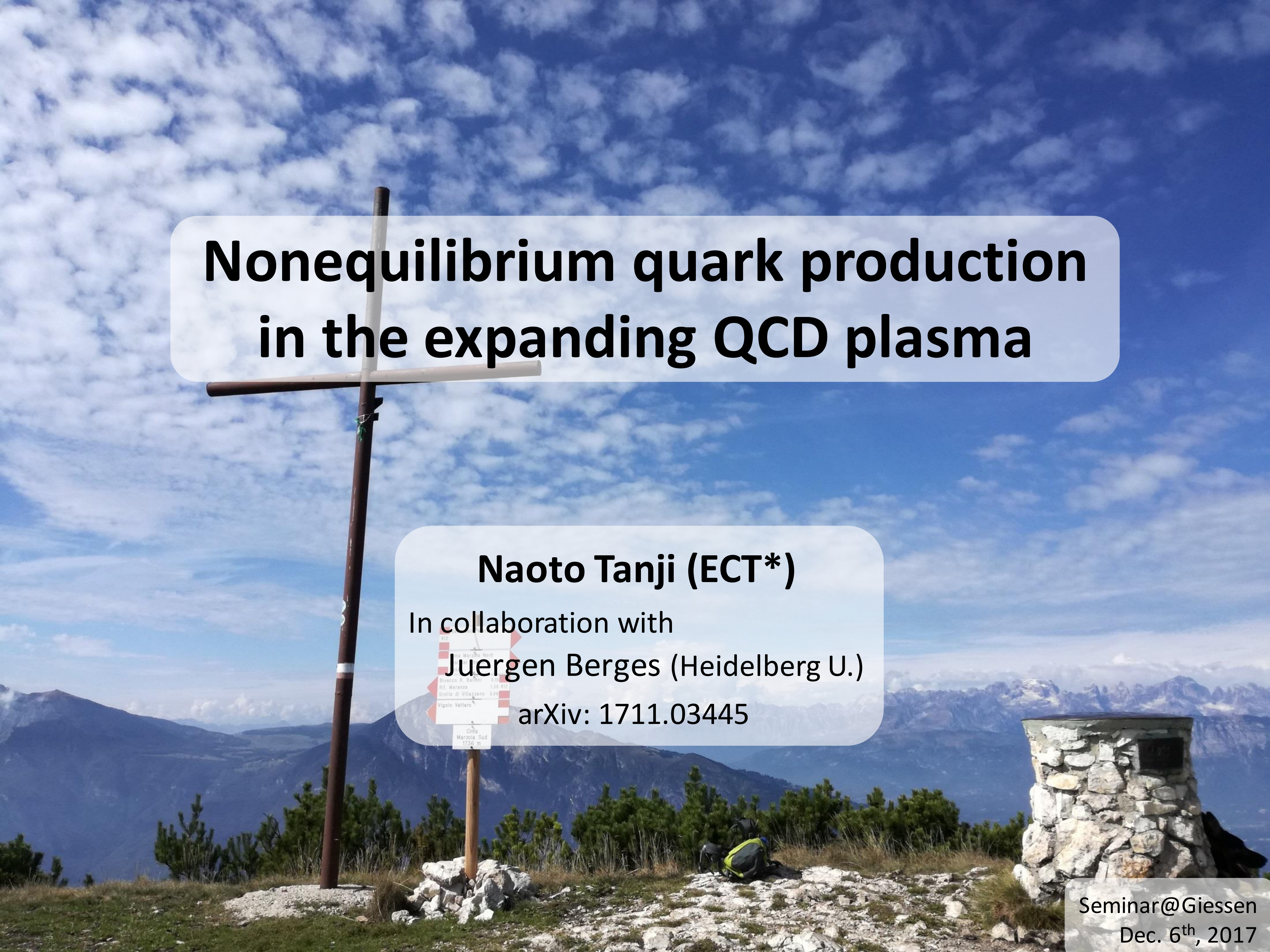


Nonequilibrium quark production in the expanding QCD plasma

Naoto Tanji (ECT*)

In collaboration with

Juergen Berges (Heidelberg U.)

arXiv: 1711.03445

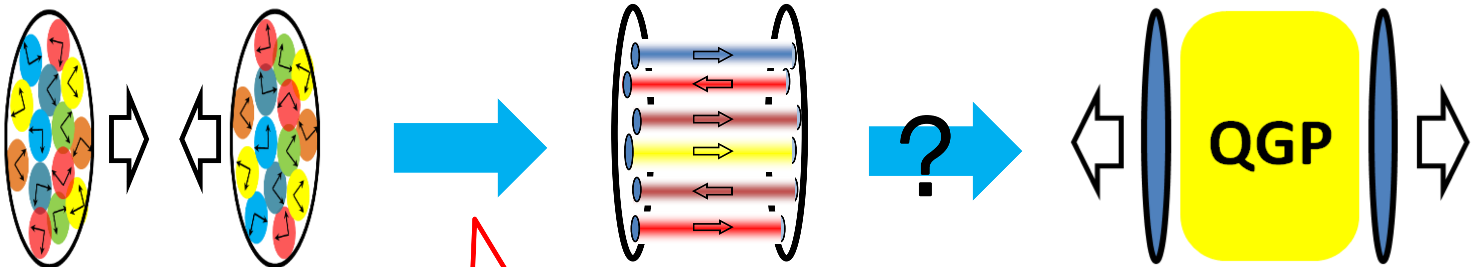
Seminar@Giessen

Dec. 6th, 2017

Outline

- Early-time dynamics in heavy-ion collisions
- Classical-statistical method for strong-field QCD
- Numerical results

Early-time dynamics in heavy-ion collisions



Color glass condensate $\boldsymbol{E}_T^a \perp \boldsymbol{B}_T^a$

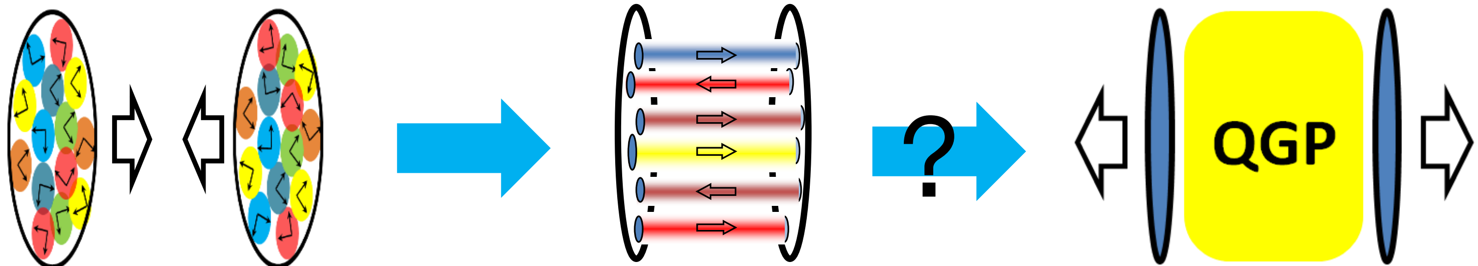
Glasma $\boldsymbol{E}_L^a \parallel \boldsymbol{B}_L^a$

