Towards a first-principles QCD phase diagram

Anton Konrad Cyrol

Ruprecht-Karls-Universität Heidelberg

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Idea:

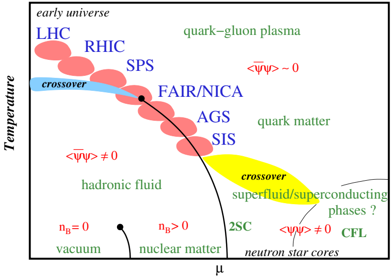
- Compute phase diagram with function methods (DSE/FRG)
- This talk: FRG

Challenges:

- Qualitative understanding
- Quantitative precision

fQCD-collaboration:

- Braun, Corell, **AKC**, Fu, Leonhardt, **Mitter**, **Pawlowski**, Pospiech, Rennecke, Schneider, **Wink**, . . .

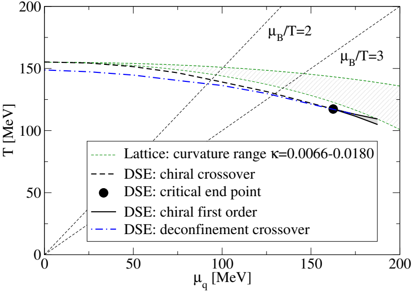


Schaefer and Wagner, Prog.Part.Nucl.Phys. 62 (2009) 381

Where are we?

DSE & model results

- Critical endpoint (CEP) found in (quark meson) models
- Also found in DSE calculations:



Fischer, Luecker, Welzbacher
Phys. Rev. D 90 (2014) 034022

cf. also Braun, Contant, Eichmann, Fister, Huber, Leonhardt, Pawlowski, Pospiech, Rennecke, Schaefer, Smekal, Williams, . . .

Outline of this talk:

1. FRG & QCD introduction
   basics, short & quick
2. Yang-Mills → remember my talk from Nov. 16
   AKC, Fister, Mitter, Pawlowski, Strodthoff; 1605.01856
3. Reconstructing the gluon
   AKC, Pawlowski, Rothkopf, Wink; 1804.00945
4. Two-Flavor QCD
   AKC, Mitter, Pawlowski, Strodthoff; 1706.06326
5. Yang-Mills at finite temperature
   AKC, Mitter, Pawlowski, Strodthoff; 1708.03482

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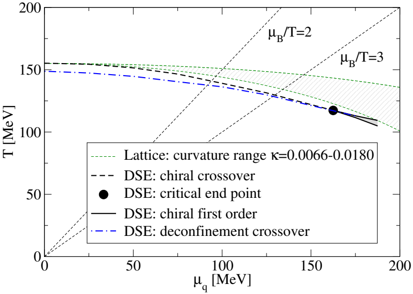

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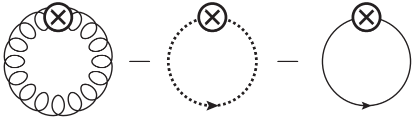
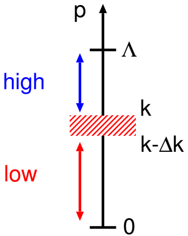
QCD from the functional renormaliztion group

- Initial condition $\Gamma_\Lambda[\Phi] = S[\Phi]$

$$\frac{\mathrm{d}\Gamma_k}{\mathrm{d}t} = \frac{1}{2}$$



- Effective action $\Gamma[\Phi] = \lim_{k\to 0} \Gamma_k[\Phi]$



- Exact equation
- ∂_t: integration of momentum shells controlled by regulator
- Full field-dependent equation with $G = (\Gamma^{(2)}[\Phi] + R)^{-1}$ on rhs

- Only fundamental QCD parameters needed:
  - $\alpha_S(\mu = \mathcal{O}(10)\text{ GeV})$
  - $m_q(\mu = \mathcal{O}(10)\text{ GeV})$

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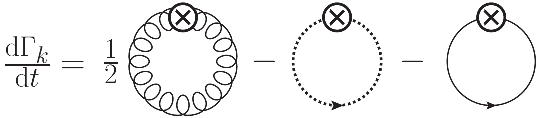

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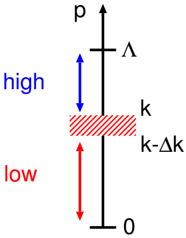

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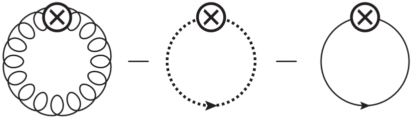

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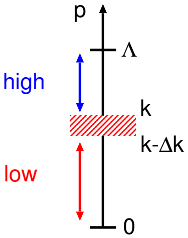

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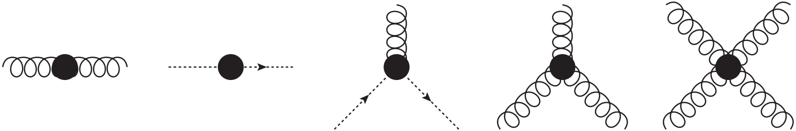
Vertex expansion

- Approximation necessary – vertex expansion:

$$\Gamma[\Phi] = \sum_n \int_{p_1,\dots,p_{n-1}} \Gamma^{(n)}_{\Phi_1\cdots\Phi_n}(p_1,\dots,p_{n-1})\Phi^1(p_1)\cdots\Phi^n(-p_1-\cdots-p_{n-1})$$

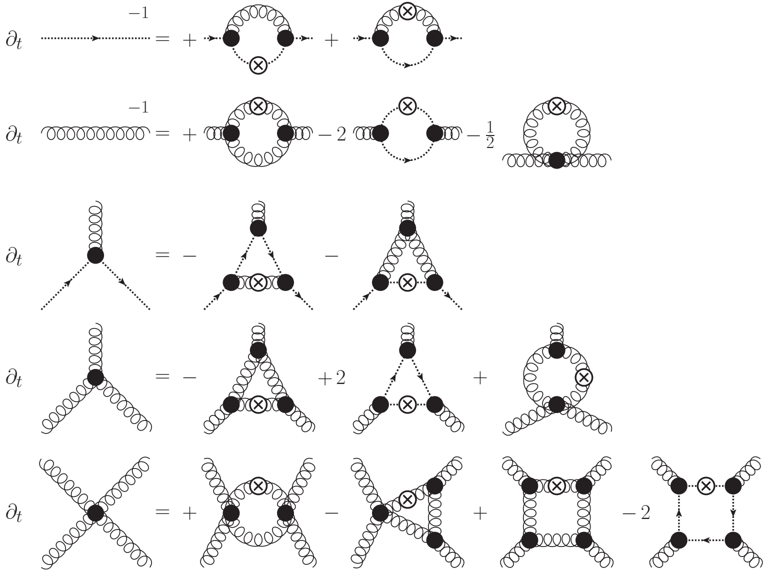
- Wanted: "apparent convergence" of $\Gamma[\Phi] = \lim_{k\to 0}\Gamma_k[\Phi]$
- Current state-of-the-start truncation in pure Yang-Mills:



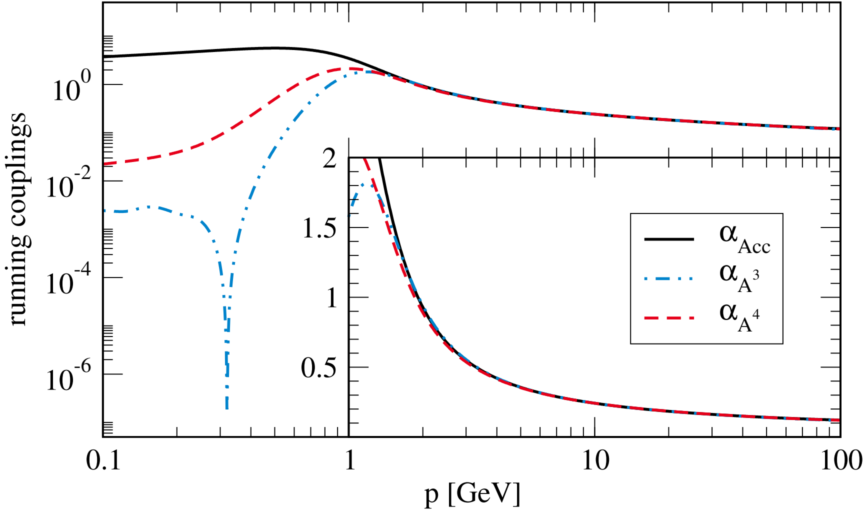
$\Gamma^{(2)}_{AA}(p)$ $\Gamma^{(2)}_{\bar{c}c}(p)$ $\Gamma^{(3)}_{A\bar{c}c}(\bar{p})$ $\Gamma^{(3)}_{A^3}(\bar{p})$ classical tensor $\Gamma^{(4)}_{A^4}(\bar{p})$ classical tensor

- Functional derivatives of $\Gamma_k[\Phi]$ with respect to fields yield equations

