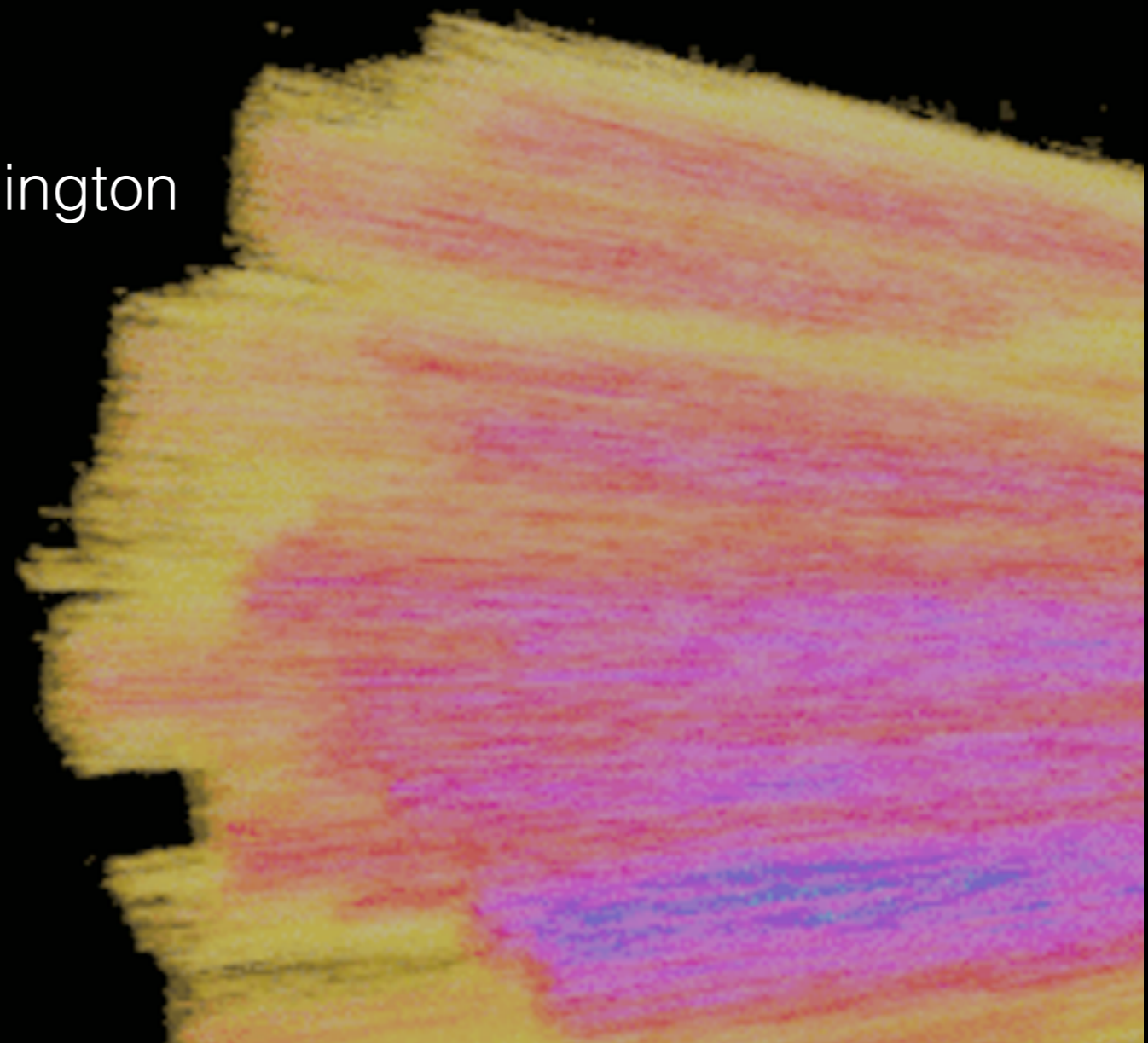


Jet fragmentation in a QCD medium: Universal q/g ratio and wave turbulence

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*Based on
SS, Y. Mehtar-Tani (in preparation)*

JLU Giessen
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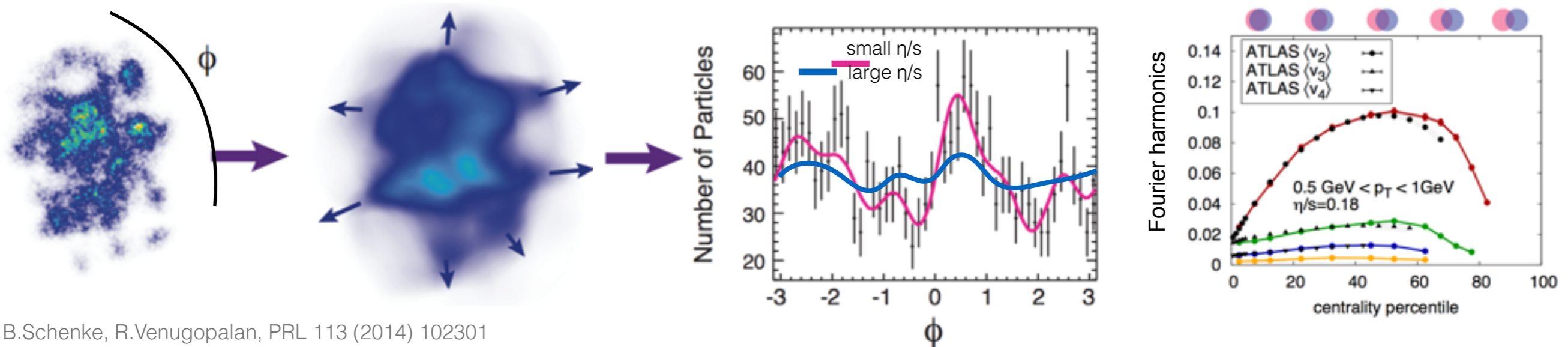
Probes of the QGP

High-energy heavy-ion collisions at RHIC, LHC produce a deconfined QGP

Goal: Characterize & understand properties of QGP phase

Various ways to probe the QGP at different scales

Typical degrees of freedom ($p \sim T$) $\lesssim 1\text{GeV}$



B.Schenke, R.Venugopalan, PRL 113 (2014) 102301

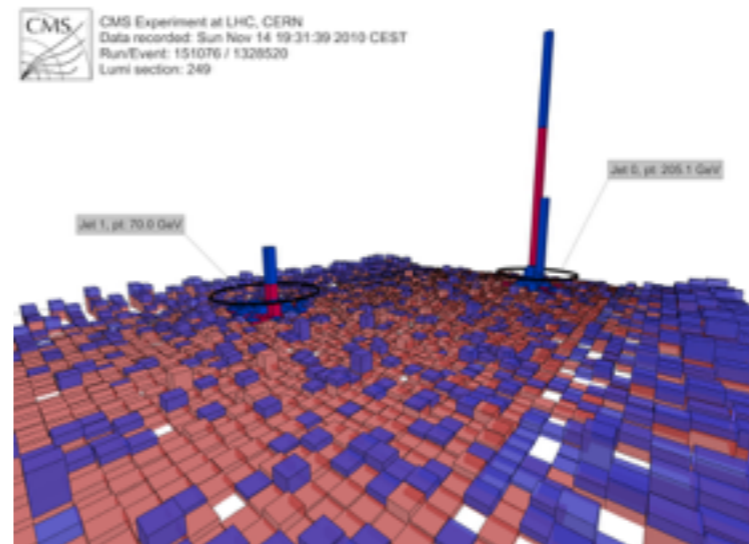
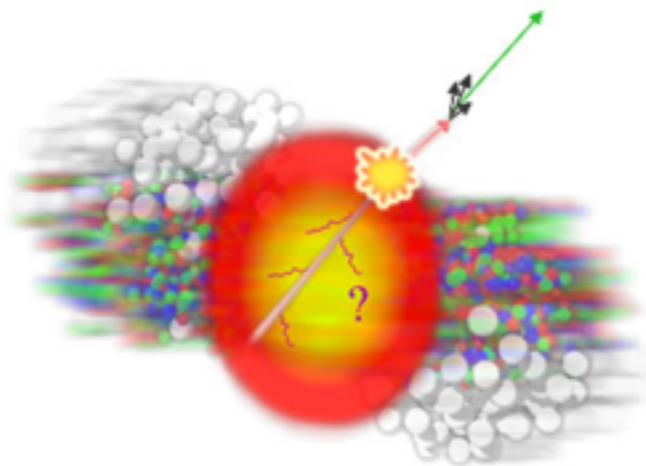
Phenomena: Elliptic flow, close to ideal fluidity ($\eta/s \ll 1$)

=> Extraction of near equilibrium/transport properties of QGP

Probes of the QGP

Various ways to probe the QGP at different scales

Hard probes ($p \gg T$) $\sim 100\text{GeV}$



Hard scale in the problem allows for perturbative control of (at least) some aspects of jet-medium interaction => Calibrated probes of QGP

High- p_T objects provide non-equilibrium probes
=> Interesting in its own right to study their properties

Note that in practice there is no sharp distinction between high p_T and low p_T ; interesting non-equilibrium physics at intermediate scales

Jets/High- p_T probes

Experimentally accessible as:

High- p_T hadrons:

always involve non-perturbative hadron production

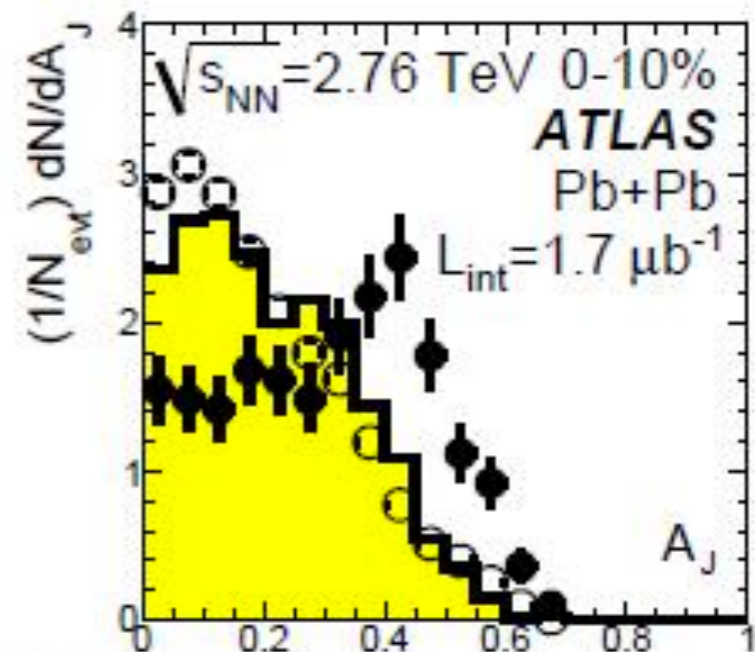
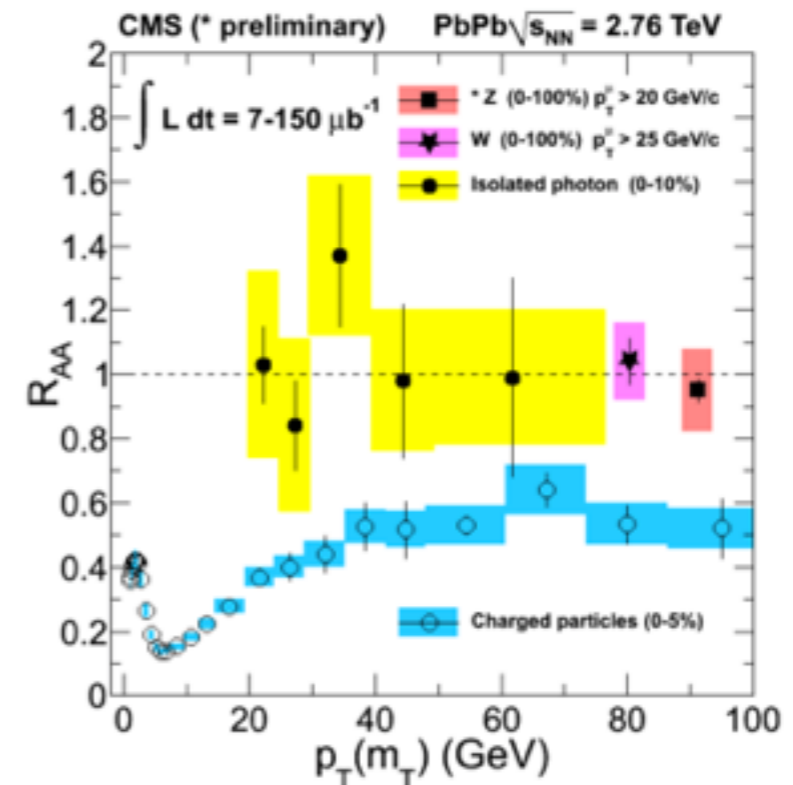
-> energy loss, hadron chemistry, azimuthal anisotropy, ...

