Quarks and pions at finite chemical potential

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in collaboration with Christian S. Fischer

Institute for Theoretical Physics
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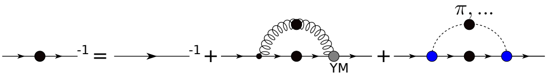
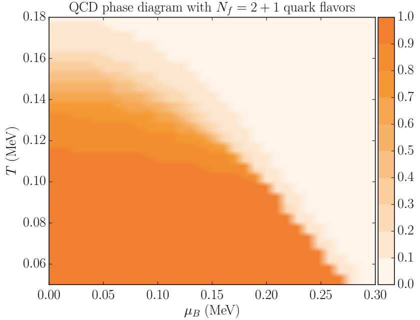
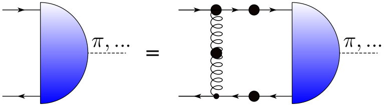
Lunch Club, May 16, 2018

Outline



1. Introduction and motivation
2. QCD phase diagram
3. Pions at finite chem. potential
4. Summary and outlook





Phase structure of QCD

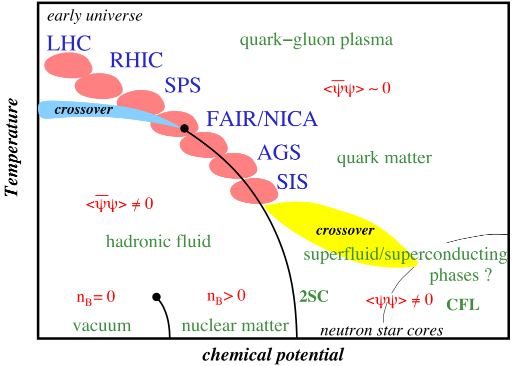


Figure: Schaefer and Wagner, Prog.Part.Nucl.Phys. 62 (2009) 381

Previous DS studies:

QCD phase diagram with $N_f = 2 + 1$ and $N_f = 2 + 1 + 1$ quark flavors:

- Luecker and Fischer, Prog.Part.Nucl.Phys. 67(2), 200–205 (2012)
- Fischer, Luecker and Welzbacher, Phys.Rev. D 90(3), 34022 (2014)

Baryon effects on the location of QCD's CEP:

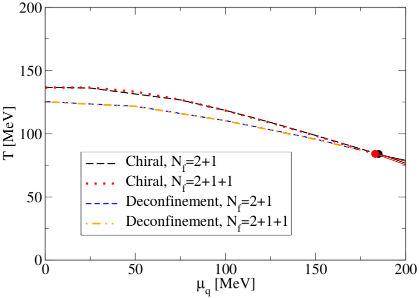
- Eichmann, Fischer and Welzbacher, Phys.Rev. D 93, 034013 (2016)

Mesonic back-coupling effects in vacuum and finite T:

- Fischer, Nickel and Wambach Phys.Rev. D 76, 094009 (2007)
- Fischer and Williams Phys.Rev. D 78, 074006 (2008)
- Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

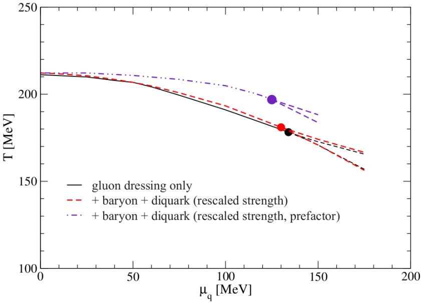
Previous studies

Including the charm quark:



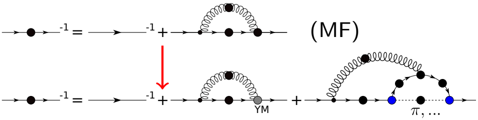
Fischer, Luecker and Welzbacher, Phys.Rev. D 90(3), 34022 (2014)

Modeling Baryon effects:

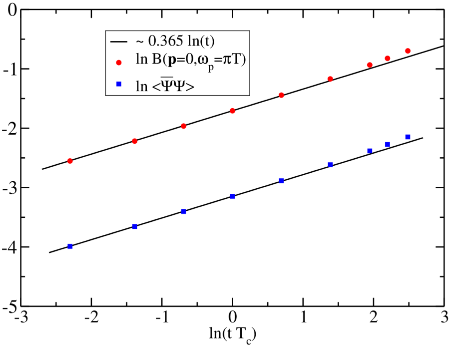


Eichmann, Fischer and Welzbacher, Phys.Rev. D 93, 034013 (2016)

Importance of mesonic back-coupling effects:



$$t = \frac{T_c - T}{T_c}$$



$T = 0$: Meson corrections in order of $\sim 10 - 20\%$

$T = T_c$: Meson corrections dominant $\longrightarrow$ critical scaling ($m_q \to 0$):

$$\langle \bar{\Psi}\Psi \rangle (t) \sim B(t) \sim t^{\nu/(2-\eta)} \sim t^{\nu/2} \sim t^{\beta} \qquad \beta = 0.365,\ \nu = 0.73$$

(Heisenberg class)

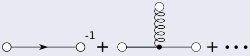
$T = 0$: Fischer, Williams, Phys.Rev. D 78, 074006 (2008)
$T = T_c$: Fischer, Mueller, Phys.Rev. D 84, 054013 (2011)