Truncation – closed set of equations

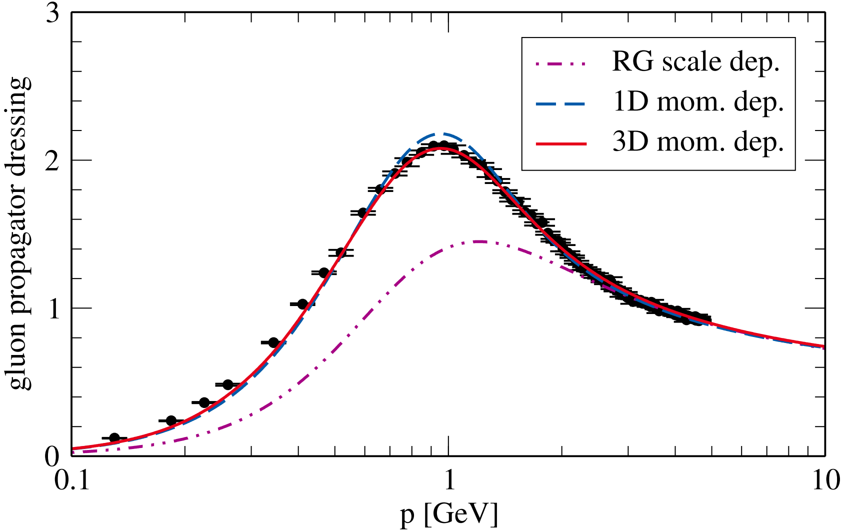


Running couplings (scaling solution)



AKC, Fister, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 94 (2016) 054005

Gluon propagator dressing



AKC, Fister, Mitter, Pawlowski, Strodthoff, 1605.01856; **Lattice**: Sternbeck et al., hep-lat/0610053

Reconstructing spectral functions [AKC, Pawlowski, Rothkopf, Wink; 1804.00945]

Källén–Lehmann representation

$$G(p_0) = \int_0^\infty \frac{\mathrm{d}\lambda}{\pi} \frac{\lambda}{\lambda^2 + p_0^2}\, \rho(\lambda)$$

Analytic Continuation

$$\rho(\omega) = 2\,\mathrm{Im}\; G(-\mathrm{i}\,(\omega + \mathrm{i}0^+))$$

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Obtain a **simple, very general relation**:

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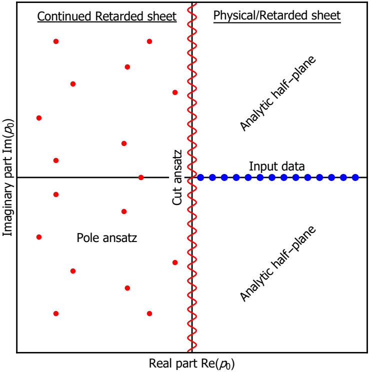
Analytic Structure



The gluon spectral function

[AKC, Pawlowski, Rothkopf, Wink; 1804.00945]

- Exploit analytic structure in fit ansatz for the propagator
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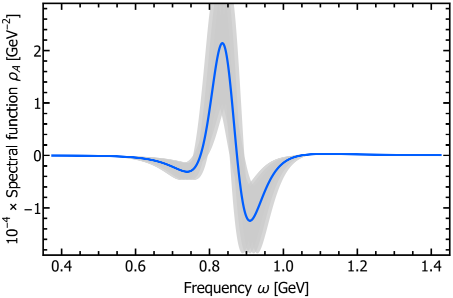
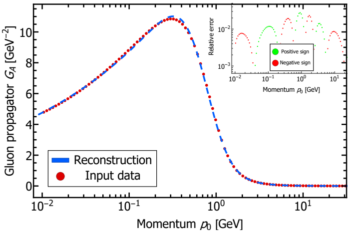
Our result for the gluon spectral function



Reconstructed propagator



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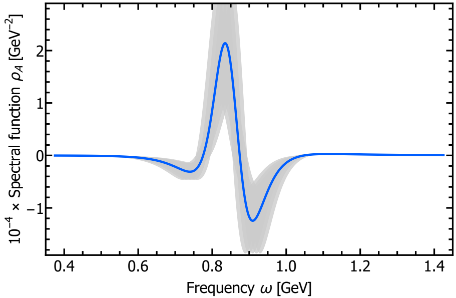

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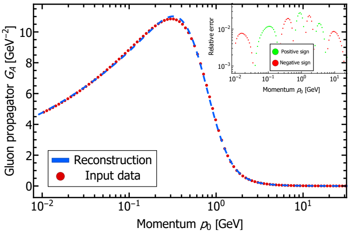
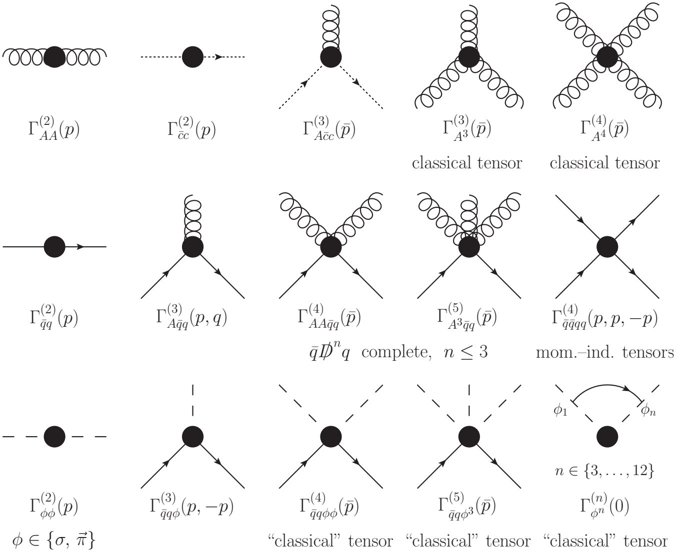


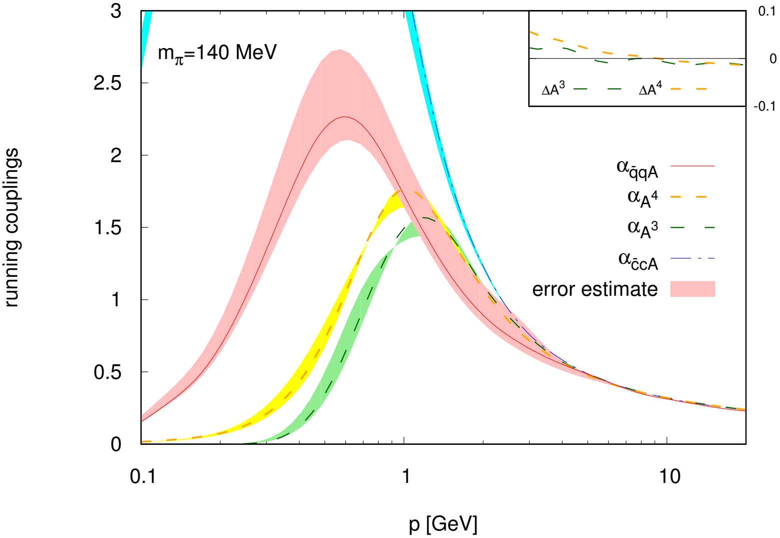
cf. talk by **Nicolas Wink** next week

Unquenched two-flavor QCD [AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97, 2018]

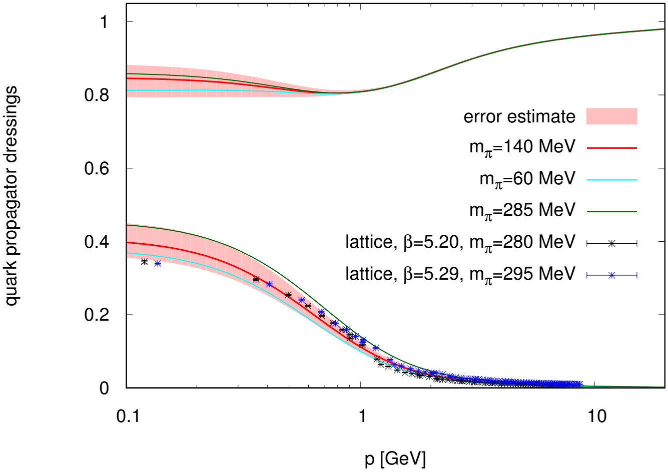


Running couplings in QCD

[AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97, 2018]

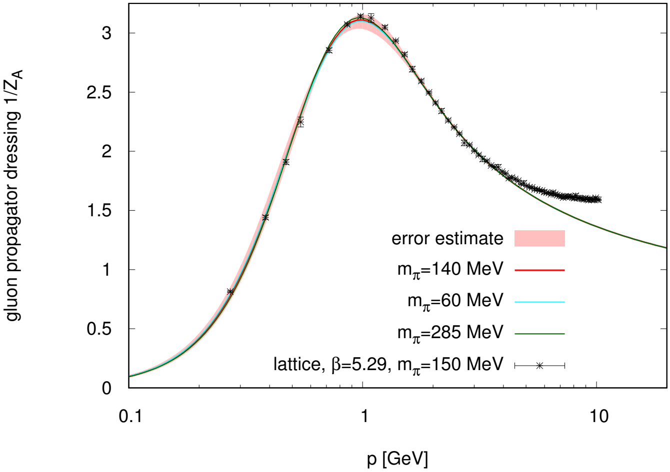


Unquenched quark propagator [AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97, 2018]



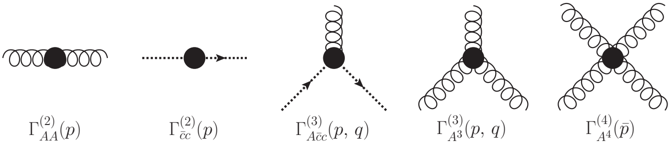
lattice data: Orlando Oliveira, Kızılersu, Silva, Skullerud, Sternbeck, Williams, arXiv:1605.09632 [hep-lat]

Unquenched gluon propagator [AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97, 2018]



lattice data: Sternbeck, Maltman, Muller-Preussker, von Smekal, arXiv:1212.2039 [hep-lat]

Introducing finite temperature: $\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \to T \sum_{\omega_n} \int \frac{\mathrm{d}^3 p}{(2\pi)^3}$



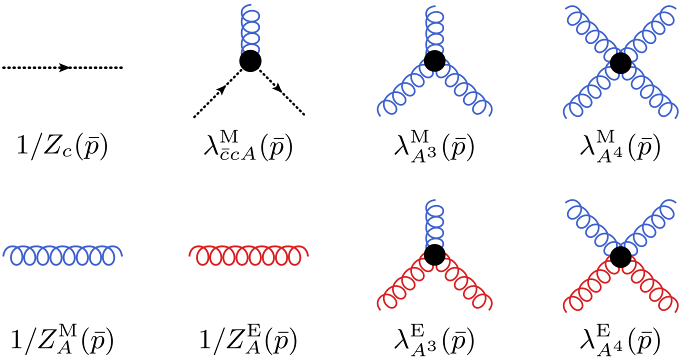
Splitting of magnetic and electric componentes necessary!

