GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Low-energy limit of the $O(4)$ quark-meson model

Jürgen Eser, Florian Divotgey, Mario Mitter, and Dirk H. Rischke

Lunch Club Seminar

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Motivation

$$U(N_f)_L \times U(N_f)_R$$

Chiral symmetry

Linear realization

Chiral partners

LSM, eLSM, QMM

[Gallas et al. '10;
Parganlija et al. '10;
Janowski et al. '11;
Parganlija et al. '13;
Eshraim et al. '15;
Olbrich et al. '16,'17;...]

Integration

FRG

?

Nonlinear realization

Goldstone bosons

Effective pion theory

Low-energy limit
of the QMM

[Eser et al. 1804.01787]

$O(4)$ quark-meson model (QMM)

- Two-flavor model, $N_f = 2$
- Euclidean action:

$$
\begin{aligned}
S = \int_x \Big\{ & \frac{1}{2} \left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{1}{2} \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} \\
& + U(\rho) - h_{\mathrm{ESB}} \sigma \\
& + \bar{\psi} \left(\gamma_\mu \partial_\mu + y \Phi_5\right) \psi \Big\}, \\
\Phi_5 = & \sigma t_0 + i \gamma_5 \vec{\pi} \cdot \vec{t}, \\
\rho = & \sigma^2 + \vec{\pi}^2
\end{aligned}
$$

- Explicit and spontaneous symmetry breaking implemented:

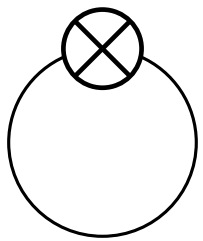
$$
\begin{aligned}
h_{\mathrm{ESB}} & \neq 0, \\
\sigma & \to \phi + \sigma
\end{aligned}
$$

Integration within the FRG

Functional renormalization group (FRG)

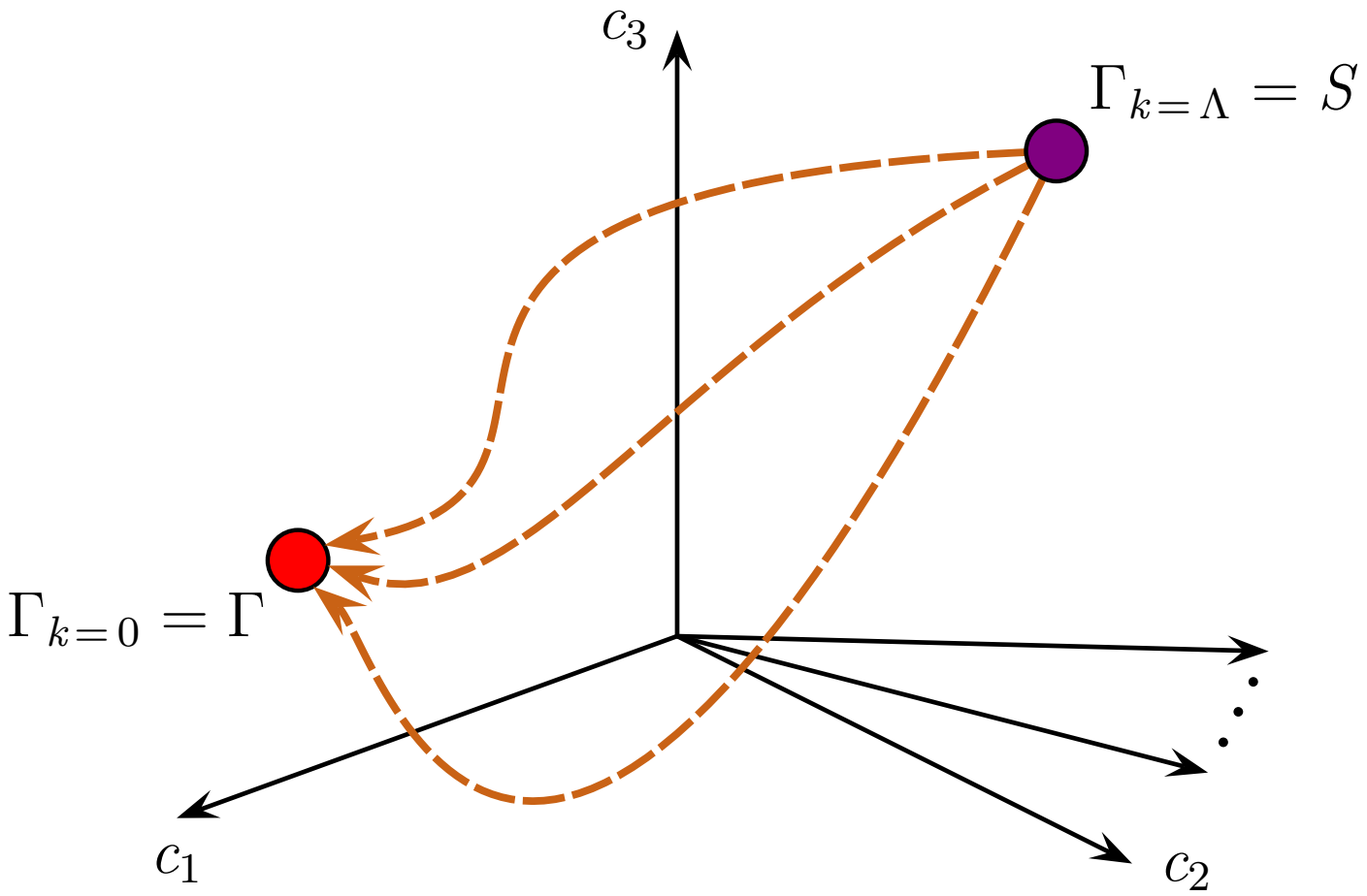
- Implementation of the Wilsonian RG idea
- Renormalization scale(k)-dependent effective action Γ_k
- FRG flow equation [Wetterich '93]:

$$\partial_k \Gamma_k = \frac{1}{2} \operatorname{tr} \left[\partial_k R_k \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right] =$$



- Regulator function R_k provides correct integration limits

Flow in theory space



Effective action

- Introduce low-energy couplings:

$$
\begin{aligned}
\Gamma_k = \int_x \Bigg\{ & \frac{Z_k^\sigma}{2} \left(\partial_\mu \sigma\right) \partial_\mu \sigma + \frac{Z_k^\pi}{2} \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} \\
& + U_k(\rho) - h_{\mathrm{ESB}} \sigma \\
& + C_{2,k} \left(\vec{\pi} \cdot \partial_\mu \vec{\pi}\right)^2 + Z_{2,k} \, \vec{\pi}^2 \left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi} \\
& - C_{3,k} \left[\left(\partial_\mu \vec{\pi}\right) \cdot \partial_\mu \vec{\pi}\right]^2 - C_{4,k} \left[\left(\partial_\mu \vec{\pi}\right) \cdot \partial_\nu \vec{\pi}\right]^2 \\
& - C_{5,k} \, \vec{\pi} \cdot \left(\partial_\mu \partial_\mu \vec{\pi}\right) \left(\partial_\nu \vec{\pi}\right) \cdot \partial_\nu \vec{\pi} \\
& - C_{6,k} \, \vec{\pi}^2 \left(\partial_\mu \partial_\nu \vec{\pi}\right) \cdot \partial_\mu \partial_\nu \vec{\pi} \\
& - C_{7,k} \left(\vec{\pi} \cdot \partial_\mu \partial_\mu \vec{\pi}\right)^2 - C_{8,k} \, \vec{\pi}^2 \left(\partial_\mu \partial_\mu \vec{\pi}\right)^2 \\
& + \bar{\psi} \left(Z_k^\psi \gamma_\mu \partial_\mu + y \Phi_5 \right) \psi \Bigg\}
\end{aligned}
$$

- $C_{1,k}(\vec{\pi}^2)^2$ is part of the effective potential U_k
- Goal: Compute the IR values of $C_{1,k}$, $C_{2,k}$, $Z_{2,k}$, and $C_{3,k}, \ldots, C_{8,k}$

Low-energy limit

- Equation of motion (renormalized quantities):

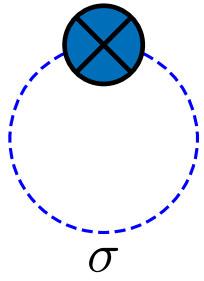
$$\frac{\delta\Gamma}{\delta\tilde{\sigma}} = 0$$

- Effective pion action:

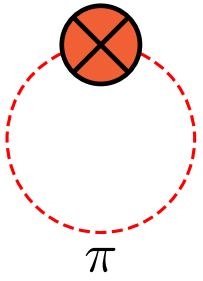
$$\begin{aligned}
\Gamma_k = \int_x \Bigg\{ & \frac{1}{2}\left(\partial_\mu \tilde{\vec{\pi}}\right)\cdot\partial_\mu\tilde{\vec{\pi}} + \frac{1}{2}M^2_{\pi,k}\tilde{\vec{\pi}}^2 - \tilde{C}^{\text{total}}_{1,k}\left(\tilde{\vec{\pi}}^2\right)^2 \\
& + \tilde{C}^{\text{total}}_{2,k}\left(\tilde{\vec{\pi}}\cdot\partial_\mu\tilde{\vec{\pi}}\right)^2 + \tilde{Z}^{\text{total}}_{2,k}\,\tilde{\vec{\pi}}^2\left(\partial_\mu\tilde{\vec{\pi}}\right)\cdot\partial_\mu\tilde{\vec{\pi}} \\
& - \tilde{C}^{\text{total}}_{3,k}\left[\left(\partial_\mu\tilde{\vec{\pi}}\right)\cdot\partial_\mu\tilde{\vec{\pi}}\right]^2 - \tilde{C}^{\text{total}}_{4,k}\left[\left(\partial_\mu\tilde{\vec{\pi}}\right)\cdot\partial_\nu\tilde{\vec{\pi}}\right]^2 \\
& - \tilde{C}^{\text{total}}_{5,k}\,\tilde{\vec{\pi}}\cdot\left(\partial_\mu\partial_\mu\tilde{\vec{\pi}}\right)\left(\partial_\nu\tilde{\vec{\pi}}\right)\cdot\partial_\nu\tilde{\vec{\pi}} \\
& - \tilde{C}^{\text{total}}_{6,k}\,\tilde{\vec{\pi}}^2\left(\partial_\mu\partial_\nu\tilde{\vec{\pi}}\right)\cdot\partial_\mu\partial_\nu\tilde{\vec{\pi}} \\
& - \tilde{C}^{\text{total}}_{7,k}\left(\tilde{\vec{\pi}}\cdot\partial_\mu\partial_\mu\tilde{\vec{\pi}}\right)^2 - \tilde{C}^{\text{total}}_{8,k}\,\tilde{\vec{\pi}}^2\left(\partial_\mu\partial_\mu\tilde{\vec{\pi}}\right)^2 \Bigg\}
\end{aligned}$$

Flow equations

$$\partial_k U_k = \mathcal{V}^{-1} \left(\frac{1}{2} \right.$$



$$+ \frac{1}{2}$$



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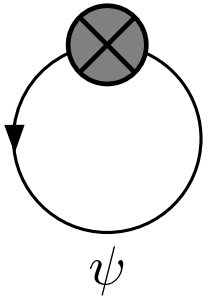
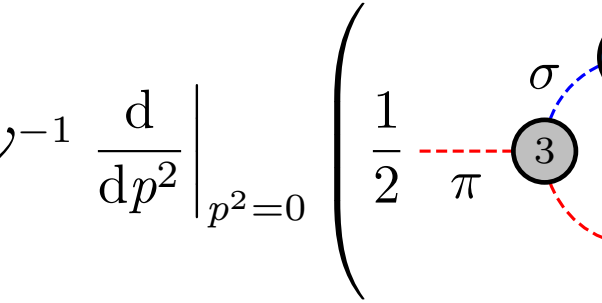
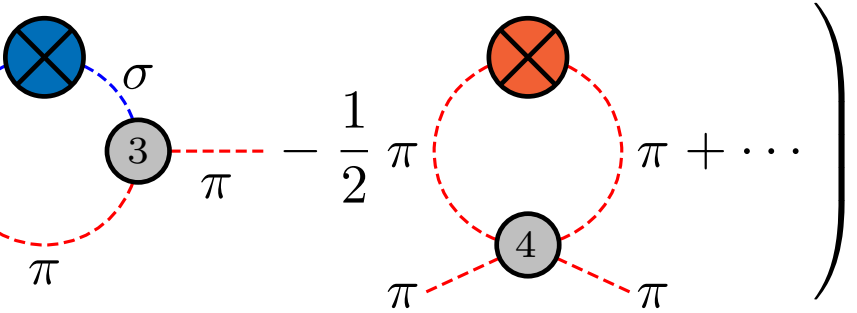


$$\left. \right) ,$$

$$\partial_k Z_k^{\pi} = \mathcal{V}^{-1} \left. \frac{\mathrm{d}}{\mathrm{d}p^2} \right|_{p^2=0} \left(\frac{1}{2} \right.$$

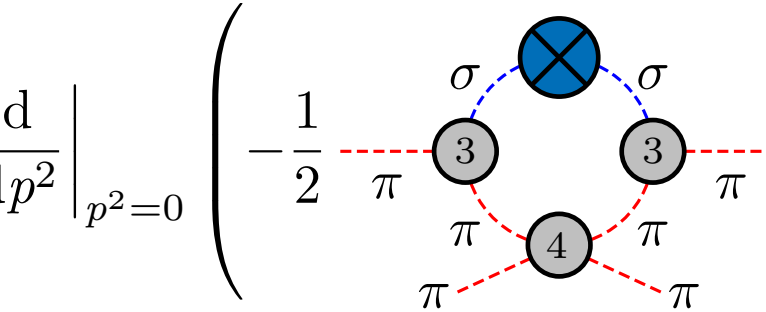


$$- \frac{1}{2}$$



$$+ \cdots \left. \right) ,$$

$$\partial_k C_{2,k} = \mathcal{V}^{-1} \left. \frac{\mathrm{d}}{\mathrm{d}p^2} \right|_{p^2=0} \left(-\frac{1}{2} \right.$$