Origin of the Dyson-Schwinger equations

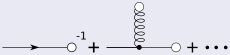
Generating functional $\xrightarrow[\text{invariance}]{\text{local translation}}$ **Master DSE**:

$$\frac{\delta\Gamma}{\delta\tilde{\varphi}(x)}[\tilde{\varphi}] = \frac{\delta S}{\delta\varphi(x)}\left[\tilde{\varphi} + \int_y \Delta_{\bullet y}[\tilde{\varphi}]\frac{i\delta}{\delta\tilde{\varphi}(y)}\right]$$

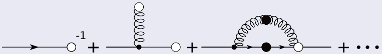
QCD classical action

$S =$ 

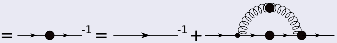
QCD action derivative

$\frac{\delta S}{\delta\bar{\Psi}} =$ 

QCD effective action derivative

$\frac{\delta\Gamma}{\delta\bar{\tilde{\Psi}}} =$ 

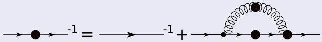
Quark propagator

$\left.\frac{\delta^2\Gamma}{\delta\bar{\tilde{\Psi}}\delta\tilde{\Psi}}\right|_{\bar{\tilde{\Psi}}=\tilde{\Psi}=0}$ 

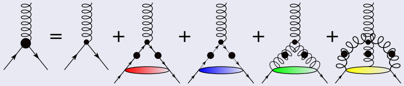
Dyson-Schwinger approach

Coupled set of Dyson-Schwinger equations:

Quark DSE:



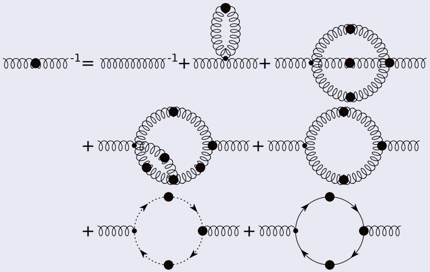
Quark-Gluon-Vertex DSE:



Further Vertex and Ghost DSEs:

...

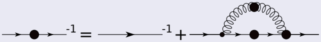
Gluon DSE:



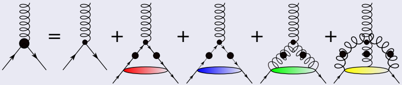
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step I):

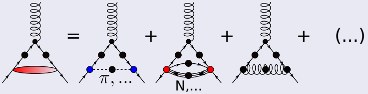
Quark DSE:



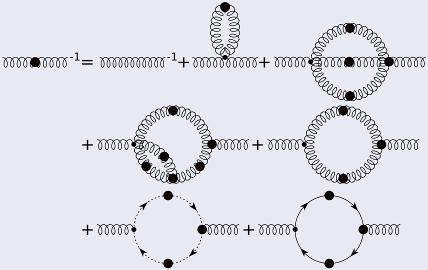
Quark-Gluon-Vertex DSE:



Skeleton expansion:



Gluon DSE:



Truncation

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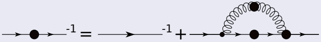
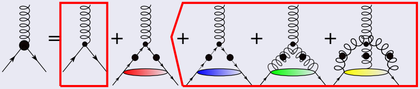
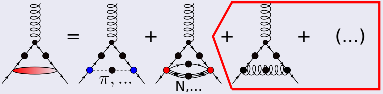
Quark DSE:



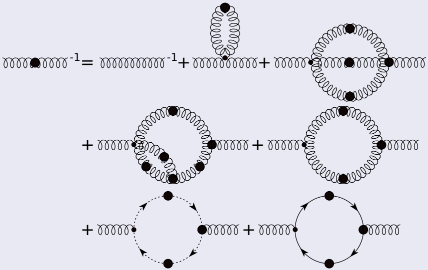
Quark-Gluon-Vertex DSE:



Skeleton expansion:



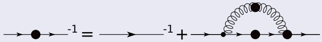
Gluon DSE:



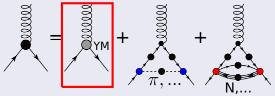
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step II):

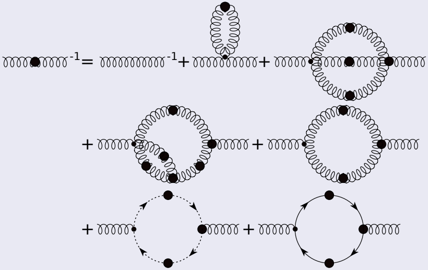
Quark DSE:



Quark-Gluon-Vertex DSE:



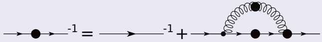
Gluon DSE:



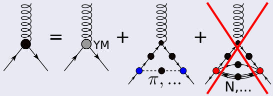
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step II):

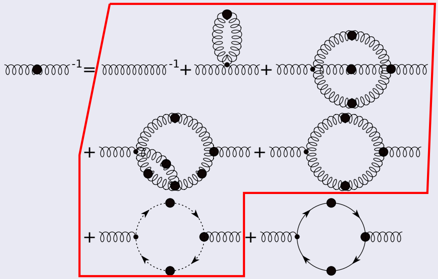
Quark DSE:



Quark-Gluon-Vertex DSE:



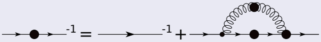
Gluon DSE:



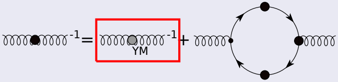
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step III):

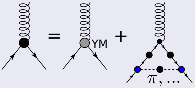
Quark DSE:



Gluon DSE:



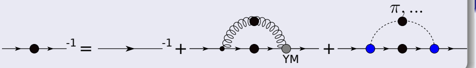
Quark-Gluon-Vertex DSE:



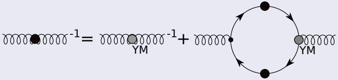
Truncation

Coupled set of **truncated** Dyson-Schwinger equations (Step IV):

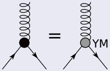
Quark DSE:



Gluon DSE:



Quark-Gluon-Vertex DSE:



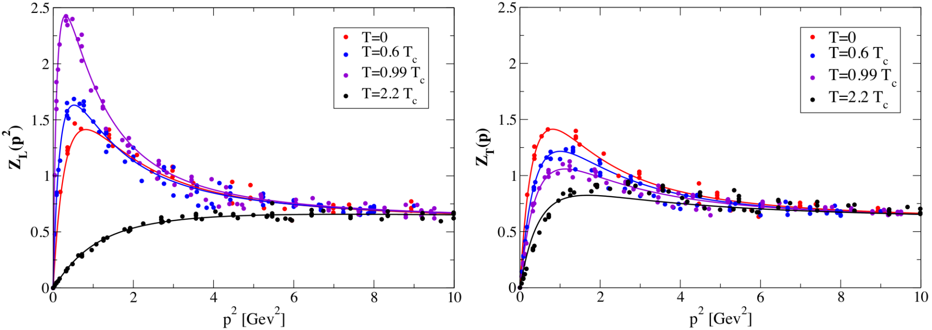
Lattice data for quenched gluon:

Fischer, Maas and Müller, Eur.Phys.J. C (2010) 68: 165

Fischer and Müller Phys.Rev. D 80, 074029 (2009)

Truncation scheme properties

T-dependent gluon propagator from quenched lattice simulations:



Crucial difference between transversal and longitudinal gluon ($T_c = 277$ MeV)

Cucchieri, Maas, Mendes, PRD 75 (2007)
CF, Maas, Mueller, EPJC 68 (2010)
Aouane et al., PRD 85 (2012) 034501
Cucchieri, Mendes, PoS FACESQCD 007 (2010)
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503
FRG: Fister, Pawlowski, arXiv:1112.5440

Truncation scheme properties

Vertex truncation: STI and perturbative behavior at large momenta constrain vertex

$$\Gamma^f_\nu(p,q,k) = \Gamma(k^2)\gamma_\nu \left(\delta_{\nu,s}\Sigma_A + \delta_{\nu,4}\Sigma_C\right)$$

$$\Gamma(k^2) = \frac{d_1}{d_2+k^2} + \frac{k^2}{\Lambda^2+k^2}\left(\frac{\beta_0\alpha(\mu'')\ln[k^2/\Lambda^2+1]}{4\pi}\right)^{2\delta}$$

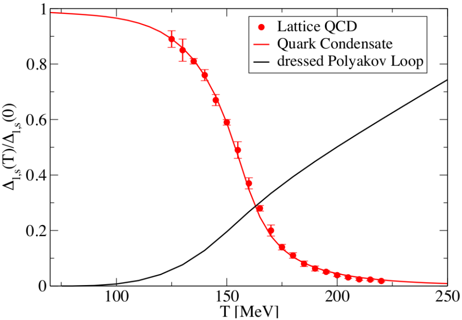
With first Ball-Chiu structure: $\Sigma_X = \frac{X(\vec{p}^2,\omega_p)+X(\vec{q}^2,\omega_q)}{2}$, $X \in \{A, C\}$

Abelian WTI: from approximated STI
Perturbation theory
Infrared ansatz: d_2 fixed to match gluon input, d_1 fixed via quark condensate

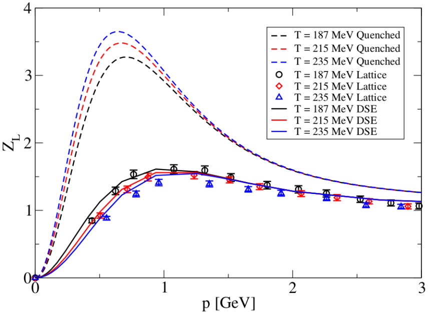
Fischer and Mueller, Phys.Rev. D 80, 074029 (2009)
Fischer, Phys.Rev.Lett. 103, 052003 (2009)

Truncation scheme properties

Determination of d_1 and prediction for unquenched gluon:



Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022



Lattice: Aouane, Burger, Ilgenfritz, Müller-Preussker, Sternbeck, PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