Introducing finite temperature: $\int \frac{d^4 p}{(2\pi)^4} \to T \sum_{\omega_n} \int \frac{d^3 p}{(2\pi)^3}$

$P^{L}_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2}$ $\quad$ $P^{T}_{\mu\nu}(p) = \delta_{\mu\nu} - P^{L}_{\mu\nu}(p)$

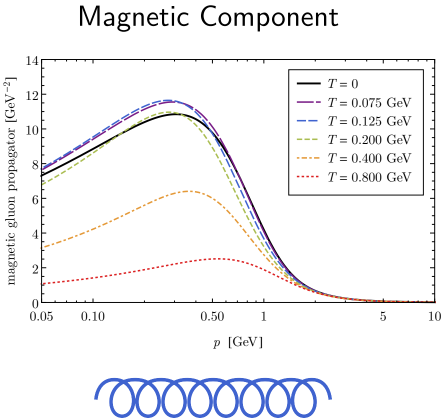
FormTracer can do that!

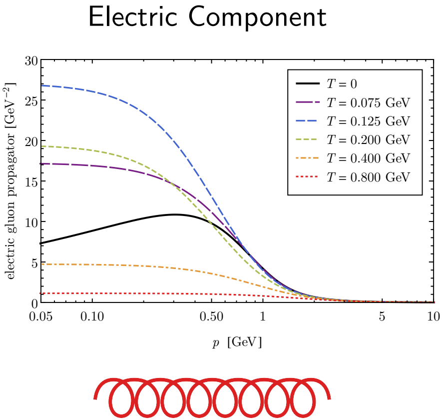
$\boldsymbol{P^{M}_{\mu\nu}(p) = (1 - \delta_{0\mu})(1 - \delta_{0\nu}) \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{\vec{p}^2}\right)}$ $\quad$ $\boldsymbol{P^{E}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - P^{M}_{\mu\nu}(p)}$



Other zeroth mode classical tensor structures are degenerate.

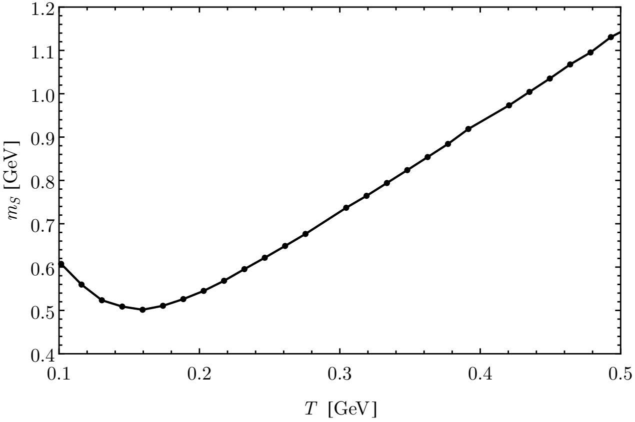
Gluon propagator at finite temperature





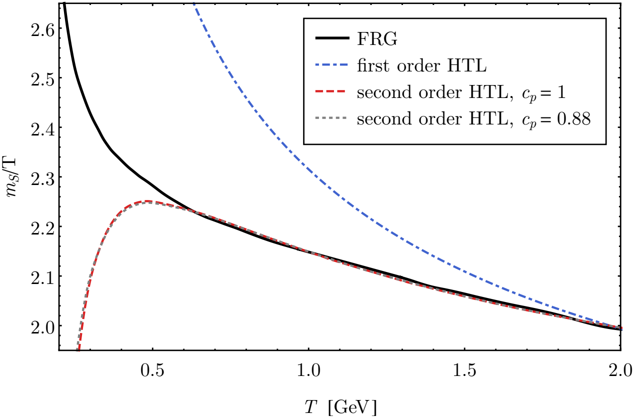
AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97 (2018) 054015

Screening/Debye mass at low temperatures



$$G^{\mathrm{E}}_T(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\, p}{2\pi}\; G^{\mathrm{E}}_T(p)\; e^{\mathrm{i}\, p\, x} \qquad \lim_{x\to\infty} G^{\mathrm{E}}_T(x) = c_e \,\exp\left(-\mathbf{m_s}\, x\right)$$

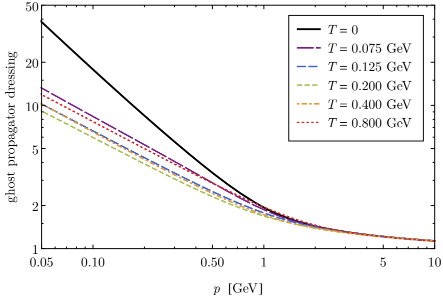
Screening/Debye mass compared to perturbation theory



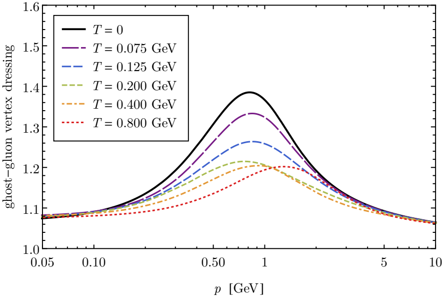
$$m_D^0 = \sqrt{\frac{N}{3}} gT; \qquad m_D = m_D^0 + \left(c_D + \frac{N}{4\pi} \ln\left(\frac{m_D^0}{g^2 T}\right)\right) g^2 T + \mathcal{O}(g^3 T)$$

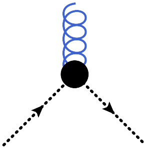
Ghost dressings

Ghost Propagator



Ghost-gluon Vertex

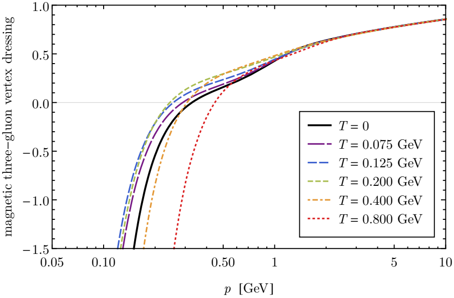


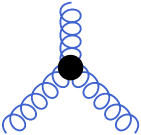


AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97 (2018) 054015

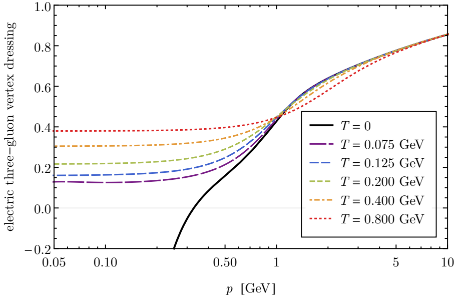
Three-gluon vertex dressings

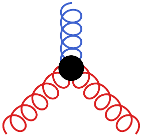
Three magnetic legs





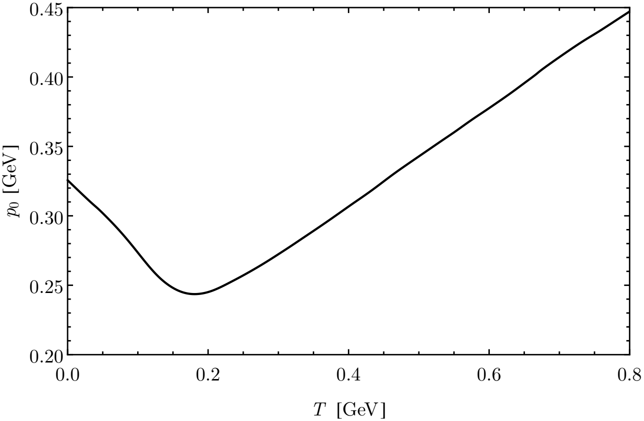
One magnetic and two electric legs





AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97 (2018) 054015

Three-gluon Vertex Zero Crossing

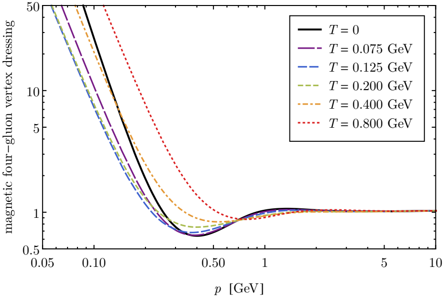


Ghost dominance is weakened at finite temperature, but the ghost still dominate the deep infrared, as expected from analytical arguments!

