Quasiparticle anisotropic hydrodynamics

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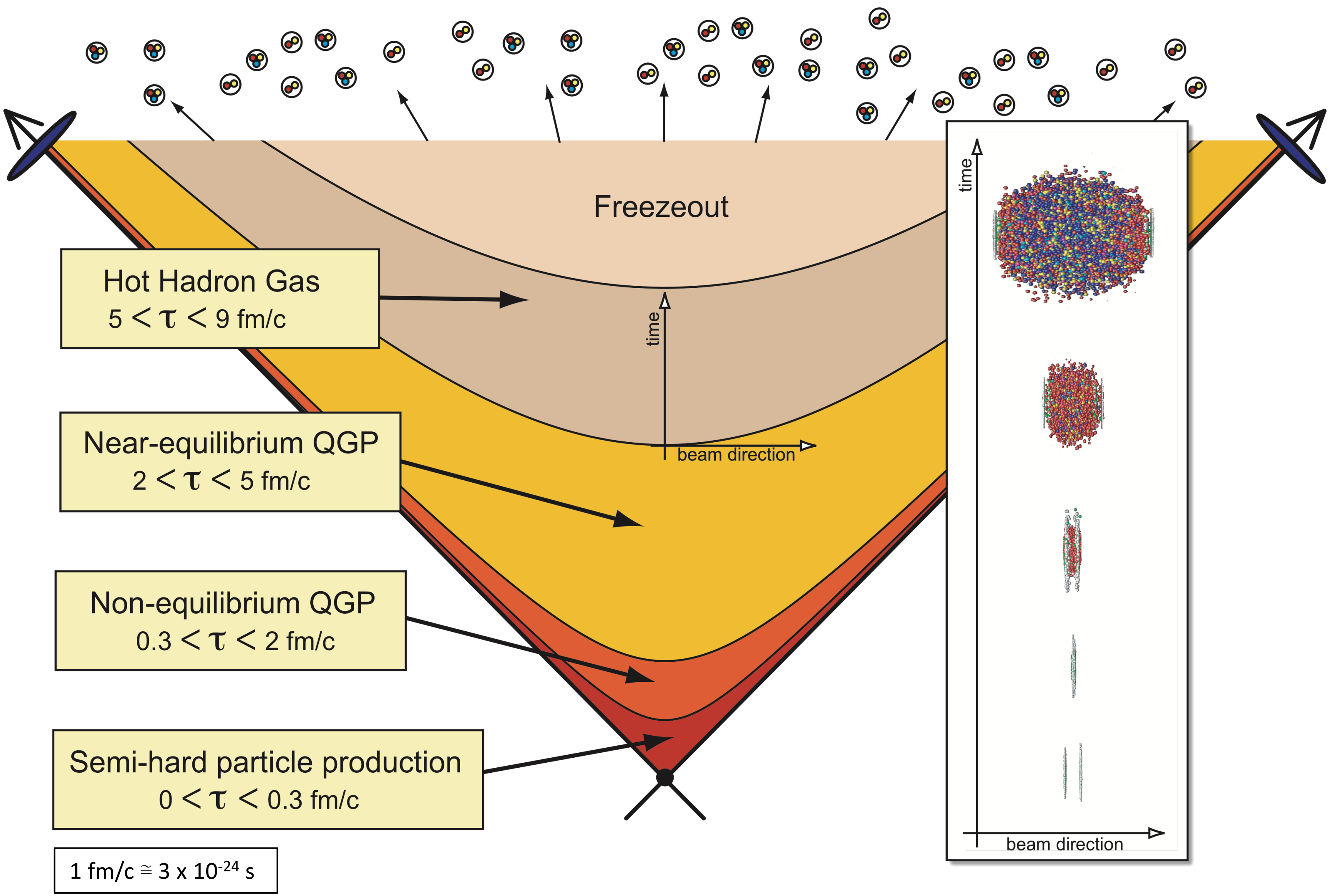
Primary References: M. Alqahtani, M. Nopoush, R. Ryblewski, and MS
1703.05808 (accepted to PRL) and 1705.10191

Theoretical Hadron Physics Lunch Seminar
Institute for Theoretical Physics, Justig-Liebig-Universität Giessen – July 5, 2017

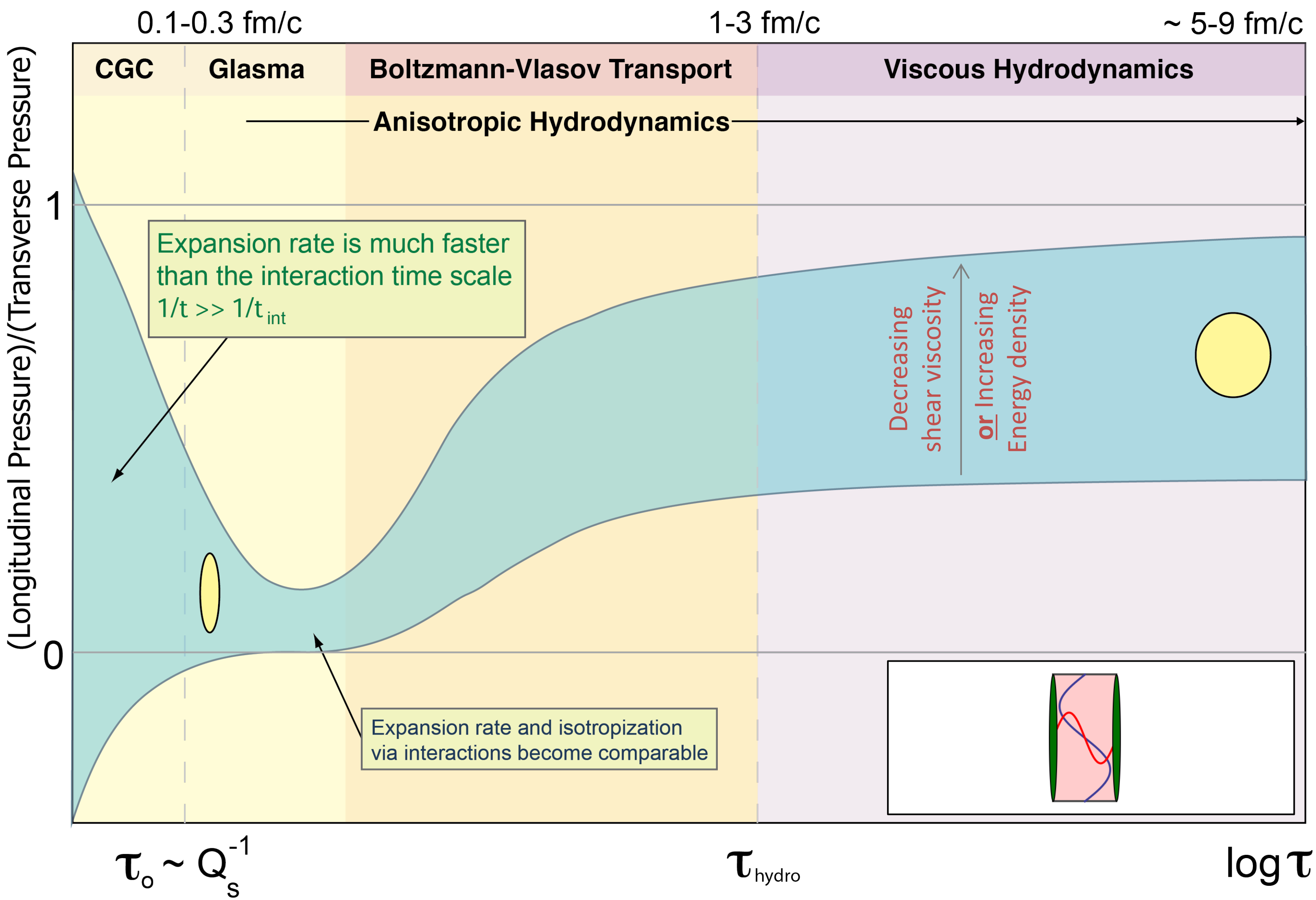
Motivation

- Relativistic viscous hydrodynamical modeling for heavy ion collisions at RHIC and LHC is now ubiquitous
- Designed to describe particle production at $p_T \lesssim 2$ GeV
- Application is justified a posteriori by the smallness of the shear viscosity of the plasma (relative to the entropy density)
- Viscous hydrodynamics is phenomenologically quite successful, however, the extreme environment generated in HICs presents a bit of a challenge to the standard formalism if you start looking closer
- The **QGP** is born into a state of rapid longitudinal expansion which drives the system **out of equilibrium**
- There are many groups now focused on improving viscous hydrodynamics itself in order to better describe systems that are **far from equilibrium**, e.g. **anisotropic hydrodynamics**

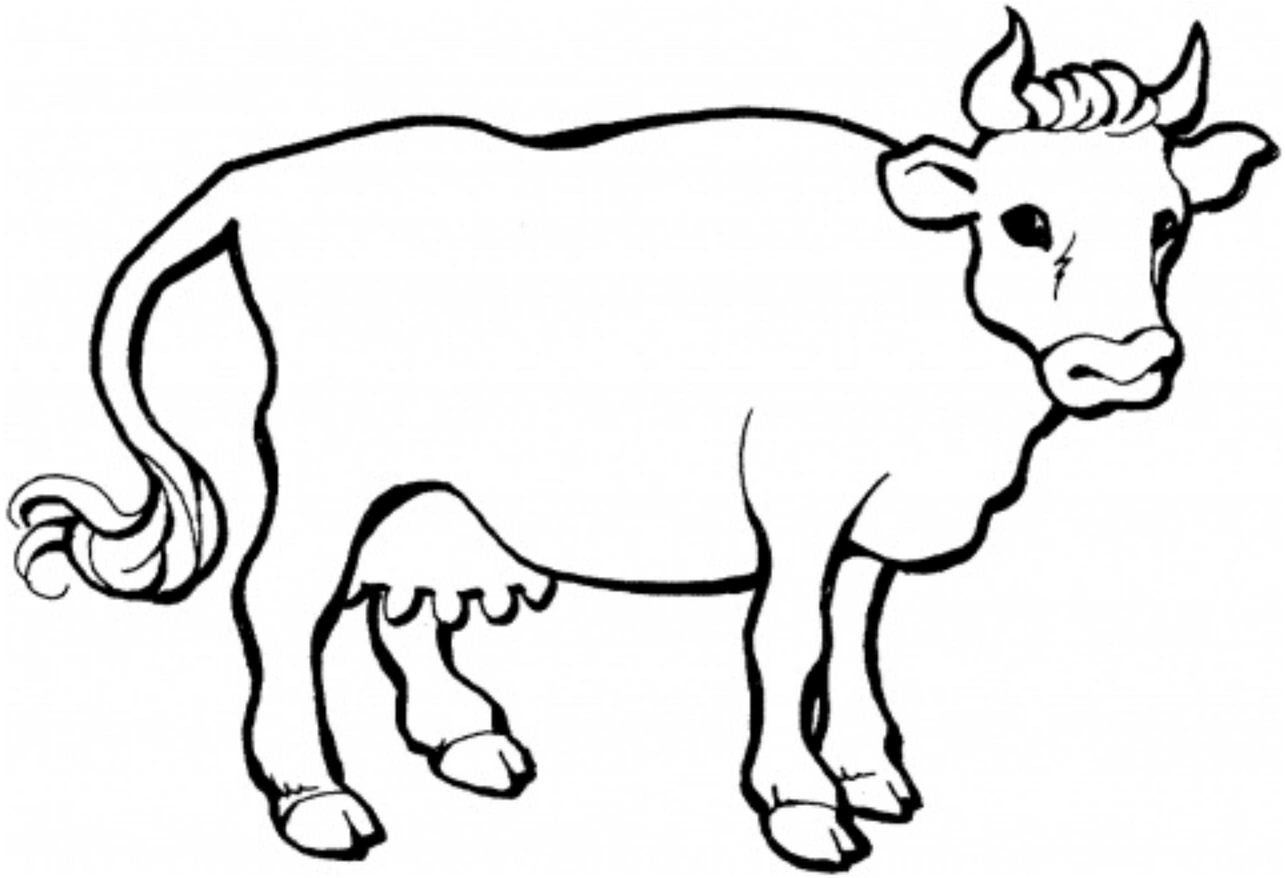
Heavy Ion Collision Timescales



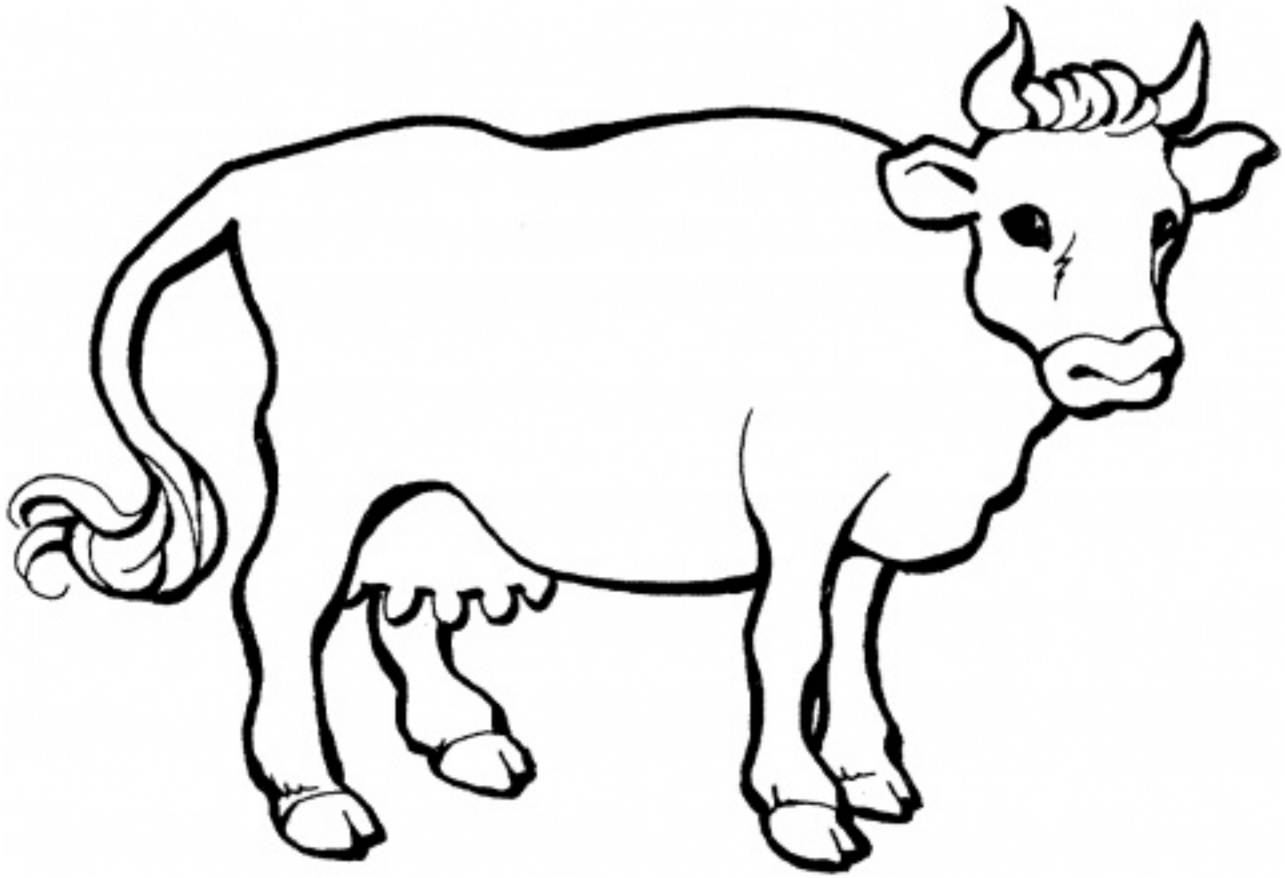
QGP momentum anisotropy cartoon



Physics 101



Cows are spheres?



