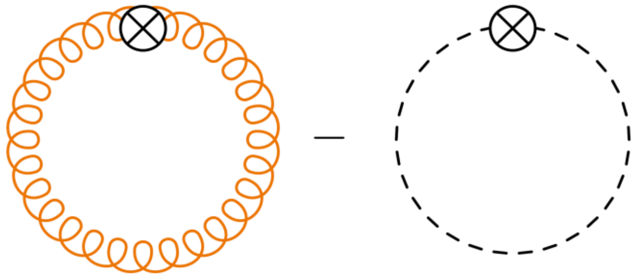
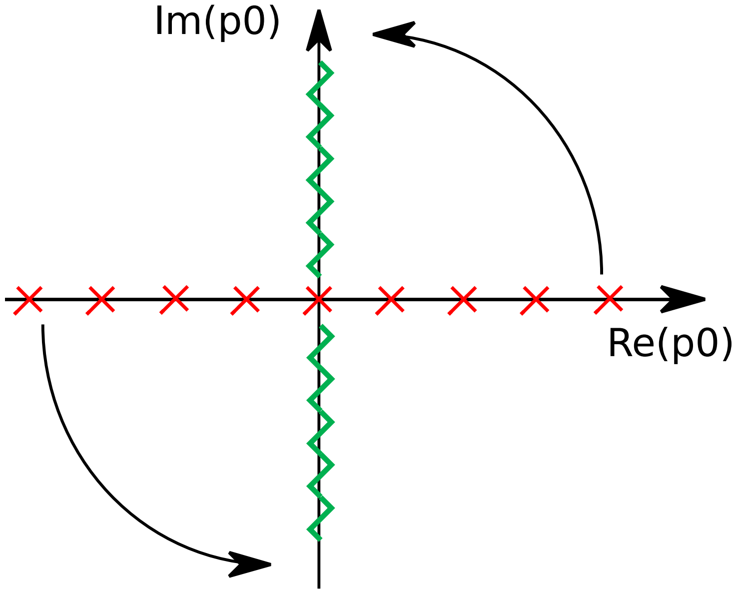
Spectral functions in QCD: Calculation and Application

