Mass Sensitivity of the QCD Phase Diagram

Masters Thesis

Simon Resch

Bernd-Jochen Schaefer, Fabian Rennecke, Konstantin Otto

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Lunch Club Seminar

Overview

Low energy QCD at high T and μ

- Effective quark-meson (QM) model
- Focus on three flavor dynamics
- Influence of masses on chiral phase transition
- Role of $U(1)_A$ anomaly

Non-perturbative effects with functional renormalization group (FRG)

Outline

Introduction

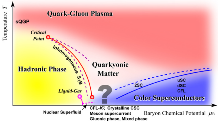
Symmetries of QCD

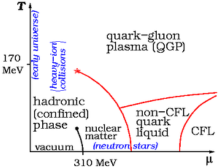
(Approximate) Symmetries of QCD with N_f light quarks

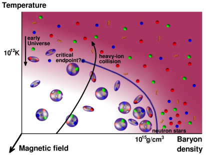
$$SU(N_c) \otimes U(1)_V \otimes SU(N_f)_V \otimes U(1)_A \otimes SU(N_f)_A$$

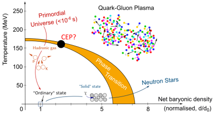
- Exact symmetries:
  - $SU(N_c)$ gauge symmetry
  - $U_V(1) \to$ baryon number conservation
- $SU(N_f)_V$ isospin symmetry $\to$ broken by $m_{q,f} - m_{q,f'} \neq 0$
- $SU(N_f)_A$ explicitly and spontaneously broken
  $\Rightarrow$ quark condensate $\langle \bar{q}q \rangle > 0$, $N_f^2 - 1$ (pseudo) Goldstone bosons
- $U(1)_A$ anomalously broken

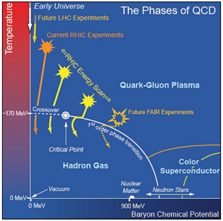
QCD Phase Diagram

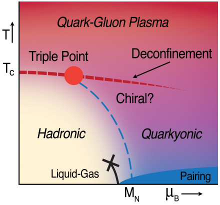


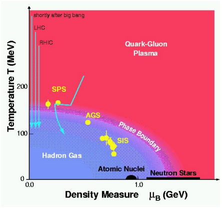


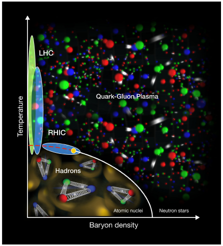


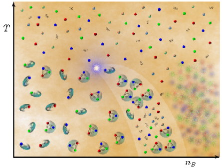


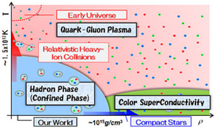


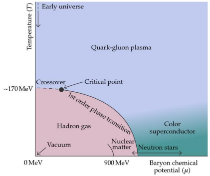


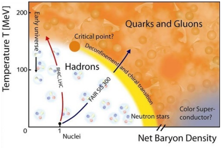












Functional Renormalization Group

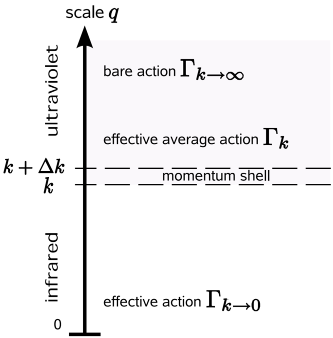
Why FRG?