Quantitative agreement: DSE results verified by lattice

Present QCD phase diagram

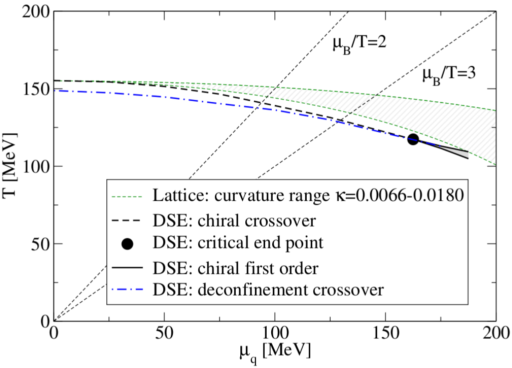
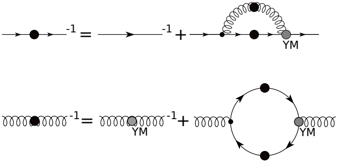


Figure: Fischer, Luecker, Welzbacher, PRD 90 (2014) 34022

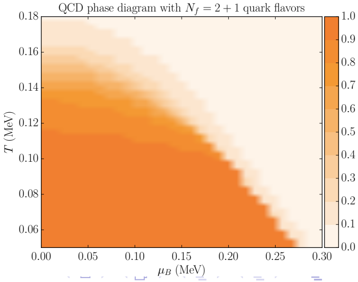
Extrapolated curvature from lattice

- Kaczmarek at al. PRD 83 (2011) 014504,
- Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001
- Cea, Cosmai, Papa, PRD 89 (2014), PRD 93 (2016)
- Bonati et al., PRD 92 (2015) 054503
- Bellwied et al. PLB 751 (2015) 559

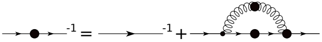


CEP at large μ_q

- CF, Luecker, PLB 718 (2013) 1036
- CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
- CF, Luecker, Welzbacher, PRD 90 (2014) 034022



Chirality:



Quark condensate

$$\left\langle \bar{\Psi}\Psi \right\rangle^{f} = -Z_m Z_2 \int_q \mathrm{tr}_{DC}\left[S^f(p)\right]$$

Regularized quark condensate

$$\Delta_{f'f} = \left\langle \bar{\Psi}\Psi \right\rangle^{f'} - \frac{m_B^{f'}}{m_B^{f}} \left\langle \bar{\Psi}\Psi \right\rangle^{f}$$

Deconfinement: $\langle L[A] \rangle \propto e^{-\frac{F_q}{T}}$, static quark free energy F_q

Dressed Polyakov loop[1]

$$\Sigma = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \left\langle \bar{\Psi}\Psi \right\rangle_{\varphi}$$

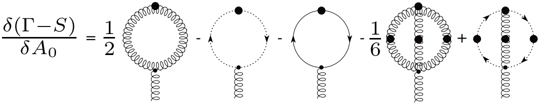
Polyakov loop potential[2]

$$L[A] := \frac{1}{N_c} \mathrm{tr}_C \left(\mathcal{P} e^{i \int d\tau A_0(\vec{x},\tau)} \right)$$

1:
Synatschke, Wipf, Wozar, PRD 75, 114003 (2007)

Bilgici, Bruckmann, Gattringer, Hagen, PRD 77 094007 (2008)

Fischer, PRL 103 052003 (2009)



2:
Braun, Gies, Pawlowski, PLB 684, 262 (2010)

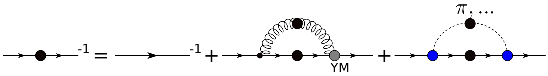
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)

Fister, Pawlowski, PRD 88 045010 (2013)

Fischer, Fister, Luecker, Pawlowski, PLB 732 (2013)

Reminder

Wanted: Influence of Pion back-coupling onto QCD phase diagram and CEP



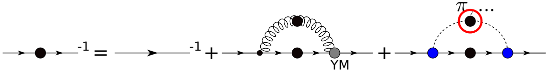
Needed: Pion propagator in medium

Pion Bethe-Salpether amplitude in medium

First step: Investigate Pion amplitudes at finite chemical potential

Reminder

Wanted: Influence of Pion back-coupling onto QCD phase diagram and CEP



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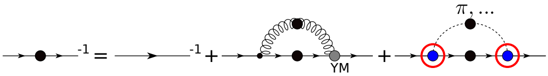

Needed: Pion propagator in medium

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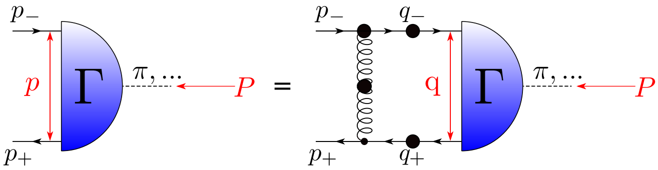
Bound states - Bethe-Salpether equations

Homogeneous BSE:



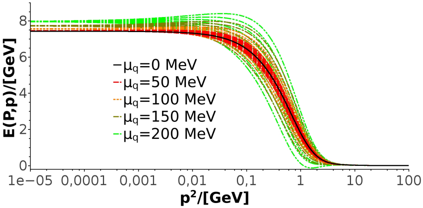
Pion amplitude in vacuum

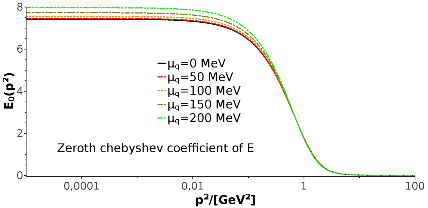
$$\Gamma_\pi(P,p) = \gamma_5 \left[-iE(P,p) + \not{P} F(P,p) + \not{p}(Pp)G(P,p) + \left[\not{P}, \not{p}\right] H(P,p)\right]$$