Quark-gluon plasma described by hydro

non-Abelian Gauss laws

$$\nabla \cdot \boldsymbol{E}^a = g f^{abc} \boldsymbol{A}^b \cdot \boldsymbol{E}^c$$
$$\nabla \cdot \boldsymbol{B}^a = g f^{abc} \boldsymbol{A}^b \cdot \boldsymbol{B}^c$$

Early-time dynamics in heavy-ion collisions



Color glass condensate
$\boldsymbol{E}_T^a \perp \boldsymbol{B}_T^a$

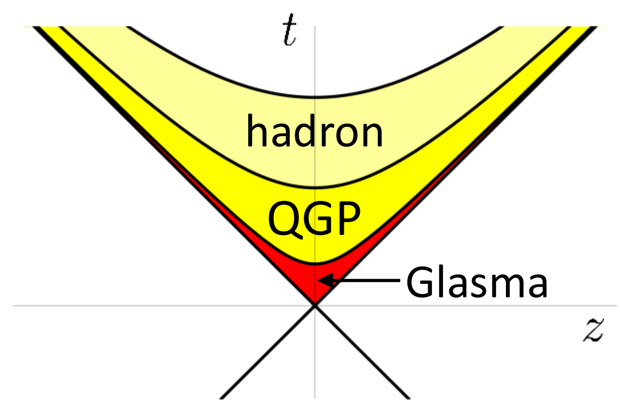
Glasma
$\boldsymbol{E}_L^a \parallel \boldsymbol{B}_L^a$

Quark-gluon plasma
described by hydro

- far from equilibrium
- boost-invariant expansion in the longitudinal direction
- strong gauge fields, highly occupied gluons

$$A \sim Q_s/g \qquad f_g \sim 1/g^2$$

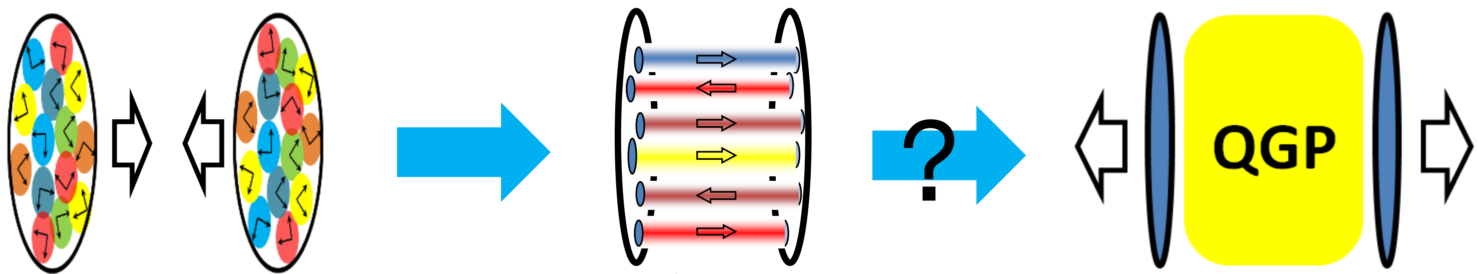
→ strongly interacting even though weak coupling



space-time evolution

$$\tau = \sqrt{t^2 - z^2}\,, \;\; \eta = \mathrm{atanh}\frac{z}{t}$$

Early-time dynamics in heavy-ion collisions



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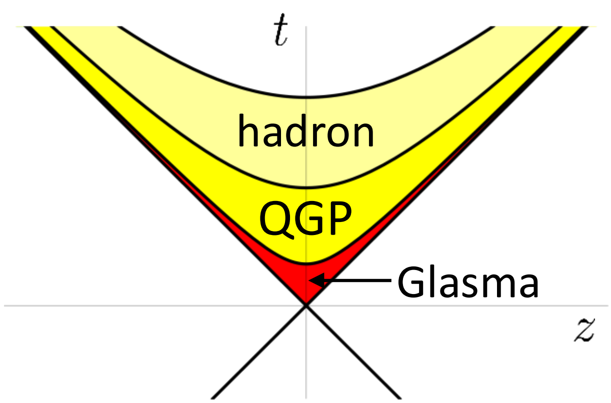


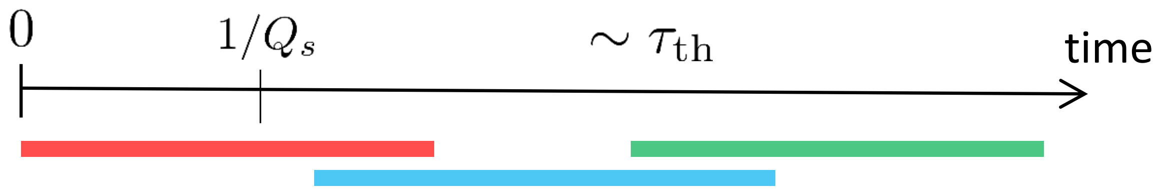
space-time evolution

$$\tau = \sqrt{t^2 - z^2}\,, \quad \eta = \mathrm{atanh}\frac{z}{t}$$

- ➢ How does the over-occupied gluon plasma evolve toward thermalization or hydrodynamization?
- ➢ Quark production?

Early-time dynamics in heavy-ion collisions

QCD-based theoretical descriptions



Classical-statistical

$f_g \gg 1$

Epelbaum, Gelis (2013)
Berges et al. (2014)

Kinetic theory

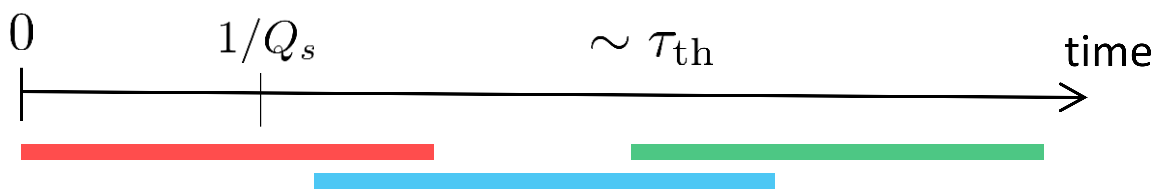
$1/\alpha_s \gg f_g$

Kurkela, Zhu (2015)
Keegan et al. (2015)

Hydro

Early-time dynamics in heavy-ion collisions

QCD-based theoretical descriptions



Classical-statistical	Kinetic theory	Hydro
$f_g \gg 1$	$1/\alpha_s \gg f_g$	
Epelbaum, Gelis (2013) Berges et al. (2014)	Kurkela, Zhu (2015) Keegan et al. (2015)	

Pure gluon systems have been extensively investigated.
Though the early stage is dominated by gluons, quarks play important roles in connection with experimental observables.