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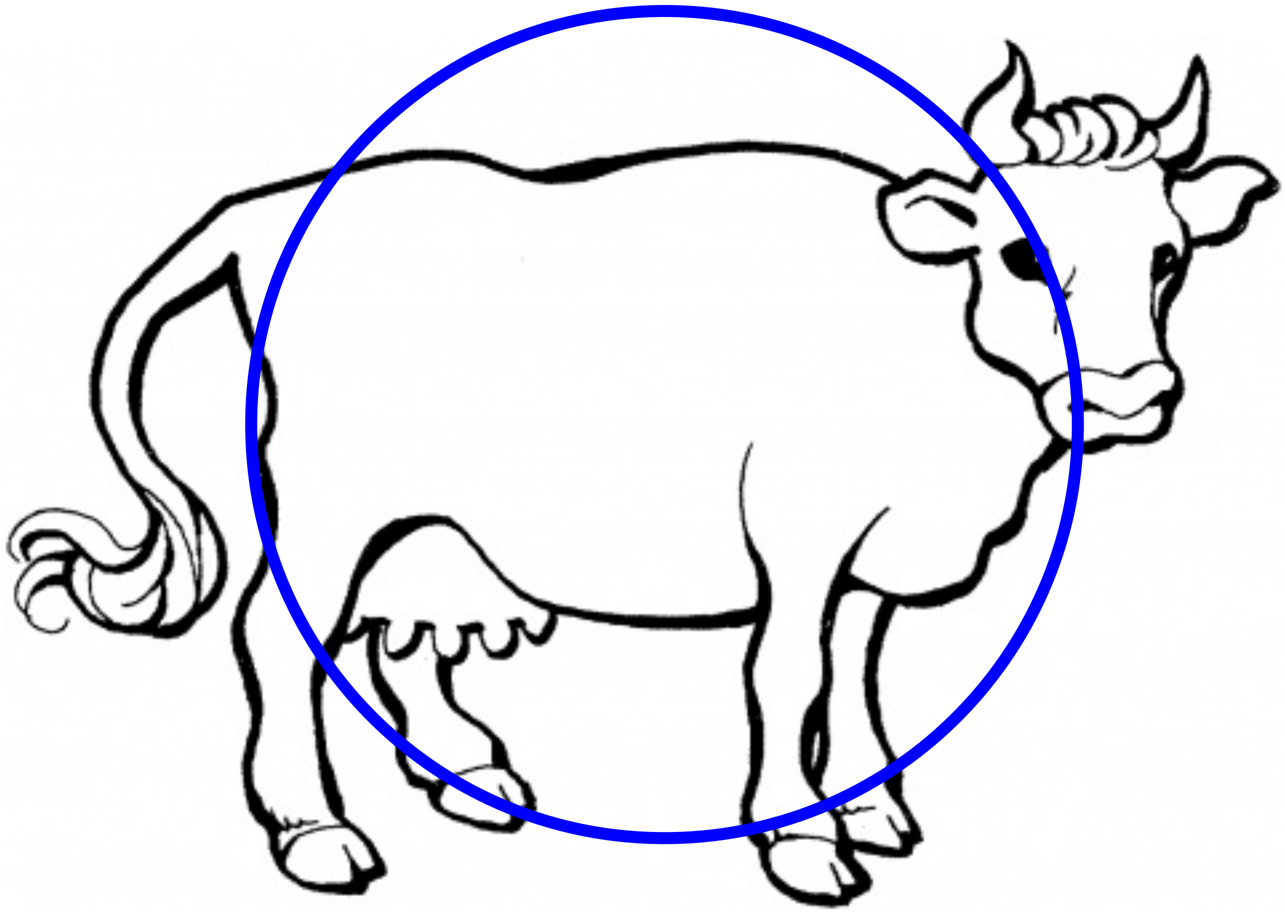
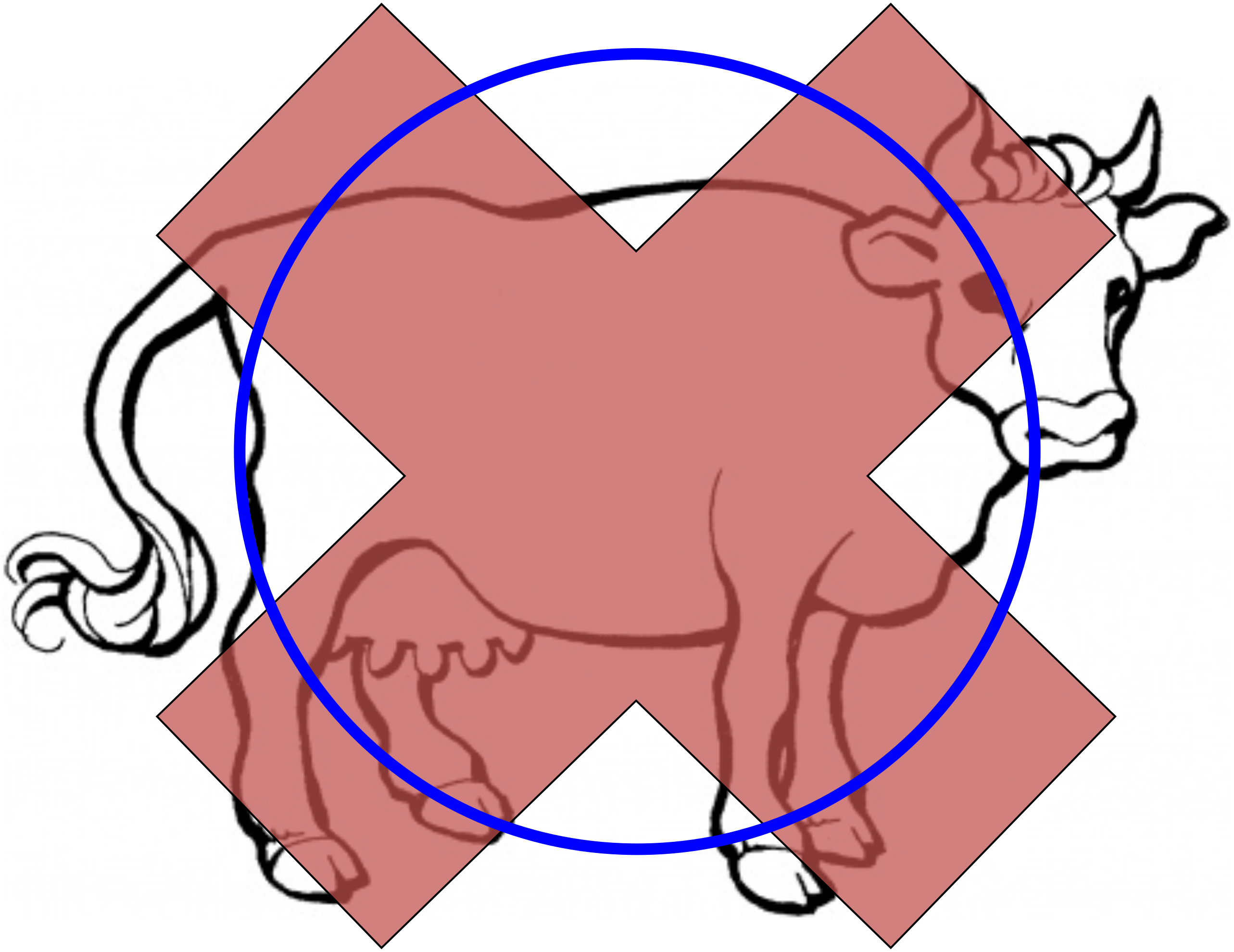
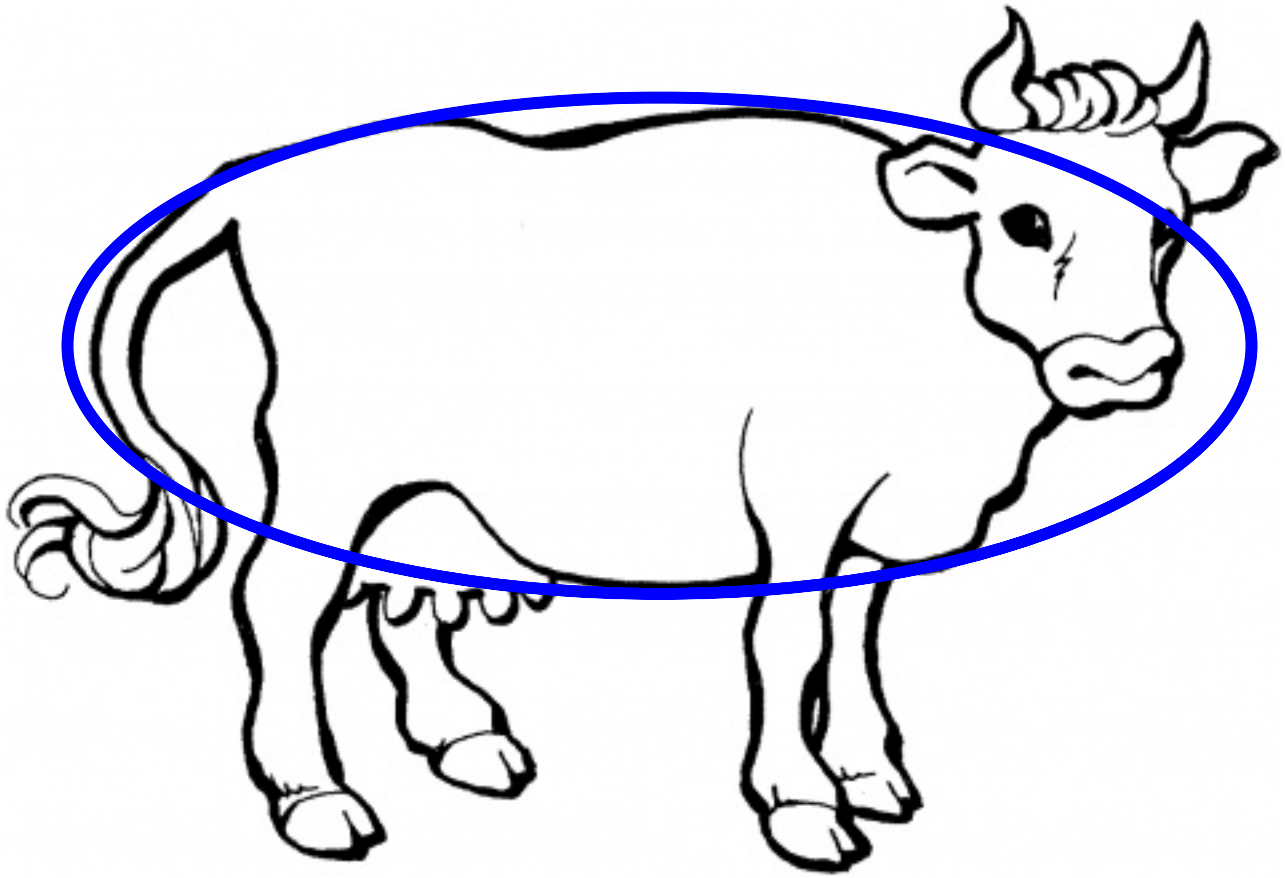


Cows are <u>not</u> spheres



Cows are more like ellipsoids!



Spheroidal expansion method

Viscous Hydrodynamics Expansion

See e.g.
- M. Martinez and MS, 1007.0889
- W. Florkowski and R. Ryblewski, 1007.0130
- D. Bazow, U. Heinz, and MS, 1311.6720
- D. Bazow, U. Heinz, and M. Martinez, 1503.07443
- E. Molnar, H. Niemi, and D. Rischke, 1602.00573; 1606.09019

$$f(\tau, \mathbf{x}, \mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p}, T(\tau, \mathbf{x}))} + \delta f$$

Isotropic in momentum space

Anisotropic Hydrodynamics (aHydro) Expansion

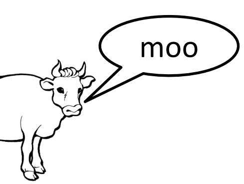
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{\rm aniso}(\mathbf{p}, \underbrace{\Lambda(\tau, \mathbf{x})}_{T_\perp}, \underbrace{\xi(\tau, \mathbf{x})}_{\rm anisotropy}\,) + \delta\tilde{f}$$

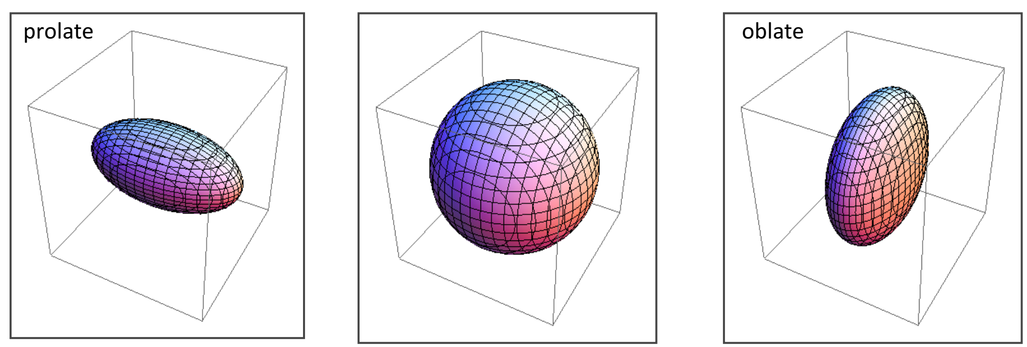
Treat this term perturbatively
→ "NLO aHydro"

→ "Romatschke-Strickland" form in LRF

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x}, \tau) p_z^2}}{\Lambda(\mathbf{x}, \tau)}\right)$$

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

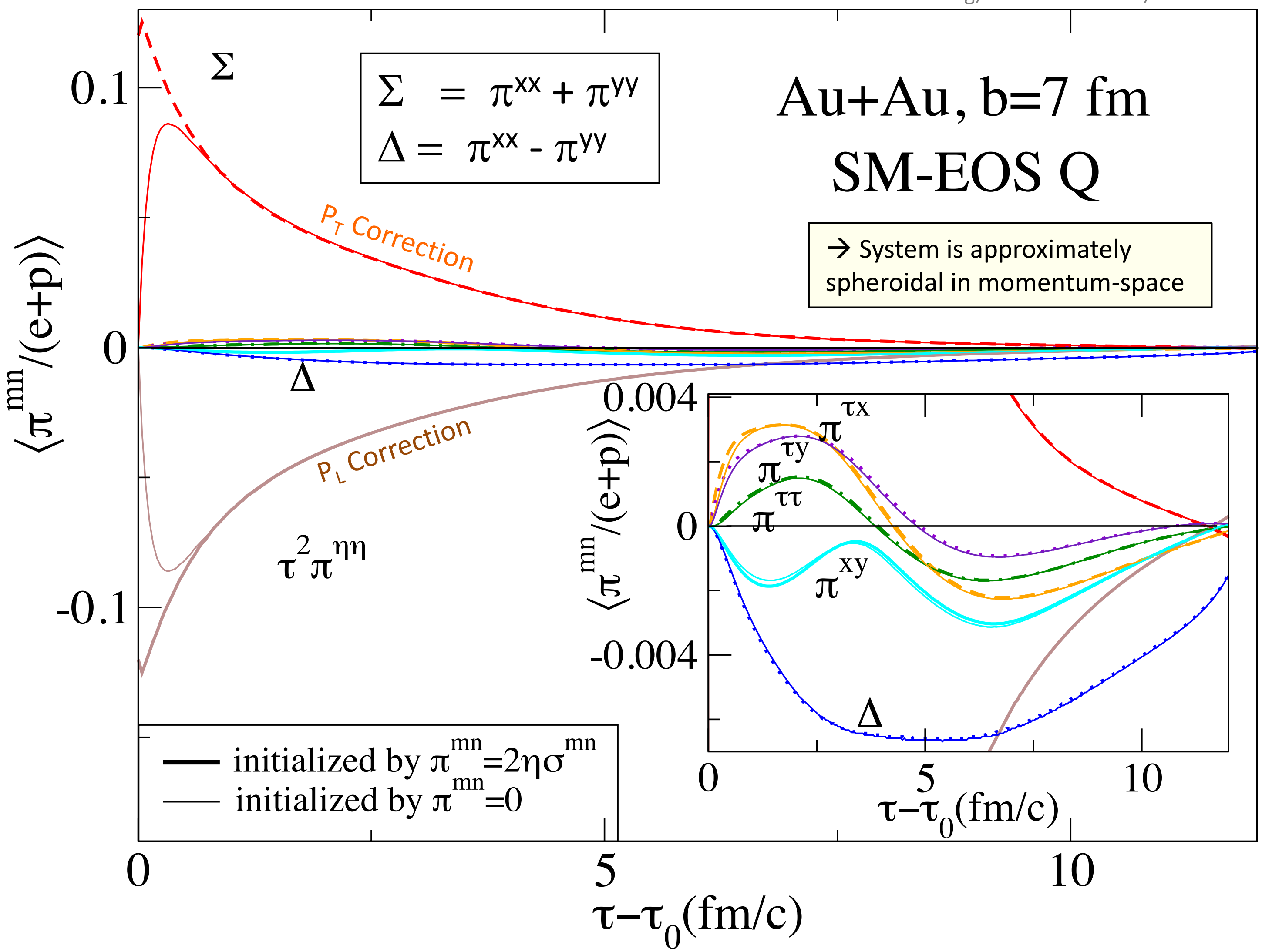




$-1 < \xi < 0$ $\qquad$ $\xi = 0$ $\qquad$ $\xi > 0$

What are the largest viscous corrections?

H. Song, PhD Dissertation, 0908.3656



→ System is approximately spheroidal in momentum-space

Why spheroidal form at LO?

- What is special about this form at leading order?