$$+ \cdots \left. \right) ,$$

$$\vdots$$

Symmetry breaking and renormalized masses

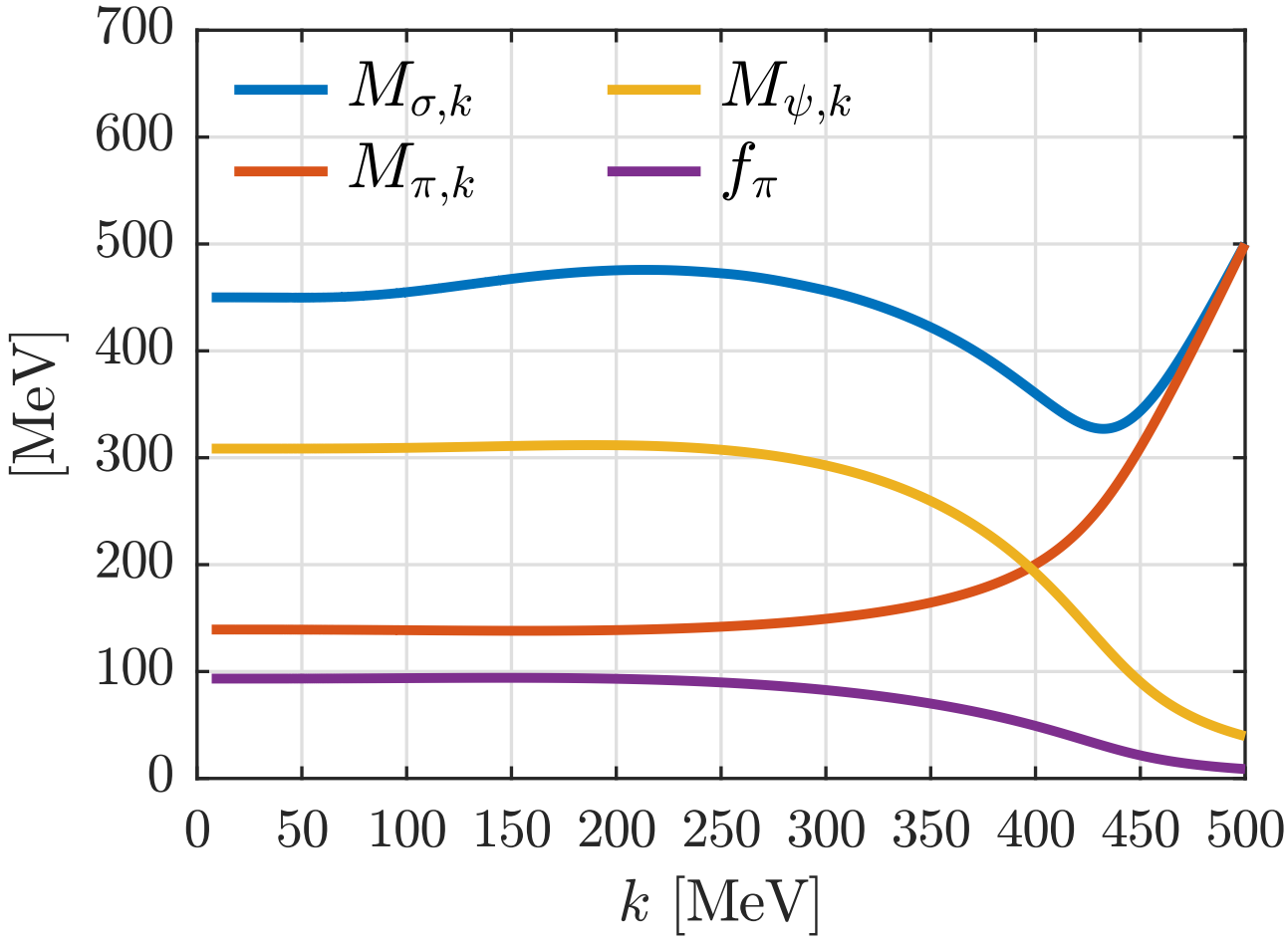


Figure 1: Scale evolution of the renormalized meson and quark masses and the pion decay constant; $k_{\mathrm{IR}} = 7\ \mathrm{MeV}$; [1804.01787].

Wave-function renormalization

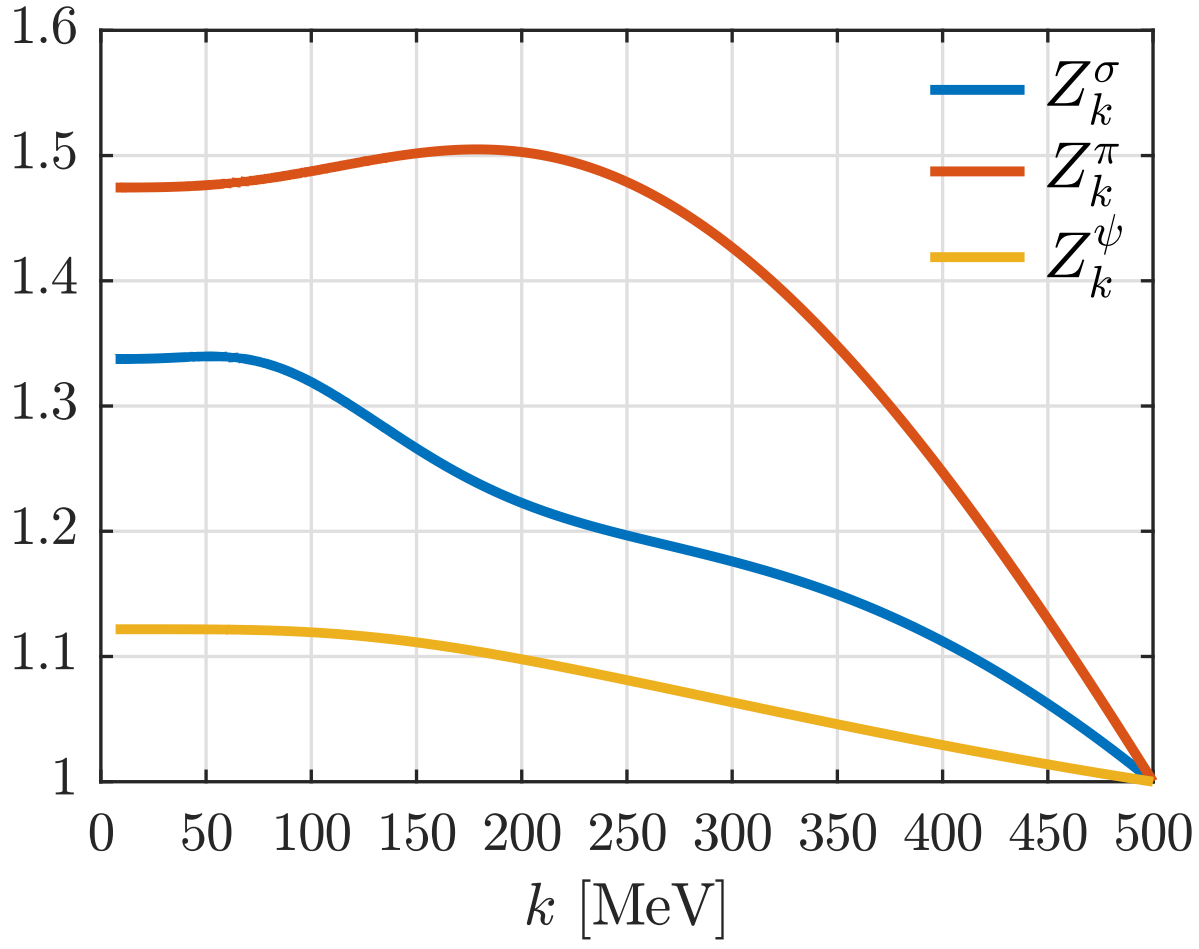


Figure 2: Scale evolution of the wave-function renormalization factors Z_k^σ, Z_k^π, and Z_k^ψ; $k_{\mathrm{IR}} = 7$ MeV; [1804.01787].

Pion self-interactions $\mathcal{O}(\partial^2)$

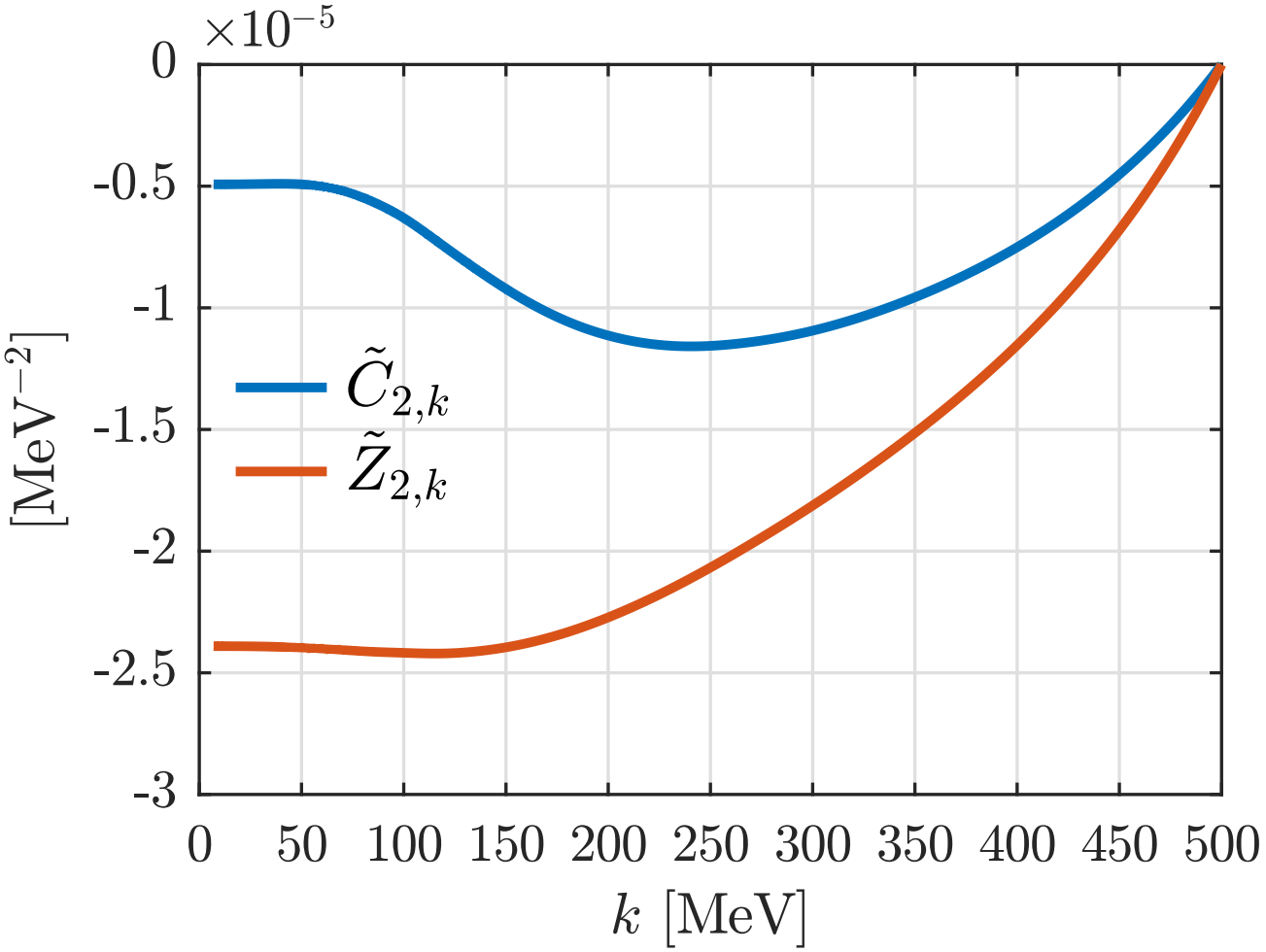


Figure 3: Scale evolution of the momentum-dependent pion self-interactions of the order $\mathcal{O}(\partial^2)$; $k_{\mathrm{IR}} = 7$ MeV; [1804.01787].