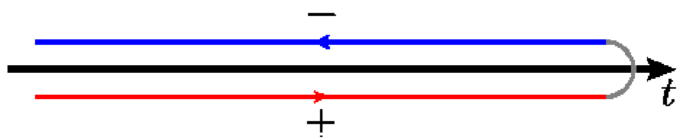
- Electromagnetic probes
- Possible signals of the chiral magnetic effect

Classical-statistical real-time lattice simulations for quark production

Schwinger-Keldysh formalism

In nonequilibrium, we are mostly interested in expectation values $\langle 0_{\rm in}|\mathcal{O}|0_{\rm in}\rangle$

→ Schwinger-Keldysh (closed time path) formalism



Generating functional

$$Z[J] = \int \mathcal{D}\mathcal{A}^{+}\mathcal{D}\mathcal{A}^{-}\ \rho_0[\mathcal{A}^{+}_{ini}, \mathcal{A}^{-}_{ini}]e^{i\int d^4x[\mathcal{L}_{\rm SK}+J^{+}\mathcal{A}^{+}-J^{-}\mathcal{A}^{-}]}$$

initial density matrix

$$\mathcal{L}^{\rm SK}_{\rm gauge}\left[\mathcal{A}^{+},\mathcal{A}^{-}\right] = \mathcal{L}_{\rm gauge}[\mathcal{A}^{+}] - \mathcal{L}_{\rm gauge}[\mathcal{A}^{-}]$$

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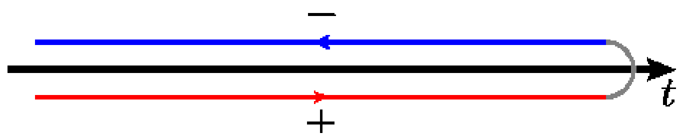

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initial density matrix

$$\mathcal{L}^{\rm SK}_{\rm gauge}[\mathcal{A}^{+},\mathcal{A}^{-}] = \mathcal{L}_{\rm gauge}[\mathcal{A}^{+}] - \mathcal{L}_{\rm gauge}[\mathcal{A}^{-}]$$

$$= [D_\mu, F^{\mu\nu}]^a\,\eta^a_\nu + \frac{ig}{4}[D_\mu,\eta_\nu]^a\,[\eta^\mu,\eta^\nu]^a$$

classical EOM

Change variables (Keldysh rotation)

$$A_\mu = \frac{\mathcal{A}^{+}_\mu + \mathcal{A}^{-}_\mu}{2}\,,\quad \eta_\mu = \mathcal{A}^{+}_\mu - \mathcal{A}^{-}_\mu$$

classical field quantum field

Classical-statistical method for strong gauge fields

Classical approximation $A \gg \eta$

$$\mathcal{L}_{\rm gauge}^{\rm SK} = [D_\mu, F^{\mu\nu}]^a\, \eta_\nu^a + \frac{ig}{4}[D_\mu, \eta_\nu]^a [\eta^\mu, \eta^\nu]^a$$

neglect higher order terms in η

The path integration over η gives

$$Z \sim \int \mathcal{D}A\, \rho_0[A_{ini}]\delta\left([D_\mu, F^{\mu\nu}]\right) e^{i\int d^4x\, JA}$$

restricts field trajectories to the classical path.

Computation procedure

1. Generate an ensemble of initial fields according to the initial density matrix.
2. Solve the classical EOM for each initial condition.
3. Take the average over the initial ensemble.

Fermions in the classical-statistical method

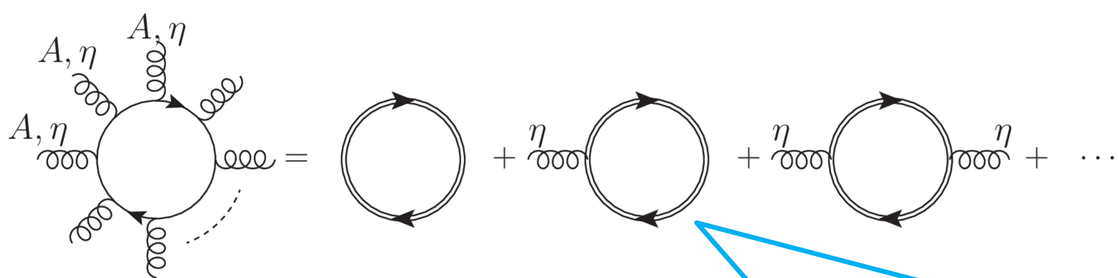
Kasper et al. PRD90 (2014)

Fermions are always quantum, must be treated exactly.

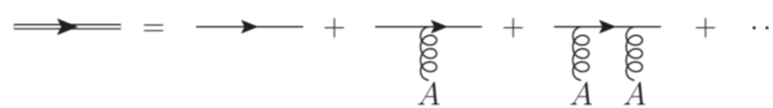
$$\mathcal{L}_{\mathrm{matter}} = \overline{\psi}(i\partial\!\!\!/ + gA\!\!\!/ - m)\psi \quad \Longrightarrow \quad \mathrm{Det}\left[i\partial\!\!\!/ + gA\!\!\!/ - m\right]$$

Quadratic fields can be integrated out.

Expand the determinant w.r.t. the quantum gauge field.



Propagator dressed by the classical field S_F



$$\mathrm{Det}\left[i\partial\!\!\!/ + gA\!\!\!/ - m\right] = \exp\left[-i\int d^4x\, J^{a,\mu}(x)\eta^a_\mu(x) + \mathcal{O}(\eta^2)\right]$$

color current $J^a_\mu(x) = g\,\mathrm{tr}\left[S_F(x,x)\gamma_\mu T^a\right]$

Fermions in the classical-statistical method

The path integration over η gives

$$Z \sim \int \mathcal{D}A \, \rho_0[A_{ini}] \delta\left([D_\mu, F^{\mu\nu}] - J^\mu\right) e^{i\int d^4x \, J_{\rm ext} A}$$

Classical Yang-Mills equation that couples to the quark current

$$[D_\mu, F^{\mu\nu}] = J^\mu$$

The color current and other quark observables can be obtained from the quark propagator $S_F(x,y)$ dressed by the classical gauge field.

$$[i\partial\!\!\!/_x + gA\!\!\!/(x) - m + i\mathrm{sgn}_{\mathcal{C}}\epsilon] \, S_F(x,y) = i\delta_{\mathcal{C}}(x,y)$$

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The propagator can be constructed by the field operator that obey the Dirac eq.

$$\left[i\partial\!\!\!/ + gA\!\!\!/(x) - m\right]\hat{\psi}(x) = 0$$

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mode expansion

$$\hat{\psi}(x) = \sum_{s,c}\int \frac{d^3p}{(2\pi)^3}\left[\psi^+_{\boldsymbol{p},s,c}(x) a_{\boldsymbol{p},s,c} + \psi^-_{\boldsymbol{p},s,c}(x) b^\dagger_{\boldsymbol{p},s,c}\right]$$

The Dirac eq. for c-number mode functions

$$[i\partial\!\!\!/ + gA\!\!\!/(x) - m]\,\psi^\pm_{\boldsymbol{p},s,c}(x) = 0$$

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The Dirac eq. for c-number mode functions