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

- Gives the ideal hydro limit when $\xi=0$ ($\Lambda \rightarrow T$)
- For longitudinal (0+1d) free streaming, the LRF distribution function is of spheroidal form; limit emerges automatically in 0+1d aHydro

$$\xi_{\rm FS}(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

- Since $f_{iso} \geq 0$, the one-particle distribution function and pressures are ≥ 0 (not guaranteed in standard 2nd-order viscous hydro)
- Reduces to 2nd-order viscous hydrodynamics in limit of small anisotropies

M. Martinez and MS, 1007.0889

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

For general (3+1d) proof of equivalence to second-order viscous hydrodynamics using generalized RS form in the near-equilibrium limit see Tinti 1411.7268.

The growing anisotropic hydrodynamics family

- There are two approaches being actively followed in the literature to address this problem

  A. Linearize around a spheroidal distribution function and treat the perturbations using standard kinetic vHydro methods ["vaHydro"]
  Bazow, Heinz, Martinez, Molnar, Niemi, Rischke, MS

  B. Introduce a generalized anisotropy tensor which replaces the entire viscous stress tensor at LO and then linearize around that instead
  Tinti, Ryblewski, Martinez, Nopoush, Alqahtani, Florkowski, Molnar, Niemi, Rischke, Schaefer, Bluhm, MS

- Each of these methods has its own advantages.
- In what I will show today, I will use the generalized method (B) at leading order.

Generalized aHydro formalism

In generalized aHydro, one assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{Traceless symmetric anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{"Bulk"}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- 3 degrees of freedom in u^μ
- 5 degrees of freedom in $\xi^{\mu\nu}$
- 1 degree of freedom in Φ
- 1 degree of freedom in λ
- 1 degree of freedom in μ

→ 11 DOFs

See e.g.
- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

Equations of Motion

- Herein the EOM are obtained from moments of the Boltzmann equation in the relaxation time approximation (RTA)

$$p^\mu \partial_\mu f = -\mathcal{C}[f] \qquad \mathcal{C}[f] = \frac{p^\mu u_\mu}{\tau_{\rm eq}}(f - f_{\rm eq})$$

- **It is relatively straightforward to use other collisional kernels (in progres)**

- 1 equation from the 0th moment [number (non-conservation)]
- 4 equations from the 1st moment [energy-momentum conservation]
- 6 equations from the 2nd moment [dissipative dynamics]
- We must also specify the relation between the equilibrium (isotropic) pressure and energy density (EoS). More on this later.

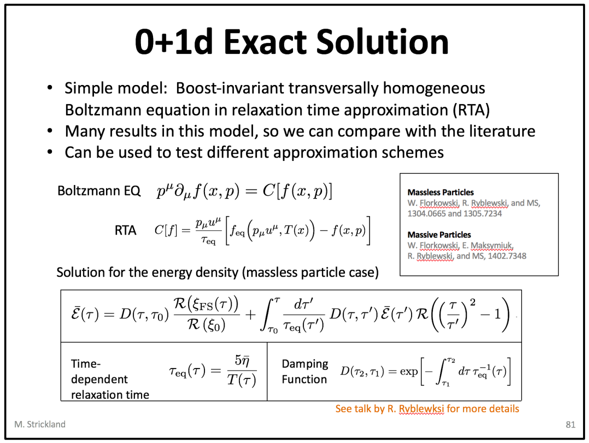
$$D_u n + n\theta_u = \frac{1}{\tau_{\rm eq}}(n_{\rm eq} - n)$$

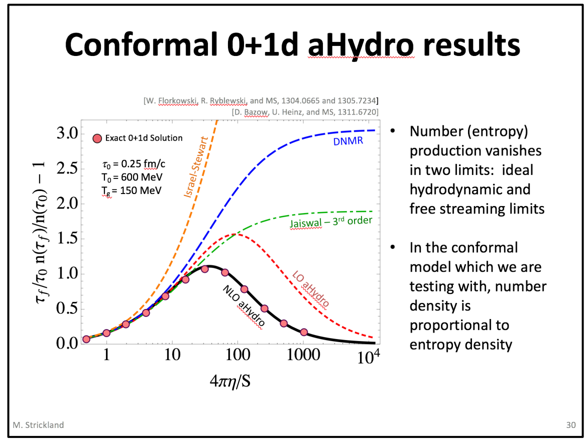
$$\partial_\mu T^{\mu\nu} = 0$$

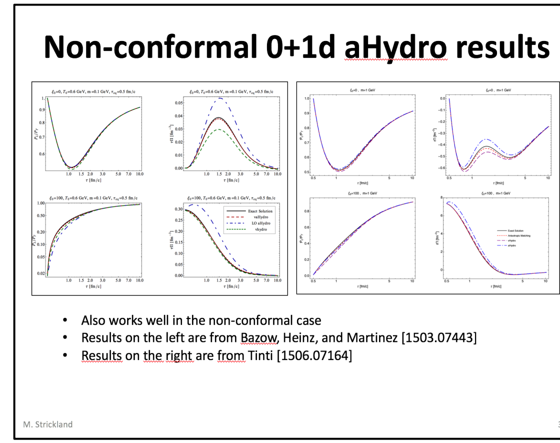
$$\partial_\mu \mathcal{I}^{\mu\nu\lambda} = \frac{1}{\tau_{\rm eq}}(u_\mu \mathcal{I}^{\mu\nu\lambda}_{\rm eq} - u_\mu \mathcal{I}^{\mu\nu\lambda}) \qquad \mathcal{I}^{\mu\nu\lambda} \equiv \int dP\, p^\mu p^\nu p^\lambda f(x,p)\,.$$

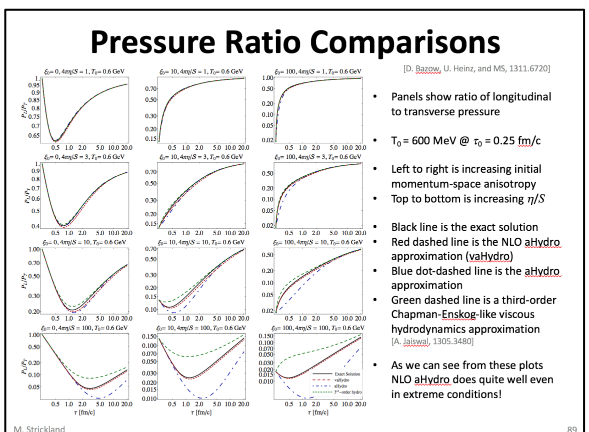
Is it really better?

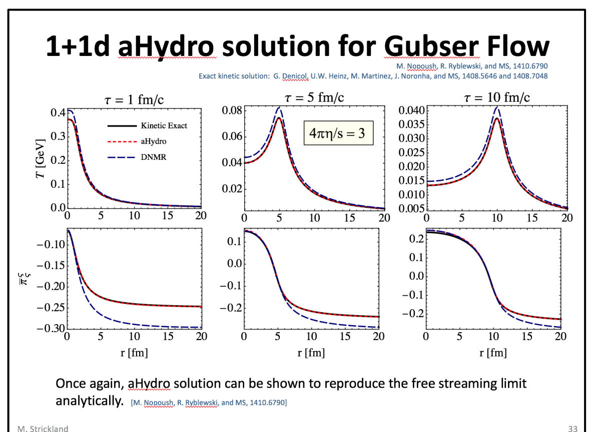
aHydro reproduces exact solutions to the Boltzmann equation in a variety of expanding backgrounds better than standard viscous Hydro.

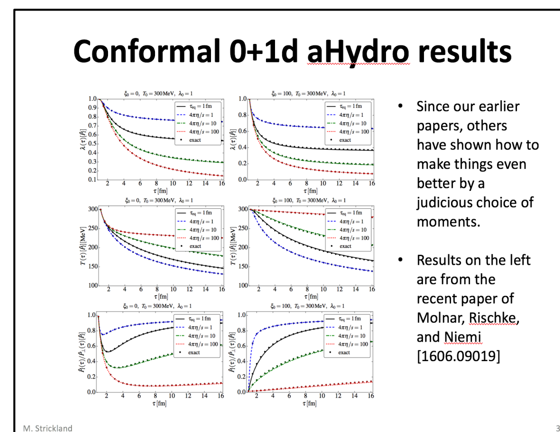




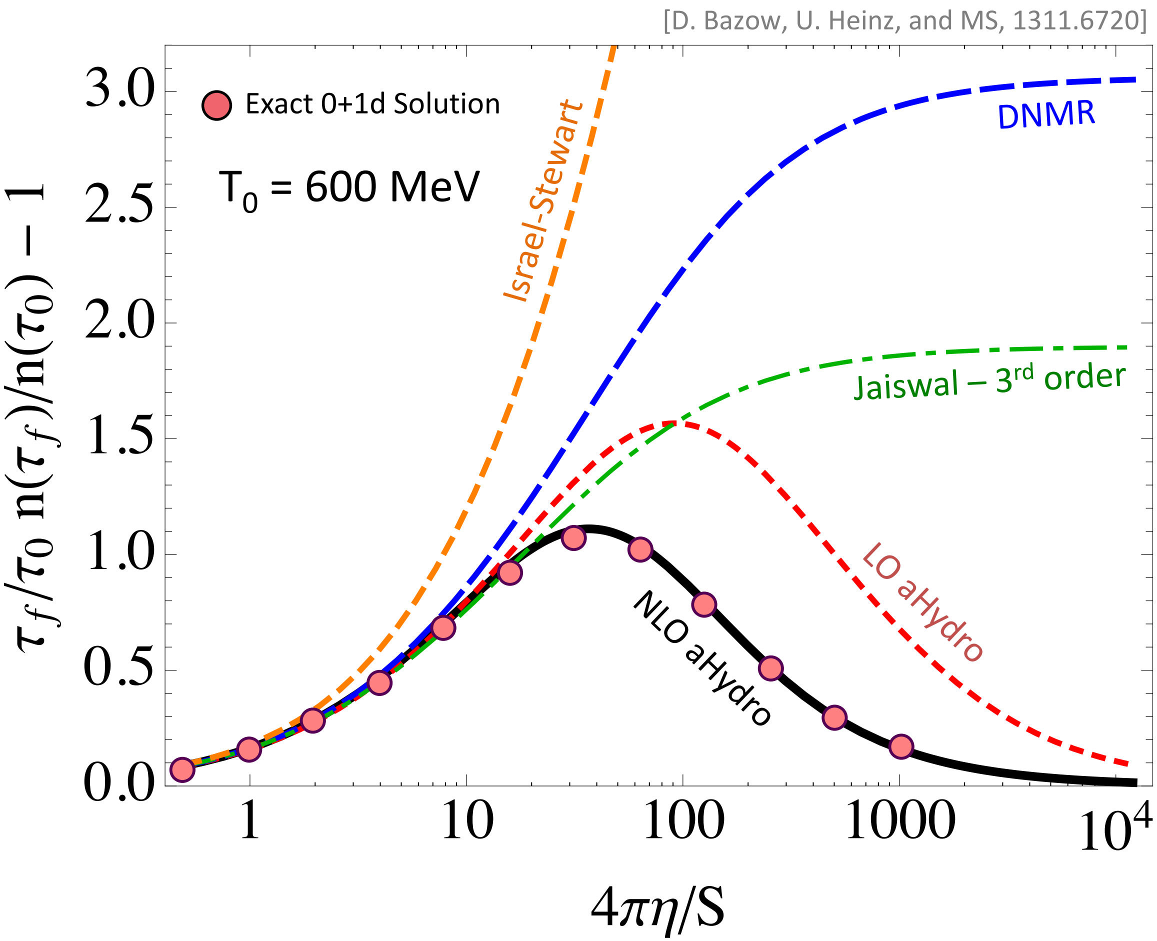






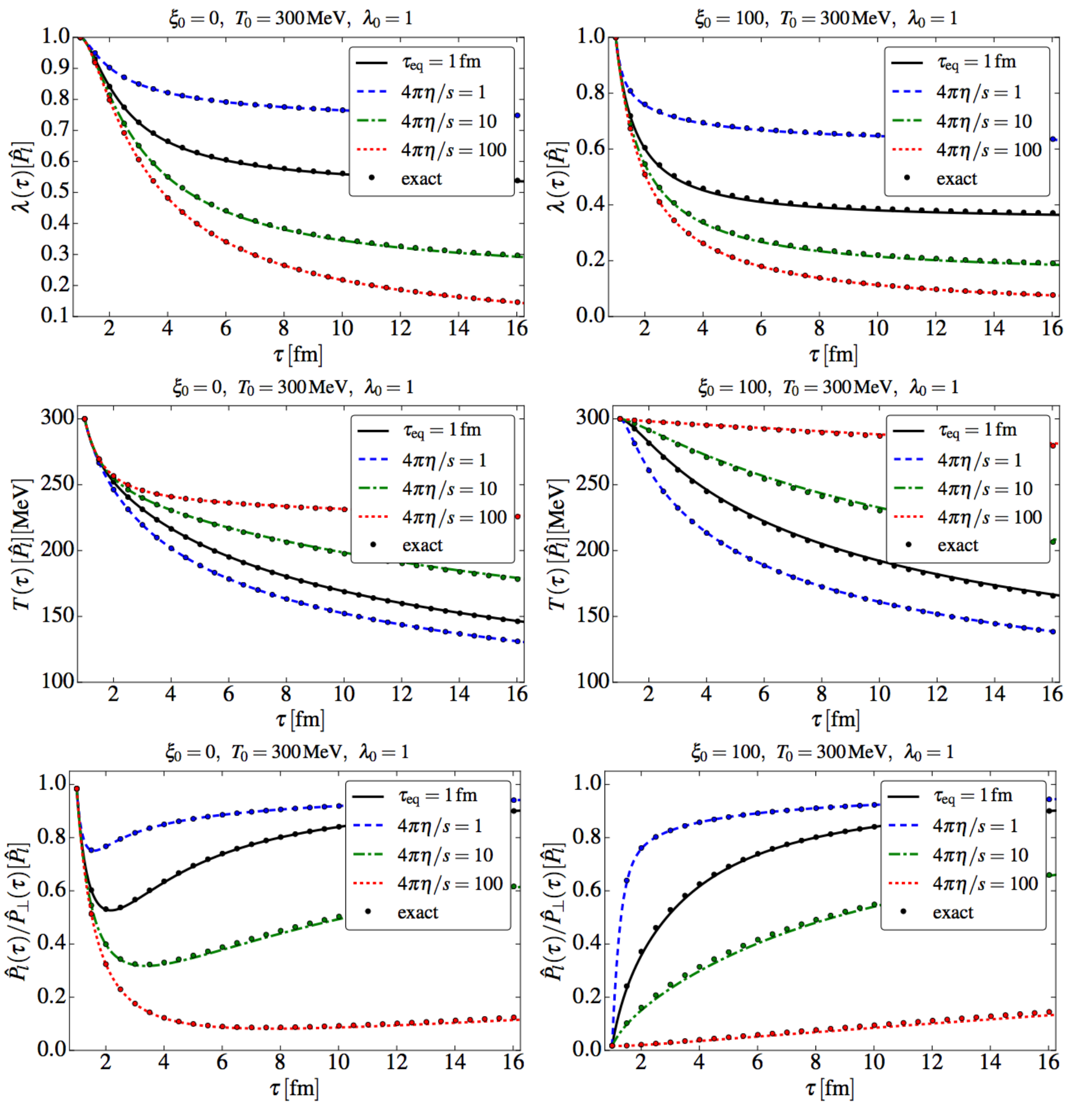


Ex. 1: Entropy Generation



- Number (entropy) production vanishes in two limits: ideal hydrodynamic and free streaming limits
- In the conformal model which we are testing with, number density is proportional to entropy density