- Non-perturbative method needed for low energy QCD
- Separation of degrees of freedom

Wetterich Equation

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right]$$

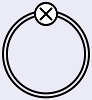
- Connects microscopic action S to full quantum effective action Γ
- Fluctuations up to RG scale k are suppressed by regulator R_k



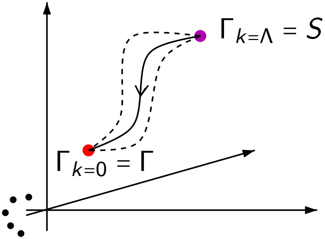
[CBM Book]

Functional Renormalization Group

One-Loop Structure

$$G_k = \left(\Gamma_k^{(2)} + R_k\right)^{-1} \quad \Rightarrow \quad \partial_t \Gamma_k = \frac{1}{2} \mathrm{STr}\left(G_k \cdot \partial_k R_k\right) = \frac{1}{2}$$ 

$\partial_k \Gamma_k^{(n)}$ also one-loop structure



[Gies,2006]

- Choice of regulator $\Rightarrow$ path in theory space
- Theory space generally infinitely dimensional

  $\Rightarrow$ needs truncating

$N_f = 2 + 1$ Quark-Meson Model

$U(N_f)_V \otimes U(N_f)_A$ Quark-Meson Action in Leading Potential Approximation (LPA)

$$\Gamma_\Lambda \equiv S[q,\bar{q},\phi] = \int_x \Big\{ \bar{q}\left(\not{\partial} + \gamma_0\mu + g\phi_5\right) q + \mathrm{Tr}(\partial_\mu\phi^\dagger\partial_\mu\phi) + U_\Lambda(\rho_1,\ldots,\rho_{N_f}) - c_A \det(\phi^\dagger + \phi) - \mathrm{Tr}\left[c_a T^a(\phi^\dagger + \phi)\right]\Big\}$$

- Scalar σ_a and pseudoscalar π_a fields collected in matrix field $\phi = T^a(\sigma_a + i\pi_a)$, $\phi_5 = T^a(\sigma_a + i\gamma_5\pi_a)$
- Yukawa interaction from bosonization of scalar-pseudoscalar four fermion vertex
- Chiral invariants $\rho_i = \mathrm{Tr}\left[(\phi^\dagger\phi)^i\right]$ with $i = 1,\ldots,N_f$
- Meson potential $U(\rho_1,\ldots,\rho_{N_f})$: meson-meson interactions of arbitrary power

$U(N_f)_V \otimes U(N_f)_A$ Quark-Meson Action in Leading Potential Approximation (LPA)

$$\Gamma_\Lambda \equiv S[q,\bar{q},\phi] = \int_x \Big\{ \bar{q}\left(\not{\partial} + \gamma_0\mu + g\phi_5\right) q + \mathrm{Tr}(\partial_\mu \phi^\dagger \partial_\mu \phi)$$
$$+ U_k(\rho_1, \ldots, \rho_{N_f}) - c_A \det(\phi^\dagger + \phi) - \mathrm{Tr}\left[c_a T^a (\phi^\dagger + \phi)\right] \Big\}$$

- $c_A \det(\phi^\dagger + \phi)$ bosonized 't Hooft determinant
  $\Rightarrow$ explicitly breaks $U(1)_A$ symmetry
- $\mathrm{Tr}\left[c_a T^a (\phi^\dagger + \phi)\right]$ explicitly breaks $SU(N_f)_V$ (follows from bosonization of quark mass matrix)

$N_f = 2 + 1$

Order parameters: $\bar{\sigma}_l$, $\bar{\sigma}_s$

- $SU(2)_V$ symmetry $\rightarrow \bar{\sigma}_3 = 0$
  $\Rightarrow$ light quarks degenerate $\left(m_{q,u} = m_{q,d} \equiv m_{q,l}\right)$
- Chiral invariants ρ_1, ρ_2, ρ_3 $\rightarrow$ drop ρ_3 for simplicity

VEV of Chiral Invariants and 't Hooft Determinant

$$\bar{\rho}_1 = \frac{1}{2}\left(\bar{\sigma}_l^2 + \bar{\sigma}_s^2\right), \quad \bar{\rho}_2 = \frac{1}{8}\left(\bar{\sigma}_l^4 + 2\bar{\sigma}_s^4\right), \quad \left\langle \det(\phi^\dagger + \phi)\right\rangle = \frac{\bar{\sigma}_s \bar{\sigma}_l^2}{2\sqrt{2}}$$