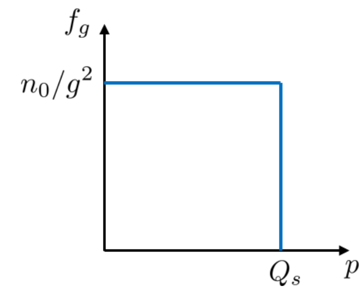
$$[i\partial\!\!\!/ + gA\!\!\!/(x) - m]\,\psi^{\pm}_{\boldsymbol{p},s,c}(x) = 0$$

Initial conditions

➢ Gauge sector

$\rho_0[A_{ini}]$: Gaussian density matrix that corresponds to the initial quasi-particle distribution

$$f_{\rm g}(\tau_0, p_\perp, p_z) = \frac{n_0}{g^2}\Theta\left(Q_s - \sqrt{p_\perp^2 + (\xi_0 p_z)^2}\right)$$



n_0 : initial over-occupancy parameter
ξ_0 : initial anisotropy parameter

Such an incoherent gluonic plasma is expected to appear after coherent Glasma fields decay by instabilities at the time scale of $\tau_0 \sim Q_s^{-1} \ln^2 \alpha_s^{-1}$.
We employ $g = 10^{-2}$ and $Q_s \tau_0 = 100$.

➢ Quark sector

perturbative vacuum $\psi^{\pm}_{\boldsymbol{p},s,c}(\tau_0, \boldsymbol{x}_\perp, \eta) = \text{free spinor}$

Momentum distribution function

statistical two-point functions → momentum distribution functions

$$\langle\{A_\mu(x), A_\nu(y)\}\rangle$$

$$\langle[\psi(x), \overline{\psi}(y)]\rangle$$

$$f_{\mathrm{g}}(\tau, p_\perp, p_z)$$

$$f_{\mathrm{q}}(\tau, p_\perp, p_z)$$

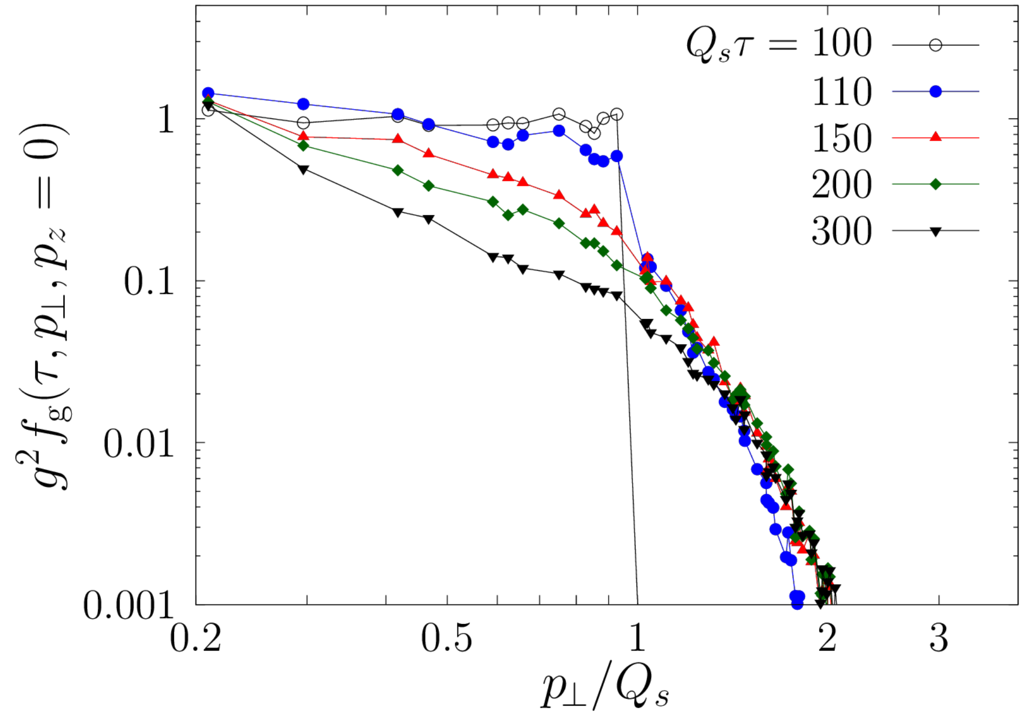
quasi-particle Fock-state projection $\langle a^\dagger_{\boldsymbol{p}} a_{\boldsymbol{p}}\rangle$

- (−) ambiguity in interacting theory
- (−) gauge dependence
- (+) provide insights into the nonequilibrium evolution
- (+) enable a direct comparison to the kinetic theory

Time evolution of the gluon distribution

$N_c = 2\,,\ g = 10^{-2}\,,\ n_0 = 1\,,\ \xi_0 = 2$

$N_\perp = 48\,,\ N_\eta = 256$
$Q_s a_\perp = 0.625\,,\ a_\eta = 1.95 \cdot 10^{-3}$



- IR and UV cascades
- Decrease due to the longitudinal expansion and momentum broadening
- At later times, it reaches non-thermal fixed point

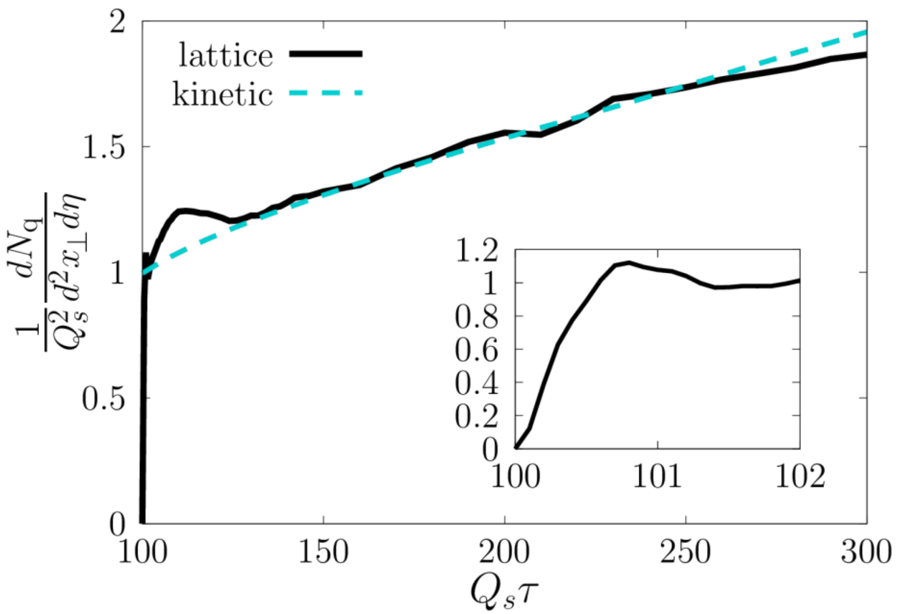
$$f_g(\tau, p_\perp, p_z) = (Q_s\tau)^{-2/3} f_S\left(p_\perp, (Q_s\tau)^{1/3} p_z\right)$$

Berges et al. PRD89 (2014)

