Symmetry breaking patterns in QCD at high density

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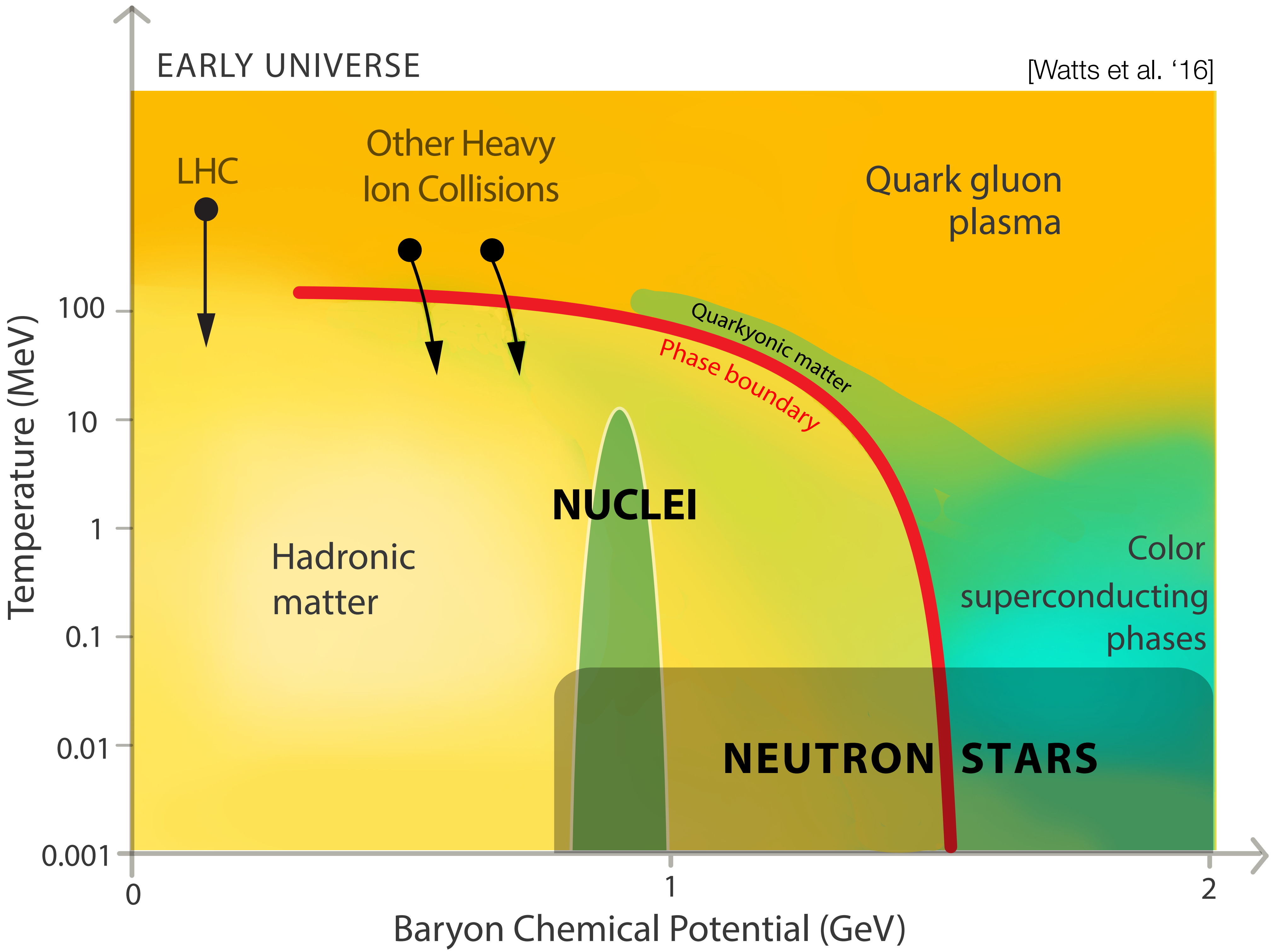
LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

HIC for FAIR
Helmholtz International Center

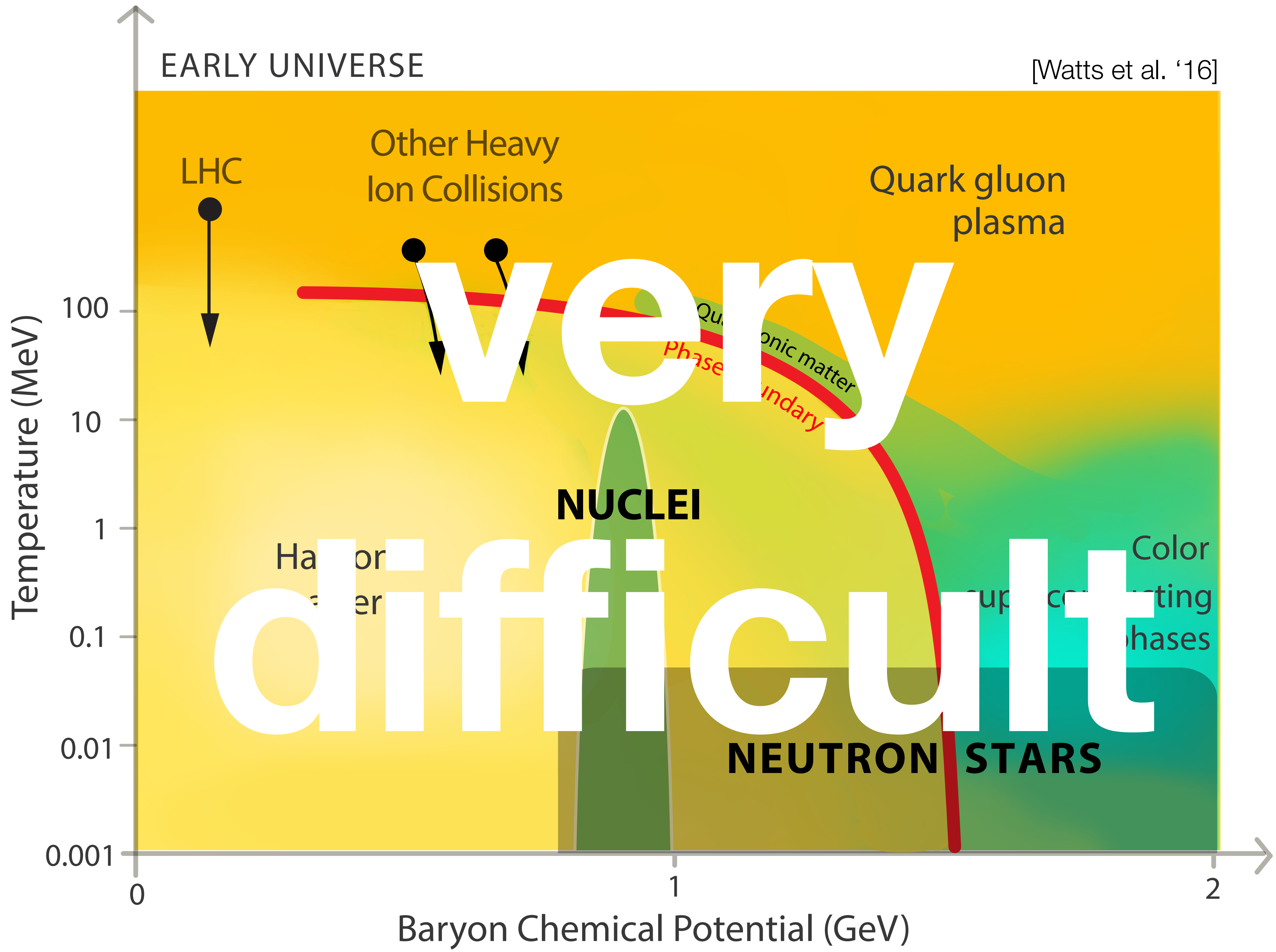
SFB 1245

[based on JB, Leonhardt, Pospiech, arXiv:1705.00074 & arXiv:1801.08338 & in prep.]

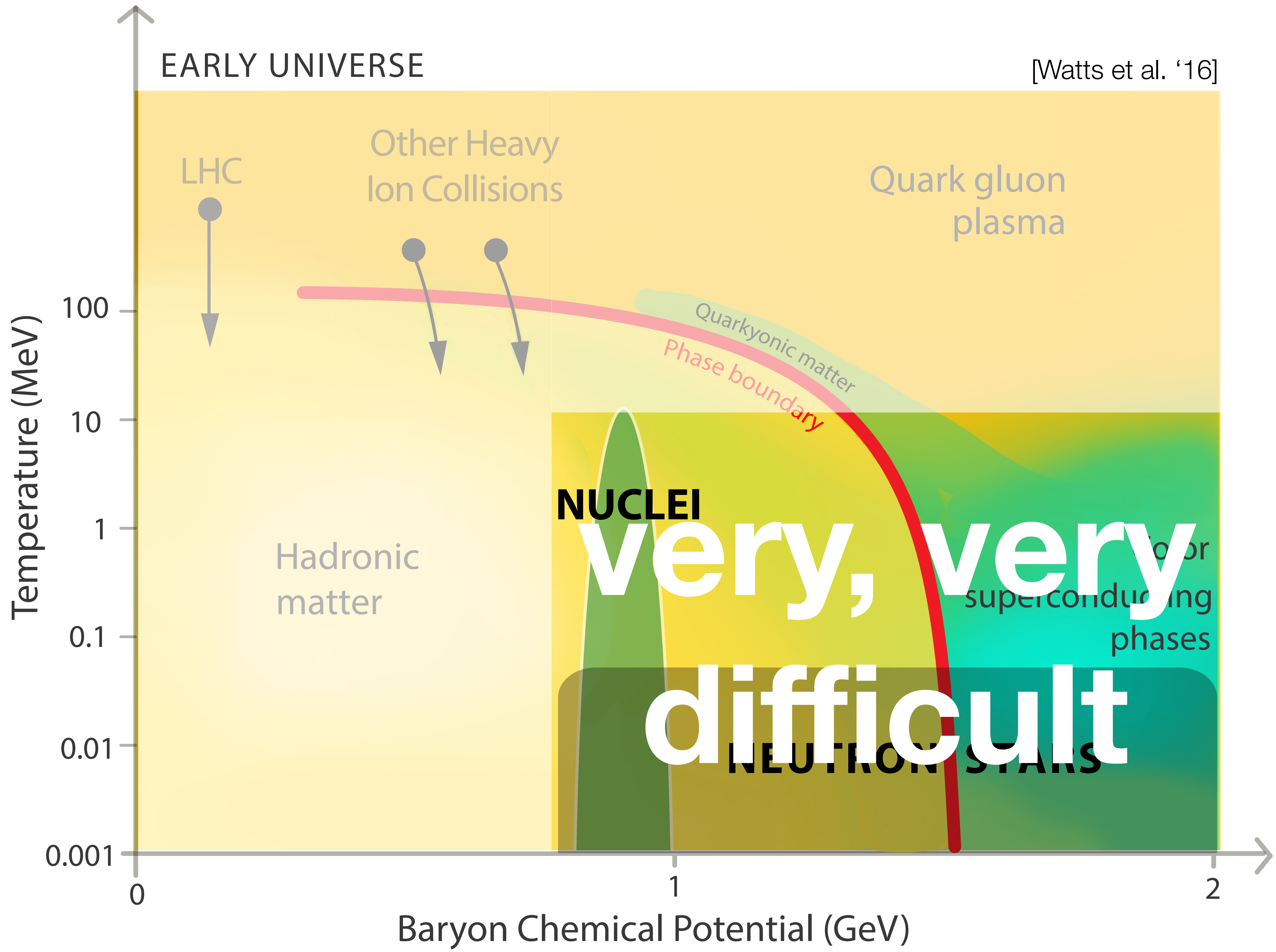
Phases of QCD



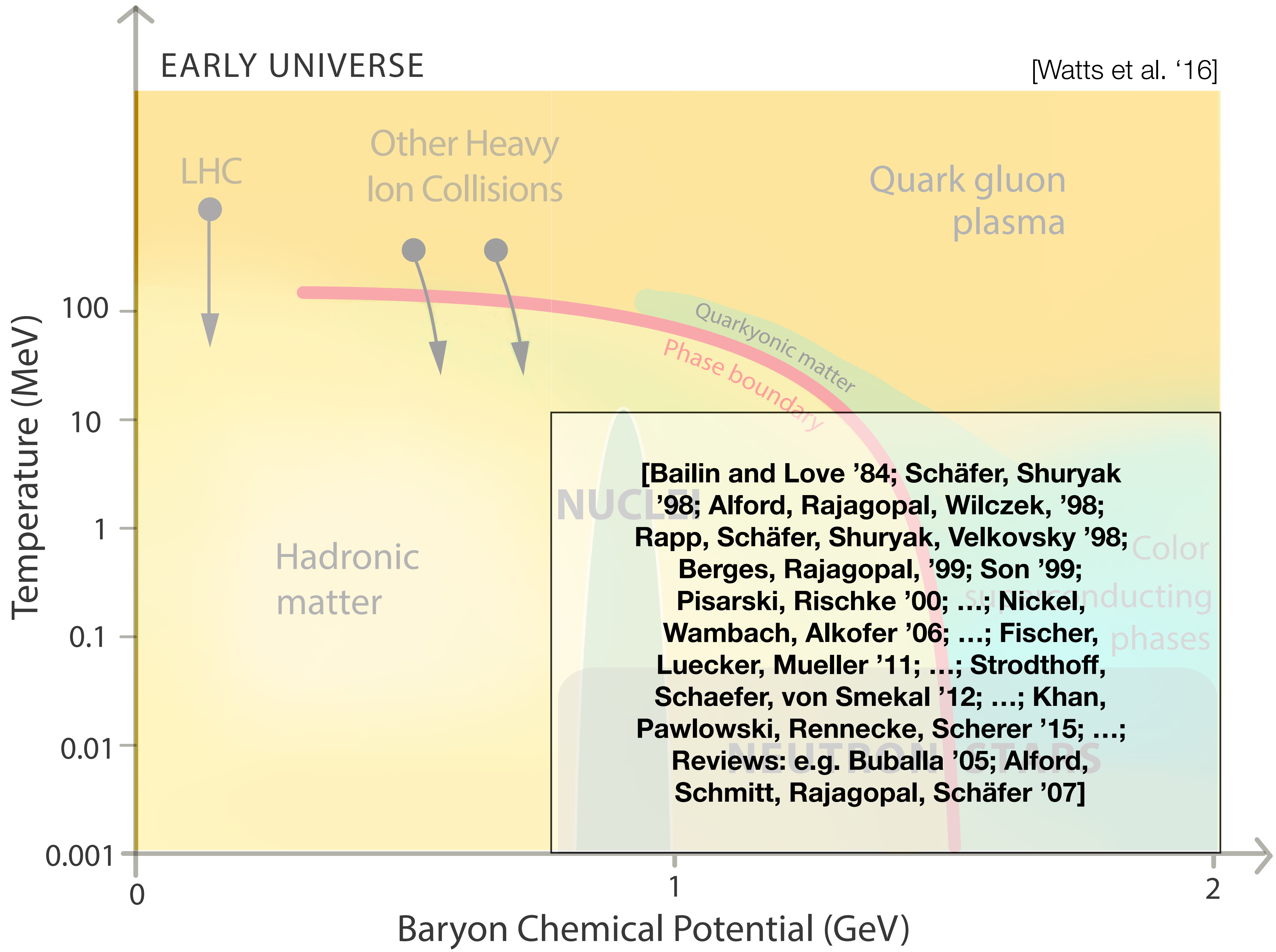
Phases of QCD



Phases of QCD

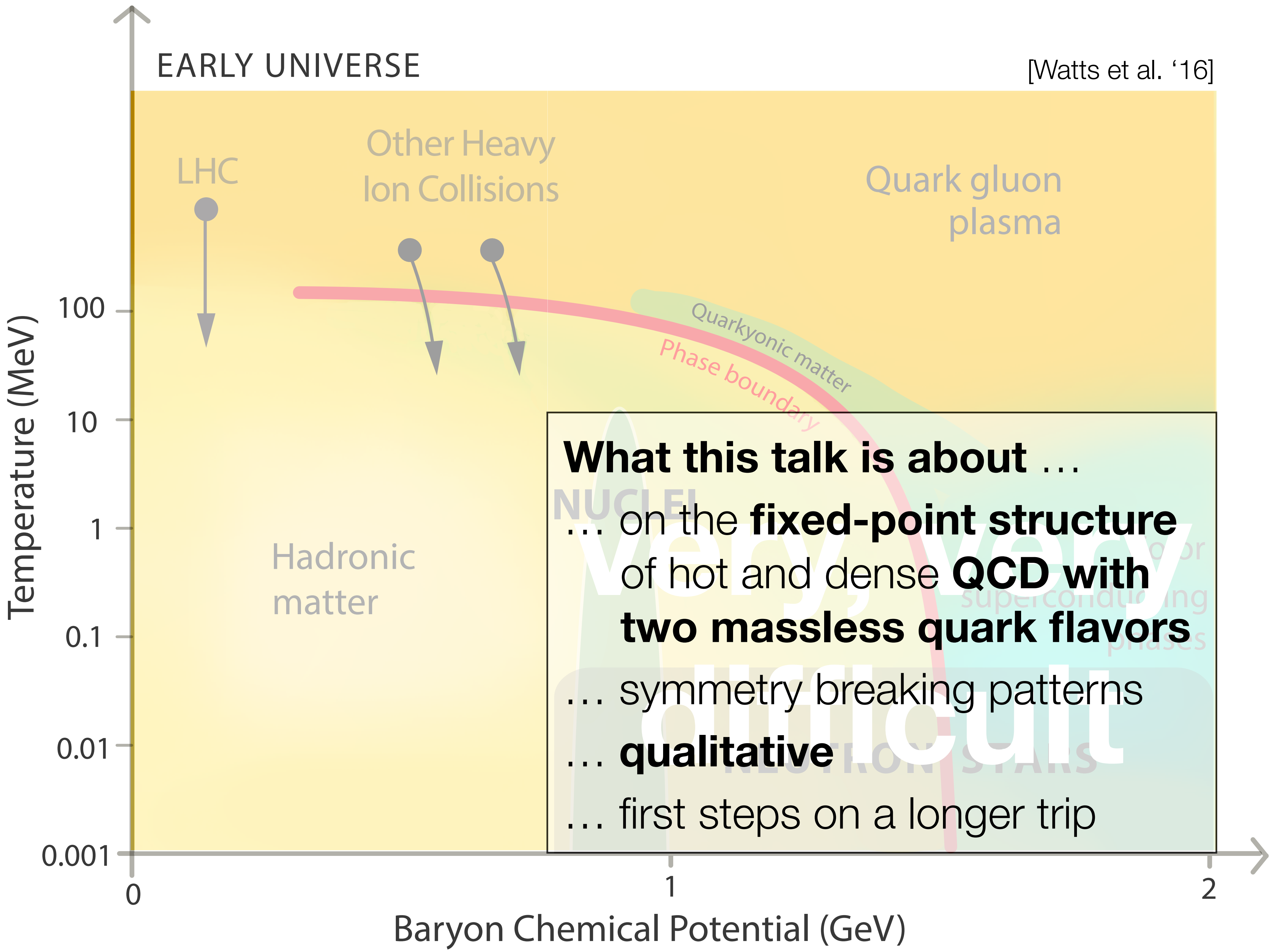


Phases of QCD



[Bailin and Love '84; Schäfer, Shuryak '98; Alford, Rajagopal, Wilczek, '98; Rapp, Schäfer, Shuryak, Velkovsky '98; Berges, Rajagopal, '99; Son '99; Pisarski, Rischke '00; …; Nickel, Wambach, Alkofer '06; …; Fischer, Luecker, Mueller '11; …; Strodthoff, Schaefer, von Smekal '12; …; Khan, Pawlowski, Rennecke, Scherer '15; …; Reviews: e.g. Buballa '05; Alford, Schmitt, Rajagopal, Schäfer '07]

Phases of QCD



What this talk is about …

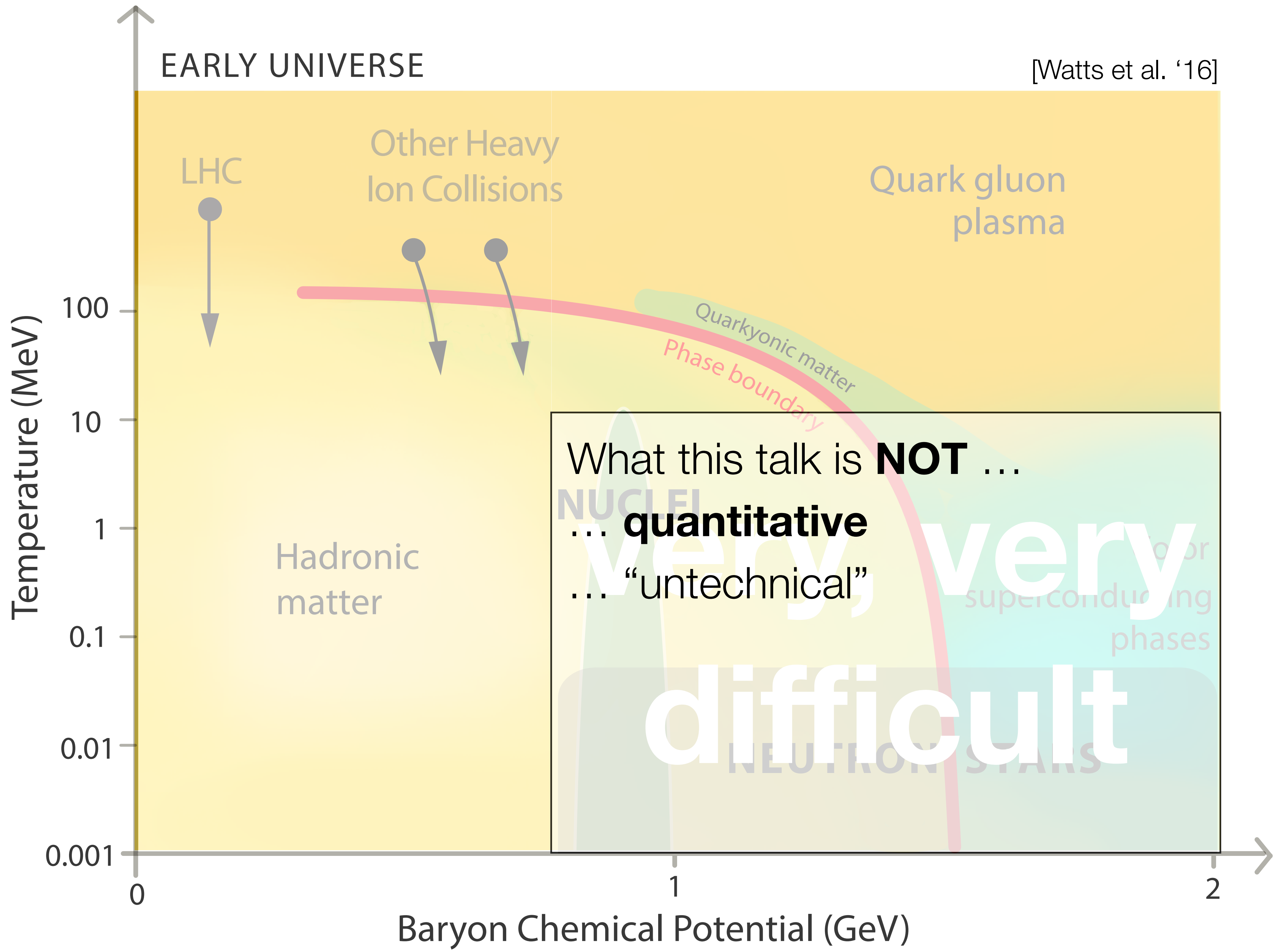
… on the **fixed-point structure** of hot and dense **QCD with two massless quark flavors**

… symmetry breaking patterns

… **qualitative**

… first steps on a longer trip

Phases of QCD



QCD: from high to low energies

- Renormalization Group (RG) flow:

$$S \simeq \Gamma_{k=\Lambda} = \int_x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{g} A\!\!\!/)\psi \right\}$$

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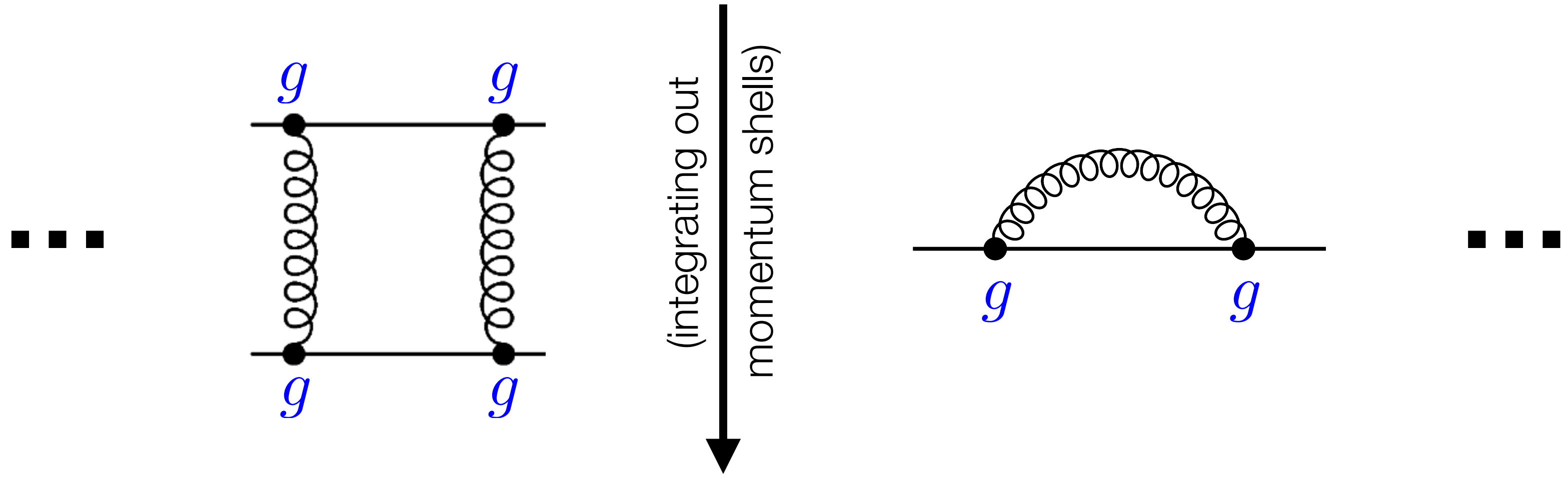

$$\Gamma_{k<\Lambda-\delta k} = \int_x \left\{ \frac{\bar{g}^2}{g^2} F_{\mu\nu} F_{\mu\nu} + \cdots + \bar{\psi} \left(\mathrm{i} Z_\psi \partial\!\!\!/ + \mathrm{i} Z_1 \bar{g} A\!\!\!/ \right) \psi \right.$$

$$\left. + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \lambda_{\alpha\beta\gamma\delta} \bar{\psi}_\alpha \psi_\beta \bar{\psi}_\gamma \psi_\delta + \ldots \right\}$$

QCD: from high to low energies

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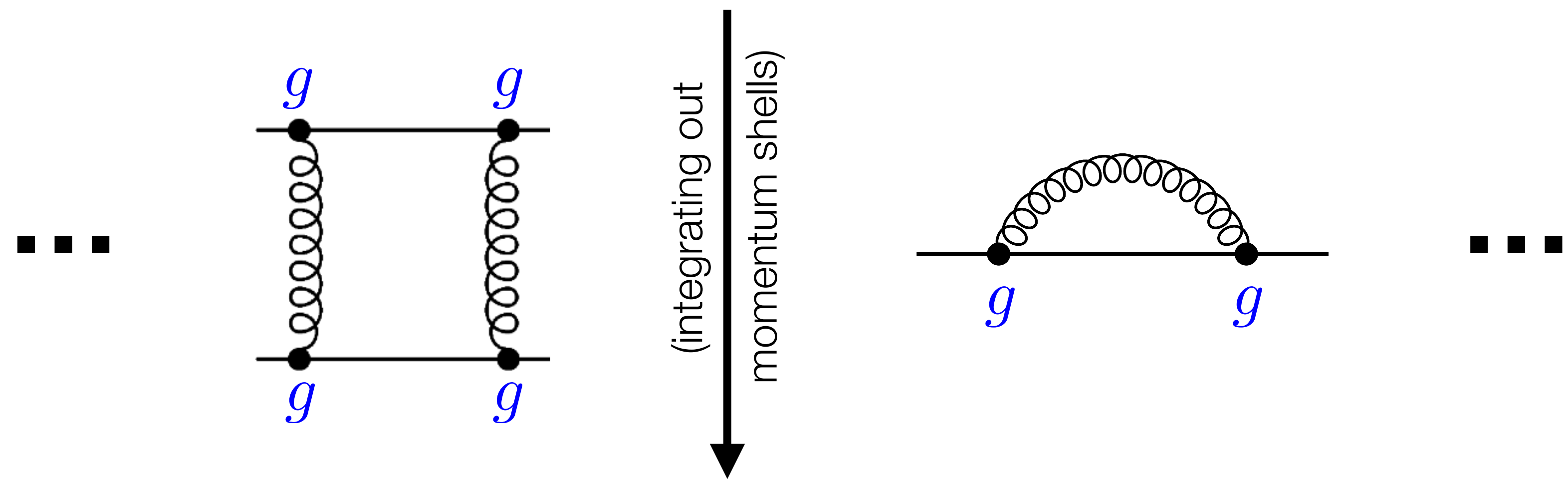

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$$\left. + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \lambda_{\alpha\beta\gamma\delta} \bar{\psi}_\alpha \psi_\beta \bar{\psi}_\gamma \psi_\delta + \dots \right\}$$

Role of gluon-induced four-fermion interactions?

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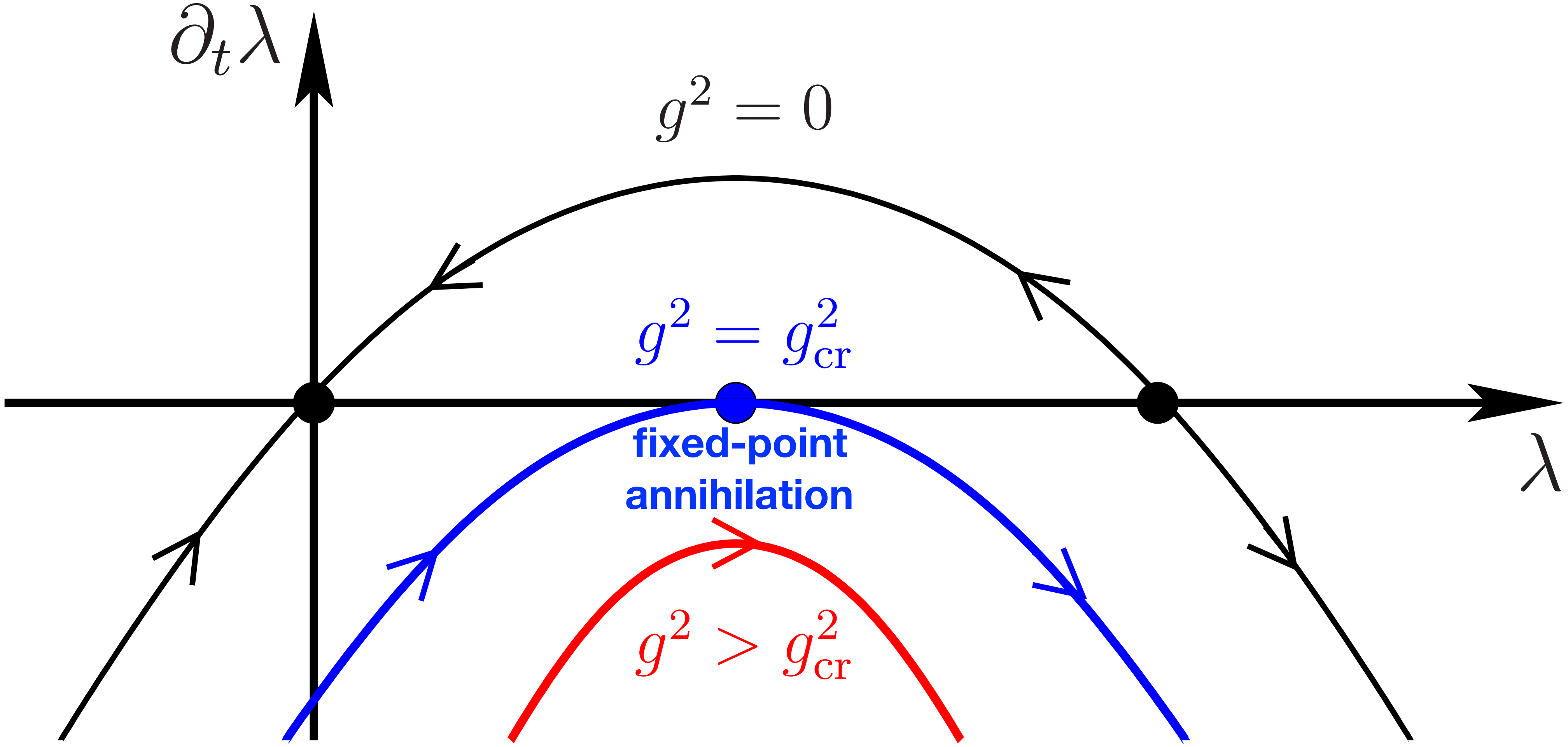
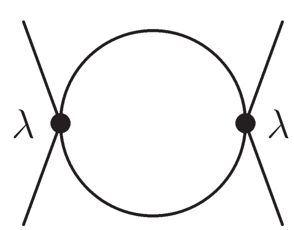
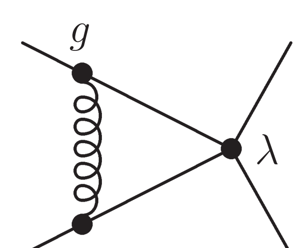
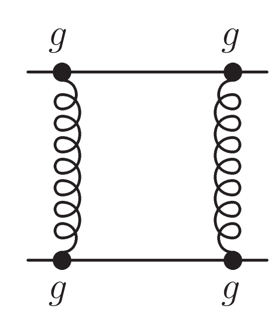
[Gies & Jaeckel ’05; JB & Gies ’05, ’06]



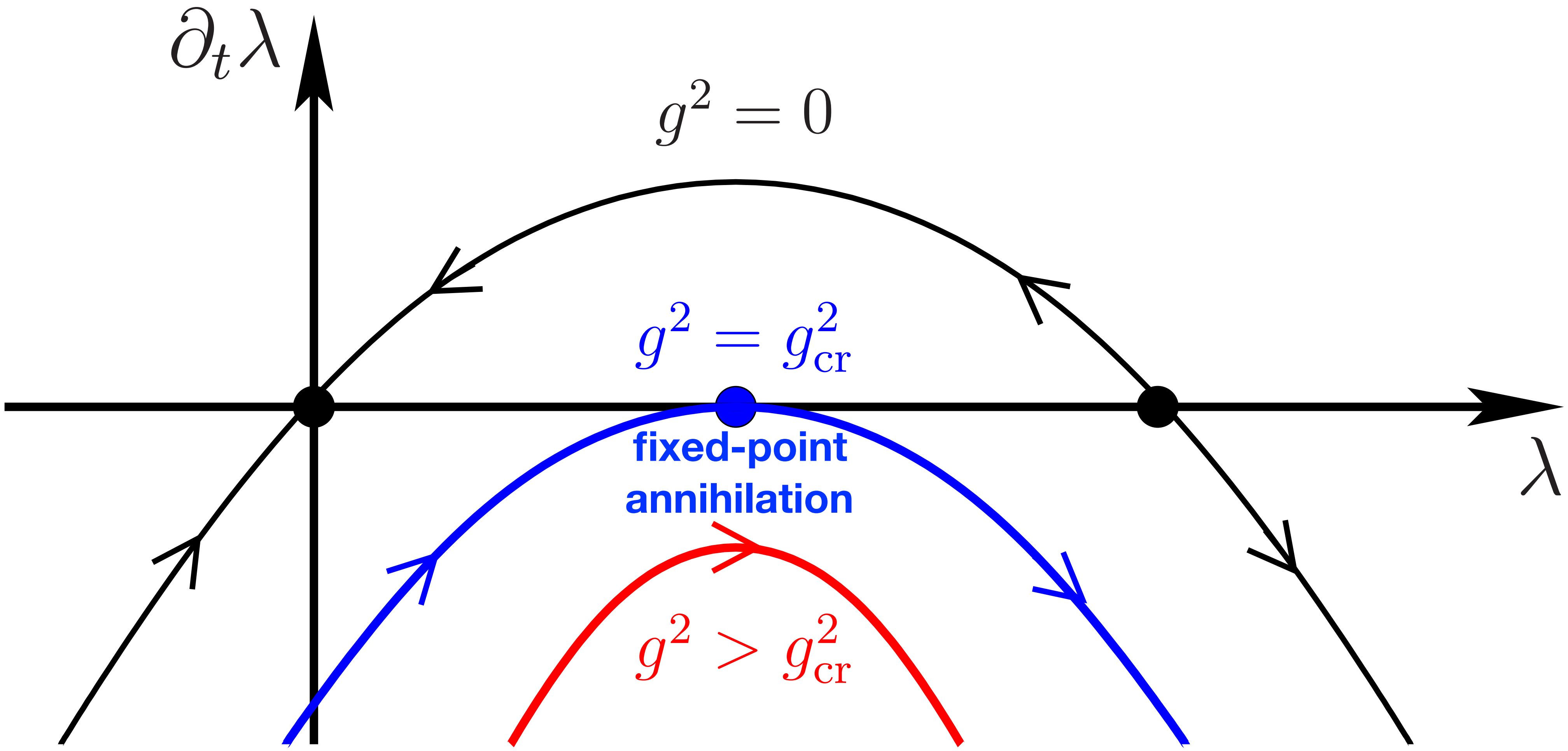
$\partial_t\lambda \equiv k\partial_k\lambda \simeq 2\lambda -$  $-$  $-$ 

$\sim \lambda^2$ $\quad\sim \lambda g^2$ $\quad\sim g^4$

k is the RG scale:
k~p (“momentum scale”)

Role of gluon-induced four-fermion interactions

[Gies & Jaeckel '05; JB & Gies '05, '06]



critical gauge coupling g^2_{cr}:

if $g^2 > g^2_{\mathrm{cr}}$ $\Longrightarrow$ **no** fixed points
$(\lambda \to \infty)$

How many four-quark channels? Symmetries!

