High Density Fluctuation in Neutron Stars

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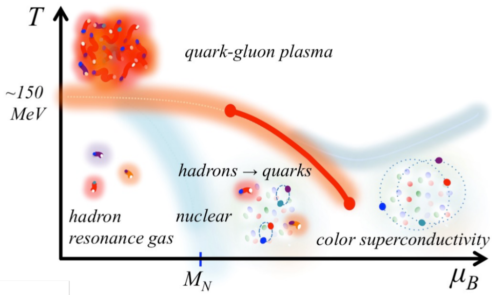
Motivation I

Phase diagram of Quantum Chromodynamics



[Baym, Hatsuda, Kojo, Powell, Song, Takatsuka '17]

Explore phase structure in cold and dense region!

Motivation II

This work:

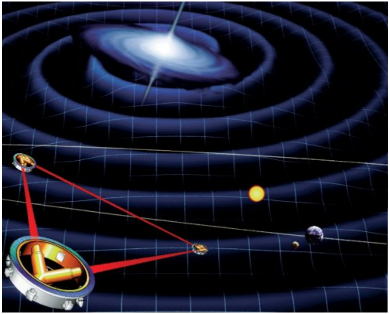
- Influence of fluctuations
  $\Rightarrow$ mean-field versus functional renormalization group (FRG)

- Role of strangeness
  $\Rightarrow$ two-flavor versus 2+1-flavor quark-meson model

Motivation III

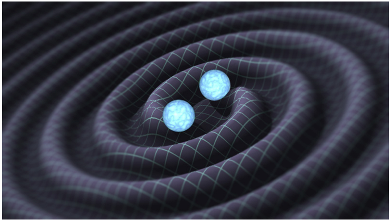
Experimental research interest:

LISA (planned: 2030s)



NICER experiment

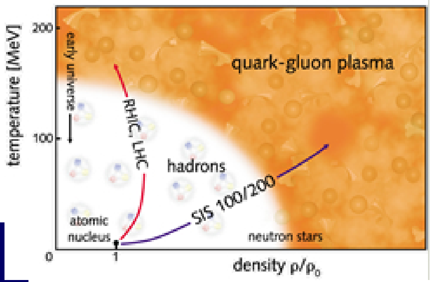
LIGO, Virgo, GEO, ...



Motivation IV

Experimental research interest:

CBM @ FAIR

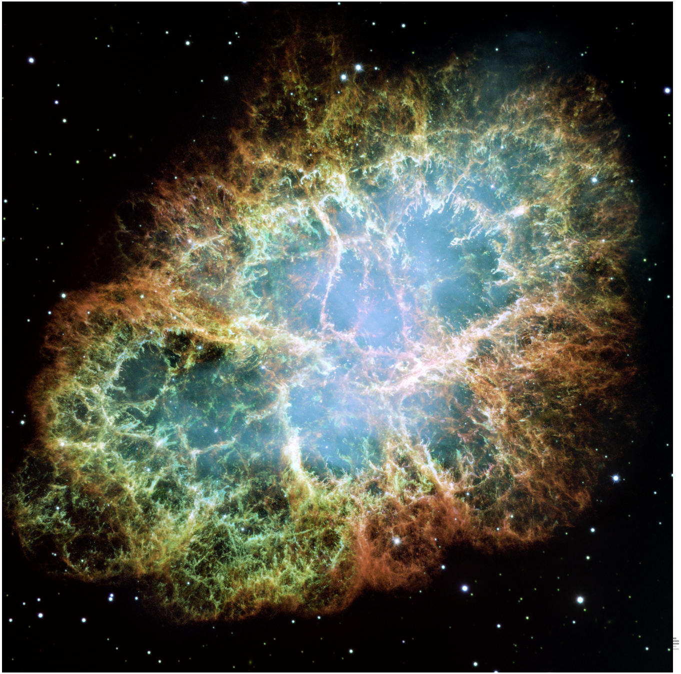


The
Compressed
Baryonic
Matter experiment

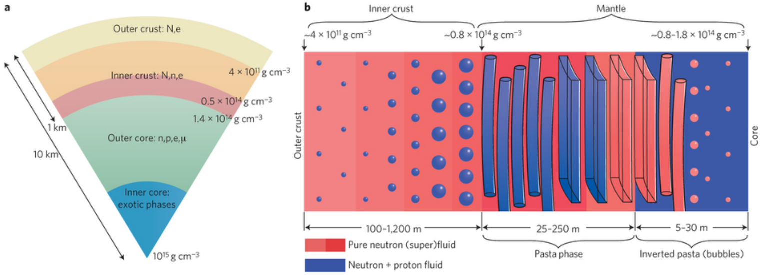
Neutron Stars – An Overview I

Properties:

- End of stellar lifecycle of stars of $\sim 10\,M_\odot$
- Mass benchmark: pulsar J0348+0432 at $\sim 2.01(4)\,M_\odot$
- Composition:
  - Crust: solid, transition to liquid nuclear matter
  - Outer core: up to density of $\sim 2n_0$
  - Inner core: quark degrees of freedom



Neutron Stars – An Overview II



[https://compstar.uni-frankfurt.de/wp-content/uploads/2015/12/pastaNewton.jpg]

Neutron Stars – An Overview III

Non-rotating stars in hydrostatic equilibrium from GR:
Tolman-Oppenheimer-Volkoff equation

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -G\frac{(\varepsilon(r) + p(r))[m(r) + 4\pi r^3 p(r)]}{r[r - 2Gm(r)]}$$

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

- Coupled ordinary differential equation (ODE)
- initial conditions $m(0) = 0$, $p(0) = p_0$
- Integrate up to $p(R) = p_{\mathrm{min}} \gtrsim 0$
  $\Rightarrow$ neutron star radius R, mass $M = m(R)$
  $\Rightarrow$ one-parameter curve $M(R)$

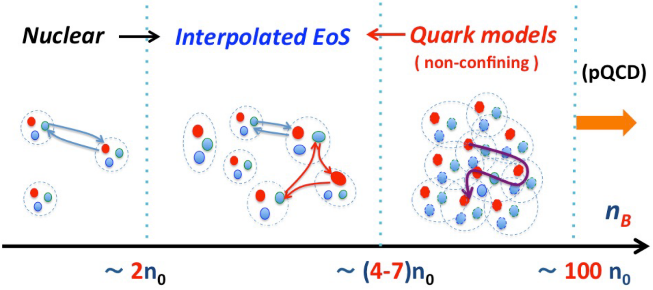
Neutron Stars – An Overview IV

- Energy density $\varepsilon(r)$ from equation of state (EoS) $p(\varepsilon)$
- Properties of matter enter here
- Constuction of EoS: combine nuclear matter and quark matter EoS
  - Nuclear models in beta equilibrium
  - Quark matter: Nambu–Jona-Lasinio (NJL) model calculations, phenomenological approaches (bag models, ...)
- either choose EoS with lower energy density for each μ_B (first-order transition) or interpolate (crossover)

Neutron Stars – An Overview V

Requirements for the stiffness of the EoS:

$$\frac{\partial n_B}{\partial \mu_B} = \frac{\partial^2 p}{\partial \mu_B^2} > 0 \quad , \quad c_s^2 = \frac{\partial p}{\partial \varepsilon} \leq 1$$



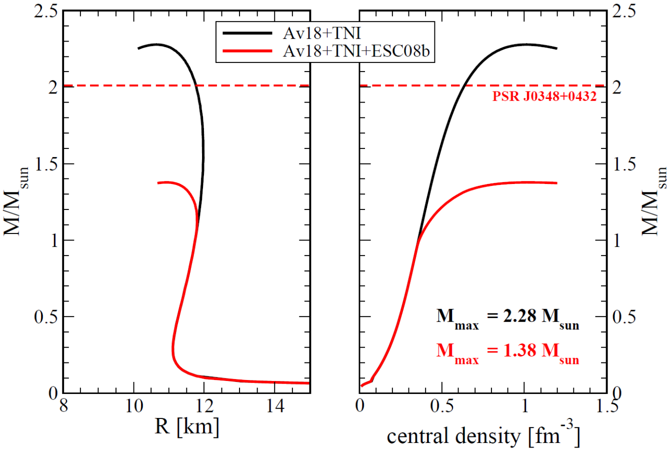
[Baym, Hatsuda, Kojo, Powell, Song, Takatsuka '17]

Neutron Stars – An Overview VI

Current problems:

- Description of inner core: QCD phase diagram
- Pure neutron stars, hybrid stars, or both?
  $\Rightarrow$ "masquerade problem"
  discussion: [Alvarez-Castillo, Blaschke '14]
- Role of strangeness
  $\Rightarrow$ "hyperon puzzle"
  hyperonic model calculations: [Djapo, Schaefer, Wambach '10], [Bombaci '16]

