Callan-Symanzik equations for infrared QCD

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Quantization of continuum Yang-Mills theory

- $SU(N)$ Yang-Mills theory in the continuum, Landau gauge
- Faddeev-Popov determinant $\Rightarrow$ ghosts (and BRST symmetry)
- Faddeev-Popov action in D-dimensional Euclidean space-time

$$S = \int d^D x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i B^a \partial_\mu A^a_\mu \right)$$

- gauge copies $\Rightarrow$ restriction of the gauge field configurations to the (first) Gribov region Ω (properly to the fundamental modular region) [Gribov 1978]
- Zwanziger's horizon function, breaks the BRST symmetry; local formulation $\Rightarrow$ additional auxiliary fields [Zwanziger 1989]
- condensates of the additional auxiliary fields: "refined Gribov-Zwanziger scenario" $\Rightarrow$ effective mass term for the gluons [Dudal et al. 2008]

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- Dyson-Schwinger equations are not affected by the restriction of the A-integral to Ω (without introducing the horizon function): the contributions from the boundary of Ω vanish because $\det(-\partial_\mu D_\mu^{ab}) = 0$ at the boundary [Zwanziger 2002]

- the perturbative expansion of the correlation functions, obtained from the iterative solution of the Dyson-Schwinger equations, is also unchanged

- what can change are the (re)normalization conditions; and the BRST symmetry is broken

How nonperturbative is IR Yang-Mills theory?

- effective description by a local renormalizable quantum field theory: include a gluonic mass term in the Faddeev-Popov action (in 4-dimensional Euclidean space-time)

$$S = \int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} A^a_\mu m^2 A^a_\mu + \partial_\mu \bar{c}^a D^{ab}_\mu c^b + iB^a \partial_\mu A^a_\mu \right)$$

  (Curci-Ferrari model, perturbatively renormalizable)

- apply straightforward one-loop perturbation theory to this action;
  adjust *two* constants, g, m^2, at some renormalization scale
  (in principle, m^2 is nonperturbatively fixed in terms of Λ_{QCD}, or g)

- result: excellent fit to the lattice data for the propagators in the IR [Tissier, Wschebor 2010]

- notations: propagators

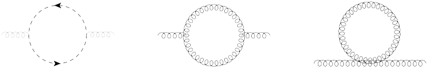
$$\langle A^a_\mu(p) A^b_\nu(-q)\rangle = G_A(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}(2\pi)^4\delta(p-q)$$

$$\langle c^a(p)\bar{c}^b(-q)\rangle = G_c(p^2)\,\delta^{ab}(2\pi)^4\delta(p-q)$$

and dressing functions

$$G_A(p^2) = \frac{F_A(p^2)}{p^2}\,, \qquad G_c(p^2) = \frac{F_c(p^2)}{p^2}$$

- one-loop contributions to the gluon self energy



and the ghost self energy

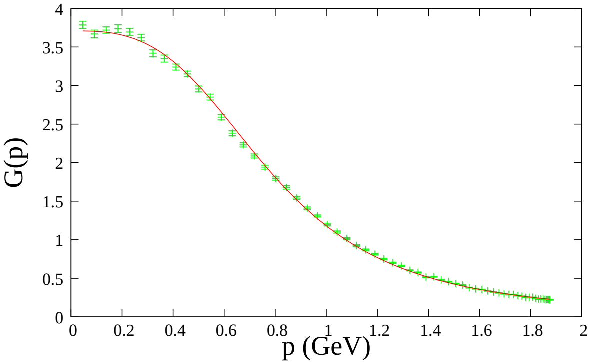
calculated with a massive gluon propagator

$$G_A(p^2) = \frac{1}{m^2 + p^2}$$

gluon propagator $G_A(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(2) lattice data: Cucchieri, Mendes 2008a

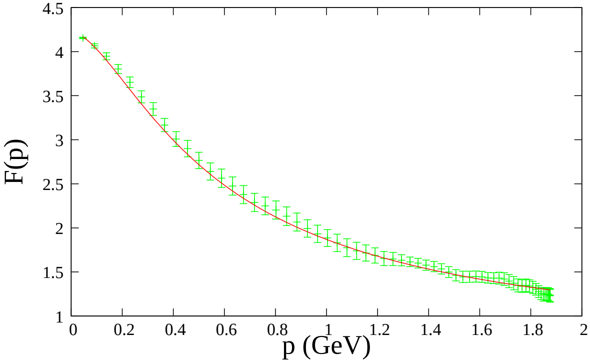


at tree level, $G_A(p^2) \propto \dfrac{1}{m^2 + p^2}$

ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(2) lattice data: Cucchieri, Mendes 2008b



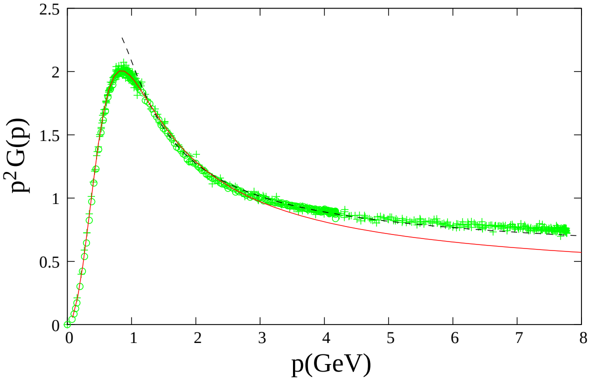
at tree level, $F_c(p^2) = 1$

gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(3) lattice data: Bogolubsky et al. 2009

