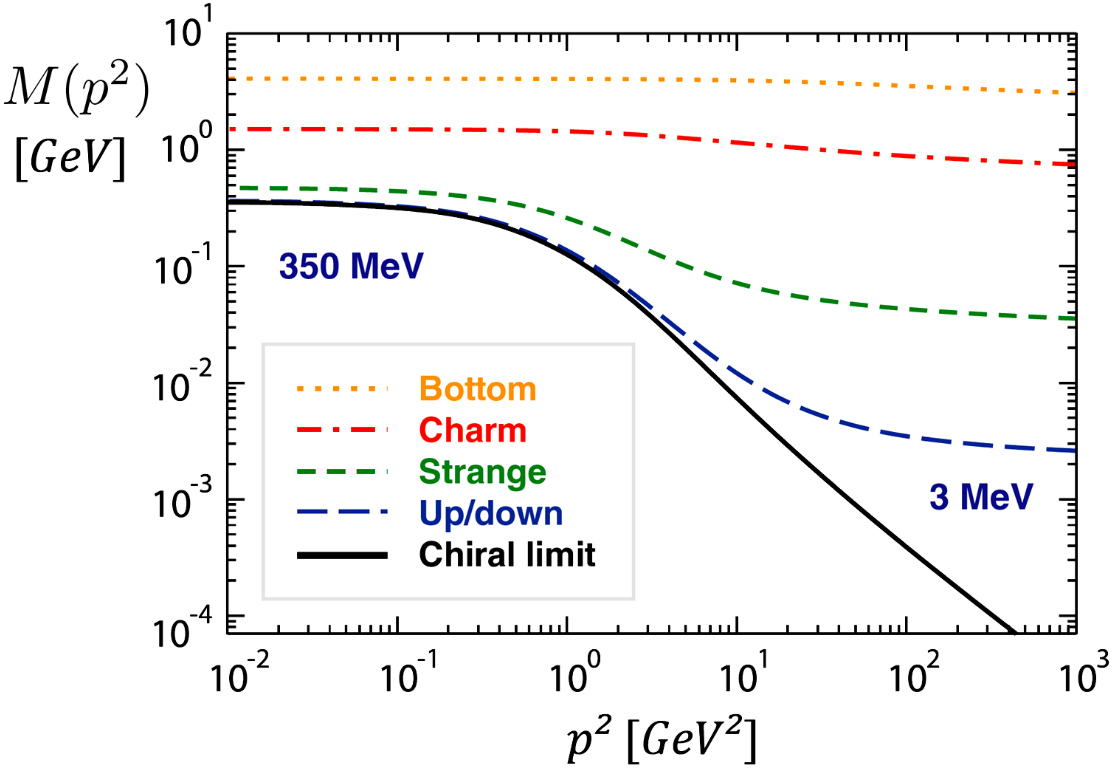
- **Degrees of freedom:** quantum numbers *I*, *J*, *P*, *C* are contained – as observed by experiment and calculated in first-principles approaches
- Suitable for dynamics → can **test structure of observed particles**
- **Historical success:** a simple chiral model predicted the sigma meson a decade before first experimental hints

Connection to QCD Degrees of Freedom



The diagram contains typical DSE solutions for the quark mass function from realistic truncations.

[G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer, Prog. Part. Nucl. Phys. 91, 1 (2016); arXiv:1606.09602 [hep-ph]]

Mesons

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$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

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$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix}$$

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- Also necessary: dilatation symmetry

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$$\mathcal{L}_0 = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2 \quad \mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2$$

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- **Glueball mixes with quarkonia; full Lagrangian → next slide**

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$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G^2}{\Lambda^2}\right| - \frac{G^4}{4}\right)$$

← **Schechter et al. (1981)**

$$\mathcal{L} = \mathcal{L}_{dil} + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - m_0^2\left(\frac{G}{G_0}\right)^2\Phi^\dagger\Phi - \lambda_2(\Phi^\dagger\Phi)^2\right] - \lambda_1(\mathrm{Tr}\left[\Phi^\dagger\Phi\right])^2$$

$$+ c[\det(\Phi^\dagger) + \det(\Phi)] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right]$$

$$+ \frac{m_1^2}{2}\left(\frac{G}{G_0}\right)^2\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu]$$