reconstructed Jets:

defined by reconstruction algorithm & kinematic cuts

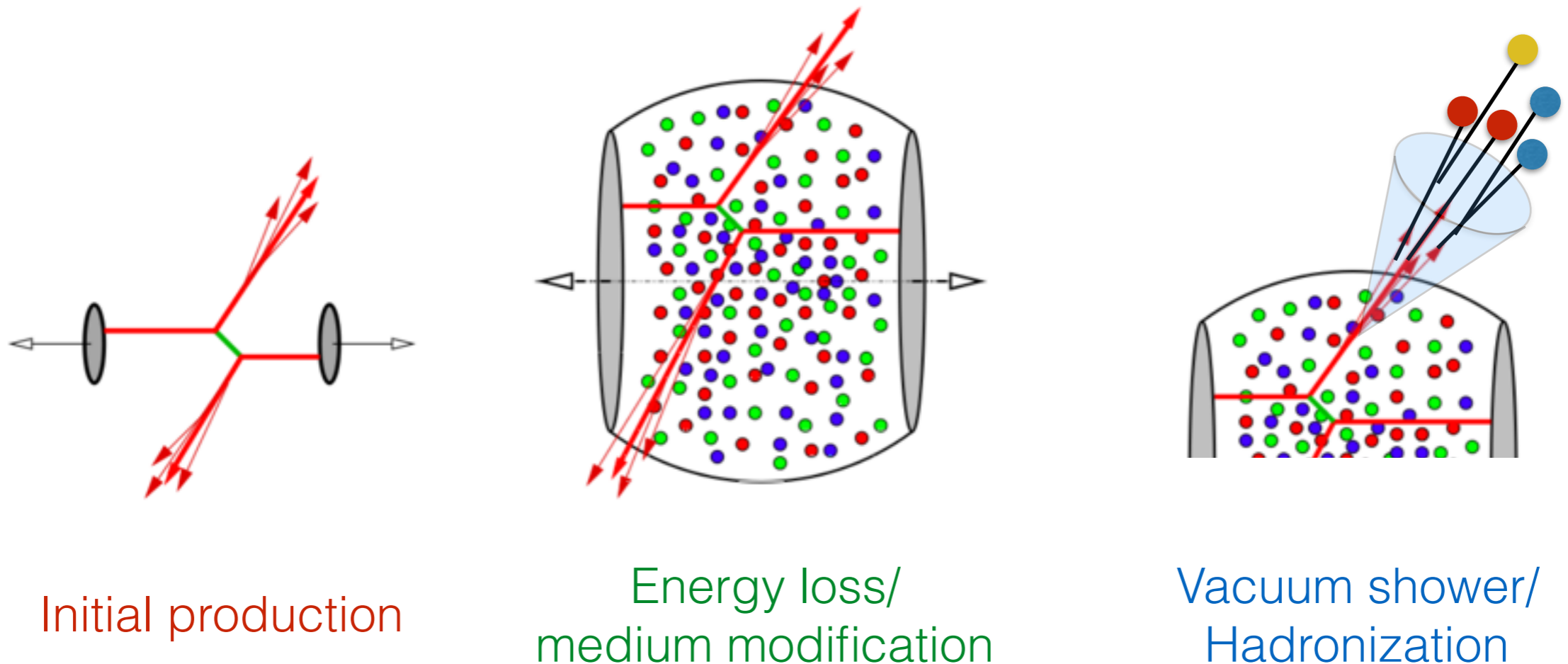
vacuum: controlled calculations in pQCD; precision studies in p+p

-> energy loss, substructure observables



Jets/High- p_T probes

Description of hard probes in HIC involves at least three different processes



even though in general not independent of each other factorization of these processes is often assumed in phenomenological treatments

Outline

Will follow focus on the evolution of the hard probe inside the medium

- How does a hard probe interact with the medium?
- How does a hard probe ($p \gg T$) lose energy to a thermal medium?
- How is the structure of the probe (e.g. chemical composition of fragments) modified through interactions with medium?

Note that there are several MC generators for HIC (JEWEL, PHSD, CoLBT, ...), designed to describe this physics. Scope of this work is somewhat different, namely to understand generic features & develop analytic insights.

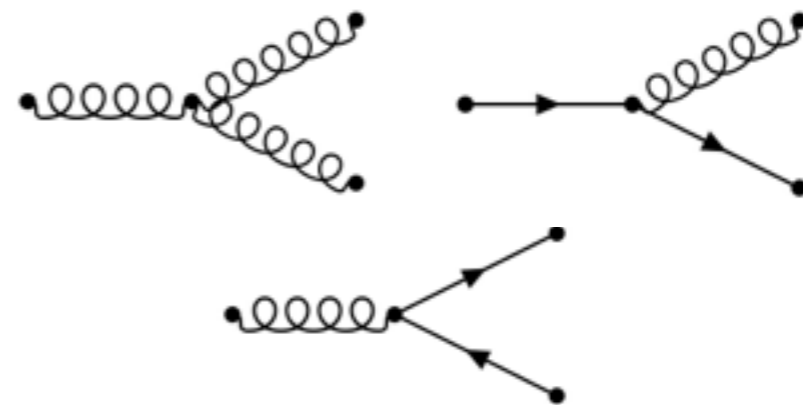
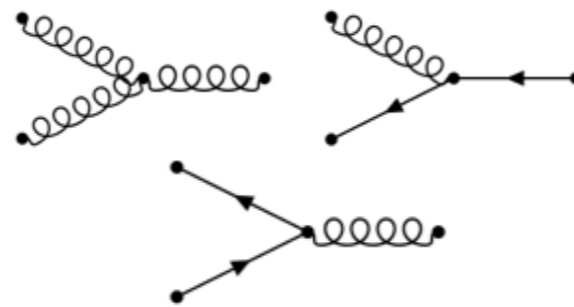
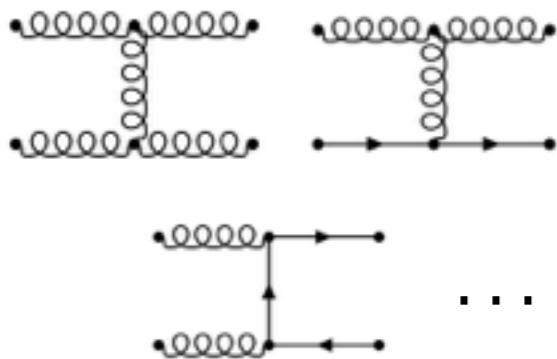
Jet-Medium interactions

Consider an ensemble of highly energetic on-shell quark/gluons ($E \gg T$) propagating in a thermal QGP

=> Elastic & Inelastic interactions with the medium can modify properties

elastic processes & inelastic merging
(2->2; eff. 2->1)

radiative branching (eff. 1->2)



Characteristic rates

$$\Gamma \sim \gamma_{Eq} \frac{T^2}{E^2}$$

$$\Gamma \sim \gamma_{Eq} \frac{T}{E}$$

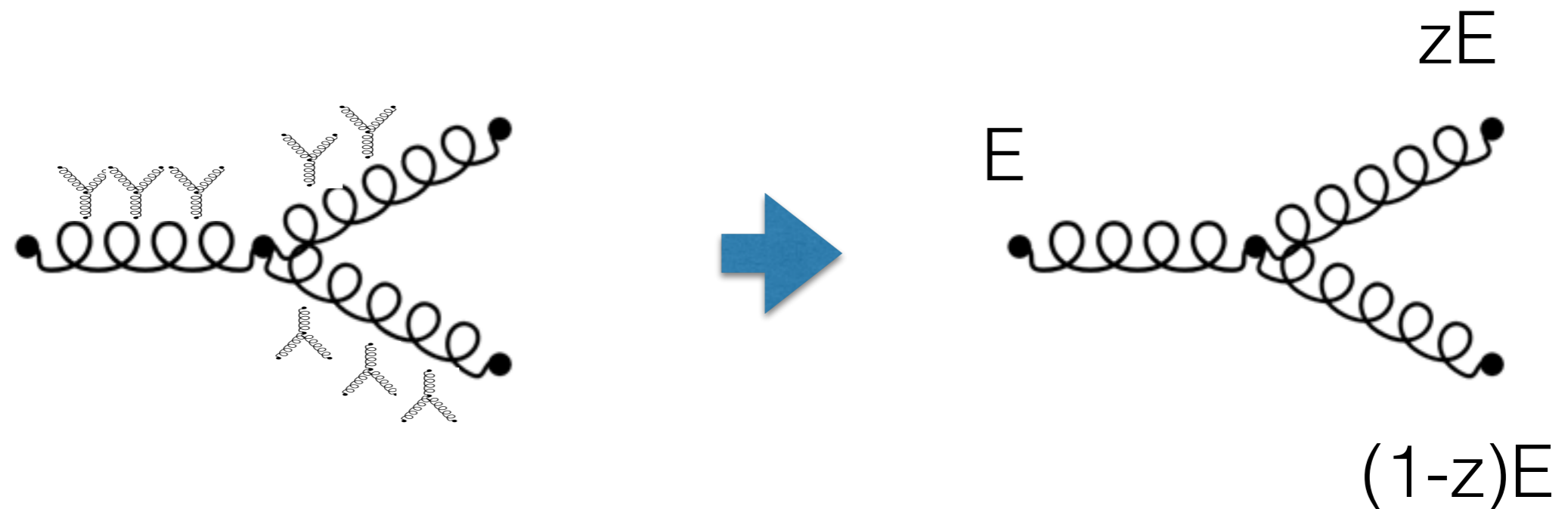
$$\Gamma \sim \gamma_{Eq} \sqrt{\frac{T}{E}}$$

=> Evolution is dominated by radiative branching up to scales $E \sim T$

(c.f. Arnold, Moore, Yaffe (LO); Ghiglieri, Moore, Teaney (NLO))

Medium induced radiation

Splitting of the hard on-shell parton in a (thermal) medium is induced by n elastic interactions with the medium



=> Clearly different from vacuum radiation from off-shell parton (DGLAP)

Since momentum transfer from the medium can occur via frequent soft interactions, need to simultaneously consider multiple re-scatterings and interference effects;