and Dudal, Oliveira, Vandersickel 2010



at tree level, $F_A(p^2) \propto \dfrac{p^2}{m^2 + p^2}$

renormalization group improvement necessary for the UV

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Extreme IR regime

- successful description of the IR fixed point for all dimensions ($D \geq 2$) with Callan-Symanzik equations in an epsilon expansion [Weber 2012 and Weber, Dall'Olio, Astorga 2016]
- normalization condition for the gluon propagator

$$G_A(p^2 = \mu^2) = \frac{1}{m^2}$$

  corresponding to a "high-temperature" fixed point
- for $D > 2$ dimensions ("decoupling solutions"), use an epsilon expansion in $D = 2 + \epsilon$ dimensions with the normalization condition for the ghost propagator

$$G_c(p^2 = \mu^2) = \frac{1}{p^2}$$

- for $D = 2$ dimensions ("scaling solution"), use an epsilon expansion in $D = 6 - \epsilon$ dimensions with the normalization condition for the ghost propagator

$$G_c(p^2 = \mu^2) = \frac{b^2}{p^4}$$

  corresponding to a "Lifshitz point" and Zwanziger's original horizon condition
- in the following, describe the crossover from the UV to the IR fixed point, and hence the complete momentum dependence of the propagators, in dimension $D = 4$

- “IR safe” renormalization scheme proposed by Tissier and Wschebor [Tissier, Wschebor 2011]: normalization conditions for the proper two-point functions

$$\Gamma_A^{\perp}(p^2)\big|_{p^2=\mu^2} = m^2 + p^2$$
$$\Gamma_A^{\parallel}(p^2)\big|_{p^2=\mu^2} = m^2$$
$$\Gamma_c(p^2)\big|_{p^2=\mu^2} = p^2$$

- $\Gamma_A^{\perp}$ and $\Gamma_A^{\parallel}$ are the transverse and longitudinal parts of the proper gluonic 2-point function

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \Gamma_A^{\perp}(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma_A^{\parallel}(p^2)\,\frac{p_\mu p_\nu}{p^2}$$

- the first two normalization conditions can be rewritten as

$$\Gamma_A^{\perp}(p^2) - \Gamma_A^{\parallel}(p^2)\big|_{p^2=\mu^2} = p^2$$
$$\Gamma_A^{\parallel}(p^2)\big|_{p^2=\mu^2} = m^2$$

- these combinations correspond to the decomposition of the 2-point function

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \left(\Gamma_A^{\perp}(p^2) - \Gamma_A^{\parallel}(p^2)\right)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma_A^{\parallel}(p^2)\delta_{\mu\nu}$$

which is analogous to the grouping of terms in the classical action

$$p^2\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + m^2\delta_{\mu\nu}$$

- renormalized coupling constant defined from the renormalized proper ghost-gluon vertex in the Taylor limit (ghost momentum $p \to 0$) where there are no loop corrections to the vertex $\Rightarrow g(\mu^2) = Z_A^{1/2}(\mu^2) Z_c(\mu^2) g_B$ (alternatively, use the symmetry point $p^2 = q^2 = k^2 = \mu^2$)
- calculate the flow functions at one-loop order; then, solve the Callan-Symanzik renormalization group equations for the propagators
- example: the μ^2-independence of the bare longitudinal proper gluonic 2-point function implies

$$\mu^2 \frac{d}{d\mu^2} \Gamma_A^{\parallel}(p^2,\mu^2) = \mu^2 \frac{d}{d\mu^2} \left(Z_A(\mu^2)\, \Gamma_A^{\parallel\, B}(p^2,\mu^2) \right)$$
$$= \left(\mu^2 \frac{d}{d\mu^2} Z_A(\mu^2) \right) \Gamma_A^{\parallel\, B}(p^2,\mu^2)$$
$$= \gamma_A(\mu^2)\, \Gamma_A^{\parallel}(p^2,\mu^2)$$

- integrating between two renormalization scales $\bar{\mu}^2$ and μ^2,

$$\Gamma_A^{\parallel}(p^2,\mu^2) = \Gamma_A^{\parallel}(p^2,\bar{\mu}^2) \exp\left(\int_{\bar{\mu}^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \gamma_A(\mu'^2) \right)$$

- setting $\bar{\mu}^2 = p^2$ and using the normalization condition,

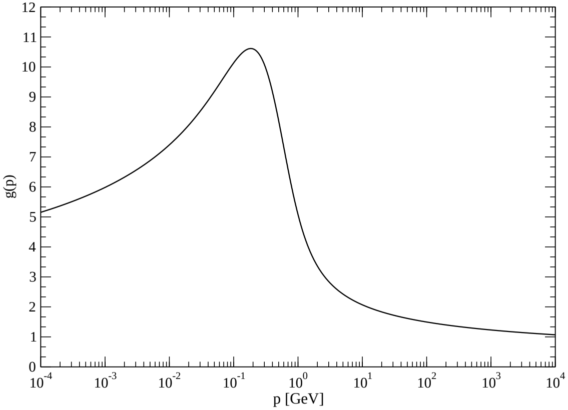
$$\Gamma_A^{\parallel}(p^2,\mu^2) = m^2(p^2) \exp\left(- \int_{\mu^2}^{p^2} \frac{d\mu'^2}{\mu'^2} \gamma_A(\mu'^2) \right)$$

- $\gamma_A(\mu^2)$ and $\gamma_c(\mu^2)$ depend on $g(\mu^2)$ and $m^2(\mu^2)$ which are obtained from the integration of the system of differential equations