Pion self-interactions $\mathcal{O}(\partial^4)$

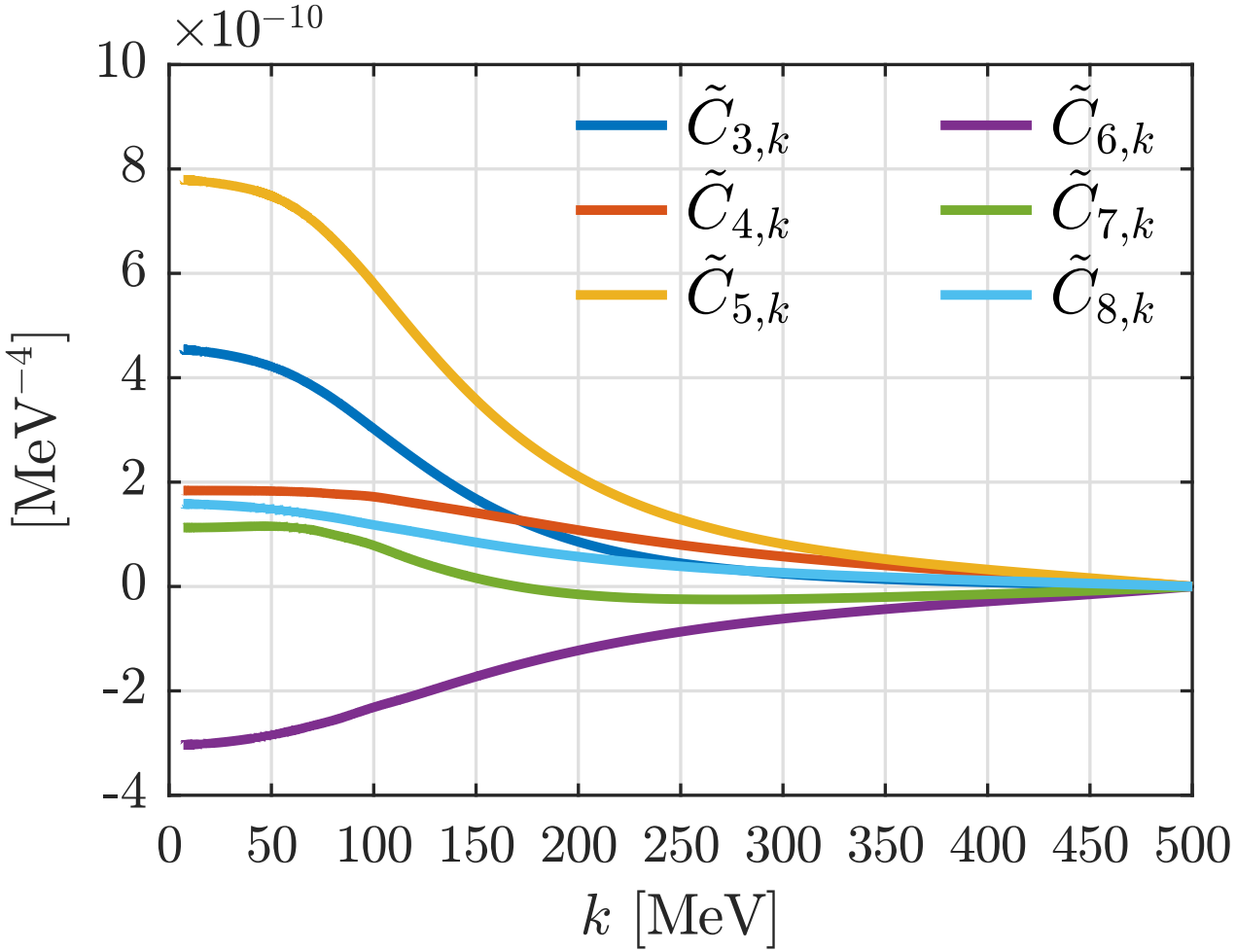


Figure 4: Scale evolution of the momentum-dependent pion self-interactions of the order $\mathcal{O}(\partial^4)$; $k_{\mathrm{IR}} = 7$ MeV; [1804.01787].

Numerical results

Table 1: Low-energy couplings ($f_\pi = 93\ \mathrm{MeV}$); [1804.01787].

		FRG		
Coupling	Tree level	tree	trunc	total
C_1	-0.2514	2.1063	-2.3550	-0.2487
$C_2\ [1/f_\pi^2]$	0.4054	0.3598	-0.0426	0.3172
$Z_2\ [1/f_\pi^2]$	–	–	-0.2068	-0.2068
$C_3\ [1/f_\pi^4] \times 10^2$	1.7311	1.5364	3.3946	4.9310
$C_4\ [1/f_\pi^4] \times 10^2$	–	–	1.3752	1.3752
$C_5\ [1/f_\pi^4] \times 10^2$	3.4621	3.0728	5.8294	8.9023
$C_6\ [1/f_\pi^4] \times 10^2$	–	–	-2.2697	-2.2697
$C_7\ [1/f_\pi^4] \times 10^2$	1.7311	1.5364	0.8439	2.3804
$C_8\ [1/f_\pi^4] \times 10^2$	–	–	1.1828	1.1828