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[Gies & Jaeckel ’05; JB & Gies ’05, ’06; Mitter, Pawlowski, Strodthoff ’14; JB, Leonhardt, Pospiech’ 18]

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\mathrm{U}_A(1)$

Poincare	✓
time reversal	✓
parity	✓
charge conjugation	✓

# of channels	4
# of fixed points	16

Fierz-complete set

How many four-quark channels? Symmetries!

[Mitter, Pawlowski, Strodthoff '14; JB, Leonhardt, Pospiech' 18]

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	~~$\mathrm{U}_A(1)$~~

Poincare	✓
time reversal	✓
parity	✓
charge conjugation	✓

# of channels	6
# of fixed points	64

Fierz-complete set

How many four-quark channels? Symmetries!

[JB, Leonhardt, Pospiech' 18]

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\mathrm{U}_A(1)$

Poincare	✗
time reversal	✓
parity	✓
charge conjugation	✗

# of channels	8
# of fixed points	256

Fierz-complete set

How many four-quark channels? Symmetries!

[JB, Leonhardt, Pospiech' 18]

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\mathrm{U}_A(1)$ ✗
Poincare	✗
time reversal	✓
parity	✓
charge conjugation	✗
# of channels	**10**
# of fixed points	**1024**

Fierz-complete set

How many four-quark channels? Symmetries!

[JB, Leonhardt, Pospiech' 18]

may be broken spontaneously

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\mathrm{U}_A(1)$ ✗

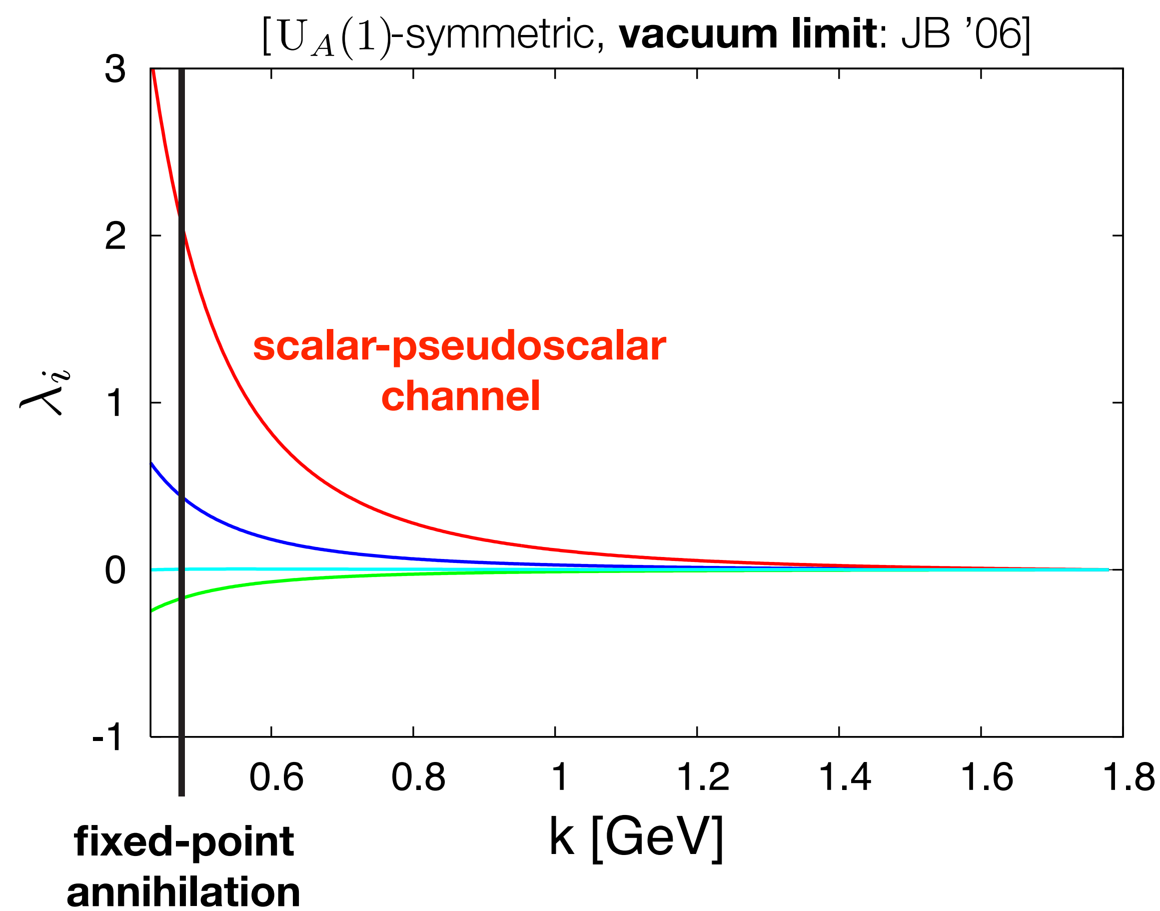
Poincare	✗
time reversal	✓
parity	✓
charge conjugation	✗

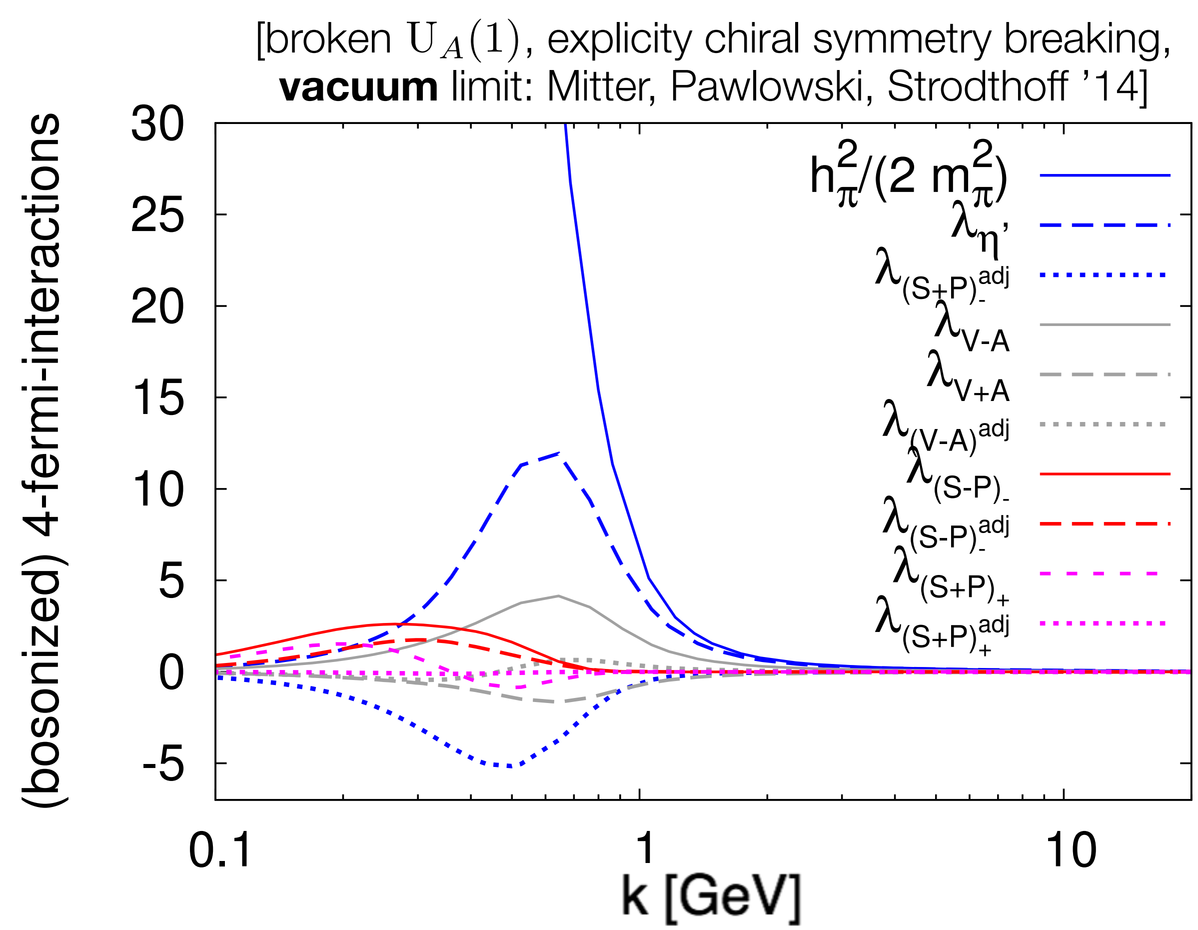
# of channels	$\gg 10$
# of fixed points	$\gg 1024$

Fierz-complete set

Running of gluon-induced four-quark interactions

[JB '06; Mitter, Pawlowski, Strodthoff '14; Springer, JB, Rechenberger, Rennecke' 16]





- scalar-pseudoscalar channel is dominantly generated at high scales
- at high scales: similar behavior at finite temperature and chemical potential

Four-quark interactions and symmetry breaking?

Aspects of the low-energy regime: brief reminder

- classical action (NJL model):

$$S = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\lambda}_{(\sigma-\pi)} \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5 \vec{\tau}\psi)^2 \right] \right\}$$

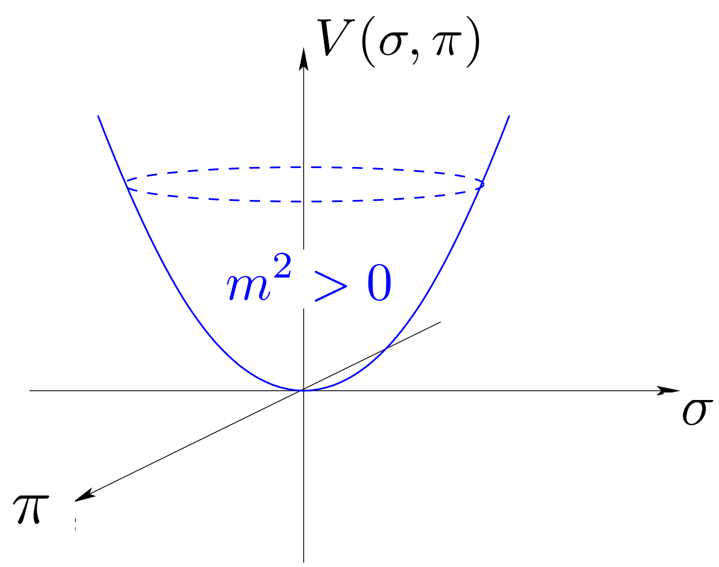
- spontaneous (chiral) symmetry breaking:

$$\langle \bar{\psi}\psi \rangle \neq 0$$

- bosonized version: $(\sigma \sim \bar{\psi}\psi\,, \quad \vec{\pi} \sim \bar{\psi}\gamma_5\vec{\tau}\psi)$

$$S = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\psi}(\sigma + \mathrm{i}\gamma_5 \vec{\tau}\cdot\vec{\pi})\psi - \bar{\lambda}^{-1}_{(\sigma-\pi)} (\sigma^2 + \vec{\pi}^2) \right\}$$

Aspects of the low-energy regime: brief reminder



no fermion mass/gap

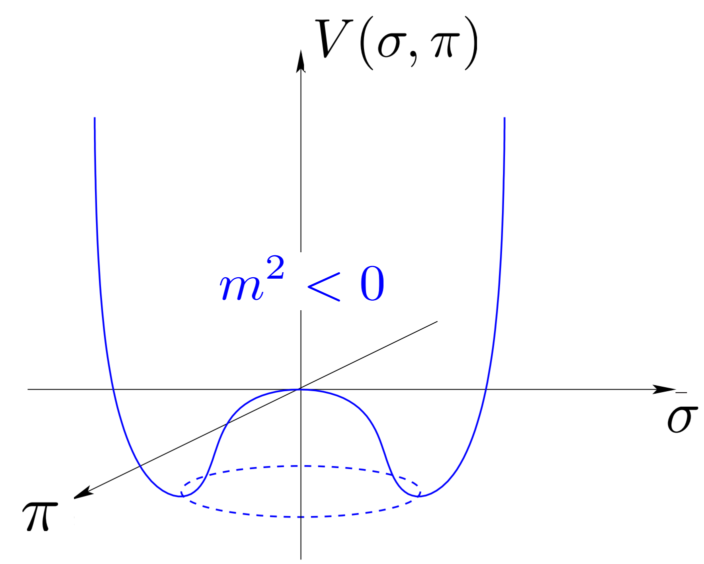
$\langle\bar{\psi}\psi\rangle = 0$

(symmetric phase)

large four-fermion coupling indicates onset of chiral symmetry breaking

$m^2 \sim 1/\bar{\lambda}_{(\sigma-\pi)}$

→



finite fermion mass/gap

$\langle\bar{\psi}\psi\rangle \neq 0$

(broken phase)

wine-bottle potential

- bosonized version: $(\sigma \sim \bar{\psi}\psi\,, \quad \vec{\pi} \sim \bar{\psi}\gamma_5\vec{\tau}\psi)$

$$S = \int_x \left\{ \bar{\psi}\mathrm{i}\partial\!\!\!/\psi + \bar{\psi}(\sigma + \mathrm{i}\gamma_5\vec{\tau}\cdot\vec{\pi})\psi - \bar{\lambda}^{-1}_{(\sigma-\pi)}(\sigma^2 + \vec{\pi}^2) \right\}$$

Effective low-energy description

[JB, Leonhardt, Pospiech '18]

- Fierz-complete ansatz for the effective action:

$$\Gamma_k = \int_x \Big\{ \bar{\psi}\left(\mathrm{i}Z_{\|}\gamma_0\partial_0 + \mathrm{i}Z_{\perp}\gamma_i\partial_i - \mathrm{i}Z_{\mu}\mu\gamma_0\right)\psi + \frac{1}{2}\bar{\lambda}_{(\sigma-\pi)}\,(\mathrm{S}-\mathrm{P}) + \frac{1}{2}\bar{\lambda}_{\mathrm{csc}}\,(\mathrm{CSC}) + \frac{1}{2}\bar{\lambda}_{(S+P)_-}\,(\mathrm{S}+\mathrm{P})_- + \frac{1}{2}\bar{\lambda}_{(S+P)_-^{\mathrm{adj}}}\,(\mathrm{S}+\mathrm{P})_+^{\mathrm{adj}} + \sum_{j=5}^{10}\frac{1}{2}\bar{\lambda}_j\left(\mathcal{O}_{\bar{\psi}\psi}\right)^{(j)} \Big\}$$

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- complicated ground state: many "wine bottles" …

Effective low-energy description

[JB, Leonhardt, Pospiech '18]

- $\mathrm{U}_A(1)$-breaking channels:

$$(\mathrm{S}-\mathrm{P}) = \left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\vec{\tau}\psi\right)^2$$

[associated with the formation of a **chiral condensate**]

$$(\mathrm{CSC}) = 4\left(\mathrm{i}\bar{\psi}\gamma_5\vec{\tau}T^a\psi^C\right)\left(\mathrm{i}\bar{\psi}^C\gamma_5\vec{\tau}T^a\psi\right)$$

[associated with the formation of a **diquark condensate**]

$$(\mathrm{S}+\mathrm{P})_- = \left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\vec{\tau}\psi\right)^2 + \left(\bar{\psi}\gamma_5\psi\right)^2 - \left(\bar{\psi}\vec{\tau}\psi\right)^2$$

$$(\mathrm{S}+\mathrm{P})_-^{\mathrm{adj}} = \left(\bar{\psi}T^a\psi\right)^2 - \left(\bar{\psi}\gamma_5\vec{\tau}T^a\psi\right)^2 + \left(\bar{\psi}\gamma_5T^a\psi\right)^2 - \left(\bar{\psi}\vec{\tau}T^a\psi\right)^2$$

- $\mathrm{U}_A(1)$-symmetric channels: the remaining **six** channels

Effective low-energy description

[JB, Leonhardt, Pospiech '18]

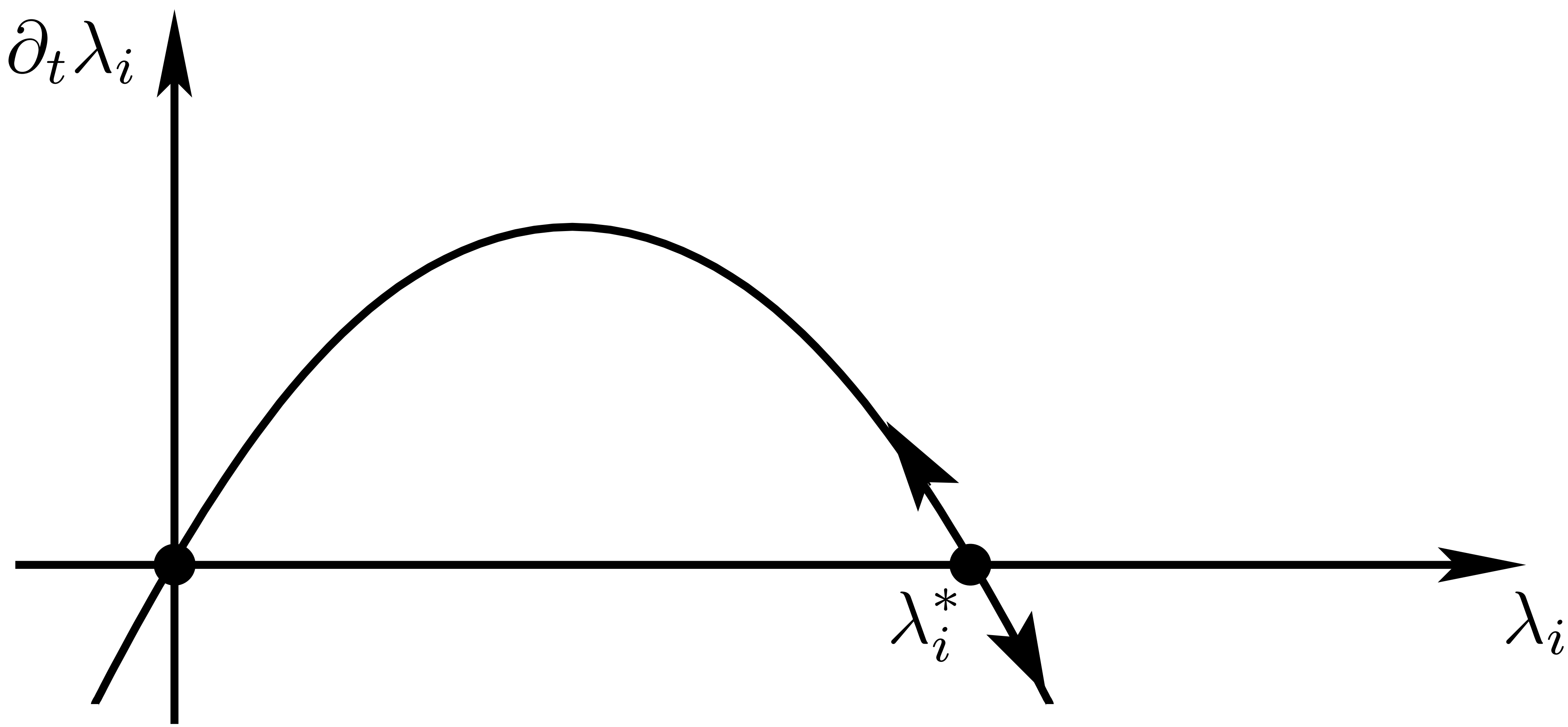
- Fierz-complete ansatz for the effective action:

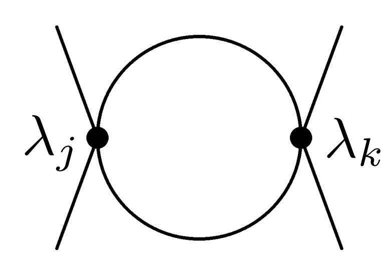
$$\Gamma_k = \int_x \Big\{ \bar{\psi}\left(\mathrm{i}Z_{\|}\gamma_0\partial_0 + \mathrm{i}Z_{\perp}\gamma_i\partial_i - \mathrm{i}Z_{\mu}\mu\gamma_0\right)\psi + \frac{1}{2}\bar{\lambda}_{(\sigma-\pi)}\,(\mathrm{S}-\mathrm{P}) + \frac{1}{2}\bar{\lambda}_{\mathrm{csc}}\,(\mathrm{CSC}) + \frac{1}{2}\bar{\lambda}_{(S+P)_-}\,(\mathrm{S}+\mathrm{P})_- + \frac{1}{2}\bar{\lambda}_{(S+P)_-^{\mathrm{adj}}}\,(\mathrm{S}+\mathrm{P})_+^{\mathrm{adj}} + \sum_{j=5}^{10}\frac{1}{2}\bar{\lambda}_j\,\left(\mathcal{O}_{\bar{\psi}\psi}\right)^{(j)} \Big\}$$

- leading order in the derivative expansion, allows to preserve Fierz-completeness

- parameter fixing **inspired** by gluon-induced four-quark flows: $\left(\bar{\lambda}_{(\sigma-\pi)}(\Lambda) \neq 0, \bar{\lambda}_{\mathrm{csc}}(\Lambda) = 0, \ldots, \bar{\lambda}_{10}(\Lambda) = 0\right)$

RG flow of four-quark interactions at low energies



$$\partial_t \lambda_i \equiv k\partial_k \lambda_i \simeq 2\lambda_i - \sum_{j,k}$$ 

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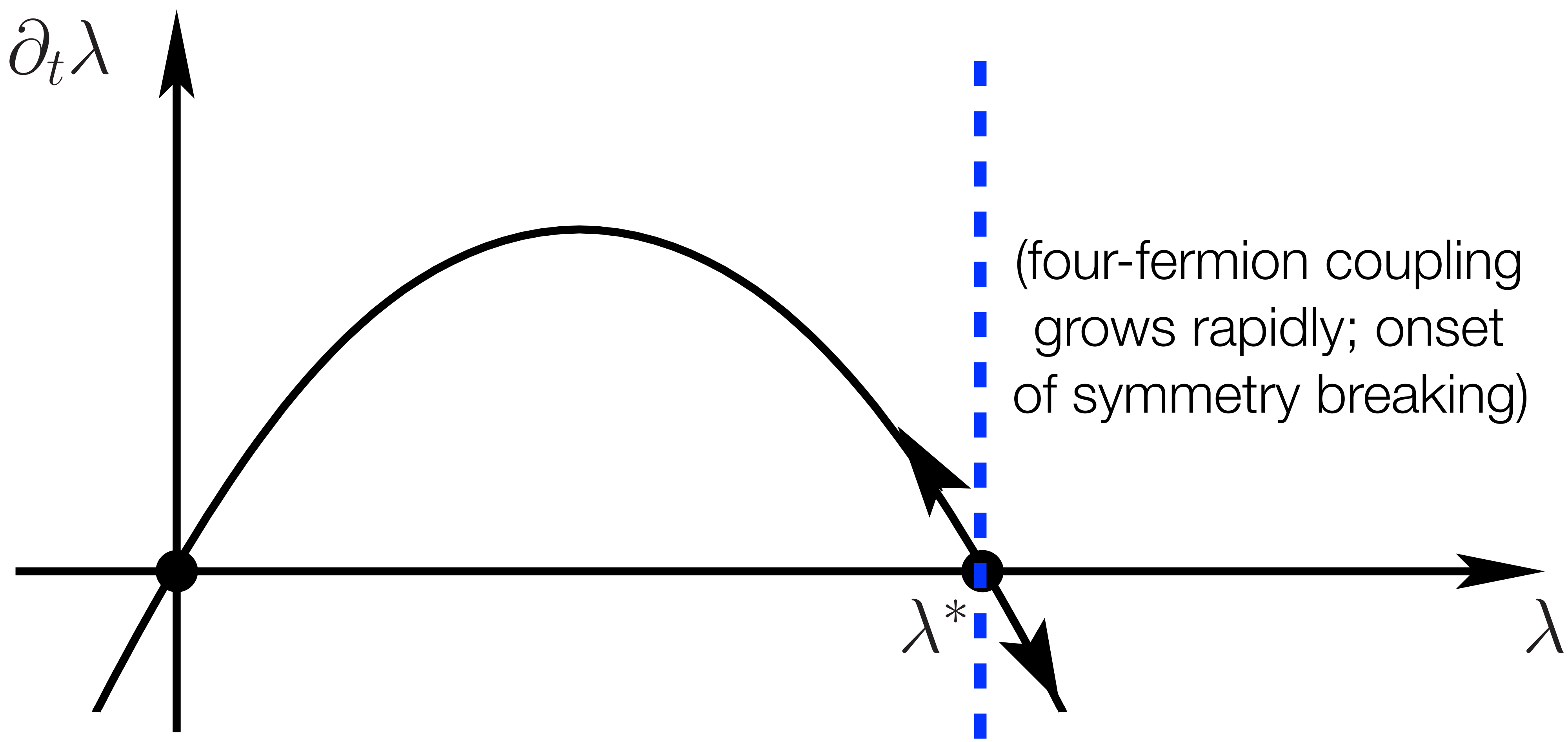
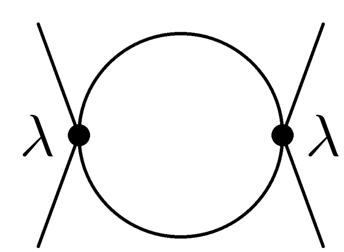


$$\partial_t \lambda \equiv k \partial_k \lambda \simeq 2\lambda -$$



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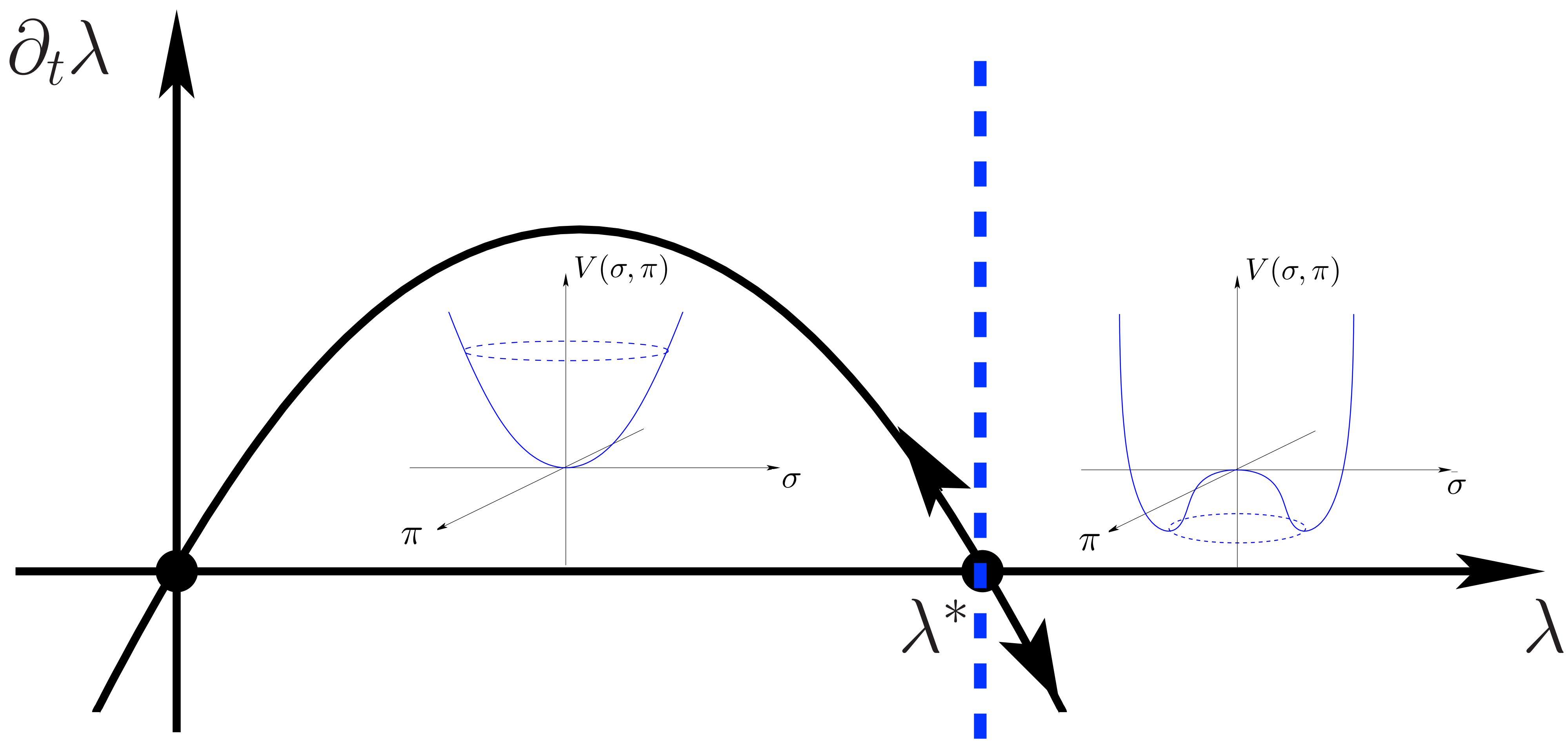
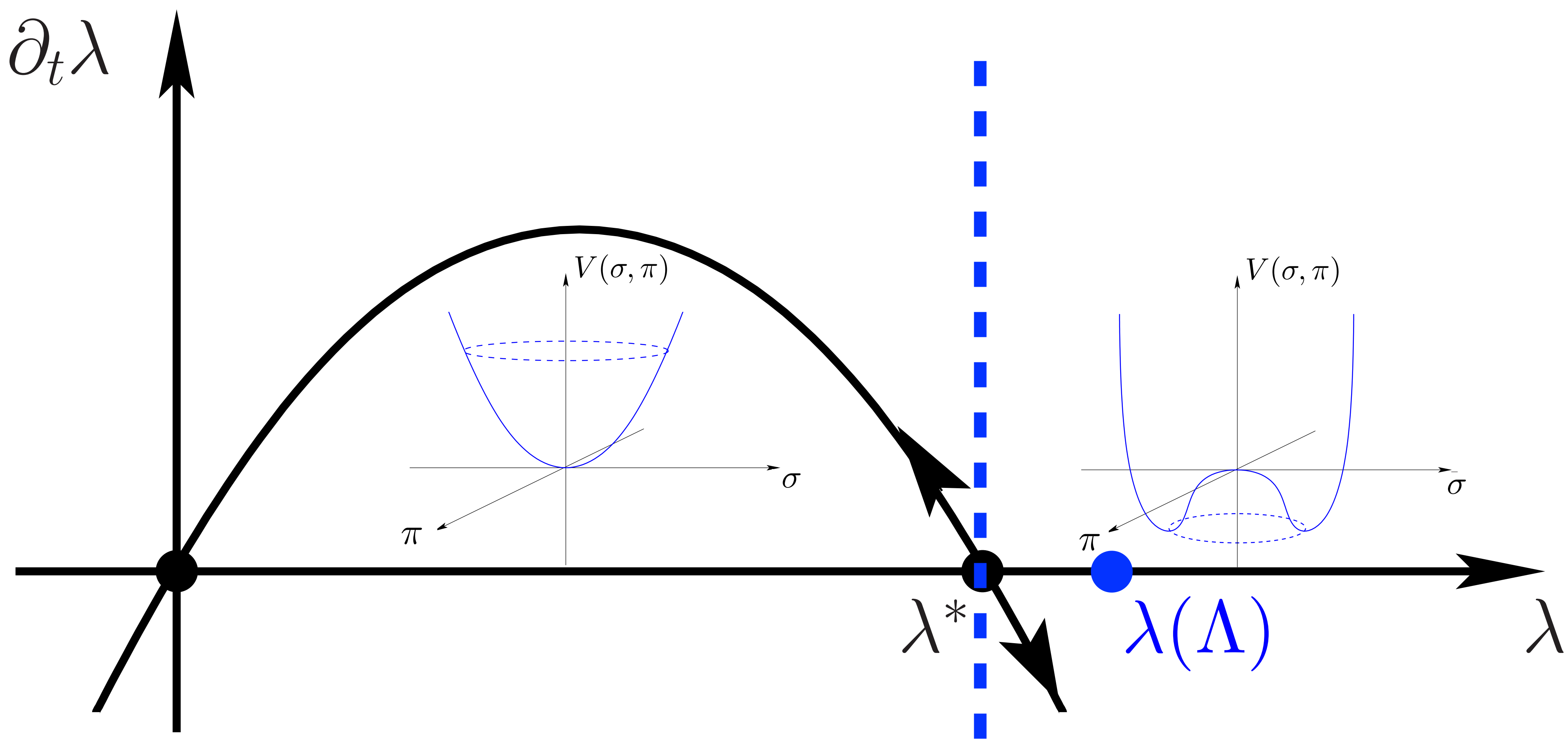


initial condition of the differential equation determines whether, e.g., chiral symmetry is spontaneously broken or not