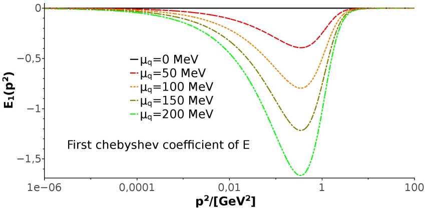
Pion amplitude in medium

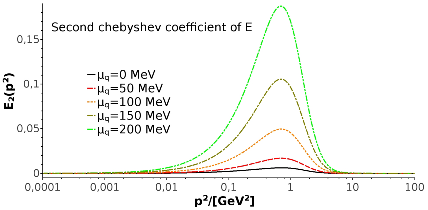
$$\Gamma_\pi(P,p) = \gamma_5 \left[-iE(P,p) + P_4\gamma_4 F_l(P,p) + \vec{P}\vec{\gamma}F_t(P,p) + \dots\right]$$

Pion amplitude





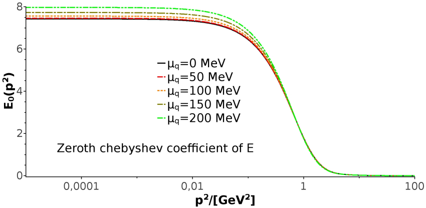


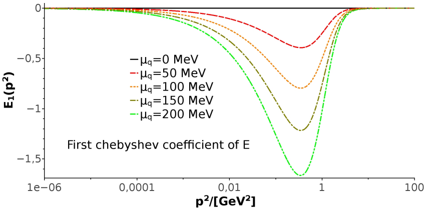


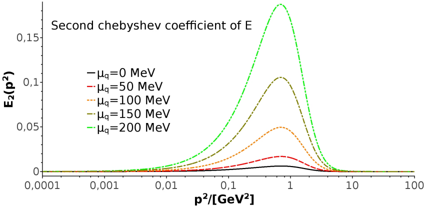
Pion amplitude

Chebyshev expansion:

$$E(P^2, p^2, \hat{P}\hat{p}) \approx$$
$$\sum_j E^j(P^2, p^2) T_j(\hat{P}\hat{p})$$







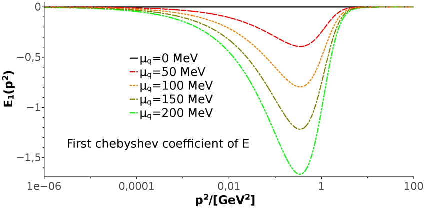
Pion amplitude

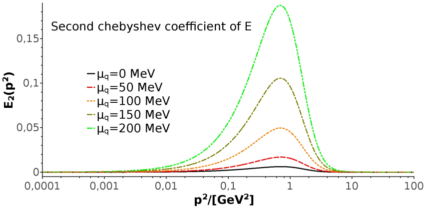
Chebyshev expansion:

$$E(P^2, p^2, \hat{P}\hat{p}) \approx \sum_j E^j(P^2, p^2) T_j(\hat{P}\hat{p})$$

Charge-conjugated pion amplitude:

$$\bar{\Gamma}_\pi(P, p) = \left[C\Gamma_\pi(P, -p)C^{-1}\right]^T$$





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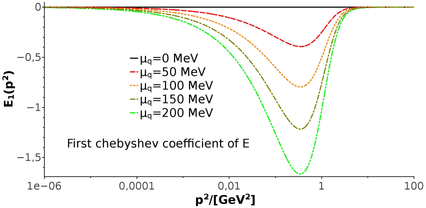


Charge-conjugated pion amplitude:

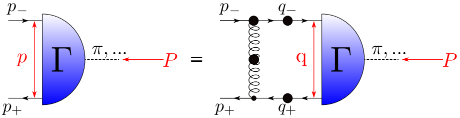
$$\bar{\Gamma}_\pi(P, p) = \left[C\Gamma_\pi(P, -p)C^{-1}\right]^T$$

Pion: $J^{PC} = 0^{-+}$

→ odd Chebyshev coefficients vanish

→ μ_q breaks C-Parity

Progress: Complex quark



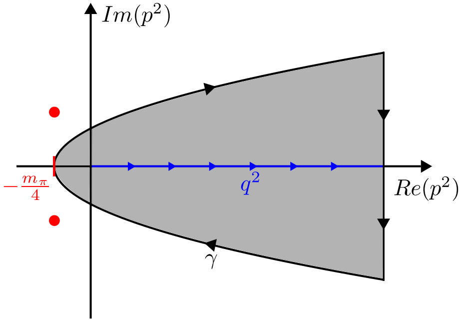
Momentum routing:

$$p_\pm^2 = \left(p \pm \frac{P}{2}\right)^2 \quad , \quad P = \left(\vec{0}, im_\pi\right)$$

$$\longrightarrow \quad p_\pm^2 = p^2 - \frac{m_\pi^2}{4} \pm im_\pi \sqrt{p^2}$$