Neutron Stars – An Overview VII



[Bombaci '16]

Quark-meson Model I

- Effective low-energy model of QCD
- Spontaneous chiral symmetry breaking and chiral phase transition
- Bosonization of effective four-quark interactions

$$(\bar{q} T_f^a q)^2 \quad , \quad (\bar{q} T_f^a \gamma_5 q)^2$$

[Braun '11]

- Four-quark interaction re-generated due to fluctuations
  (future work: dynamical hadronization)

Quark-meson Model II

$$\mathcal{L}_{\mathrm{QM}} = \bar{q}[\partial_\mu \gamma_\mu + g T_f^a(\sigma_a + \mathrm{i}\gamma_5 \pi_a)]q + \frac{1}{2}(\partial_\mu \sigma_a)^2 + \frac{1}{2}(\partial_\mu \pi_a)^2 + U_\Lambda$$

- $U_\Lambda(\rho_1, \ldots, \rho_{N_f})$ function of the N_f chiral invariants

$$\rho_n := \mathrm{Tr}\left[\left(\Phi^\dagger \Phi\right)^n\right] \quad , \quad \Phi := T_f^a(\sigma_a + \mathrm{i}\pi_a)$$

- Explicit chiral symmetry breaking

$$-\mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] = -h_a \sigma_a \quad , \quad H := h_a T_f^a$$

- $U(1)_A$ axial symmetry breaking ('t Hooft determinant)

$$-c\left[\mathrm{Det}\left(\Phi^\dagger\right) + \mathrm{Det}\left(\Phi\right)\right]$$

Quark-meson Model III

2 flavors:

- $T_f^a = \tau_a/2$ (τ_a: Pauli matrices)
- $-c\left[\det\left(\Phi^\dagger\right) + \det\left(\Phi\right)\right] = -\frac{c}{2}(\sigma_0^2 + \vec{\pi}^2 - \pi_0^2 - \vec{\sigma}^2)$
  $\Rightarrow$ mass splitting, drop heavy π_0 and $\vec{\sigma}$!
- $\varphi := (\sigma, \vec{\pi})^T \Rightarrow O(4)$ symmetry!
- One independent chiral invariant: $\rho \sim \vec{\varphi}^2$
- Chiral condensate $\langle\sigma\rangle$ order parameter for χSB

Quark-meson Model IV

2+1 flavors:

- $T_f^a = \frac{\lambda_a}{2}$ (λ_a: Gell-Mann matrices)
- T_0, T_3 and T_8 are diagonal
- Rotate in flavor space

$$\begin{pmatrix} \sigma_l \\ \sigma_s \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_8 \end{pmatrix}$$

  and set $\langle \sigma_3 \rangle \equiv 0$ to preserve isospin symmetry
  $\Rightarrow$ two chiral condensates, $\langle \Phi \rangle = \mathrm{diag}\left(\frac{\langle \sigma_l \rangle}{2}, \frac{\langle \sigma_l \rangle}{2}, \frac{\langle \sigma_s \rangle}{\sqrt{2}} \right)$
  $\Rightarrow$ two chiral invariants ρ_1 and ρ_2

- $U(1)_A$ breaking at EV: $-\frac{c}{2\sqrt{2}} \langle \sigma_l \rangle^2 \langle \sigma_s \rangle$
  $\Rightarrow$ only quantity that goes with field cubed!

Mean-field Approximation

- Meson fields as static, constant background fields
- Quark contribution to grand potential:

$$\Omega_q = \frac{N_c}{\pi^2} T \sum_f \int_0^\infty \mathrm{d}p\, p^2 \left[\ln(1 - n_f) + \ln(1 - \bar{n}_f)\right]$$

with

$$n_f := \frac{1}{1 + \exp\left(\frac{E_f - \mu}{T}\right)} \quad , \quad \bar{n}_f := \frac{1}{1 + \exp\left(\frac{E_f + \mu}{T}\right)}$$