RG flow of four-quark interactions at low energies

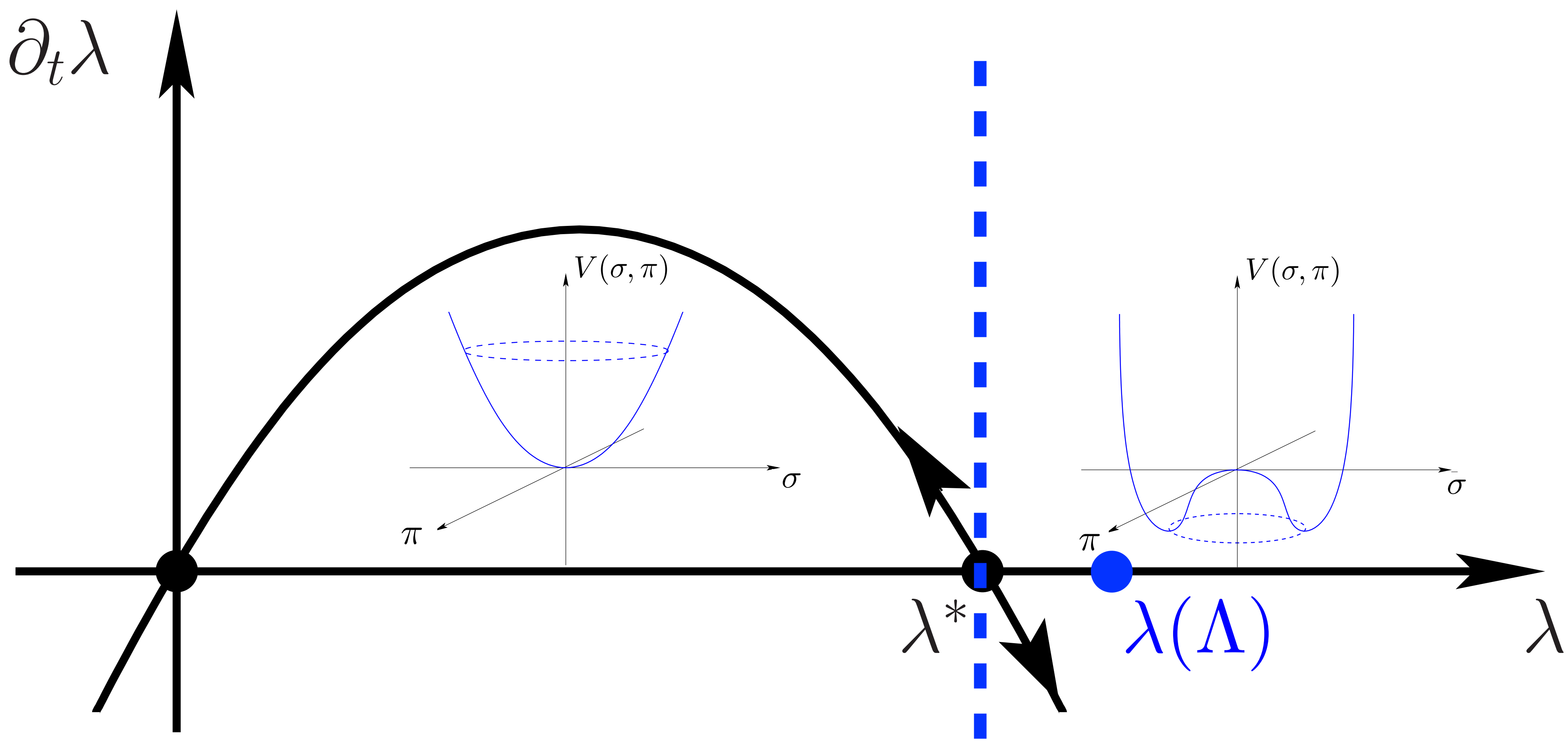
[JB' 11]



no direct access to low-energy observables but the scale for low-energy observables $\mathcal{O}$ is set by the scale k_0 at which the four-fermion coupling **diverges**, $1/\lambda(k_0) = 0$:

$$\mathcal{O} \sim k_0 \sim |\lambda^* - \lambda_\Lambda|^{\frac{1}{\Theta}}$$

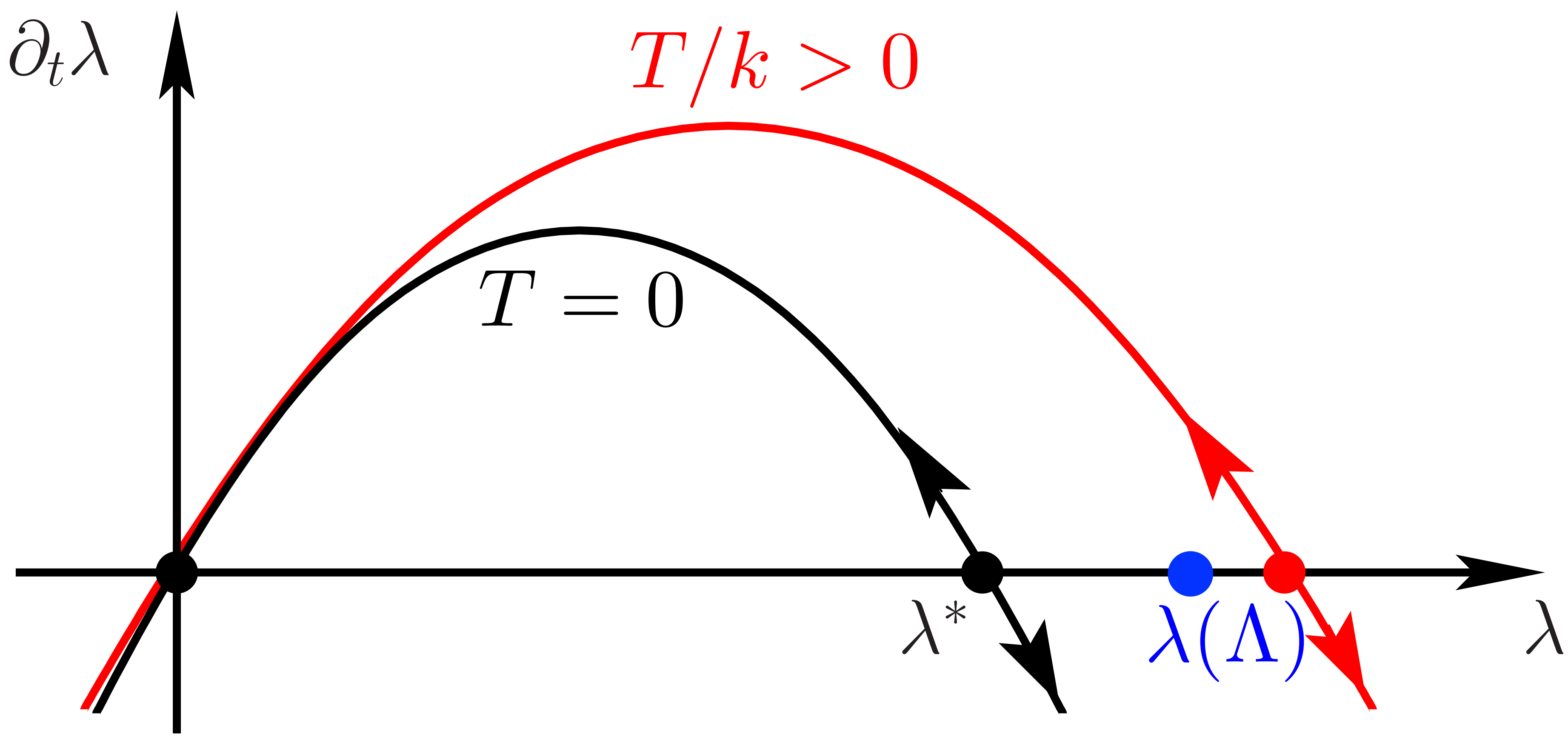
RG flow of four-quark interactions at low energies



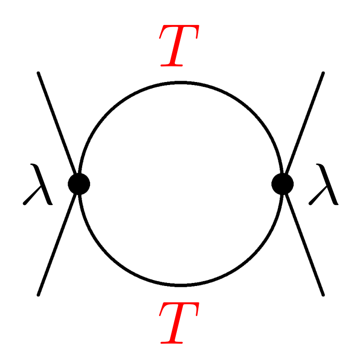
(fine-)tuning of the initial condition
“mimics” the effect of the gauge degrees of
freedom in a “gluon-free” low-energy
parametrization of QCD

RG flows: finite temperature (but zero density)

[JB, Janot '11; JB, Herbst' 12]



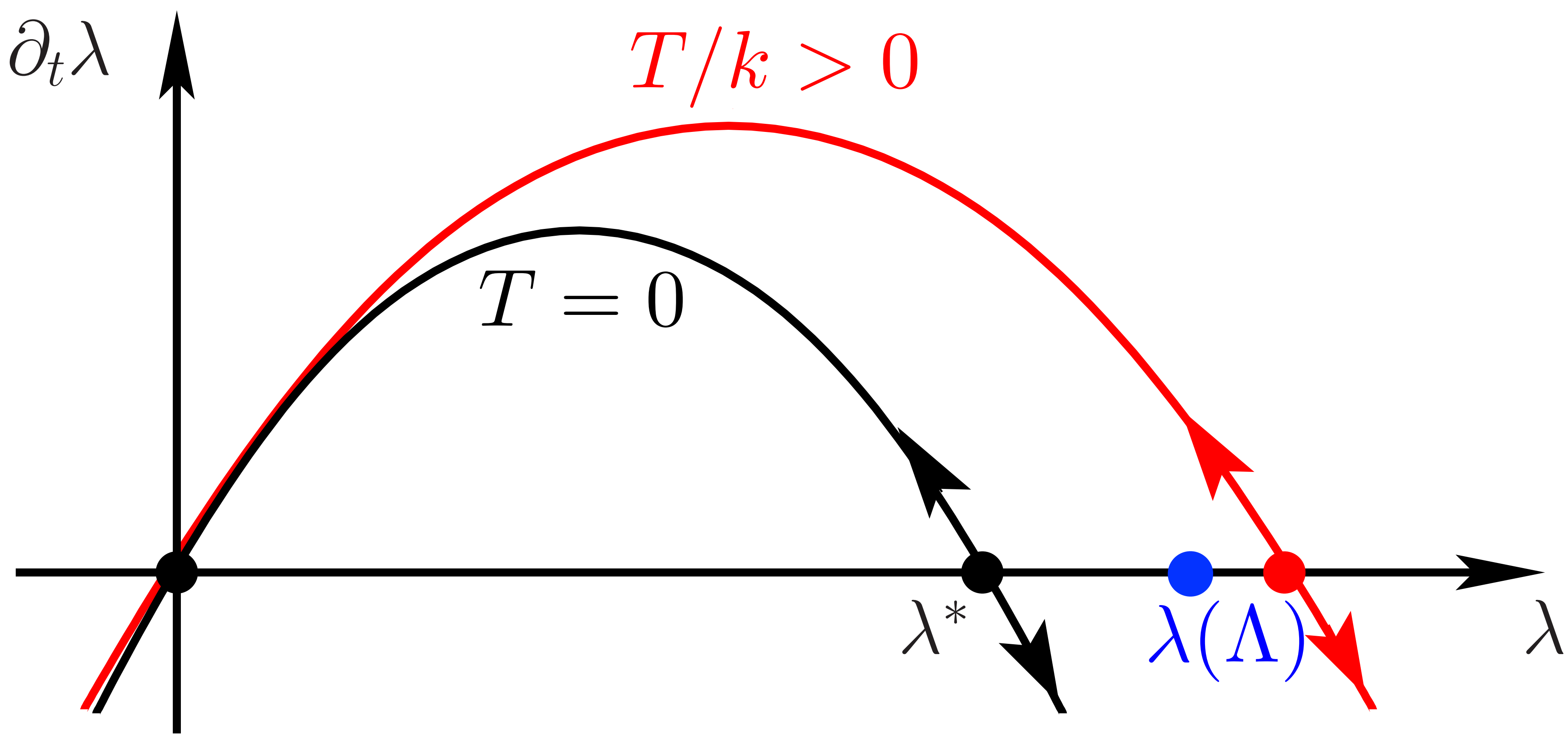
$$\partial_t \lambda \equiv k \partial_k \lambda \simeq 2\lambda -$$



(decreases with increasing temperature)

RG flows: finite temperature (but zero density)

[JB, Janot '11; JB, Herbst' 12]

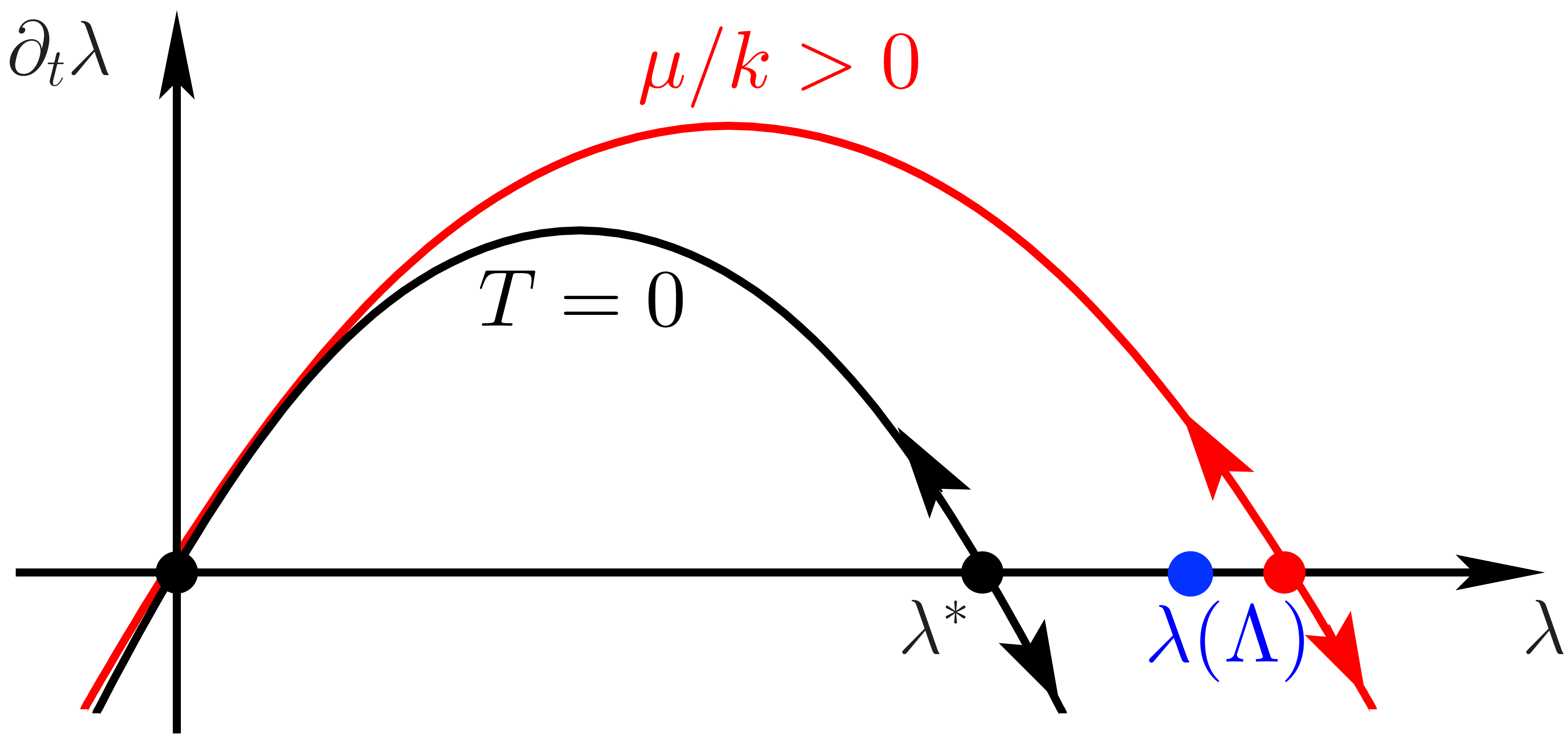


scale for low-energy observables
now depends on the temperature:

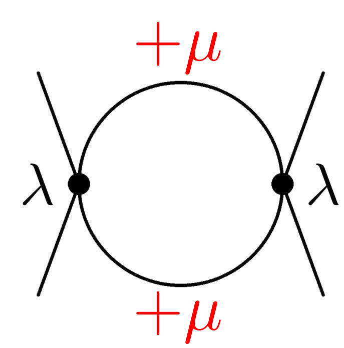
$$k_T \sim k_0 \sqrt{\text{const.} - T^2}$$

RG flows: finite density I (but zero temperature)

[JB, Leonhardt, Pospiech ’17]



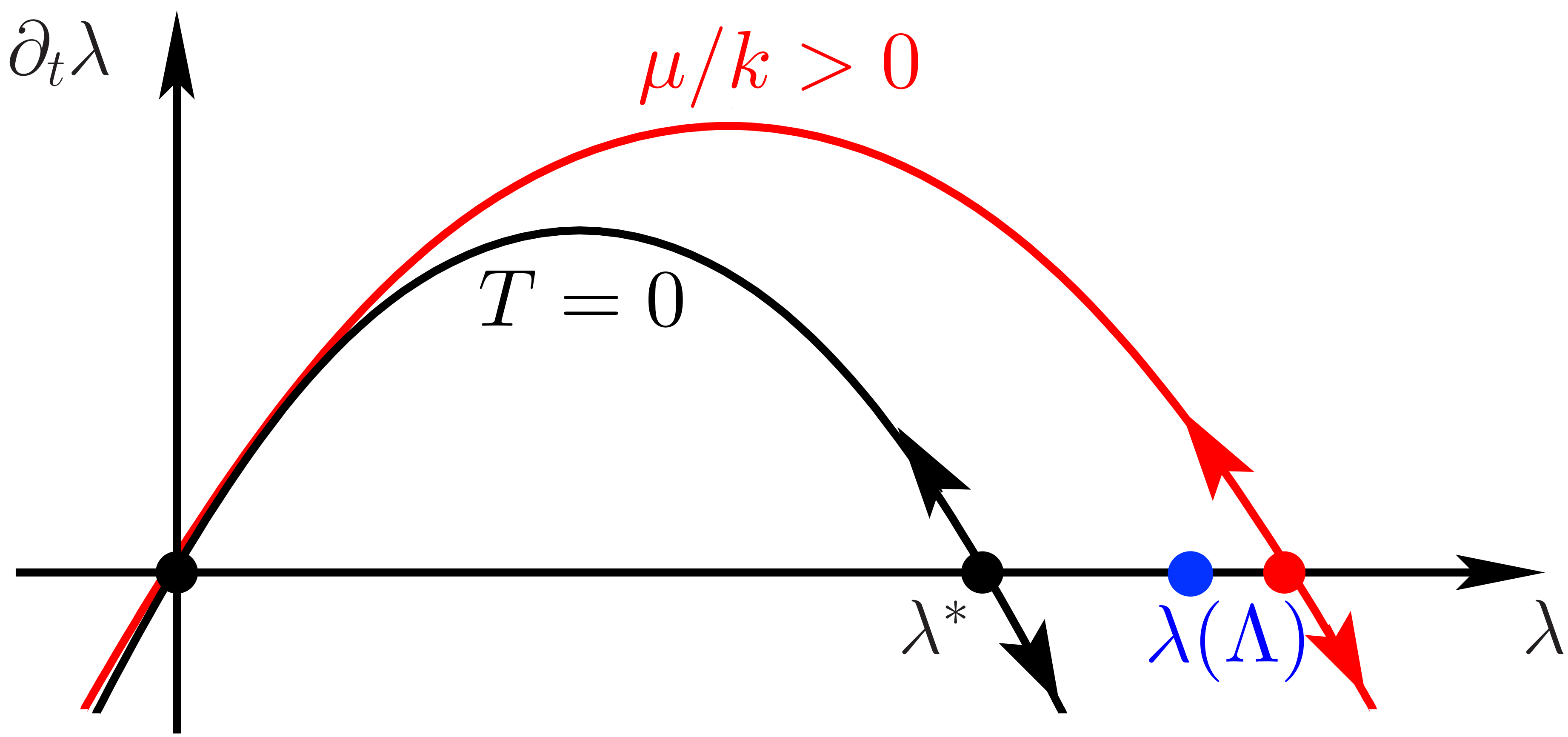
$$\partial_t \lambda \equiv k\partial_k \lambda \simeq 2\lambda -$$



(decreases with increasing chemical potential)

RG flows: finite density I (but zero temperature)

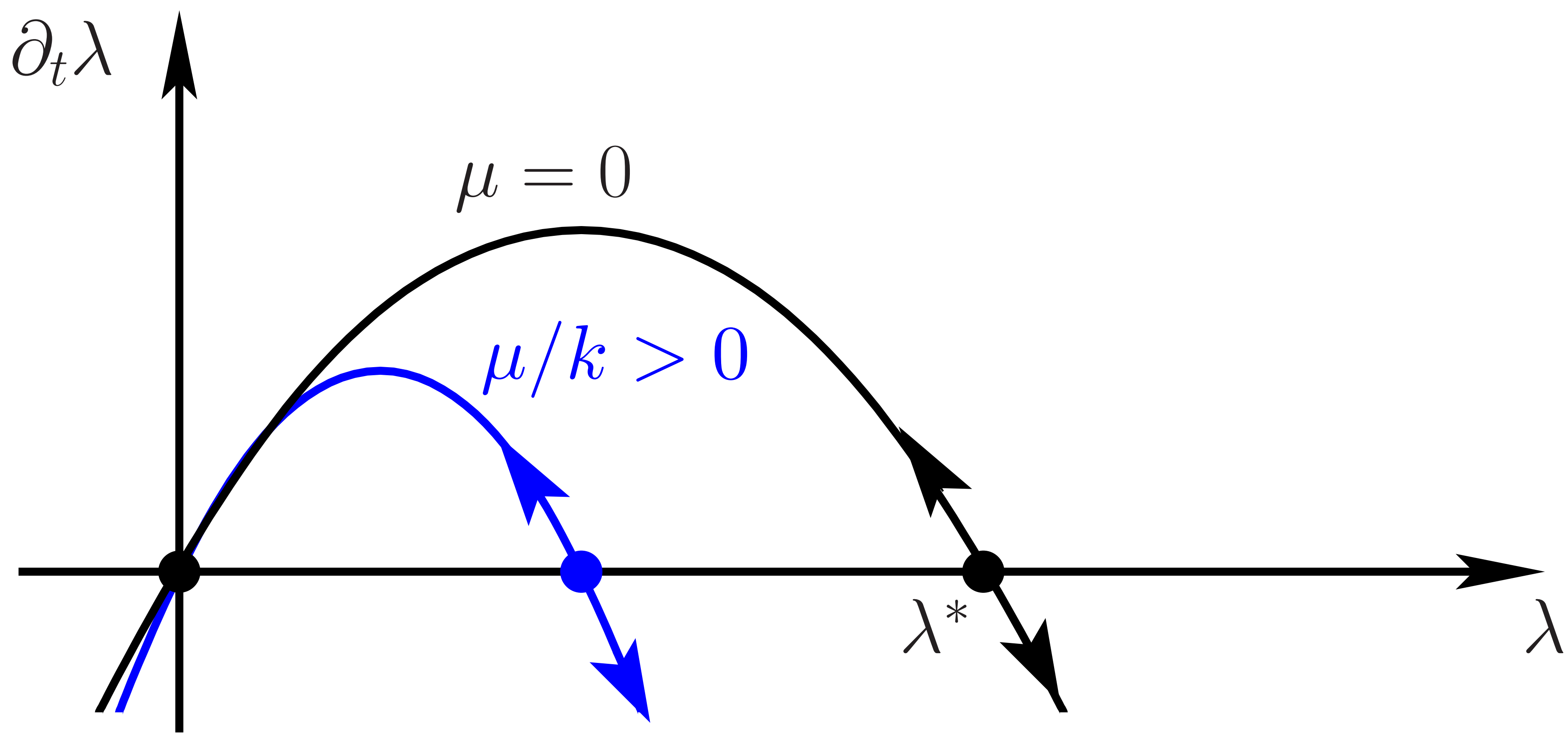
[JB, Leonhardt, Pospiech ’17]



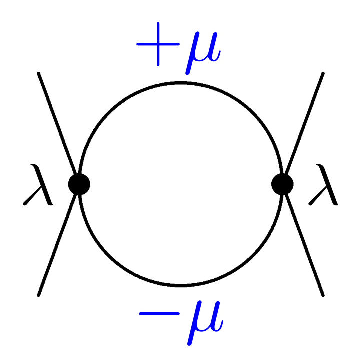
similar symmetry restoration effect
as at finite temperature

RG flows: finite density II (but zero temperature)

[JB, Leonhardt, Pospiech ’17]

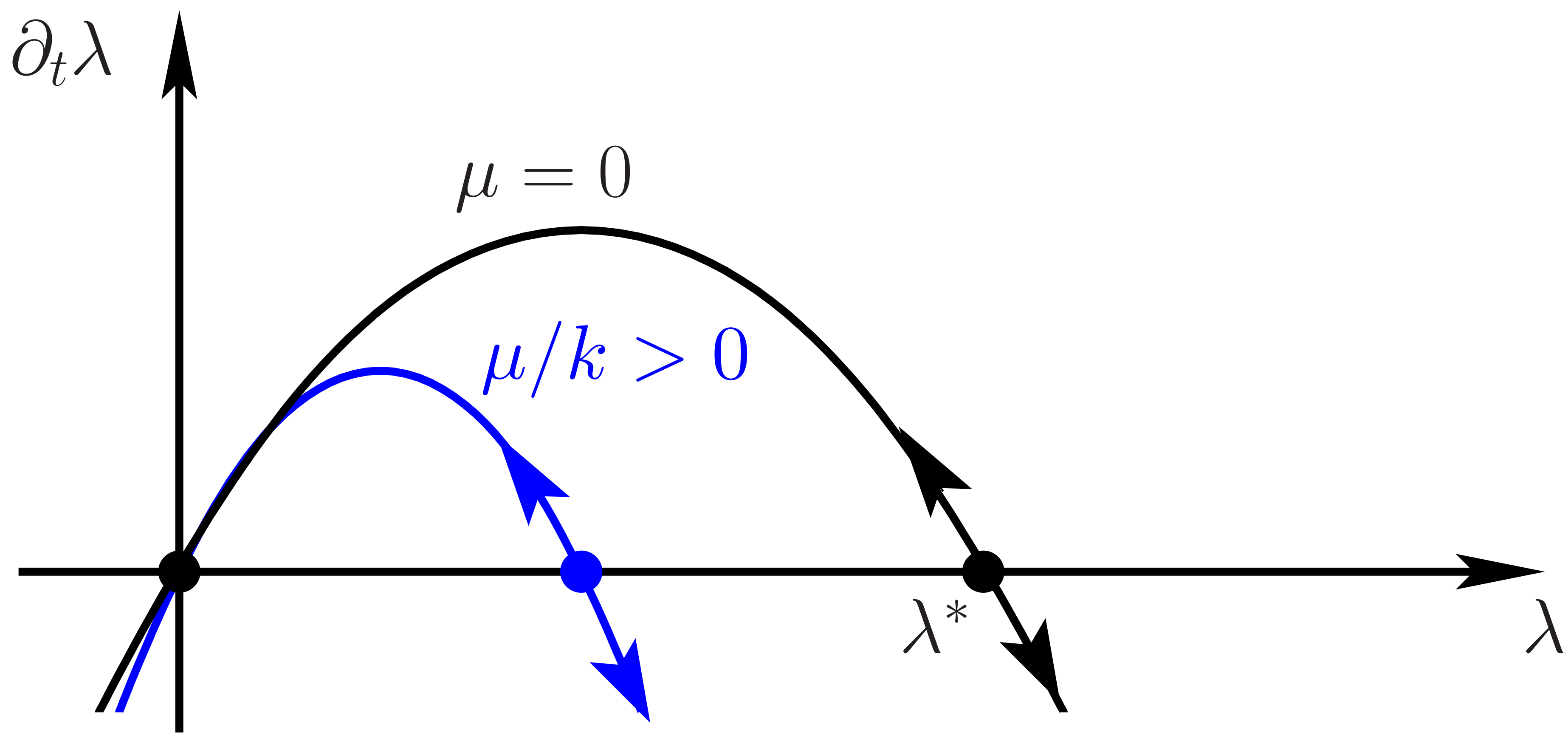


$$\partial_t \lambda \equiv k \partial_k \lambda \simeq 2\lambda -$$



RG flows: finite density II (but zero temperature)

[JB, Leonhardt, Pospiech ’17]



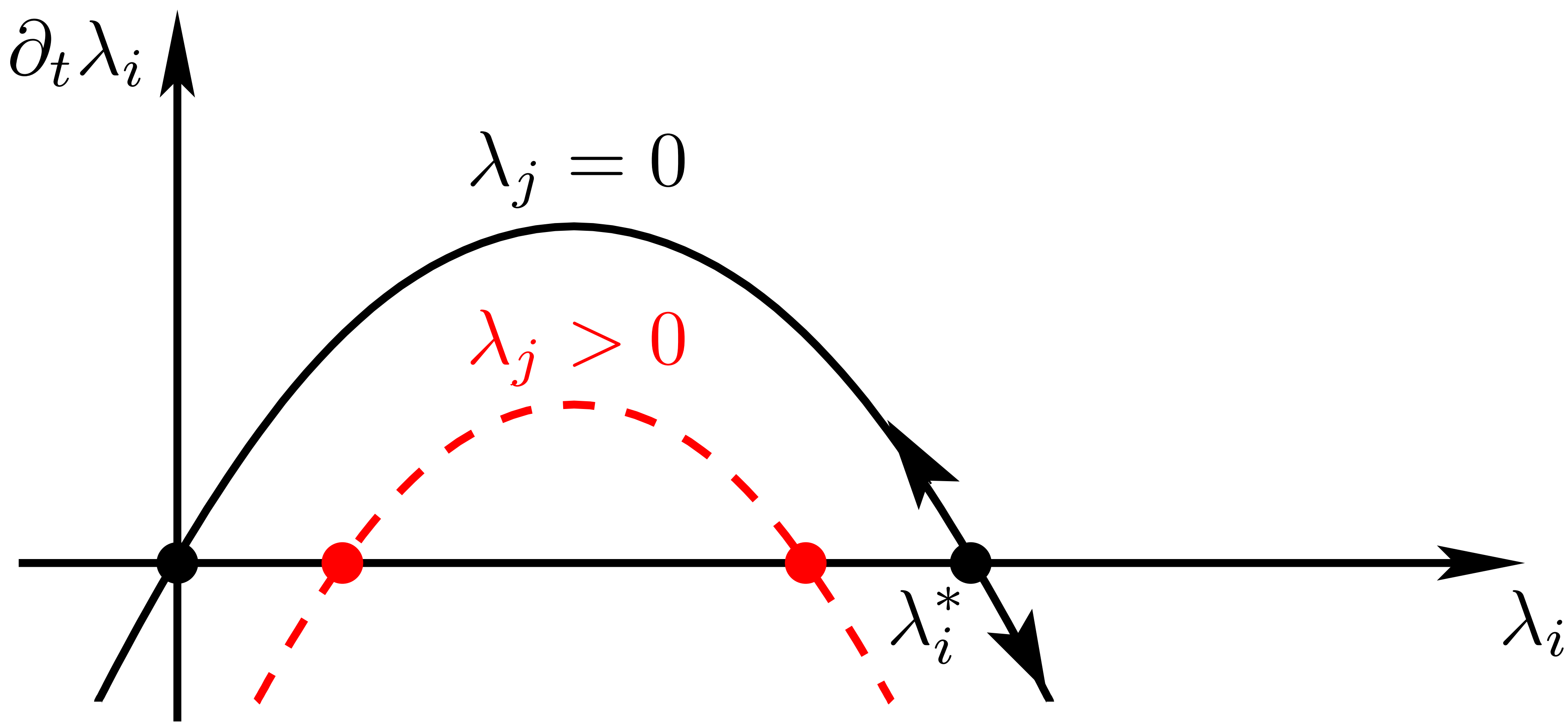
“BCS instability”,

symmetry breaking for **any** value of the chemical potential

$$k_\mu \sim \exp\left(-\text{const.}/\mu^2\right)$$

BUT: more than one coupling — competing effects

[JB' 11]



$$\partial_t \lambda_i \equiv k \partial_k \lambda_i \simeq 2\lambda_i - c_i \lambda_i^2 - c_j {\lambda_j}^2$$

(e.g. fixed-point annihilation may be induced by competing effects)

Fixed-point structure and phases: 1 channel

Fixed-point structure and phases: 1 channel

[JB, Leonhardt, Pospiech '18]

- effective action:

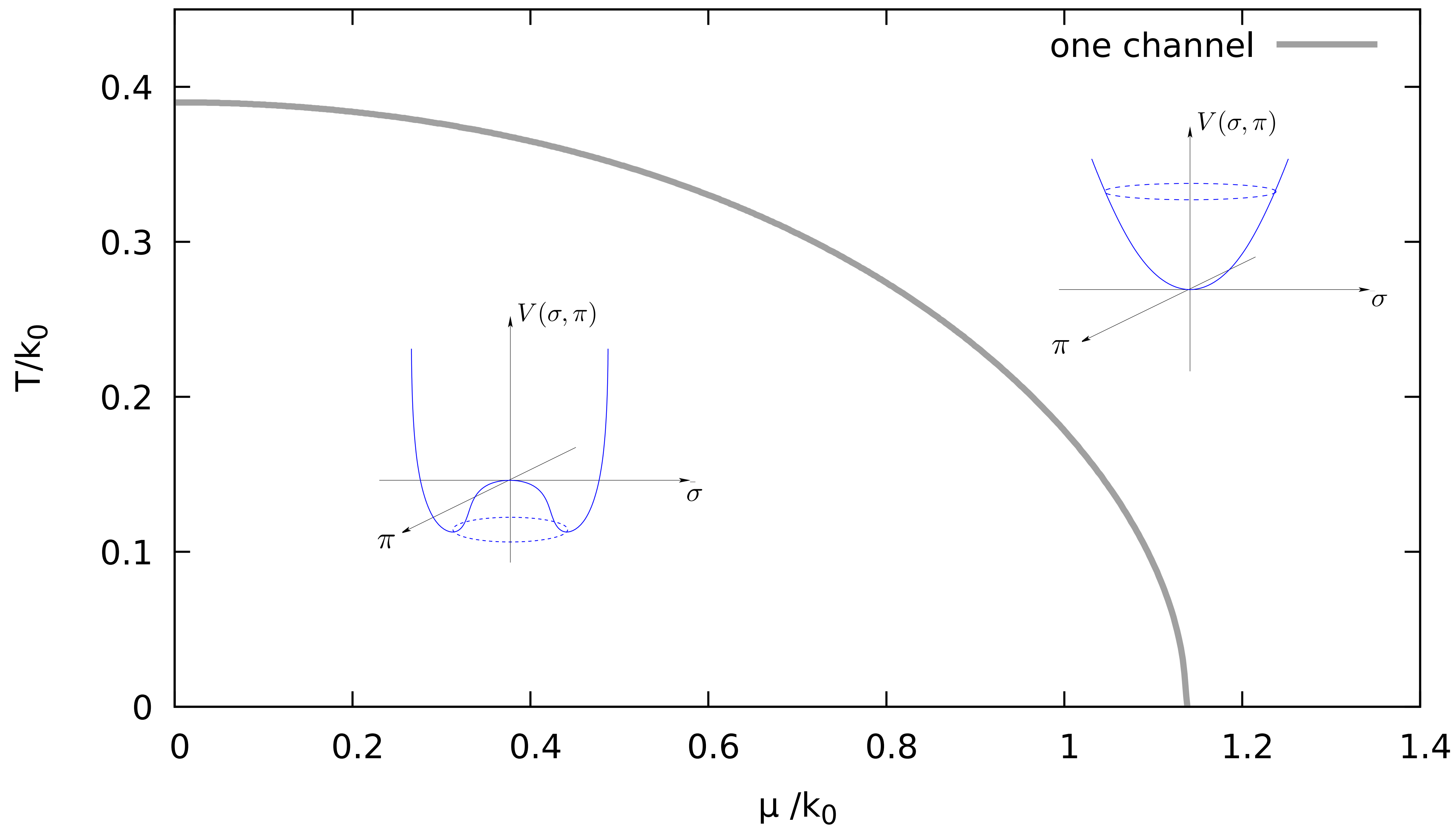
$$\Gamma_k = \int_x \Big\{ (\text{kinetic term}) + \frac{1}{2}\bar{\lambda}_{(\sigma-\pi)}\,(\mathrm{S}-\mathrm{P}) \Big\}$$

- **scale-fixing procedure**: adjust the initial condition of the scalar-pseudoscalar coupling such that a given value for the symmetry breaking scale k_0 is obtained in the **vacuum limit**

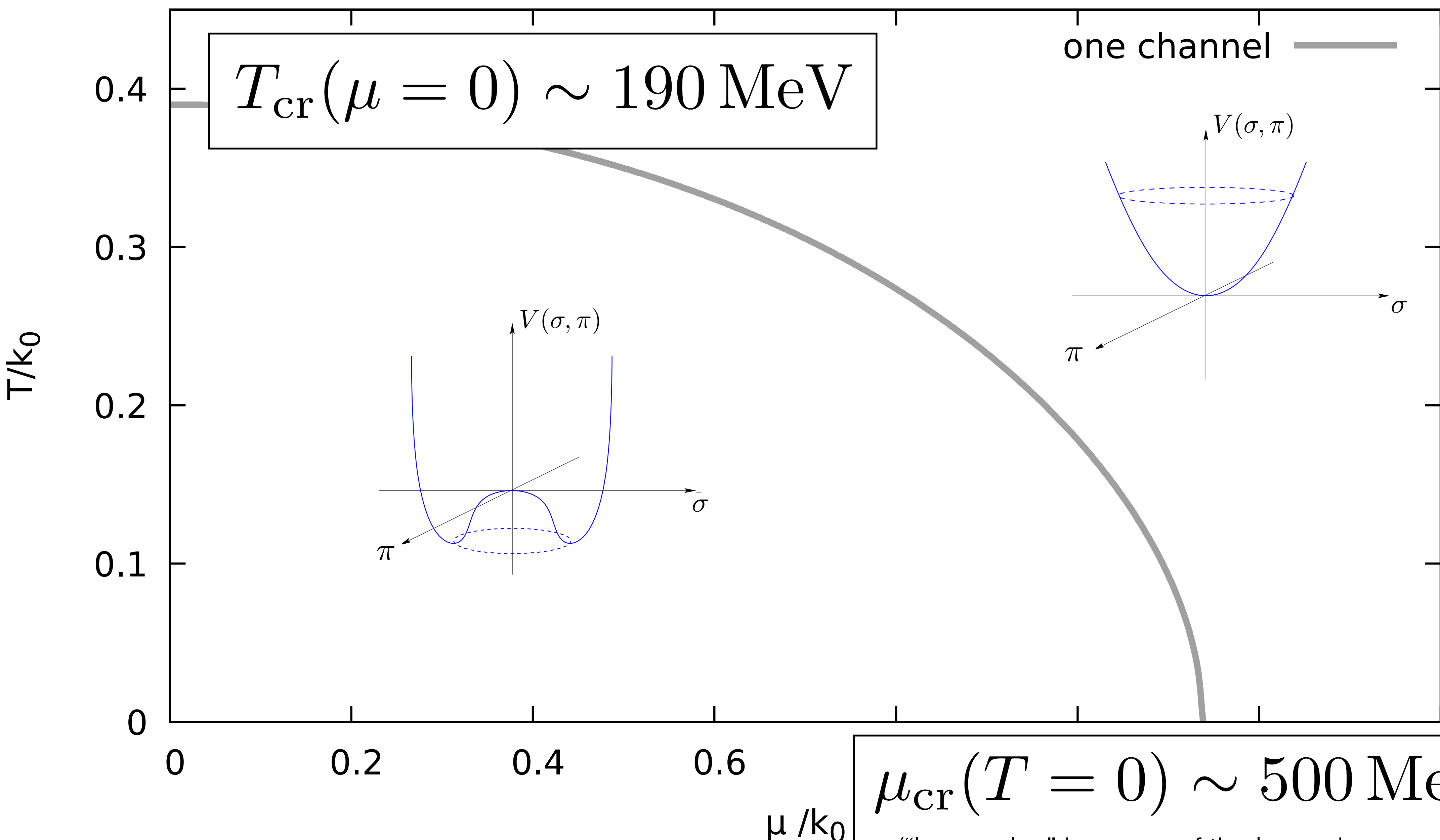
- note: RG flow of the **1-channel** approximation can be mapped on the **mean-field gap equation** for the quark mass: [JB ' 11]

$$m_{\mathrm{q}}(k_0) \approx 300\,\mathrm{MeV}\,, \quad m_\sigma(k_0) \approx 800\,\mathrm{MeV}$$