Lunch Club Seminar 2018

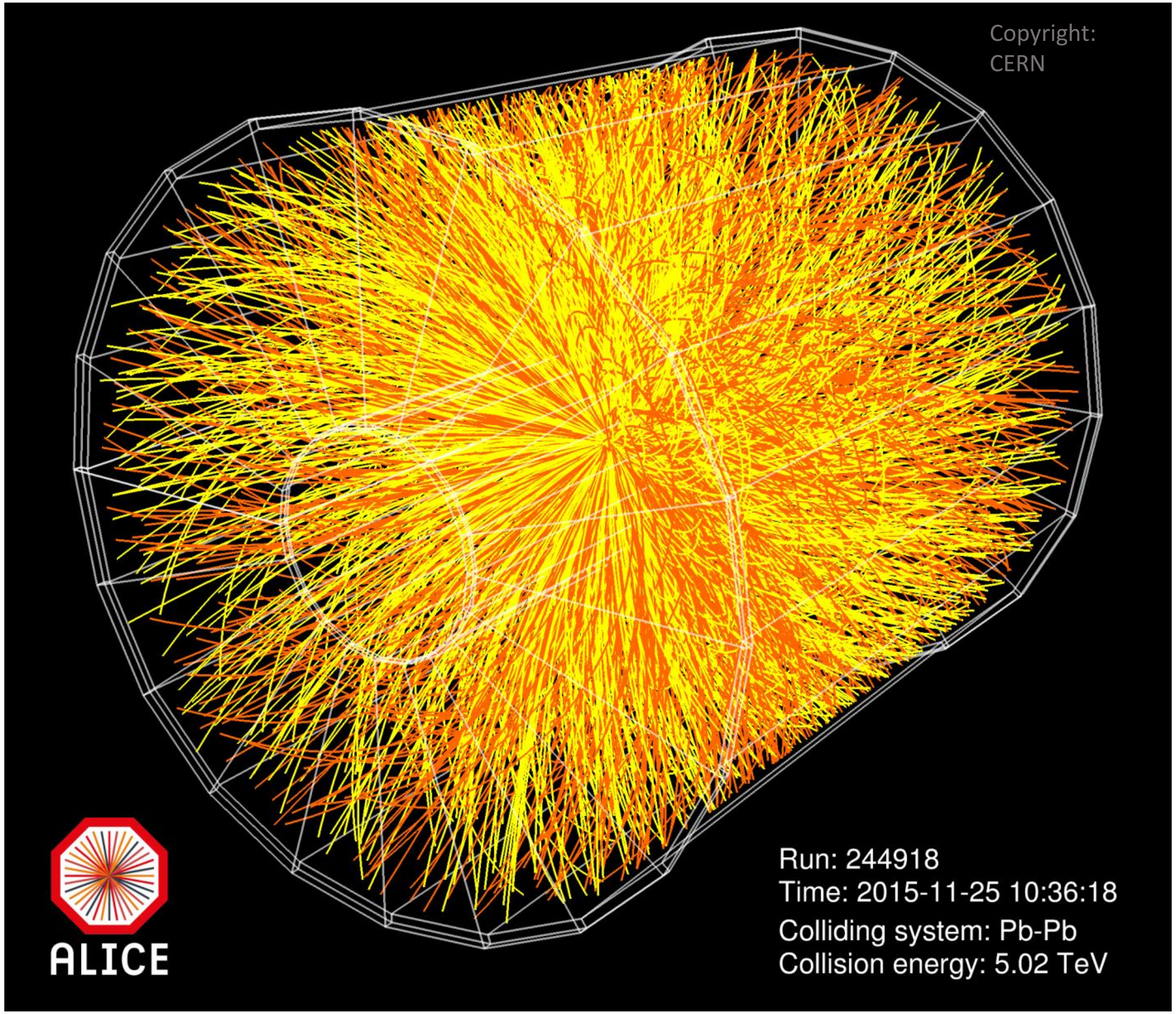
Nicolas Wink

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$ 

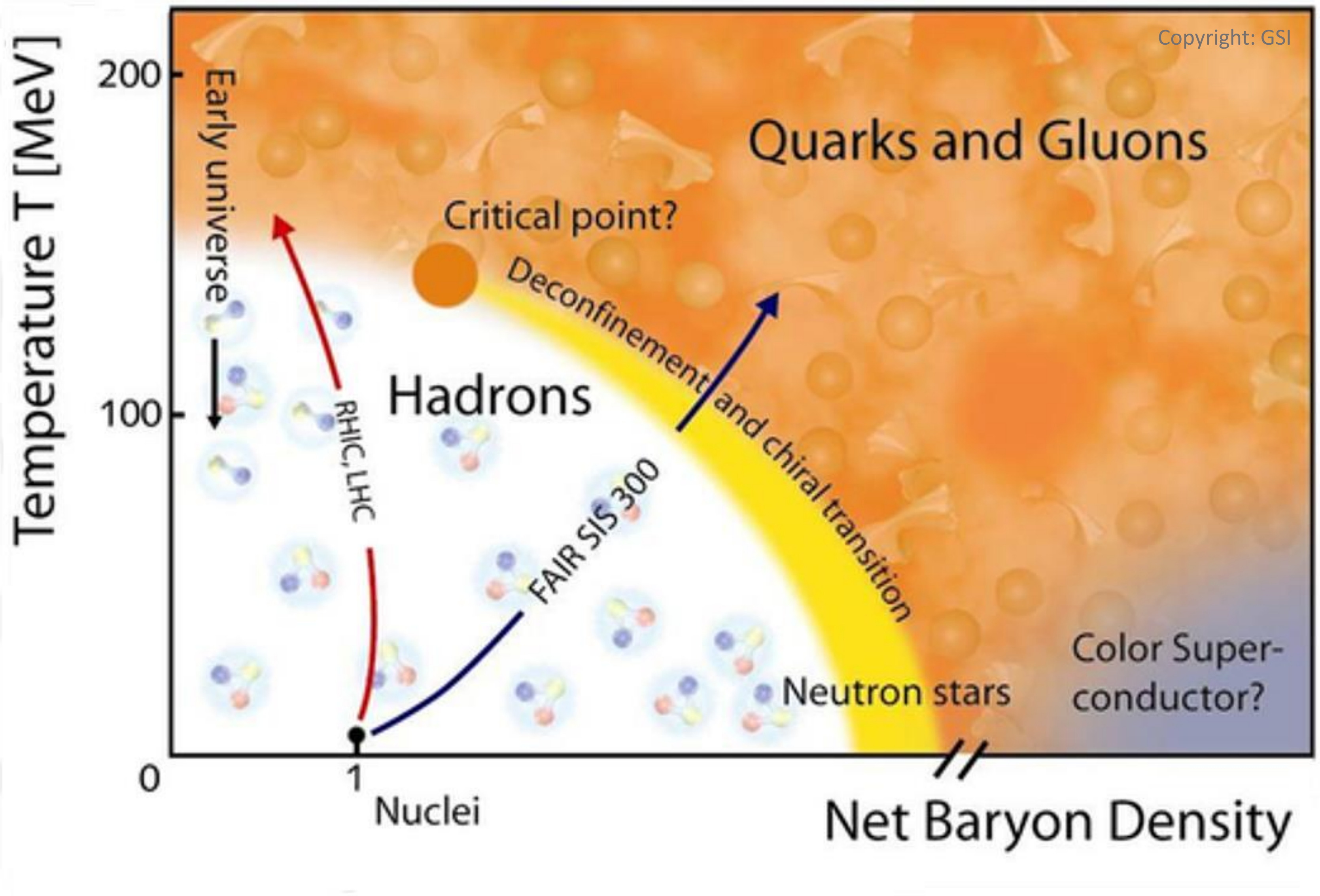


Work in collaboration with:

M. Bluhm, A. K. Cyrol, Y. Jiang, M. Mitter, M. Nahrgang,
J. M. Pawlowski, F. Rennecke, A. K. Rothkopf

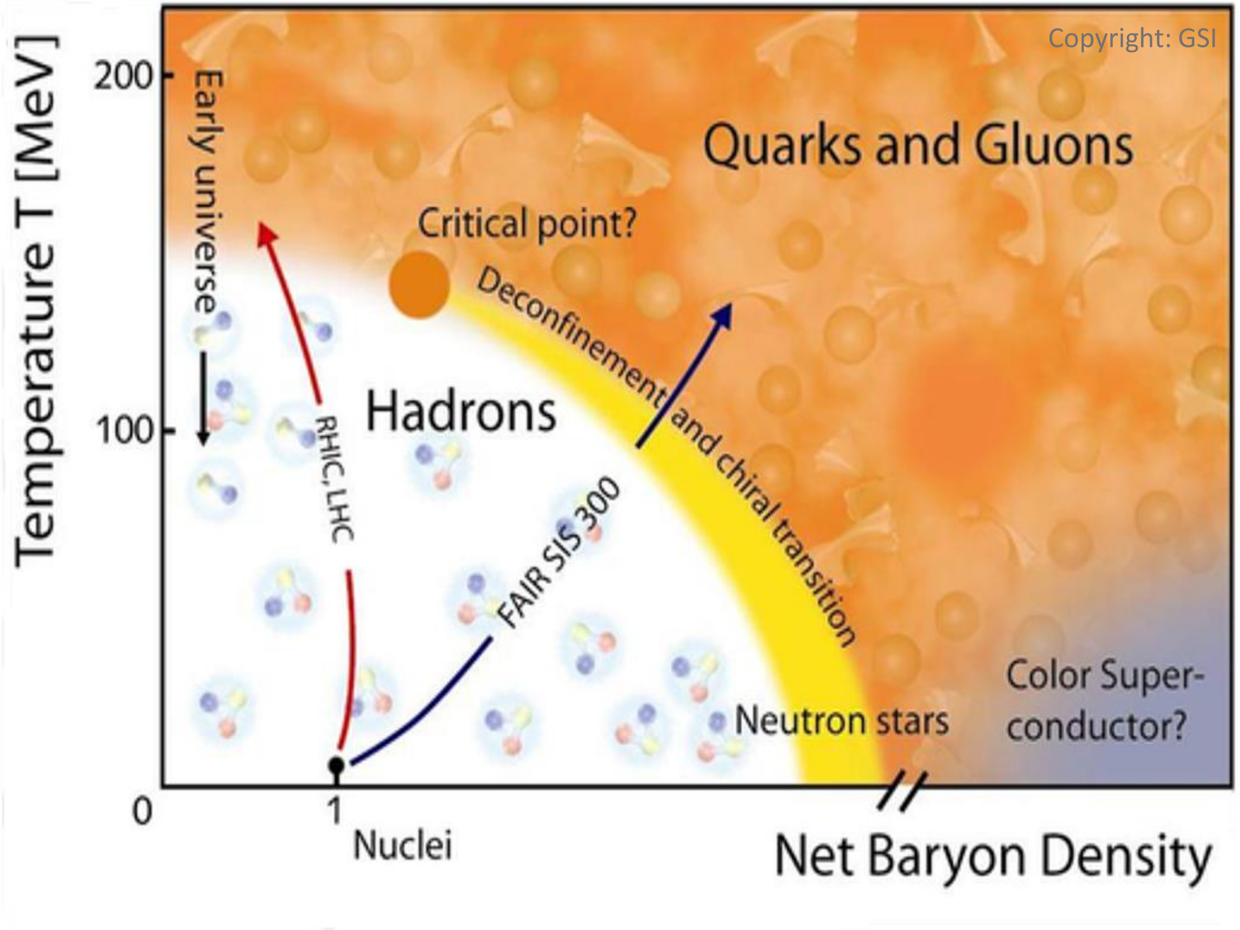


Connection to the phase structure of QCD?



c.f. talk by A. K. Cyrol last week

Dynamical effects matter



- Is there a critical point?
- Connection between theoretical equilibrium results and Heavy Ion Collisions?