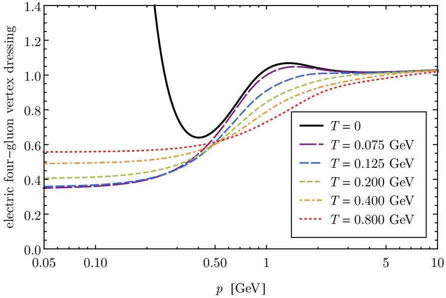
AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97 (2018) 054015

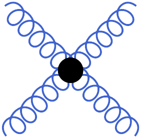
Four-gluon vertex dressings

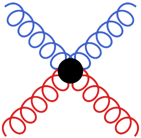
Four magnetic legs



Two magnetic and two electric legs



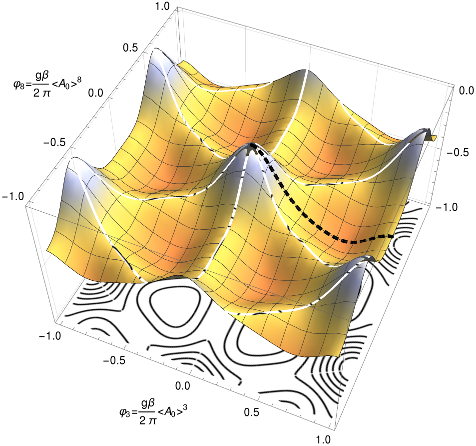




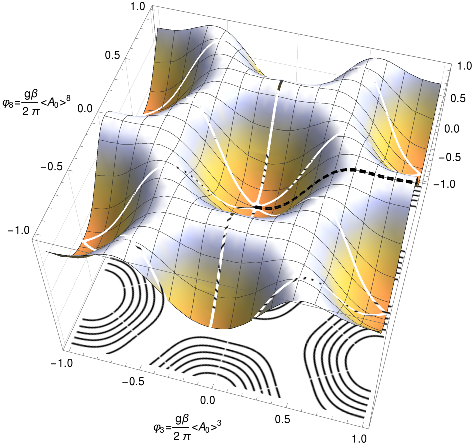
AKC, Mitter, Pawlowski, Strodthoff, Phys. Rev. D 97 (2018) 054015

Background field $\langle A_0 \rangle$ and glue potential $V(\phi_3, \phi_8)$

Confined Phase



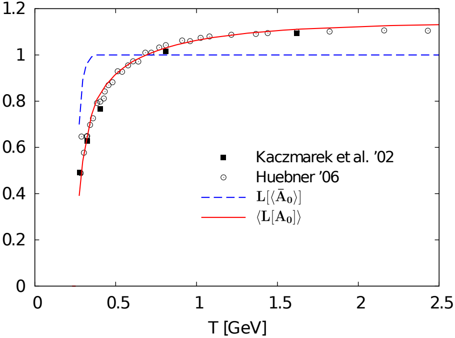
Deconfined Phase



Images stolen from Herbst, Luecker, Pawlowski, arXiv:1510.03830 [hep-ph]

Order Parameters

Polyakov loop



Herbst, Luecker, Pawlowski,
arXiv:1510.03830 [hep-ph]

Background Propagators:

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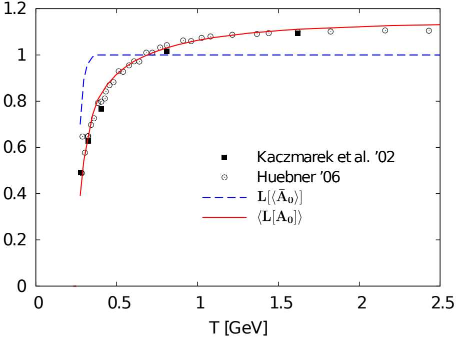
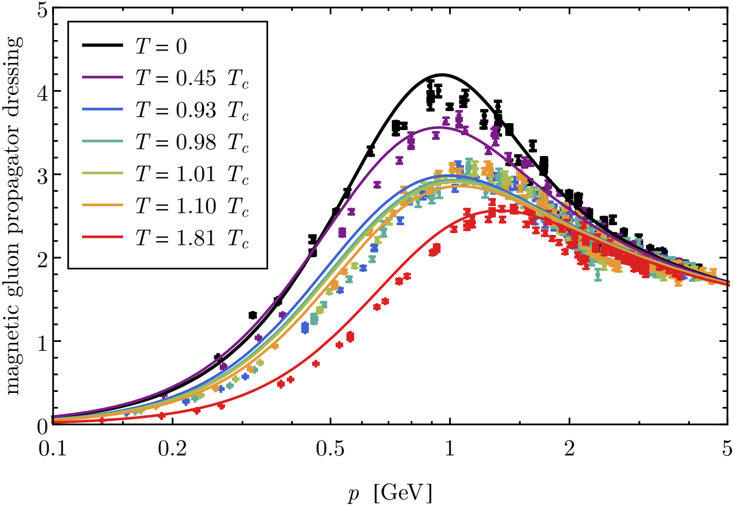


Herbst, Luecker, Pawlowski, arXiv:1510.03830 [hep-ph]

Background Propagators:

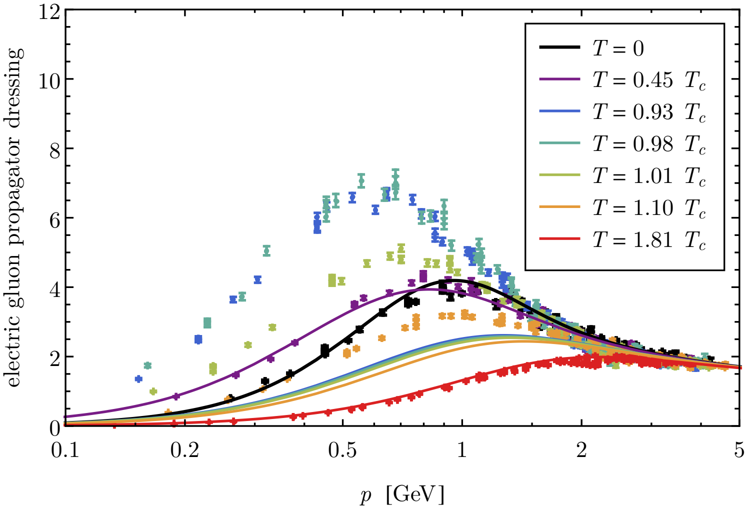
- distinguished color direction
- work in Cartan-Weyl basis
- . . .
- . . . many more components
- . . .
- . . . non-zero Matsubaras wanted
- . . .
- . . . longer equations
- . . .
- work in progress ... stay tuned!

Magnetic gluon propagator dressing



Lattice: Maas, Pawlowski, Smekal, Spielmann, 2011

Electric gluon propagator dressing



Lattice: Maas, Pawlowski, Smekal, Spielmann, 2011

Conclusion

- Gluon spectral function obtained from Euclidean propagator!
- FRG first principal approach to QCD
- Much progress, but still not enough!
- Promising results for $T > 0$ and the unquenched system.

Outlook

- Background fields $\Rightarrow$ this year
- Transport coefficients $\Rightarrow$ this year
- YM trace anomaly, pressure & EoS $\Rightarrow$ this year?
- Critical endpoint from the FRG $\Rightarrow$ as soon as possible

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Thank you for your attention!

FormTracer – Mathematica tracing package using FORM

- Mathematica: very powerful, flexible and convenient
- FORM: very fast and efficient

FormTracer uses FORM while it keeps the usability of Mathematica:

- Lorentz/Dirac traces in arbitrary dimensions
- Arbitrary number of group product spaces
- Intuitive, easy-to-use and highly customizable Mathematica frontend
- Support for finite temperature/density applications
- Support for FORM's optimization algorithm
- Convenient installation and update procedure within Mathematica:

Preprint: AKC, Mitter, Strodthoff; arXiv:1610.09331 [hep-ph]

Open source: `https://github.com/FormTracer/FormTracer`

FormTracer – installation and usage

FormTracer.nb - Wolfram Mathematica 11.0

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Installing

```
Import["https://raw.githubusercontent.com/FormTracer/FormTracer/master/src/FormTracerInstaller.m"]
```

Tracing

Space-Time

Define syntax for space-time

```
DefineLorentzTensors[δ[μ, ν] (*Kronecker delta*), vec[p, μ] (*vector*), p.q (*inner product*)];
```

Take traces:

```
FormTrace[vec[p + 2 r, μ] δ[μ, ν] vec[s, ν]]
FormTrace[δ[α, ν] (δ[ν, ρ] + δ[ν, ρ] δ[σ, σ]) δ[ρ, α]]
FormTrace[δ[1, ν] vec[s, ν]]
```