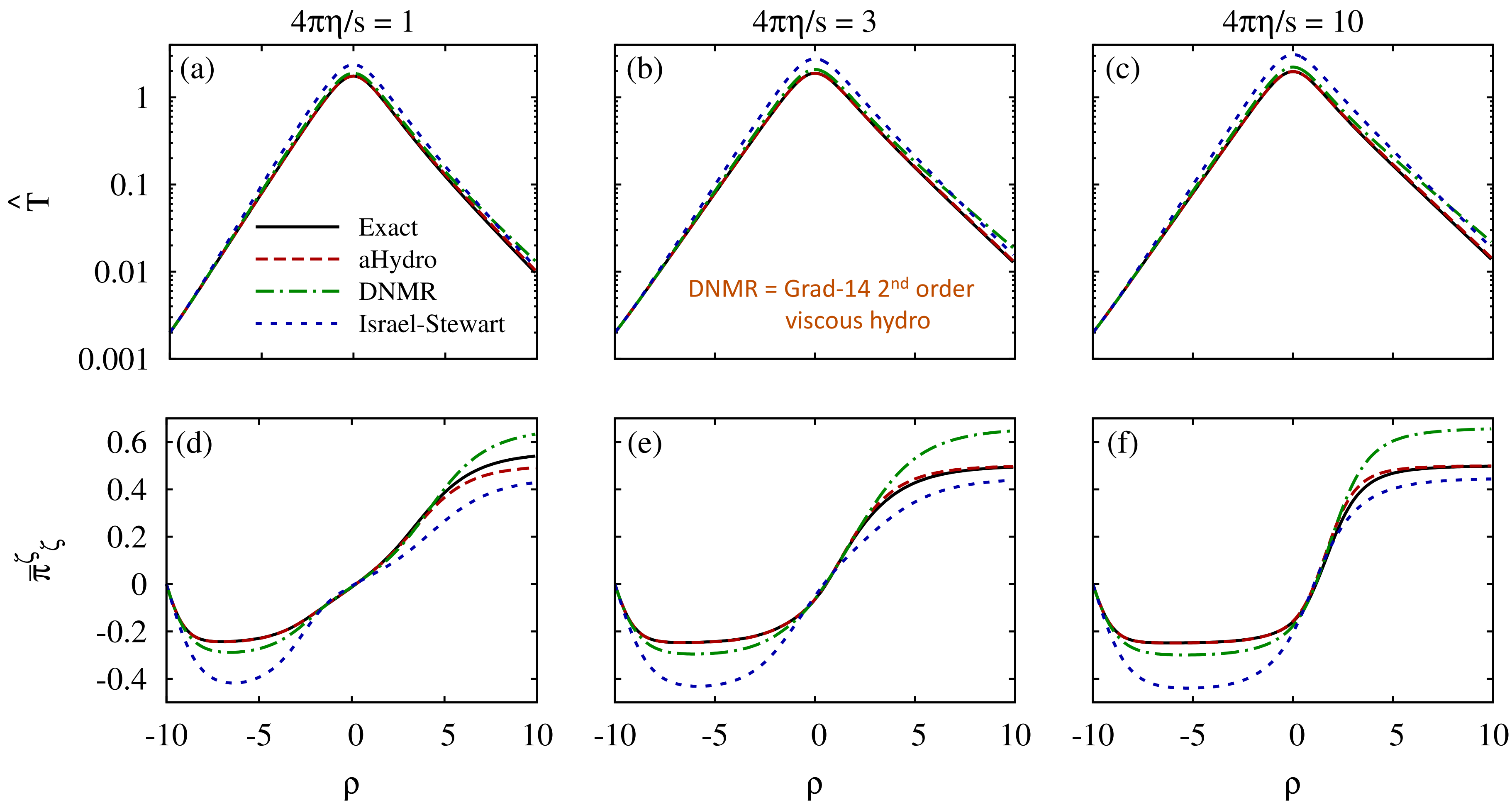
Ex. 2: Conformal 0+1d aHydro results



- aHydro results (lines) on the left are from the recent paper of Molnar, Rischke, and Niemi [1606.09019]
- Exact result is shown by dots [W. Florkowski, R. Ryblewski, and MS, 1304.0665 and 1305.7234]

Ex 3: LO aHydro for Gubser flow

M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact Solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048



Isotropic initial conditions

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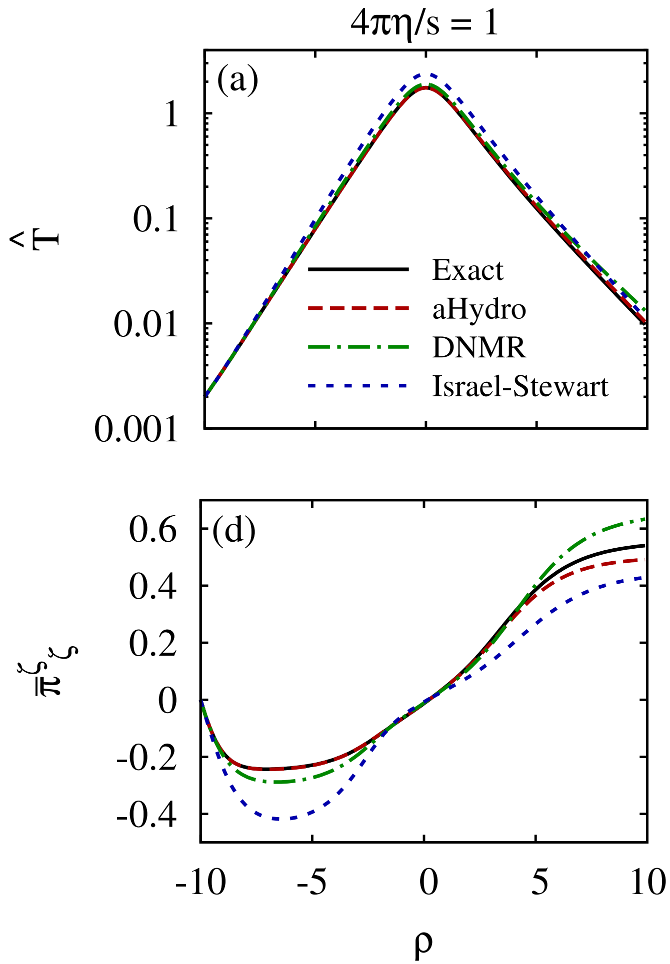
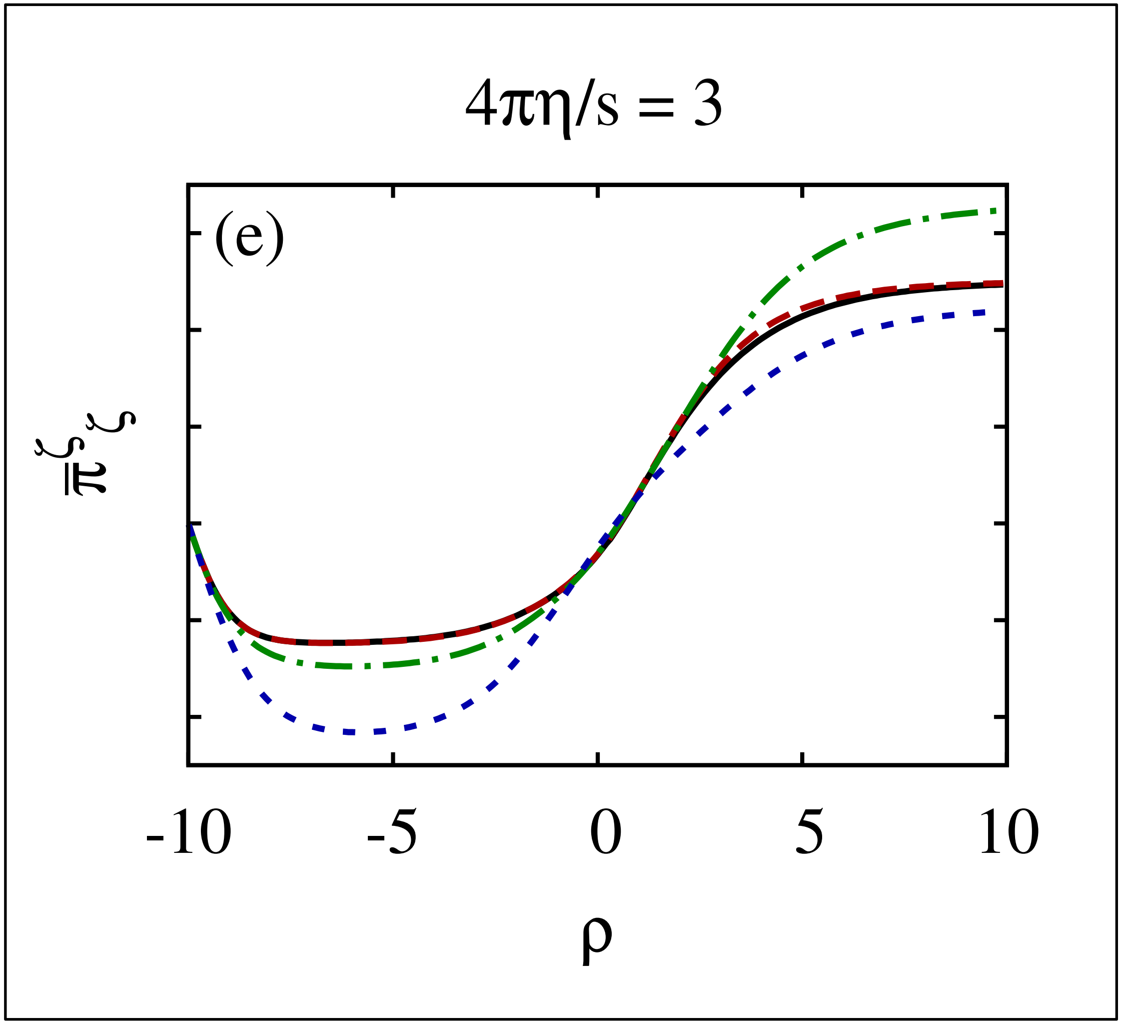
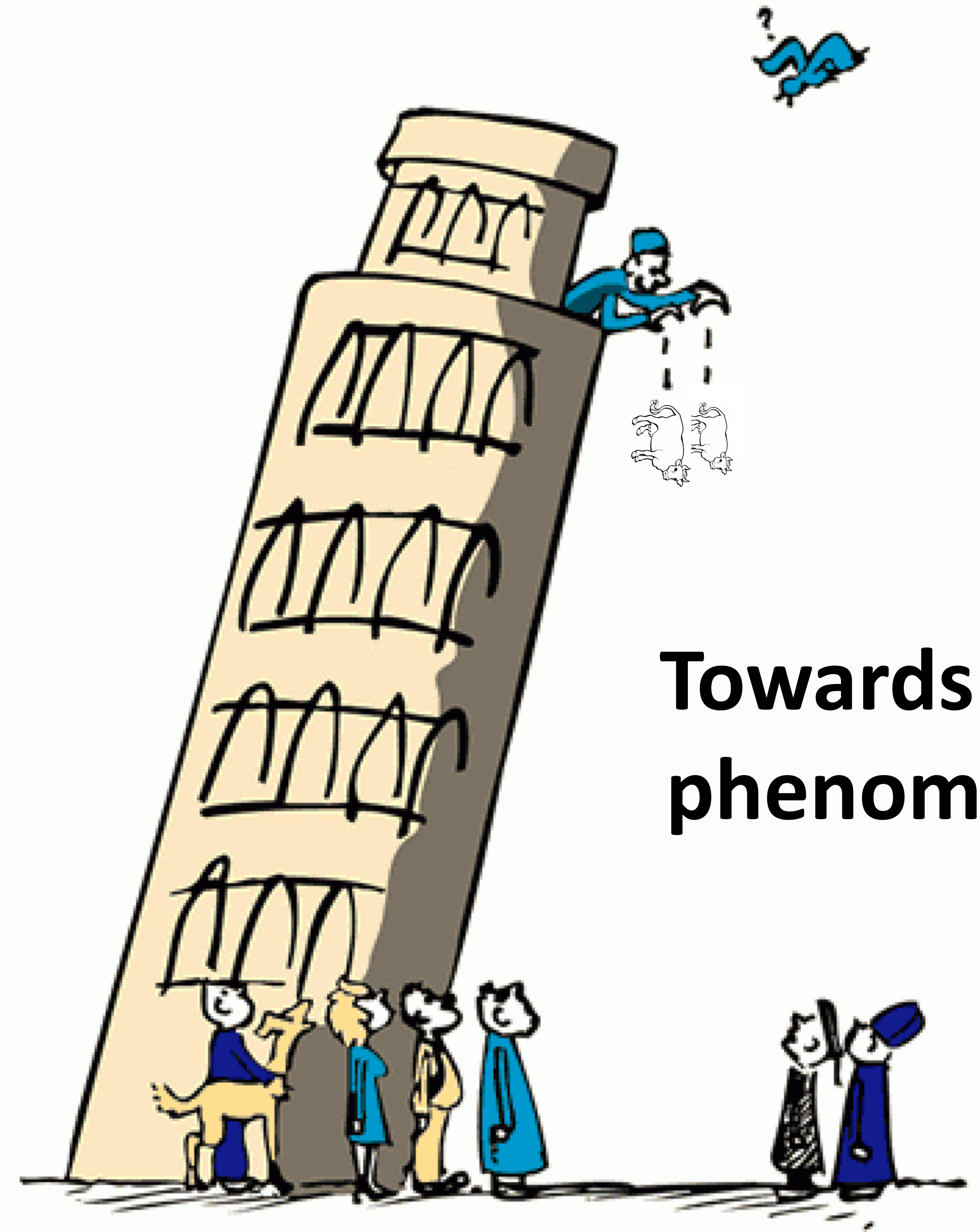




See also Martinez, McNelis, Heinz, 1703.10955 for NLO aHydro for Gubser Flow



Towards realistic phenomenology

3+1d aHydro Equations of Motion

Assuming an ellipsoidal form for the anisotropy tensor (ignoring off-diagonal components for now), one has seven degrees of freedom ξ_x, ξ_y, ξ_z, u_x, u_y, u_z, and λ which are all fields of space and time.

$$
\begin{aligned}
D_u\mathcal{E} + \mathcal{E}\theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu &= 0\,,\\
D_x\mathcal{P}_x + \mathcal{P}_x\theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu &= 0\,,\\
D_y\mathcal{P}_y + \mathcal{P}_y\theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu &= 0\,,\\
D_z\mathcal{P}_z + \mathcal{P}_z\theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu &= 0\,.
\end{aligned}
$$

First Moment

$$
\mathcal{I}^{\mu\nu\lambda} \equiv \int dP\, p^\mu p^\nu p^\lambda f(x,p)\,.
$$

$$
\begin{aligned}
\mathcal{I}_i &= \alpha\, \alpha_i^2\, \mathcal{I}_{\rm eq}(\lambda, m)\,,\\
\mathcal{I}_{\rm eq}(\lambda, m) &= 4\pi \tilde{N} \lambda^5 \hat{m}^3 K_3(\hat{m})\,,
\end{aligned}
$$

$$
\begin{aligned}
D_u\mathcal{I}_x + \mathcal{I}_x(\theta_u + 2u_\mu D_x X^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_x)\,,\\
D_u\mathcal{I}_y + \mathcal{I}_y(\theta_u + 2u_\mu D_y Y^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_y)\,,\\
D_u\mathcal{I}_z + \mathcal{I}_z(\theta_u + 2u_\mu D_z Z^\mu) &= \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_z)\,.
\end{aligned}
$$

Second Moment

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Implementing the equation of state

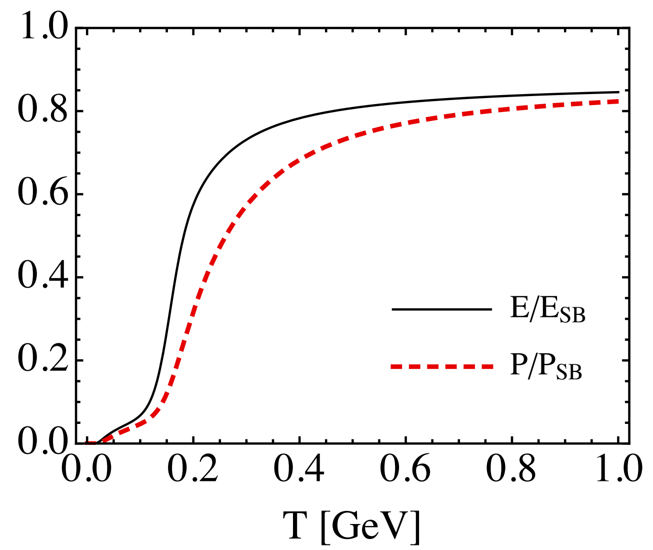
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Quasiparticle Method

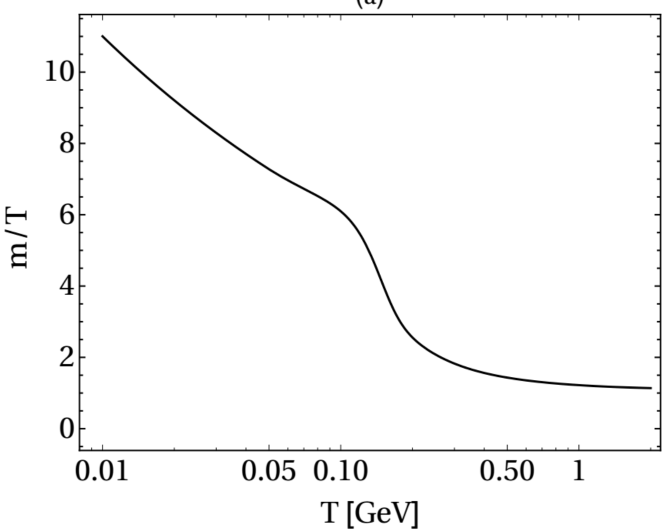
$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