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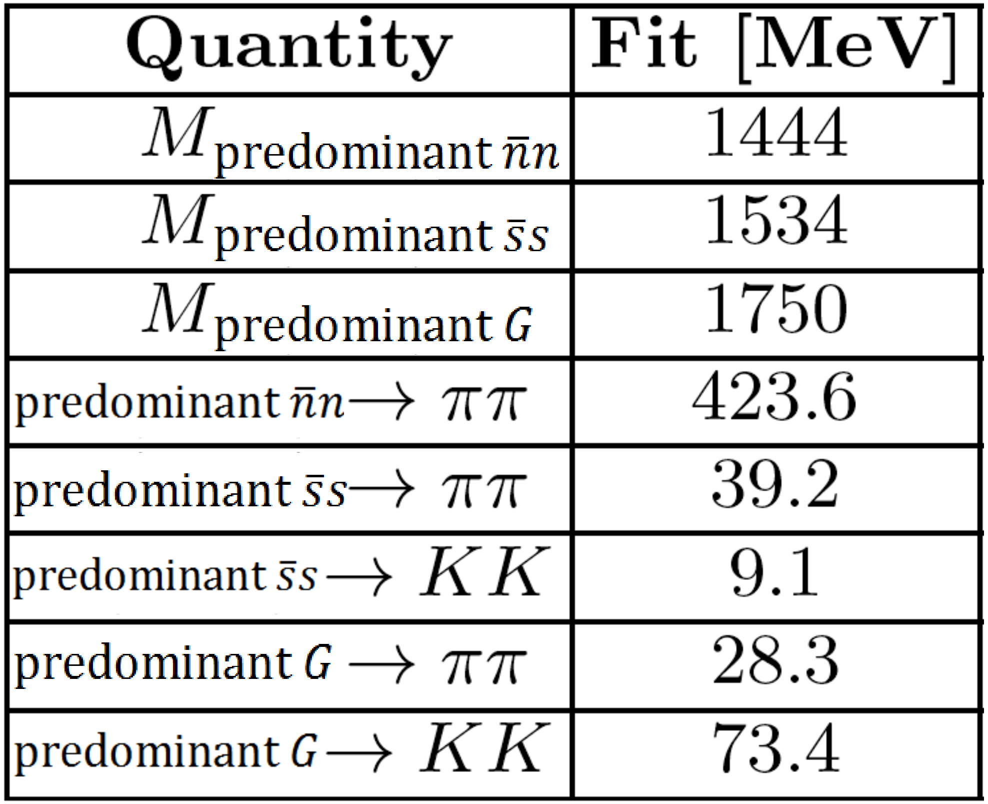
↑ ↑

contain vectors and axial-vectors

Results



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[S. Janowski, F. Giacosa and D. H. Rischke, Phys. Rev. D 90, no. 11, 114005 (2014) arXiv: 1408.4921 [hep-ph]]

Glueballs and Experiment

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Absent/suppressed in $\gamma\gamma$:

$f_0(1500)$ [L3; ALEPH]

$f_0(1710)$ [ALEPH]

[Belle: contrary statement]

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The exact number of f_0 resonances is not known - need more data!

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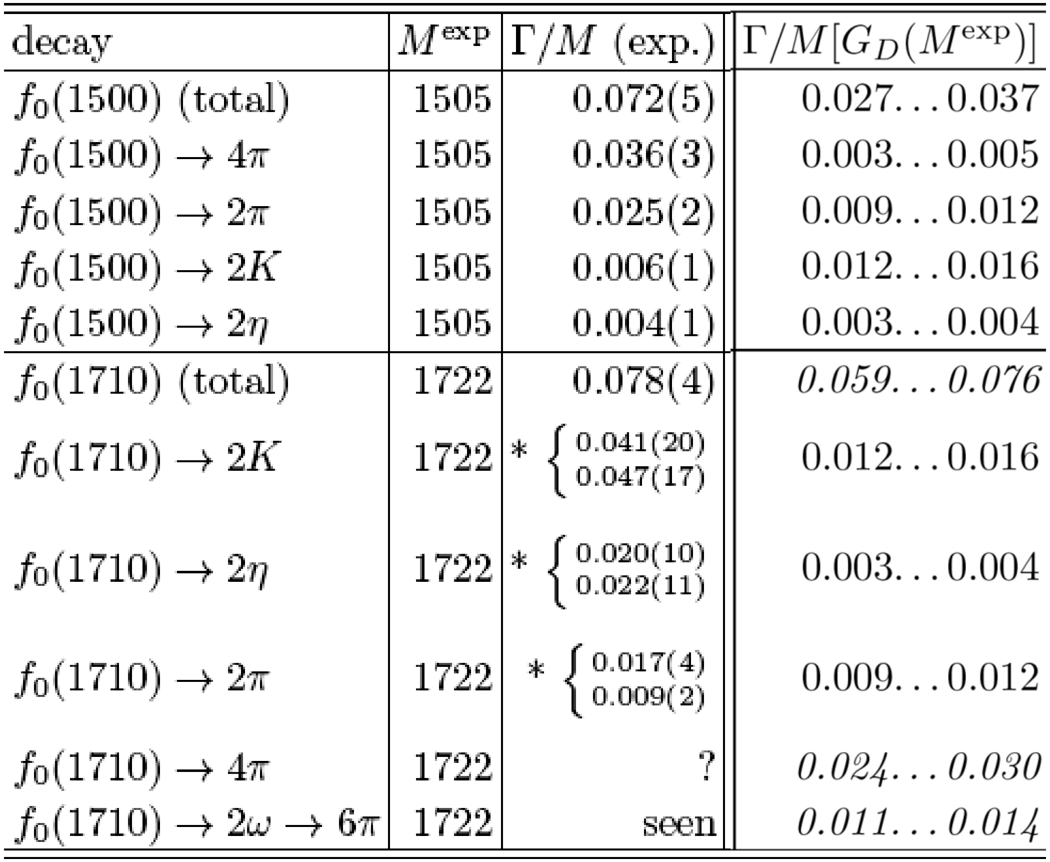