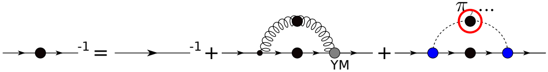
Cauchy method:

$$f(p_0^2) = \frac{\oint_\gamma \frac{f(p^2)}{p^2 - p_0^2} dp^2}{\oint_\gamma \frac{1}{p^2 - p_0^2} dp^2}$$



Reminder

Wanted: Influence of Pion back-coupling onto QCD phase diagram and CEP



Needed: Pion propagator in medium

Pion Bethe-Salpether amplitude in medium

Second step: Investigate Pion properties at finite chemical potential

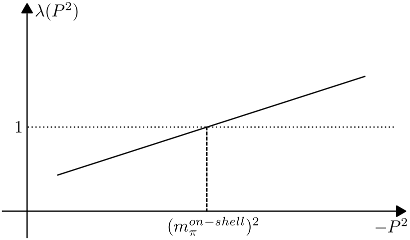
Pion properties at finite chemical potential

Eigenvalue equation:

$$K(P^2)\Gamma(P^2,p^2) = \lambda(P^2)\Gamma(P^2,p^2)$$

On-shell condition:

$$P^2 = -M_j^2 \quad \Rightarrow \quad \lambda(P^2) = 1$$

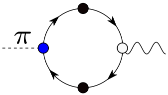


$\Rightarrow$ Pion mass m_π extraction

Power method:

$$\hat{K}(P^2)\Gamma_n(P^2,p^2) = \lambda_n(P^2)\Gamma_{n+1}(P^2,p^2), \qquad \lambda_n(P^2) \xrightarrow{n\to\infty} \lambda(P^2)$$

Pion properties at finite chemical potential

$$f_\pi P_\mu \propto$$ 

Pion decay constant in vacuum

$$f_\pi P^\mu = Z_2 N_c \int_q tr_D \left[\gamma_5 \gamma^\mu S(q_+) \Gamma_\pi(q,P) S(-q_-)\right]$$

From vacuum to finite chemical potential:

$$f_\pi P^\mu \xrightarrow{\mu_q > 0} \left(f_\pi^t P_{\mu\nu}^{\mathscr{L}}(v) + f_\pi^s P_{\mu\nu}^{\mathscr{T}}(v) \right) P^\nu$$

Long. $P_{\mu\nu}^{\mathscr{L}}(v)$ and trans. proj. $P_{\mu\nu}^{\mathscr{T}}(v)$ with assigned direction $v = (\vec{0}, 1)$

Pion propagator

Pion velocity:

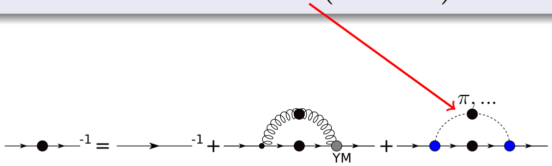
$$u = \frac{f_\pi^s}{f_\pi^t}$$

Pion dispersion relation:

$$\omega^2 = u^2 \left(\vec{P}^2 + m_\pi^2 \right)$$

Pion propagator:

$$D_\pi(P) = \frac{1}{P_4^2 + u^2 \left(\vec{P}^2 + m_\pi^2 \right)}$$



Son and Stephanov Phys.Rev. D, 66(7) (2002)
Fischer and Mueller Phys.Rev. D 84, 054013 (2011)

Silver Blaze property

Definition:

T. D. Cohen, Phys. Rev. Lett. 91 , 222001 (2003)
T. D. Cohen, arXiv:hep-ph/0405043 (2004)

$\mu_q <$ mass gap of the system δ and $T = 0$

$\Rightarrow$ Partition function and observables independent from μ_q

$$\delta = \frac{m_B}{3} \qquad m_B = \text{lightest baryon}$$

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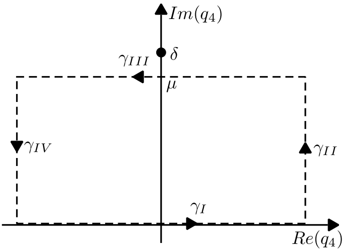


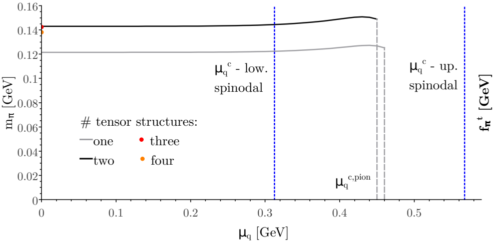
Substitution:

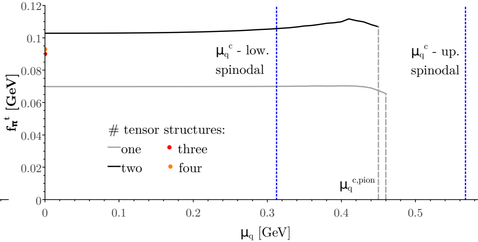
$$\langle \bar{\Psi}\Psi \rangle \sim \int_q S(\vec{q}^2, q_4 + i\mu_q)$$

$$\overset{q_4 \to q_4 + i\mu_q}{=} \int_q S(\vec{q}^2, q_4) \sim \langle \bar{\Psi}\Psi \rangle_{vac}$$