Time evolution of the quark number density

Integrated quark number density per unit transverse area and per unit rapidity

$$\frac{dN_\mathrm{q}}{d^2x_\perp d\eta} = \nu_\mathrm{q}\,\tau \int \frac{d^2p_\perp dp_z}{(2\pi)^3} f_\mathrm{q}(\tau, p_\perp, p_z) \qquad \nu_\mathrm{q} = 2{\cdot}2N_f N_c$$



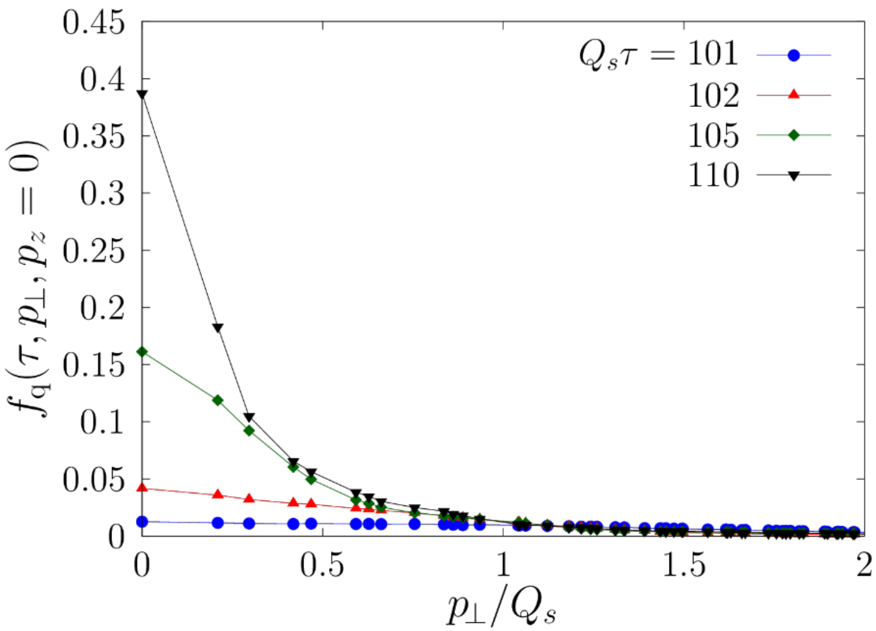
$m/Q_s = 0.1$

$N_f = 1$

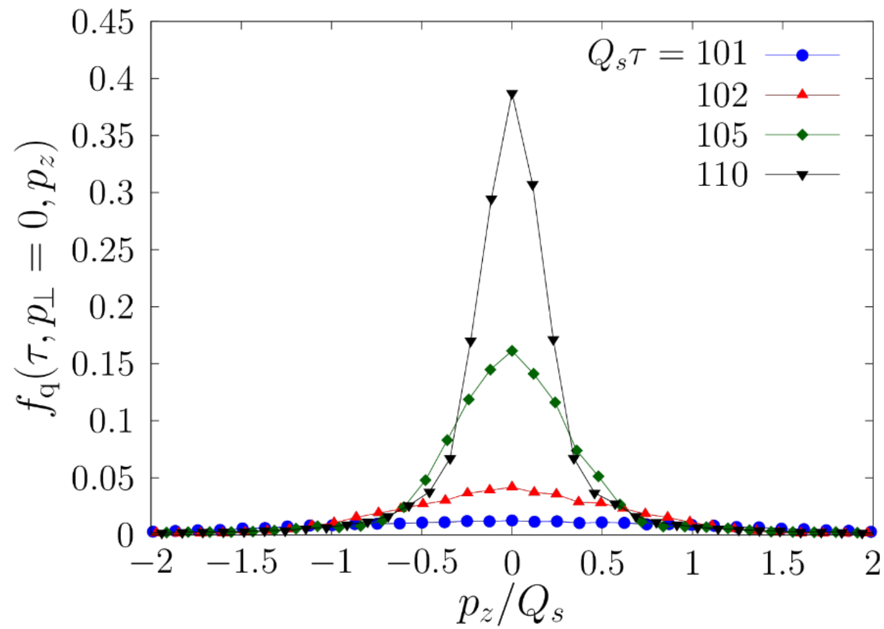
- rapid increase at earlier times ➡ non-perturbative production or initial quench
- nearly linear increase at later times ➡ well explained by the kinetic theory

Time evolution of the quark distribution

Earlier times



transverse momentum



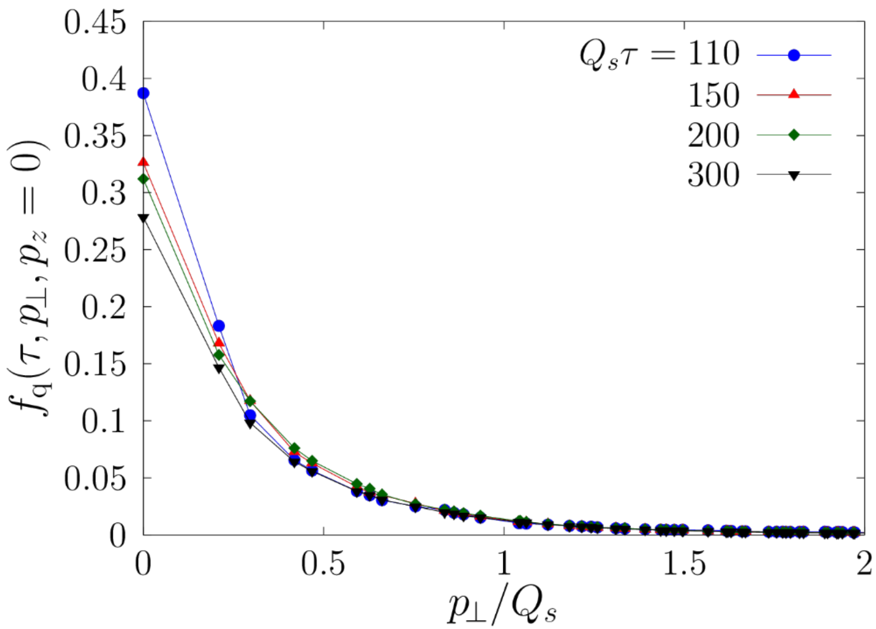
longitudinal momentum

- Occupation number of the order of one is developed at this stage.
- Quark production is an order-one effect even in weak coupling.