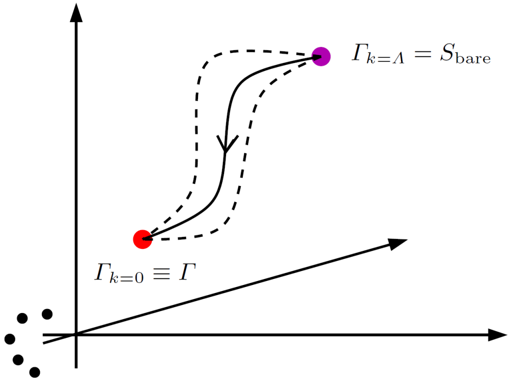
- Divergent vacuum term

$$-\frac{N_c}{\pi^2} \sum_f \int_0^\infty \mathrm{d}p\, p^2 \sqrt{p^2 + m_f^2}$$

missing, "standard mean-field approximation" (sMFA)

Funtional Renormalization Group

- Successively integrate out fluctuations in momentum shells
- Modified scale-dependent effective action $\Gamma_k[\phi]$



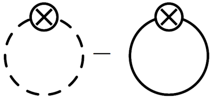
[Gies '06]

FRG – Wetterich Equation

Wetterich equation:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}_B\left[\partial_t R_k^B\left(\Gamma_k^{(2)} + R_k^B\right)^{-1}\right] - \mathrm{Tr}_F\left[\partial_t R_k^F\left(\Gamma_k^{\{2\}} + R_k^F\right)^{-1}\right]$$

$$\partial_t \Gamma_k = \frac{1}{2}$$



[Rennecke, PhD thesis '15]

3-d Litim regulators

$$R_k^B(p^2) = (k^2 - \vec{p}^2)\,\theta\left(1 - \frac{\vec{p}^2}{k^2}\right)\ ,\ R_k^F(p^2) = \mathrm{i}\vec{p}\cdot\vec{\gamma}\left(\sqrt{\frac{k^2}{\vec{p}^2}} - 1\right)\theta\left(1 - \frac{\vec{p}^2}{k^2}\right)$$

FRG – Local Potential Approximation

LPA truncation:

$$\Gamma_k[\phi] = \int \mathrm{d}^4 x \left[U_k(\phi) + \frac{1}{2} Z_k (\partial_\mu \phi)^2 + \mathcal{O}(\partial^4) \right]$$

with $Z_k \equiv 1$.

Spatially homogeneous VEV ϕ_0:

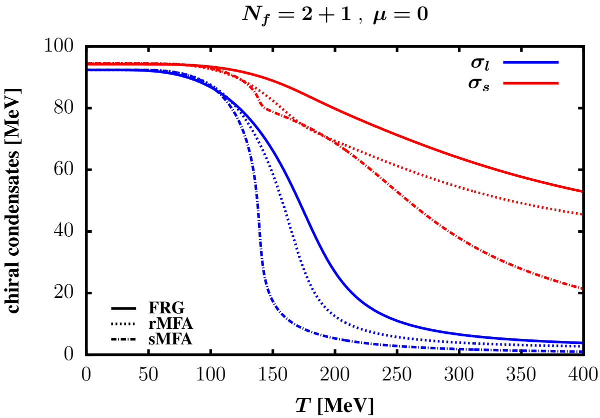
$$\Omega = -\frac{1}{\beta V} \ln Z = \frac{1}{\beta V} \Gamma_{k=0}[\phi_0] = U_{k=0}(\phi_0)$$

FRG – Application to QM Model

$$\partial_t U_k = \frac{k^5}{12\pi^2}\left\{\sum_b \frac{1}{E_b}\coth\left(\frac{E_b}{2T}\right) -2N_c \sum_f \frac{1}{E_f}\left[\tanh\left(\frac{E_f-\mu}{2T}\right)+\tanh\left(\frac{E_f+\mu}{2T}\right)\right]\right\}$$

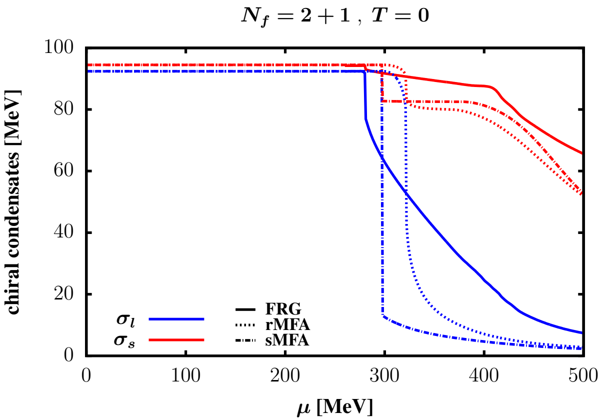
- Only integrate out the quark loop
  $\Rightarrow$ mean-field with renormalized vacuum contributions (rMFA)
- Solve full FRG flow on 1-d ($N_f = 2$) and 2-d ($N_f = 2+1$) grid
  $\Rightarrow$ Meson fluctuations included!
- UV cutoff: 1 GeV