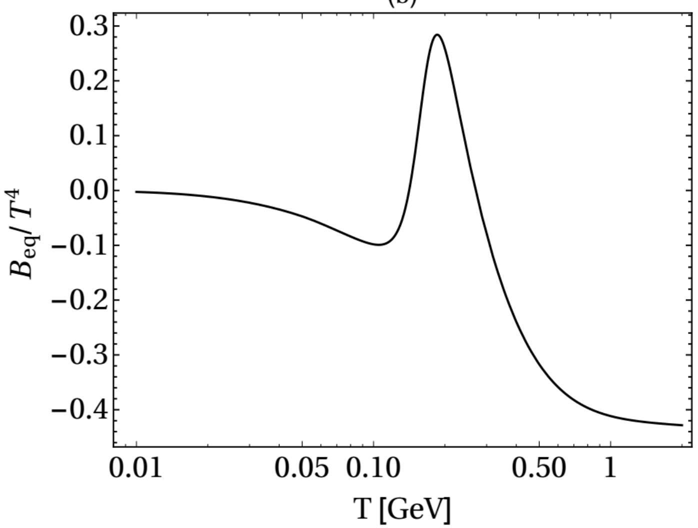
$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP f(x,p)$$



(a)



(b)



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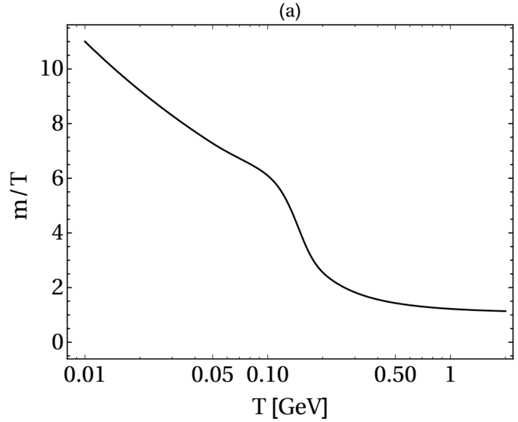
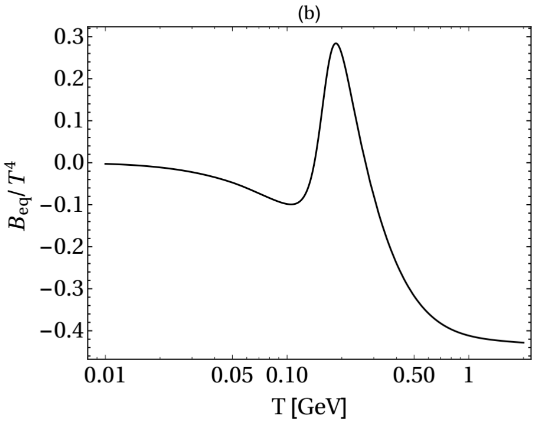




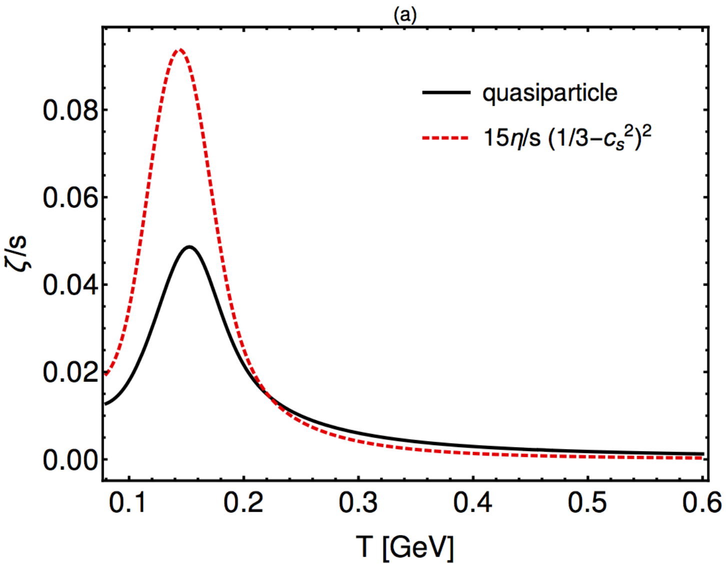
Bulk viscosity

$$\frac{\zeta}{\tau_{\text{eq}}} = \frac{5}{3T} I_{3,2} - c_s^2(\mathcal{E}+\mathcal{P}) + T\hat{m}^3 \frac{dm}{dT} I_{1,1}$$

$$I_{3,2}(x) = \frac{N_{\text{dof}} T^5 x^5}{30\pi^2}\left[\frac{1}{16}\Big(K_5(x) - 7K_3(x) + 22K_1(x)\Big) - K_{i,1}(x)\right],$$

$$K_{i,1}(x) = \frac{\pi}{2}\left[1 - xK_0(x)\mathcal{S}_{-1}(x) - xK_1(x)\mathcal{S}_0(x)\right],$$

$$I_{1,1} = \frac{g\, m^3}{6\pi^2}\left[\frac{1}{4}(K_3 - 5K_1) + K_{i,1}\right]$$



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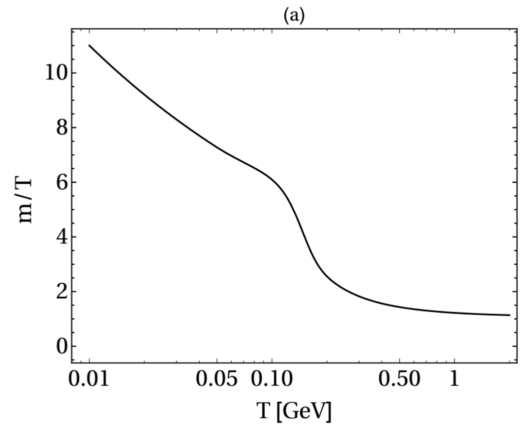
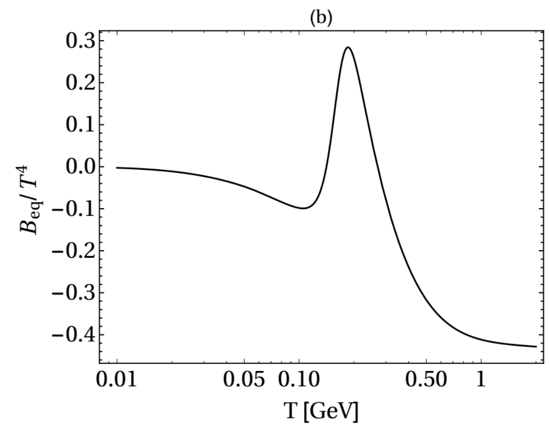


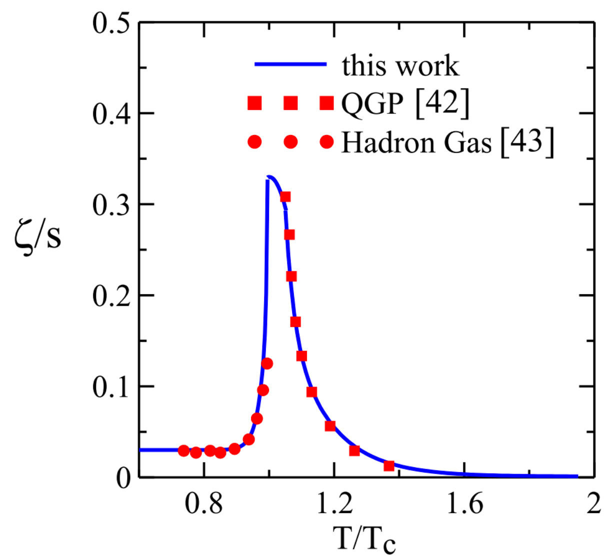


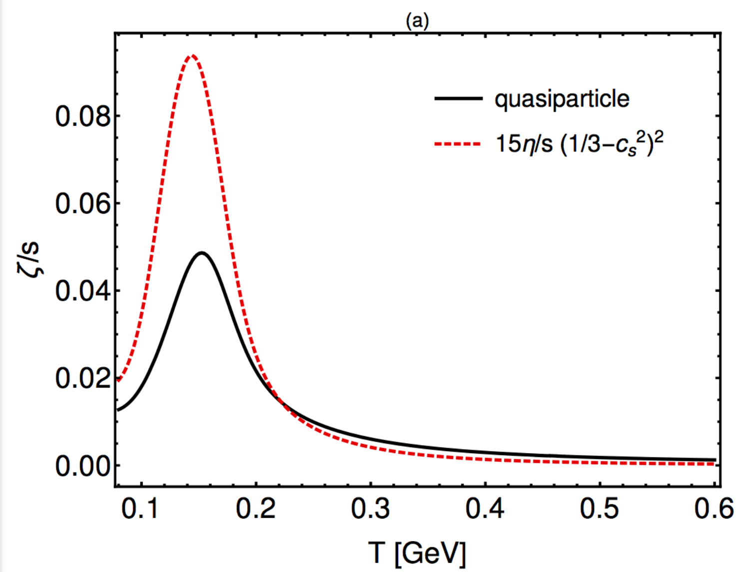
Bulk viscosi

$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I$$

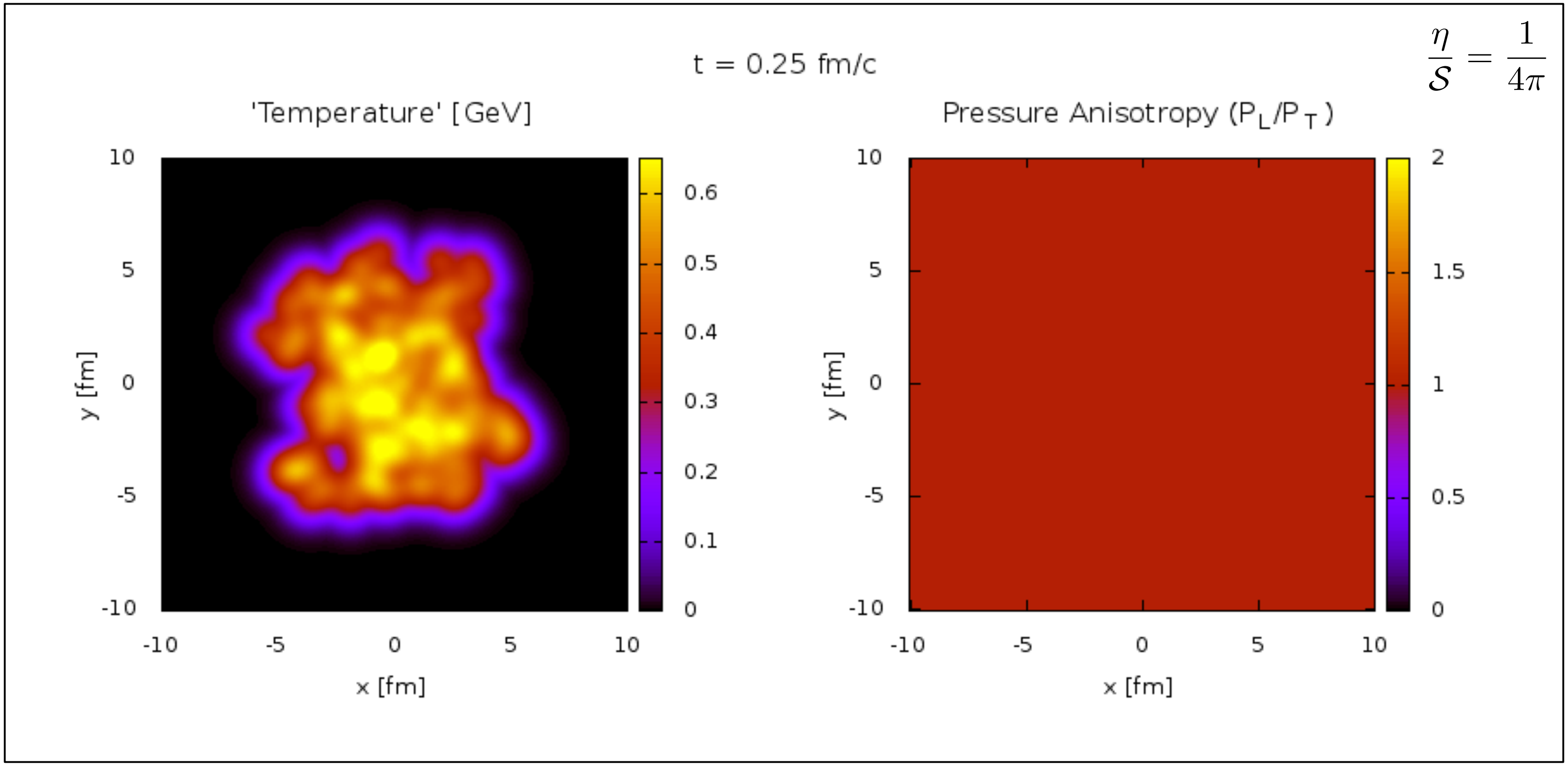
$$I_{1,1} = \frac{g\, m^3}{6\pi^2}$$

Ryu et al, PRL 115 (2015) no.13, 132301





Spatiotemporal Evolution



- Pb-Pb, b = 7 fm collision with Monte-Carlo Glauber initial conditions T_0 = 600 MeV @ τ_0 = 0.25 fm/c
- Left panel shows temperature and right shows pressure anisotropy

Anisotropic Cooper-Frye Freezeout

D. Bazow, U. Heinz, M. Martinez, M. Nopoush, R. Ryblewski, MS, 1506.05278
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

- Use same ellipsoidal form for "anisotropic freeze-out" at LO.
- Form includes both shear and bulk corrections to the distribution function.
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ → e.g. $T_{\rm eff,FO}$ = 150 MeV

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Phi\Delta^{\mu\nu}$$

isotropic ($u^\mu u^\nu$), anisotropy tensor ($\xi^{\mu\nu}$), bulk correction ($\Phi\Delta^{\mu\nu}$)

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0, \xi_x, \xi_y, \xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\rm eq}\left[1 + (1 - a f_{\rm eq}) \frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon + P)T^2}\right]$$

$$f_{\rm eq} = 1/[\exp(p\cdot u/T) + a] \qquad a = \text{-}1,\ +1,\ \text{or}\ 0$$

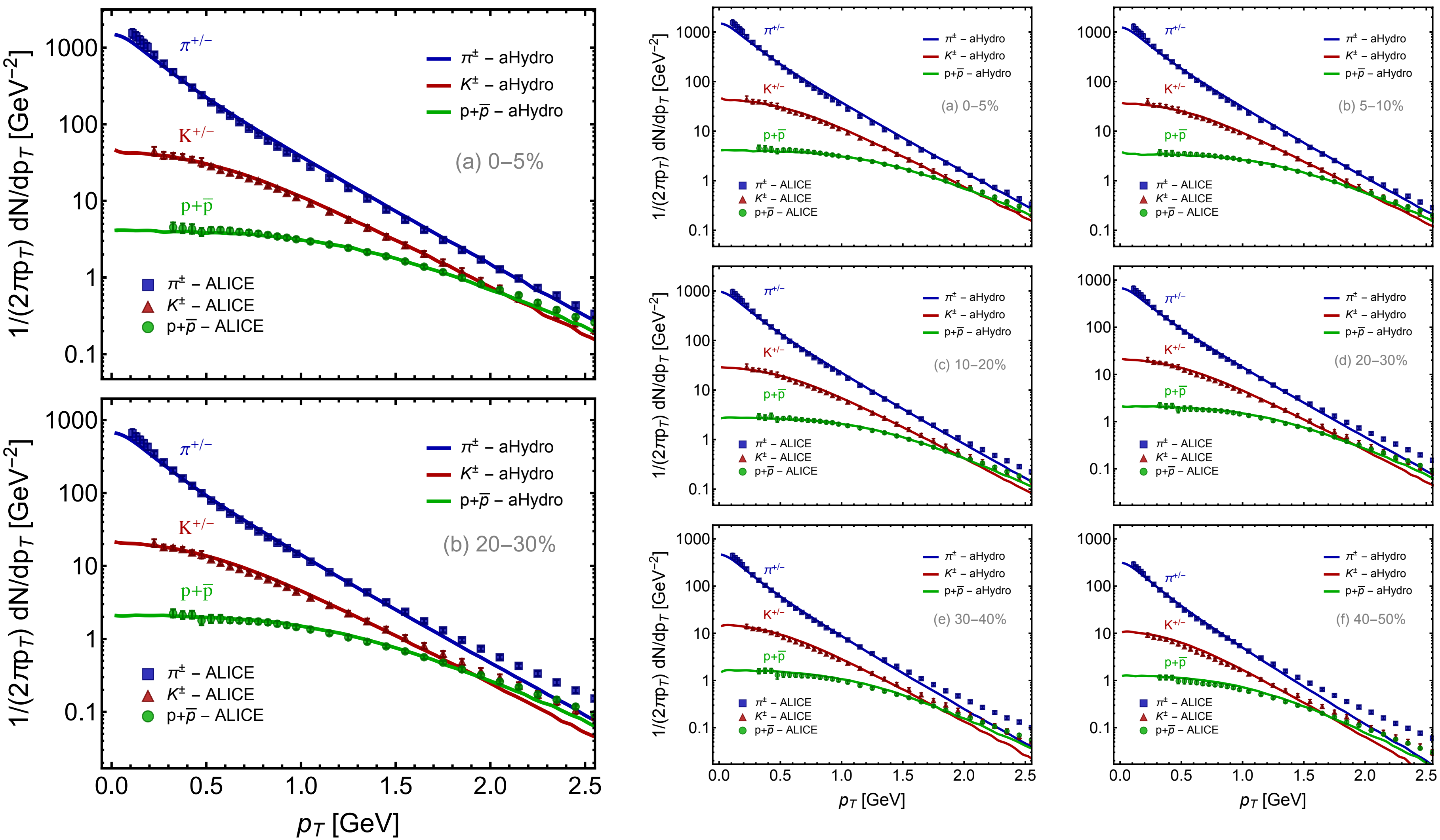
- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- **Problem becomes worse when including the bulk viscous correction**.