$$gA \sim \mathcal{O}(1)$$

Time evolution of the quark distribution

Later times



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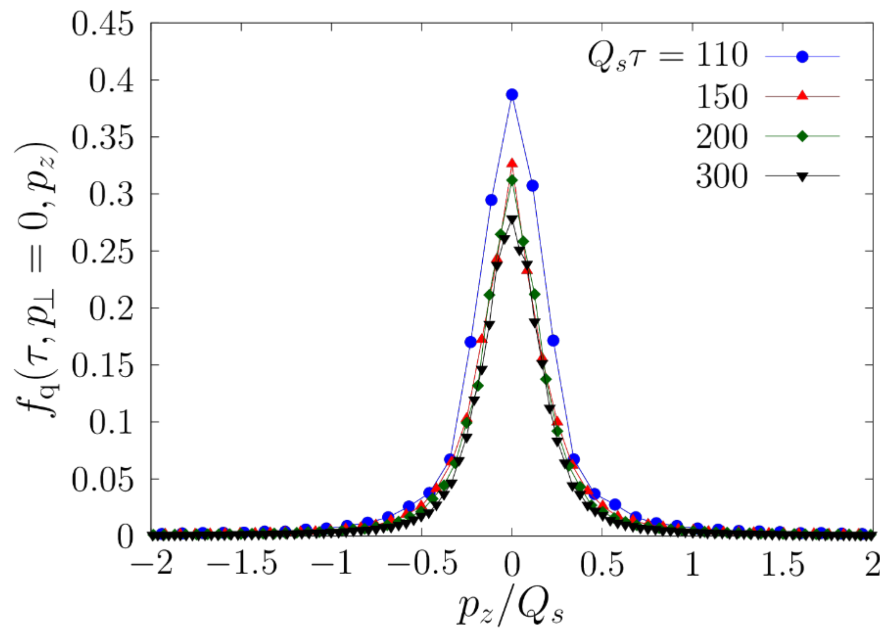


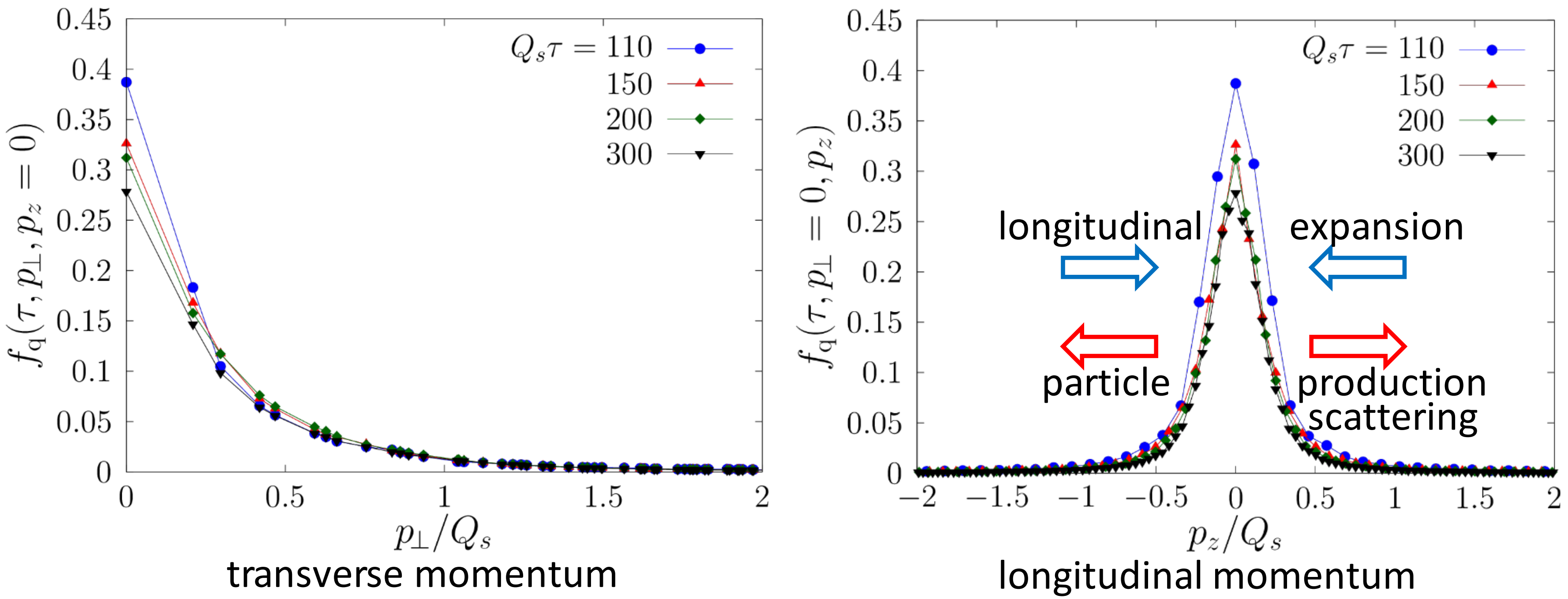
longitudinal momentum

- Occupation numbers show slight decrease due to the expansion of the system.
- The width of the longitudinal distribution is almost constant.

In the case of the free streaming (free particles in the expanding system), the longitudinal momentum decreases as $p_z \sim 1/\tau$.

Time evolution of the quark distribution

Later times



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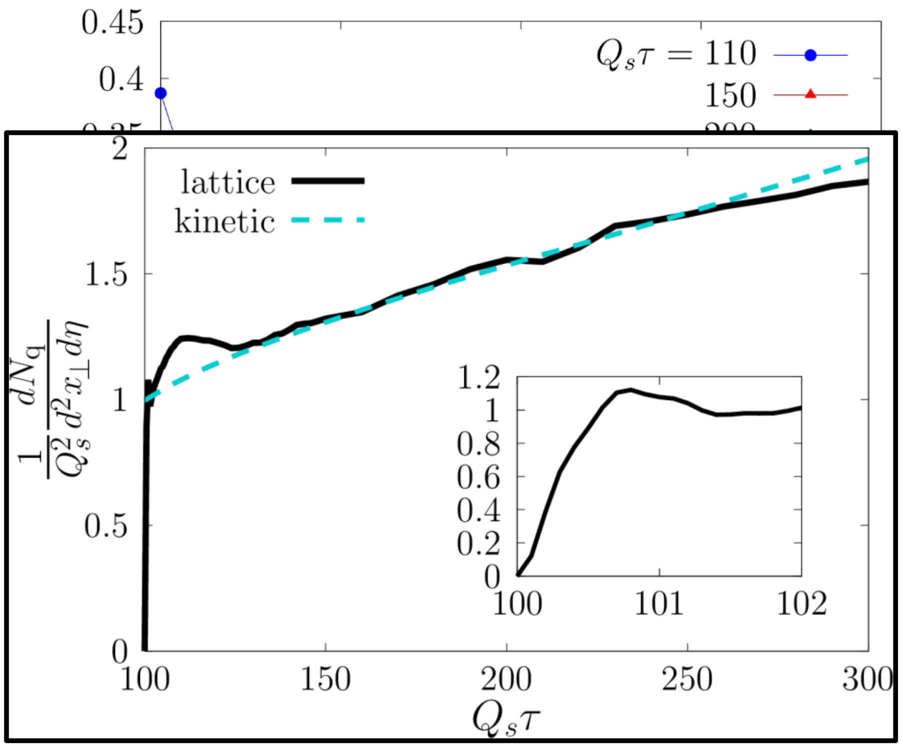
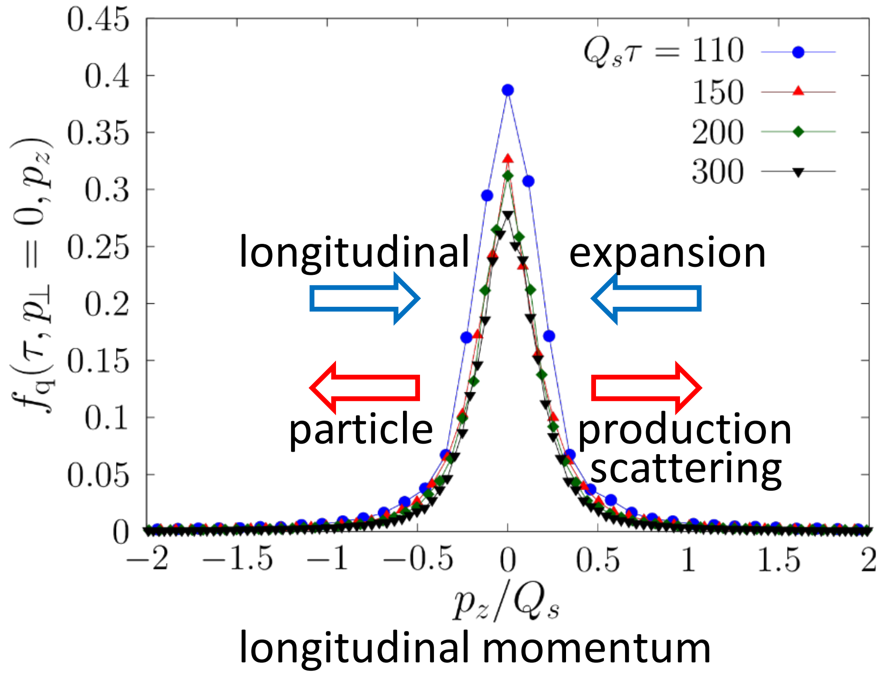
Time evolution of the quark distribution