Results – Chiral Phase Transition I



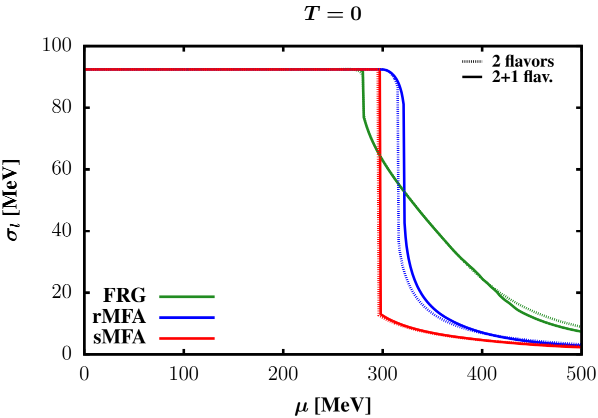
- Smooth crossover along T axis

Results – Chiral Phase Transition II



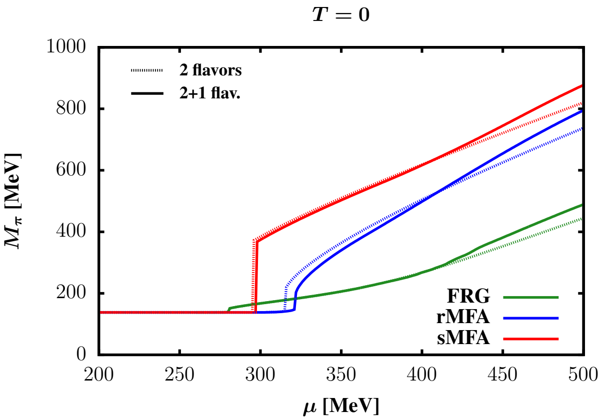
- First-order along μ axis
- Chiral phase transition in strange sector for $\mu \gtrsim m_s$

Results – Chiral Phase Transition III

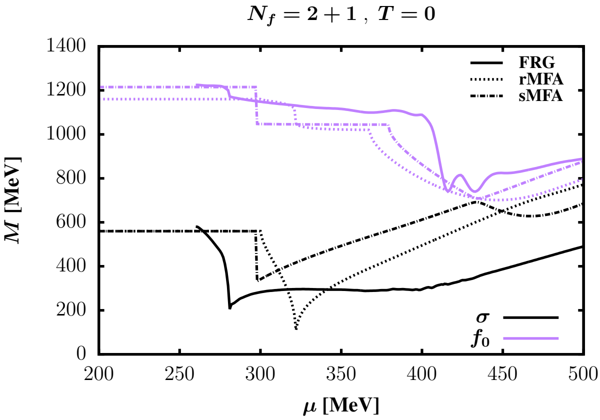


- Strangeness effects only above $\mu \gtrsim m_s$

Results – Chiral Phase Transition IV

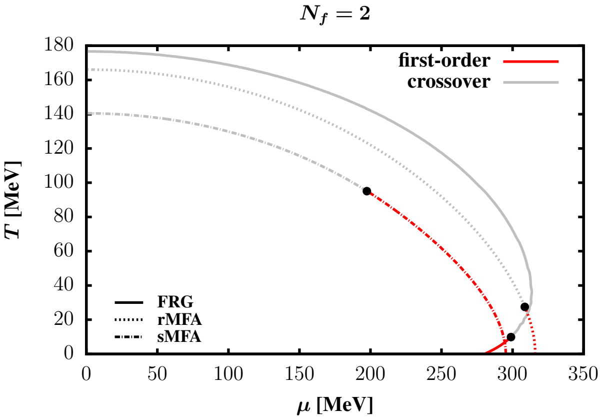


Results – Chiral Phase Transition V



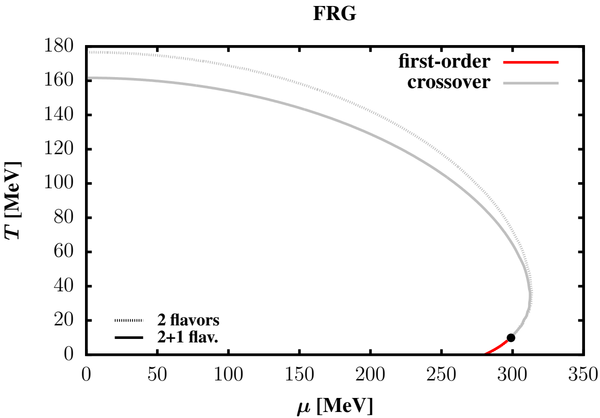
- m_σ moves before chiral phase transition: grid artifact