decay	M^{exp}	Γ/M (exp.)	$\Gamma/M[G_D(M^{\text{exp}})]$
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$f_0(1710) \to 4\pi$	1722	?	*0.024 . . . 0.030*
$f_0(1710) \to 2\omega \to 6\pi$	1722	seen	*0.011 . . . 0.014*

[F. Brünner, D. Parganlija and A. Rebhan, Phys.Rev. D91 (2015) no.10, 106002; E: Phys.Rev. D93 (2016) no.10, 109903 arXiv: 1501.07906 [hep-ph]]

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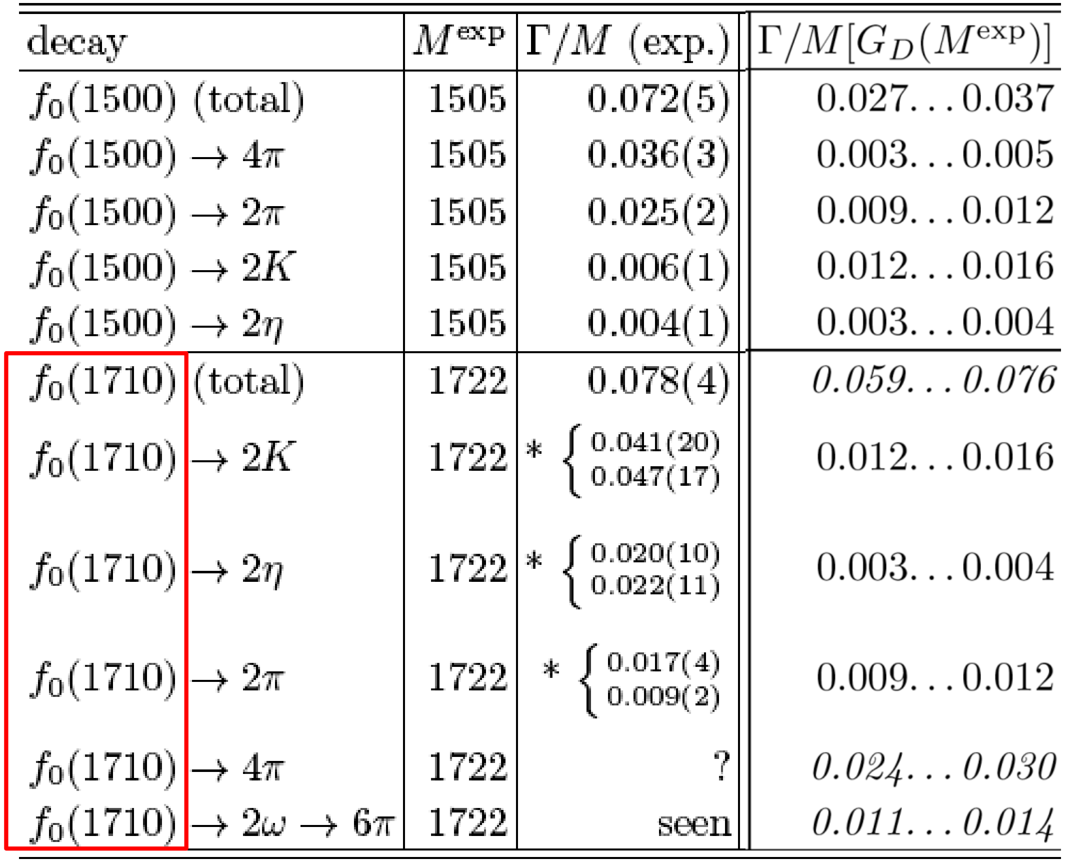

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Summary

- Glueballs are a promise of the strong interaction
- Experimental identification complicated because of overlap with quarkonia
- Ground state may have been discovered: $f_0(1710)$
- More data and more theoretical effort are needed to identify the other ones!