Later times





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In the case of the free streaming (free particles in the expanding system), the longitudinal momentum decreases as $p_z \sim 1/\tau$.

Comparison to a kinetic estimate

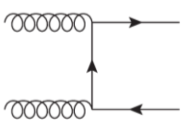
Boltzmann equation in the expanding geometry

$$\left(\frac{\partial}{\partial\tau} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f_{\mathrm{g/q}}(\tau,\boldsymbol{p}) = C_{\mathrm{g/q}}[f_{\mathrm{g}}, f_{\mathrm{q}}]$$

Total production rate

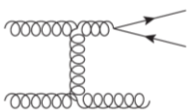
$$\frac{dN_{\mathrm{q}}}{d\tau d^2x_\perp d\eta} = \nu_{\mathrm{q}}\,\tau \int \frac{d^2p_\perp dp_z}{(2\pi)^3} C_{\mathrm{q}}$$

Consider only 2-2 scattering processes



Due to IR/collinear enhancement, 2-3 scatterings contribute to the leading order.

Arnold, Moore, Yaffe (2003)



Small-angle approximation

$$\frac{dN_{\mathrm{q}}}{d\tau d^2x_\perp d\eta} = \frac{g^4}{4\pi}\frac{(N_c^2-1)^2}{N_c} N_f \mathcal{L} I_c\,\tau \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[f_{\mathrm{g}}(1-2f_{\mathrm{q}}) - f_{\mathrm{q}}\right]$$

$$I_c = \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[f_{\mathrm{g}} + f_{\mathrm{q}}\right]$$

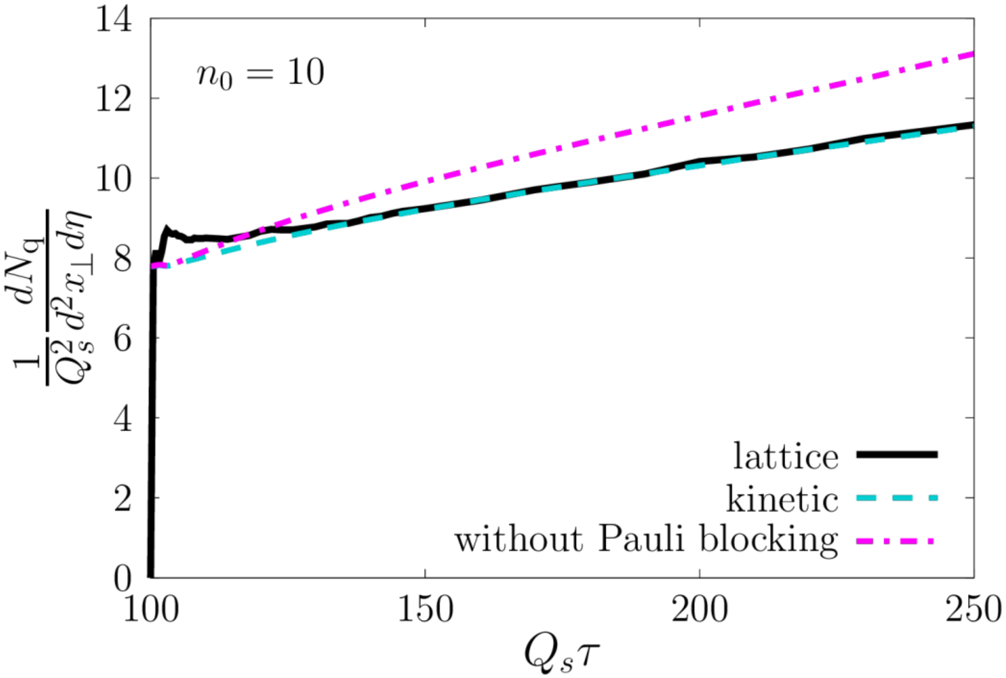
$$\mathcal{L} = \int_{m_D}^{Q_s} \frac{dq}{q} \qquad m_D^2 = 4g^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[N_c f_{\mathrm{g}} + N_f f_{\mathrm{q}}\right]$$

Comparison to a kinetic estimate

Substitute $f_{\mathrm{g/q}}$ obtained from the lattice calculations into the kinetic formula

$$\frac{dN_{\mathrm{q}}}{d\tau d^2x_\perp d\eta} = \frac{g^4}{4\pi}\frac{(N_c^2-1)^2}{N_c} N_f \mathcal{L} I_c \tau \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[f_{\mathrm{g}}(\underline{1-2f_{\mathrm{q}}}) - f_{\mathrm{q}}\right]$$