$N_f = 2 + 1$

ϕ decomposed in scalar and pseudoscalar mesons

$$T^a\pi_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} & \bar{K}^0 \\ K^+ & K^0 & \frac{\pi_0}{\sqrt{3}} - \frac{2\pi_8}{\sqrt{3}} \end{pmatrix}$$

$$T^a\sigma_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & a_0^- & \kappa^- \\ a_0^+ & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & \bar{\kappa}^0 \\ \kappa^+ & \kappa^0 & \frac{\sigma_0}{\sqrt{3}} - \frac{2\sigma_8}{\sqrt{3}} \end{pmatrix}$$

18 mesons!

Meson Curvature Masses

Eigenvalues of Hesse Matrix define meson curvature masses

Hesse Matrix

$$\left(M^2_{k,\varphi}\right)_{ij} = \frac{\partial^2}{\partial\varphi_j \partial\varphi_i}\left(U_k(\rho_1, \rho_2) - c_A \det(\phi^\dagger + \phi)\right)$$

$$\text{with} \quad \varphi = (\sigma_l, \sigma_1, \ldots, \sigma_7, \sigma_s, \pi_0, \ldots, \pi_8)$$

- Explicit symmetry breaking (SB) of $SU(N_f)_V$ does not appear in meson masses ($\sim \varphi$)
- 't Hooft determinant enters in meson masses ($\sim \varphi^3$)
- Scalar and pseudoscalar mixing angles from diagonalization of Hesse matrix

$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = R(\phi_S) \cdot \begin{pmatrix} \sigma_l \\ \sigma_s \end{pmatrix}, \quad \begin{pmatrix} \eta \\ \eta' \end{pmatrix} = R(\theta_P) \cdot \begin{pmatrix} \pi_0 \\ \pi_8 \end{pmatrix}$$

Flow Equation for Effective Potential

QM Flow Equation in Leading Potential Approximation

$$\partial_k \mathcal{U}_k(T, \mu_f, \phi)\big|_{\phi=\langle\phi\rangle} = \frac{k^4}{12\pi^2}\Bigg[\sum_{i=1}^{2N_f^2} \frac{1}{E_i}\coth\left(\frac{E_i}{2T}\right)$$

$$-2N_c \sum_{f=1}^{N_f} \frac{1}{E_{q,f}}\left(\tanh\left(\frac{E_{q,f}+\mu_f}{2T}\right) + \tanh\left(\frac{E_{q,f}-\mu_f}{2T}\right)\right)\Bigg]$$

$$E_i = \sqrt{k^2 + m_i^2}, \quad E_{q,l} = \sqrt{k^2 + \left(g\frac{\sigma_l}{2}\right)^2}, \quad E_{q,s} = \sqrt{k^2 + \left(g\frac{\sigma_s}{\sqrt{2}}\right)^2}$$

$m_i^2 \sim \partial_{\varphi_i}^2 \mathcal{U}_k \Rightarrow$ partial differential equation

Numerical Solution Techniques

Global Methods

- Solve $\partial_k \mathcal{U}_k(\rho)$ on *grid* of field values $\rho \to \rho_i$
- Derivatives $\partial_\rho \mathcal{U}$, $\partial_\rho^2 \mathcal{U}$ e.g. from finite difference formulas or interpolation methods
- 1^{st} order can be determined, but computationally expensive