LPA' masses

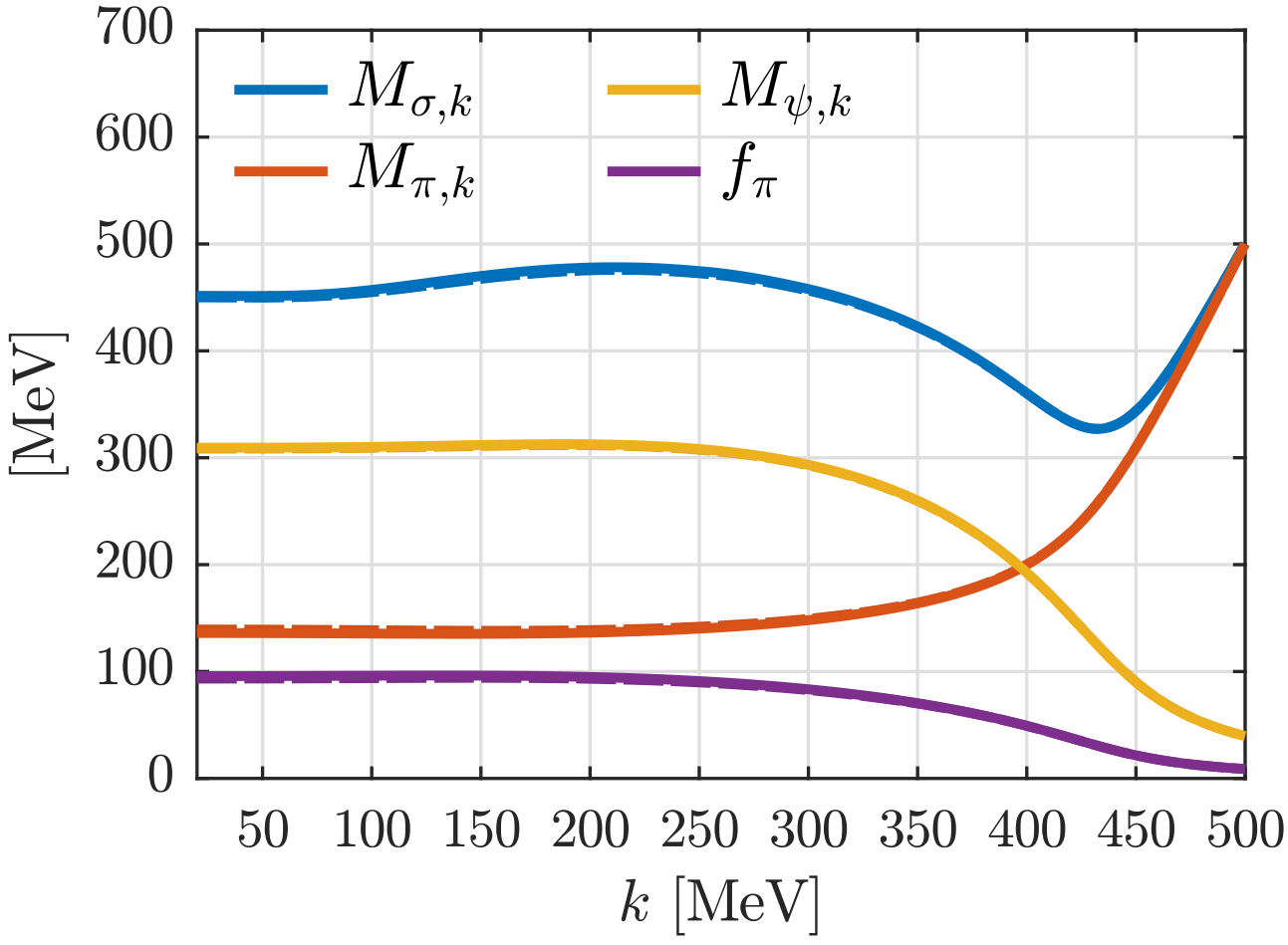


Figure 5: Scale evolution of the renormalized meson and quark masses within the LPA' truncation. The dashed lines show the related results from Fig. 1; $k_{\mathrm{IR}} = 20$ MeV; [1804.01787].

LPA' wave-function renormalization

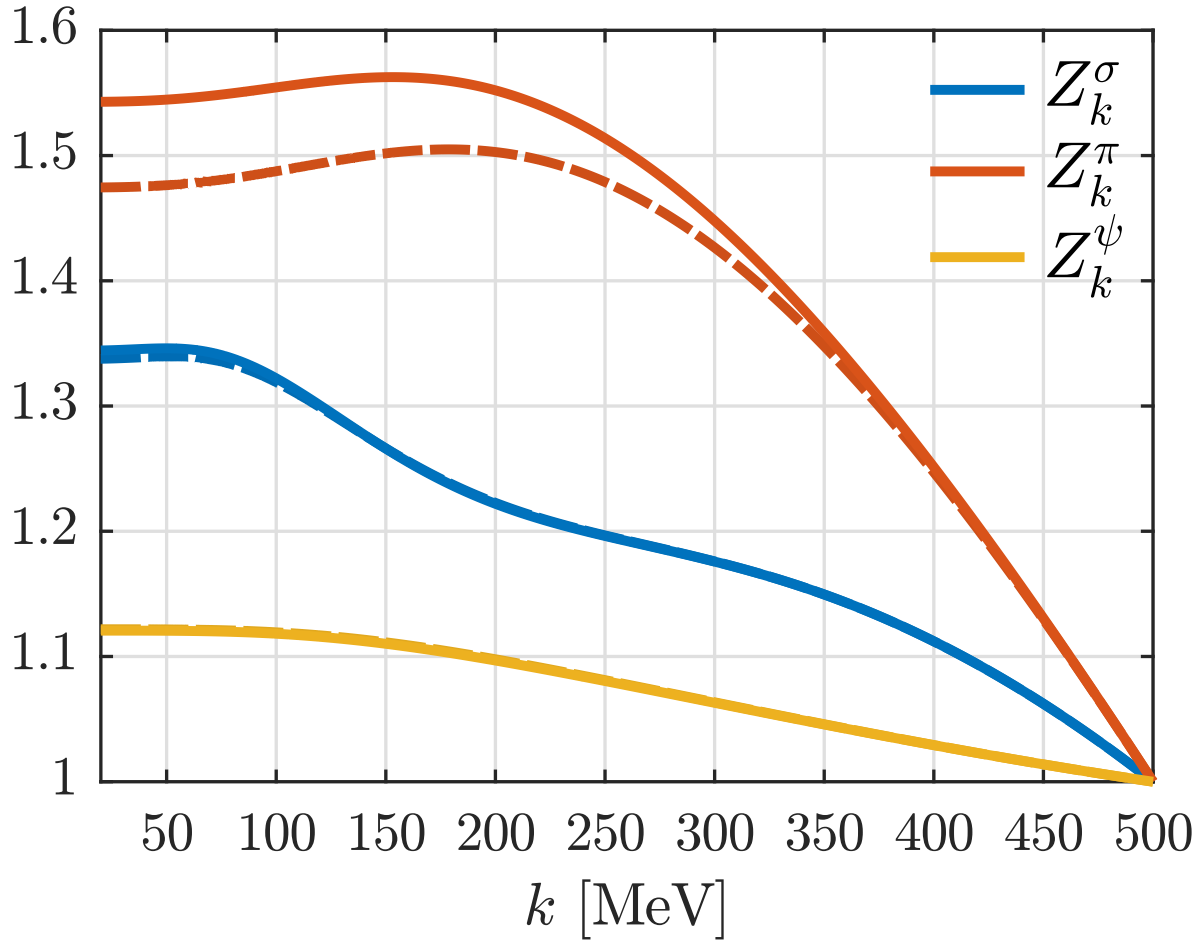


Figure 6: Scale evolution of the wave-function renormalization factors Z_k^σ, Z_k^π, and Z_k^ψ within the LPA' truncation. The dashed lines show the related results from Fig. 2; $k_{\mathrm{IR}} = 20\ \mathrm{MeV}$; [1804.01787].