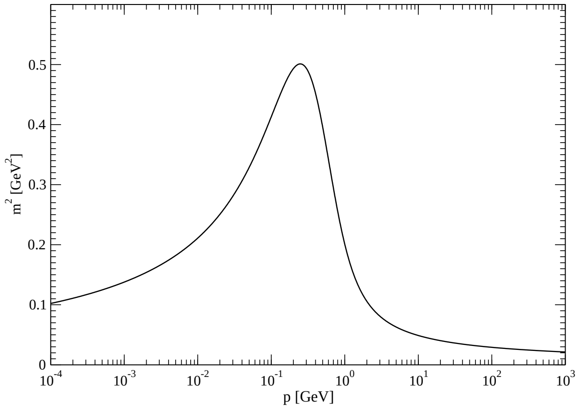
$$\mu^2 \frac{d}{d\mu^2}\, m^2(\mu^2) = \mu^2 \frac{d}{d\mu^2}\; \Gamma_A^{\|}(\mu^2,\mu^2)\Big|_{g_B,m_B^2} = \beta_{m^2}\big(g(\mu^2), m^2(\mu^2), \mu^2\big)$$
$$\mu^2 \frac{d}{d\mu^2}\, g(\mu^2) = \mu^2 \frac{d}{d\mu^2}\; g(\mu^2)\Big|_{g_B,m_B^2} = \beta_g\big(g(\mu^2), m^2(\mu^2), \mu^2\big)$$

- all these equations are exact if the flow functions are exact; here, calculate the flow functions to one-loop order and integrate the Callan-Symanzik equations with these approximate flow functions: “renormalization group improvement” of perturbation theory
- adjust $g(\mu_0), m^2(\mu_0)$ at some renormalization scale to fit the lattice data;
  note that the lattice propagators are not normalized and thus can be arbitrarily rescaled (field rescalings)
- fitting strategy: fix $g(\mu_0), m^2(\mu_0)$ by adjusting to the data for the ghost propagator and the ghost dressing function;
  comparison to the data for the gluon propagator and the gluon dressing function then shows how successful the renormalization scheme is in reproducing the lattice data
- in all of the following, the SU(2) lattice data are from Cucchieri and Mendes [Cucchieri, Mendes 2008a, 2008b]

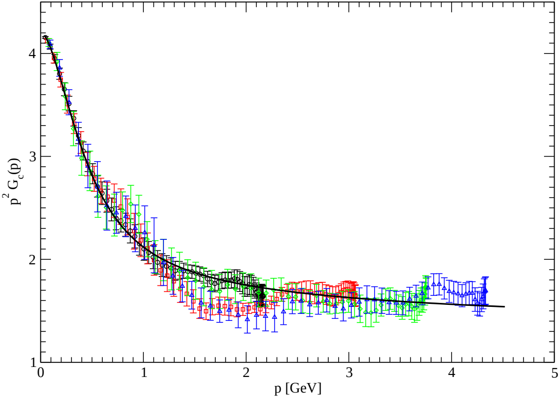
running coupling constant $g(\mu)$ in $D = 4$ dimensions



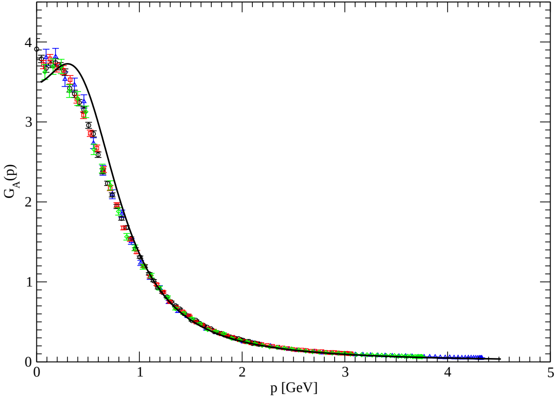
running mass parameter $m^2(\mu)$ in $D = 4$ dimensions



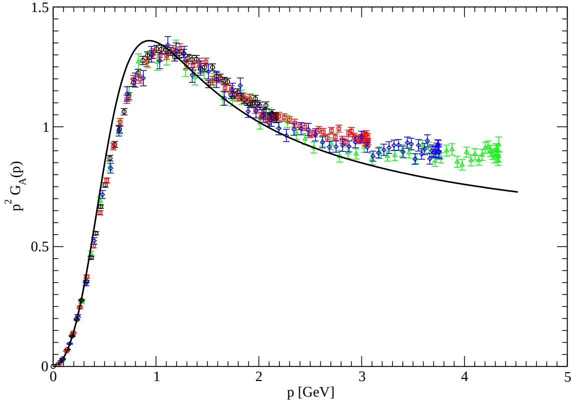
ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$, Taylor scheme



gluon propagator function $G_A(p^2)$, Taylor scheme



gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$, Taylor scheme



Derivative schemes

- in the decomposition of the gluonic 2-point function

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \left(\Gamma^{\perp}_{A}(p^2) - \Gamma^{\parallel}_{A}(p^2)\right)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma^{\parallel}_{A}(p^2)\delta_{\mu\nu}$$

  replace the normalization condition

$$\Gamma^{\perp}_{A}(p^2) - \Gamma^{\parallel}_{A}(p^2)\big|_{p^2=\mu^2} = p^2$$

  with

$$\frac{d}{dp^2}\left(\Gamma^{\perp}_{A}(p^2) - \Gamma^{\parallel}_{A}(p^2)\right)\Big|_{p^2=\mu^2} = 1$$