=> first understood for QCD by BDMPS-Z

Medium induced radiation

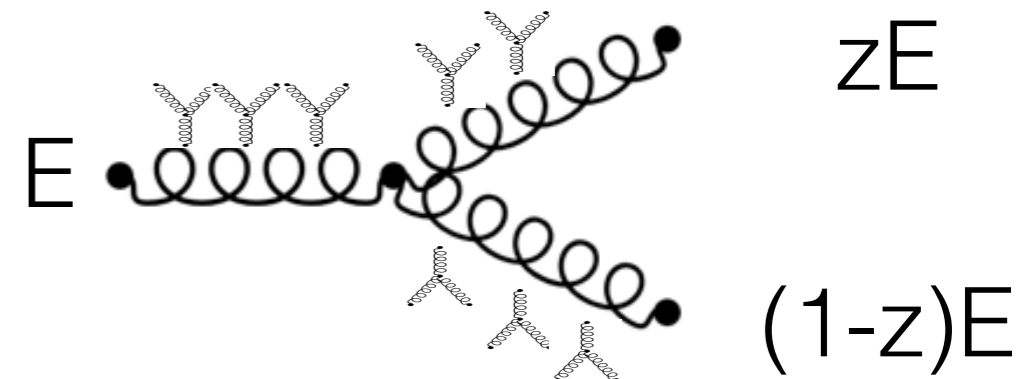
Basic picture emerging from calculation

Emission rate is controlled by formation time

$$t_{form} \sim \frac{z(1-z)E}{k_T^2}$$

which is in turn controlled by rate momentum broadening

$$k_T^2 \sim \hat{q} t_{form}$$



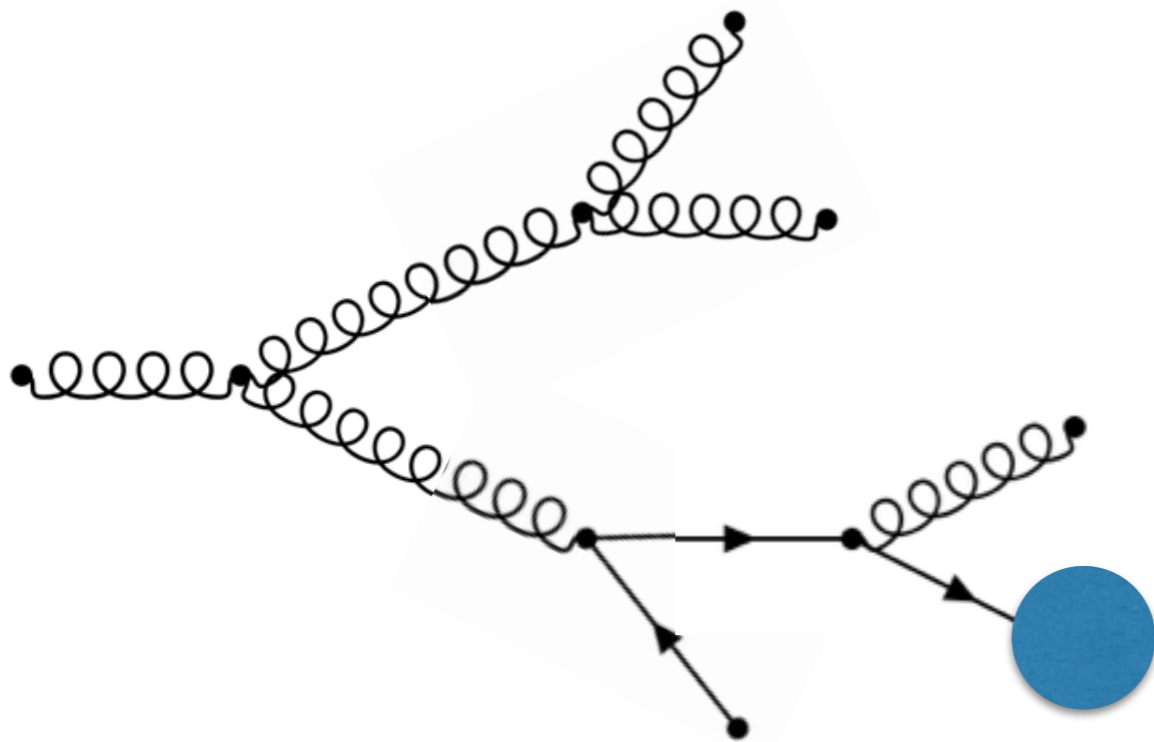
multiple scattering centers emit coherently during formation time $t_{form}^{-1} \sim \sqrt{\frac{\hat{q}}{z(1-z)E}}$

Emission rates (LL): $\Gamma_{fi}^{split}(E, zE, (1-z)E) = \bar{\alpha} \sqrt{\frac{\hat{q}}{E}} K_{fi}(z)$

$$\begin{aligned} \mathcal{K}_{gg}(z) &= \frac{1}{2} 2C_A \frac{[1 - z(1-z)]^2}{z(1-z)} \sqrt{\frac{(1-z)C_A + z^2 C_A}{z(1-z)}}, & \mathcal{K}_{qq}(z) &= \frac{1}{2} C_F \frac{1 + (1-z)^2}{z} \sqrt{\frac{(1-z)C_A + z^2 C_F}{z(1-z)}} \\ \mathcal{K}_{qg}(z) &= \frac{1}{2} 2N_f T_R (z^2 + (1-z)^2) \sqrt{\frac{C_F - z(1-z)C_A}{z(1-z)}}, & \mathcal{K}_{gq}(z) &= \frac{1}{2} C_F \frac{1 + z^2}{(1-z)} \sqrt{\frac{zC_A + (1-z)^2 C_F}{z(1-z)}}. \end{aligned}$$

Multiple branchings

Successive branchings



We will keep track of distribution of fragments in terms of in-medium fragmentation function

$$D_i(x, \tau) \equiv x \frac{dN_i}{dx}$$

which measures distribution of $i=q, g$ fragments after some evolution in the medium

Subsequent splittings are independent of each other and quasi-instantaneous => Effective kinetic equation for in-medium FF

In-Medium fragmentation

Decomposing into
flavor singlet/
non-singlet:

$$D_S \equiv \sum_{i=1}^{N_f} (D_{q_i} + D_{\bar{q}_i})$$

$$D_{NS}^{(i)} \equiv D_{q_i} - D_{\bar{q}_i},$$

Defining scaled
time variable:

$$\tau = \sqrt{\frac{\alpha^2 \hat{q}}{\pi^2 E}} t$$

Kinetic equations for in-medium fragmentation function:

$$\begin{aligned} \frac{\partial}{\partial \tau} D_g(x, \tau) &= \int_0^1 dz \mathcal{K}_{gg}(z) \left[\sqrt{\frac{z}{x}} D_g\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_g(x) \right] - \int_0^1 dz K_{qg}(z) \frac{z}{\sqrt{x}} D_g(x) \\ &+ \int_0^1 dz K_{gq}(z) \sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right), \end{aligned}$$

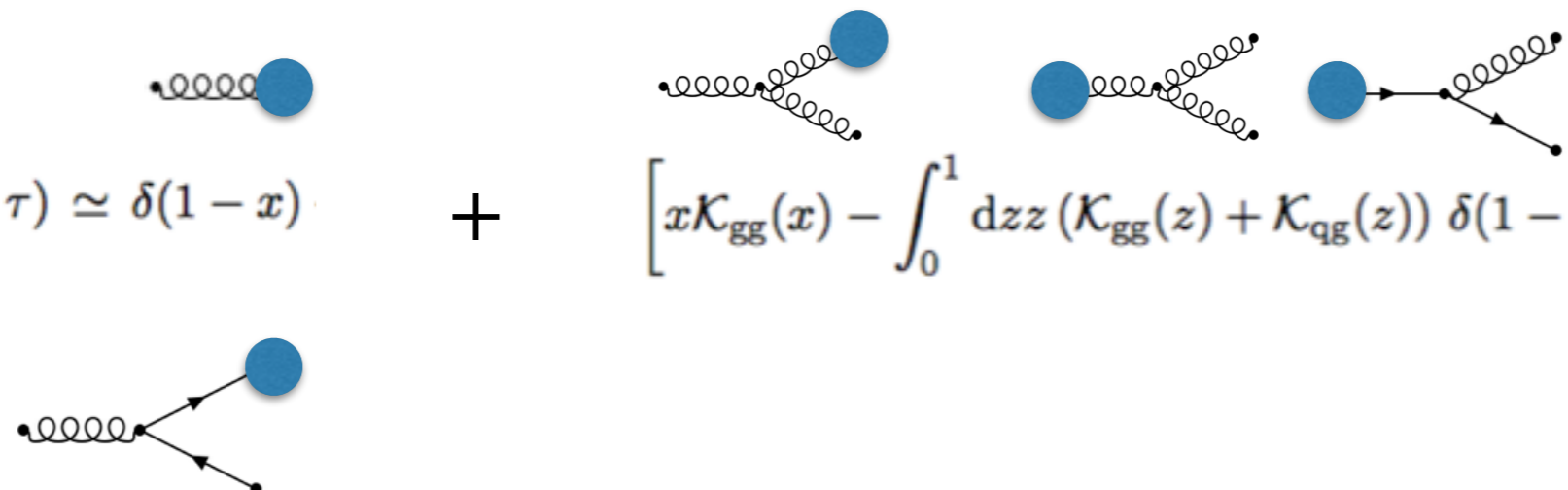
$$\frac{\partial}{\partial \tau} D_S(x, \tau) = \int_0^1 dz \mathcal{K}_{qq}(z) \left[\sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_S(x) \right] + \int_0^1 dz \mathcal{K}_{qg}(z) \sqrt{\frac{z}{x}} D_g\left(\frac{x}{z}\right)$$

$$\frac{\partial}{\partial \tau} D_{NS}^{(i)}(x, \tau) = \int_0^1 dz \mathcal{K}_{qq}(z) \left[\sqrt{\frac{z}{x}} D_{NS}^{(i)}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{NS}^{(i)}(x) \right]$$

In-Medium fragmentation

Solution to leading order/single splitting:

gluon jet: $D_g(x, \tau) \simeq \delta(1-x) + \left[x\mathcal{K}_{gg}(x) - \int_0^1 dz z (\mathcal{K}_{gg}(z) + \mathcal{K}_{qg}(z)) \delta(1-x) \right] \tau$



$D_S(x, \tau) \simeq xK_{qg}(x)\tau$ $D_{NS}(x) = 0$.

However this is only meaningful in a limited regime of (τ, x) :

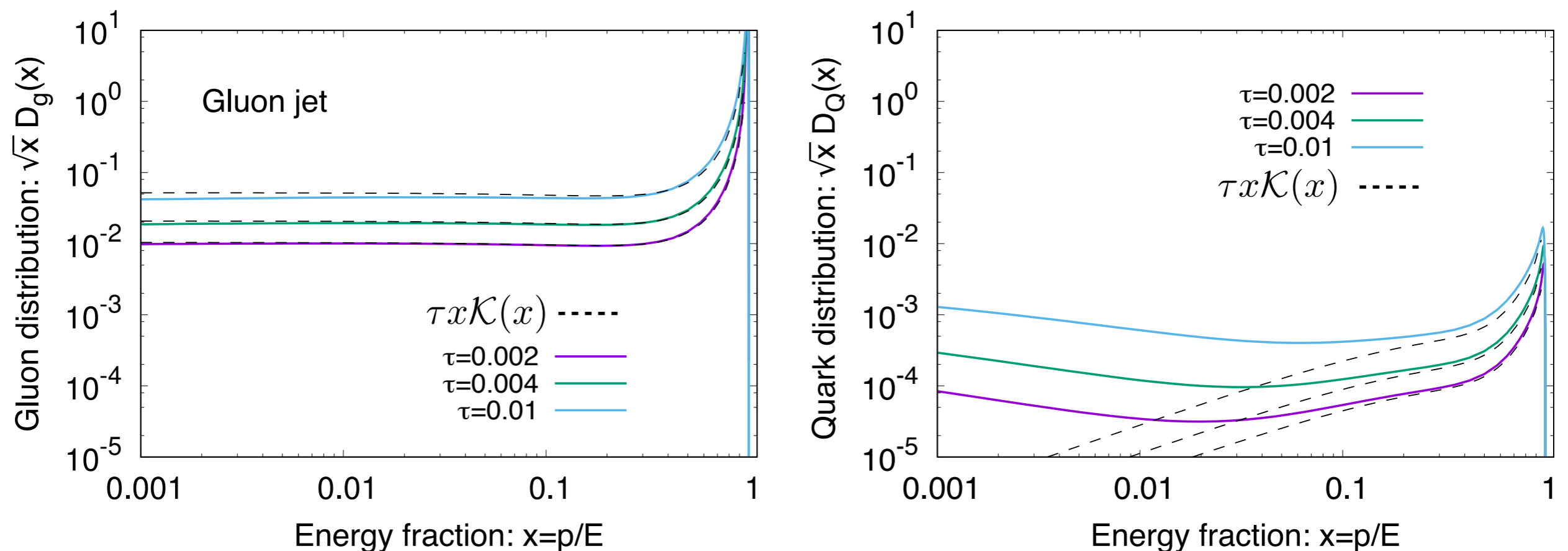
large x: Singularly at large x requires more elaborate treatment

small x: Emission rates at small x enhanced by $1/\sqrt{x}$

Scale $x_c \sim \tau^2$ below which probability for subsequent splitting is $O(1)$ for any $t > 0$