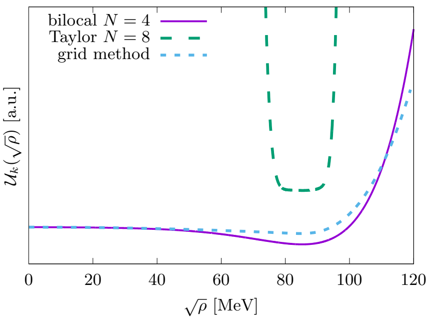
Taylor Expansion

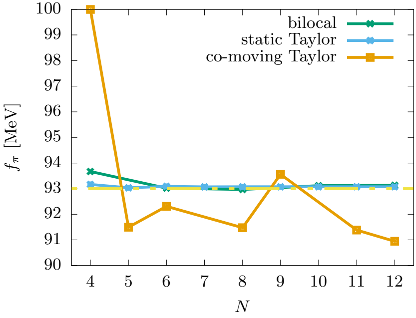
- Taylor expansion of $\mathcal{U}_k(\rho) = \sum_{n=0}^{N} \frac{a_{n,k}}{n!}(\rho - \rho_0)^n$
  $\Rightarrow$ coupled ODE for $\partial_k a_{n,k}$
- Co-moving: $\rho_0 \equiv \rho_0(k)$ follows the minimum of $\mathcal{U}_k(\rho)$
- Static: ρ_0 is fixed $\to$ better convergence!
- Fast, but hard to resolve 1^{st} order

Numerical Solution Techniques

Bilocal Expansion

- 2 coupled Taylor expansions
- Closer to global solution
  $\Rightarrow$ 1st order can be determined with only 4 couplings ($N = 4$)





[Resch et al.,2017] to be published

Numerical Implementation and UV Parameters

Numerical Setup

- $\mathcal{U}$ approximated on a 2D grid in coordinates $x = \sigma_l^2$, $y = 2\sigma_s^2 - \sigma_l^2$
  $\Rightarrow$ reliable identification of 1st order phase transition
- Derivatives in m_{φ_i} by cubic spline interpolation of $\mathcal{U}_k(x, y)$
- Resulting ordinary differential equation solved with semi-implicit time stepping algorithm

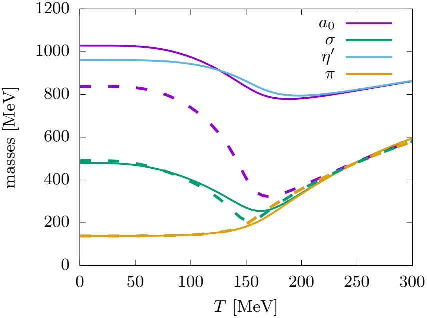
Parameter Fixing

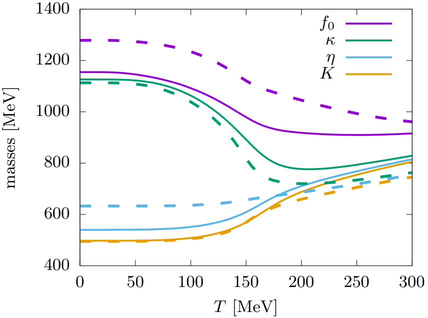
Ansatz:

$$\mathcal{U}_\Lambda = a_{10}\rho_1 + \frac{a_{20}}{2}\rho_1^2 + a_{01}\tilde{\rho}_2 - c_A \det(\phi + \phi^\dagger) - c_l\sigma_l - c_s\sigma_s$$

Parameters fixed at $\Lambda = 700, 1000$ MeV to experimental values m_π, f_π, m_K, f_K, $m_\eta + m'_\eta$, $m_{q,l}$ and $m_\sigma = 400, \ldots, 560$ MeV in IR.

Meson Masses





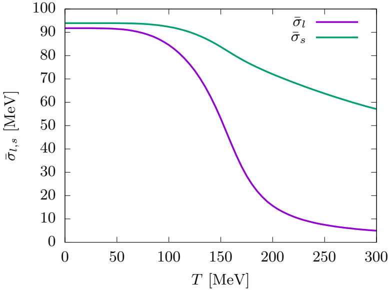
Solid: $c_A \neq 0$, dashed: $c_A = 0$.

cf. [Mitter,Schaefer,2014]

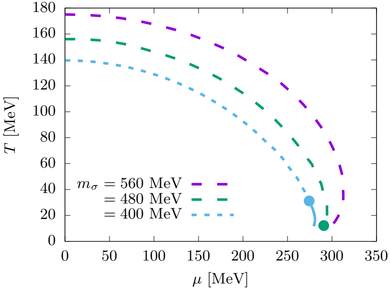
Without $U(1)_A$ breaking

- η' becomes a (pseudo) Goldstone boson, degenerates with π
- a_0 and π degenerate for $T > T_c$