- complement with the normalization conditions

$$\Gamma^{\parallel}_{A}(p^2)\big|_{p^2=\mu^2} = m^2$$

$$\frac{d}{dp^2}\Gamma_c(p^2)\Big|_{p^2=\mu^2} = 1$$

  $\Rightarrow$ quantitatively, almost no change

- generalize to the decomposition

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \left(\Gamma^{\perp}_{A}(p^2) - \zeta\Gamma^{\parallel}_{A}(p^2)\right)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma^{\parallel}_{A}(p^2)\left(\zeta\delta_{\mu\nu} + (1-\zeta)\frac{p_\mu p_\nu}{p^2}\right)$$

  and impose the normalization condition

$$\frac{d}{dp^2}\left(\Gamma^{\perp}_{A}(p^2) - \zeta\Gamma^{\parallel}_{A}(p^2)\right)\Big|_{p^2=\mu^2} = 1$$

- integrating the Callan-Symanzik equation for the proper gluonic 2-point function yields

$$\frac{\partial}{\partial p^2}\left(\Gamma_A^{\perp}(p^2,\mu^2)-\zeta\Gamma_A^{\parallel}(p^2,\mu^2)\right)=\exp\left(-\int_{\mu^2}^{p^2}\frac{d\mu'^2}{\mu'^2}\gamma_A(\mu'^2)\right)$$

  then integrate over p^2 with the initial condition inferred by locality

$$\Gamma_A^{\perp}(p^2=0,\mu^2)=\Gamma_A^{\parallel}(p^2=0,\mu^2)$$

- in the IR limit $\mu^2 \ll m^2$,

$$\beta_g=\mu^2\frac{d}{d\mu^2}\,g=\frac{g}{2}(\gamma_A+2\gamma_c)\approx\frac{g}{2}\,\gamma_A=\frac{g}{2}\,\mu^2\frac{d}{d\mu^2}\ln Z_A$$

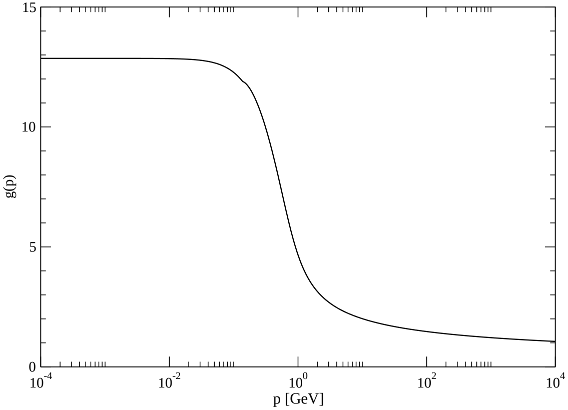
  and to 1-loop order

$$\mu^2\frac{d}{d\mu^2}\ln Z_A=\frac{Ng^2}{(4\pi)^2}\left(-\frac{1}{12}+\frac{\zeta}{4}\right)$$

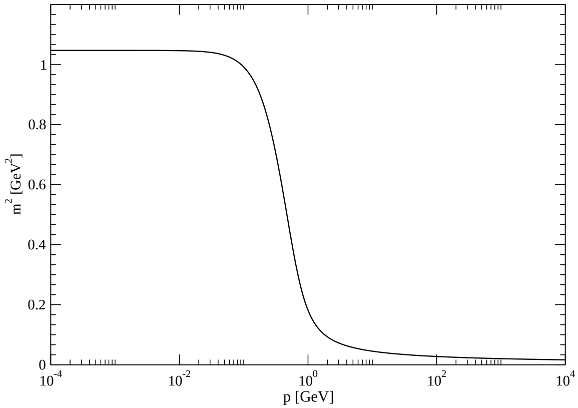
- IR safety ($\beta_g>0$) for $\zeta>1/3$, the simple derivative scheme corresponds to $\zeta=1$; the positivity of the beta function arises from the momentum dependence of the longitudinal part $\Gamma_A^{\parallel}(p^2)$!