Condition: No singularity in γ

Pion properties at finite chemical potential results





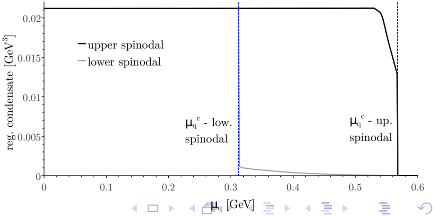
- Silver Blaze property fulfilled
- No pion solution above $\mu_q^{c,pion}$

Qualitative agreement with (simpler) truncations:

Roberts, Phys.Part.Nucl. 30:223-257 (1999)

Roberts and Schmidt, Prog.Part.Nucl.Phys. 45(1) 1-103 (2000)

Liu, Chao, Chang and Wei, Chin. Phys. Lett., Vol 22, Nr 1



Summary and Outlook

Summary:

- Present QCD phase diagram for $N_f = 2 + 1$ quark flavors
- Pion properties $(m_\pi,\ f_\pi)$ at $\mu_q \neq 0$, two tensor structures, effective interaction
  - $\mu_q \neq 0$ breaks C-Parity of pions ✓
  - m_π, f_π fulfill Silver Blaze property ✓
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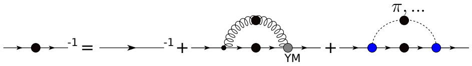
Outlook:

- Improve Pion truncation
- Include temperature
- Calculate Pion back-coupling



Summary and Outlook

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Thank you for your attention!

Outlook:

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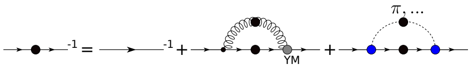
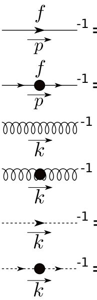
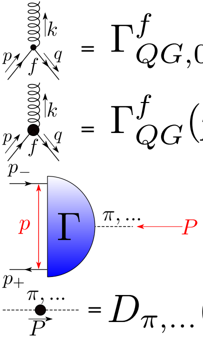


Backups

Feynman rules

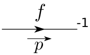


$$= S_{f,0}^{-1}(p)$$
$$= S_{f}^{-1}(p)$$
$$= D_{0}^{-1}(k)$$
$$= D^{-1}(k)$$
$$= G_{0}^{-1}(k)$$
$$= G^{-1}(k)$$

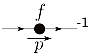


$$= \Gamma_{QG,0}^{f}(p,q,k)$$
$$= \Gamma_{QG}^{f}(p,q,k)$$
$$= \Gamma_{\pi,\ldots}(P,p)$$
$$= D_{\pi,\ldots}(P)$$

Quark propagator



$= S_{f,0}^{-1}(p)$



$= S_f^{-1}(p)$

Bare quark propagator (vacuum)

$$S_0^{-1}(p) = Z_2(i\not{p} + \mathbb{1} Z_m m_r)$$

Dressed propagator (vacuum)

$$S^{-1}(p) = i\not{p}A(p) + \mathbb{1}B(p)$$

Scalar B and vector A dressing function

Introducing the medium:

Heat bath: T and μ_q introduce assigned direction $u = \left(\vec{0}, 1\right)$

- Quark vector dressing function splits up in spatial A and heat bath C part ($\not{p} \rightarrow \vec{p}\vec{\gamma}, \tilde{\omega}_p\gamma_4$):

Dressed quark propagator (medium)

$$S^{-1}(p) = i\vec{p}\vec{\gamma}A(\omega_p, \vec{p}) + i\tilde{\omega}_p\gamma_4 C(\omega_p, \vec{p}) + \mathbb{1}B(\omega_p, \vec{p}) + \vec{p}\vec{\gamma}\tilde{\omega}_p\gamma_4 D(\omega_p, \vec{p})$$

Gluon propagator

Dressed gluon propagator (vacuum, Landau gauge)

$$D_{\sigma\nu}(k) = P^{\mathscr{T}}_{\sigma\nu}(k)\frac{Z(k)}{k^2}$$

Projector:

$$P^{\mathscr{T}}_{\sigma\nu}(k) = \left(\delta_{\sigma\nu} - \frac{k_\sigma k_\nu}{k^2}\right)$$

Introducing the medium:

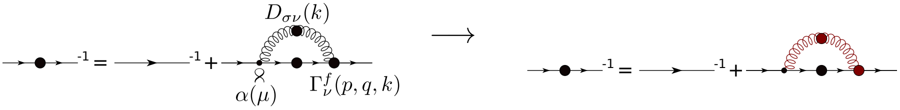
Heat bath: T and μ_q introduce assigned direction $u = \left(\vec{0}, 1\right)$

- Gluon splits up into a part transversal and a part longitudinal to heat bath ($P^{\mathscr{T}}_{\sigma\nu}(k) \rightarrow P^{T}_{\sigma\nu}(k)$, $P^{L}_{\sigma\nu}(k)$)

Dressed gluon propagator (medium)

$$D_{\sigma\nu}(k;T) = \left(P^{T}_{\sigma\nu}(k)\frac{Z_T(k;T)}{k^2} + P^{L}_{\sigma\nu}(k)\frac{Z_L(k;T)}{k^2}\right)$$

Effective interaction truncation



Combining vertex Γ and gluon Z to renormalization-group invariant effective coupling

$$\alpha(\mu)D_{\sigma\nu}(k)\Gamma^f_\nu(p,q,k) \propto \alpha(k^2)\frac{P^{\mathscr{T}}_{\sigma\nu}(k)}{k^2}\gamma_\nu$$