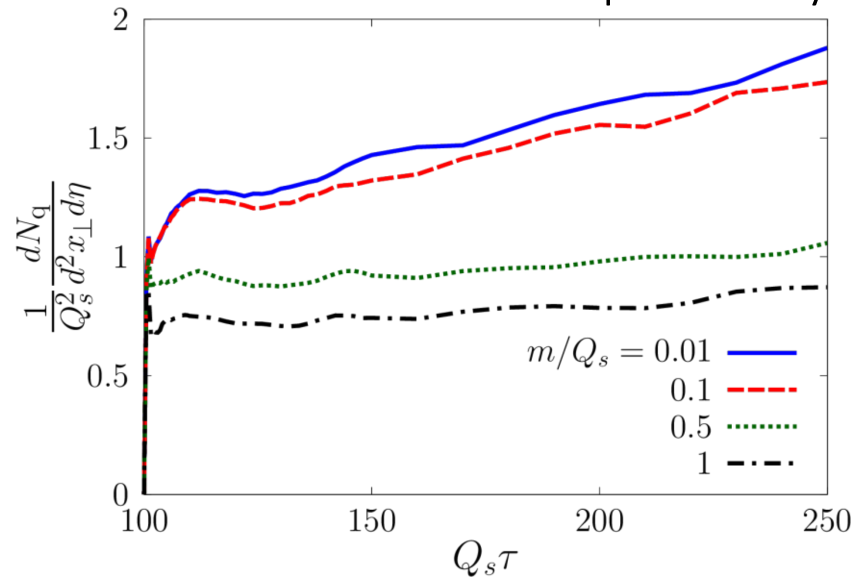
Pauli blocking



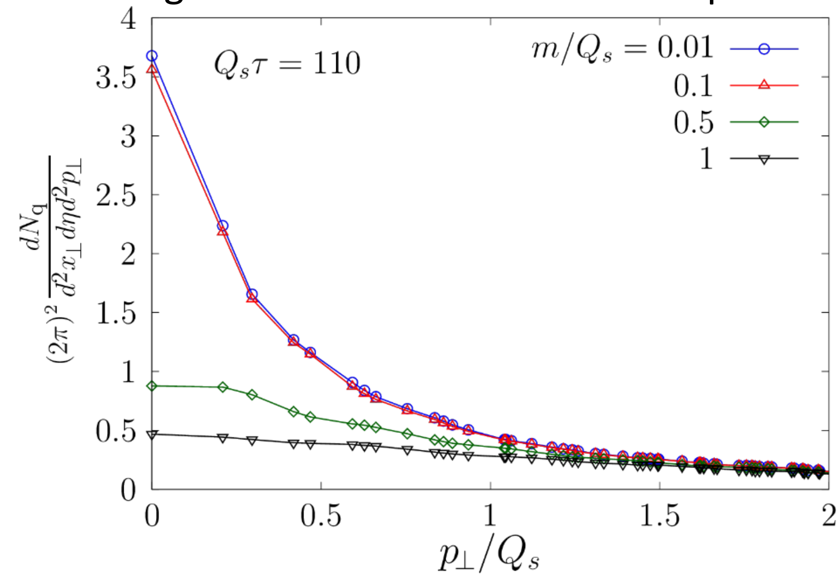
- Good agreement even for the initial gluon occupancy of $10/g^2$.
- The kinetic theory is normally not justified for such high density.
- The Pauli blocking is correctly described by the lattice calculations.

Quark mass dependence

Time evolution of the total quark density



Integrated transverse momentum spectrum

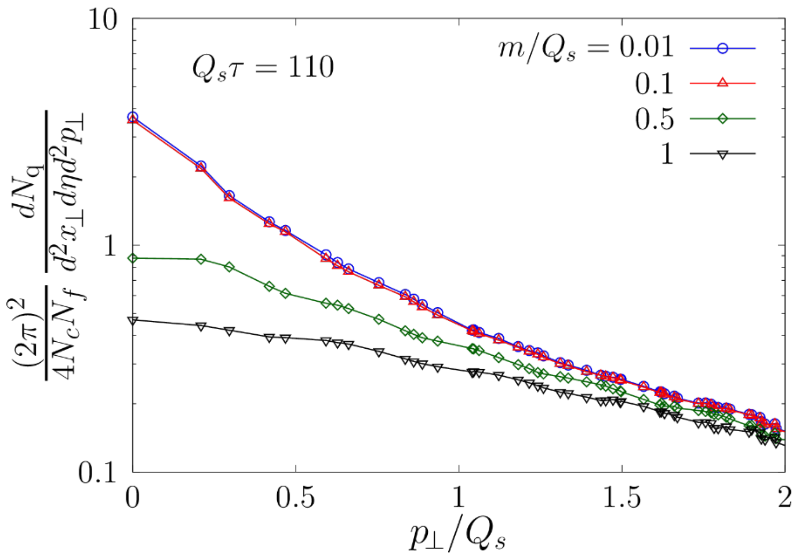


$$Q_s \sim 1\ \text{GeV} \quad \Rightarrow \quad \begin{aligned} m/Q_s &= 0.01 \gtrsim \text{u,d} \\ &= 0.1 \quad \sim \text{s} \\ &= 1 \quad\;\; \sim \text{c} \end{aligned}$$

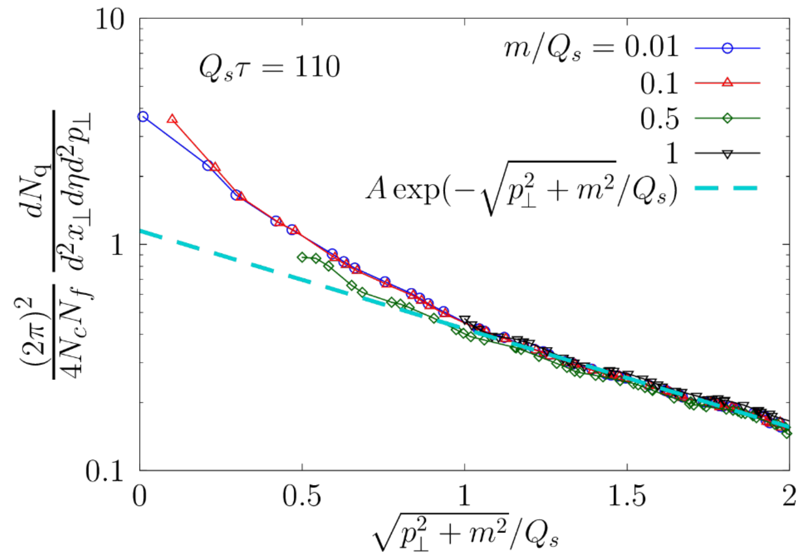
- Natural mass ordering.
- Lighter quarks $m/Q_s \leq 0.1$ are almost degenerated.

mT scaling

Integrated transverse momentum spectrum



as a function of the transverse momentum



as a function of the transverse mass

$$m_T = \sqrt{p_\perp^2 + m^2}$$

- For $m_T \gtrsim Q_s$, all the spectra for different masses lie on top of each other.
- It shows an exponential form $\exp(-m_T/Q_s)$,
  which resembles the Boltzmann distribution.

Summary

- The early stage of high-energy heavy-ion collisions is a nonequilibrium state of high-density gluons expanding to the longitudinal direction.
- The nonequilibrium evolution of quark fields and overoccupied gauge fields can be studied by classical-statistical real-time lattice computations.
- The lattice results for quark production nicely agree with the kinetic estimate that includes only 2-2 scattering processes, although the kinetic theory is not a priori justified in such a dense system.
- The transverse momentum spectra of quarks satisfy the mT scaling at high mT tails, and they show the Boltzmann-like exponential shape.