- in the following, consider only the critical case $\zeta=1/3$

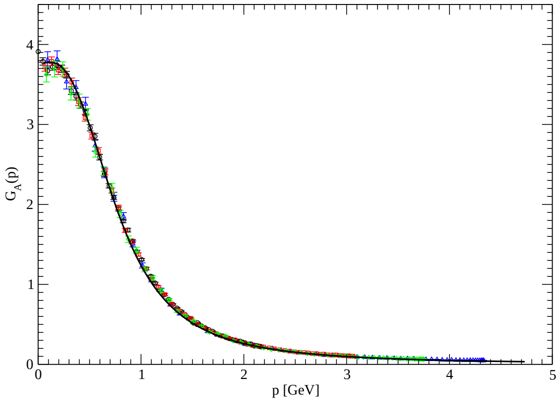
running coupling constant $g(\mu)$



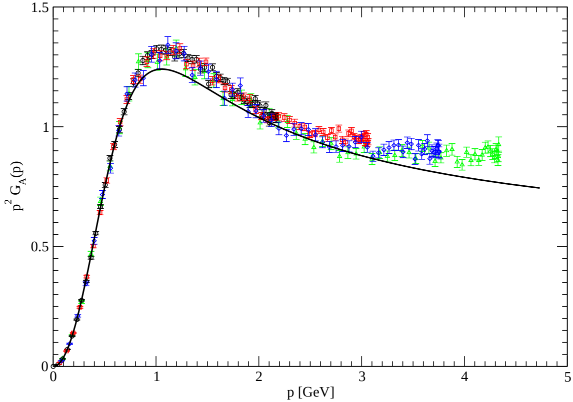
running mass parameter $m^2(\mu)$



gluon propagator function $G_A(p^2)$, critical derivative scheme



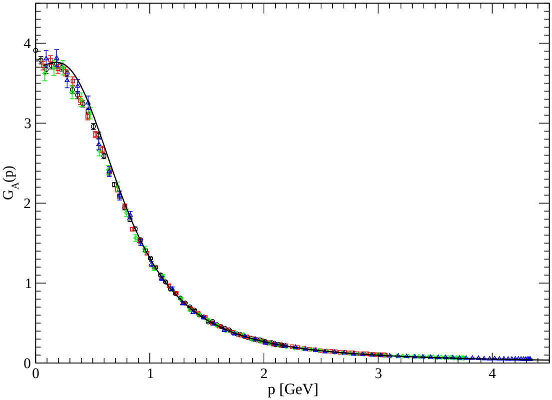
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$, critical derivative scheme



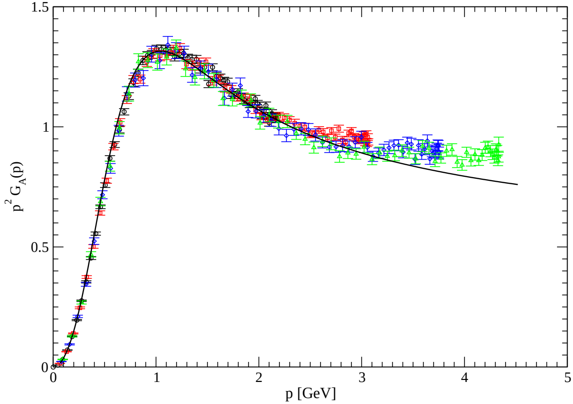
Independent running couplings

- breaking of BRST symmetry destroys the relation between the different renormalized coupling constants
- define the renormalized ghost-gluon coupling constant defined from the renormalized proper ghost-gluon vertex as before, define the renormalized three-gluon coupling constant from the renormalized proper three-point vertex at the symmetry point $p_1^2 = p_2^2 = p_3^2 = \mu^2$
- renormalized four-gluon coupling constant set equal to the renormalized three-gluon coupling constant for the time being
- integration of the Callan-Symanzik equations with two independently running coupling constants and a running mass parameter: BRST symmetry and usual non-massive behavior recovered in the UV, only two adjustable parameters (fine tuning condition)

gluon propagator $G_A(p^2)$ in 4 dimensions,
two coupling constants

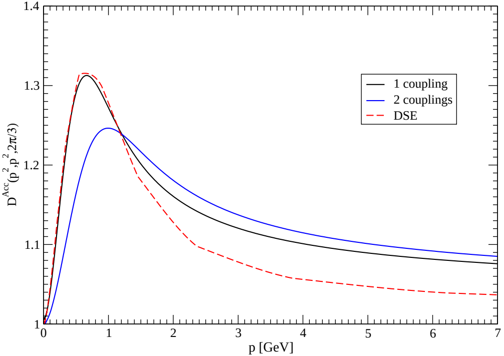


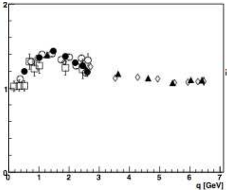
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ in 4 dimensions, two coupling constants



ghost-gluon vertex function at the symmetry point $p^2 = q^2 = k^2$
in 4 dimensions, two coupling constants,
compared to an approximate solution of the Dyson-Schwinger equations

SU(2) lattice data: Cucchieri, Maas, Mendes 2008

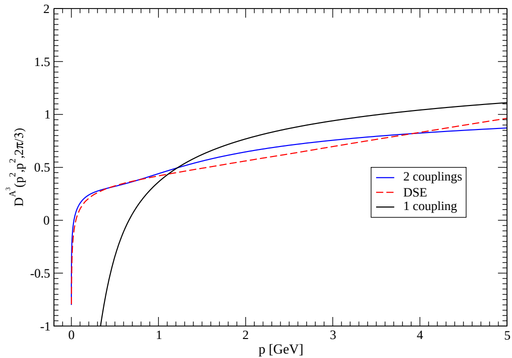


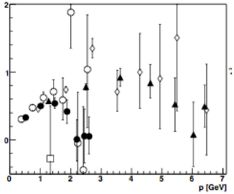


at tree level, the vertex function is equal to one

three-gluon vertex function at the symmetry point $p_1^2 = p_2^2 = p_3^2$
in 4 dimensions, two coupling constants,
compared to an approximate solution of the Dyson-Schwinger equations

SU(2) lattice data: Cucchieri, Maas, Mendes 2008





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Dynamical quarks

- success in Yang-Mills theory motivates the use of one-loop Callan-Symanzik renormalization group equations in full QCD;
  hope to describe dynamical mass generation by introducing running quark mass parameters, without a representation of its dynamical origin (chiral condensate)
- first study by Peláez, Tissier and Wschebor [Peláez, Tissier, Wschebor 2014]: reasonable representation of the mass function $M(p^2)$, but not of the dressing function (field renormalization) $Z(p^2)$ of the full quark propagator