Realistic description of non-equilibrium dynamics is crucial!

see e.g. :

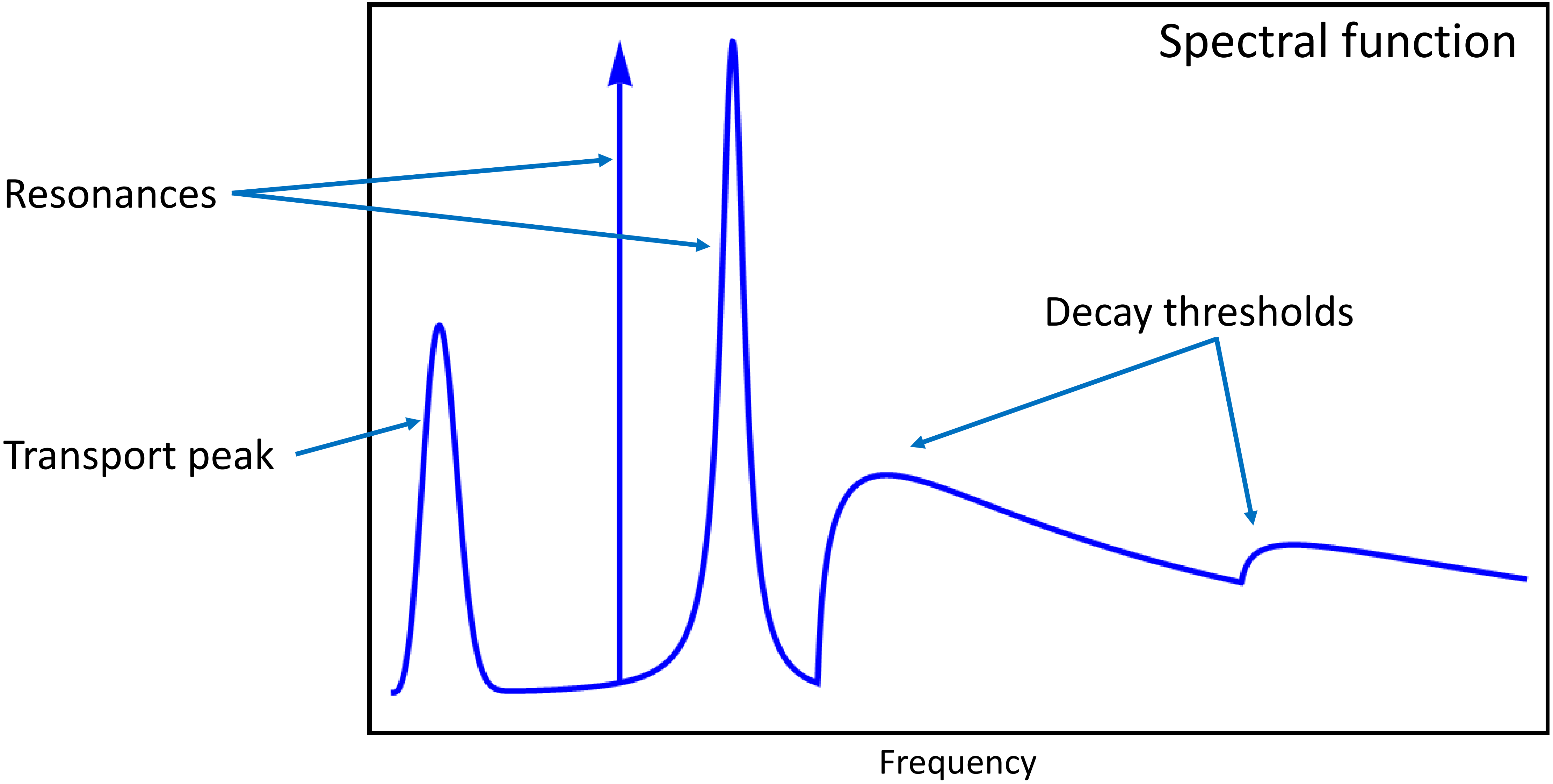
Stephanov, Rajagopal, Shuryak PRL81 (1998)
Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)
Mukherjee, Venugopalan, Yin PRC92 (2015)
Herold, Nahrgang, Yan, Kobdaj PRC93 (2016)
Nahrgang, Bluhm, Schäfer, Bass arXiv:1804.05728