Condensates and Phase Diagram



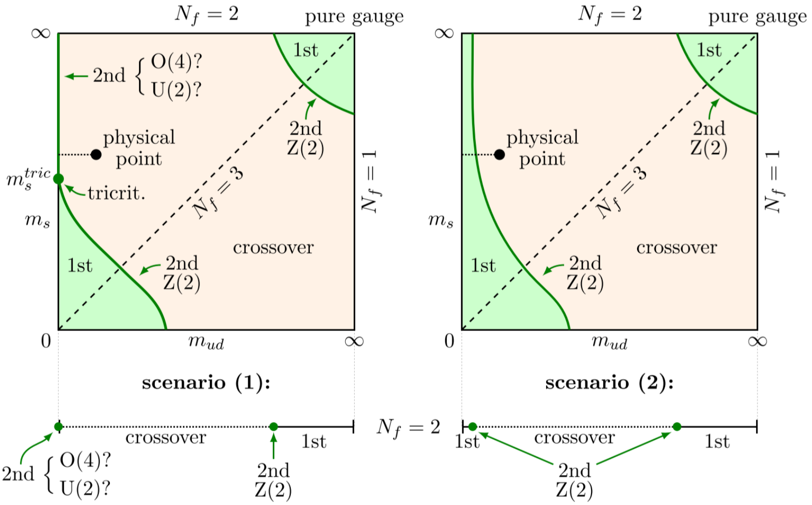
- Crossover at $\mu = 0$
- Light chiral symmetry restored first
- Strange condensate melts relatively slow



- T_c increases with sigma mass
- Slightly smaller 1st order region compared to $N_f = 2$

Mass Sensitivity of the Phase Diagram

Two Scenarios for the Columbia Plot



[Brandt,Philipsen,et al.,2016]

Limits of the Columbia Plot

- 1^{st} order phase transition for $N_f \geq 3$ massles quarks [Pisarski,Wilczek,1984]
- 1^{st} or 2^{nd} order depending on $U(1)_A$ anomaly at T_c for $N_f = 2$
- 1^{st} order for infinitely heavy quarks → $N_c = 3$ Yang–Mills theory

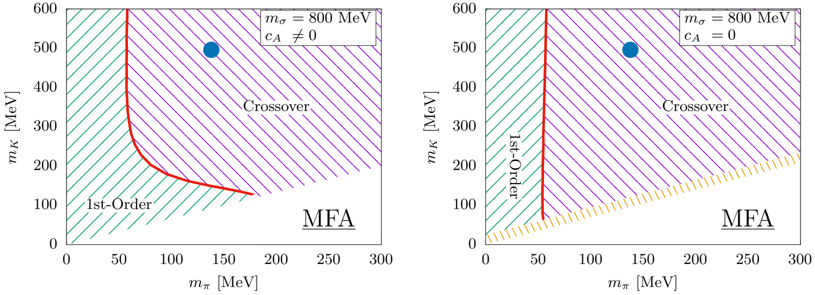
Is there a tricritical point on the $m_l = 0$ axis?

- Lattice simulations expensive for light fermions
- Possible universality classes: $Z(2)$, $O(4)$, $U(2)$ with critical exponents $\delta\beta \approx 1.56, 1.86, 1.85$ (too close to call)

  ⇒ No conclusive answer from Lattice calculations yet

MFA Columbia Plot

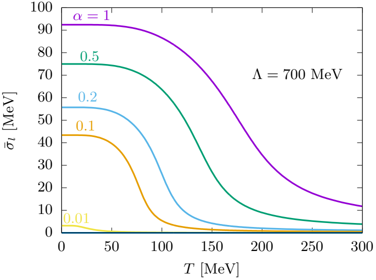
Order of chiral phase transition in (m_π, m_K) plane. No meson fluctuations and quark vacuum term (MFA)