Fixed-point structure and phases: 1 channel



Fixed-point structure and phases: 1 channel

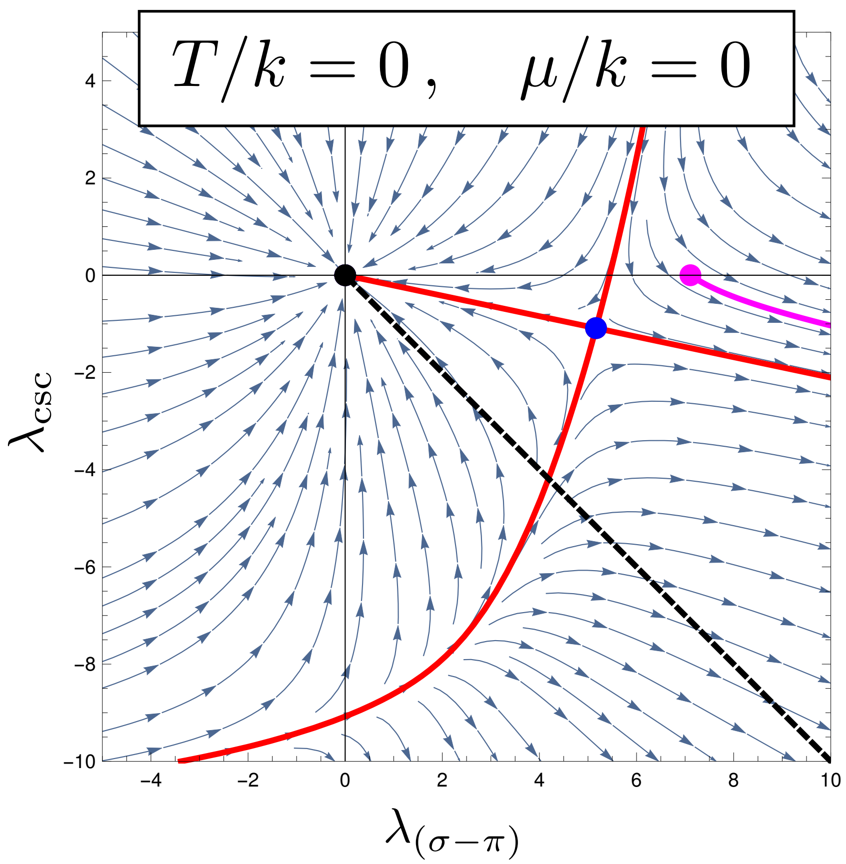


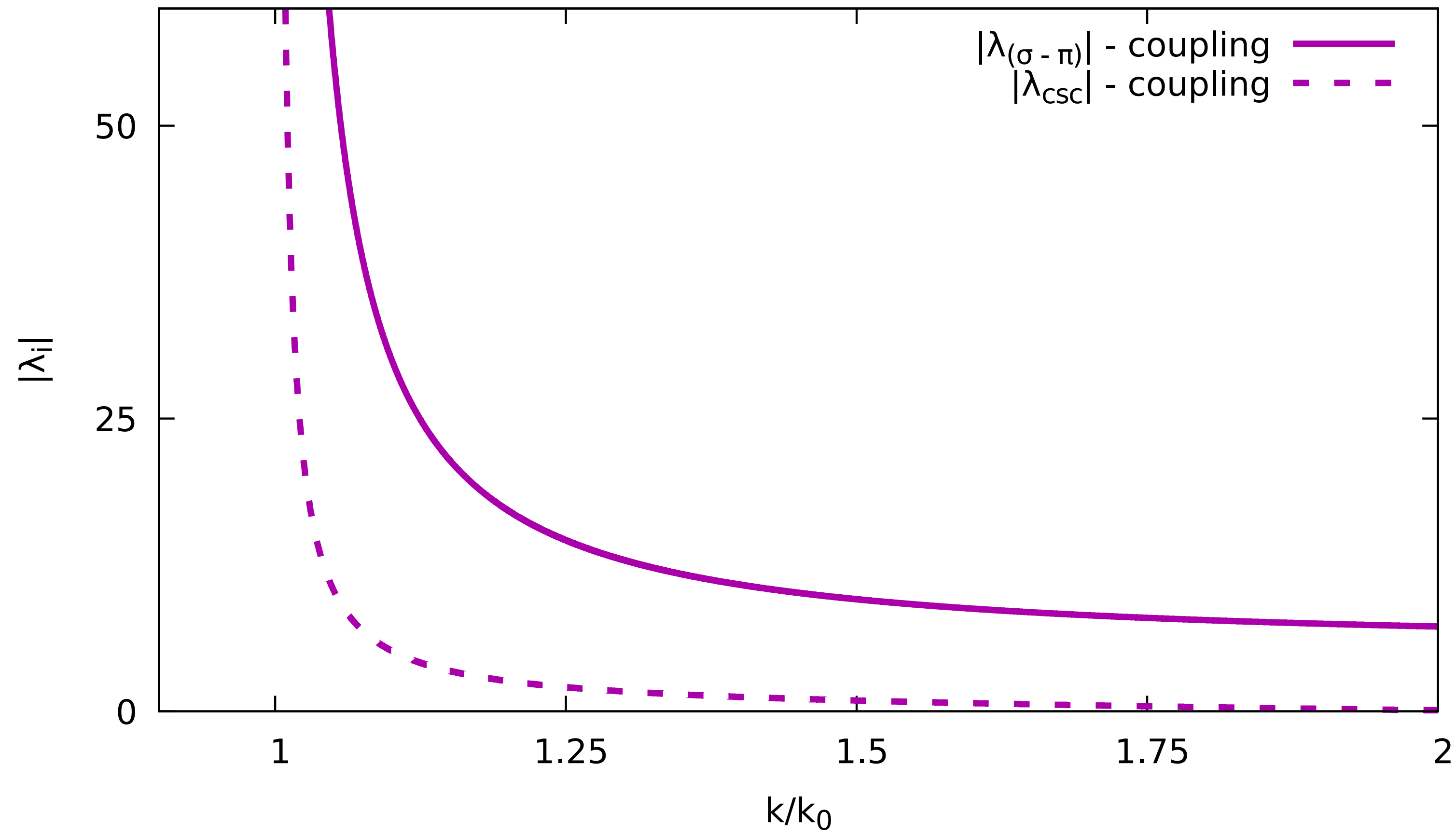
$$T_{\mathrm{cr}}(\mu=0)\sim 190\,\mathrm{MeV}$$

$$\mu_{\mathrm{cr}}(T=0)\sim 500\,\mathrm{MeV}$$

("large value" because of the large sigma mass,
cf. also [Schaefer, Wagner '08])

Fixed-point structure and phases: 2 channels



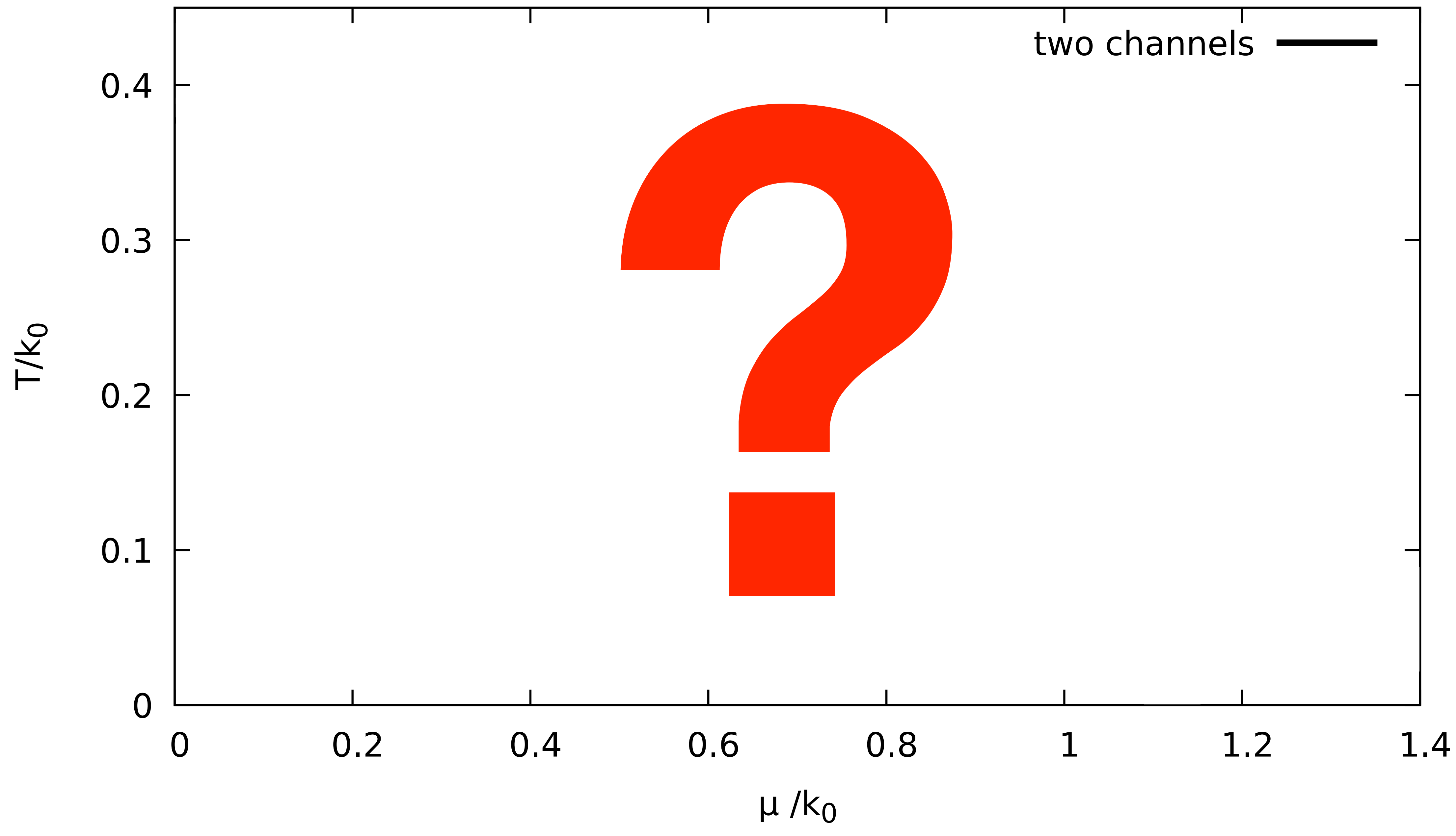


- effective action:

$$\Gamma_k = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2}\bar{\lambda}_{(\sigma-\pi)}\,(\text{S}-\text{P}) + \frac{1}{2}\bar{\lambda}_{\rm csc}\,(\text{CSC}) \right\}$$

- same **scale-fixing procedure** (also same k_0) as in the 1-channel approximation: $(\bar{\lambda}_{(\sigma-\pi)}(\Lambda) \neq 0,\, \bar{\lambda}_{\rm csc}(\Lambda) = 0)$

Fixed-point structure and phases: 2 channels



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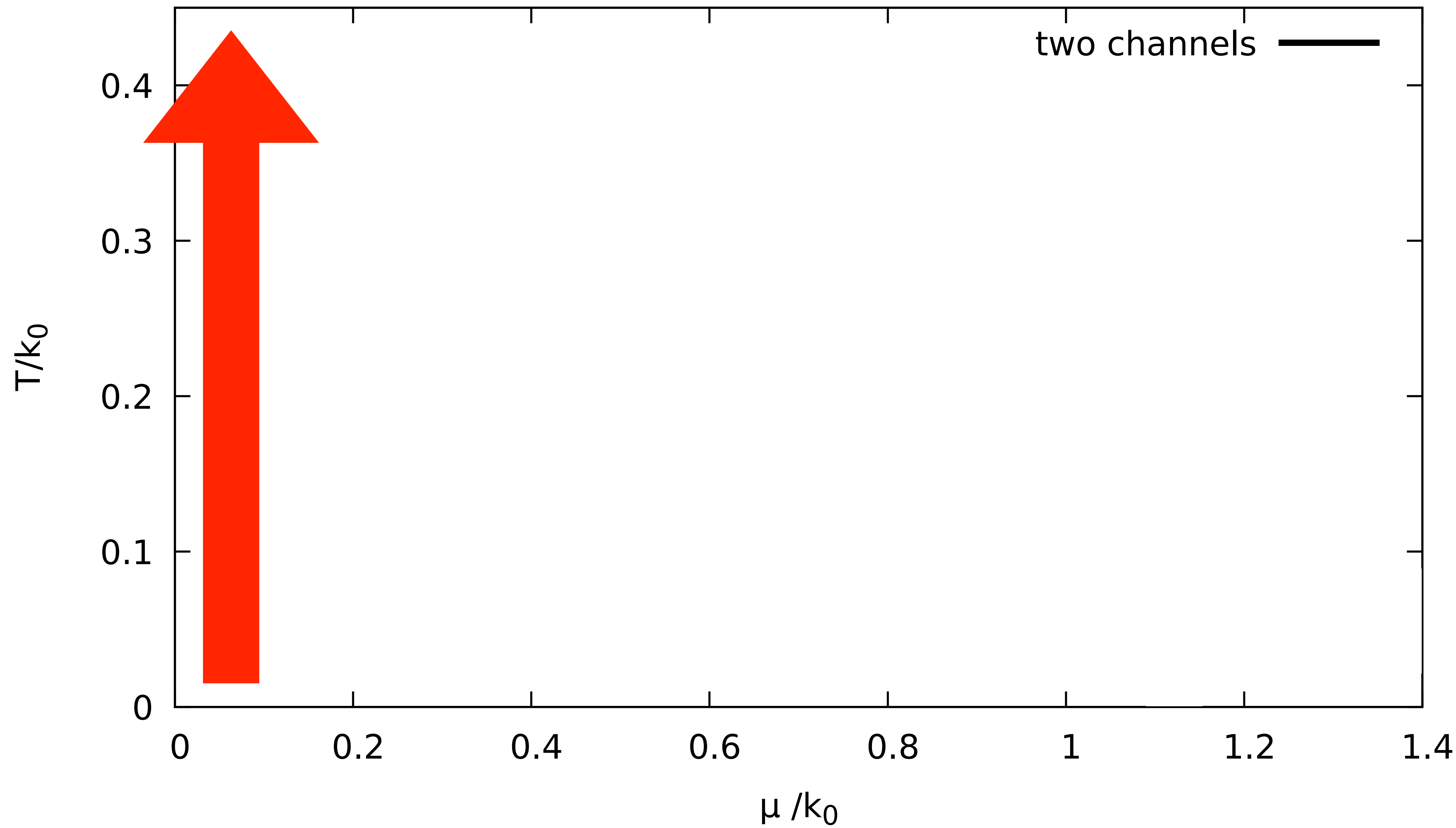

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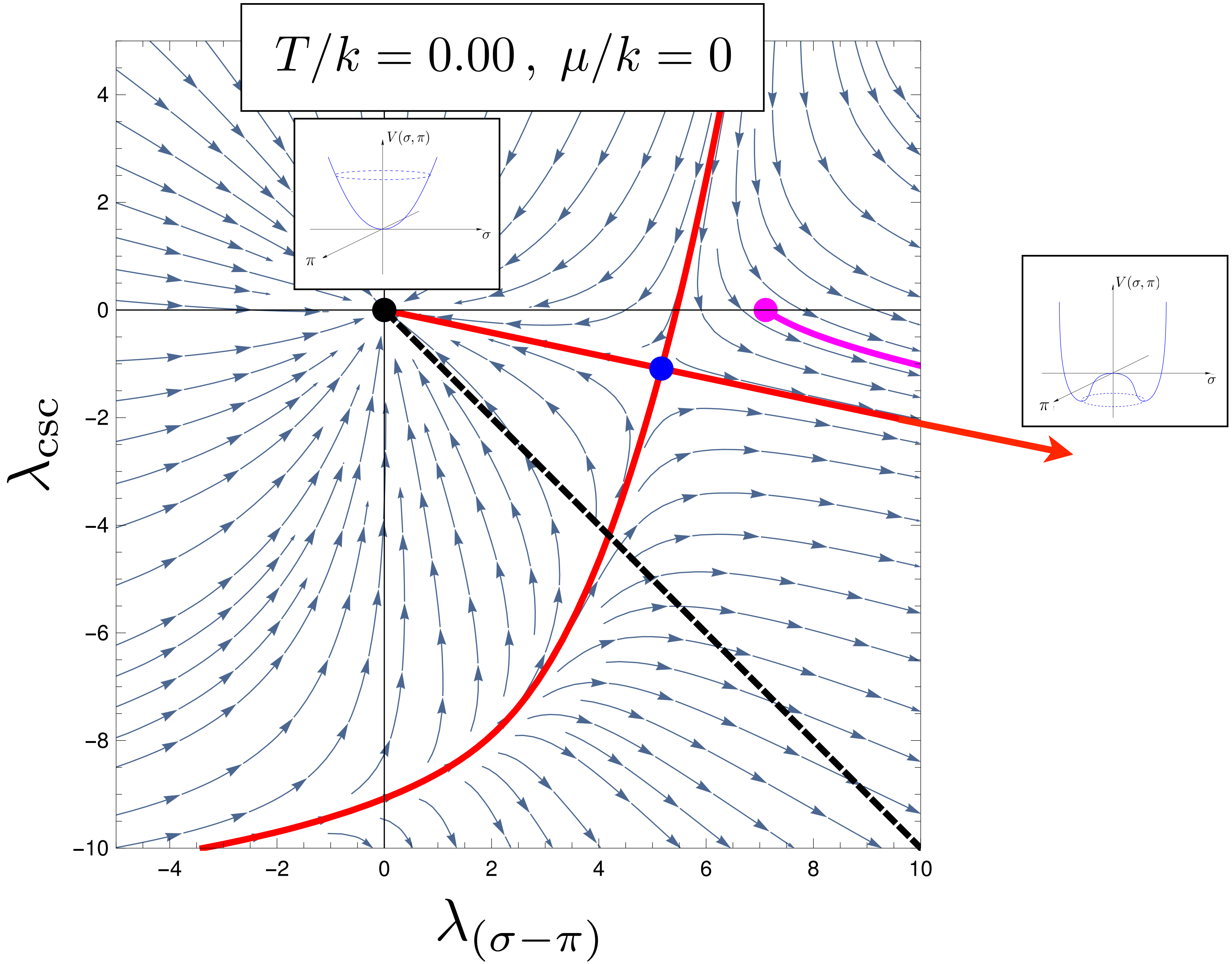

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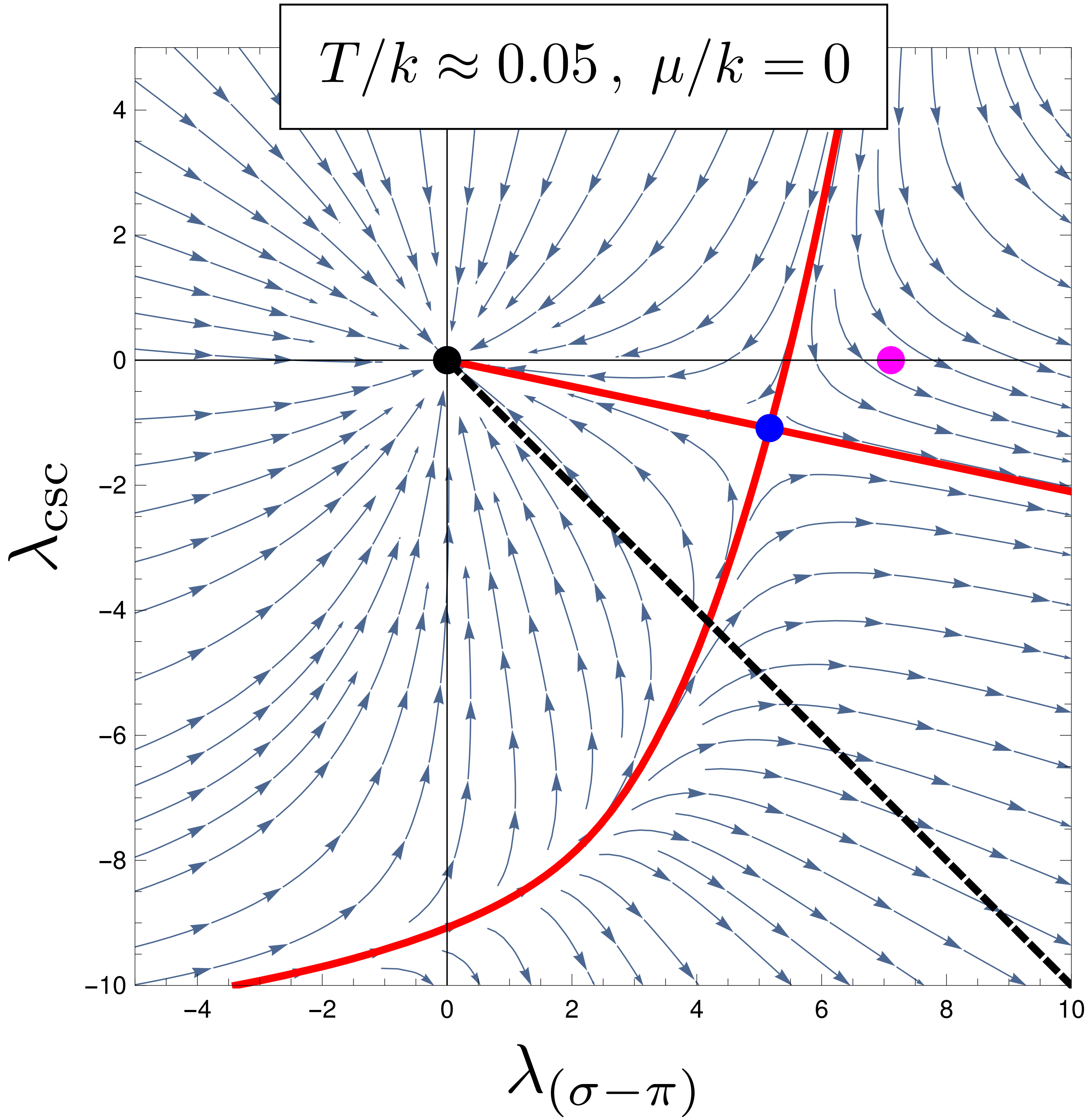

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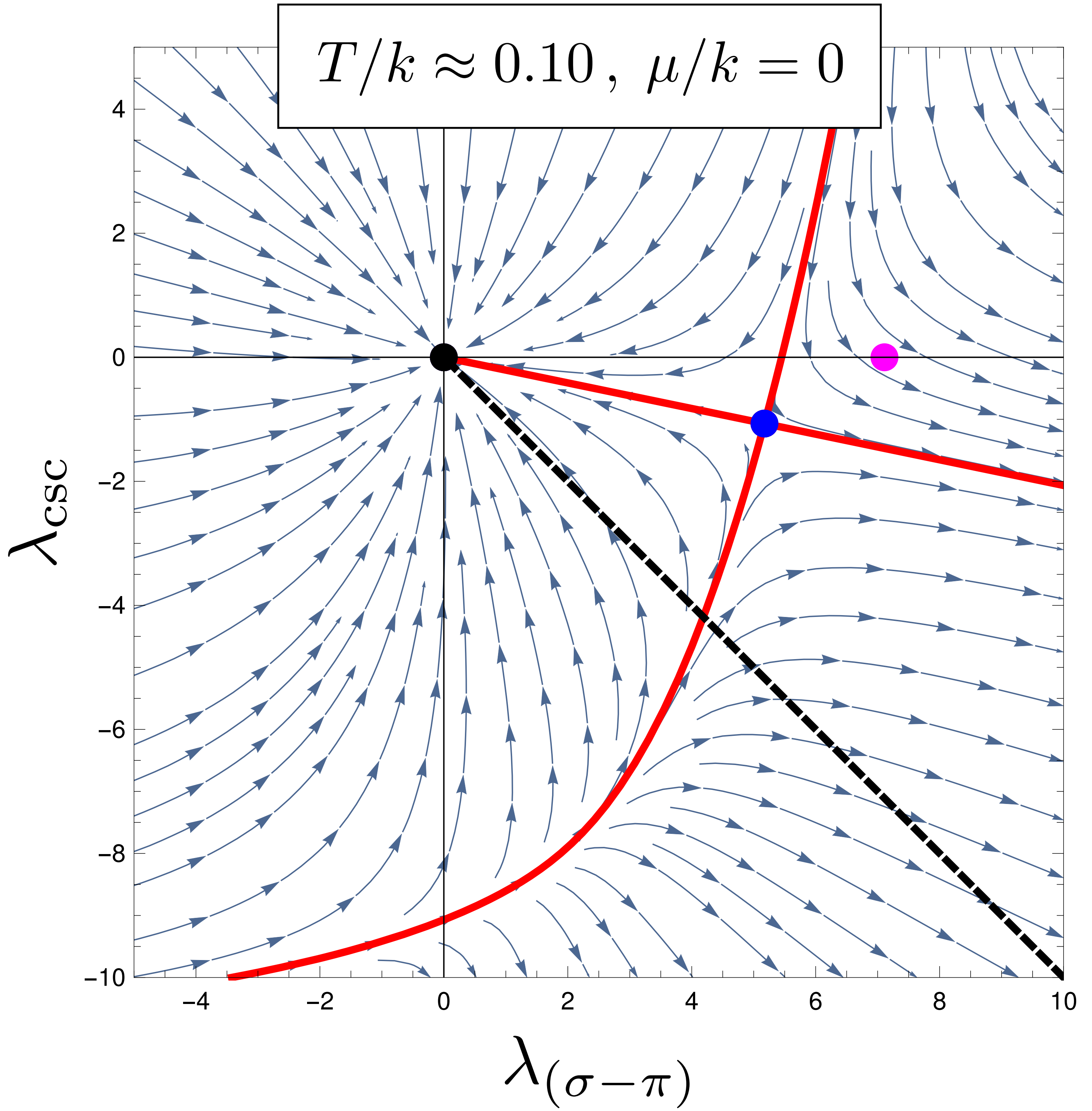

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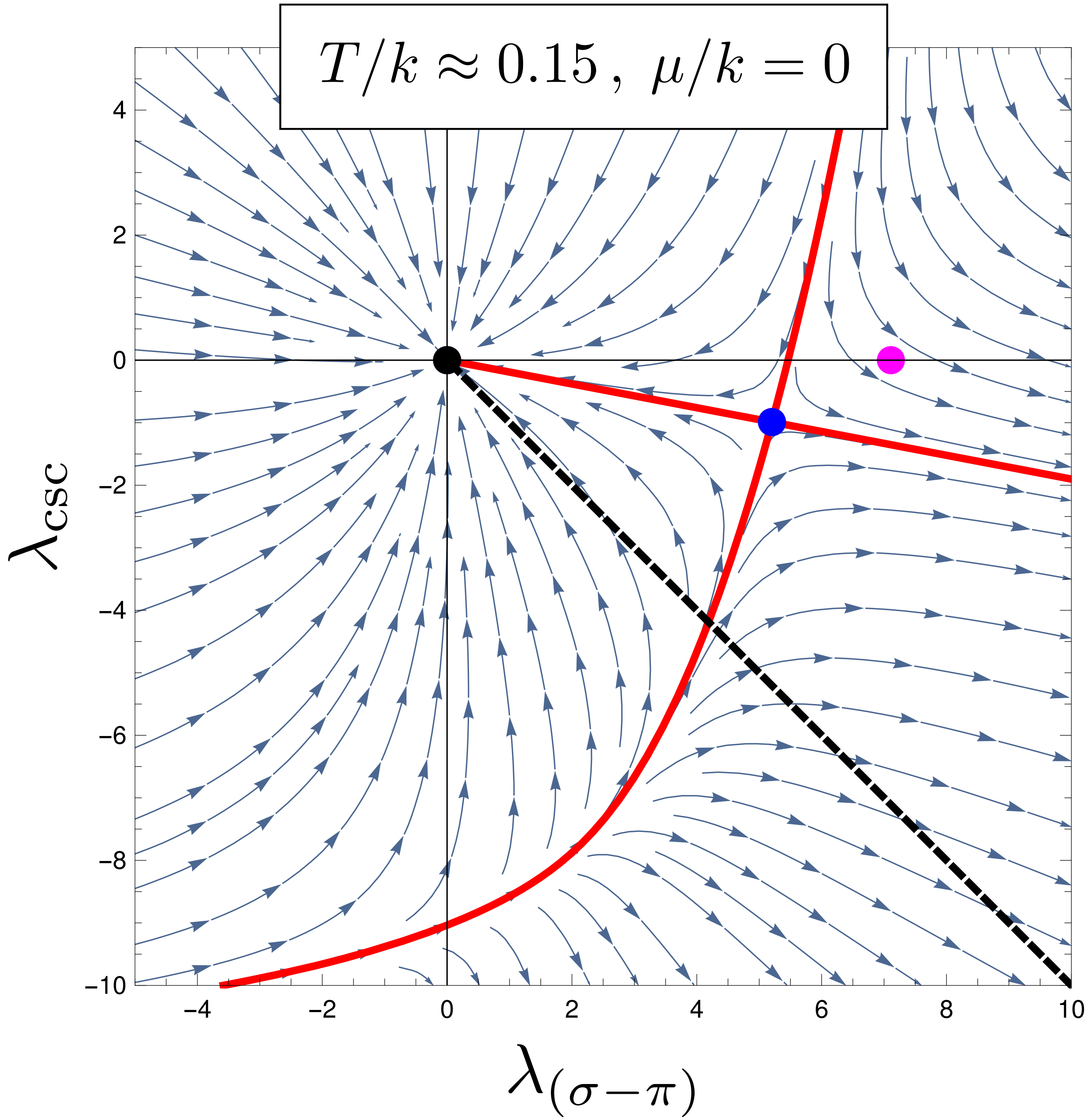

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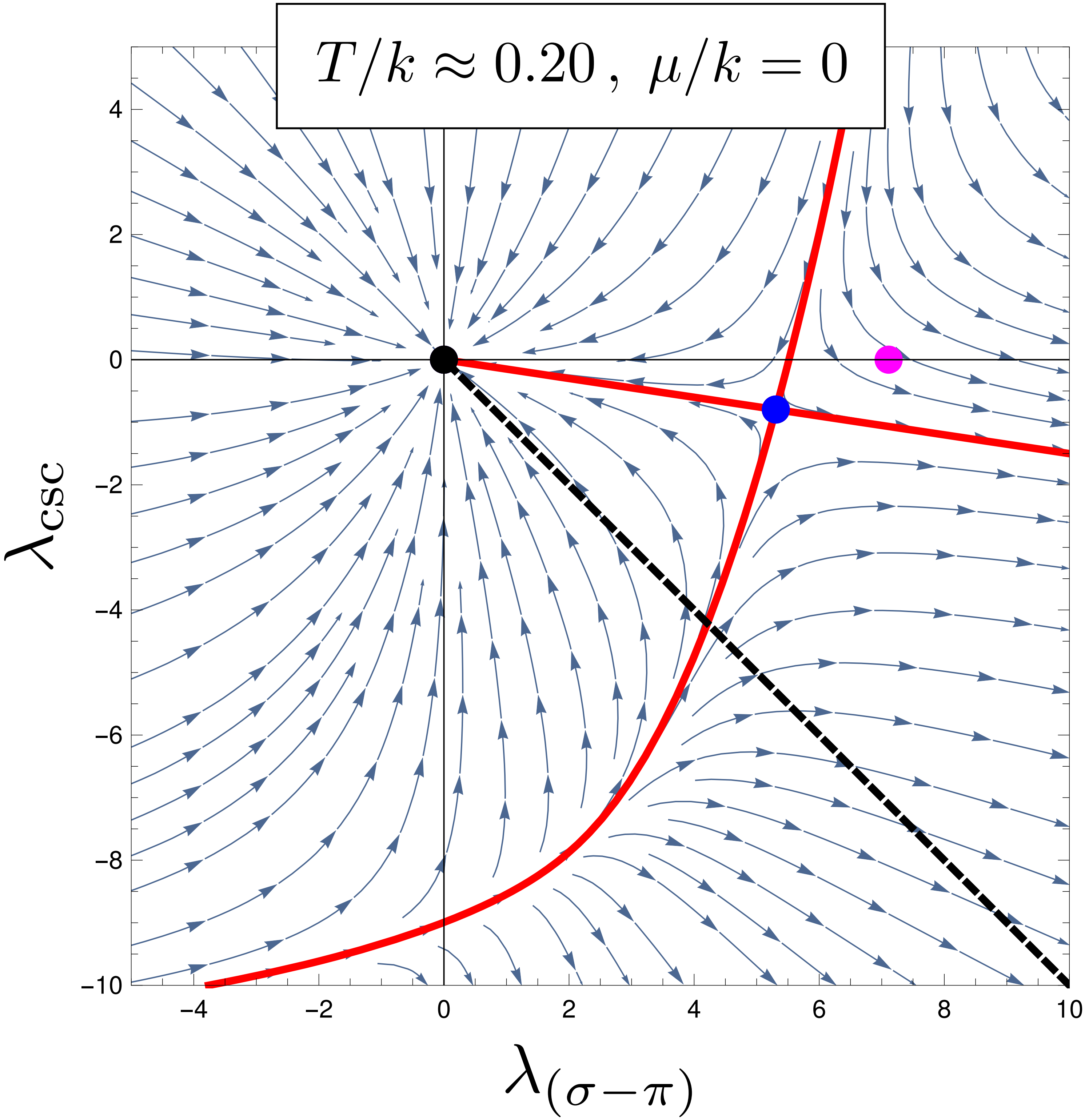

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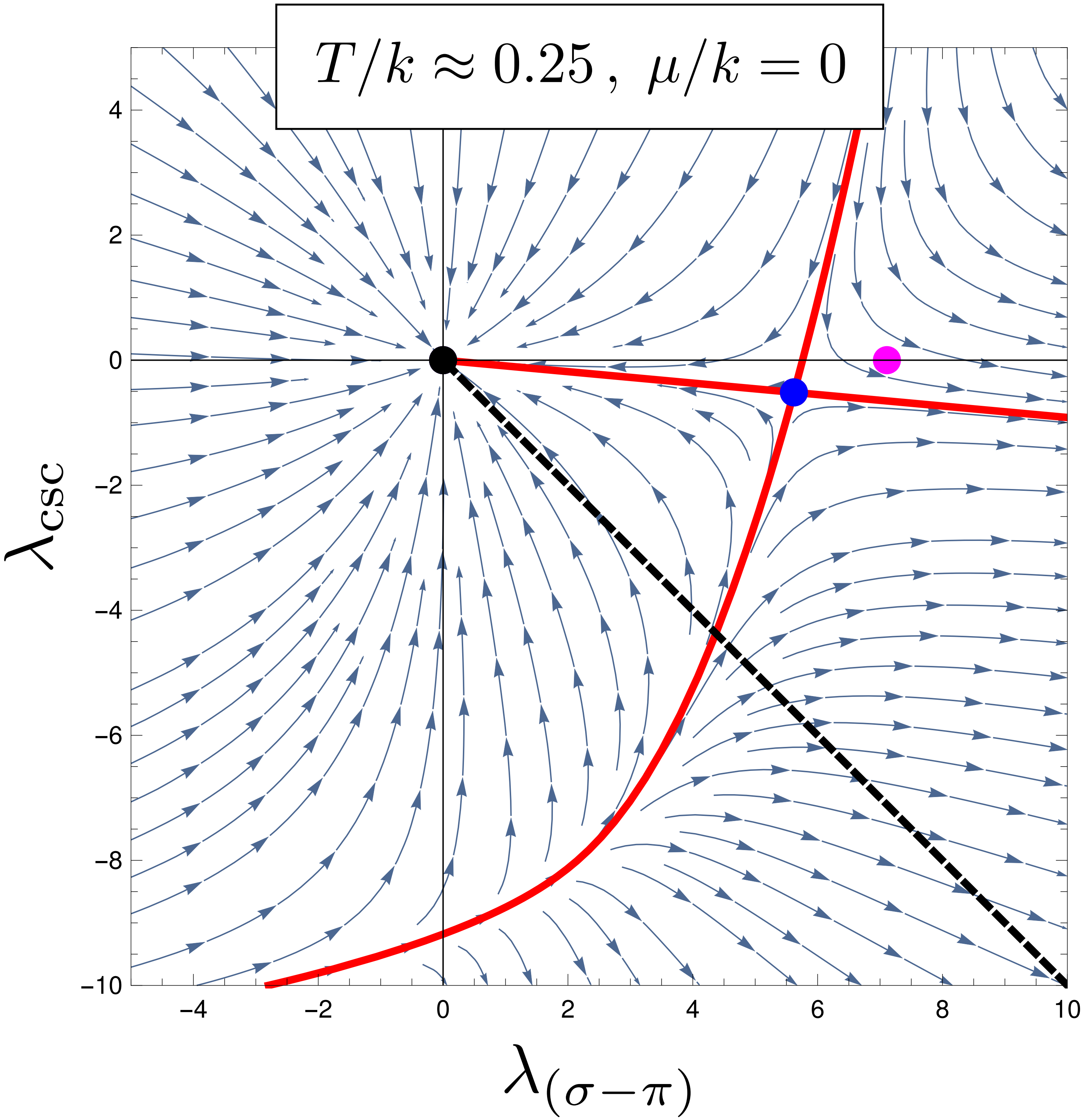