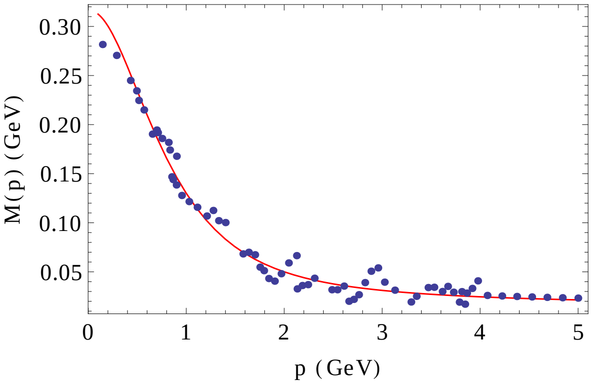
$$\frac{Z(p^2)}{i\not{p} + M(p^2)}$$

  (two-loop contributions important?)
- no dynamical mass generation in the chiral limit [Peláez, Tissier, Wschebor 2015]

quark mass function $M_{u,d}(p^2)$ in 4 dimensions for $N_f = 2 + 1$ flavors,
one-loop renormalization group improved:
optimized fit, leading to a less satisfactory fit of the unquenched gluon propagator

Peláez, Tissier, Wschebor 2014

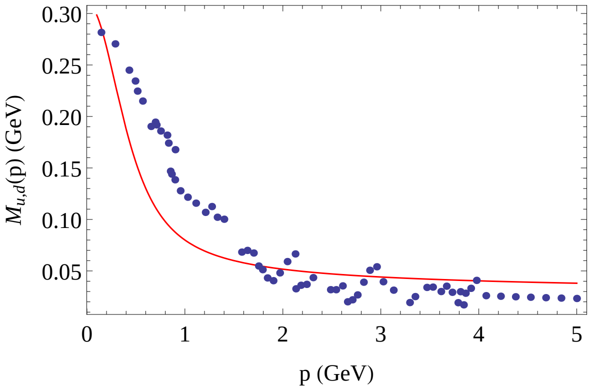
lattice data: Bowman et al. 2004, 2005



quark mass function $M_{u,d}(p^2)$ in 4 dimensions for $N_f = 2 + 1$ flavors,
one-loop renormalization group improved:
parameters adjusted for an optimized fit to the unquenched gluon propagator

Peláez, Tissier, Wschebor 2014

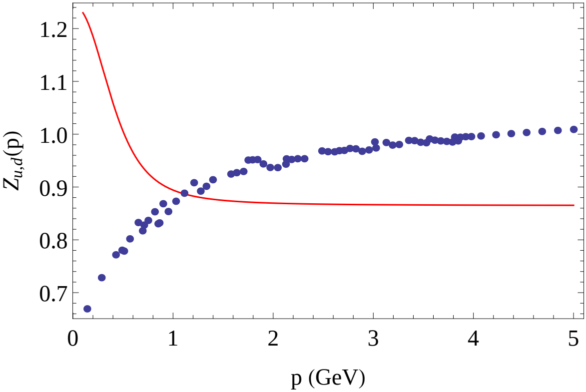
lattice data: Bowman et al. 2004, 2005



quark dressing function $Z_{u,d}(p^2)$ in 4 dimensions for $N_f = 2 + 1$ flavors, one-loop renormalization group improved:
parameters adjusted for an optimized fit to the quark mass function

Peláez, Tissier, Wschebor 2014

lattice data: Bowman et al. 2005



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General strategy

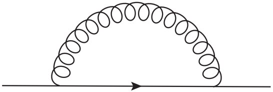
- analyze the description of dynamical mass generation through the Callan-Symanzik equations in a simpler gauge theory:
  QED in $D = 3$ Euclidean space-time dimensions in the Landau gauge (with Juan Pablo Gutiérrez)

- for a direct comparison with Dyson-Schwinger equations, use the quenched and the rainbow ladder approximations (tree-level photon propagator and electron-photon vertex; lead to a closed gap equation for the electron mass function)

- note: these approximations are not entirely consistent for the renormalization of the theory

Self-energy

- begin with the one-loop renormalization of QED_3 in the quenched and rainbow ladder aproximations
- one-loop electron self-energy in 3 dimensions



convergent for $p^2 > 0$

- for a massless photon (quenched approximation),

$$\Sigma(p) = \frac{e_0^2}{2\pi} \frac{M_0}{p} \arctan\left(\frac{p}{M_0}\right)$$

with $p = \sqrt{p^2}$, M_0 the bare electron mass and e_0 the bare coupling constant

Renormalization

- the bare one-loop electron propagator is

$$\left[i\not{p} + M_0 + \frac{e_0^2}{2\pi} \frac{M_0}{p} \arctan\left(\frac{p}{M_0}\right)\right]^{-1}$$

- implement as a normalization condition that the renormalized electron propagator for momenta p with $p^2 = \mu^2$ (μ the renormalization scale) is

$$\frac{Z(\mu)}{i\not{p} + M(\mu)}$$

- this condition defines $Z(\mu) = 1$ and the renormalized mass

$$M(\mu) = M_0 + \frac{e_0^2}{2\pi} \frac{M_0}{\mu} \arctan\left(\frac{\mu}{M_0}\right) + \mathcal{O}(e_0^4)$$

- as a consequence of the rainbow ladder approximation and $Z(\mu) = 1$, the renormalized coupling constant

$$e(\mu) = e_0 \equiv e$$

- as part of the renormalization procedure, all quantities are expressed in terms of the renormalized mass and coupling constant (as the — in principle — experimentally accessible quantities), in particular

$$M_0 = M(\mu) - \frac{e^2}{2\pi} \frac{M(\mu)}{\mu} \arctan\left(\frac{\mu}{M(\mu)}\right) + \mathcal{O}(e^4)$$