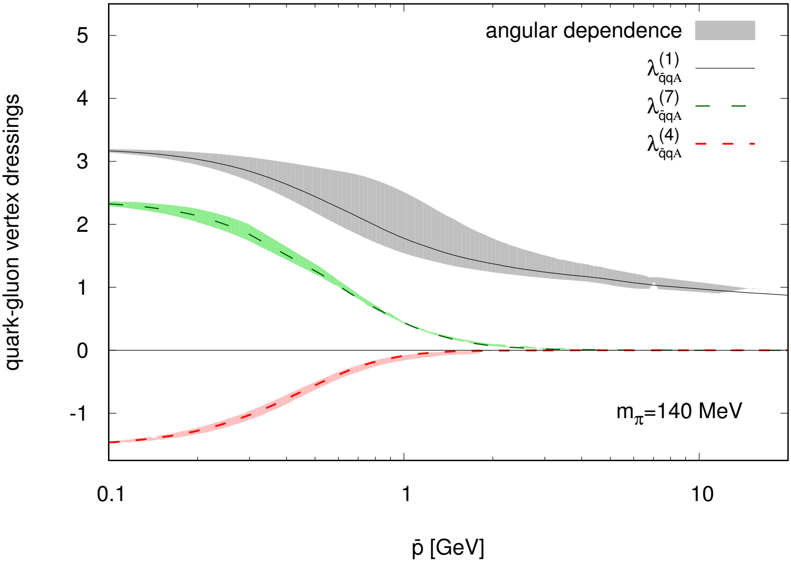
```
s.(p + 2 r)
```

```
20
```

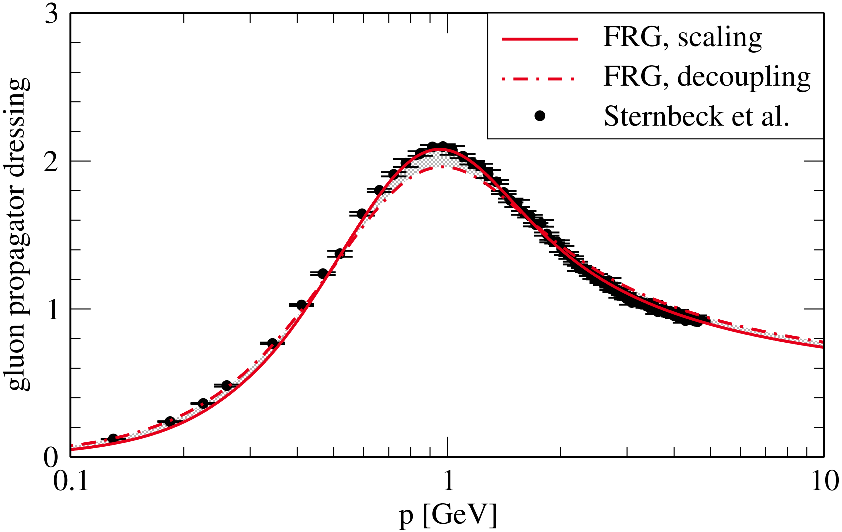
```
vec[s, 1]
```

AKC, Mitter, Strodthoff; arXiv:1610.09331 [hep-ph]

Unquenched quark-gluon vertex [AKC, Mitter, Pawlowski, Strodthoff, in preparation]

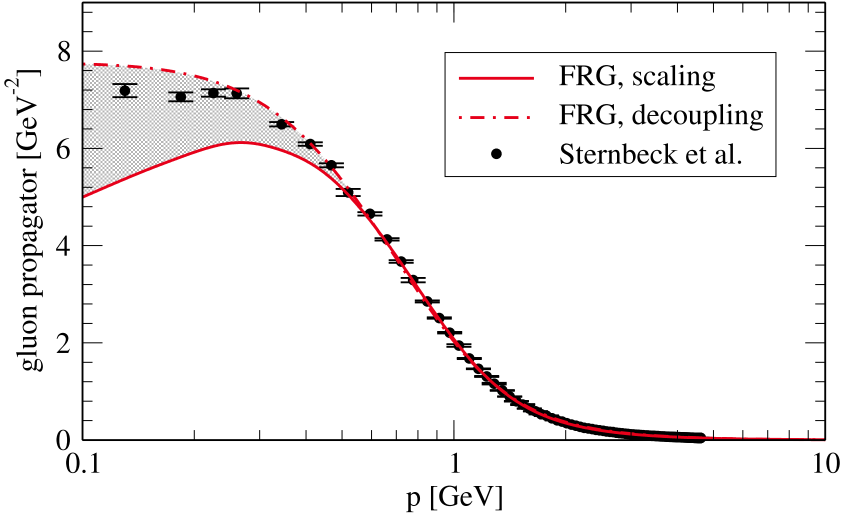


Gluon propagator dressing



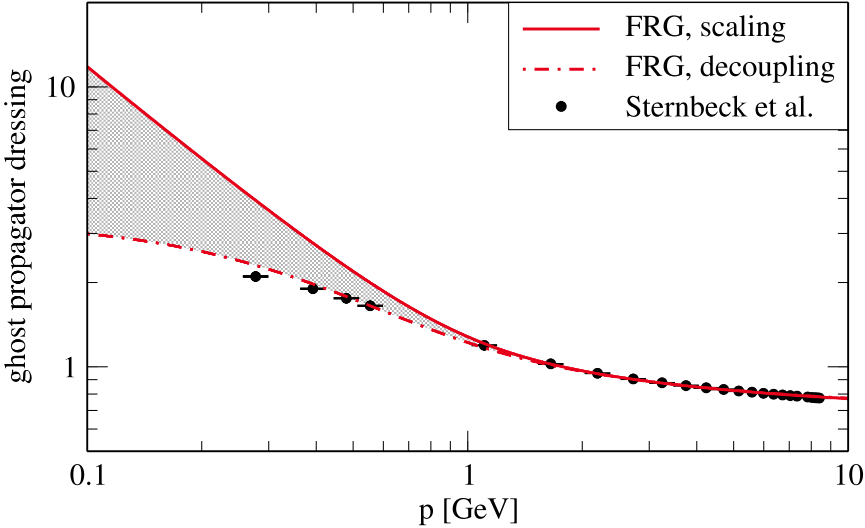
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; **Lattice**: Sternbeck et al. 2006

Gluon propagator



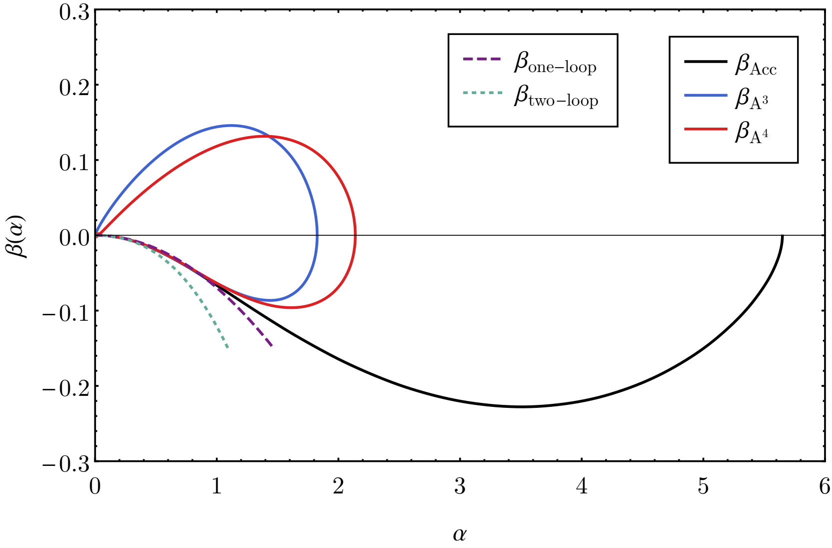
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; **Lattice**: Sternbeck et al. 2006

Ghost propagator dressing

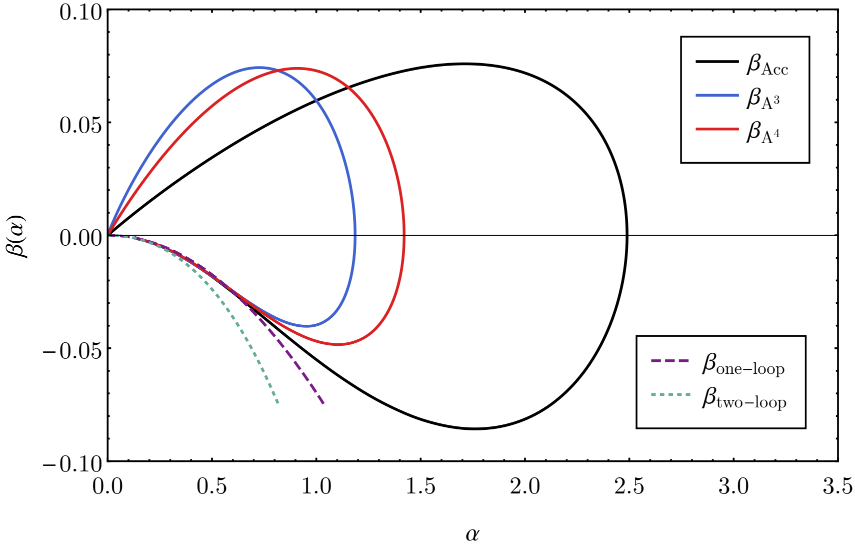


AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

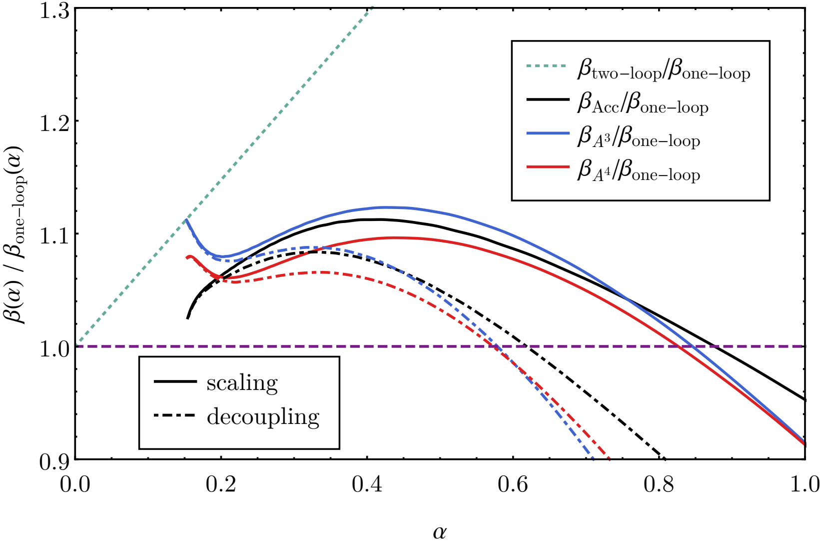
$\beta = \frac{\mu^2}{4\pi}\frac{\mathrm{d}\,\alpha}{\mathrm{d}\,\mu^2}$ Functions – Scaling