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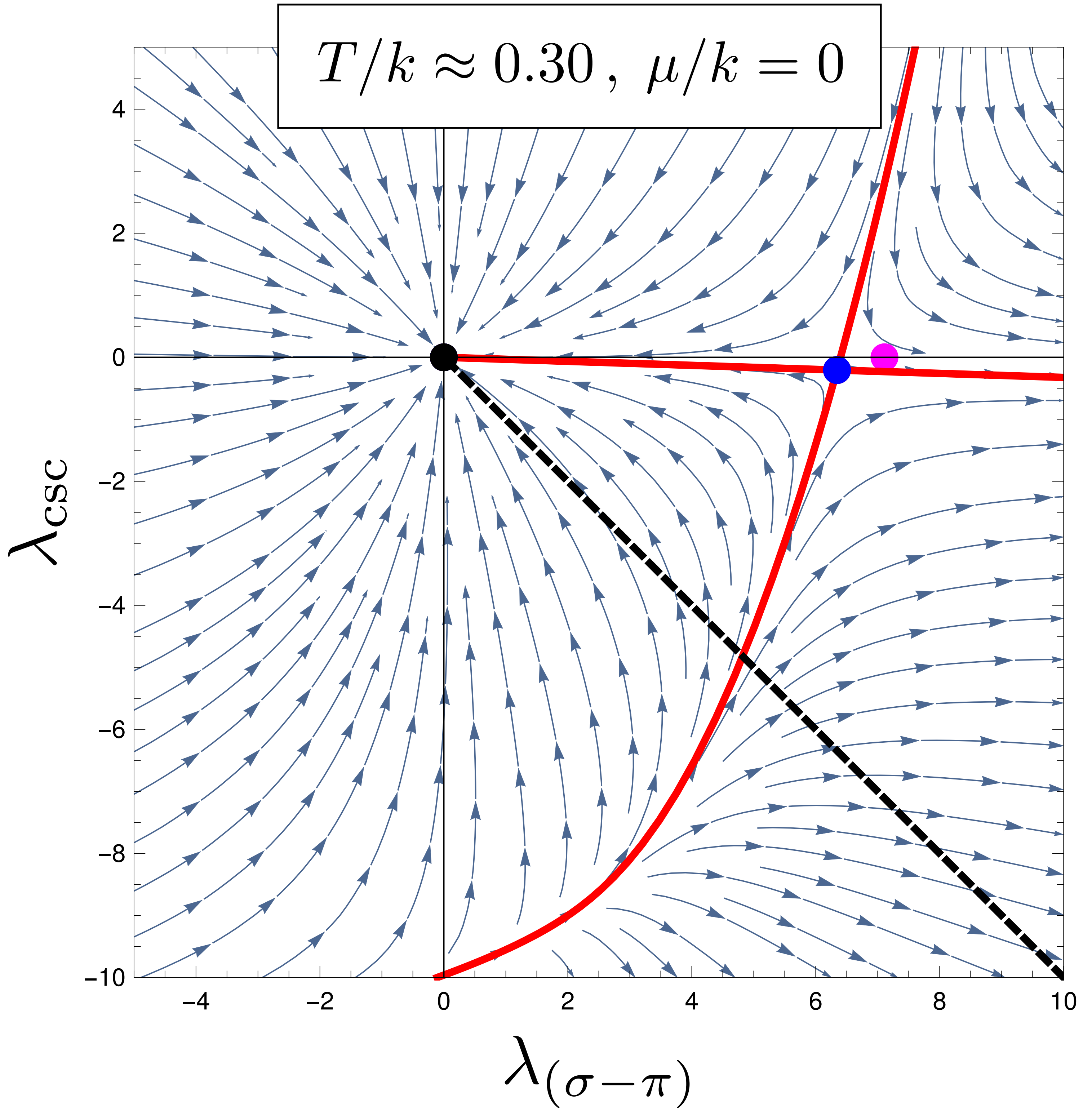

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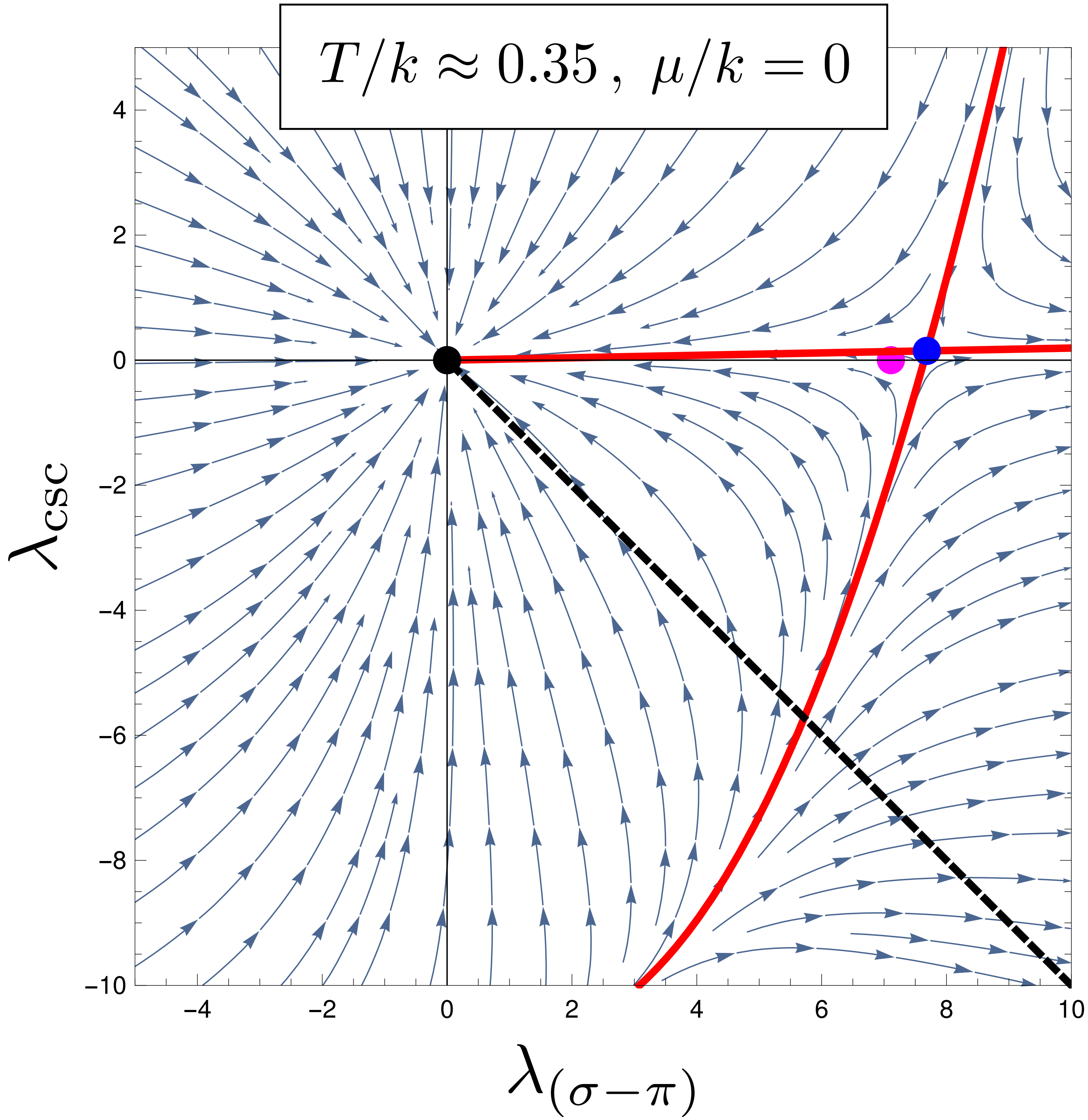

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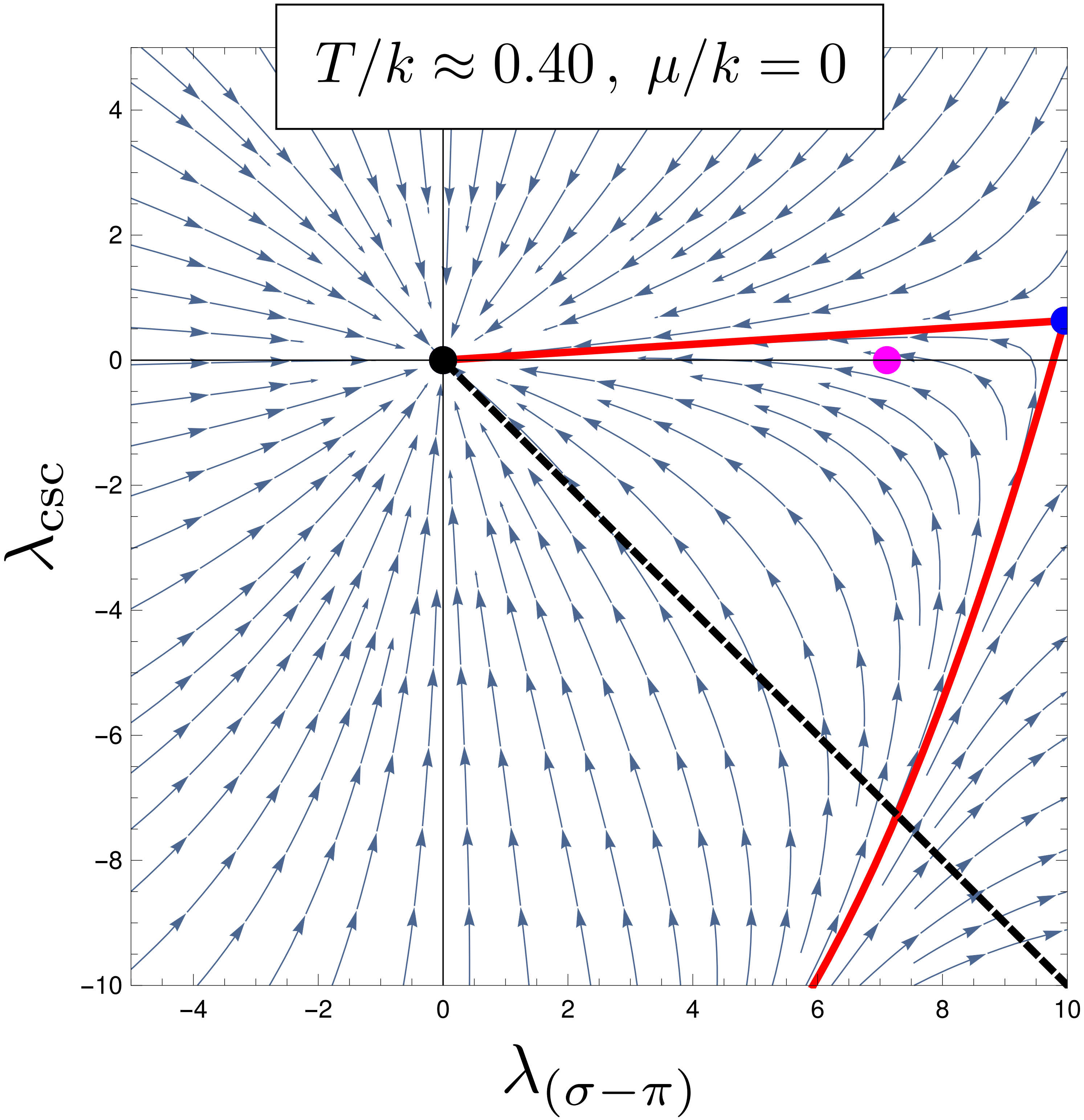

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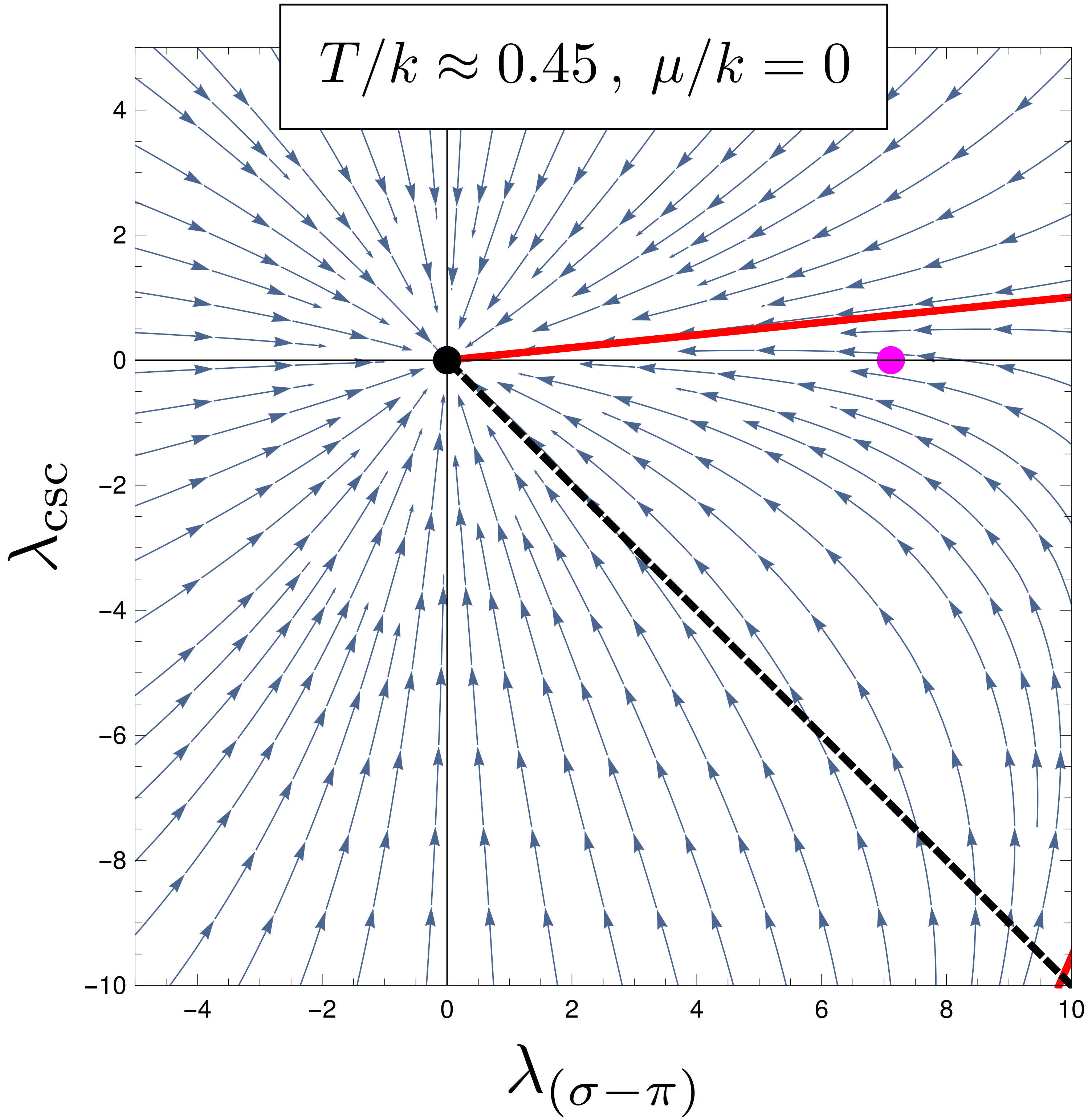

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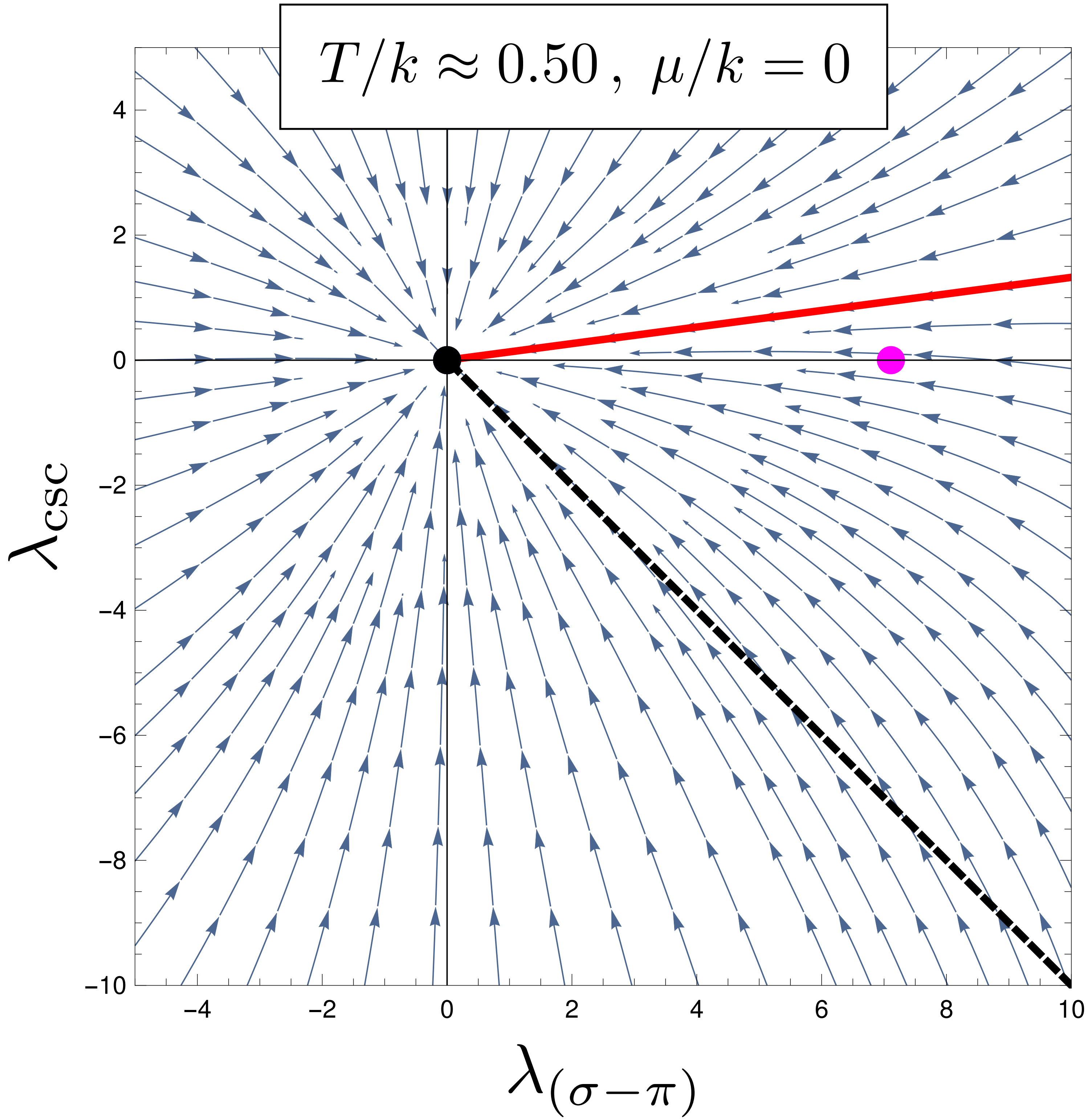

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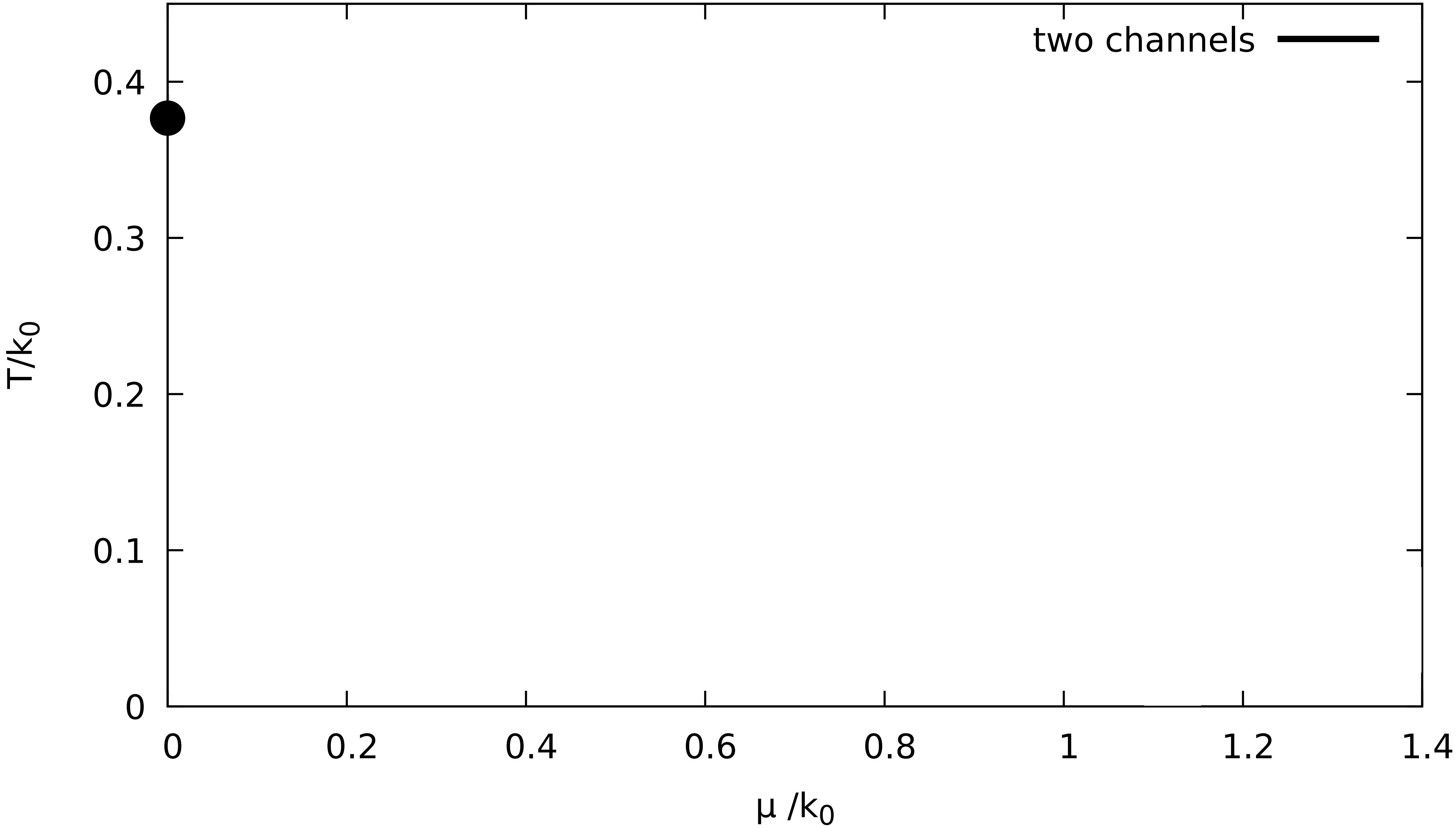

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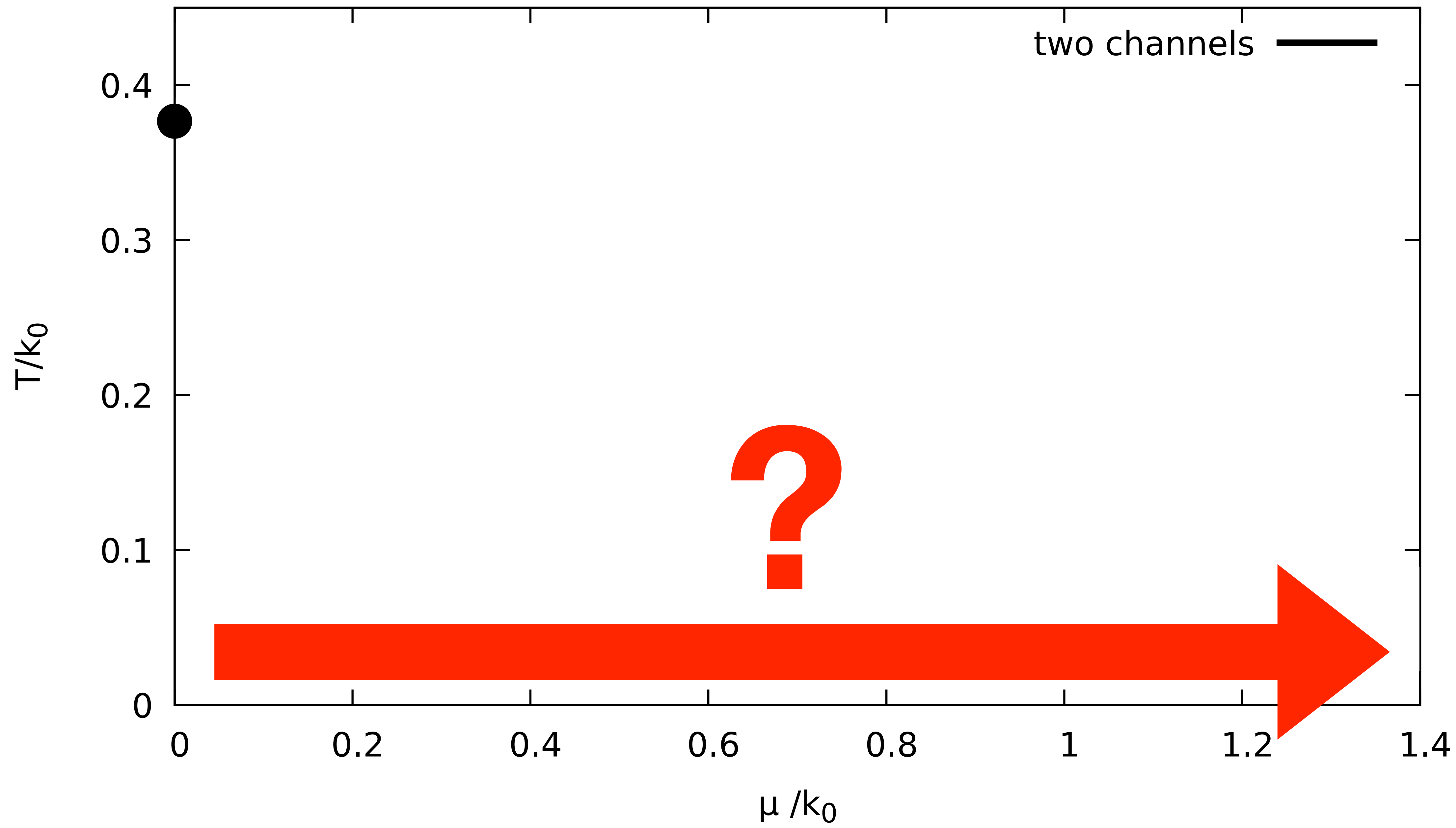

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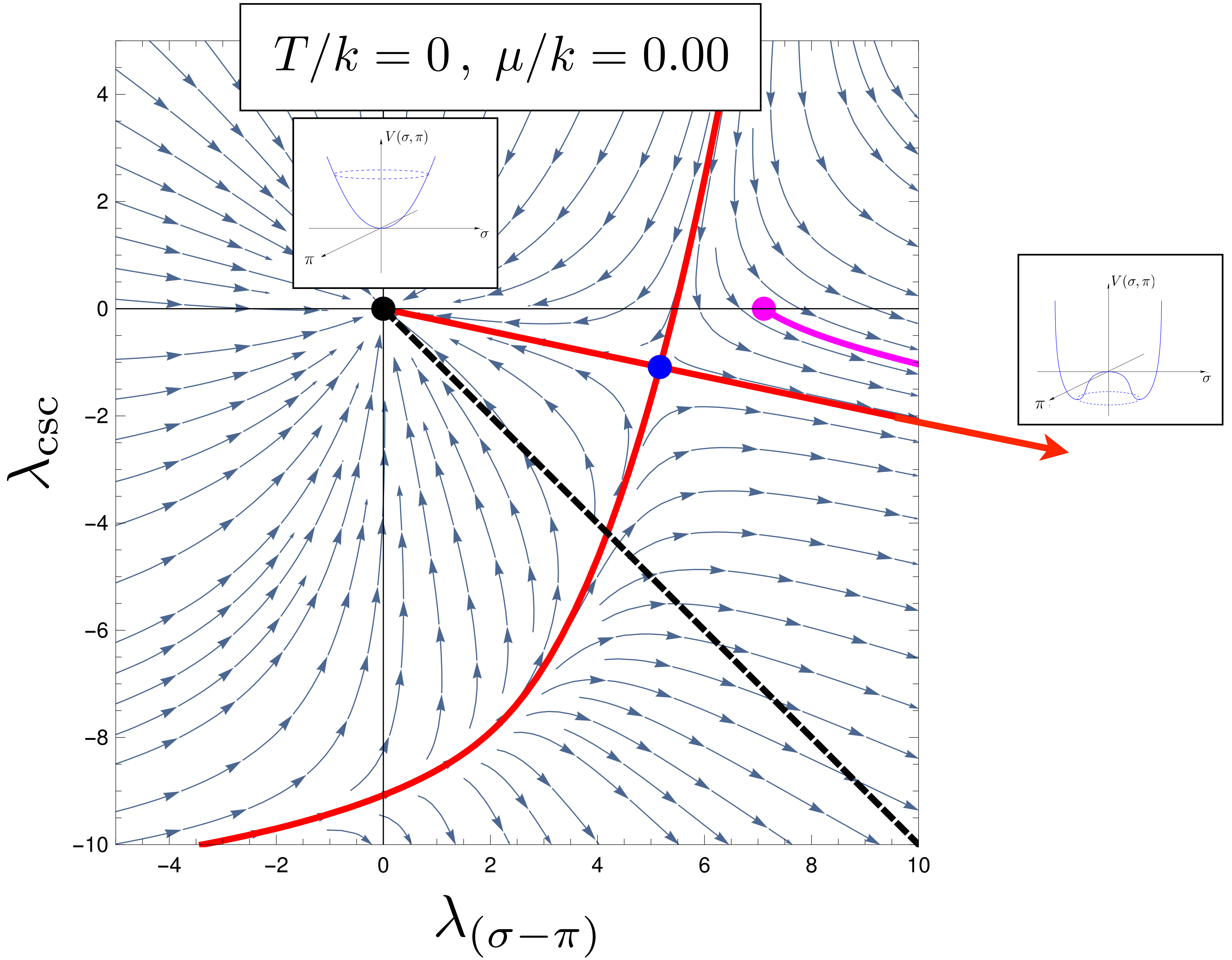

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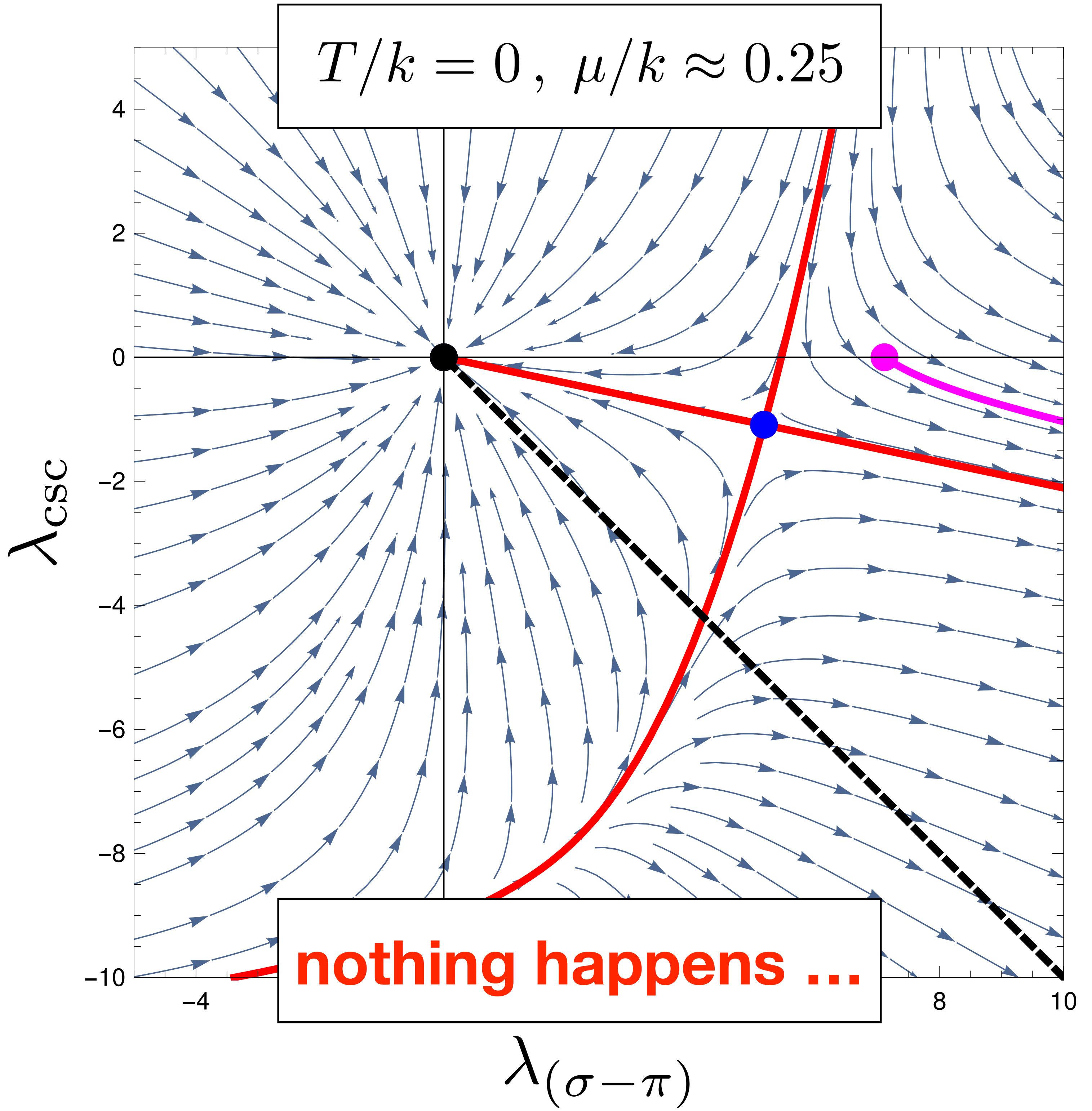

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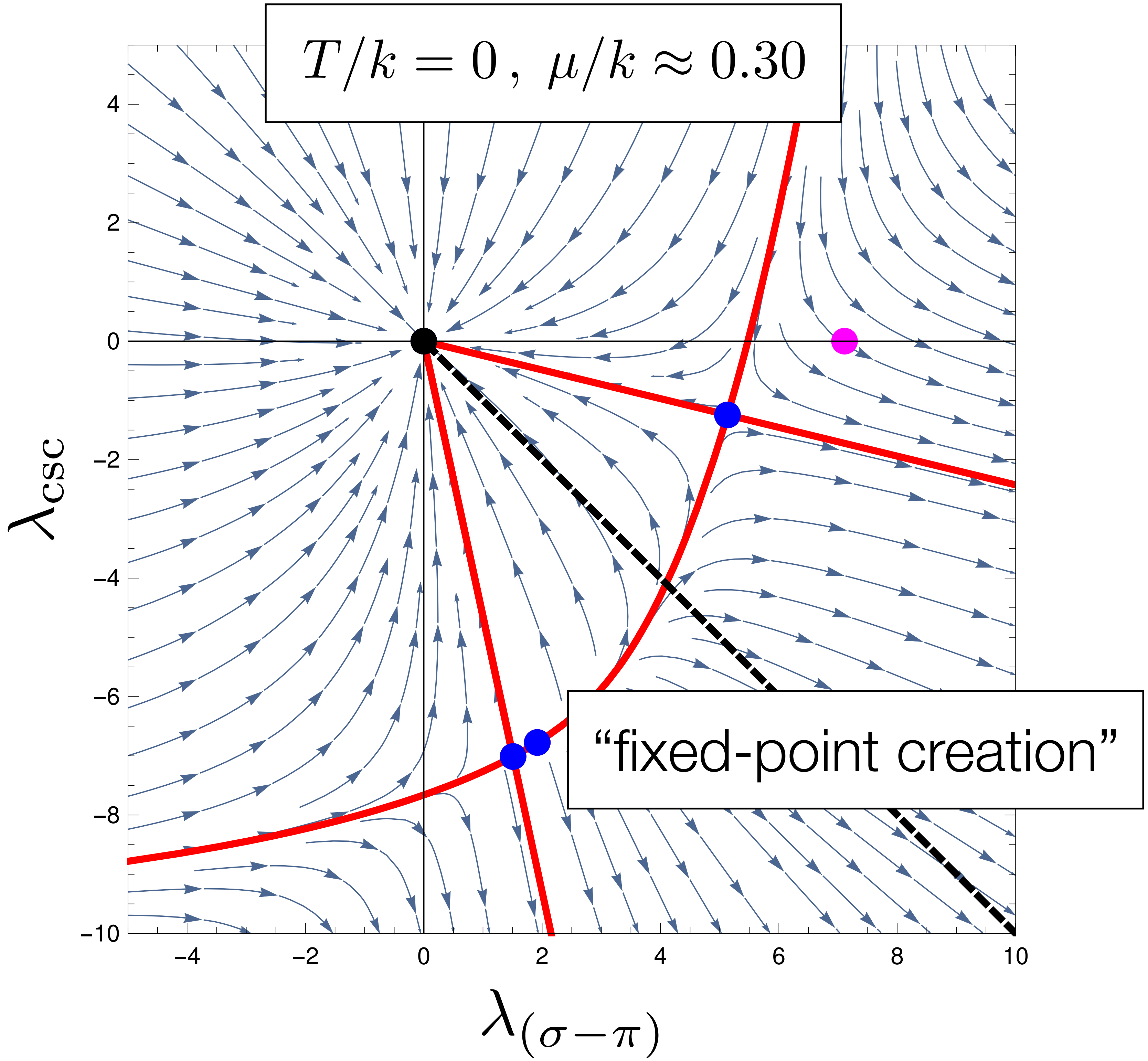