In-Medium fragmentation

Numerical solution of coupled evolution equations for gluon jet ($\tau \ll 1$)



Emergence of “non-perturbative” scale x_c clearly visible in numerical solution of evolution equations

Stationary solution

Since splitting rates become of $O(1)$ at small x , expect solution to become insensitive to initial conditions and approach fixed point of kinetic equation

Stationary non-equilibrium solution: $D_g(x) = \frac{G}{\sqrt{x}}$, $D_S = \frac{Q}{\sqrt{x}}$

$$\begin{aligned} \frac{\partial}{\partial \tau} D_g(x, \tau) &= \int_0^1 dz \mathcal{K}_{gg}(z) \left[\sqrt{\frac{z}{x}} D_g\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_g(x) \right] - \int_0^1 dz K_{qg}(z) \frac{z}{\sqrt{x}} D_g(x) \\ &+ \int_0^1 dz K_{gq}(z) \sqrt{\frac{z}{x}} D_S\left(\frac{x}{z}\right), \end{aligned}$$

Chemistry of fragments fixed by balance of $g \rightarrow qq$ and $q \rightarrow gq$ processes

$$\frac{Q}{G} = \frac{\int_0^1 dz z \mathcal{K}_{qg}(z)}{\int_0^1 dz z K_{gq}(z)} \approx 0.14 N_f$$

Existence of solution does not rely on detailed form of $K(z)$ but only on the fact that emission rates behave as $1/\sqrt{E}$

Energy cascade

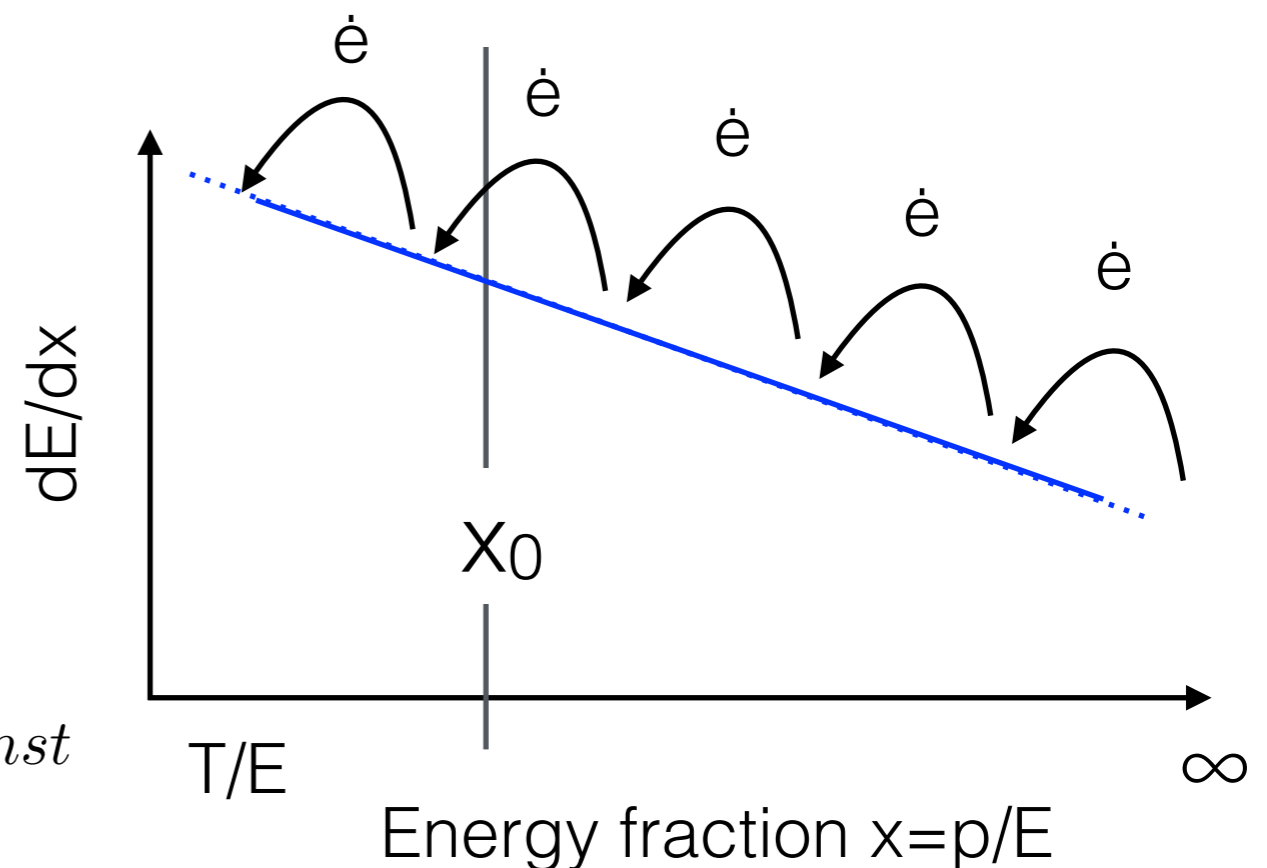
Solution is analogous to Kolmogorov-Zhakarov spectrum in weak wave turbulence

Even though the solution is stationary

$$\partial_\tau D_g(x) = \partial_\tau D_S(x) = 0,$$

it is associated with a finite energy flux

$$\dot{\epsilon}(x_0) = \int_{x_0}^1 dx [\partial_\tau D_g(x) + \partial_\tau D_S(x)] = \text{const}$$



Scale invariance of energy flux ensures that energy flows from $x \gg 1$ to $x \sim T/E$ where it is absorbed by the thermal medium

-> analogous to Richardson cascade in wave-turbulence

Energy cascade

Naive scaling solution requires $D(x) \sim 1/\sqrt{x}$ for all $x \in (0, \infty)$ to satisfy stationarity condition

However even if we limit the support of the distribution to the physical range $(0, 1)$, the energy flux becomes scale invariant $x \ll 1$ and is given by

$$\dot{\epsilon}(x_0 \ll 1) \simeq -\gamma_g G - \gamma_q Q$$
$$\gamma_q = \int_0^1 dz z \left(2N_f K_{qq}(z) + K_{gq}(z) \right) \log \left(\frac{1}{z} \right) \approx 23.19 N_f$$
$$\gamma_g = \int_0^1 dz z \left(\mathcal{K}_{gg}(z) + \mathcal{K}_{qg}(z) \right) \log \left(\frac{1}{z} \right) \approx 25.78 + 0.177 N_f .$$

=> locality of interactions implies that scaling solution can be realized within an inertial range of momenta $T/E \ll x \ll 1$

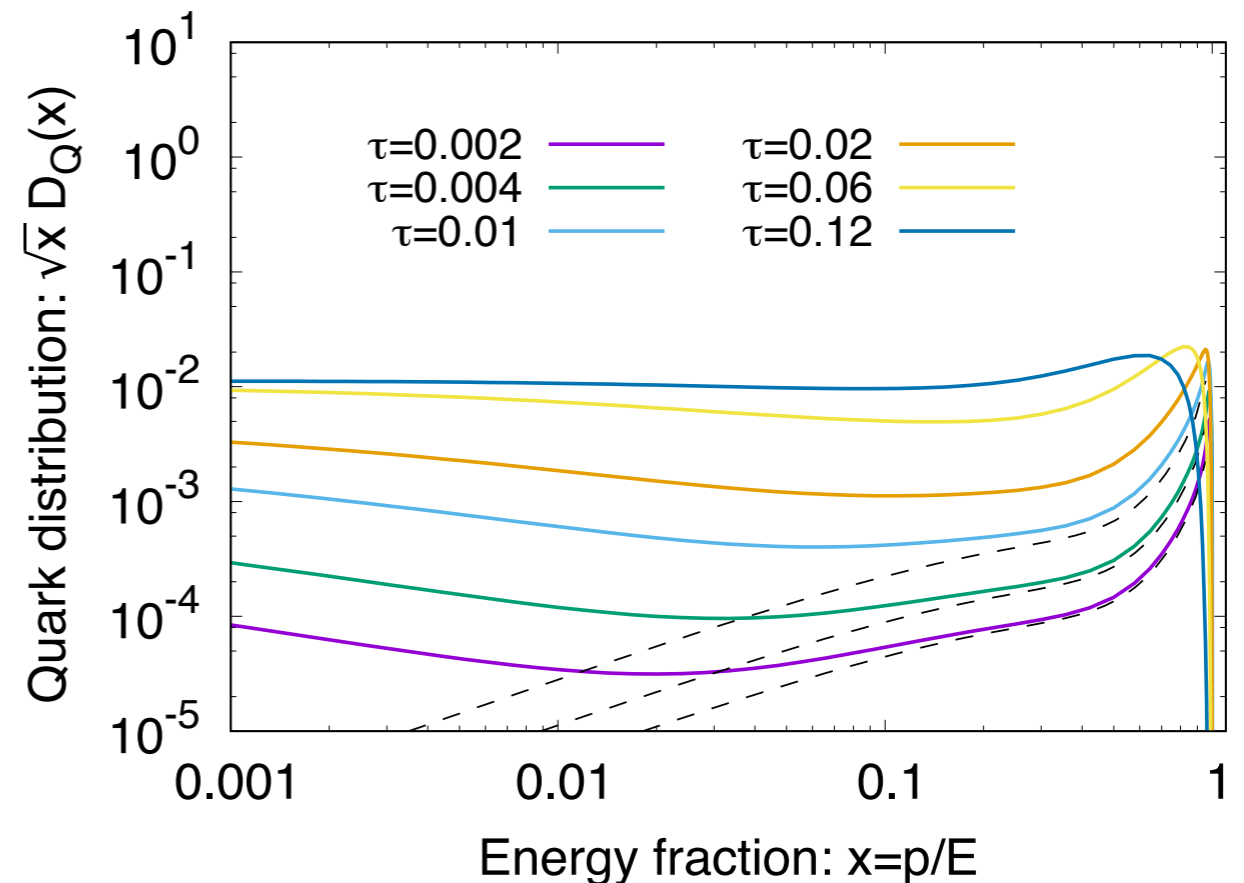
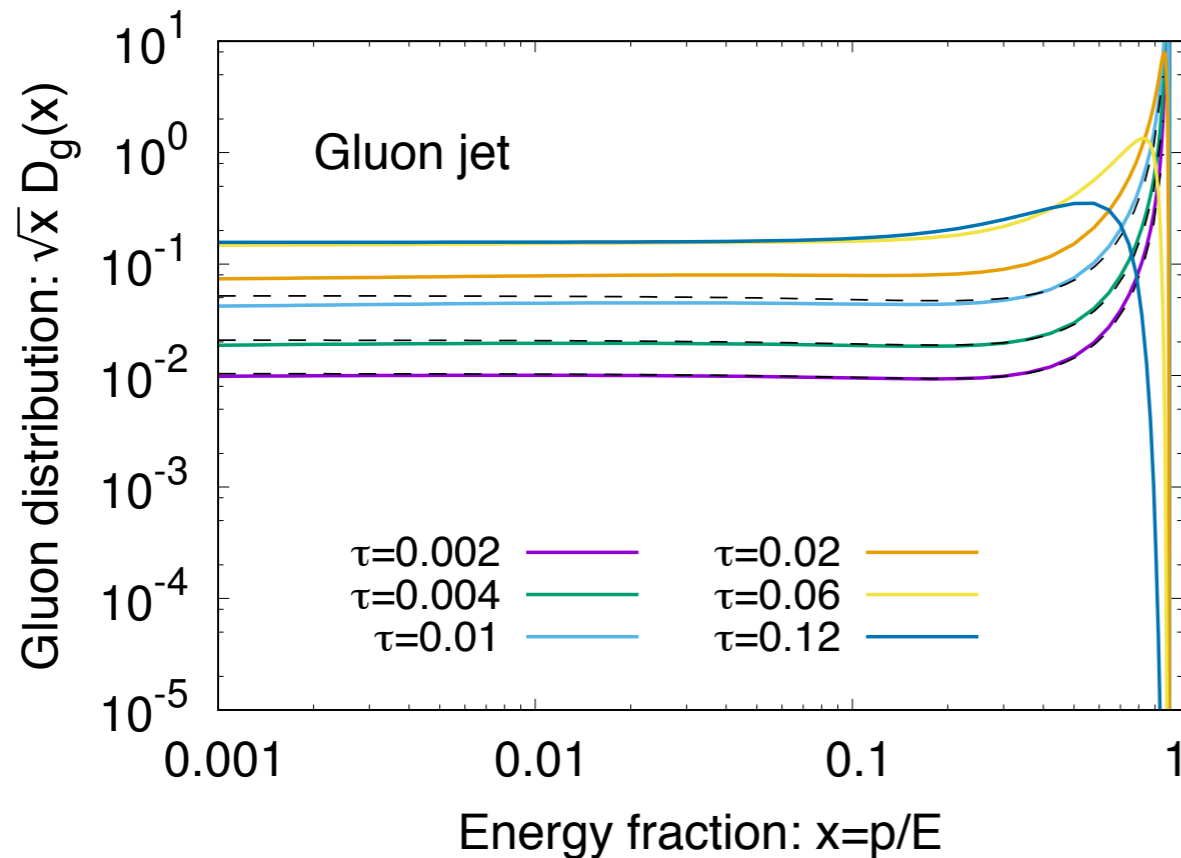
Based on universal Q/G ratio energy loss rate is