The NJL argument

- usually, one is not interested in the value of the bare mass M_0 as a function of the renormalized mass $M(\mu)$,

$$M_0 = M(\mu) - \frac{e^2}{2\pi}\frac{M(\mu)}{\mu}\arctan\left(\frac{\mu}{M(\mu)}\right) \qquad (1)$$

(at one-loop level in the renormalized theory, using the quenched and rainbow ladder approximations), but particularly in the chiral limit $M_0 \to 0$, equation (1) is relevant

- the same equation (1) follows from the original argument of Nambu and Jona-Lasinio (NJL) [Nambu, Jona-Lasinio 1961] applied to QED_3:
add a contribution δM to the bare mass M_0 such that the electron self-energy calculated with the new mass $M = M_0 + \delta M$ and including the counterterm $(-\delta M)$ vanishes,

$$\Sigma(p) = -\delta M + \frac{e_0^2}{2\pi}\frac{M}{p}\arctan\left(\frac{p}{M}\right) = 0$$

- unlike in the theory originally considered by NJL, this is only possible at one (arbitrarily chosen) momentum scale μ (on the other hand, no momentum cutoff needs to be introduced here), thus

$$M_0 + \frac{e_0^2}{2\pi}\frac{M}{\mu}\arctan\left(\frac{\mu}{M}\right) = M\,,$$

which is equation (1) with $e_0 = e$ and $M = M(\mu)$

Solving for $M(\mu)$

- in the chiral limit, equation (1) becomes

$$0 = M_0 = M(\mu)\left[1 - \frac{e^2}{2\pi\mu}\arctan\left(\frac{\mu}{M(\mu)}\right)\right]$$

- there is a trivial solution, $M(\mu) = 0$, and a nontrivial one,

$$M(\mu) = \frac{\mu}{\tan(2\pi\mu/e^2)}$$

  as long as

$$\frac{e^2}{2\pi\mu}\,\frac{\pi}{2} \geq 1 \quad \Leftrightarrow \quad \mu \leq \frac{e^2}{4}\,;$$

  in particular,

$$M(\mu = 0) = \frac{e^2}{2\pi}$$

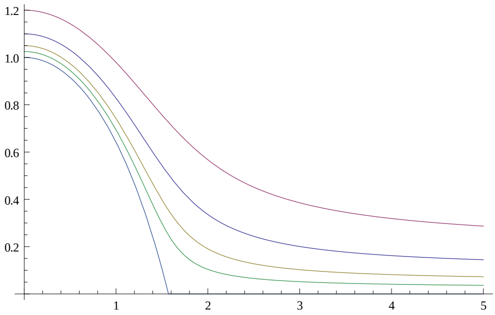
- at $\mu = e^2/4$, the nontrivial solution coincides with the trivial solution; for $\mu > e^2/4$ only the trivial solution exists

Varying M_0

- for $M_0 > 0$, equation (1) can only be solved numerically; however, two limits can be established analytically:

$$M(\mu = 0) = M_0 + \frac{e^2}{2\pi}\,, \qquad M(\mu \to \infty) = M_0$$

- a plot of $M(\mu)$ for several values of M_0, in units of $e^2/2\pi$ (both μ and M):



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Linear approximation

- Dyson-Schwinger equation for the mass function $M(p^2)$ in QED$_3$ in the Landau gauge, using the quenched and the rainbow ladder approximations:

$$M(p^2) = M_0 + 2e^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{(p-k)^2} \frac{M(k^2)}{k^2 + M^2(k^2)}$$

- in the chiral limit $M_0 \to 0$ with the (additional) linear approximation

$$\frac{M(k^2)}{k^2 + M^2(k^2)} \approx \frac{M(k^2)}{k^2 + M^2(0)} ,$$

one finds the exact solution

$$M(p^2) = \frac{e^2}{4\pi} \frac{1}{1 + (4\pi p/e^2)^2} ,$$

- in the extreme limits,

$$M(p^2 = 0) = \frac{e^2}{4\pi} , \qquad M(p^2 \to \infty) \propto \frac{1}{p^2}$$

Angular approximation

- alternatively, one may use (for $M_0 \geq 0$) the angular approximation to evaluate the k-integral

$$\int \frac{d^3k}{(2\pi)^3} \frac{M(k^2)}{k^2 + M^2(k^2)} \frac{1}{(p-k)^2}$$
$$= \frac{1}{(2\pi)^2} \int_0^\infty dk \, \frac{k^2 M(k^2)}{k^2 + M^2(k^2)} \int_{-1}^{1} \frac{d\cos\theta}{k^2 + p^2 - 2p\,k\cos\theta}$$
$$= \frac{1}{(2\pi)^2 p} \int_0^\infty dk \, \frac{k\,M(k^2)}{k^2 + M^2(k^2)} \ \ln\frac{k+p}{|k-p|}$$

- now

$$\text{for } k \gg p\,, \quad \ln\frac{k+p}{|k-p|} \approx \frac{2p}{k}\,;$$
$$\text{for } k \ll p\,, \quad \ln\frac{k+p}{|k-p|} \approx \frac{2k}{p}\,,$$