$\beta = \frac{\mu^2}{4\pi}\frac{\mathrm{d}\,\alpha}{\mathrm{d}\,\mu^2}$ Functions – Decoupling

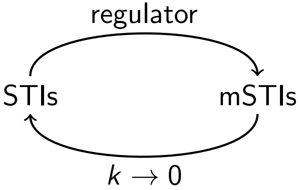


$\beta = \frac{\mu^2}{4\pi}\frac{\mathrm{d}\,\alpha}{\mathrm{d}\,\mu^2}$ Functions at Small Couplings



Regulator breaks BRST symmetry

- Breaking BRST symmetry $\to$ modified STIs
- mSTIs reduce to STIs at $k = 0$
- $\Longrightarrow$ solve mSTIs to get initial action at $k = \Lambda$
- More practical solution: choose $\Gamma_\Lambda \approx S$ such that STIs are fulfilled $k = 0$



$$\alpha_{A\bar{c}c}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z^2_{A\bar{c}c}(p)}{Z_A(p)\, Z_c^2(p)}$$

$$\alpha_{A^3}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z^2_{A^3}(p)}{Z_A^3(p)}$$

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Select

$$Z^{k=\Lambda}_{A\bar{c}c}(p) = \text{const.}$$
$$Z^{k=\Lambda}_{A^3}(p) = \text{const.}$$
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such that

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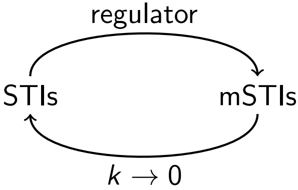

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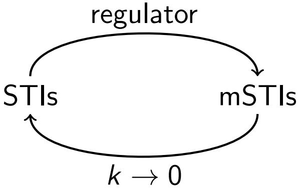

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Gluon mass gap

Scaling solution

$$\lim_{p\to 0} Z_c(p^2) \propto (p^2)^{\kappa}$$

$$\lim_{p\to 0} Z_A(p^2) \propto (p^2)^{-2\,\kappa}$$

Decoupling solution

$$\lim_{p\to 0} Z_c(p^2) \propto 1$$

$$\lim_{p\to 0} Z_A(p^2) \propto (p^2)^{-1}$$

- **Landau Gauge** gluon STI requires longitudinally mass term to vanish:

$$p_\mu \left([\Gamma^{(2)}_{AA,\mathrm{L}}]^{ab}_{\mu\nu}(p) - [S^{(2)}_{AA,\mathrm{L}}]^{ab}_{\mu\nu}(p) \right) = 0$$

- Splitting between longitudinal and transverse mass term necessary
- Splitting occures "naturally" for scaling solution
- Decoupling solution requires irregular vertices, e.g. a pole in the longitudinal sector
- Unphysical gluon mass parameter present at $k = \Lambda$, $\Longrightarrow$ can be uniquely determined

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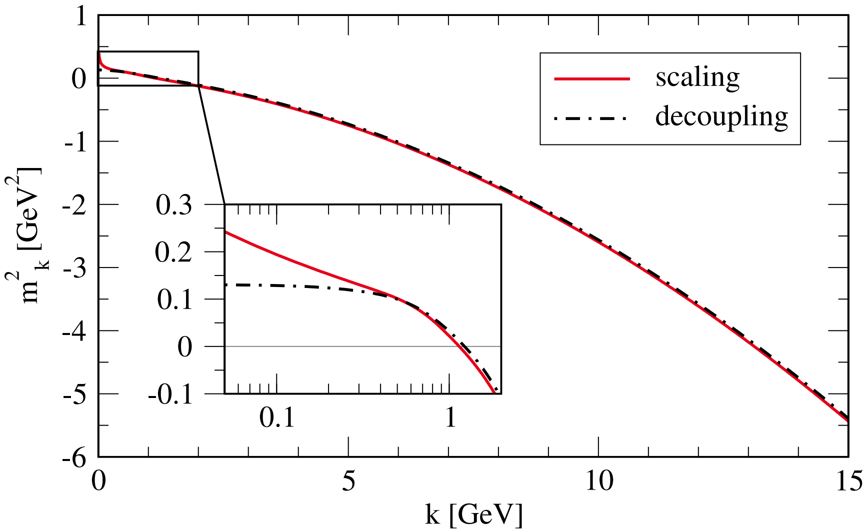
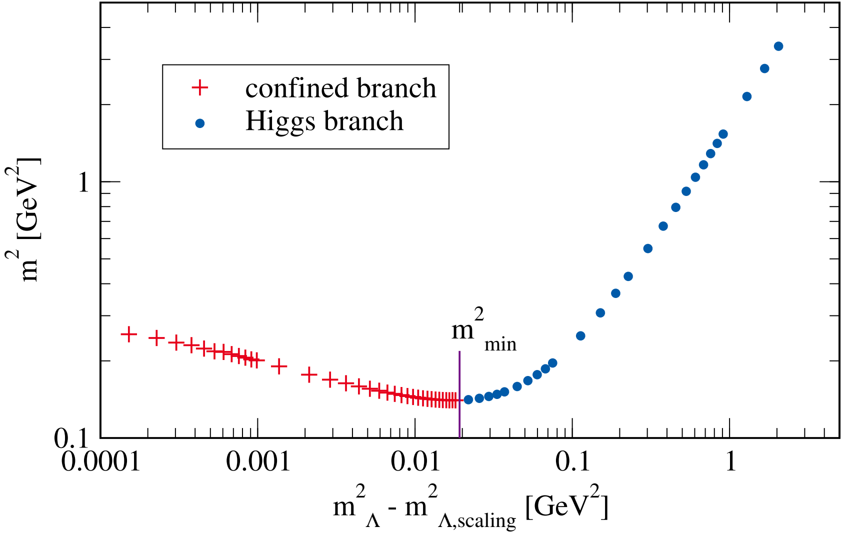
Running of the gluon mass parameter



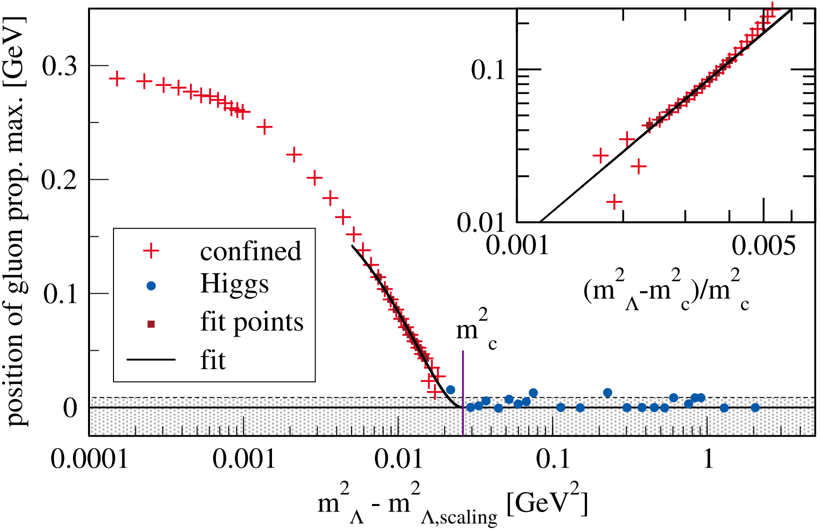
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Dynamical mass generation



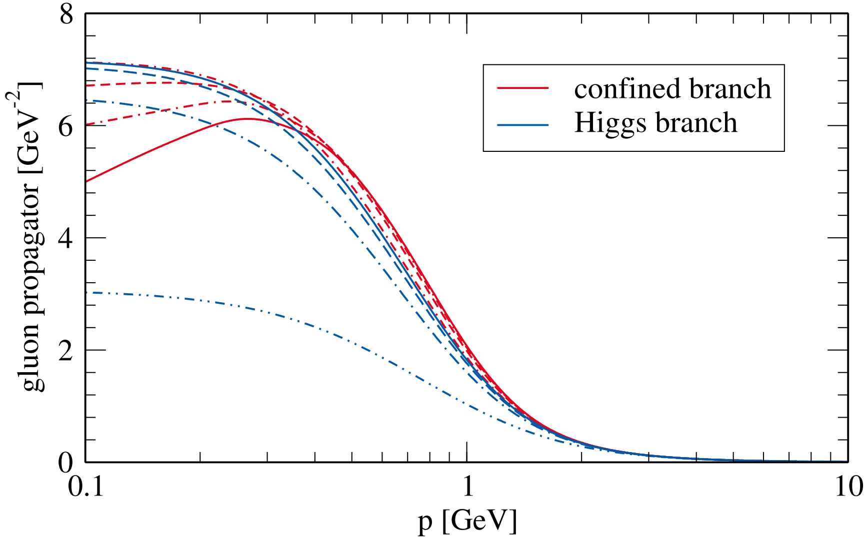
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Gluon propagator maximum over UV mass parameter