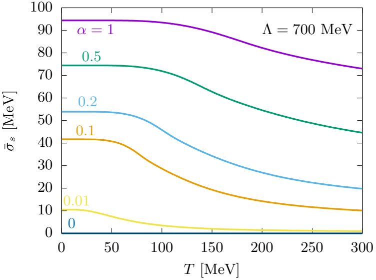


cf. [Schaefer,Wagner,2009]

- m_π, m_K adjusted with explicit SB c_l, c_s
- No tricritical point at $m_\pi = 0$ independent of $U(1)_A$ breaking
- m_σ high because $\bar{\sigma}_l$ vanishes in chiral limit for $m_\sigma \lesssim 700$ MeV

FRG Analysis





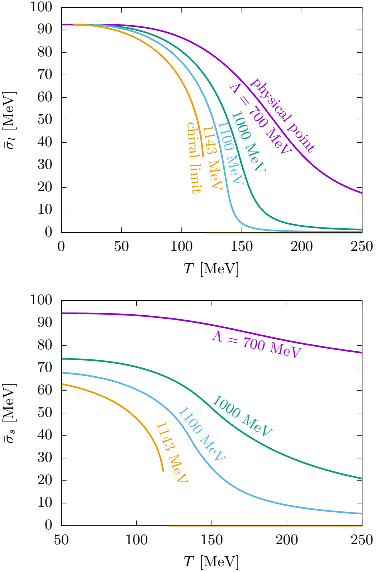
Condensates towards the chiral limit

- Parameter α interpolates between $N_f = 3$ chiral limit and physical point

$$\begin{pmatrix} c_l \\ c_s \end{pmatrix} = \alpha \cdot \begin{pmatrix} c_{l,phys} \\ c_{s,phys} \end{pmatrix}$$

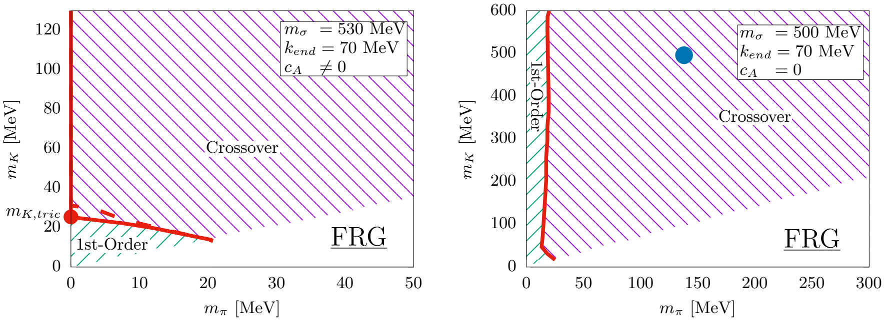
- $\bar{\sigma}_l$, σ_s vanish in chiral limit ($\alpha \to 0$) at $T = 0$!
  $\Rightarrow$ No spontaneous SB in chiral limit?

FRG Analysis



- Lattice with $N_f = 8$ light quarks: clear sign of spontaneous SB [Appelquist et al.,2014]
- Likely a result of parameter fixing procedure not valid in chiral limit
- Idea: adjust Λ to fix $\bar{\sigma}_l(T = 0) = f_\pi = 93$ MeV independent of c_l, c_s; other parameters unchanged
  $\Rightarrow$ forces spontaneous SB in chiral limit
- Constituent quark mass $m_{q,l} = g\bar{\sigma}_l/2$ almost entirely generated by spontaneous SB