Outlook

QCD input

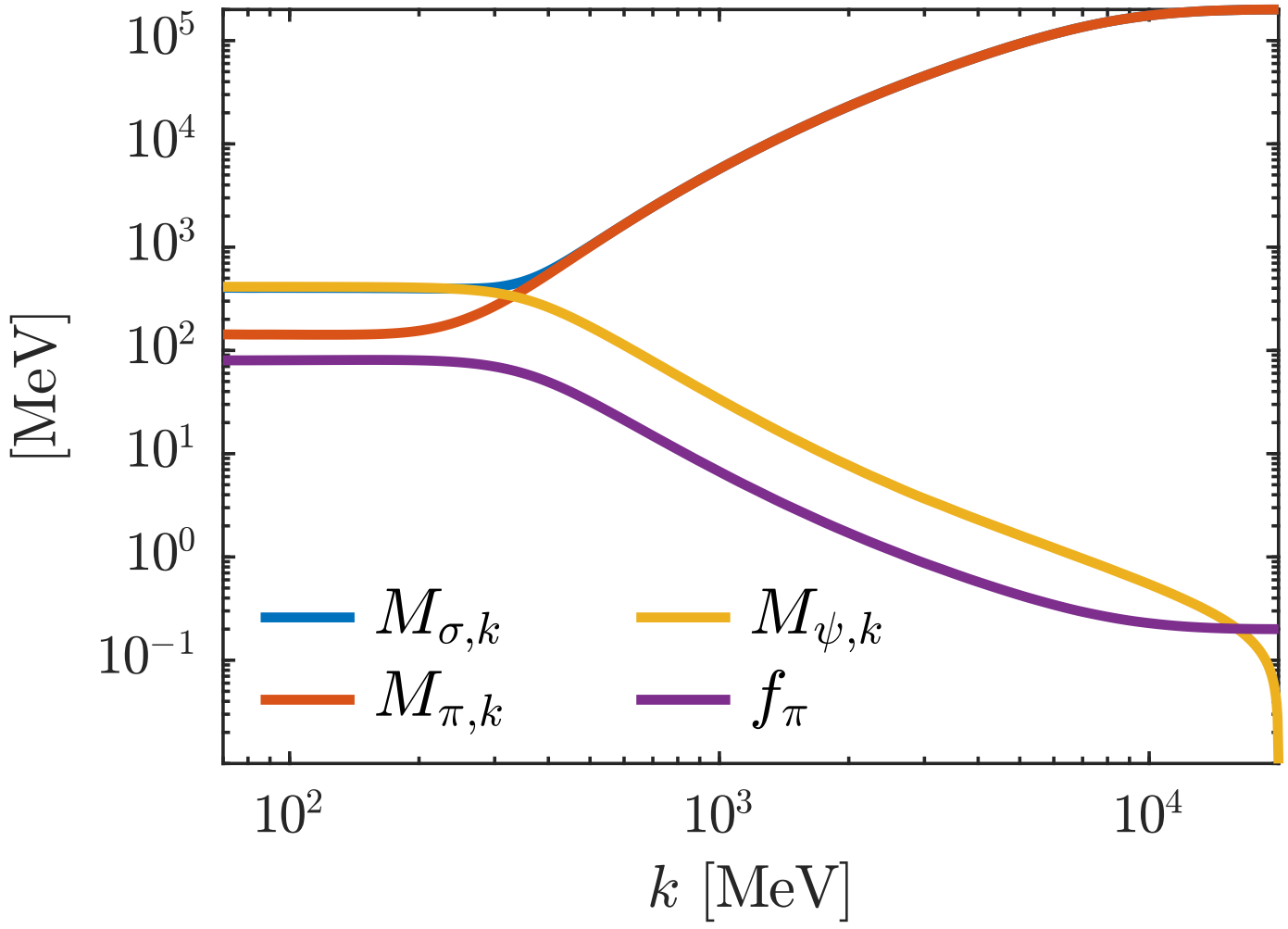


Figure 7: Scale evolution of the renormalized meson and quark masses and the pion decay constant; $k_{\mathrm{IR}} = 70\ \mathrm{MeV}$.

QCD input

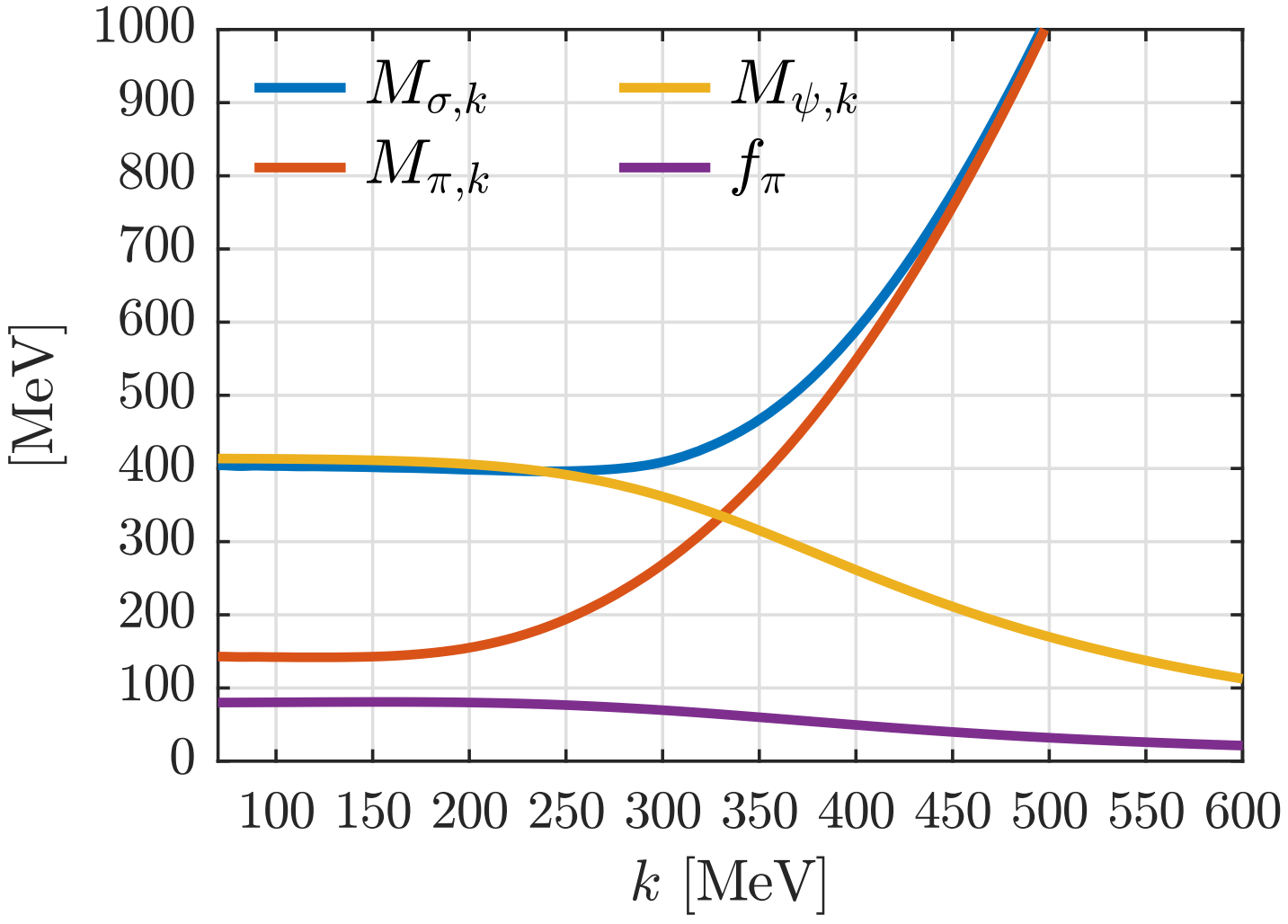


Figure 8: Scale evolution of the renormalized meson and quark masses and the pion decay constant; $k_{\mathrm{IR}} = 70\ \mathrm{MeV}$.

QCD wave-function renormalization

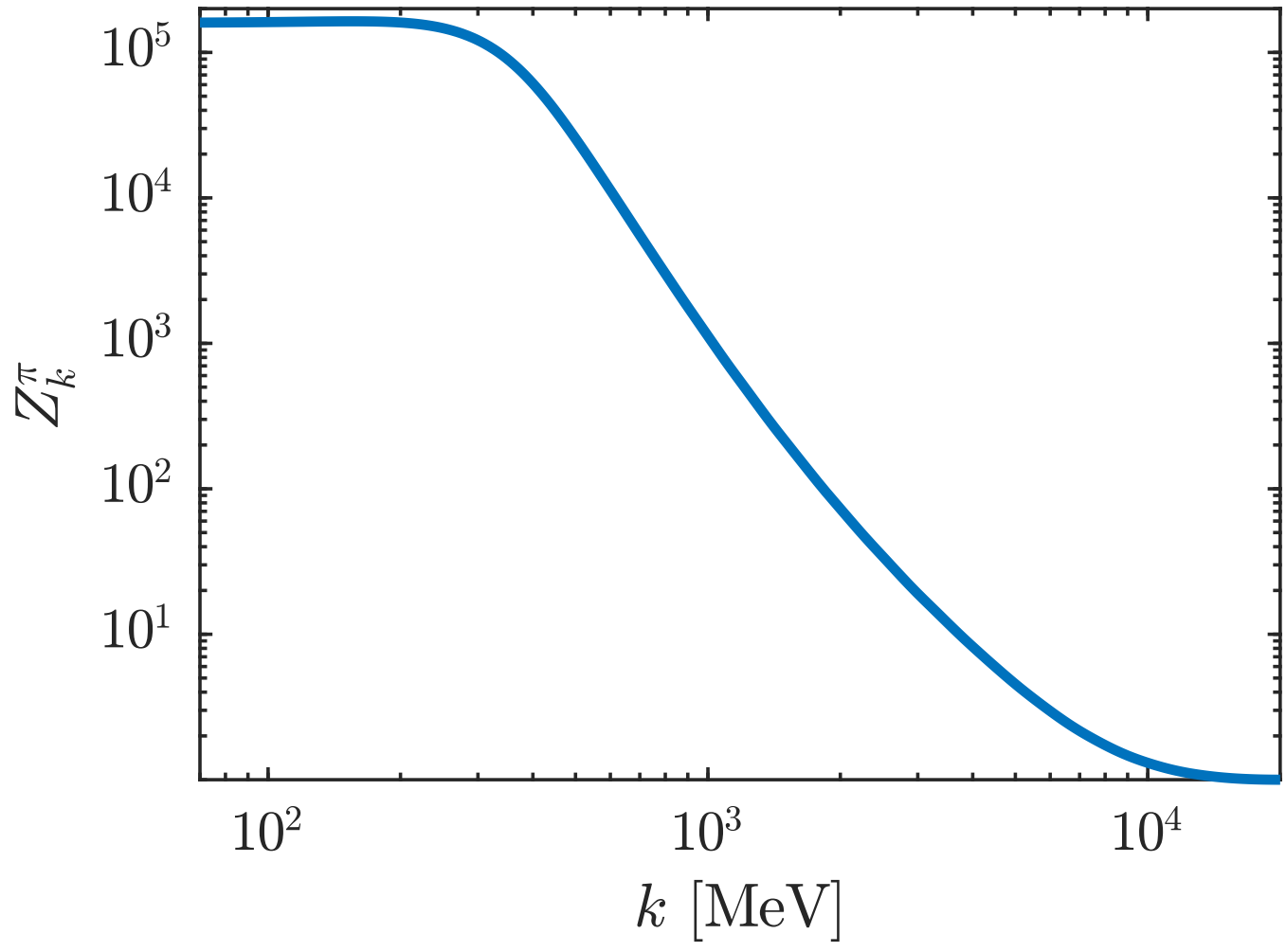


Figure 9: Scale evolution of the pion wave-function renormalization; $k_{\mathrm{IR}} = 70$ MeV.