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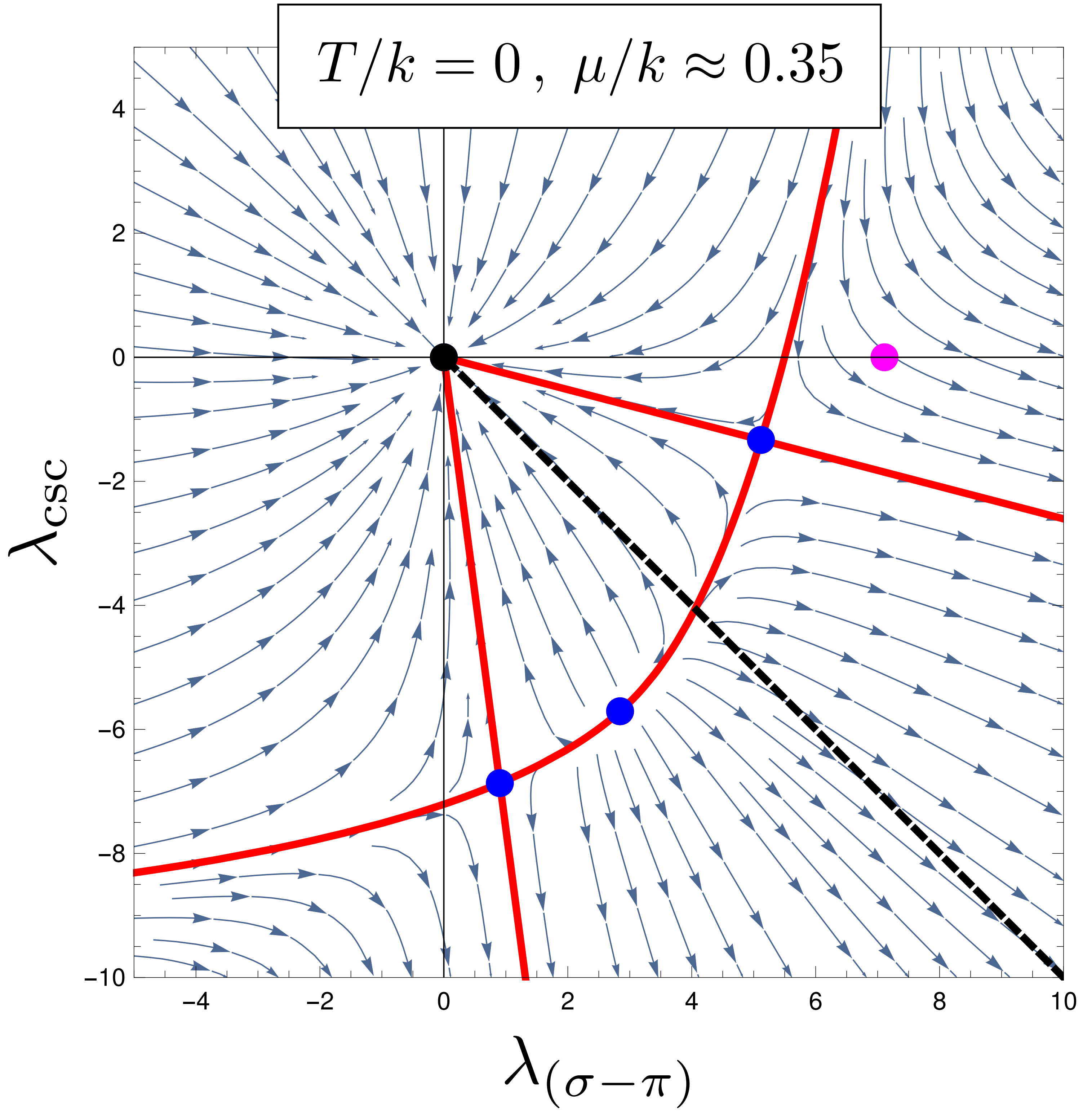

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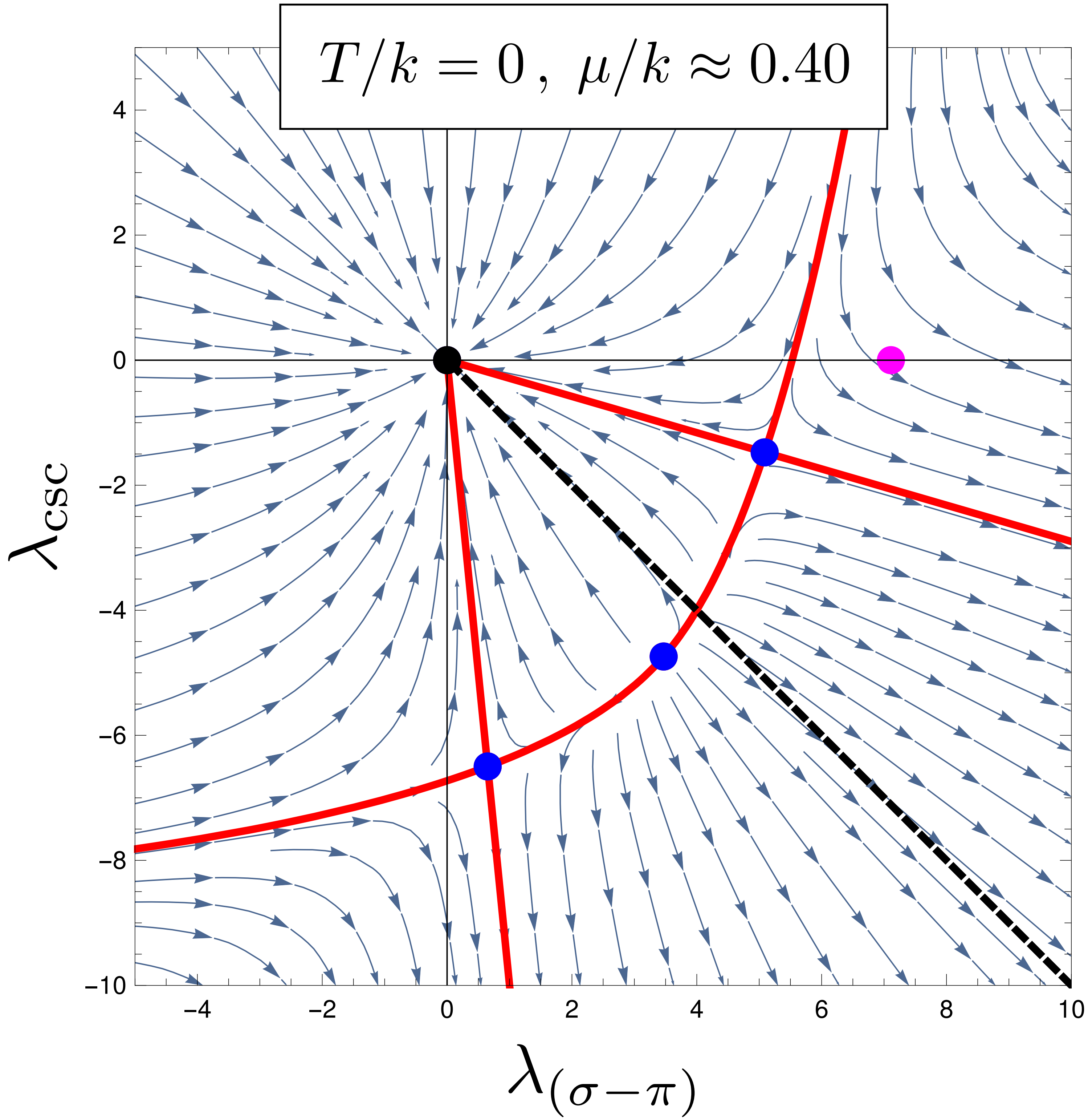

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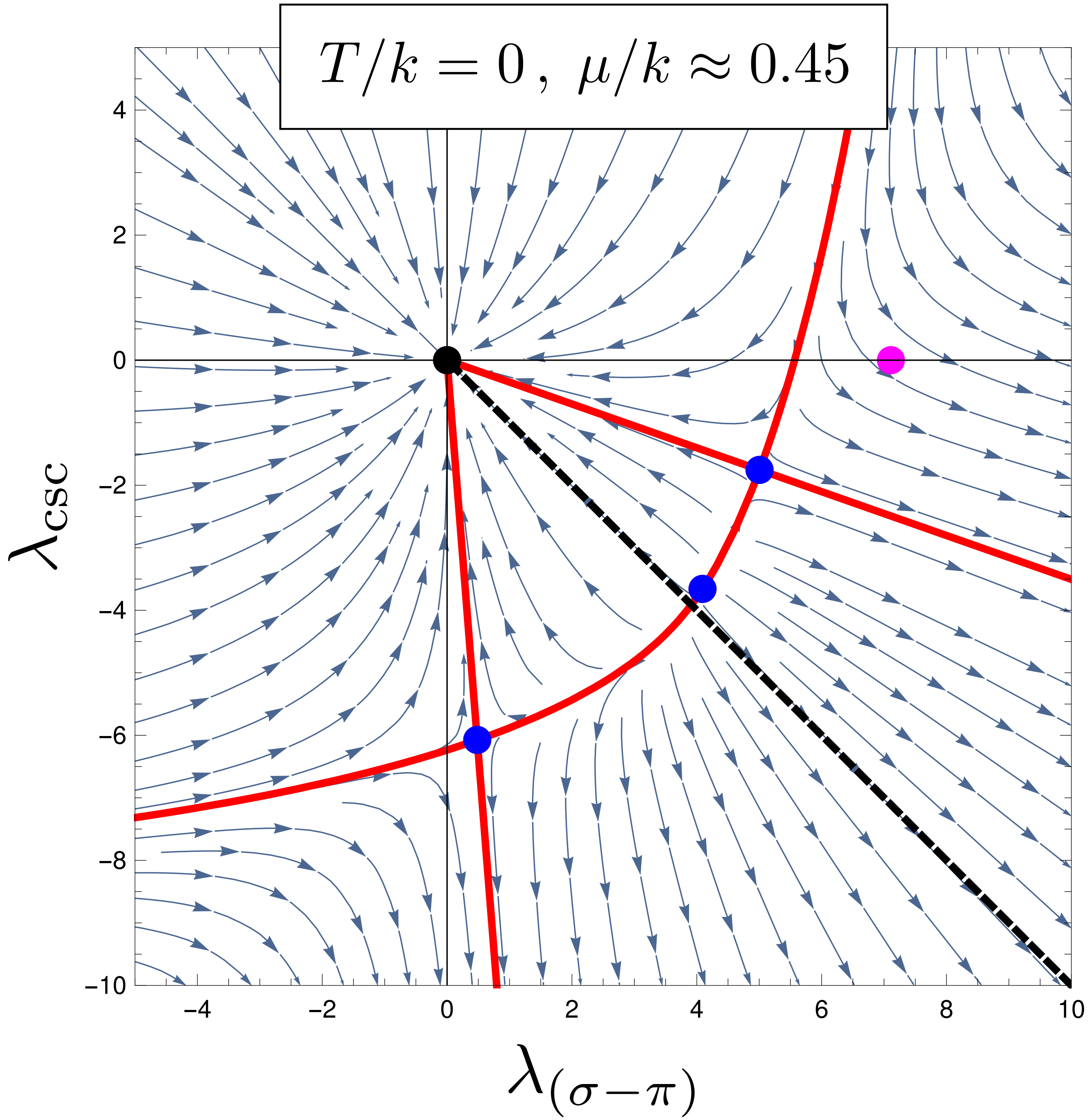

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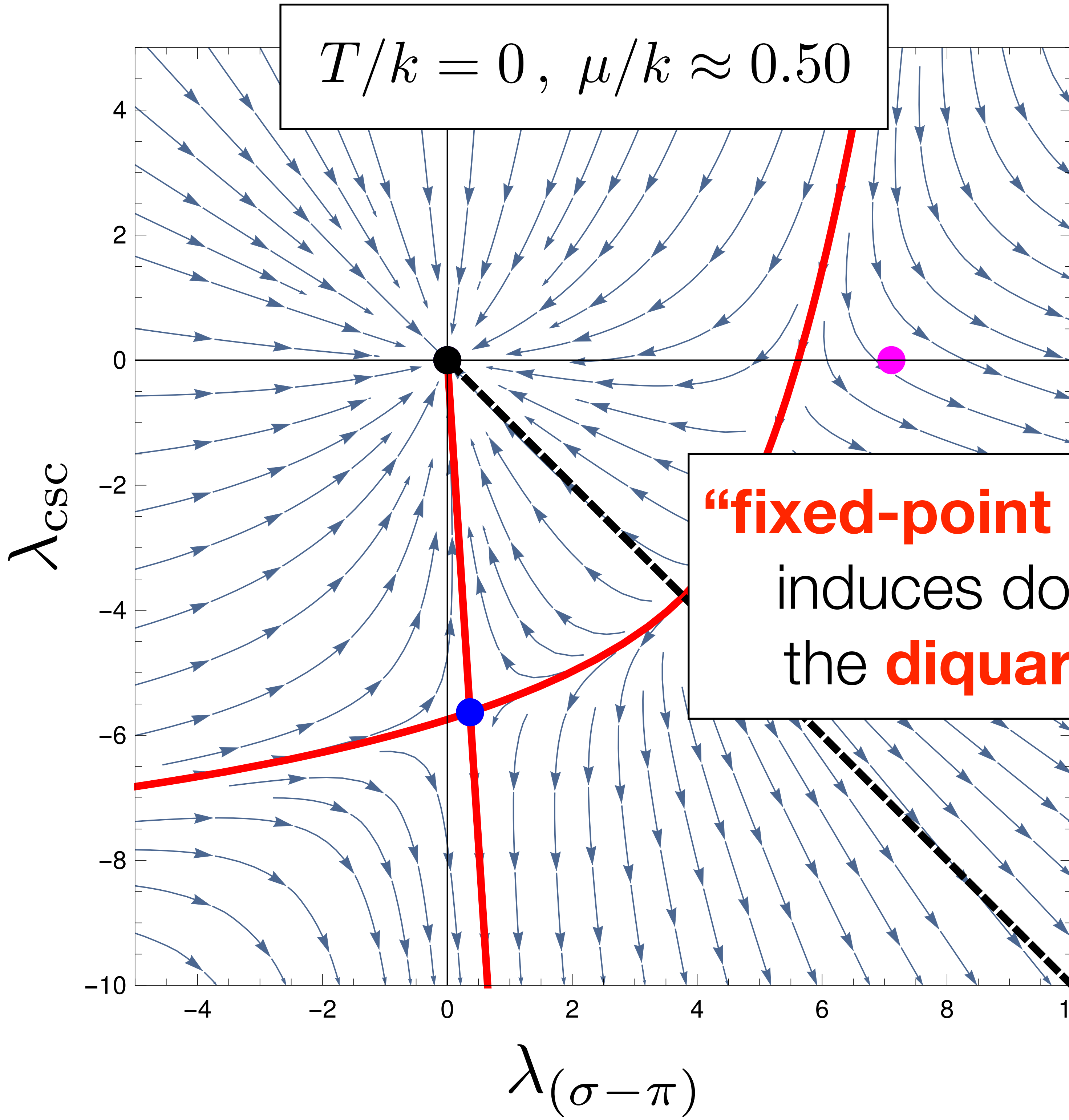
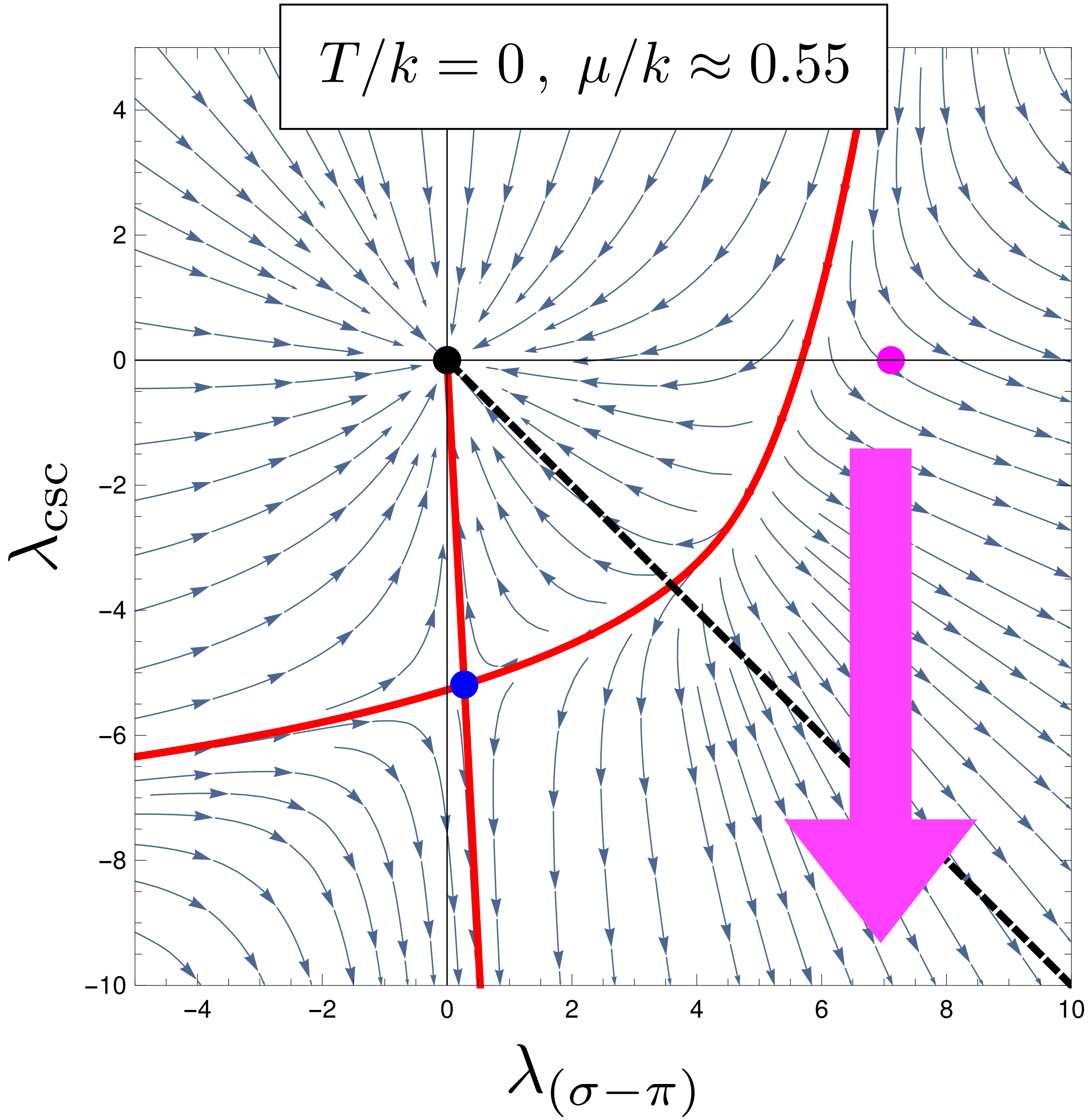


"fixed-point annihilation" induces dominance of the **diquark channel**

Fixed-point structure and phases: 2 channels



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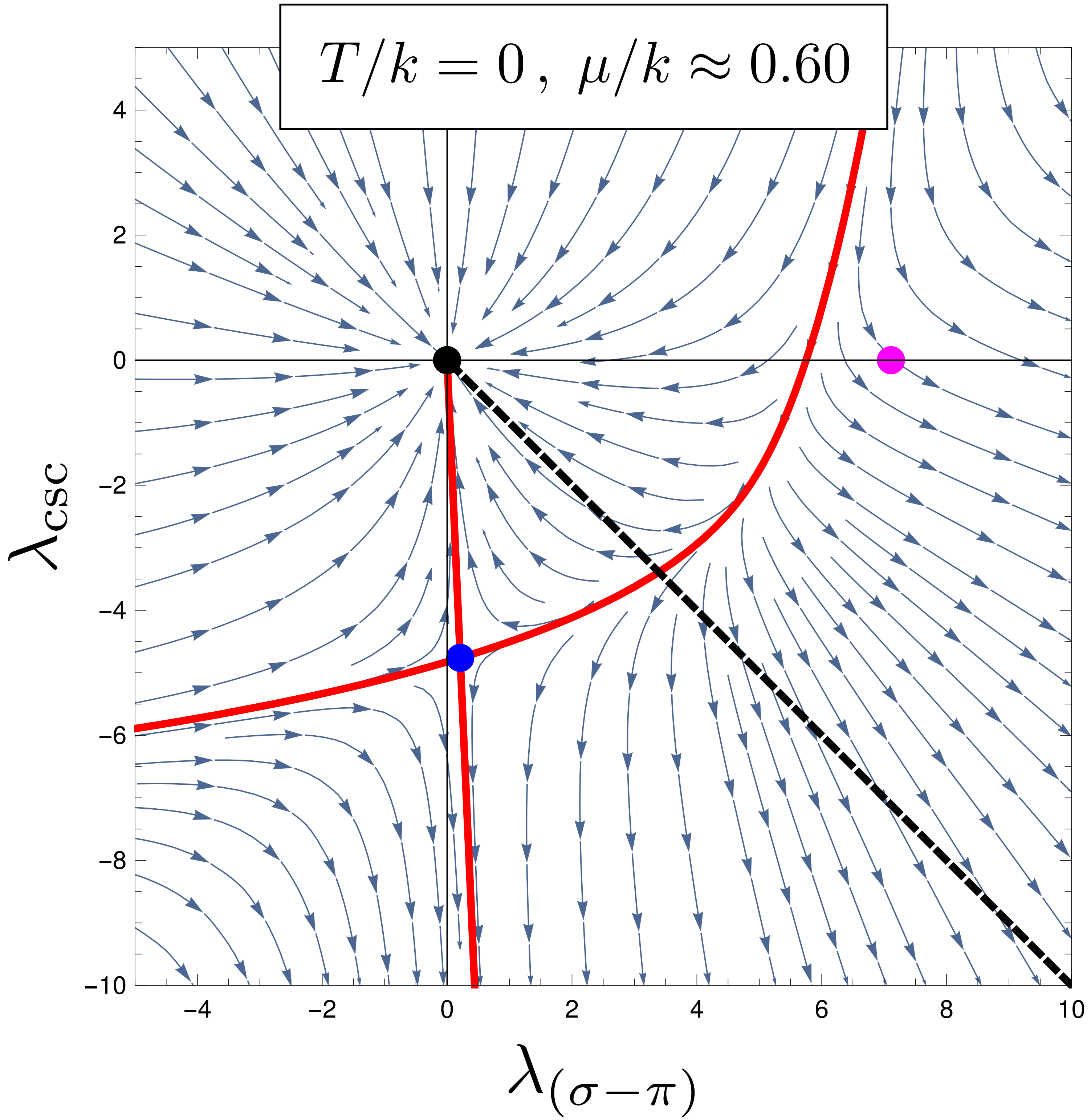

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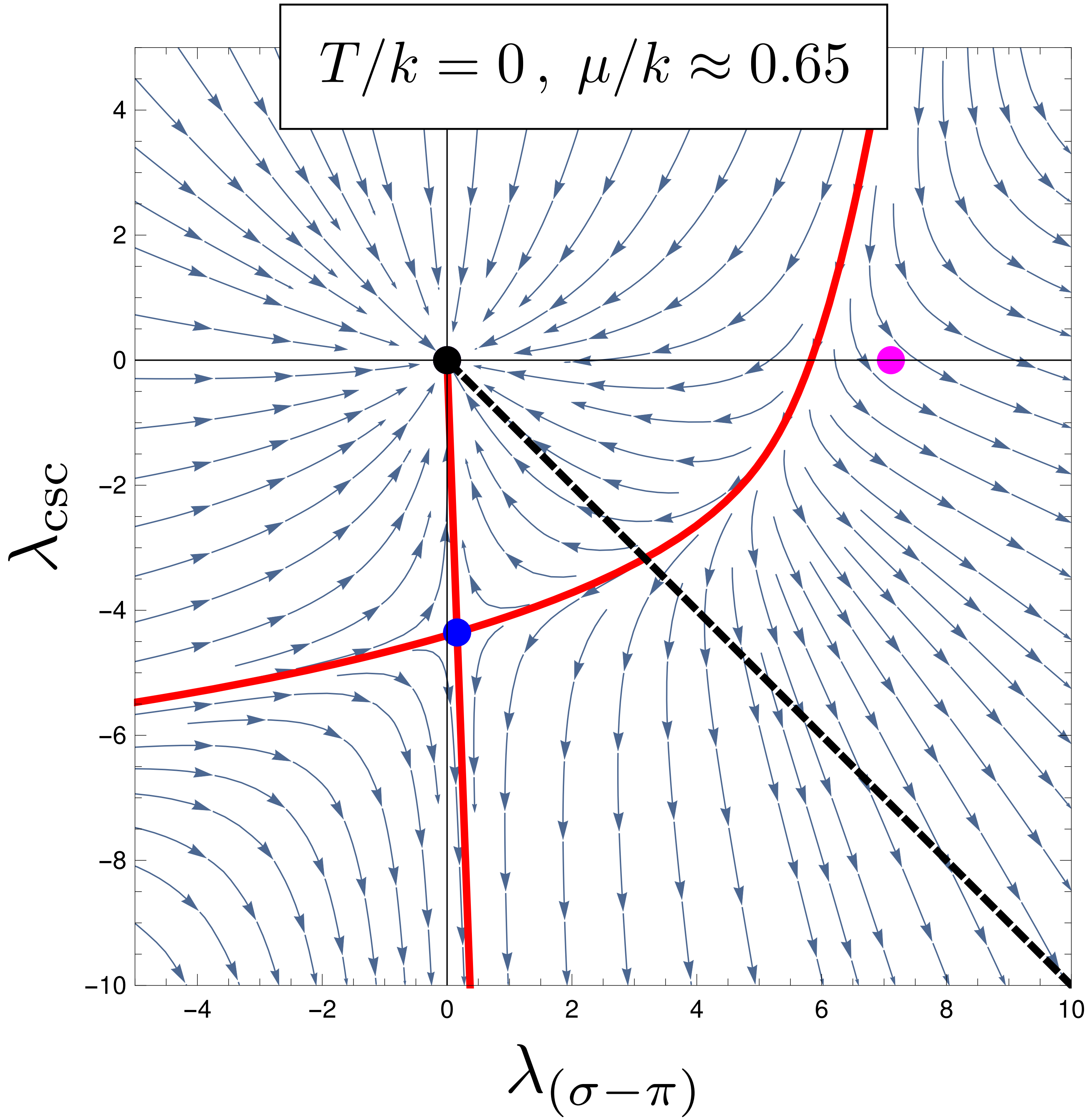

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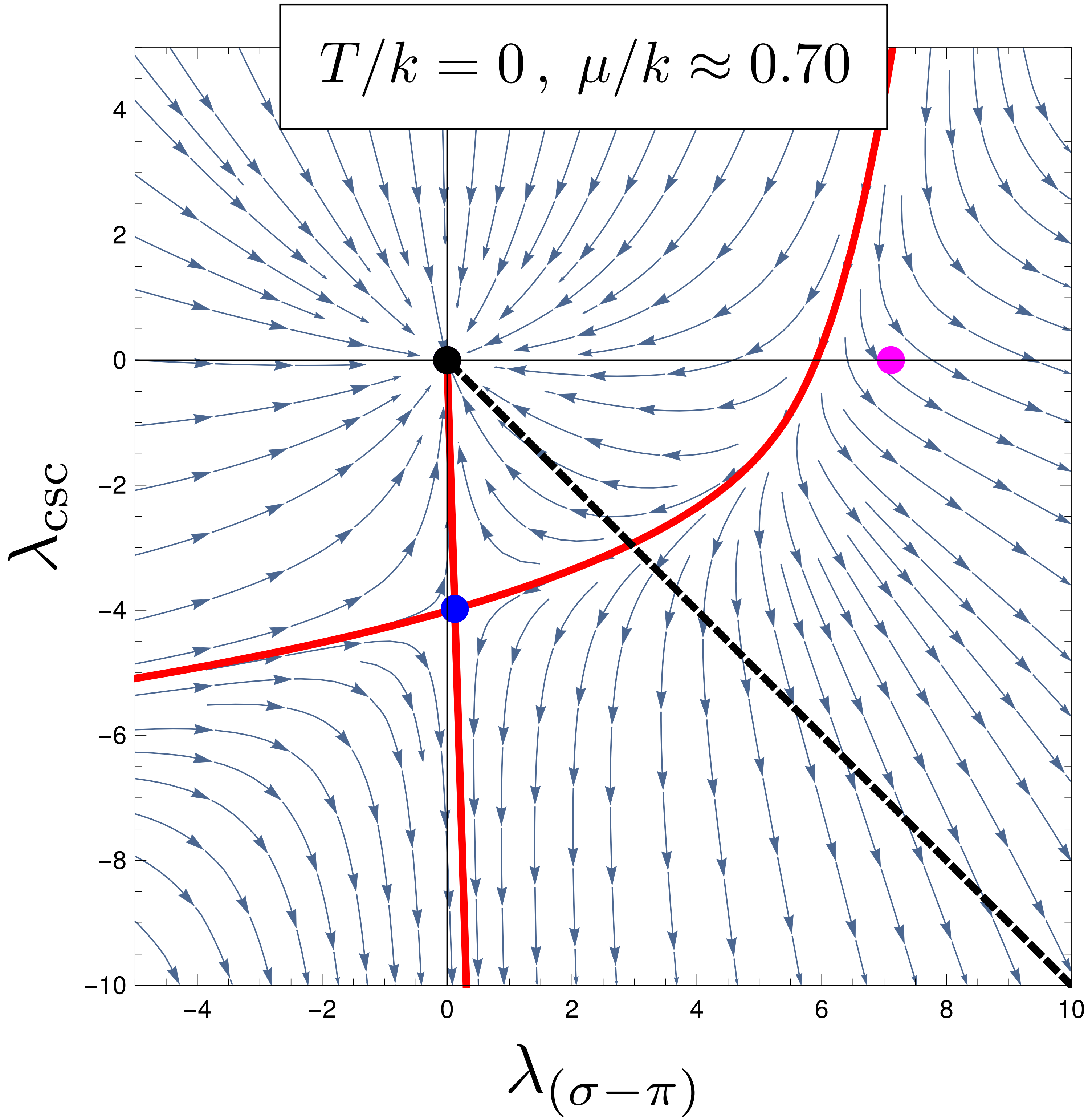

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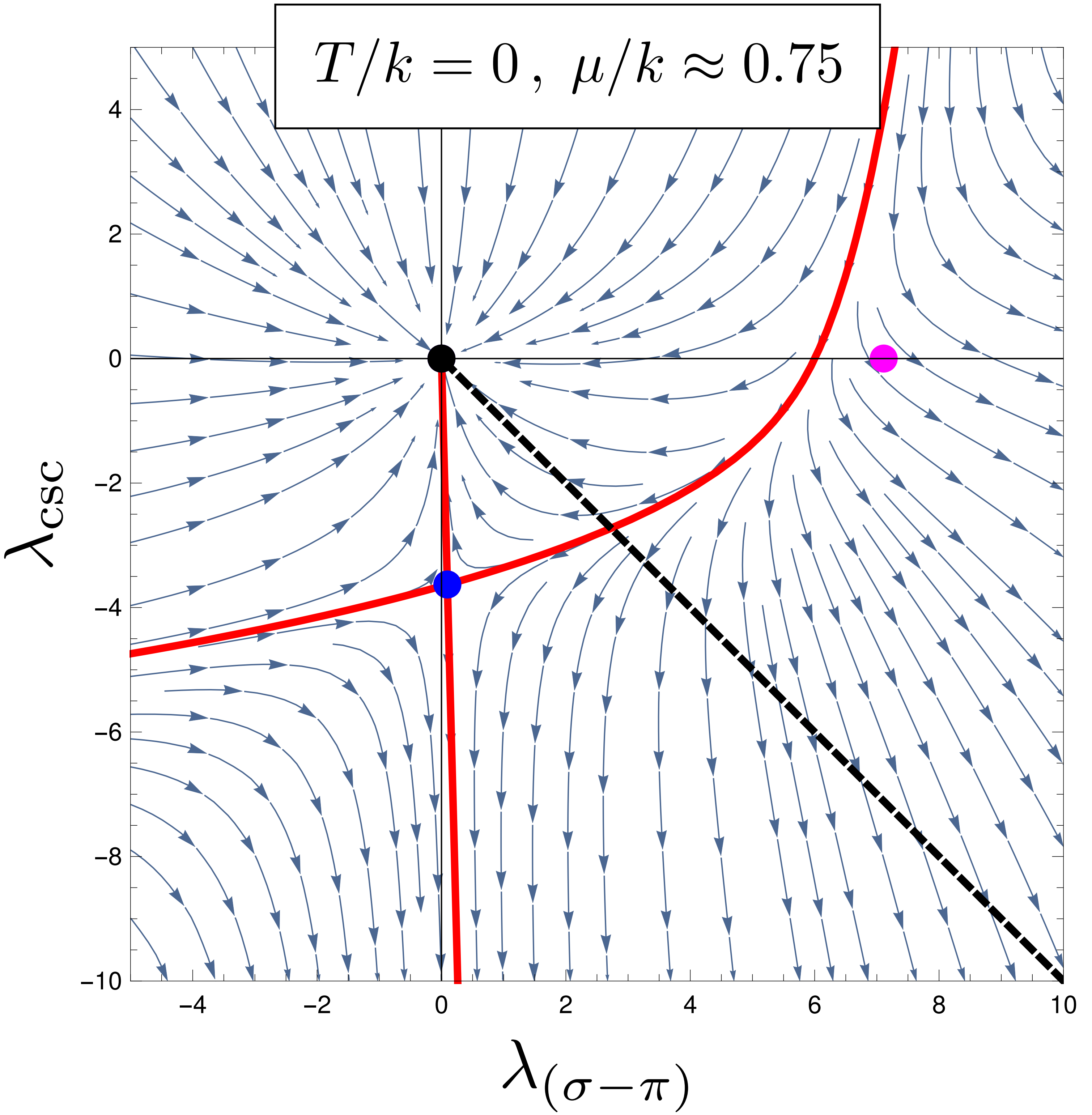

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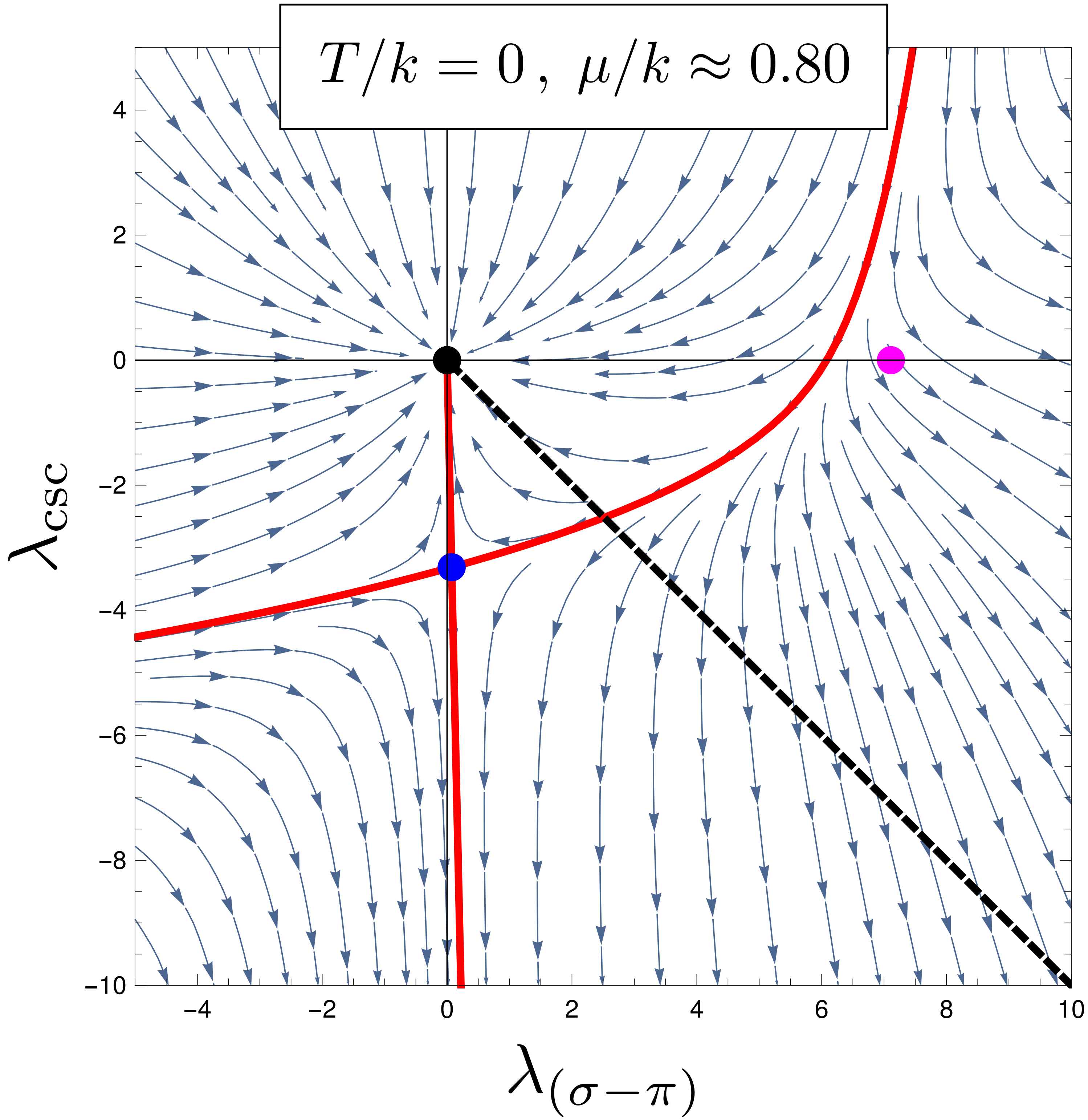