The phenomenological setup

- Use simple model using smooth optical Glauber initial conditions.
- For initial conditions we use a mixture of wounded nucleon and binary collision profiles with a binary mixing fraction of 0.15 (empirically suggested from prior viscous hydro studies).
- In the rapidity direction, we use a rapidity profile with a "tilted" central plateau and Gaussian "wings".
- We take all anisotropy parameters to be 1 initially (isotropic IC).
- We then run the code and extract the freeze-out hypersurface.
- The primordial particle production is then Monte-Carlo sampled using the Therminator 2. [Chojnacki, Kisiel, Florkowski, and Broniowski, arXiv:1102.0273]
- Therminator also takes care of all resonance feed downs.
- All data shown are from the **ALICE collaboration**.

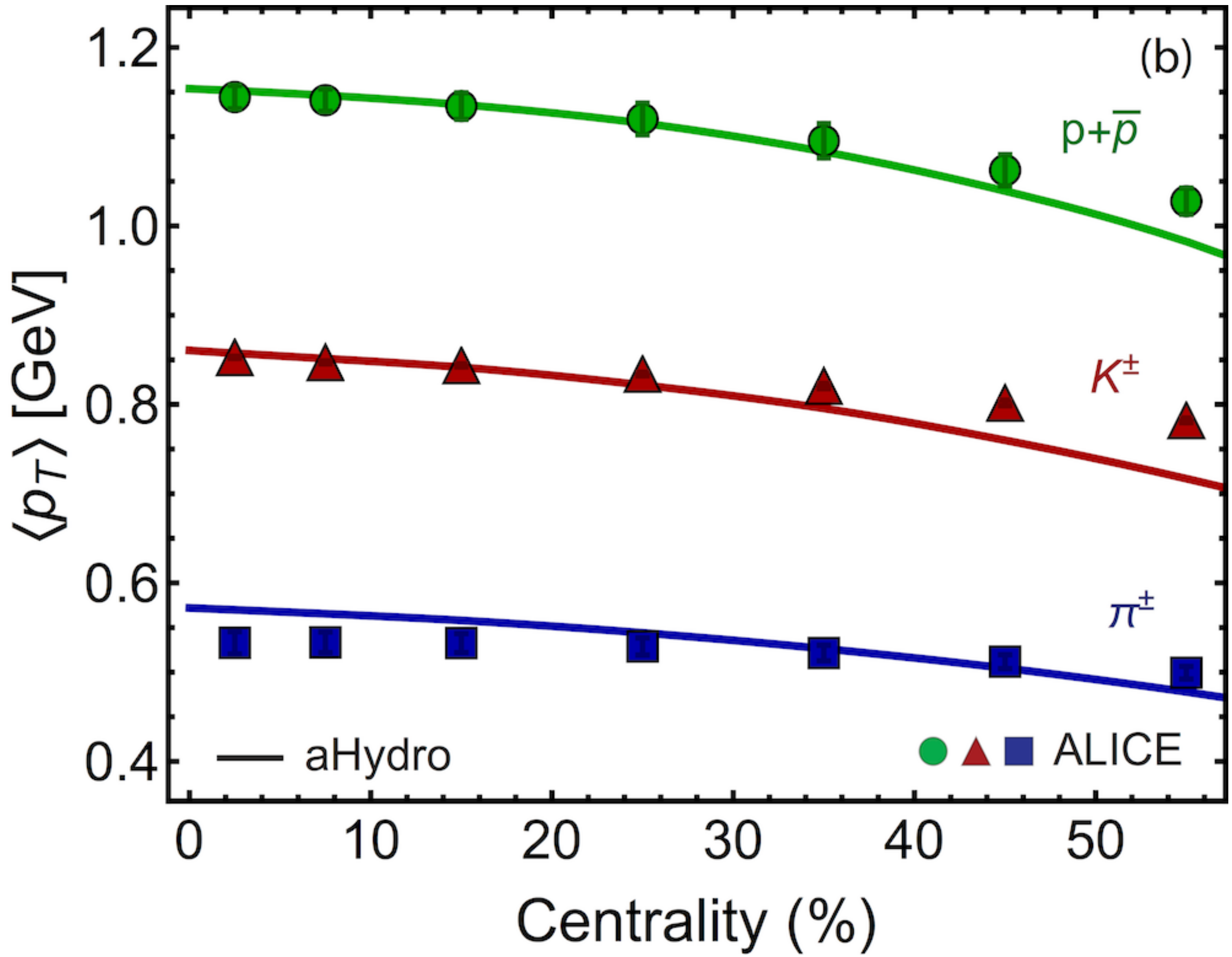
Identified particle spectra

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



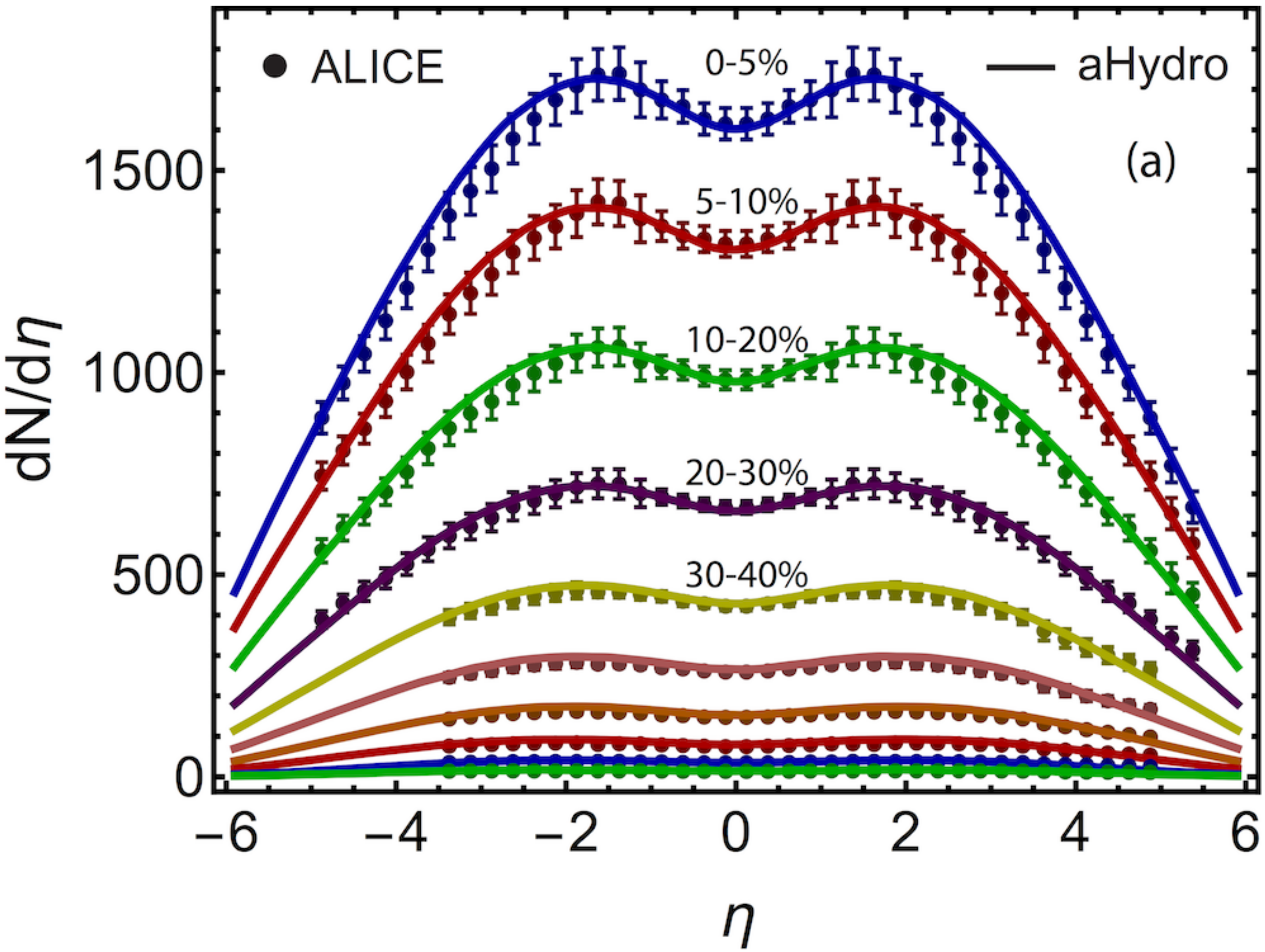
Identified particle average p_T

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



Charged particle multiplicities

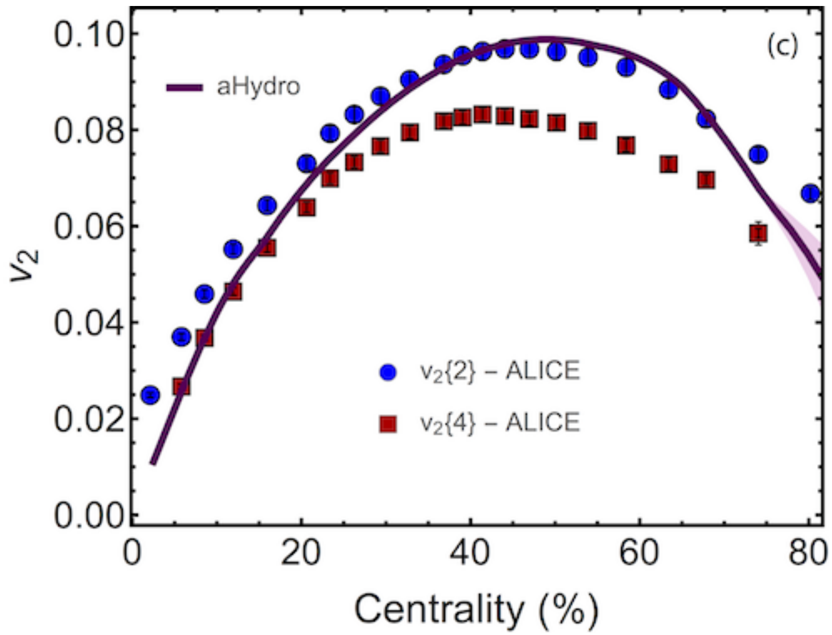
Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191

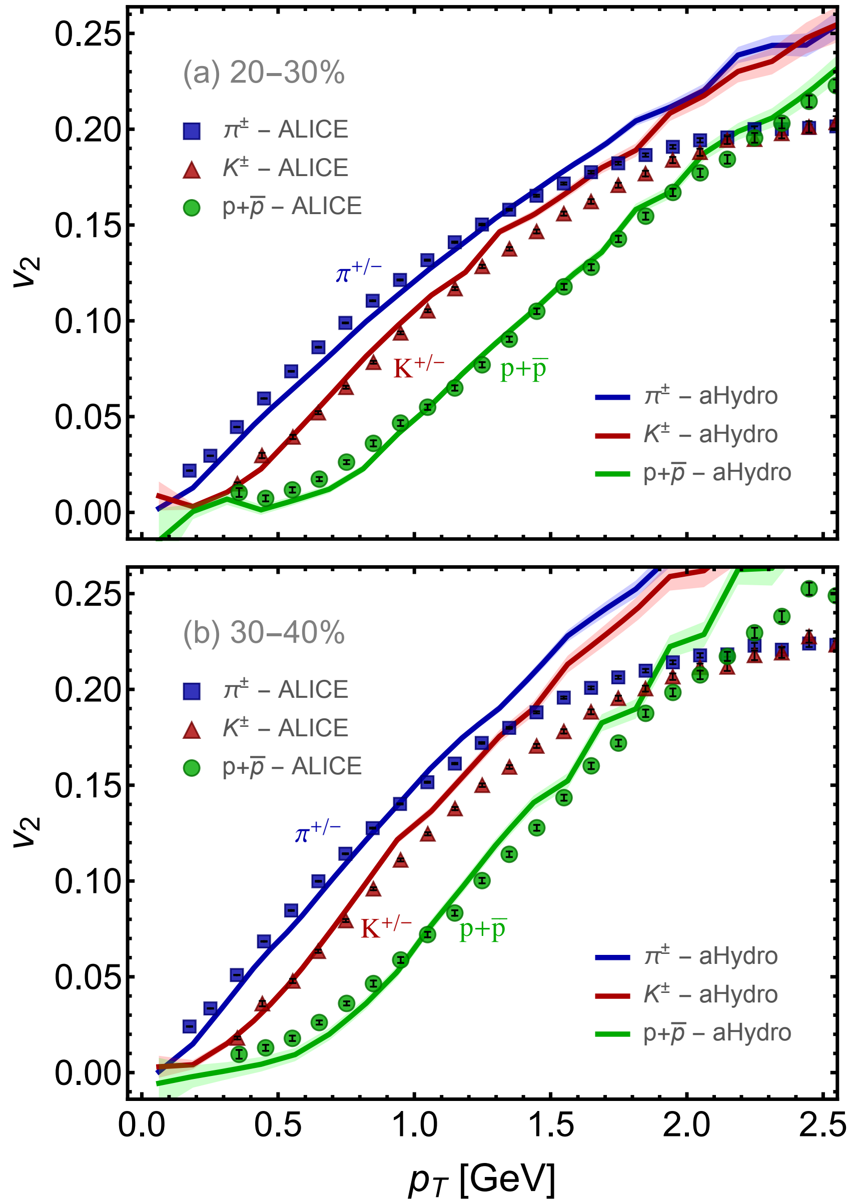


Elliptic flow

- Quite good description of elliptic flow as well
- Problems for central collisions but this is to be expected since we have not included fluctuating initial conditions yet

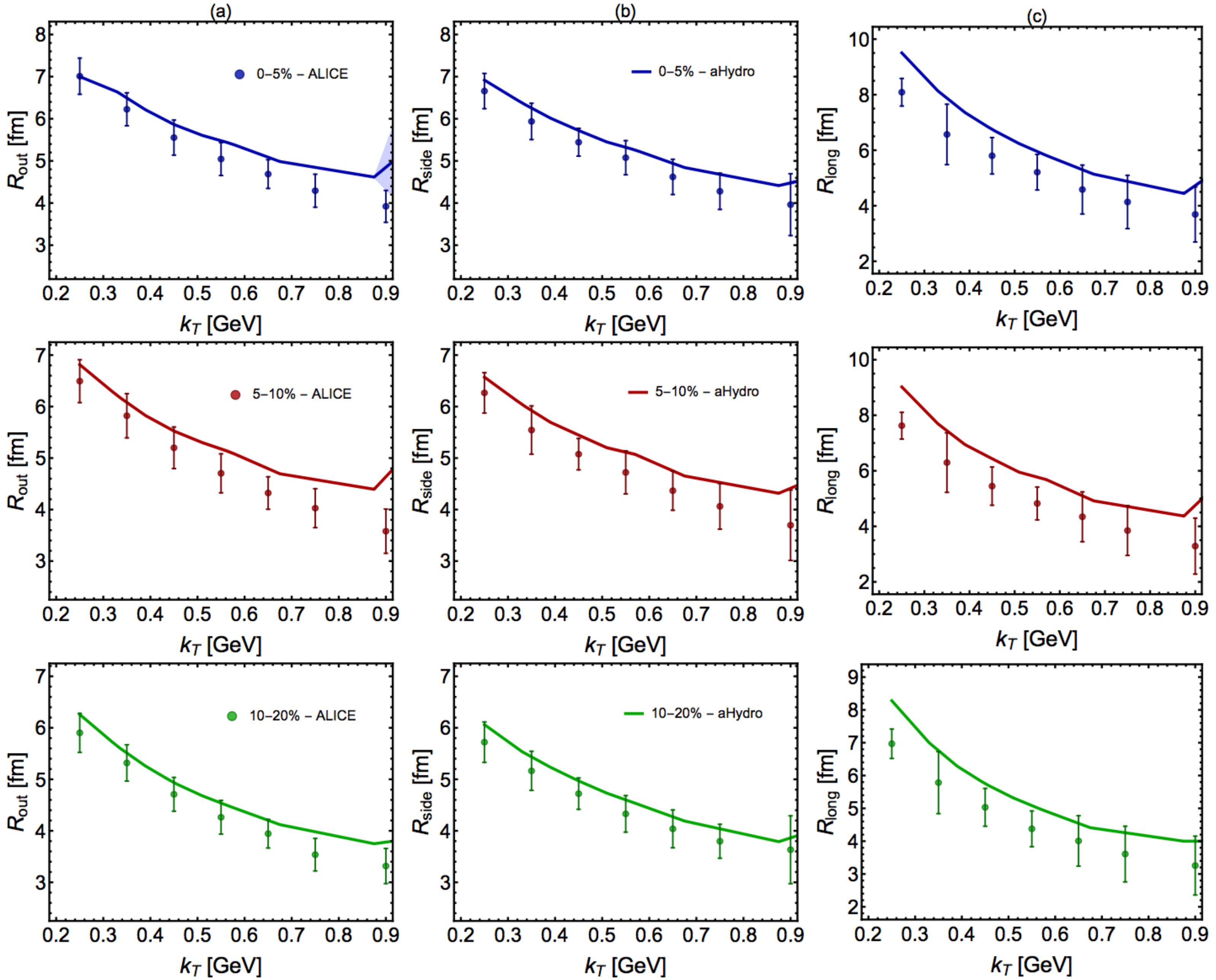
Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191