$$\dot{\epsilon}(x_0 \ll 1) \simeq (25.78 + 1.8N_f)G$$

Energy loss rate is dominated by $g \rightarrow gg$. Contributions from $q \rightarrow qq$ and $g \rightarrow qg$ give 15% (0.06%) correction per active flavor

In-Medium fragmentation of

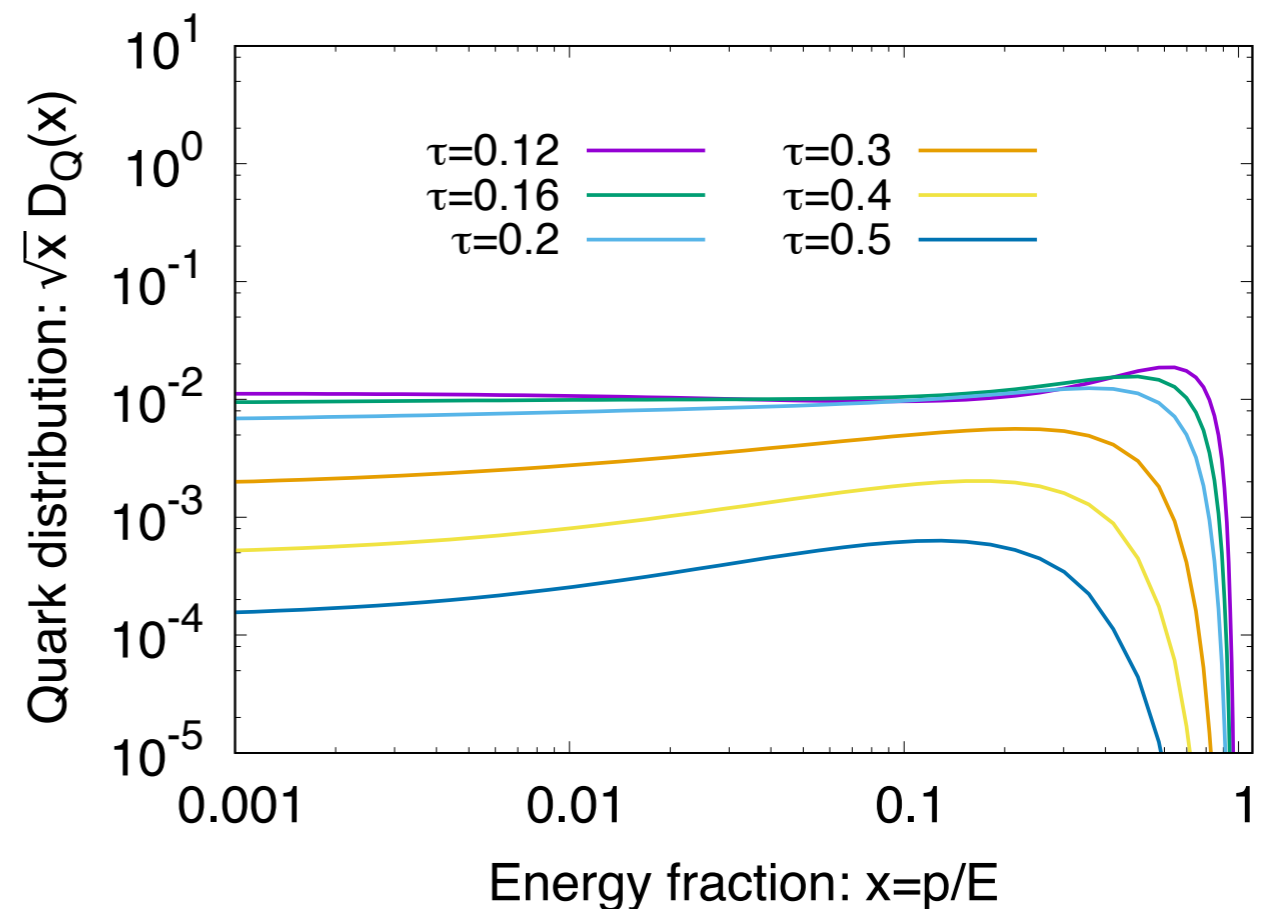
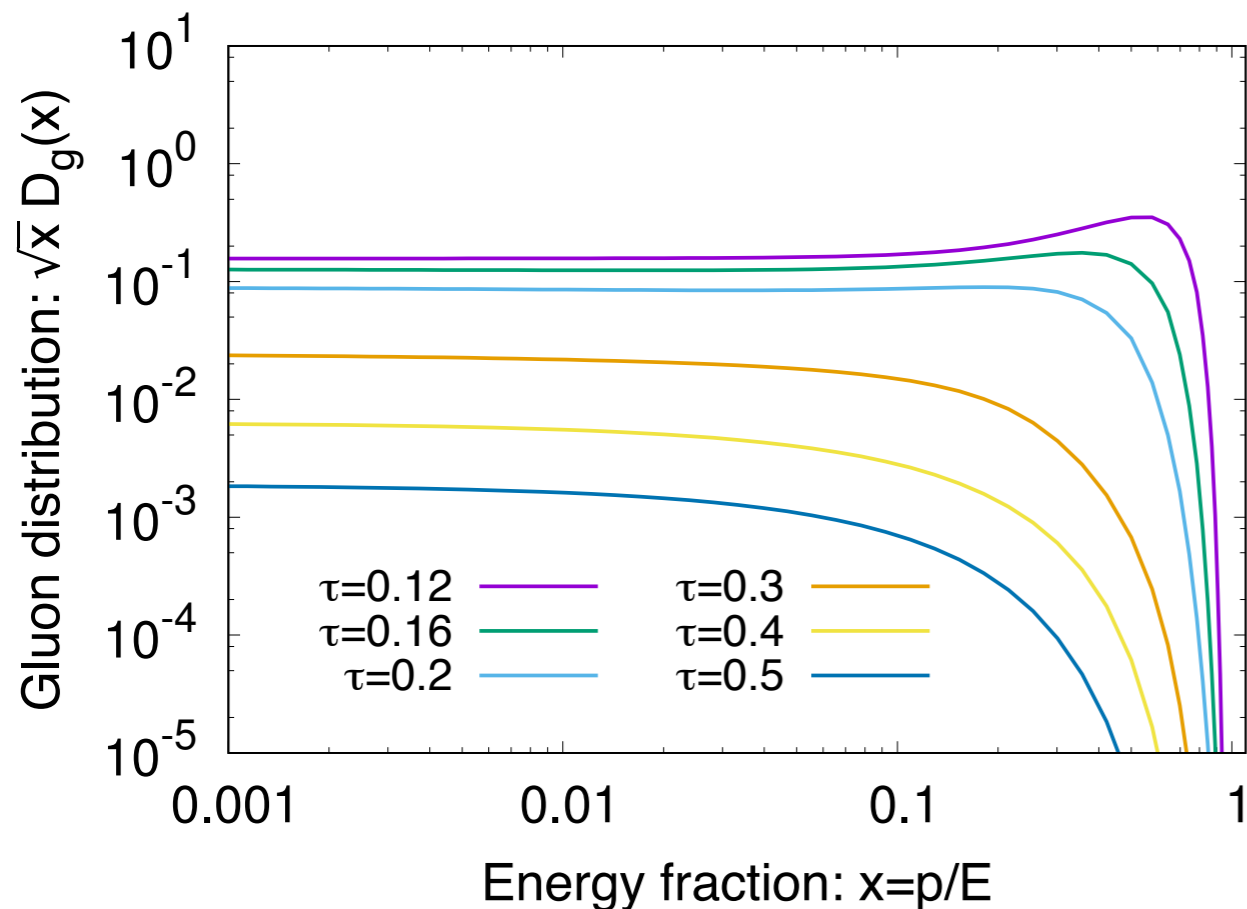
Numerical solution of coupled evolution equations for gluon jet



Enhanced splitting rates at small x lead to approach a non-equilibrium steady state, characterized by $D(x) \sim 1/\sqrt{x}$ behavior of gluon & quark distribution at small x