QCD wave-function renormalization

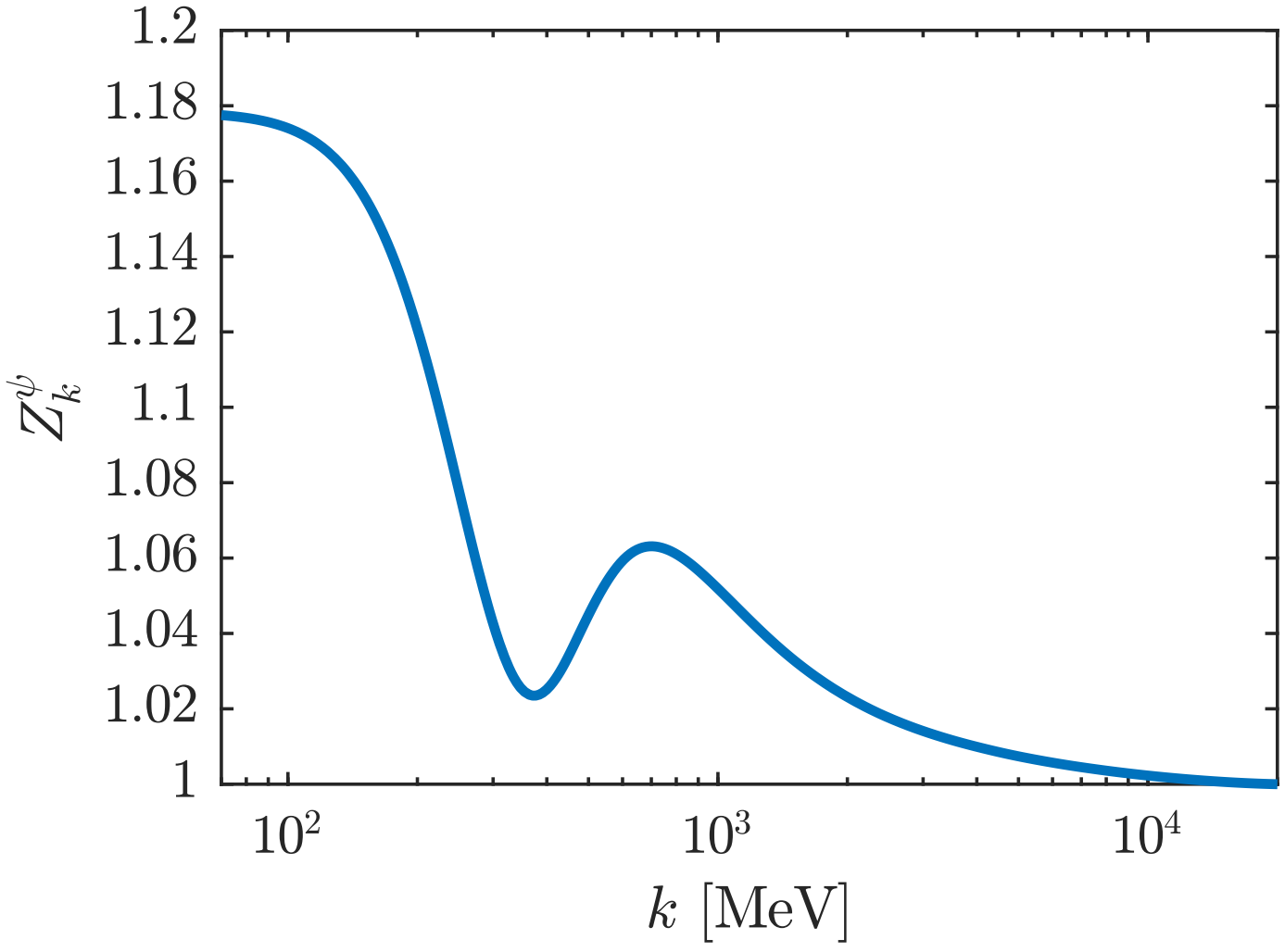


Figure 10: Scale evolution of the quark wave-function renormalization; $k_{\mathrm{IR}} = 70$ MeV.

Pion self-interactions from QCD

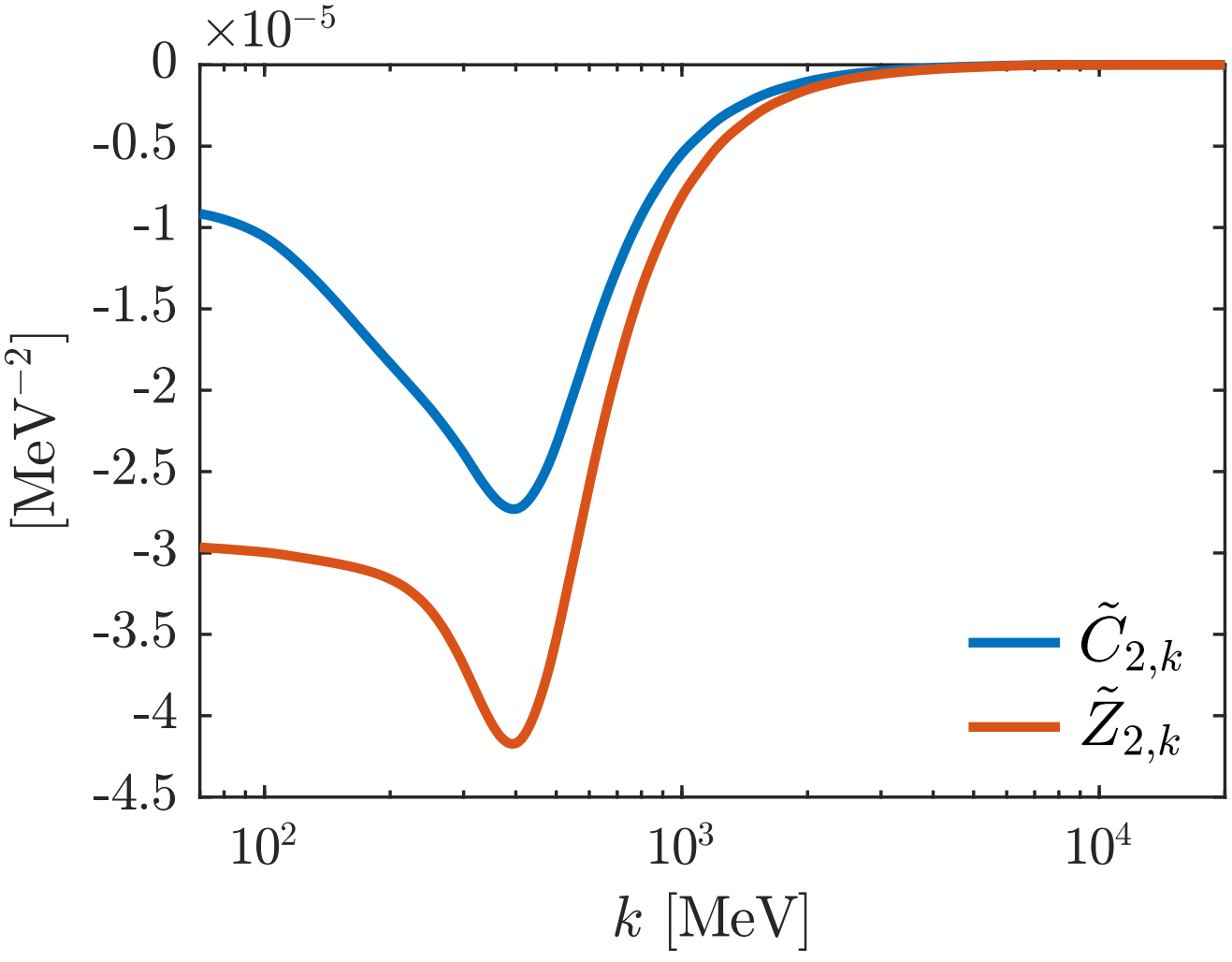


Figure 11: Scale evolution of the momentum-dependent pion self-interactions of the order $\mathcal{O}(\partial^2)$; $k_{\mathrm{IR}} = 70$ MeV.