FRG Columbia Plot

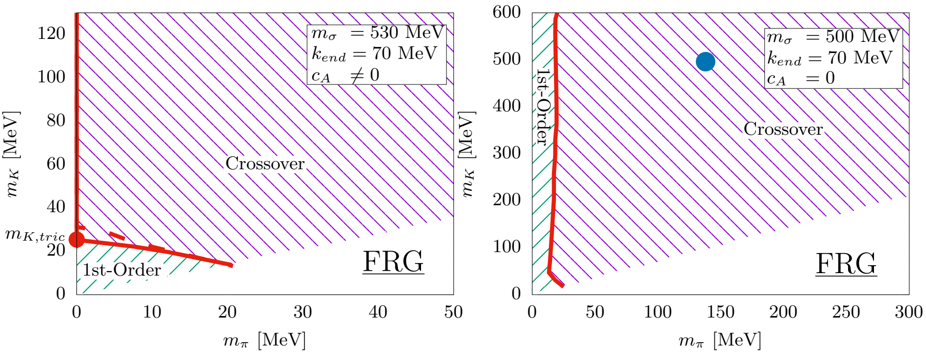


[Resch et al.,2017] to be published

With explicit breaking of $U(1)_A$ (left):

- Small 1st order region around chiral limit — cf. [Pisarski,Wilczek,1984]
- Tricritical point at $m_{K,\text{tric}} \sim 25$ MeV
- 2nd order line connects to $SU(2)_R \otimes SU(2)_L$ model for $m_K \to \infty$

FRG Columbia Plot



[Resch et al.,2017] to be published

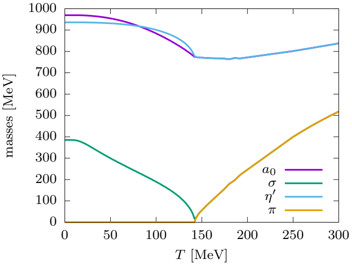
Without breaking of $U(1)_A$ (right):

- Tricritical point in the $N_f = 2$ limit

Restoration of $U(1)_A$ in the QM Model

What constitutes restoration of $U(1)_A$?

- In action: $\sim c_A \bar{\sigma}_l^2 \bar{\sigma}_s$
  $\Rightarrow$ small for $T > T_c$ because of $\bar{\sigma}_l^2$
- Meson spectrum: $m_{a_0}^2 - m_\pi^2 \sim c_A \bar{\sigma}_s$
  $\Rightarrow$ mass gap persists for $T > T_c$ due to slowly vanishing $\bar{\sigma}_s$



Light chiral limit $c_l = 0$

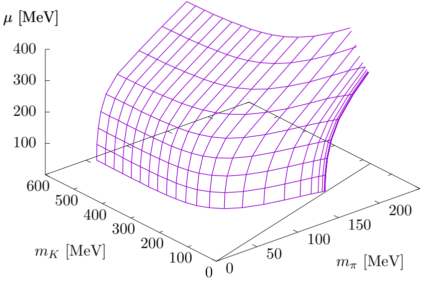
Recent Lattice Results

$\Delta M_{PS}^{m_{ud}=0}(T = T_c) = -81(282)$ MeV by chiral extrapolation [Brandt,Philipsen et al.,2016]

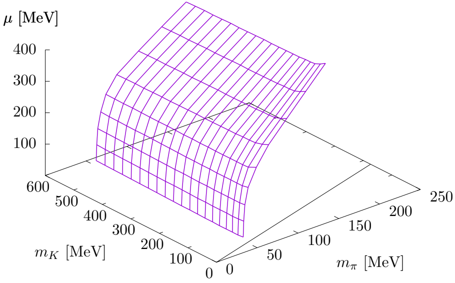
$\Rightarrow U(1)_A$ anomaly weak at T_c?

$c_A(T, k)$ needed in QM model? External input?

MFA Columbia Plot (finite μ)



(a) $c_A \neq 0$



(b) $c_A = 0$

cf. [Schaefer,Wagner,2009]

- Standard scenario: positive curvature
- Should also hold in FRG (CEP at physical m_π, m_K)

Outlook

- Finite μ curvature of chiral critical line with FRG
- Temperature and fluctuation dependence of 't Hooft determinant
- Increase truncation beyond LPA
  - Mixing angles sensitive to CEP? [Rennecke,Schaefer,2016]
- Polyakov loop → deconfinement transition

End goal: Full QCD flow.