In-Medium fragmentation

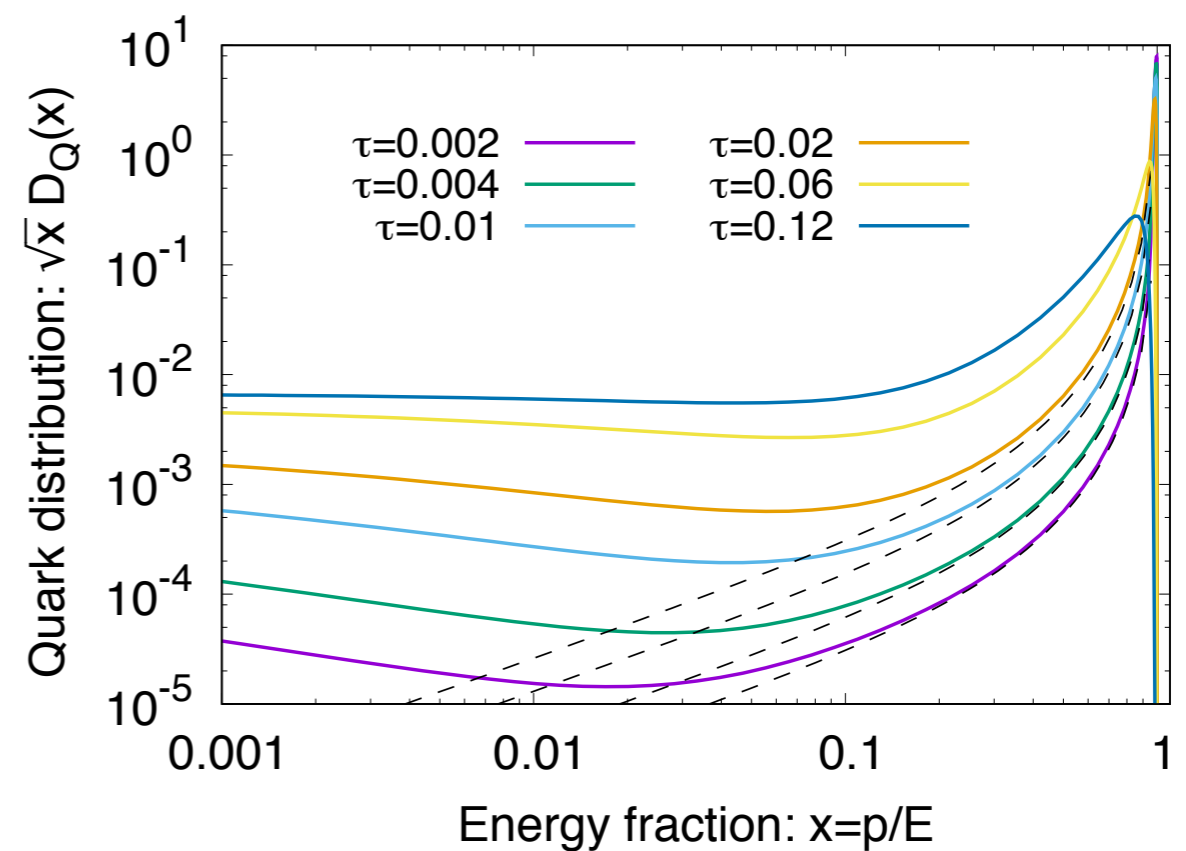
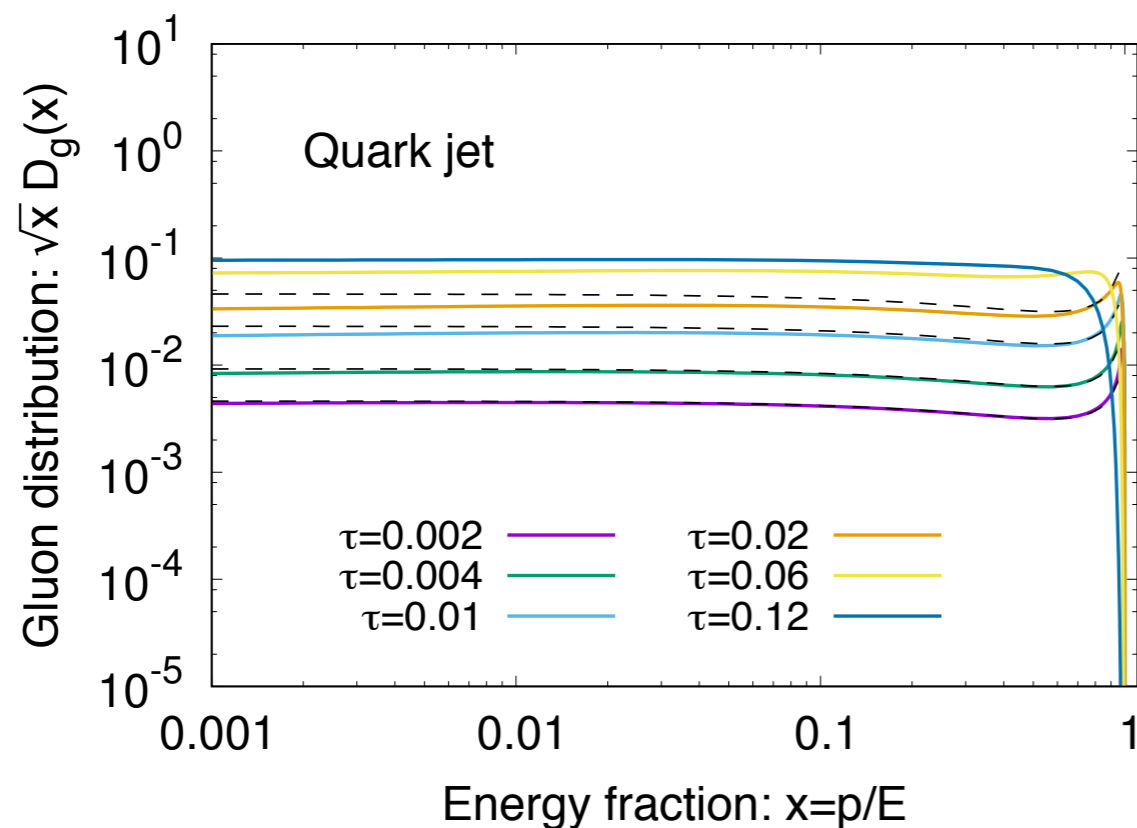
Numerical solution of coupled evolution equations for gluon jet



Kolmogorov spectrum at small x persists throughout the evolution, even when the jet has lost a significant amount of energy

In-Medium fragmentation

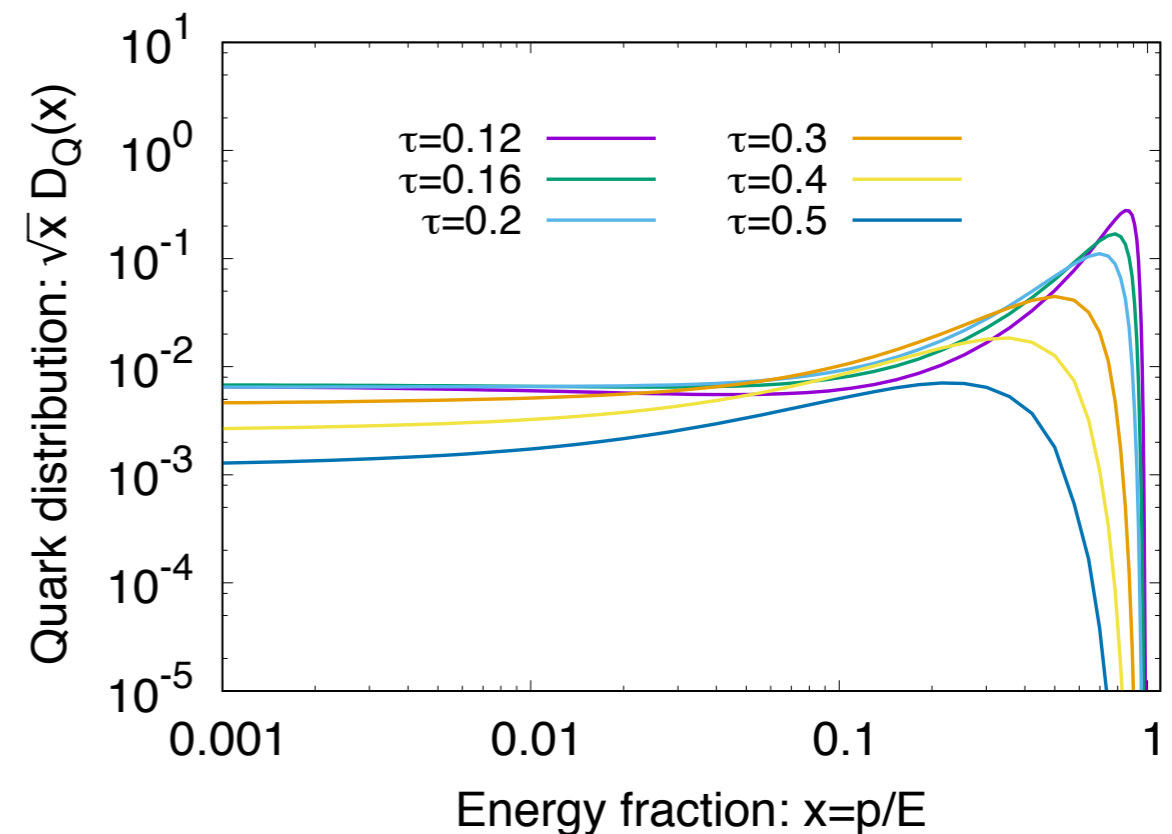
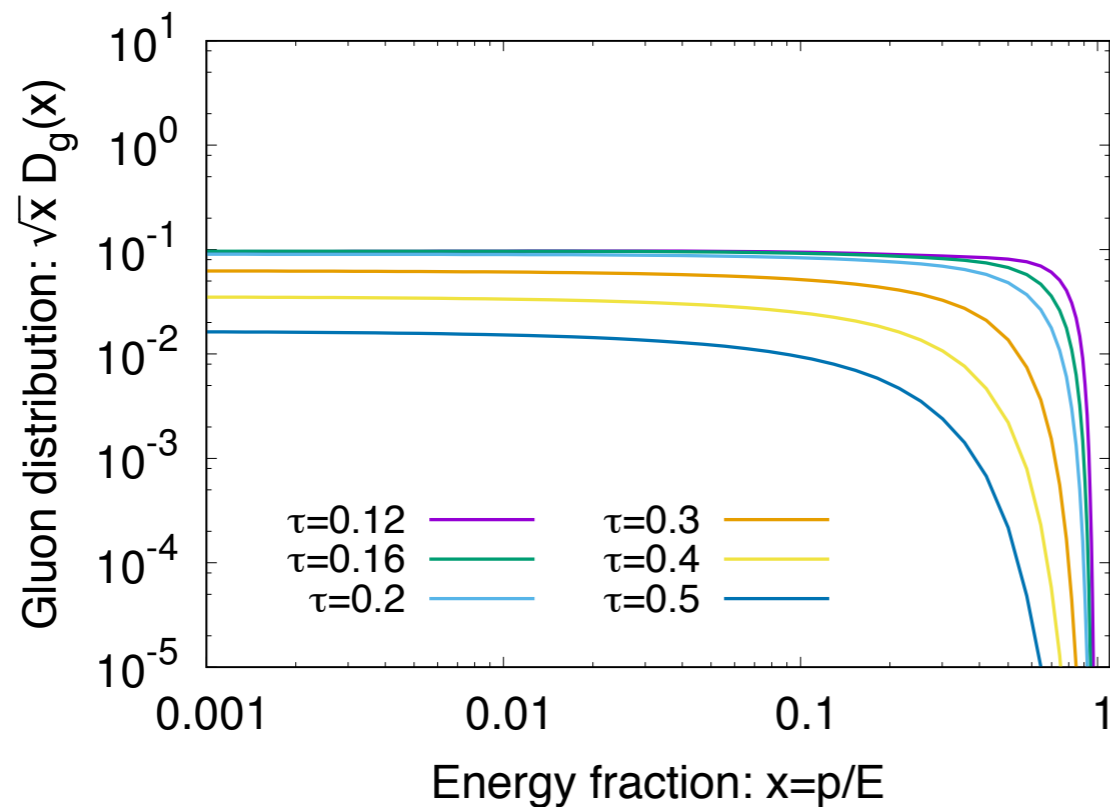
Numerical solution of coupled evolution equations for quark jet



Enhanced splitting rates at small x lead to approach a non-equilibrium steady state, characterized by $D(x) \sim 1/\sqrt{x}$ behavior of gluon & quark distribution at small x