Results – Thermodynamics I



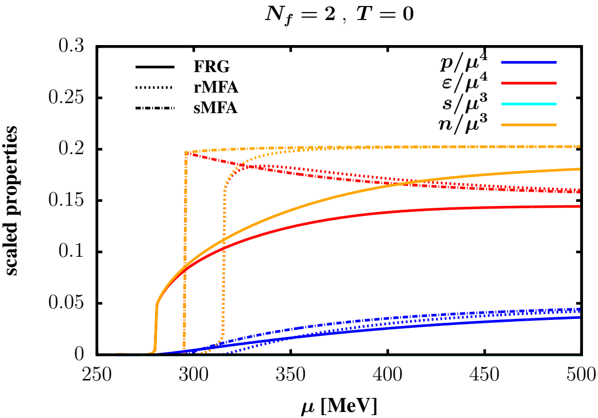
- Fluctuations drive CEP towards lower T
- Back-bending behavior in FRG

Results – Thermodynamics II



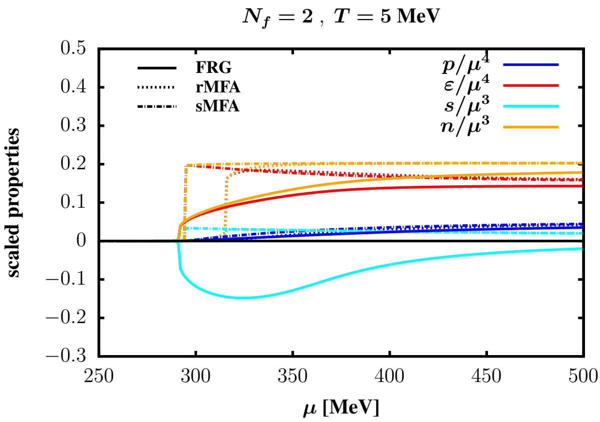
- Strangeness effects vanish close to first-order transition

Results – Thermodynamics III



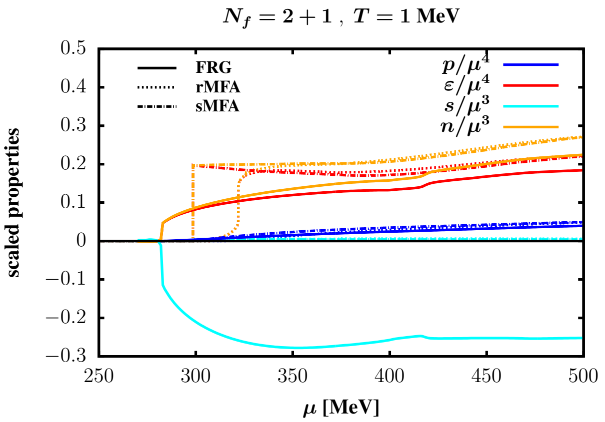
- Only FRG solution deviates from scaling behavior in n

Results – Thermodynamics IV



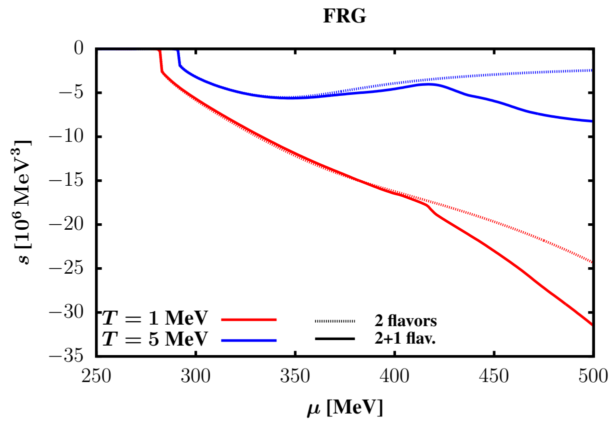
- Negative entropy density at small, finite T