Pion self-interactions from QCD

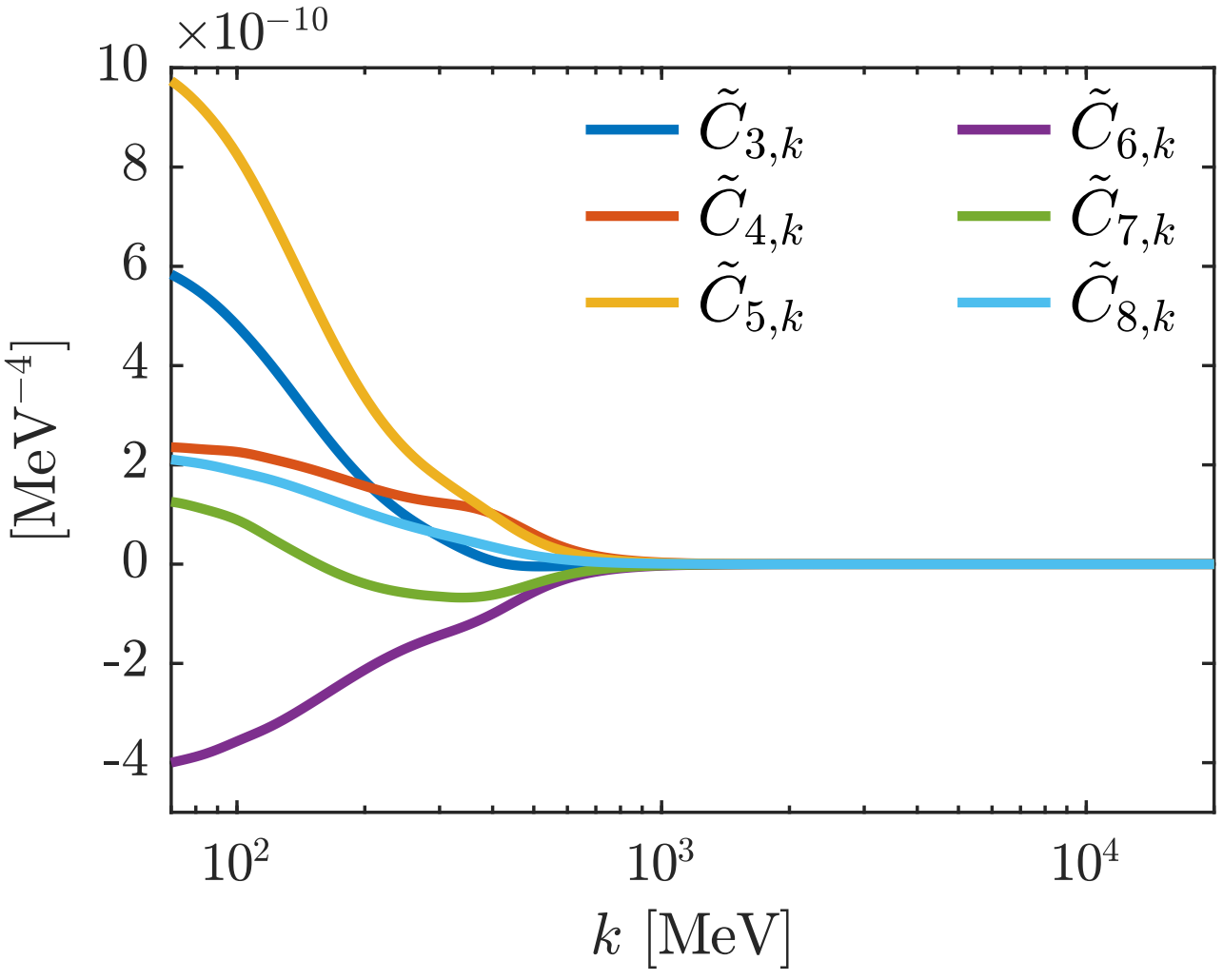


Figure 12: Scale evolution of the momentum-dependent pion self-interactions of the order $\mathcal{O}(\partial^4)$; $k_{\mathrm{IR}} = 70$ MeV.

Numerical results from QCD

Table 2: Low-energy couplings from QCD input.

		FRG	
Coupling	Tree level	$O(4)$	QCD
C_1	-0.2514	-0.2487	-0.3479
$C_2\ [1/f_\pi^2]$	0.4054	0.3172	0.4372
$Z_2\ [1/f_\pi^2]$	–	-0.2068	-0.2565
$C_3\ [1/f_\pi^4] \times 10^2$	1.7311	4.9310	7.1016
$C_4\ [1/f_\pi^4] \times 10^2$	–	1.3752	1.7625
$C_5\ [1/f_\pi^4] \times 10^2$	3.4621	8.9023	12.7534
$C_6\ [1/f_\pi^4] \times 10^2$	–	-2.2697	-2.9918
$C_7\ [1/f_\pi^4] \times 10^2$	1.7311	2.3804	3.6749
$C_8\ [1/f_\pi^4] \times 10^2$	–	1.1828	1.5770

Chiral perturbation theory (ChPT)

Compare low-energy couplings of the extended LSM (eLSM):

- Chiral expansion of the QCD generating functional (ChPT [Gasser, Leutwyler '84,'85; Leutwyler '94;...]):

$$\begin{aligned}\mathcal{L}_{\mathrm{ChPT}} = & \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^{\,2} + C_{1,\mathrm{ChPT}} \left(\vec{\pi}^{\,2}\right)^2 + C_{2,\mathrm{ChPT}} \left(\vec{\pi} \cdot \partial_\mu \vec{\pi}\right)^2 \\ & + C_{3,\mathrm{ChPT}} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2 + C_{4,\mathrm{ChPT}} \left(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi}\right)^2 \\ & + \mathcal{O}\left(\pi^6, \partial^6\right)\end{aligned}$$

- Low-energy limit of the eLSM at tree level [Divotgey et al. '18]:

$$\begin{aligned}\mathcal{L}_{\mathrm{eLSM}} = & \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - \frac{1}{2} m_\pi^2 \vec{\pi}^{\,2} + C_{1,\mathrm{eLSM}} \left(\vec{\pi}^{\,2}\right)^2 + C_{2,\mathrm{eLSM}} \left(\vec{\pi} \cdot \partial_\mu \vec{\pi}\right)^2 \\ & + C_{3,\mathrm{eLSM}} \left(\partial_\mu \vec{\pi}\right)^2 \left(\partial_\nu \vec{\pi}\right)^2 + C_{4,\mathrm{eLSM}} \left(\partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi}\right)^2 \\ & + \mathcal{O}\left(\pi^6, \partial^6\right)\end{aligned}$$

eLSM at tree level

Table 3: Comparison of the low-energy couplings at tree level, cf. [Bijnens, Ecker '14; Divotgey et al. '18].

Coupling	ChPT	eLSM
C_1	-0.29 ± 0.34	-0.268 ± 0.021
$C_2 \times 10^5$ MeV^2	5.882 ± 0.013	5.399 ± 0.081
$C_3 \times 10^{11}$ MeV^4	-5.57 ± 0.88	$-9.30 \pm 0.59 + \text{corr.}$
$C_4 \times 10^{11}$ MeV^4	2.58 ± 0.29	$9.45 \pm 0.59 + \text{corr.}$

- How do these values change if we include loop corrections?
- Can we produce similar values within the FRG formalism?

Summary

Summary

Summary:

- Low-energy limit of the $O(4)$ quark-meson model within the FRG confronted with a tree-level estimate
- Discrepancy between these approaches observed
- Low-energy couplings of the eLSM at tree level in good agreement with ChPT,
  cf. [Bijnens, Ecker '14; Divotgey et al. '18]

Upcoming:

- Extend analysis to the full $O(4)$-symmetric scenario
- Include vector mesons and scale-dependent Yukawa coupling
- Compute low-energy limit from first-principles QCD
- Confront the FRG calculation with ChPT