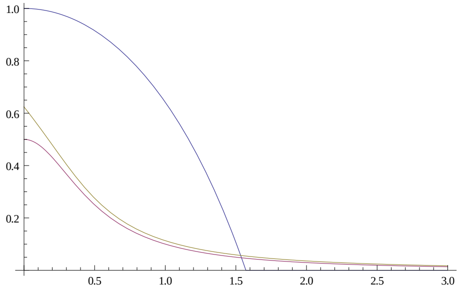
and the angular approximation is

$$\ln\frac{k+p}{|k-p|} \approx \frac{2p}{k}\Theta(k-p) + \frac{2k}{p}\Theta(p-k)$$

under the k-integral

Comparison in the chiral limit

- in the angular approximation, the gap equation can be converted (exactly) into a differential equation which is subsequently numerically solved; for large momenta, one finds analytically
$$M(p^2 \to \infty) = M_0 \,,$$
and particularly in the chiral limit $M_0 \to 0$,
$$M(p^2 \to \infty) \propto \frac{1}{p^2}$$
- Comparison of the three mass functions, $M(\mu)$ from the one-loop renormalization or the NJL argument, and $M(p^2)$ from the Dyson-Schwinger equation in the linear and the angular approximations, in the chiral limit (in units of $e^2/2\pi$):



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Callan-Symanzik equations

- the bare parameters and the bare n-point functions cannot depend on the renormalization scale μ, in particular, at the one-loop level,

$$\mu \frac{dM(\mu)}{d\mu} = \mu \frac{\partial}{\partial \mu} \left[\frac{e_0^2}{2\pi} \frac{M_0}{\mu} \arctan \left(\frac{\mu}{M_0} \right) \right] \Bigg|_{M_0, e_0 \text{ fixed}}$$
$$= \frac{e^2}{2\pi} \left[\frac{M^2(\mu)}{\mu^2 + M^2(\mu)} - \frac{M(\mu)}{\mu} \arctan \left(\frac{\mu}{M(\mu)} \right) \right], \tag{2}$$

expressing the bare quantities in terms of the renormalized ones in the last step (renormalization group improvement)

- using the normalization condition for the renormalized electron propagator

$$\frac{Z(\mu)}{i\not{p} + M(\mu)}$$

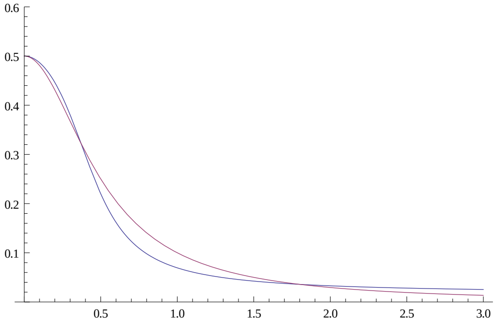
at $\mu = p$, the renormalization group improved propagator is

$$\frac{1}{i\not{p} + M(p)}$$

where $M(p)$ is the function obtained by integrating the differential equation (2)

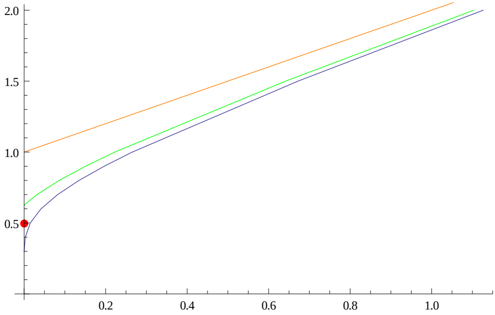
Results

- no mass is generated dynamically in the chiral limit through the integration of the Callan-Symanzik equation (2)
- Comparison of the analytical solution of the Dyson-Schwinger gap equation in the chiral limit with the solution of the Callan-Symanzik equation with the same initial value $M(p^2 = 0)$:



$M(p^2 = 0)$ as a function of $M(p^2 \to \infty) = M_0$

in the one-loop renormalization of the theory or the NJL argument,
the angular and the linear approximations of the Dyson-Schwinger gap equation
and the Callan-Symanzik renormalization group equation



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Summary

- quasi-analytic and systematic description of the correlation functions of Landau gauge Yang-Mills theory: introduce a gluonic mass term, solve the Callan-Symanzik equations
- this success motivates to try the same approach for the description of dynamical quark mass generation in QCD; a first exploration by Peláez, Tissier and Wschebor is encouraging, but not entirely successful
- look at a simpler theory first: QED in 3 space-time dimensions in the quenched and rainbow ladder approximations (for comparison to the solution of the Dyson-Schwinger gap equation)
- the standard one-loop renormalization of the theory leads to the same equation for the effective mass as Nambu-Jona-Lasinio's original argument
- an effective (or renormalized) mass is generated even in the chiral limit, but its dependence on the momentum scale is not satisfactory
- applying Callan-Symanzik renormalization group equations leads to a much better momentum scale dependence, judging from a comparison with the solution of the Dyson-Schwinger gap equation (here: in the linear and angular approximations)
- the value of the mass $M(p^2 = 0)$ generated by the renormalization group is quite similar to the one generated by the Dyson-Schwinger equation (in the angular approximation) for not too small bare masses ($M_0 \gtrsim 0.1 e^2/(2\pi)$), but there is no dynamical mass generation in the chiral limit by the Callan-Symanzik equations

Outlook

- Yang-Mills theory: current calculations in 3 space-time dimensions
- continue work on the vertex functions, compare to new lattice and Dyson-Schwinger results
- proceed to two-loop level (with renormalization group improvement)
- extend the formalism to 2 space-time dimensions (scaling solutions)
- dynamical fermion mass generation: to complete the comparison to the Dyson-Schwinger gap equation, look at the numerical solutions of the full equation
- before moving on to QCD, one may consider QED_3 without the quenched and rainbow ladder approximations in the renormalization group approach (those are not consistent approximations in the renormalization of the theory)
- consider alternative renormalization schemes, different space-time dimensions and epsilon expansions

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