Maris-Tandy ansatz:

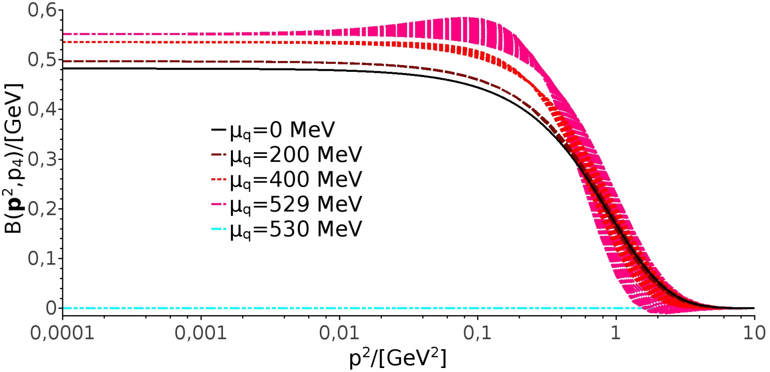
Simple ansatz, quark flavor decouple

$$\alpha(k^2) = \alpha_{IR}(k^2) + \alpha_{UV}(k^2)$$

Maris and Tandy, Phys.Rev. C 60, 055214 (1999)

Results for quark propagator at finite chemical potential

Scalar dressing function:



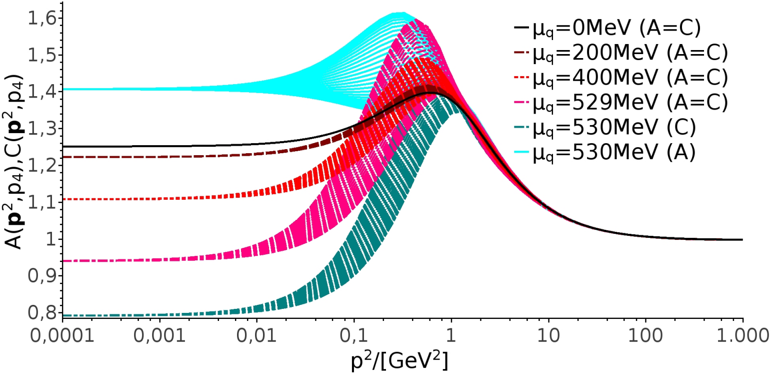
$$S^{-1}(p) = i\vec{p}\vec{\gamma}A(\vec{p}^2, p_4) + i\tilde{p}_4\gamma_4 C(\vec{p}^2, p_4) + \mathbb{1}B(\vec{p}^2, p_4)$$

$$\langle \bar{\Psi}\Psi \rangle \propto \int_q \frac{B(\vec{p}^2, p_4)}{D(\vec{p}^2, p_4)}$$

Chiral phase transition point $\mu_q^c = 530\,\text{MeV}$

Results for quark propagator at finite chemical potential

Vector dressing functions:

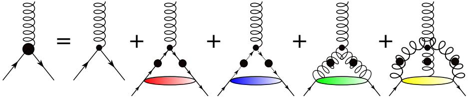


$$S^{-1}(p) = i\vec{p}\vec{\gamma}A(\vec{p}^2, p_4) + i\tilde{p}_4\gamma_4 C(\vec{p}^2, p_4) + \mathbb{1}B(\vec{p}^2, p_4)$$

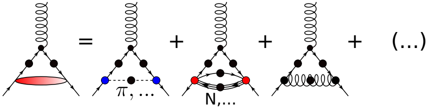
Degeneration of vector dressing function only in chiral limit

Skeleton expansion

DSE of the fully dressed quark-gluon vertex:



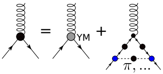
Skeleton expansion in terms of hadronic contributions:



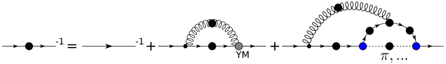
$\longrightarrow$ Separation of hadronic terms and Yang-Mills terms

Skeleton expansion

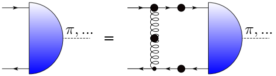
Only mesonic contributions:



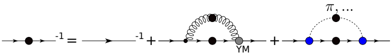
Inserting vertex into quark:

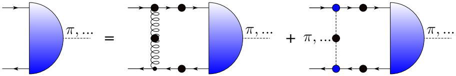


BSE:



Assumption: Only Yang-Mills part present in BSE ⇒ rewrite Quark DSE by inserting DSE into second diagram





Approximation justified if BSE vertex function with and without pion interaction term do not differ strongly

Fischer, Nickel and Wambach, Phys.Rev. D 76(9) (2007)

Curvature

Curvature κ = first coefficient in taylor series expansion of transition line in terms of $\frac{\mu_q}{T}$

$$\frac{T^c(\mu_q)}{T_0^c} = 1 - \kappa \left(\frac{\mu_q}{T_0^c}\right)^2 + O\left[\left(\frac{\mu_q}{T_0^c}\right)^4\right]$$

T_0^c = transition temperature for $\mu_q = 0$

Remark: Curvature depends on choice of pesudo-critical temperature definition in crossover region