In-Medium fragmentation

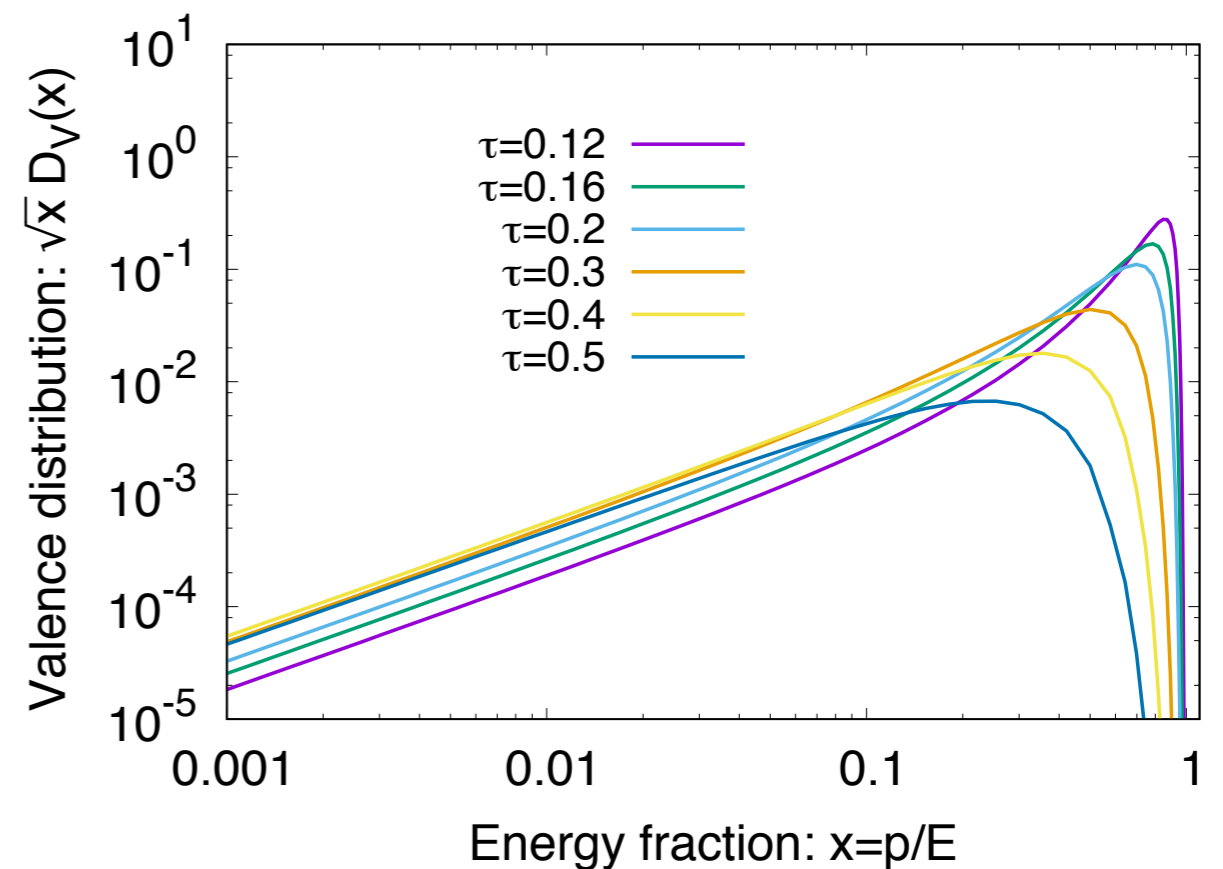
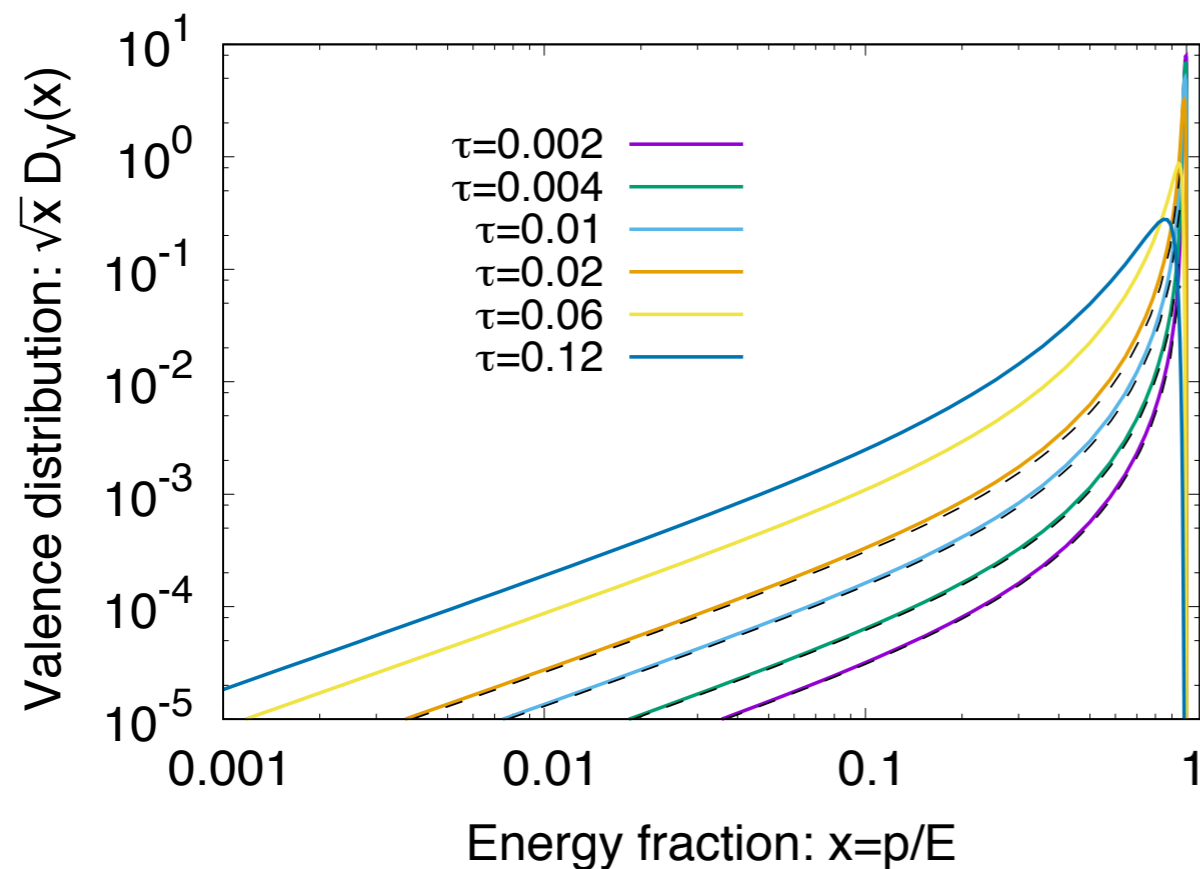
Numerical solution of coupled evolution equations for quark jet



Kolmogorov spectrum at small x persists throughout the evolution, even when the jet has lost a significant amount of energy

In-Medium fragmentation

Beside energy quark jets also carry a valence flavor (u,d,s) which gives rise to a non-vanishing NS distribution



Energy cascade: gluon + flavor singlet quark channel

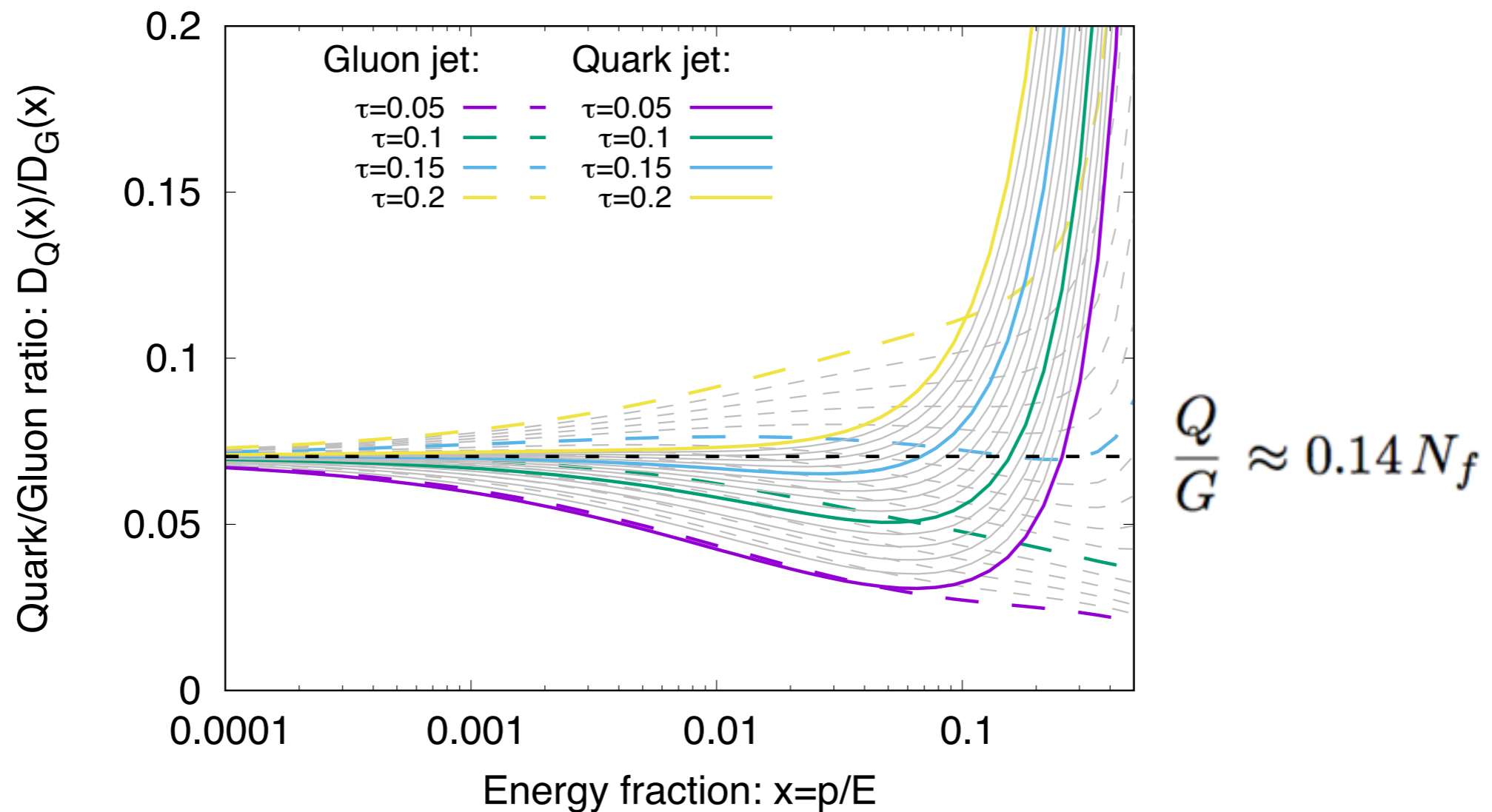
$$D_{g/s}(x) \sim 1/\sqrt{x}$$

Valence particle number cascade: flavor non-singlet quark channel

$$D_{NS}(x) \sim 1/x^{3/2}$$

In-medium jet chemistry

Balance of the $g \rightarrow qq$ and $q \rightarrow qg$ processes at the non-equilibrium steady state ($x \ll 1$) uniquely determine chemistry



Universal Kolmogorov ratio approximately realized over a substantial range of momentum fractions x and evolution times τ

Energy loss

Single emission off the original hard parton create a $1/\sqrt{x}$ gluon spectrum at small x with amplitude

$$G \simeq \tau C_A^{1/2} C_R$$

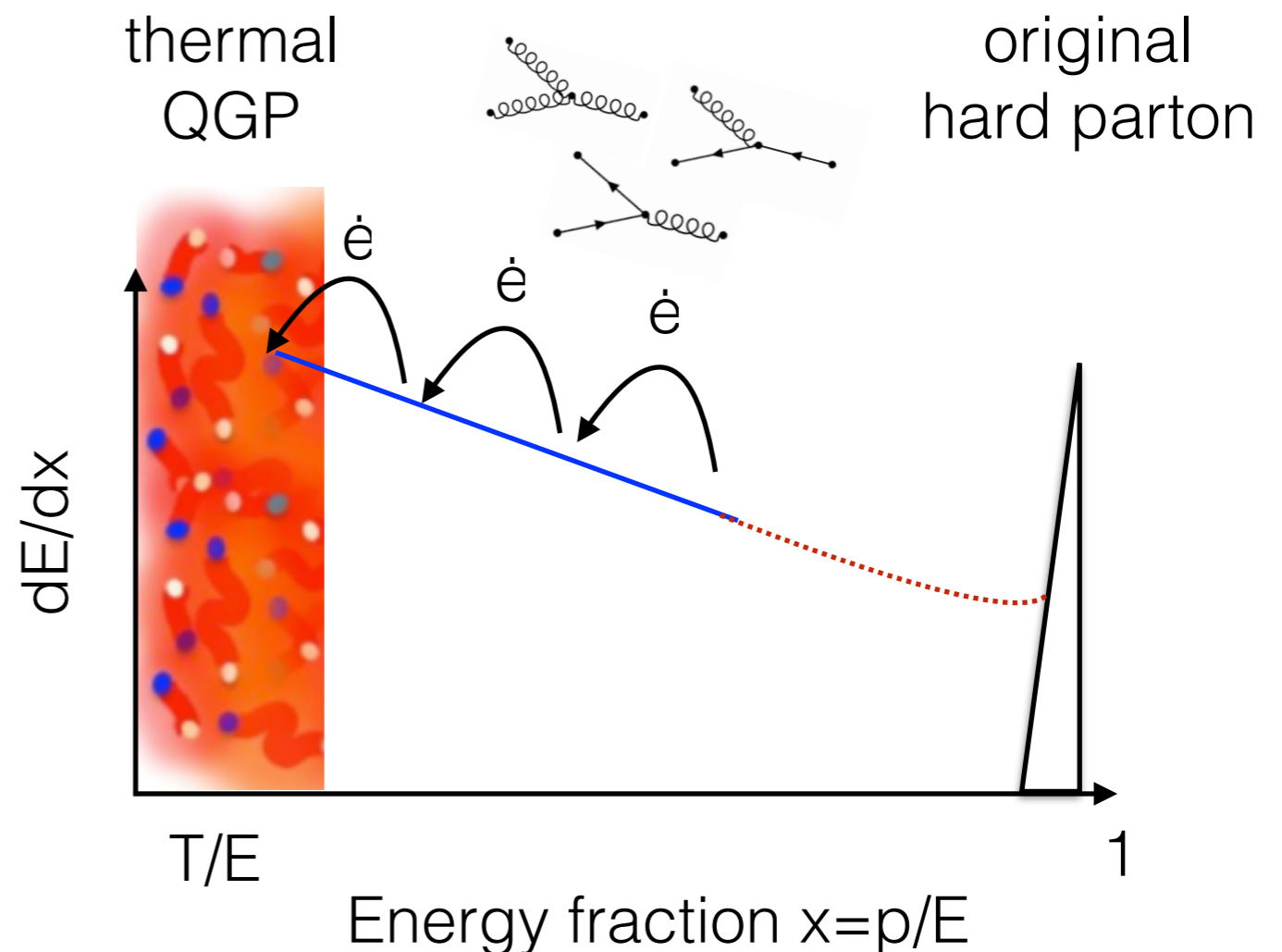
Multiple emissions off soft fragments create turbulent energy flux