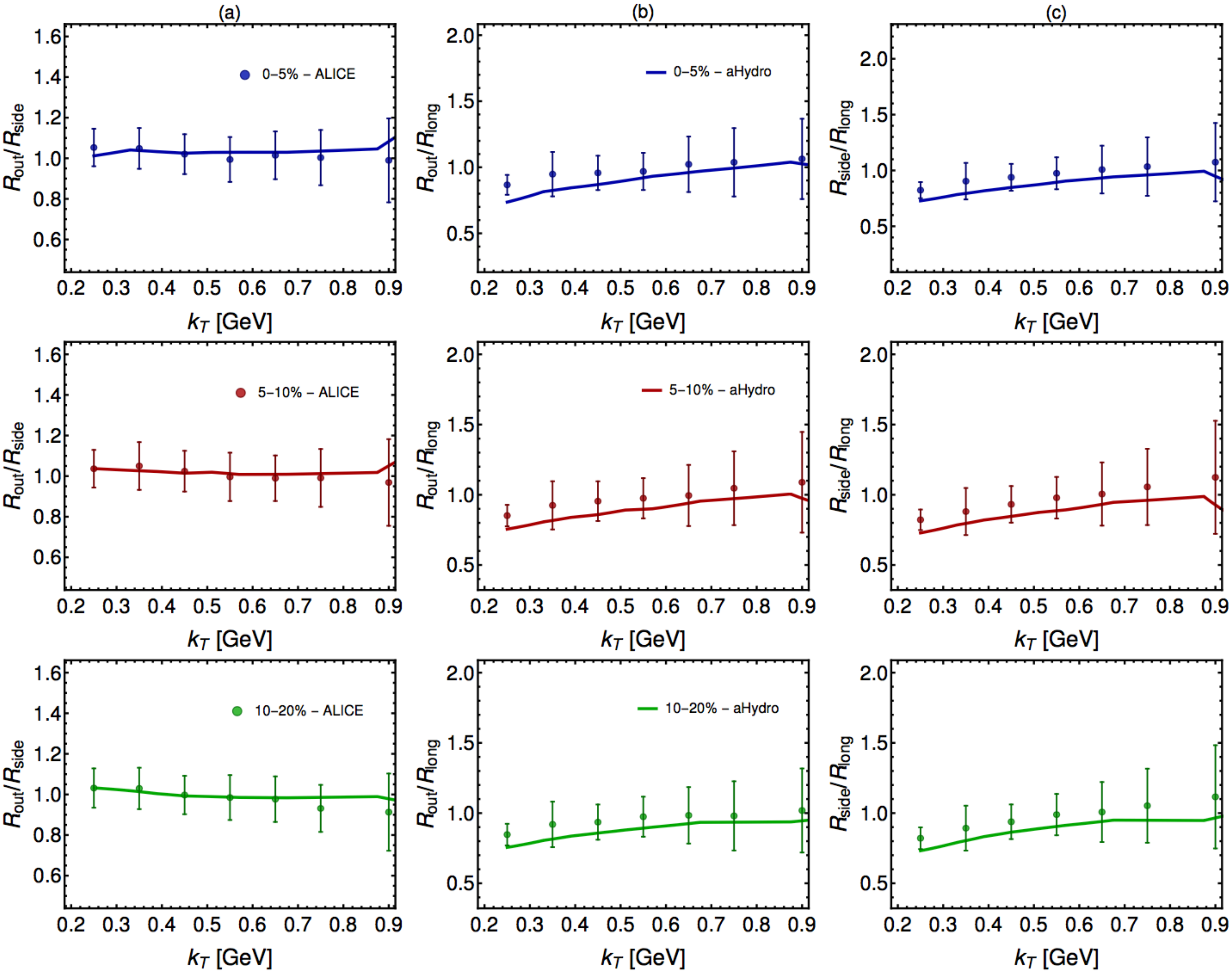
HBT Radii

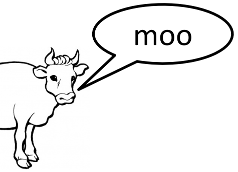
Alqahtani, Nopoush, Ryblewski, MS, 1705.10191



HBT Radii Ratios

Alqahtani, Nopoush, Ryblewski, MS, 1705.10191



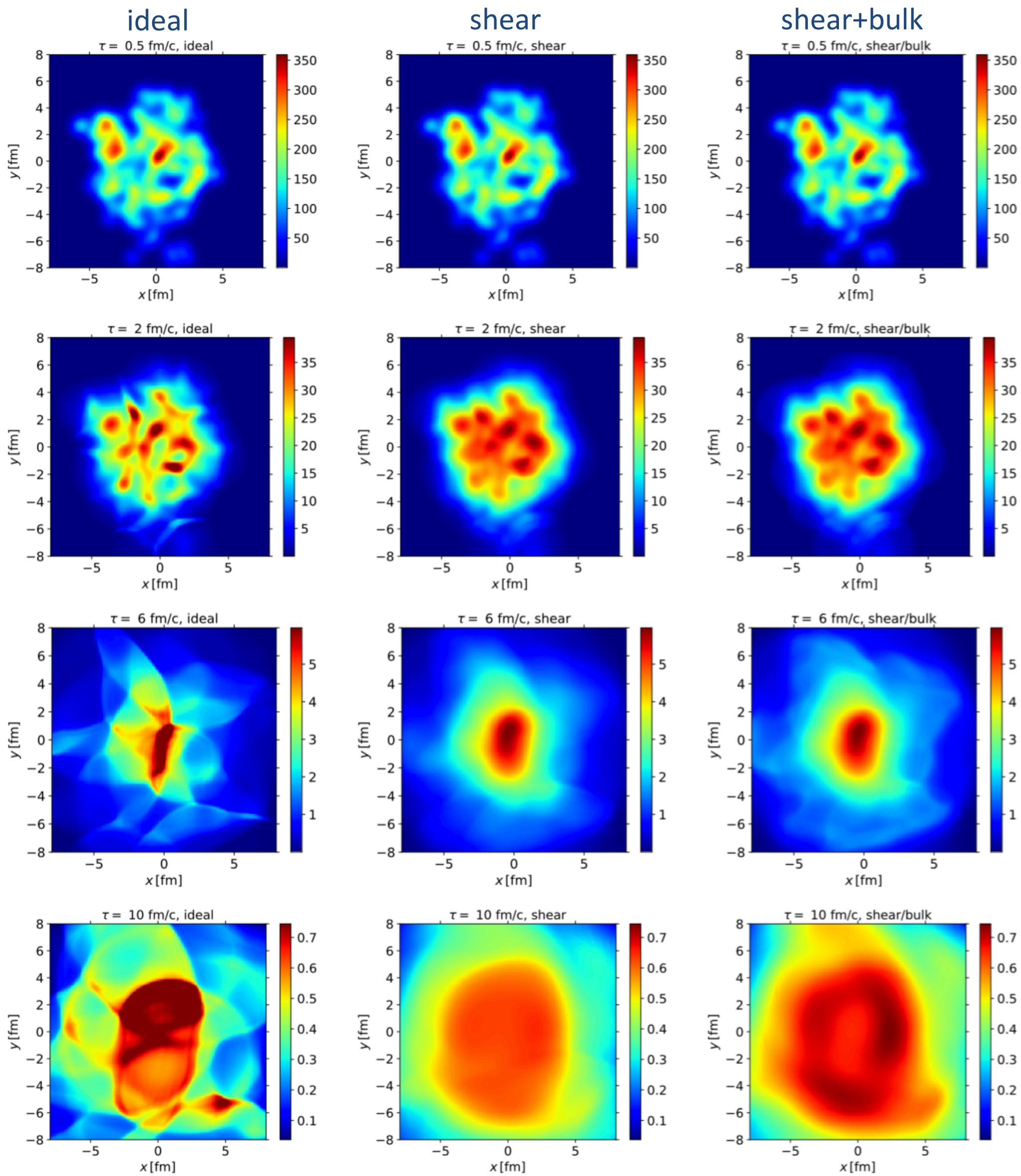


Conclusions and Outlook

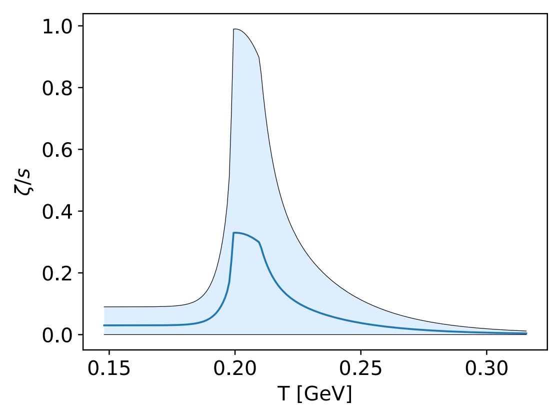
- Anisotropic hydrodynamics builds upon prior advances in relativistic hydrodynamics in an attempt to create a more quantitatively reliable model of QGP evolution.
- It incorporates some "facts of life" specific to the conditions generated in relativistic heavy ion collisions and, in doing so, optimizes the dissipative hydrodynamics approach.
- We now have a running 3+1d aHydro code with realistic EoS, anisotropic freeze-out, and fluctuating initial conditions.
- Our preliminary fits to experimental data using smooth Glauber initial conditions look quite nice.
- Also need to add the off-diagonal anisotropies and turn on the fluctuating initial conditions . . . Lots of work yet to do.

Backup slides

Some pretty pictures from 3d viscous hydro

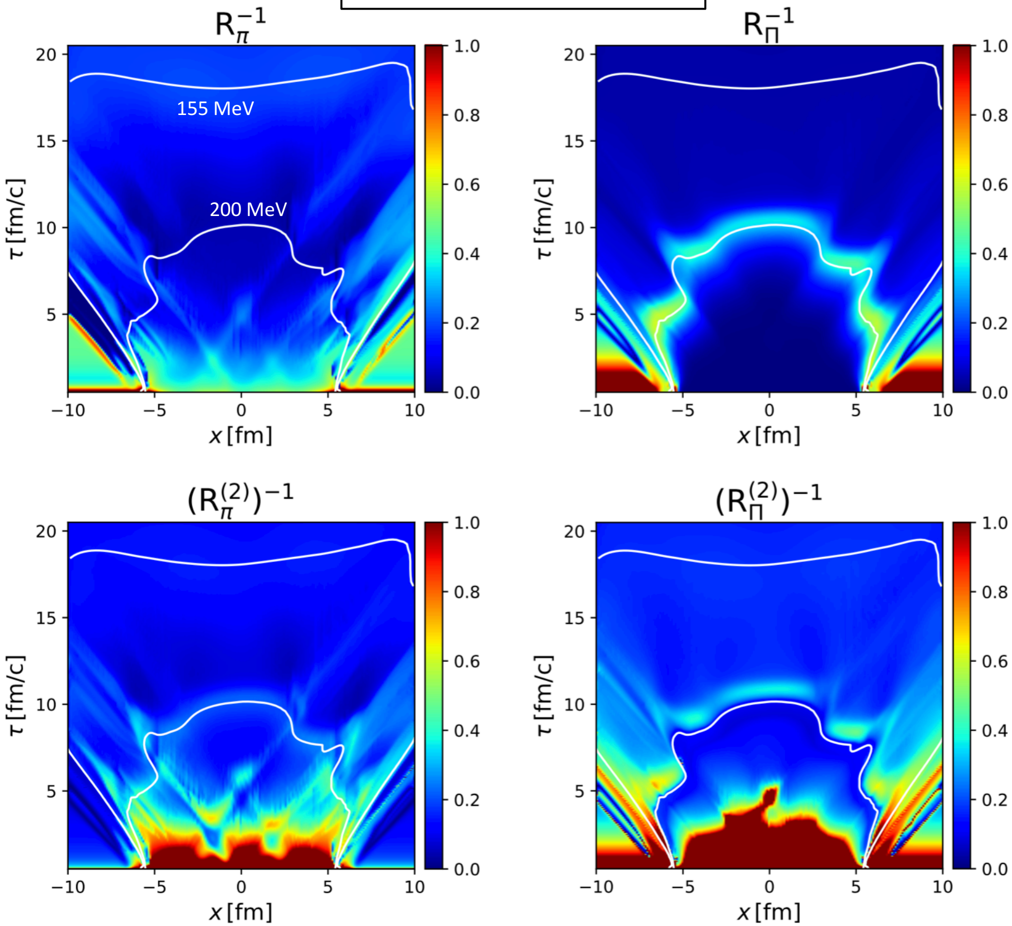


- Left panels show output from the Ohio State/Kent State GPU-based viscous hydro code [Bazow, Heinz, and MS, 1608.06577]
- Solves the non-conformal DNMR (Denicol, Niemi, Molnar, Rischke) equations with a realistic EoS
- Parameterized ζ/s (plot below)
- $\eta/s = 0.2$
- $T_0 = 600$ MeV @ $t_0 = 0.5$ fm/c



Pb-Pb @ 2.76 TeV - Don't worry, be happy

$$R_\pi^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathscr{P}_0}, \quad R_\Pi^{-1} \equiv \frac{|\Pi|}{\mathscr{P}_0}.$$



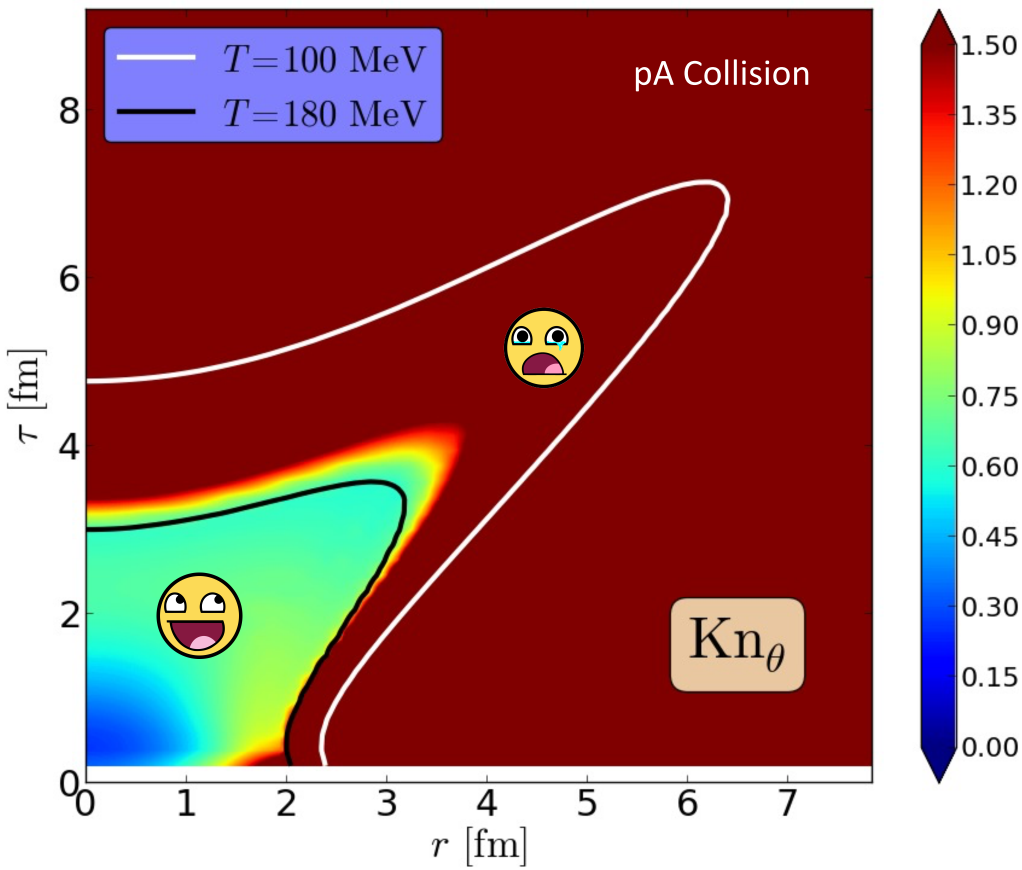
$$(R_\pi^{(2)})^{-1} \equiv \frac{\sqrt{\mathscr{J}^{\mu\nu}\mathscr{J}_{\mu\nu}}}{2\eta\sqrt{\sigma^{\mu\nu}\sigma_{\mu\nu}}}, \quad (R_\Pi^{(2)})^{-1} \equiv \frac{|\mathscr{J}|}{\zeta|\theta|}.$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathscr{J} + \mathscr{K} + \mathscr{R},$$
$$\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa I^\mu + \mathscr{J}^\mu + \mathscr{K}^\mu + \mathscr{R}^\mu,$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathscr{J}^{\mu\nu} + \mathscr{K}^{\mu\nu} + \mathscr{R}^{\mu\nu}.$$