Results – Thermodynamics V



- Onset of strangeness gradual in MFA, more sudden in FRG

Results – Thermodynamics VI



- s becomes more negative close to $T = 0$

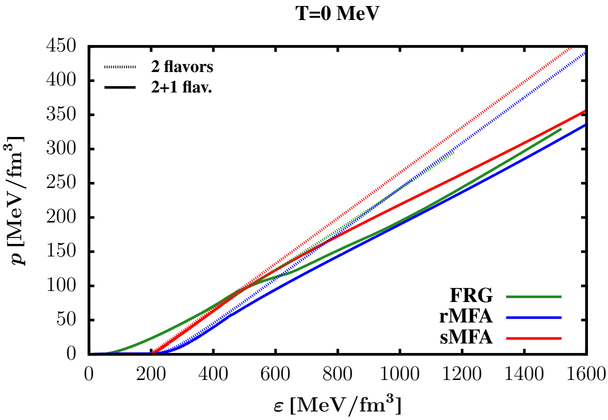
Details on Negative Entropy

Possible reasons for negative entropy:

- Cutoff effect
- Truncation effect
- Inhomogeneous phases
- Color superconductivity (diquark condensate)

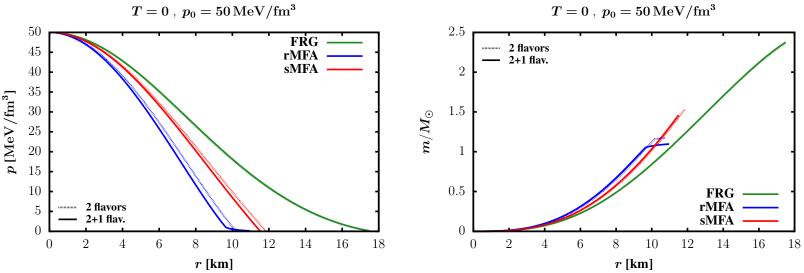
[Tripolt, Schaefer, Smekal, Wambach '17]

Numerical Results – Equation of State



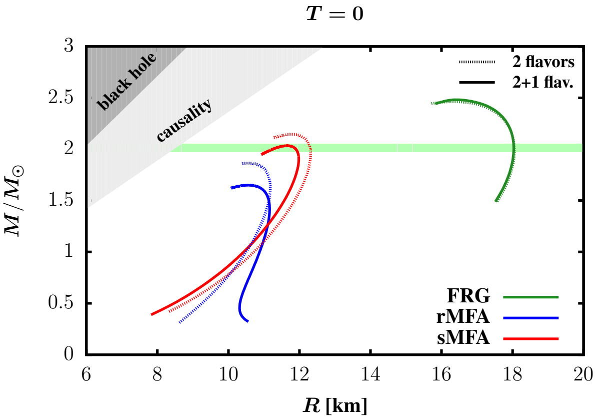
- Strange degrees of freedom soften EoS

Numerical Results – Neutron Stars I



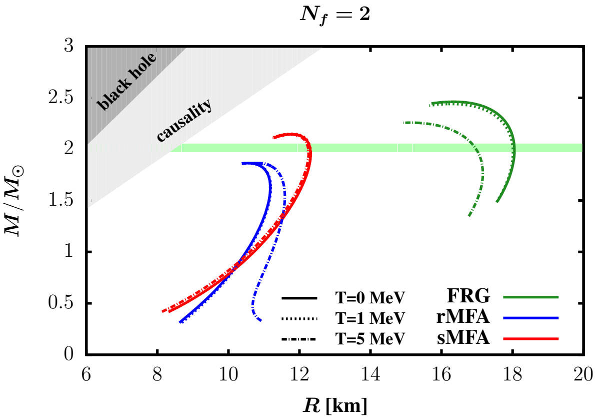
- Kink in rMFA p and m curve: initial meltdown of condensate
- Integration down to $p_{\min} = 0.1\,\mathrm{MeV/fm^3}$

Numerical Results – Neutron Stars II



- Significant impact of fluctuations, no impact of strangeness in FRG

Numerical Results – Neutron Stars III



- Thermal fluctuations only relevant well above $T = 1\,\mathrm{MeV}$

Outlook

In preparation:
[KO, Micaela Oertel, Mario Mitter, Bernd-Jochen Schaefer]

- Probe results at lower m_σ in vacuum
- Extend to PQM model
- Combine with nucleonic EoS (if possible)

Future works:

- Improve truncation
- Dynamical hadronization