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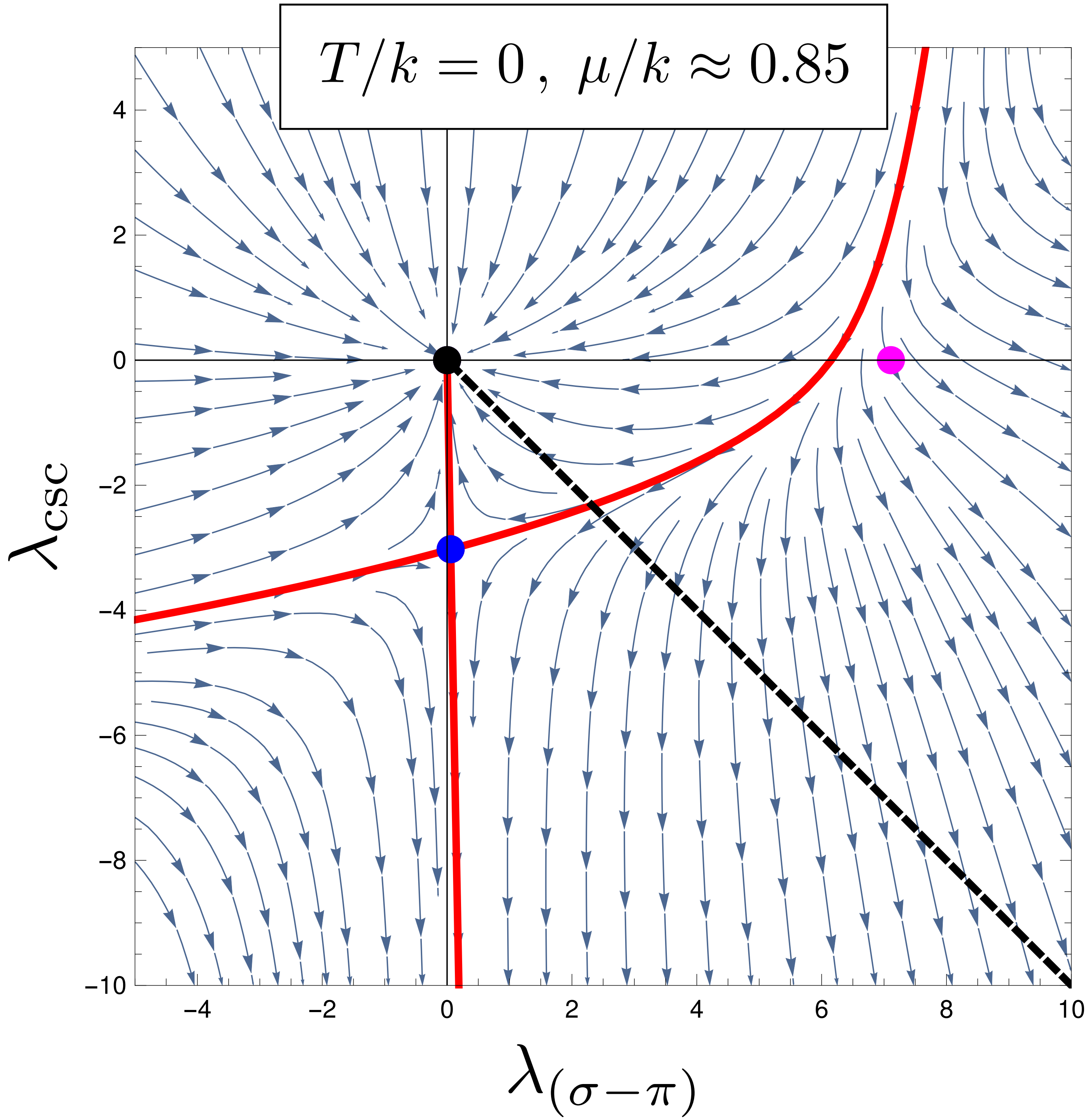

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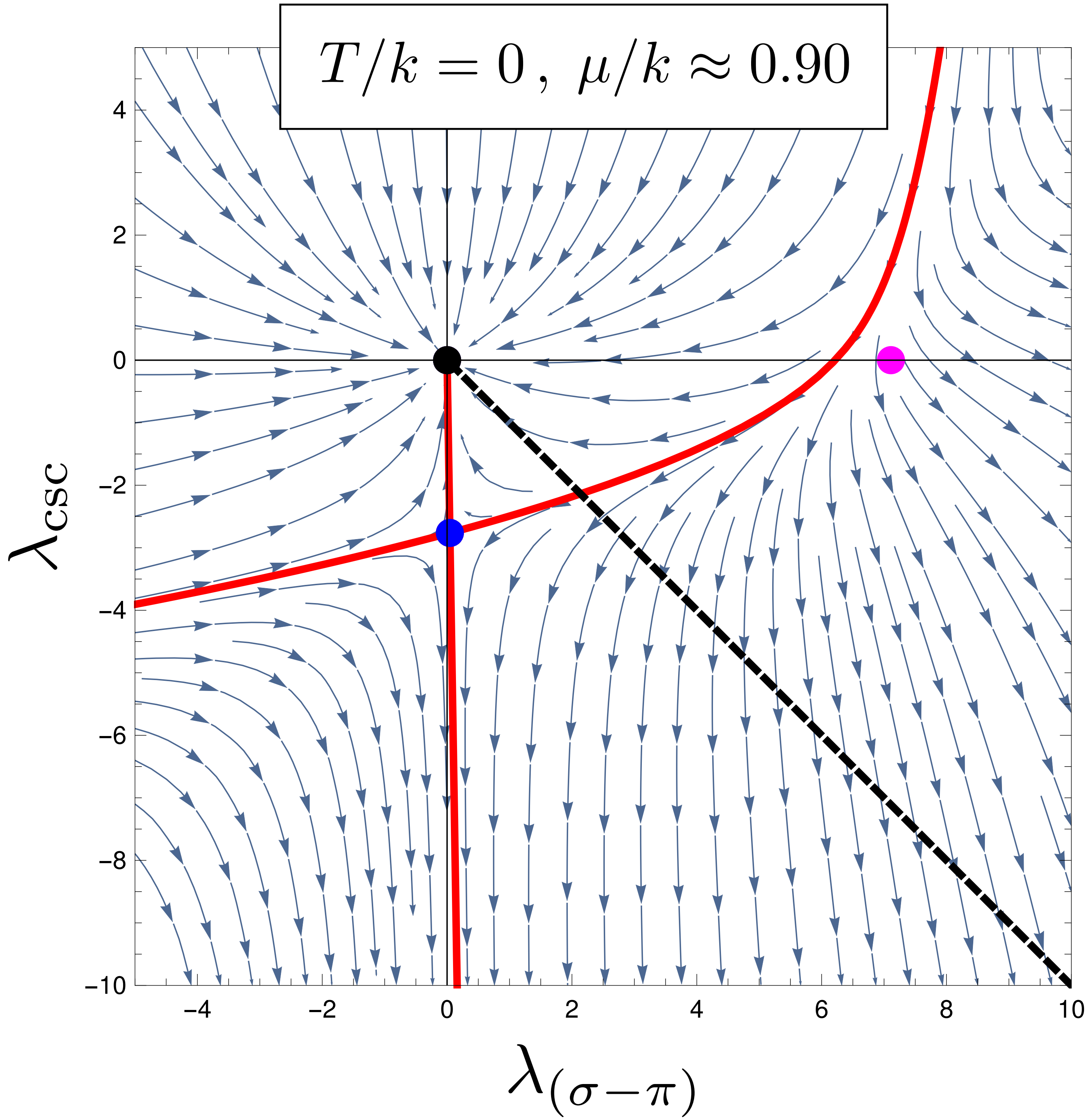

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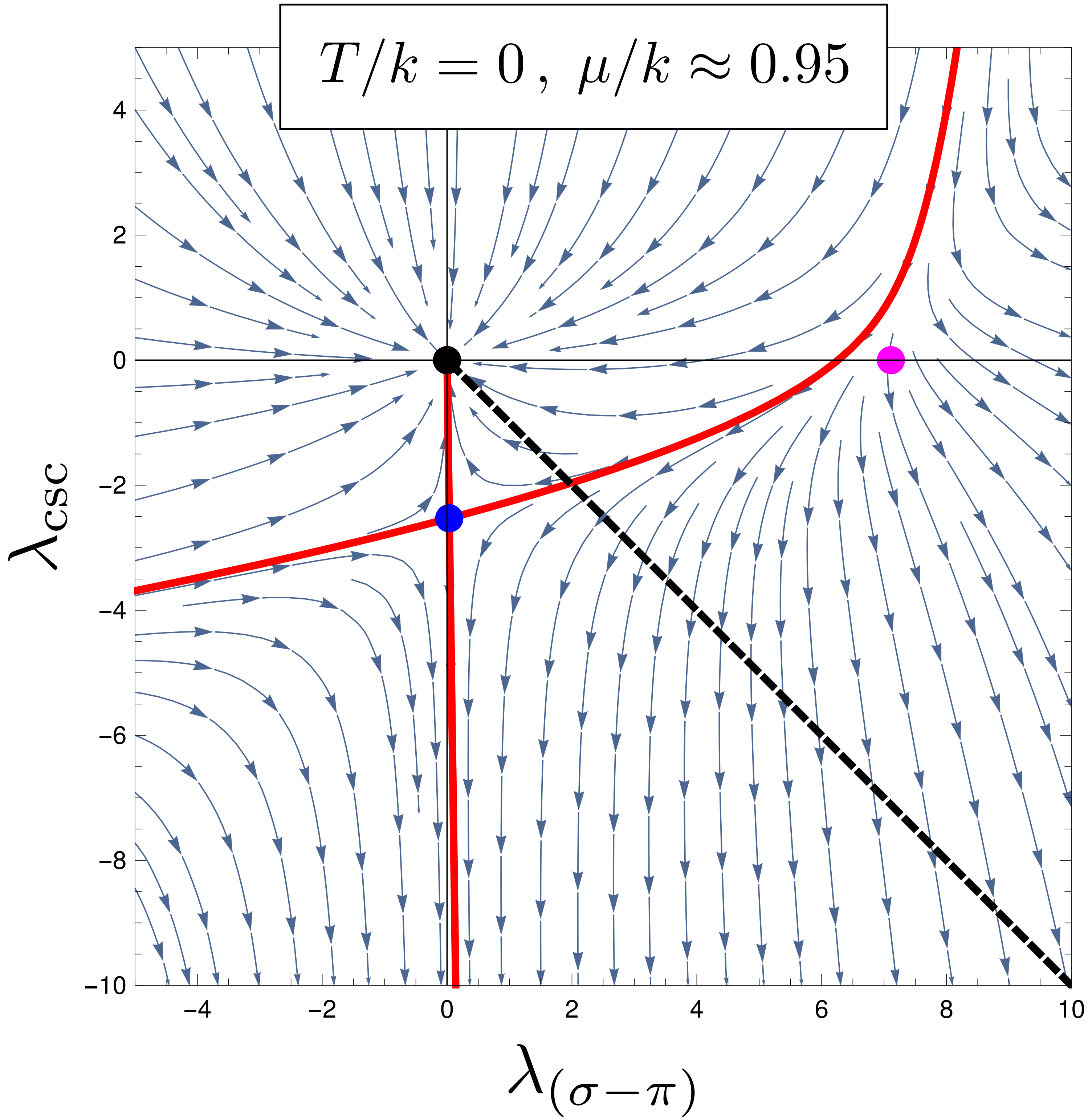

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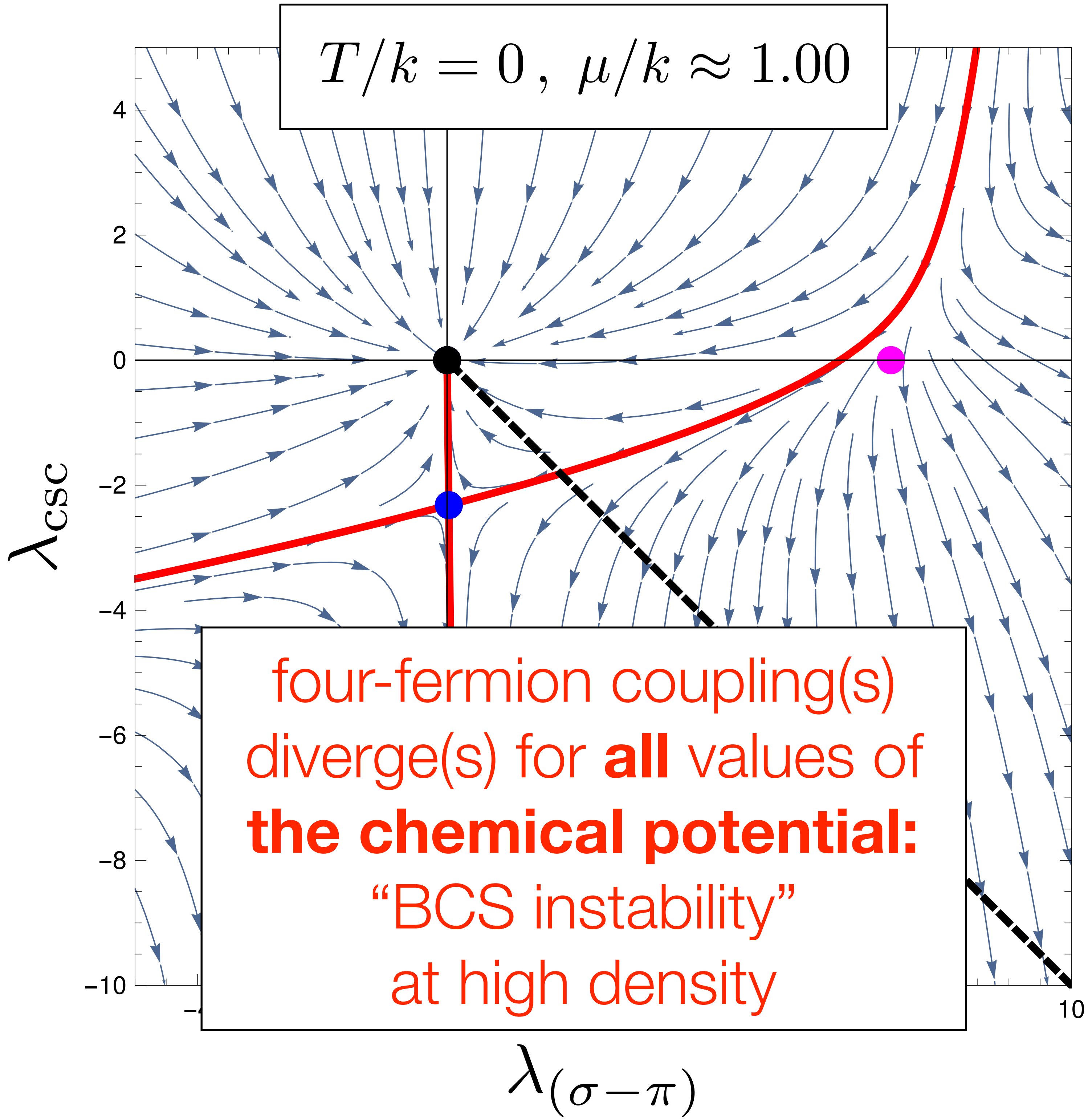

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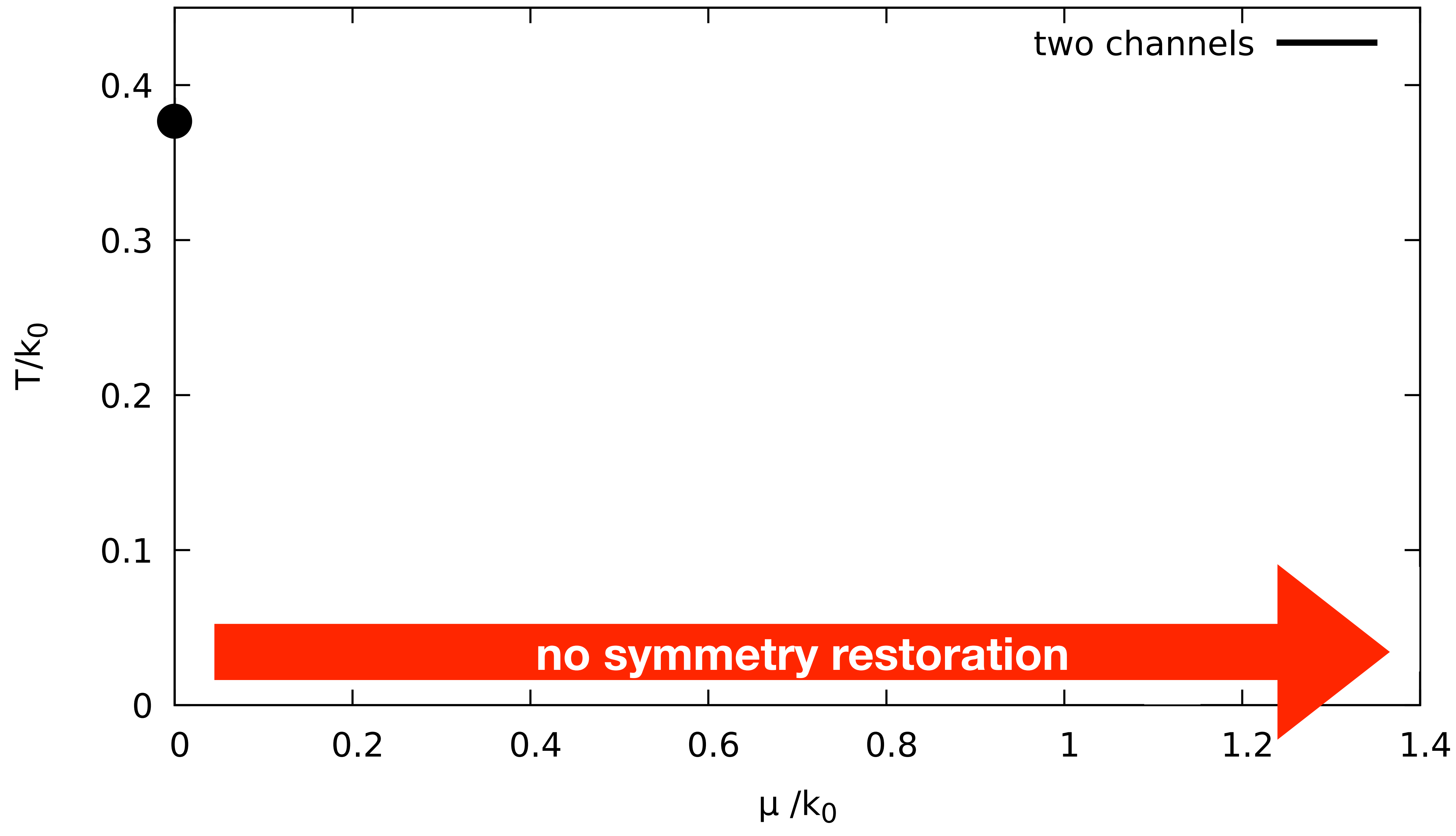

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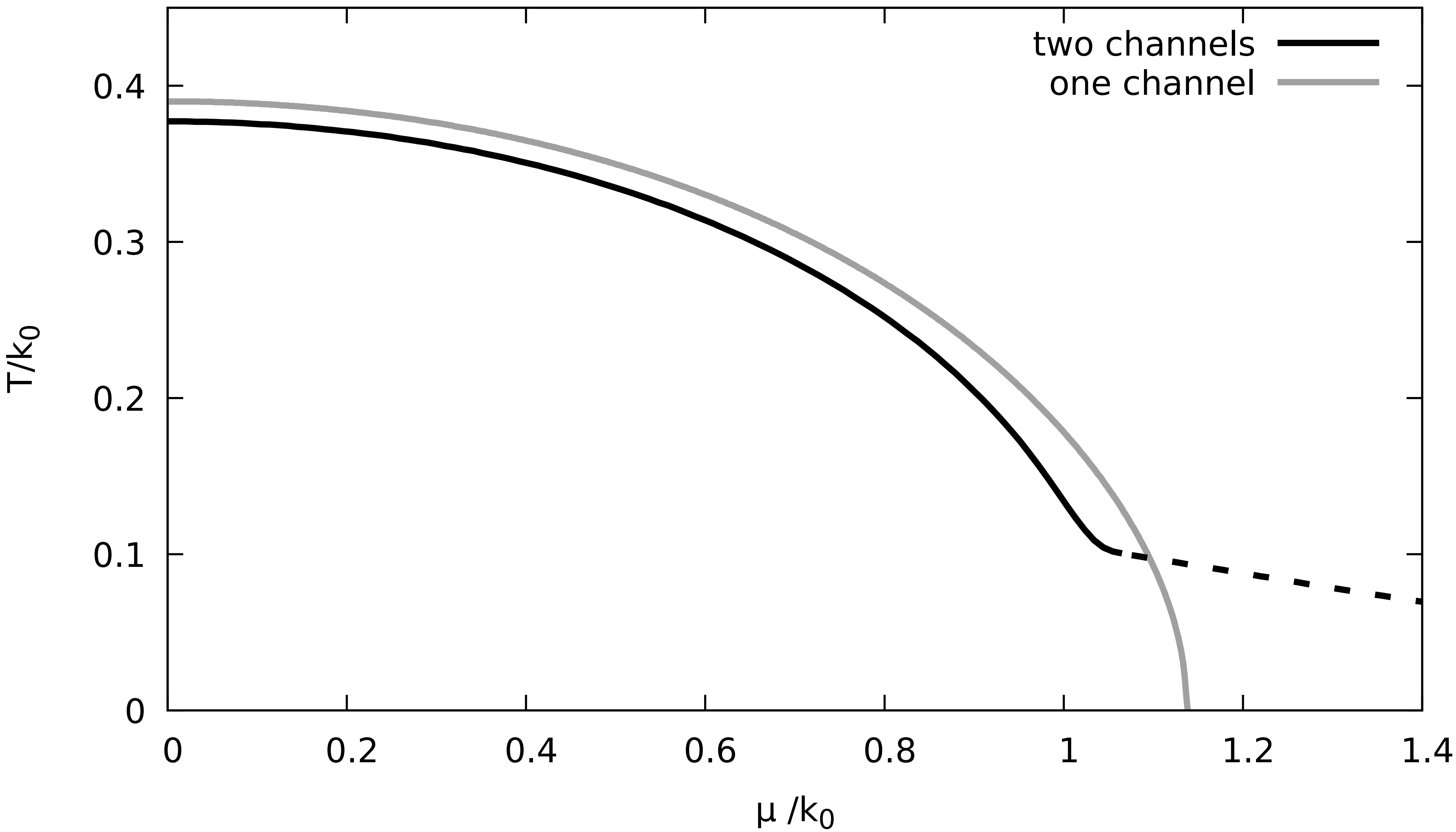

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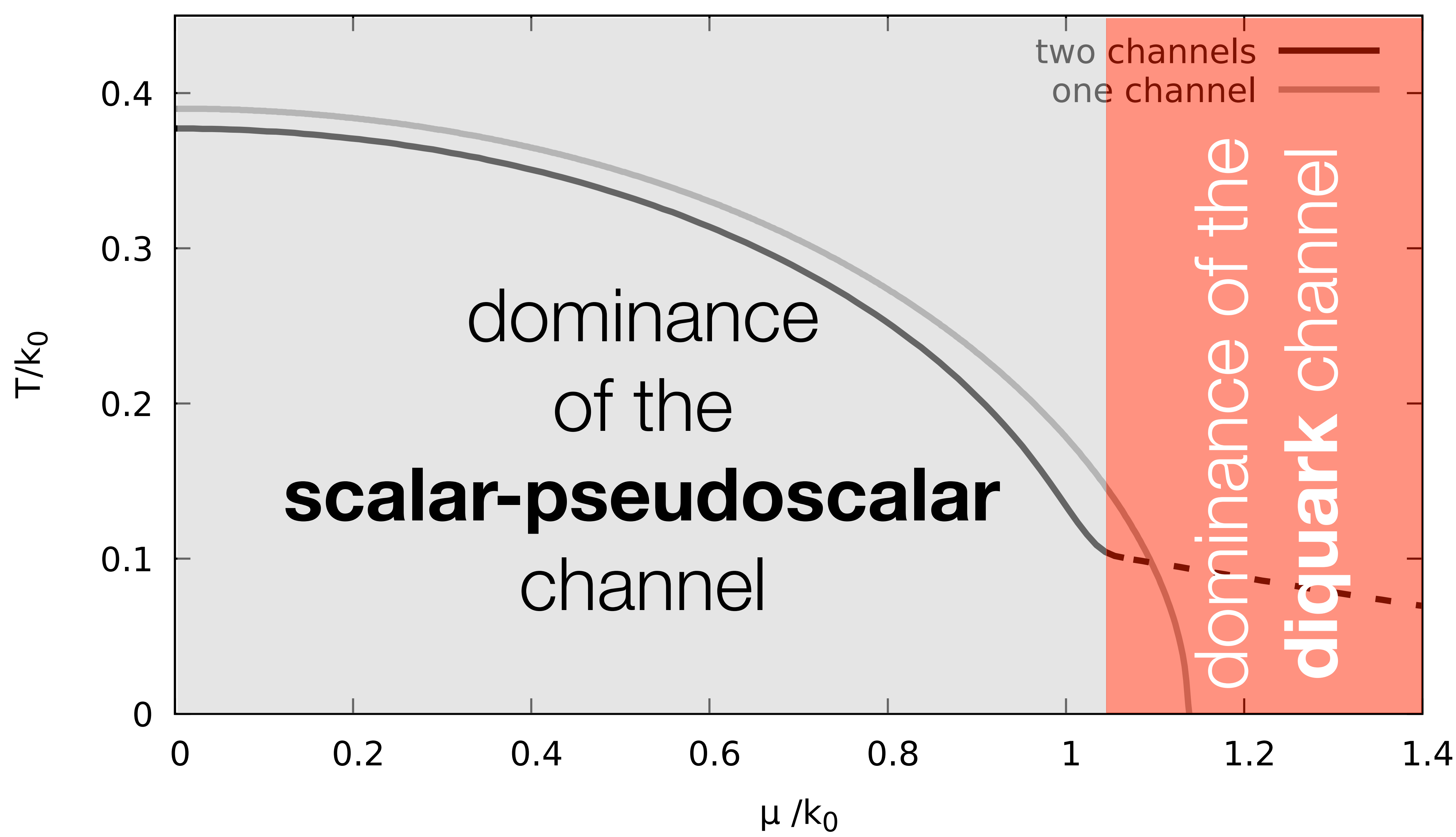

Fixed-point structure and phases: 6 channels

(Fierz-complete at $T = 0$ and $\mu = 0$)

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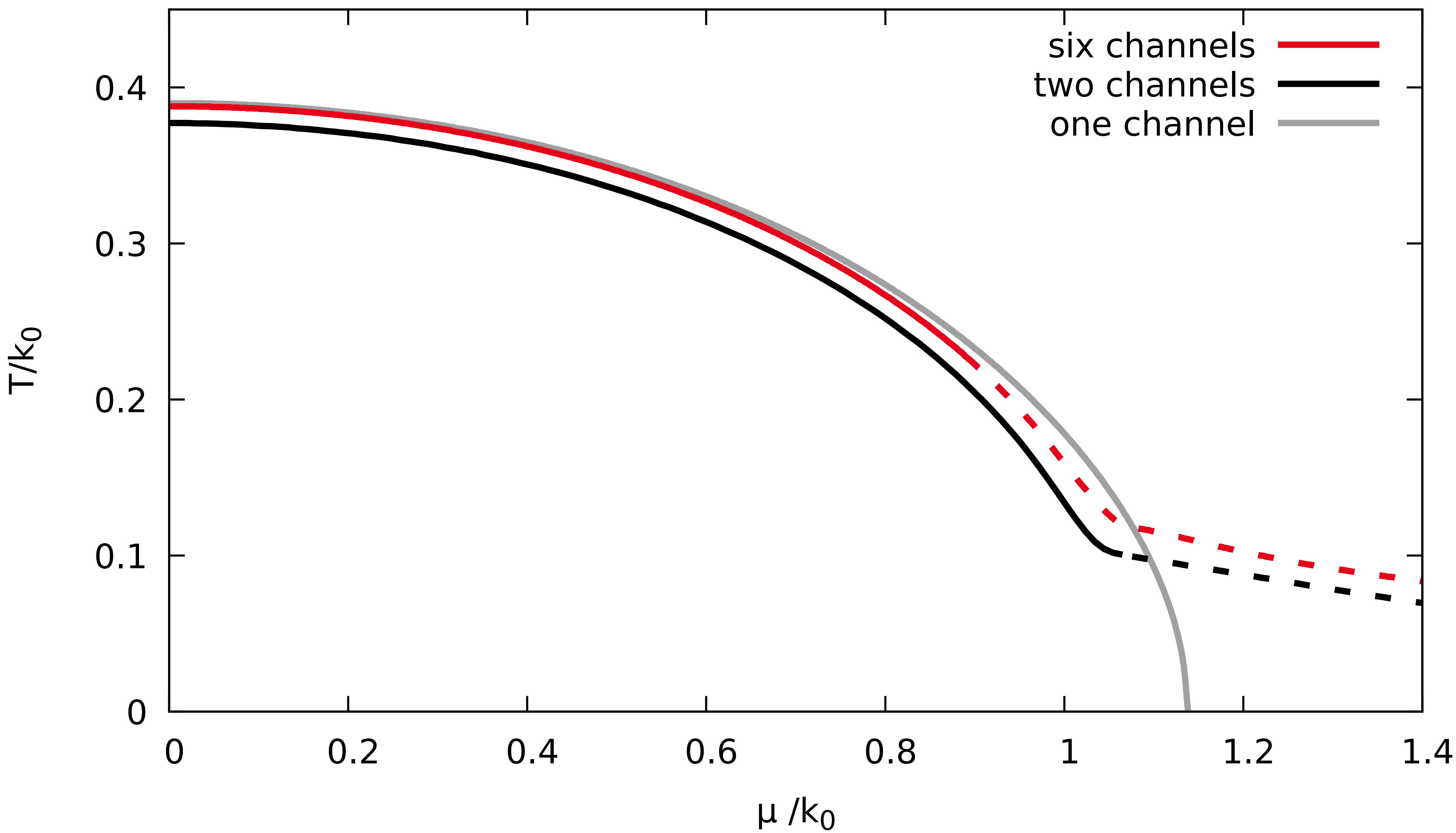

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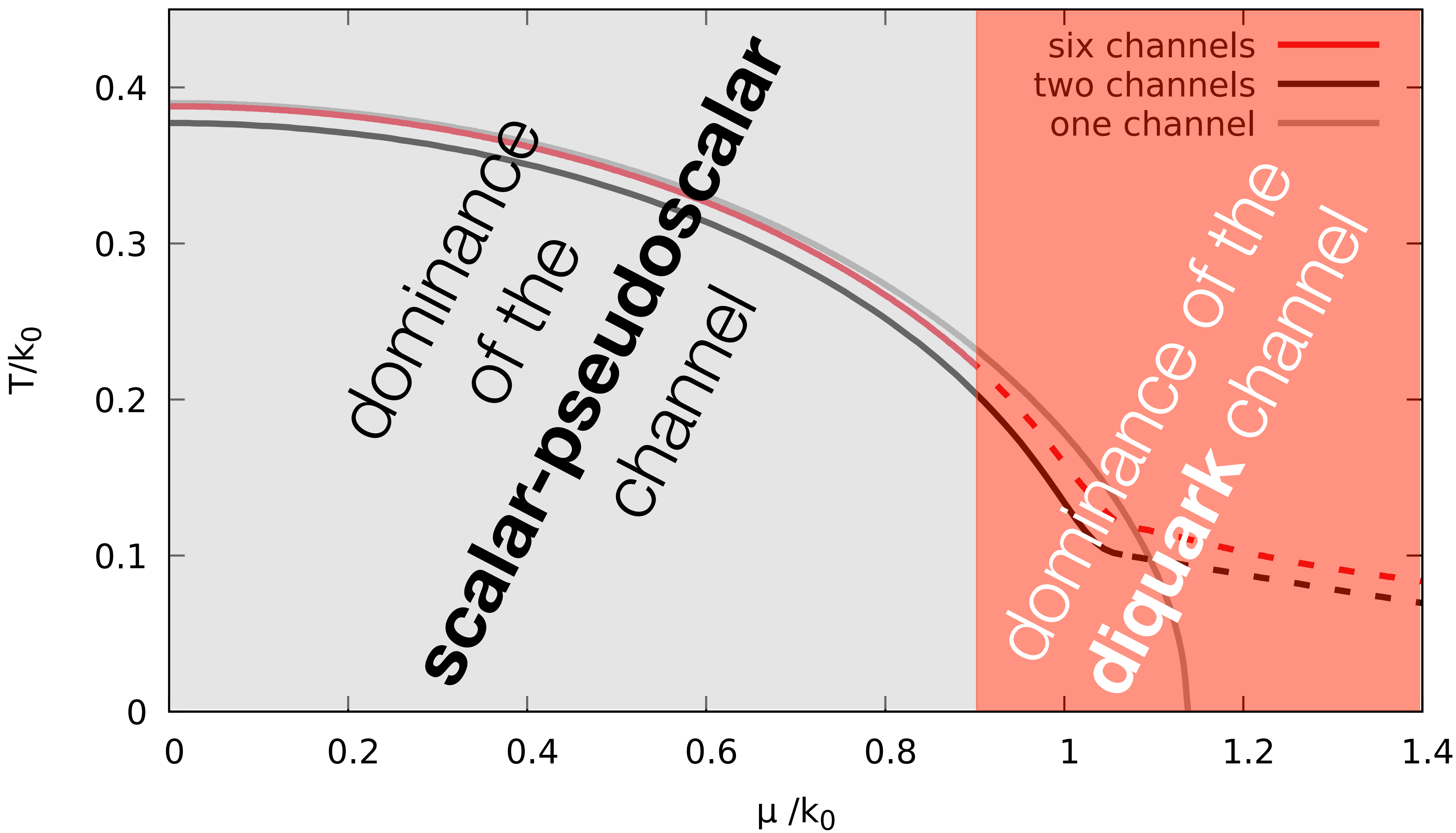

Fixed-point structure and phases: Fierz-complete

(10 coupled channels)

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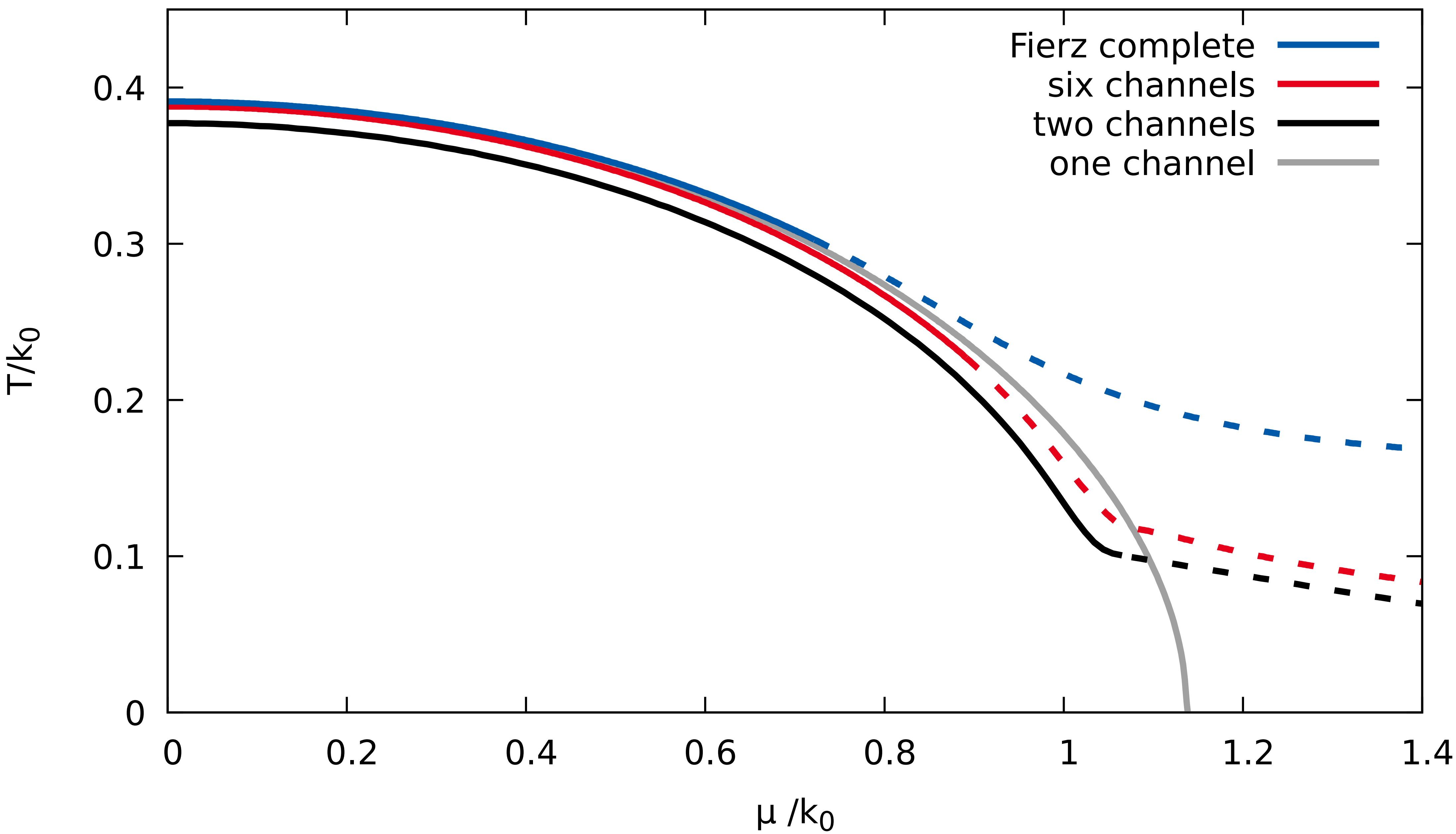