- $\mathcal{J}, \mathcal{J}^\mu$, and $\mathcal{J}^{\mu\nu}$ are O(Kn R^{-1})
- $\mathcal{K}, \mathcal{K}^\mu$, and $\mathcal{K}^{\mu\nu}$ are O(Kn^2)
- $\mathcal{R}, \mathcal{R}^\mu$, and $\mathcal{R}^{\mu\nu}$ are O(R^{-2})
- DNMR derivation assumes that Kn ~ R^{-1}
- For this to be a reasonable approx, the 2nd order terms should be smaller than the O(Kn) Navier-Stokes terms
- **In order for code to run stably, it is necessary to "dynamically regulate" the viscous corrections**

p-A @ 2.76 TeV - Don't be happy, worry

Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327



$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathscr{J} + \mathscr{K} + \mathscr{R} ,$$
$$\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu = \kappa I^\mu + \mathscr{J}^\mu + \mathscr{K}^\mu + \mathscr{R}^\mu ,$$
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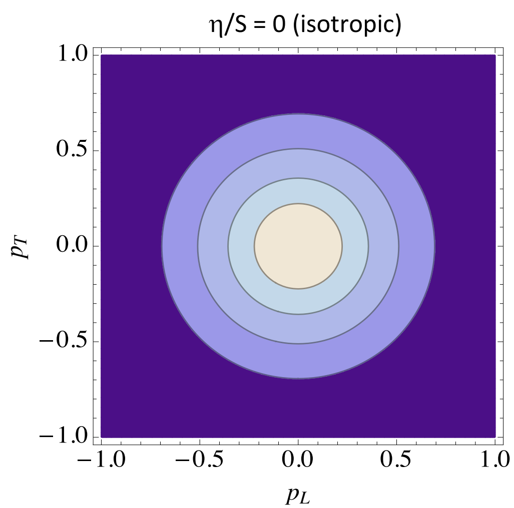
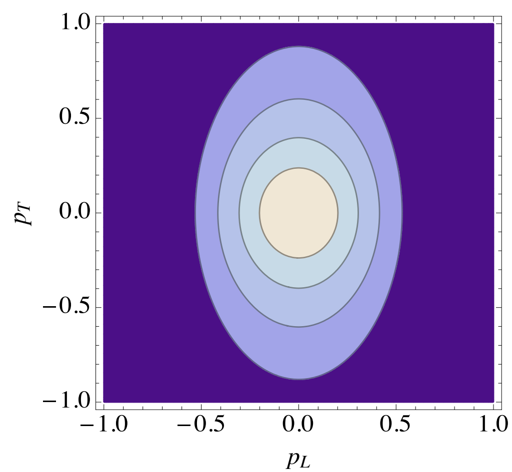
1^{st} Order Hydro – 0+1d

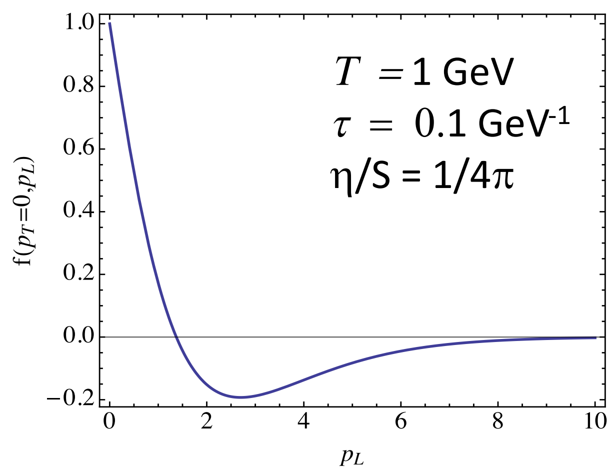
Additionally one finds for the first order distribution function

$$f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)\left[1+\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right] \longrightarrow f_{\rm eq}\left(\frac{E}{T}\right)\left[1+\frac{\eta}{\mathcal{S}}\frac{p_x^2+p_y^2-2p_z^2}{3\tau T^3}\right]$$

- Distribution function becomes anisotropic in momentum space
- There are also regions where $f(x,p) < 0$
- Anisotropy and regions of negativity increase as τ or T decrease OR η/S increases







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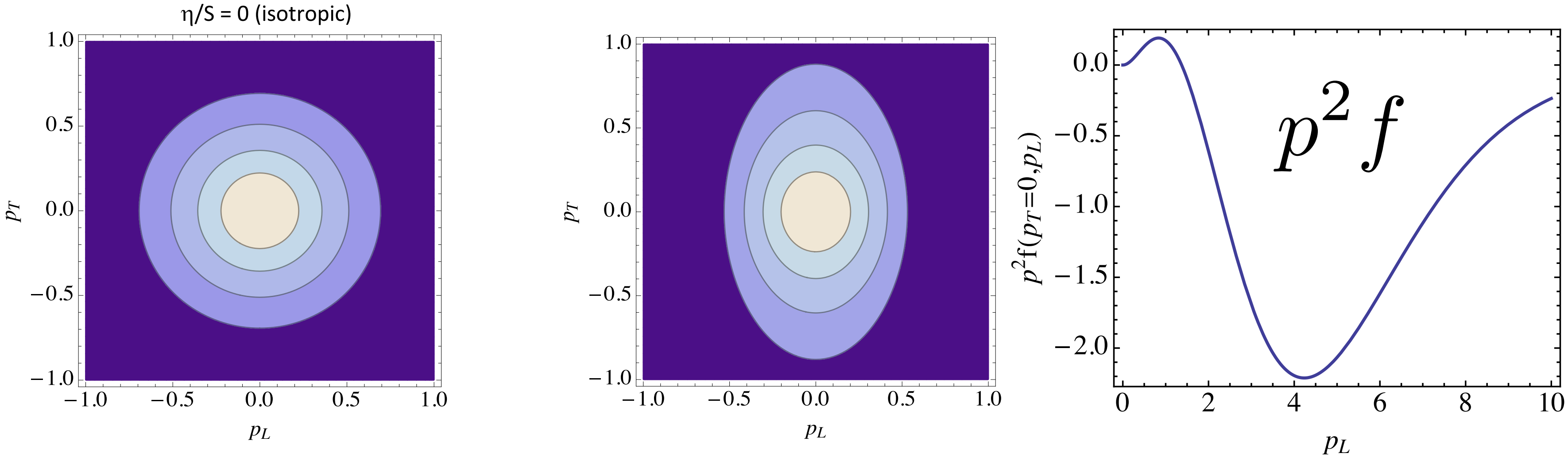
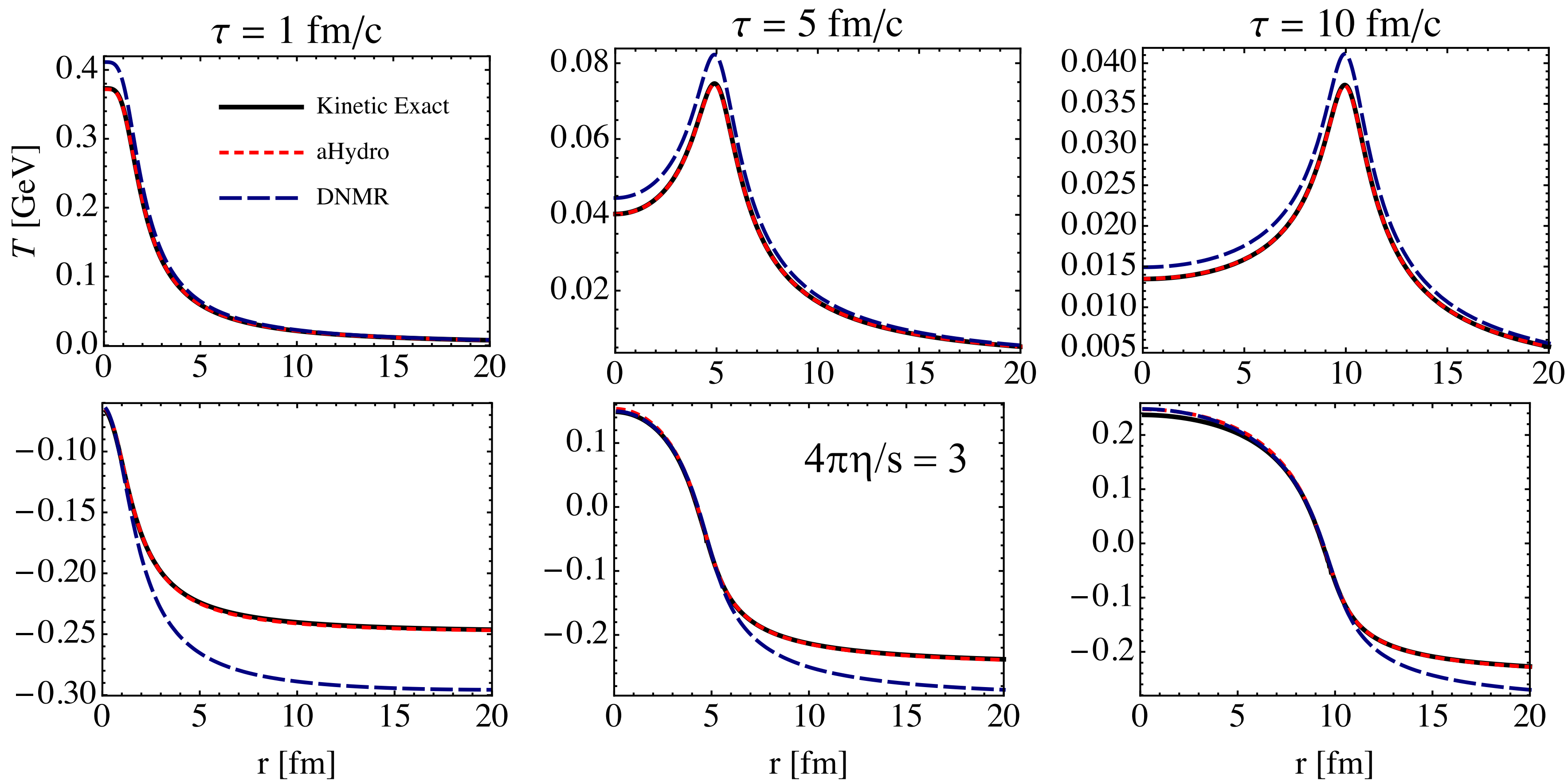


M. Strickland

Ex 3: aHydro for Gubser flow

M. Nopoush, R. Ryblewski, and MS, 1410.6790
Exact Solution: G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048



Technicalities - A numerical challenge

- One of the most daunting challenges faced by the quasiparticle approach is that one has to evaluate a bunch of "H" functions, e.g.

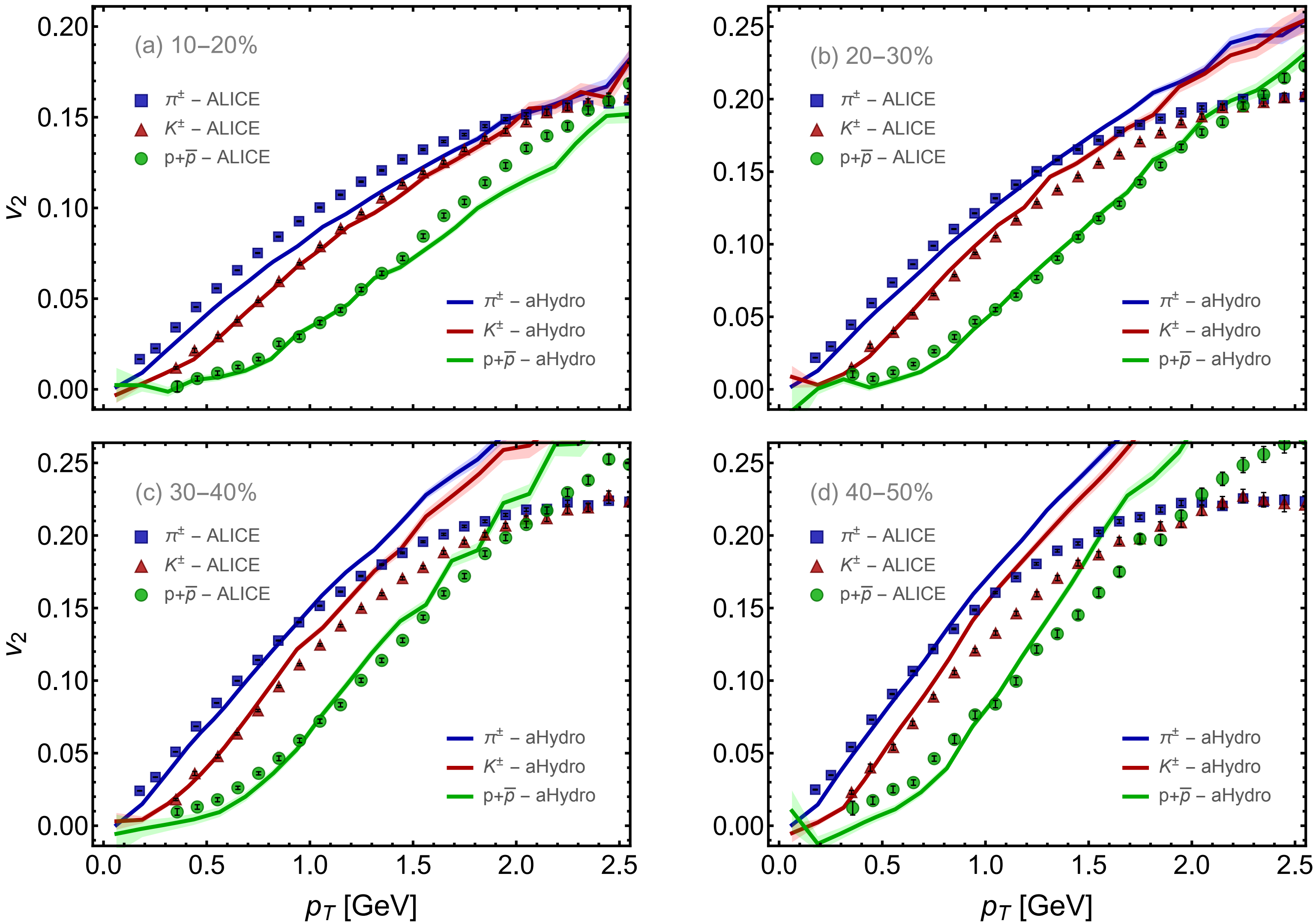
$$\mathcal{E} = \mathcal{H}_3(\boldsymbol{\alpha}, \hat{m})\, \lambda^4 + B$$

$$\mathcal{H}_3(\boldsymbol{\alpha}, \hat{m}) = \tilde{N}\alpha \int d^3\hat{p}\, \mathcal{R}(\boldsymbol{\alpha}, \hat{m})\, f_{\rm eq}\!\left(\sqrt{\hat{p}^2 + \hat{m}^2}\right)$$

$$\mathcal{R}(\boldsymbol{\alpha}, \hat{m}) = \sqrt{\alpha_x^2\, p_x^2 + \alpha_y^2\, p_y^2 + \alpha_z^2\, p_z^2 + m^2}$$

- We evaluate these efficiently by expanding the integrand around the diagonal in anisotropy space up to 12^{th} order.
- We do this around two points (1,1,1) and (2,2,2) and switch between these two expansions smoothly.
- With this method we were able to accelerate the evaulation of H functions by a factor of 10^5 while achieving < 0.1% accuracy.

More figures #1



More figures #2