$$\dot{\epsilon} = -\gamma_g G$$

all the way to T/E where energy is absorbed by the thermal QGP

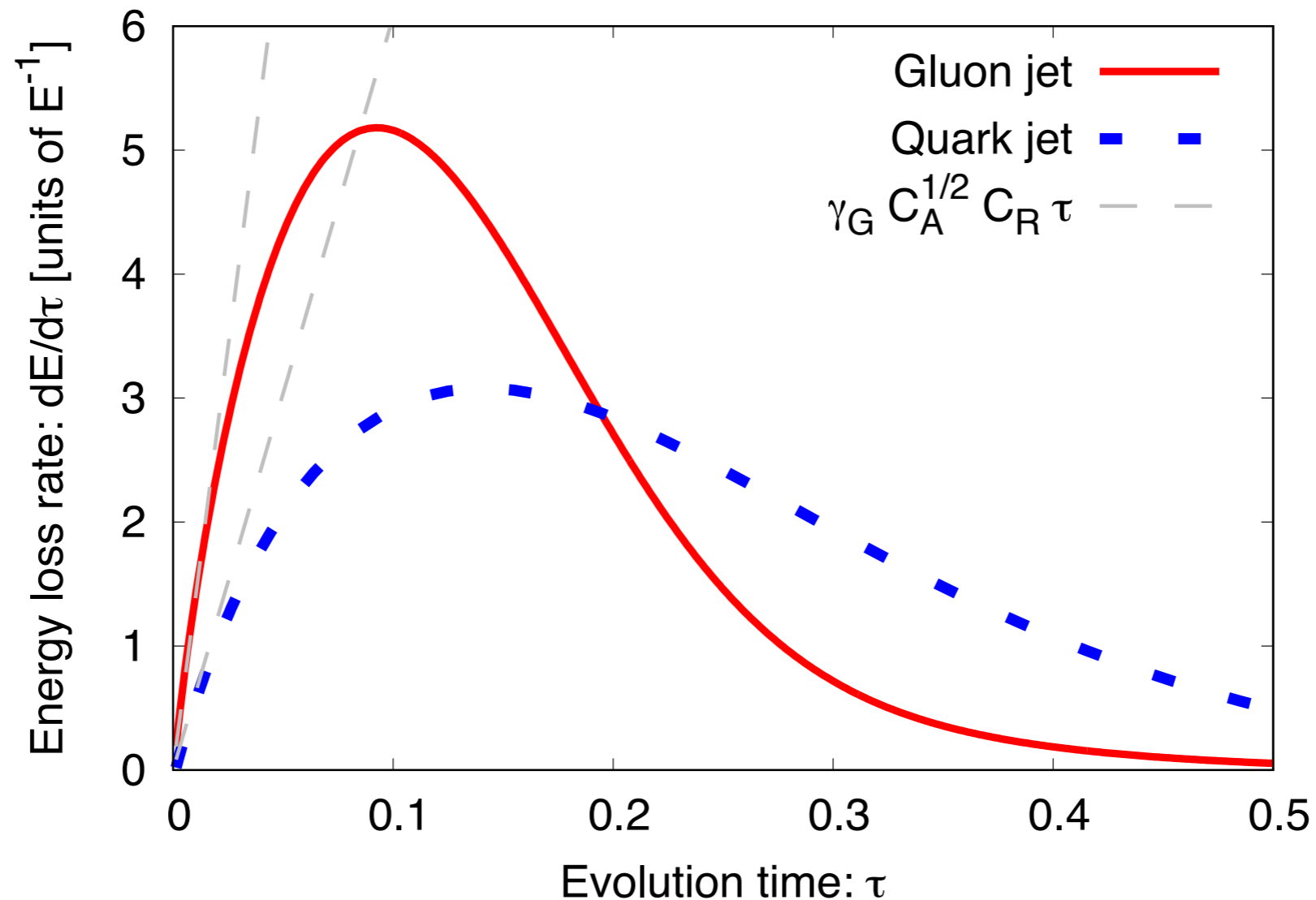
Energy loss rate at early times given by

$$\frac{1}{E} \frac{dE}{d\tau} \Big|_{\tau \ll 1} \approx -\gamma_g C_A^{1/2} C_R \tau$$



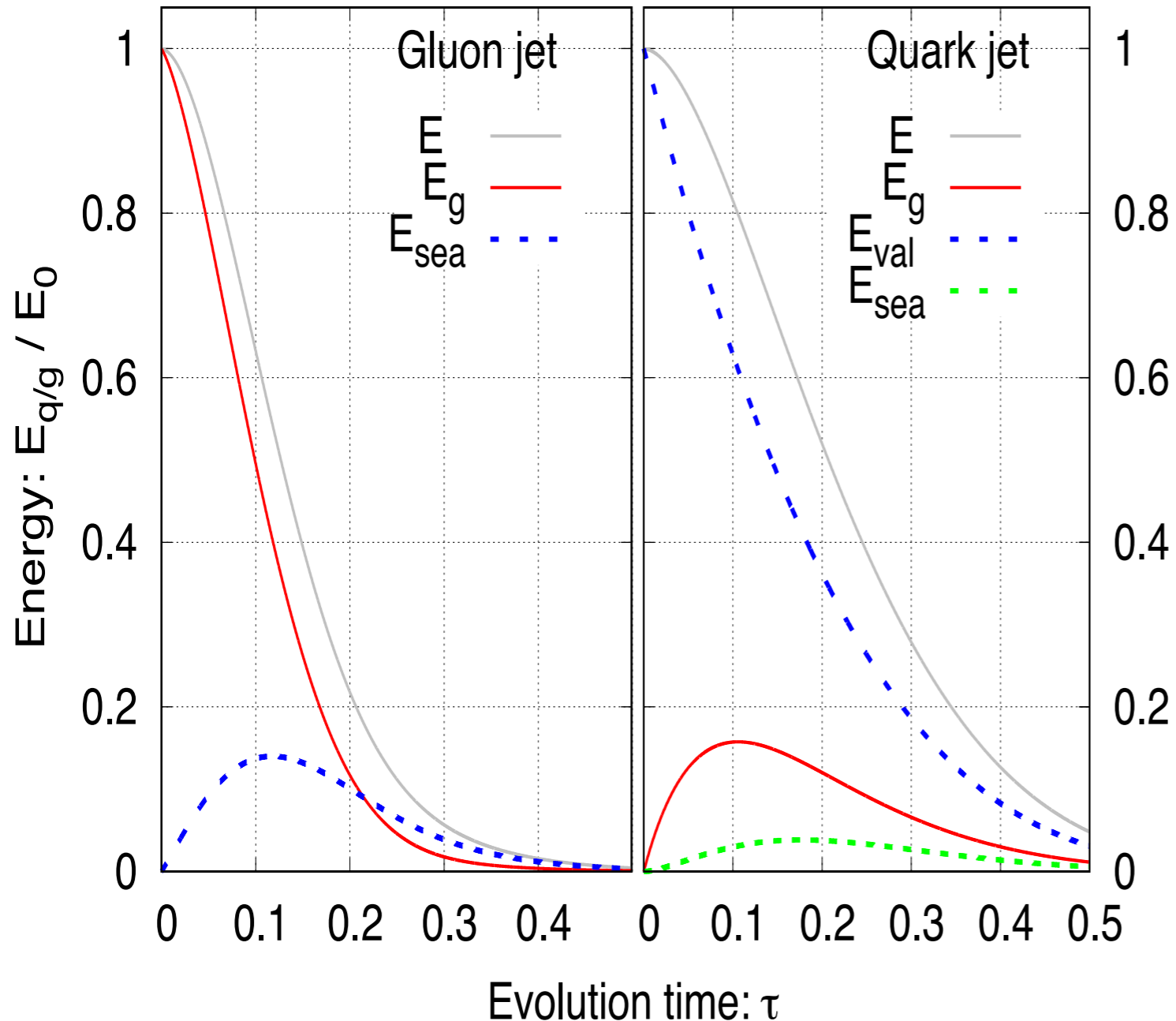
Energy loss of quark & gluon jets

Energy loss at early times follows expected behavior from turbulence analysis



Breakdown of Casimir scaling of energy loss at late times
chemistry of fragments is strongly modified

Energy loss of quark & gluon jets



medium filtering:

Since large x gluons lose energy faster than large x quarks, the large x distribution at late times is always dominated by quark d.o.f.

quark jet:

Energy always dominated by valence quarks

gluon jet:

Energy dominated by sea quarks once jet has lost $\sim 80\%$ of total energy

Phenomenological implications

So far connection to experiment is loose, but assuming factorization

$$d\sigma_{\gamma,h/jet} \sim f_a \times f_b \times H_{ab \rightarrow \gamma c} \times D_{c \rightarrow d}^{\text{Medium}} \times D_{d \rightarrow h/jet}^{\text{Vacuum}}$$

can speculate on phenomenological consequences of our findings

Universal q/g ratio for small x fragments & medium filtering

=> strangeness enhancement at small x (respectively large x)

Could be observable by looking at

- ratios of identified particles (e.g. K/ π or Λ/π) inside jets
- ratios of identified particles (e.g. K/ π or Λ/π) in backward hemisphere of high- p_T trigger particle

in A+A collisions relative to p+p reference

Ideally one would have an estimate of energy loss calibrate (x, τ)
e.g. by photon tagging

Detailed predictions will require to combine medium evolution with
initial production and hadronization (work in progress)

Conclusion & Outlook

Energy loss is realized via a turbulent cascade, associated with scale independent energy flux from energy scales ($p \sim E$) all the way to the energy scale of the medium ($p \sim T$)

=> allows for analytic predictions of interesting features

$$\frac{D_Q(x)}{D_G(x)} = \frac{\int_0^1 dz z K_{qg}(z)}{\int_0^1 dz z \mathcal{K}_{gq}(z)} \approx 0.07$$

Interesting phenomenological consequences for jet chemistry

=> could be interesting to extend analysis to heavy (c,b) flavors

Ultimately, would like to extend study to include all processes relevant at lower scales $x \sim T/E$ to obtain a more complete picture of jet-energy loss & address chemical equilibration of the QGP at early times