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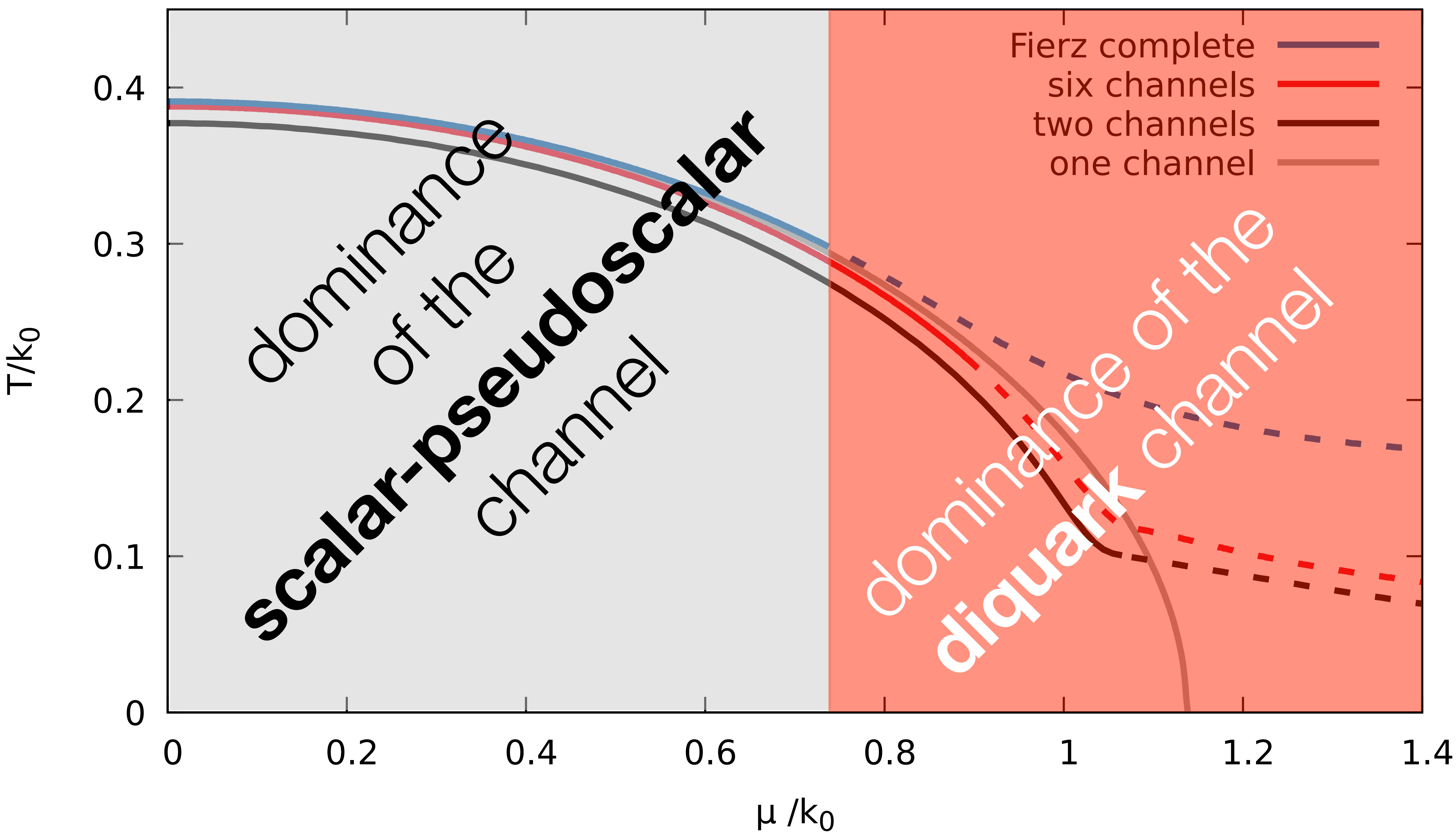
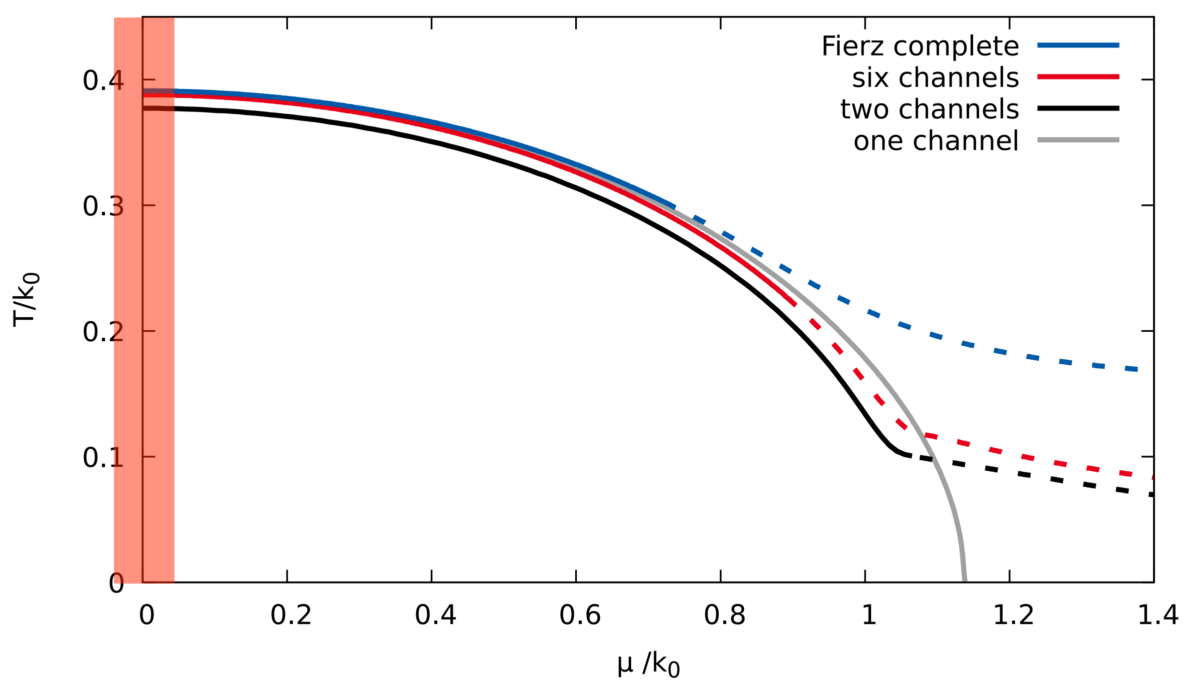
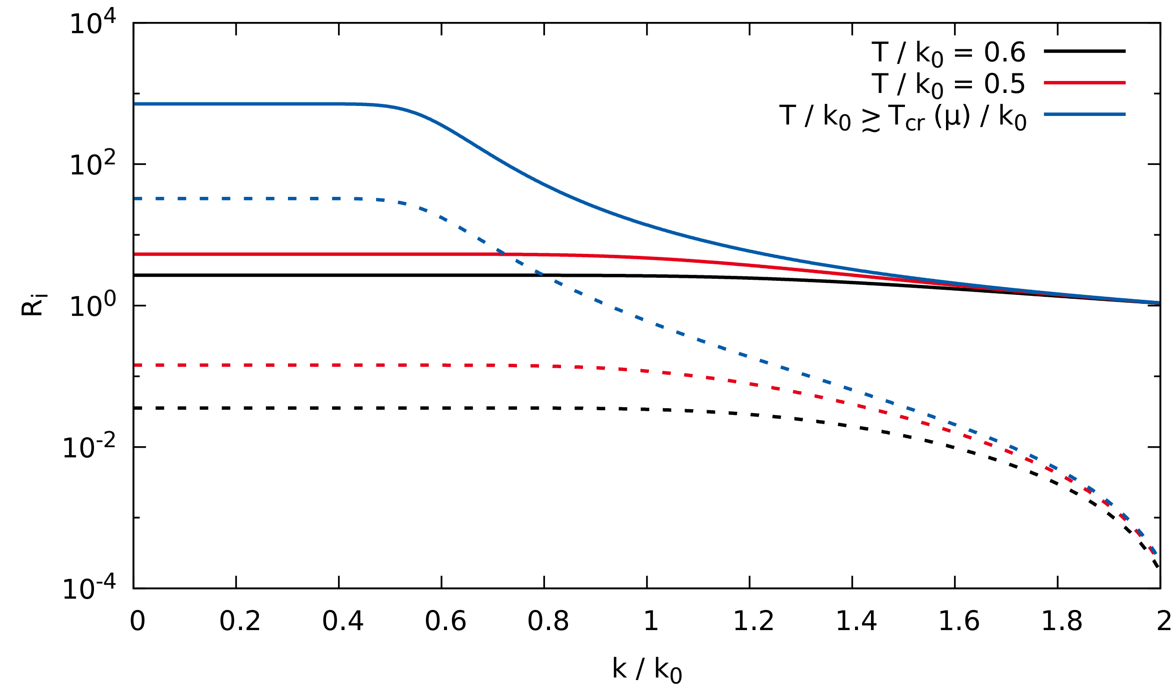


Fixed-point structure and phases: $\mathrm{U}_A(1)$ breaking

[JB, Leonhardt, Pospiech '18]





- Fierz-complete 10-channel basis can be mapped onto a Fierz-complete $\mathrm{U}_A(1)$-symmetric 8-channel basis
- "measure" strength of $\mathrm{U}_A(1)$ breaking with sum rules:

$$R_1 \sim \bar{\lambda}_{\mathrm{csc}} + \bar{\lambda}_{(S+P)_-^{\mathrm{adj}}} = 0$$ **[dashed lines]**

$$R_2 \sim \bar{\lambda}_{(S+P)_-} - \frac{2}{3}\bar{\lambda}_{\mathrm{csc}} + \bar{\lambda}_{(\sigma\text{-}\pi)} = 0$$ **[solid lines]**

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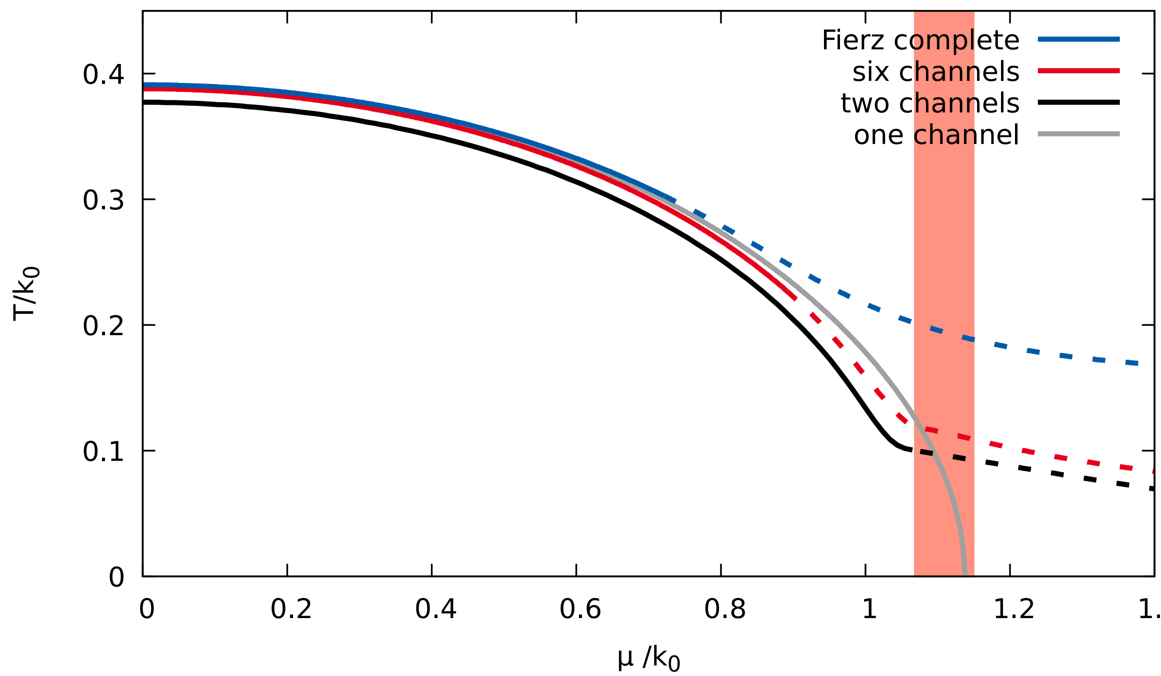

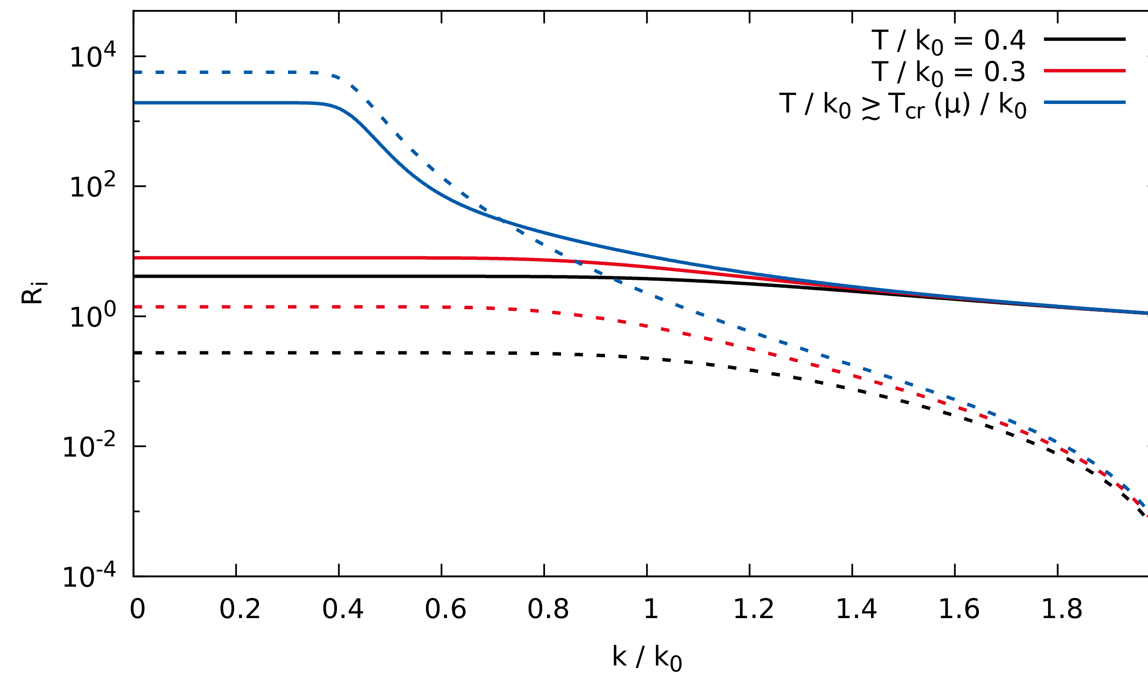

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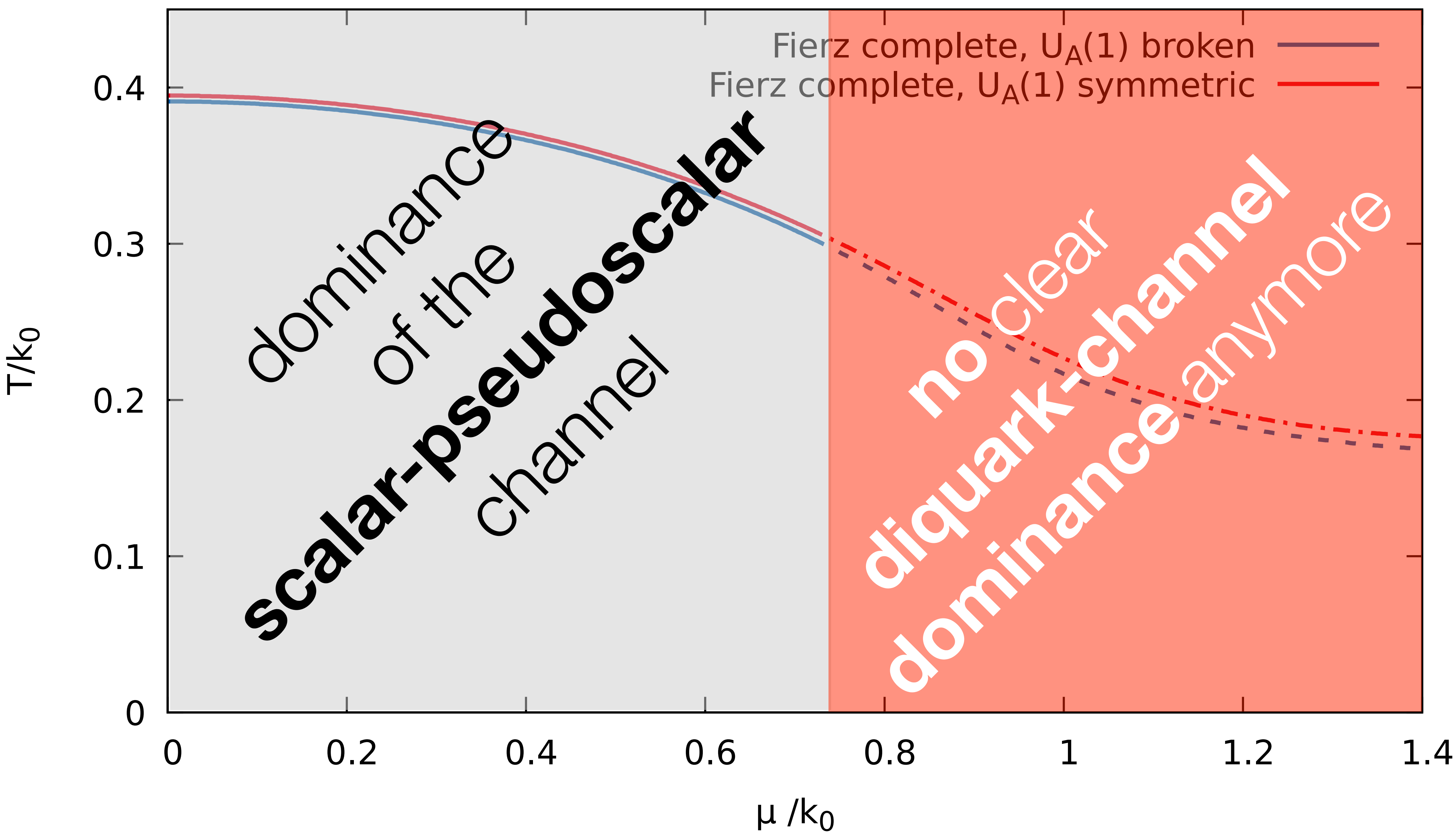
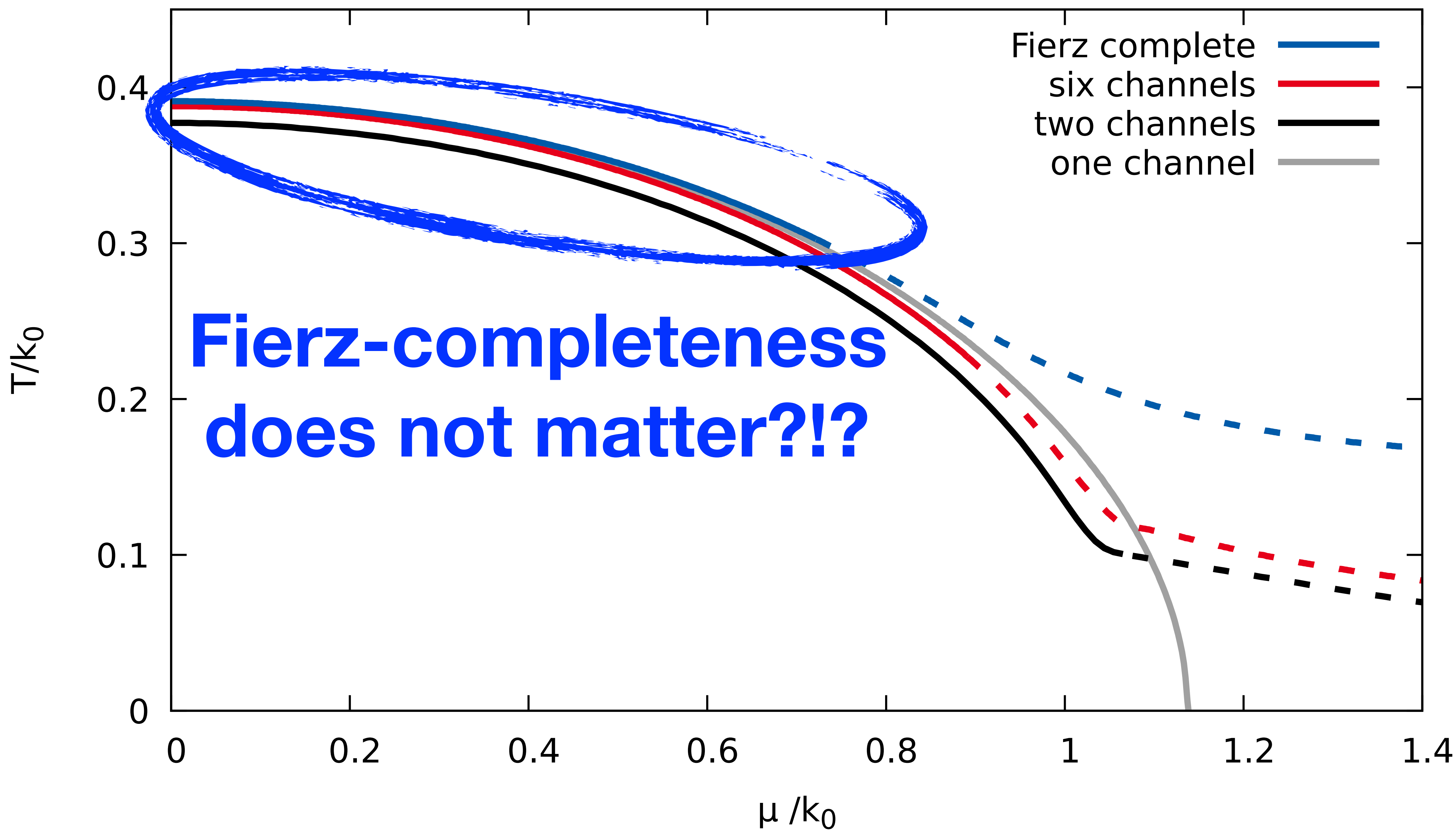
But what about a $\mathrm{U}_A(1)$-symmetric world?

(Fierz-complete: 8 coupled channels)



Fixed-point structure and phases: Fierz-complete

(10 coupled channels)



Fixed-point structure and phases: many colors

[JB, Leonhardt, Pospiech '18]

- general structure of RG flow equations for many colors:

$$\partial_t \lambda_{(\sigma-\pi)} \sim 2\lambda_{(\sigma-\pi)} - c_{(\sigma-\pi)}\lambda^2_{(\sigma-\pi)} + \sum_{i\neq(\sigma-\pi)} c^{(i)}_{(\sigma-\pi)}\lambda_i^2 + \dots$$

$$\partial_t \lambda_j \sim 2\lambda_j + \sum_{i\neq(\sigma-\pi)} c^{(i)}_j \lambda_i^2 + \dots$$

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- Using $(\lambda_{(\sigma-\pi)}(\Lambda) \neq 0, \lambda_2(\Lambda) = 0, \dots, \lambda_{10}(\Lambda) = 0)$, the set of flow equations reduces to:

$$\partial_t \lambda_{(\sigma-\pi)} = 2\lambda_{(\sigma-\pi)} - c_{(\sigma-\pi)}\lambda^2_{(\sigma-\pi)}$$

$$\partial_t \lambda_j = 0$$

NJL trajectory: "NJL coupling lives for itself" in the limit of many colors

Fixed-point structure and phases: many colors

[JB, Leonhardt, Pospiech '18]

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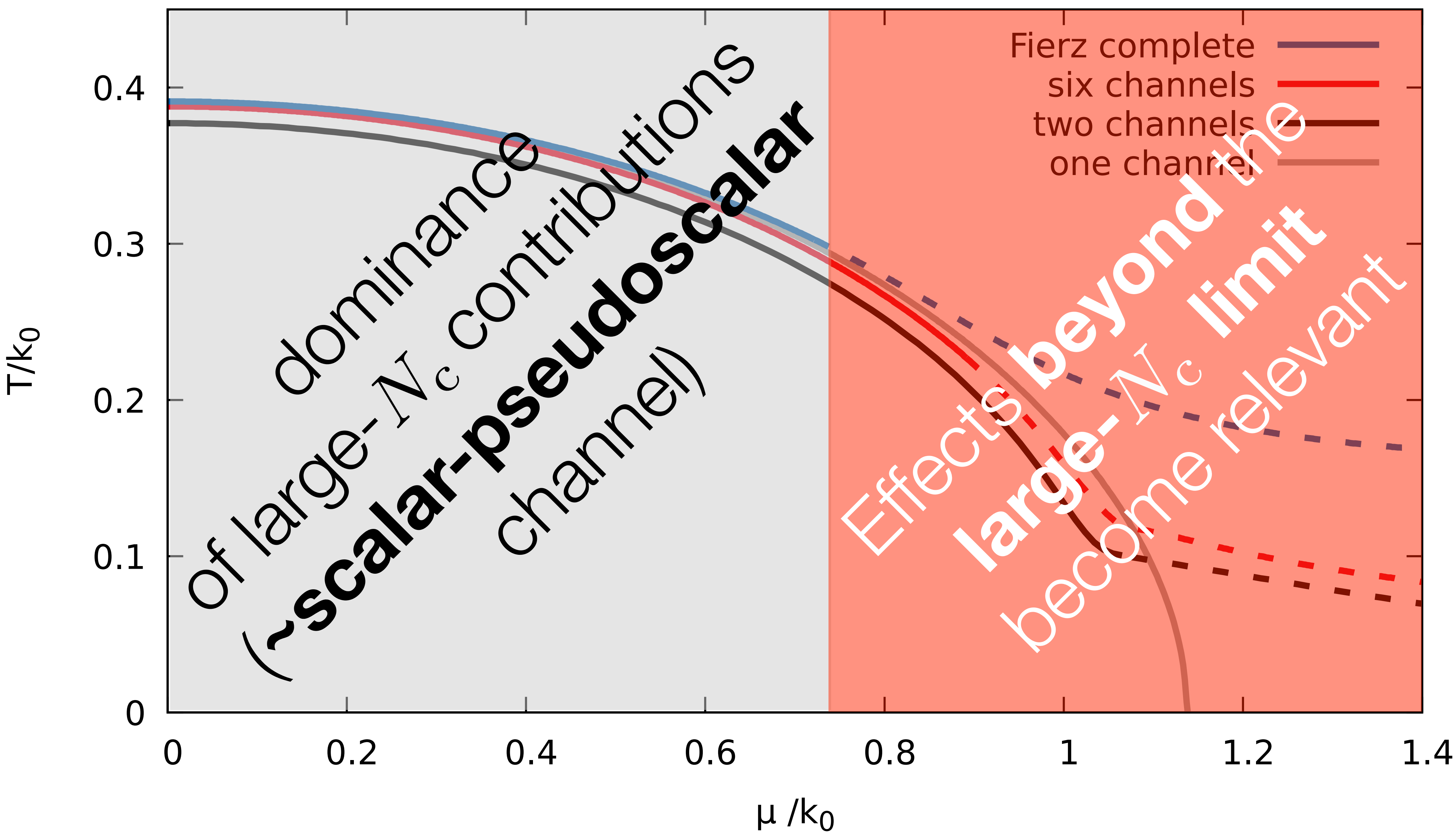
NJL fixed point (9 attractive & 1 repulsive direction) controls the dynamics

Fixed-point structure and phases: Fierz-complete

(10 coupled channels)



Conclusions and outlook

- Fierz-complete analysis of the fixed-point structure of the QCD low-energy sector
- along the phase boundary only two dominant channels are observed: scalar-pseudoscalar channel (at small chemical potential) and diquark channel (at large chemical potential), other channels are subdominant
- phase boundary can be forced to assume many shapes (“almost any”) in Fierz-**in**complete studies, even when the same scale-fixing procedure is used
- in progress: dynamical inclusion of gauge degrees of freedom in the analysis of the fixed-point structure
  [following earlier finite-temperature studies: JB, Gies ’05,’06; JB ’11]

Conclusions and outlook

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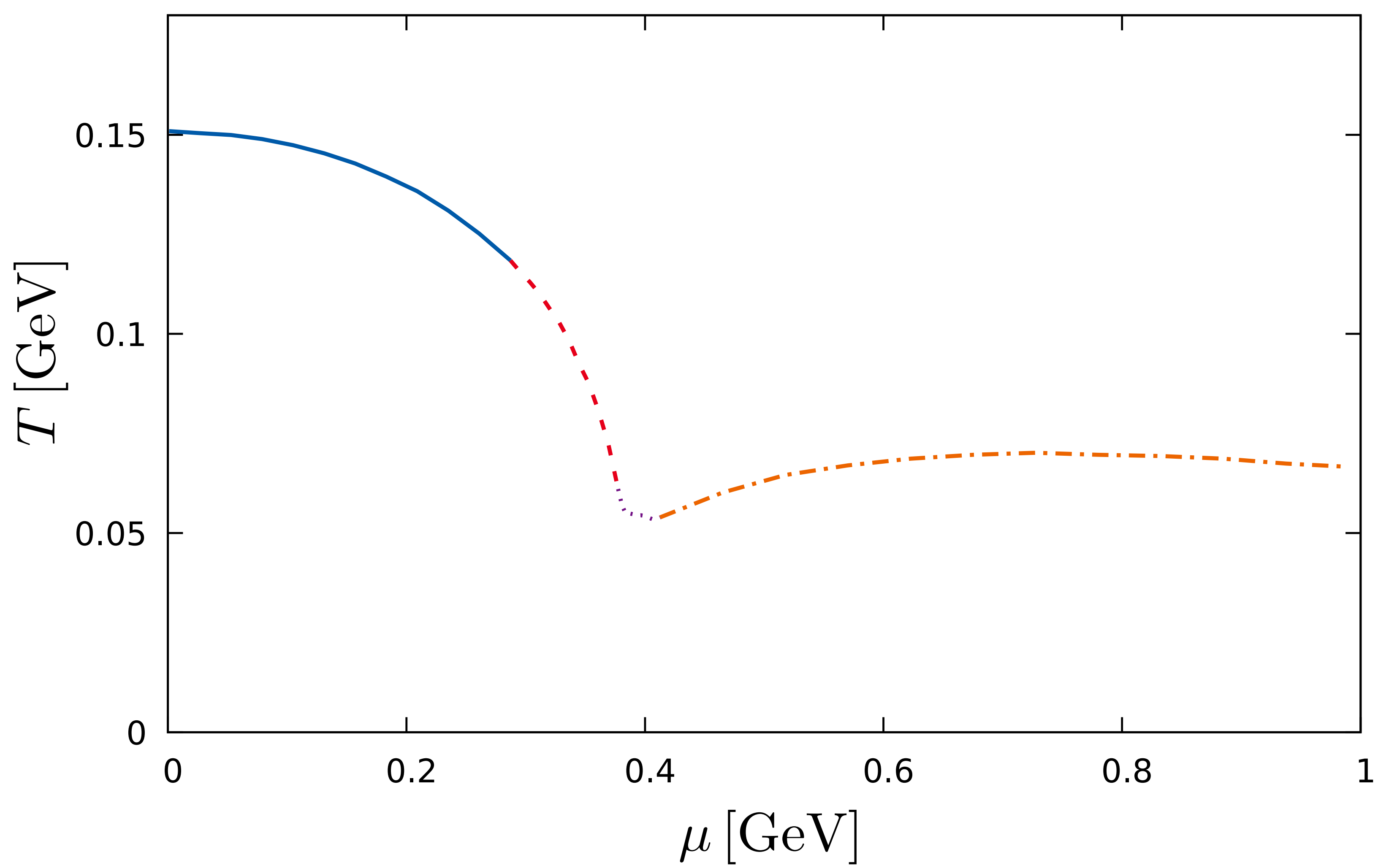
[JB, Leonhardt, Pospiech, **in prep.**]



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Conclusions and outlook

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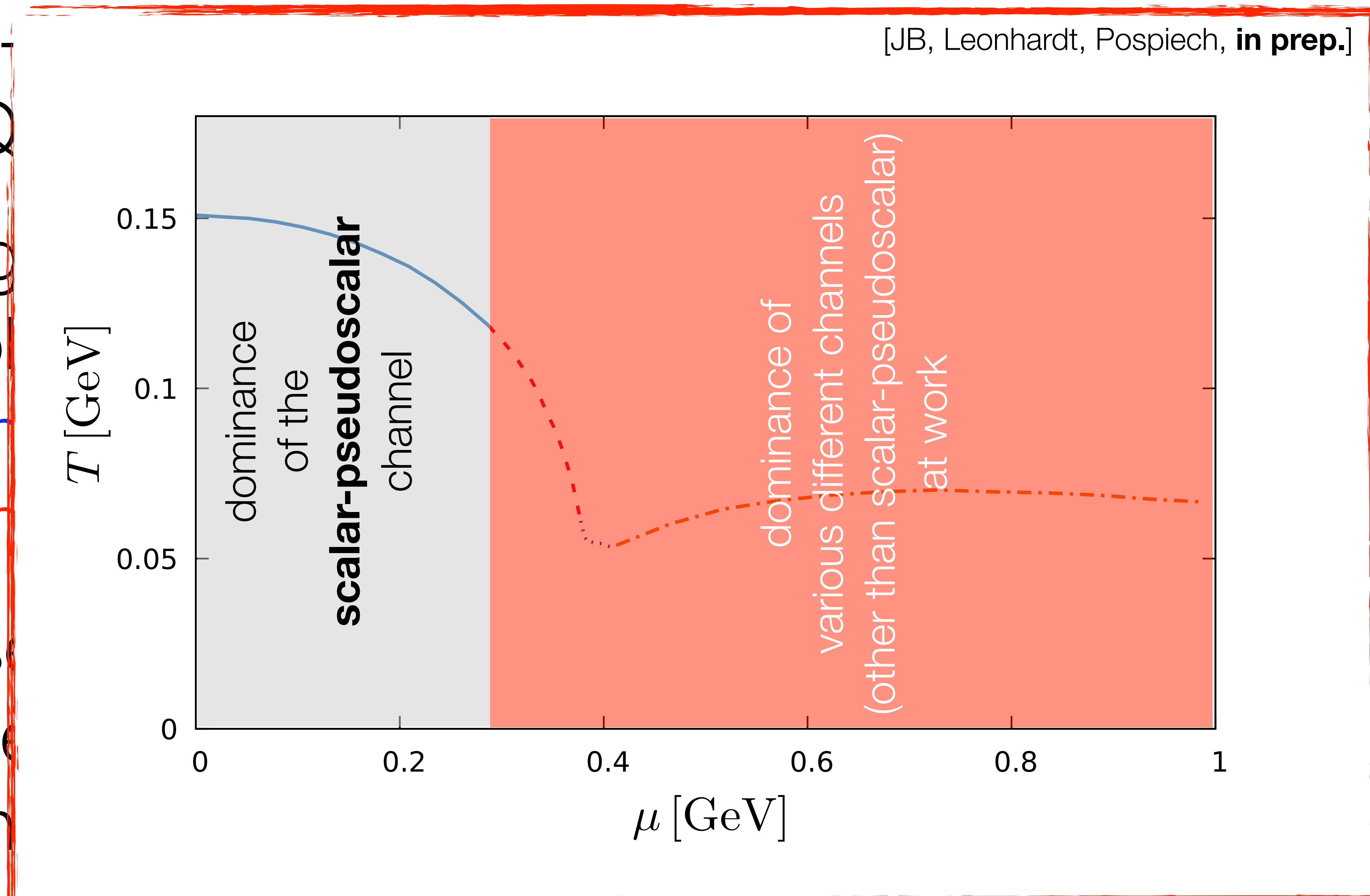

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