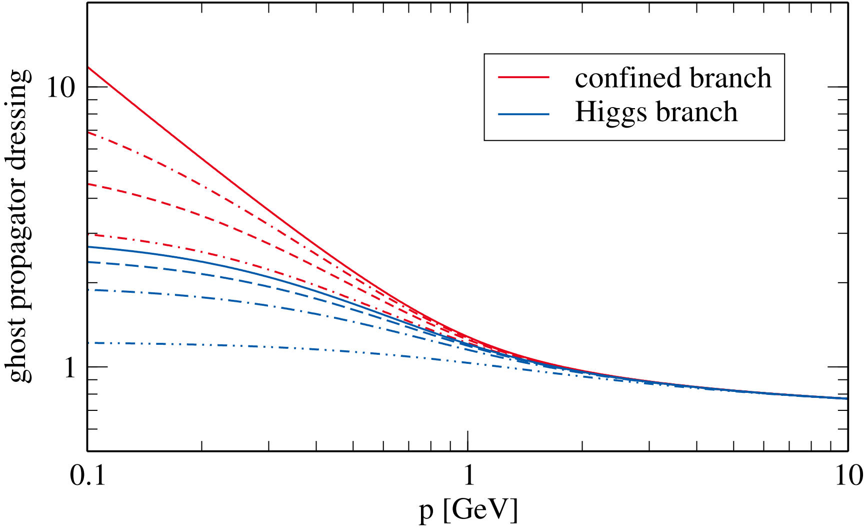
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Gluon propagator



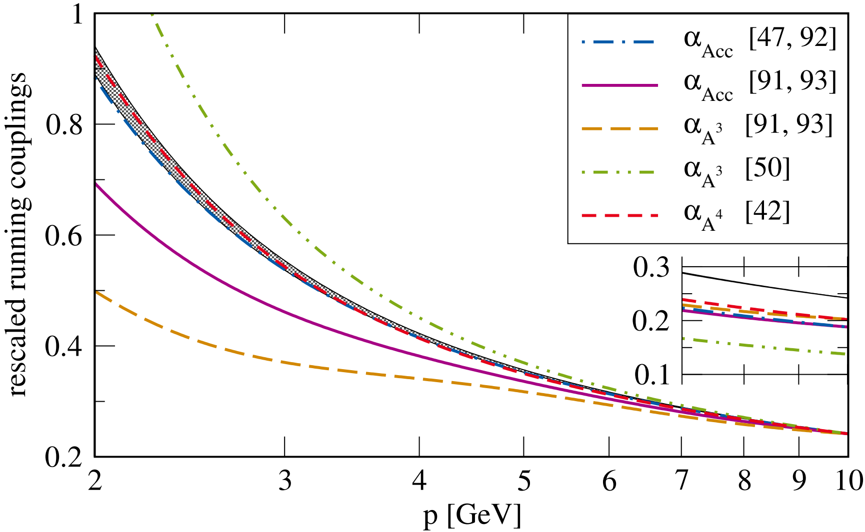
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Ghost propagator dressing



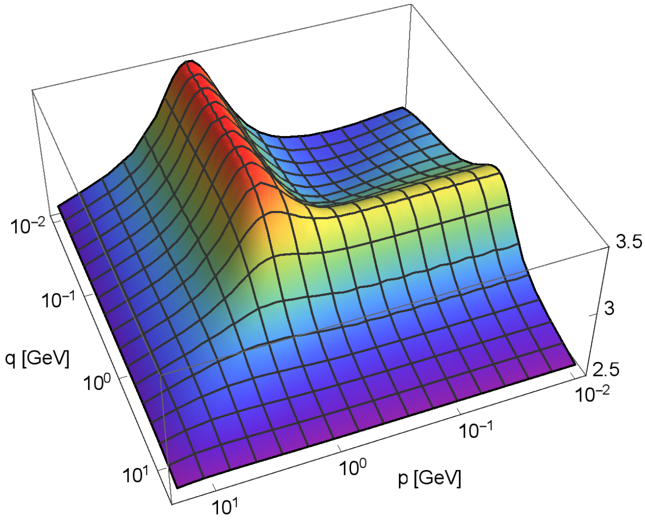
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Running couplings in comparison with DSE results



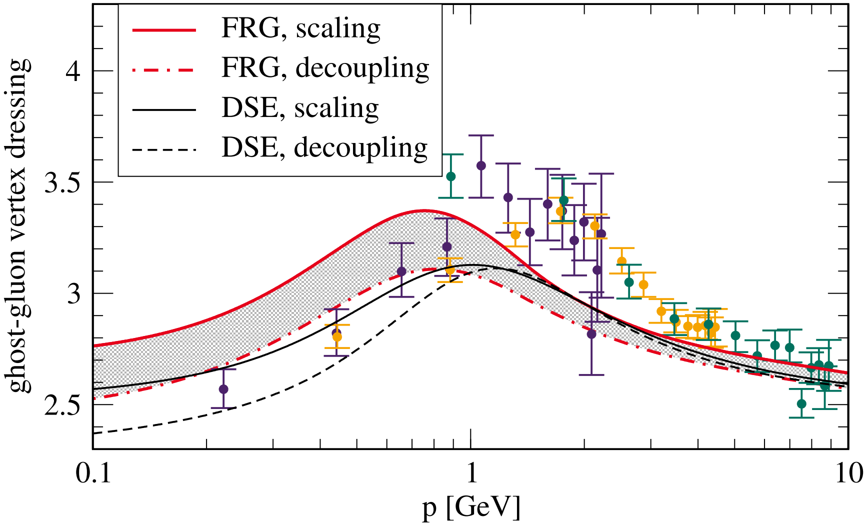
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Momentum dependence of the ghost-gluon vertex



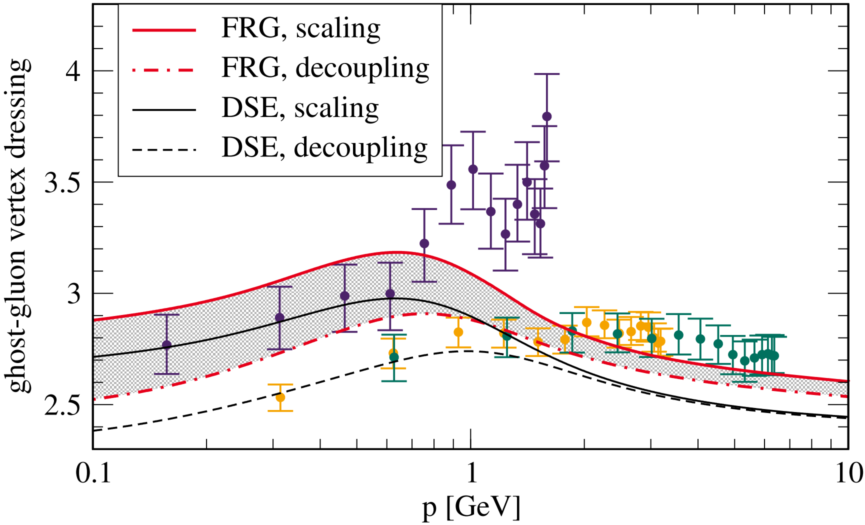
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Ghost-gluon vertex at the symmetric point



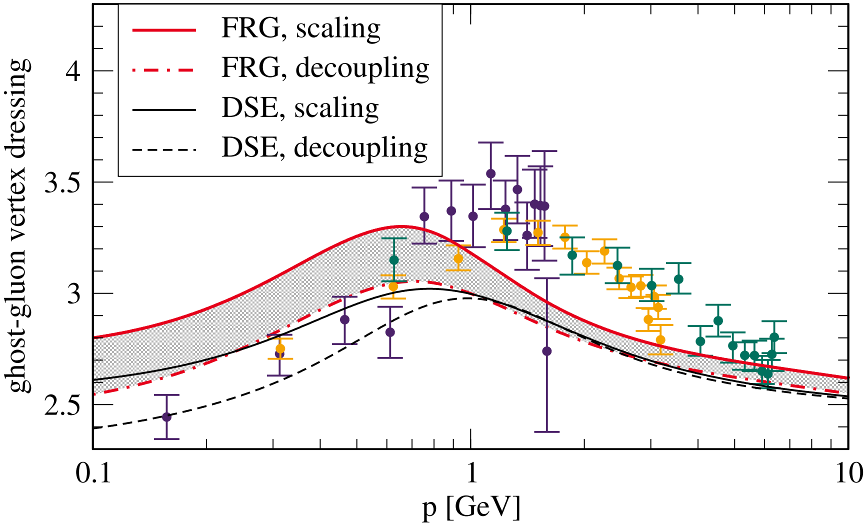
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Ghost-gluon vertex with vanishing gluon momentum



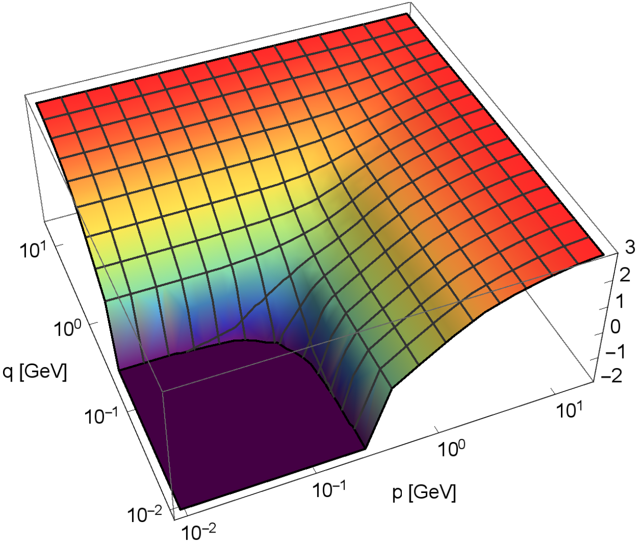
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Ghost-gluon vertex with orthogonal momenta



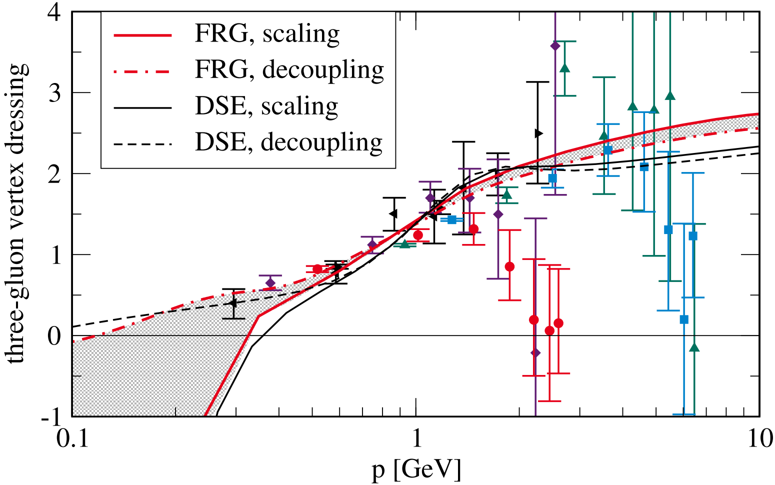
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Momentum dependence of the three-gluon vertex



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

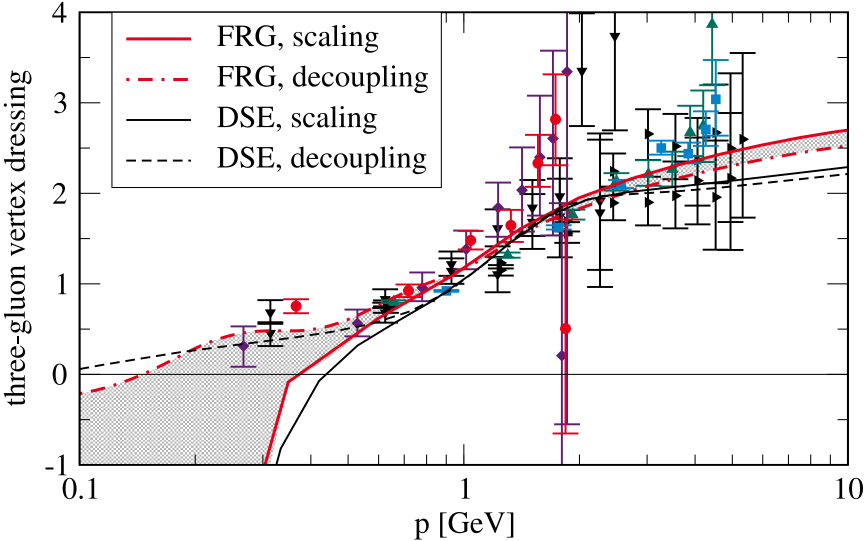
Three-gluon vertex dressing (symmetric point)



- Zero crossing between 0.1 GeV to 0.33 GeV

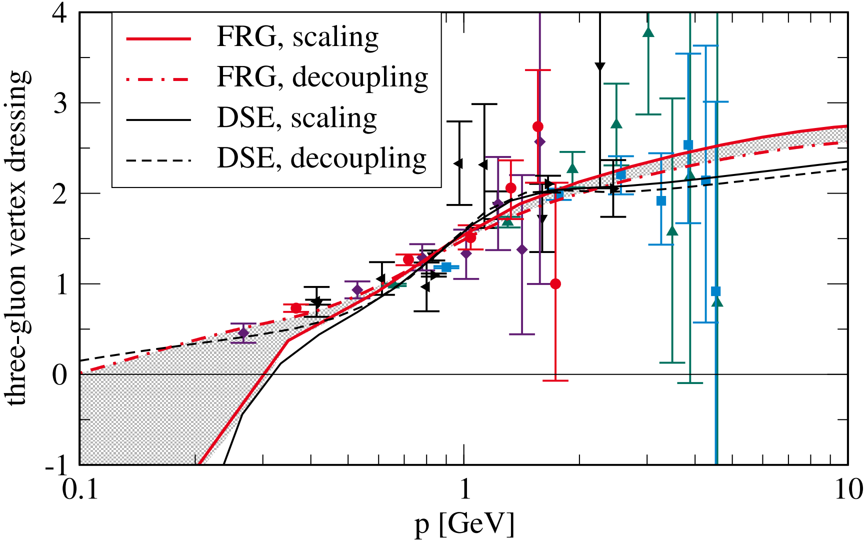
DSE: Blum, Huber, Mitter, Smekal, 2014; Lattice: Cucchieri, Maas, Mendes, 2008

Three-gluon vertex with vanishing gluon momentum



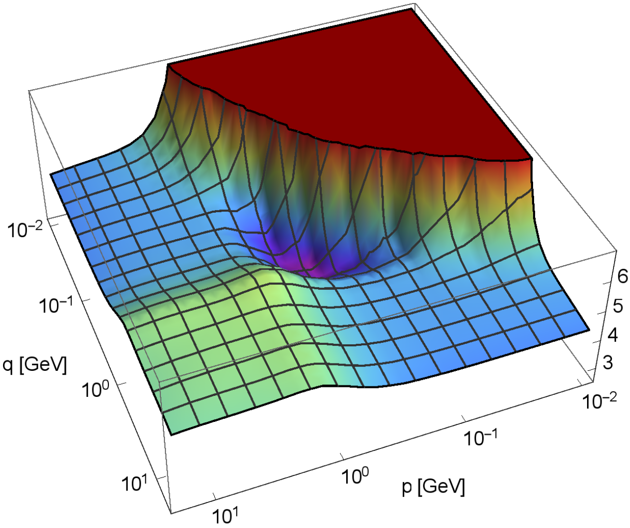
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Three-gluon vertex with orthogonal momenta



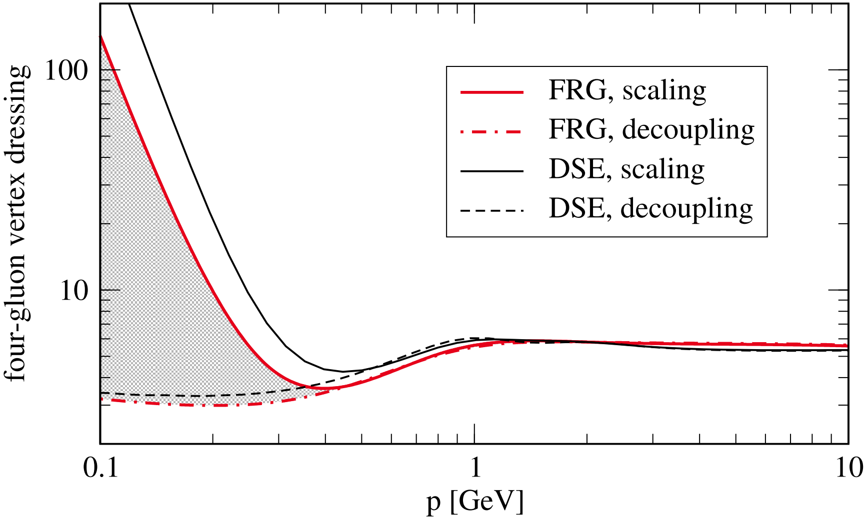
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Momentum dependence of the four-gluon vertex



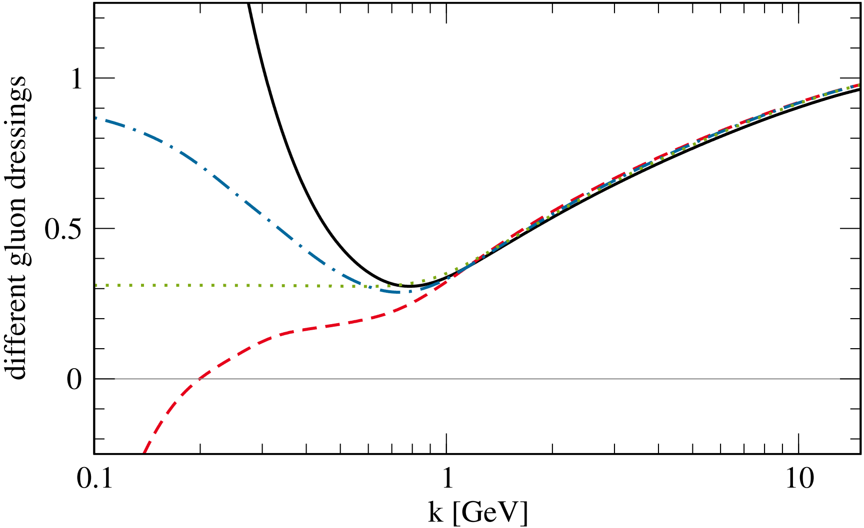
AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Four-gluon vertex at the symmetric point



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

Regulator dressing



AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016