Utilize linear response function

Connect dynamics to equilibrium QCD

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

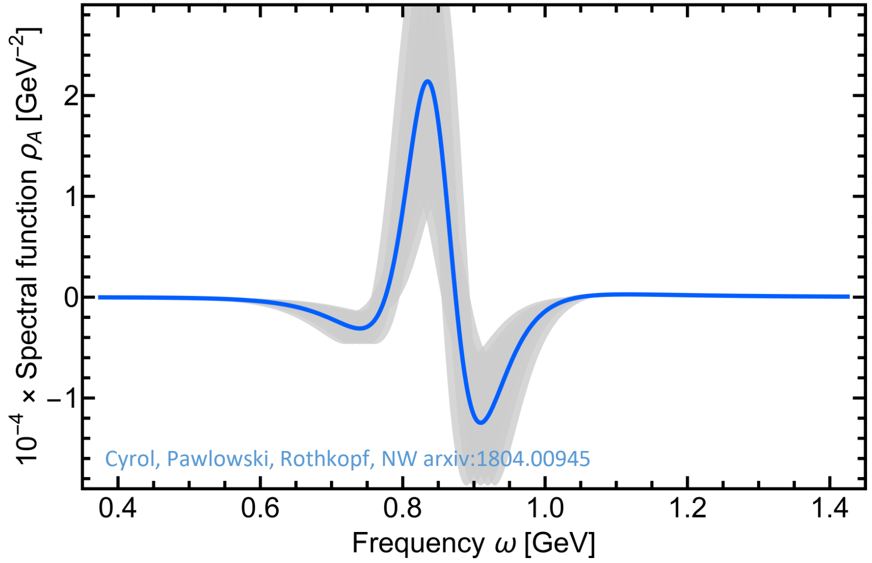
Spectral functions in QCD



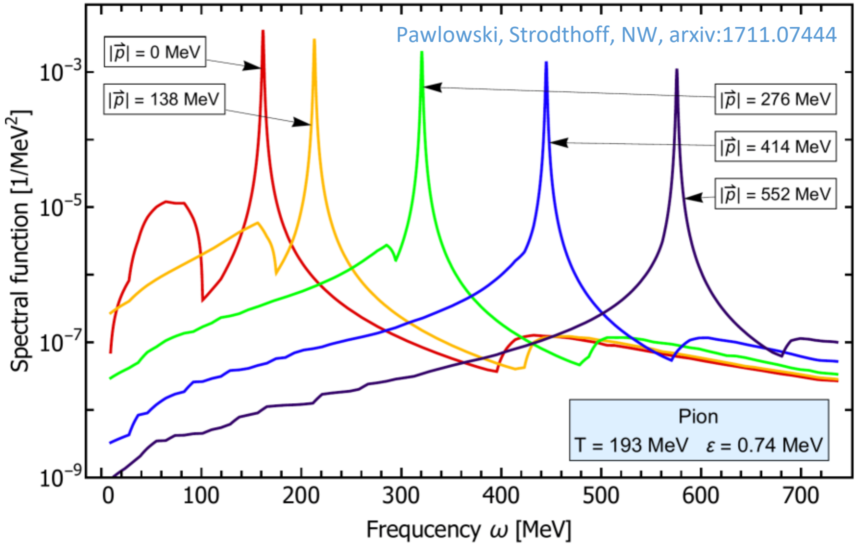
Spectral functions in QCD

How to get non-perturbative correlation functions in Minkowski space-time

Reconstruction from Euclidean data



Direct calculation



→ Applications

Spectral functions

Spectral representation

What are spectral functions

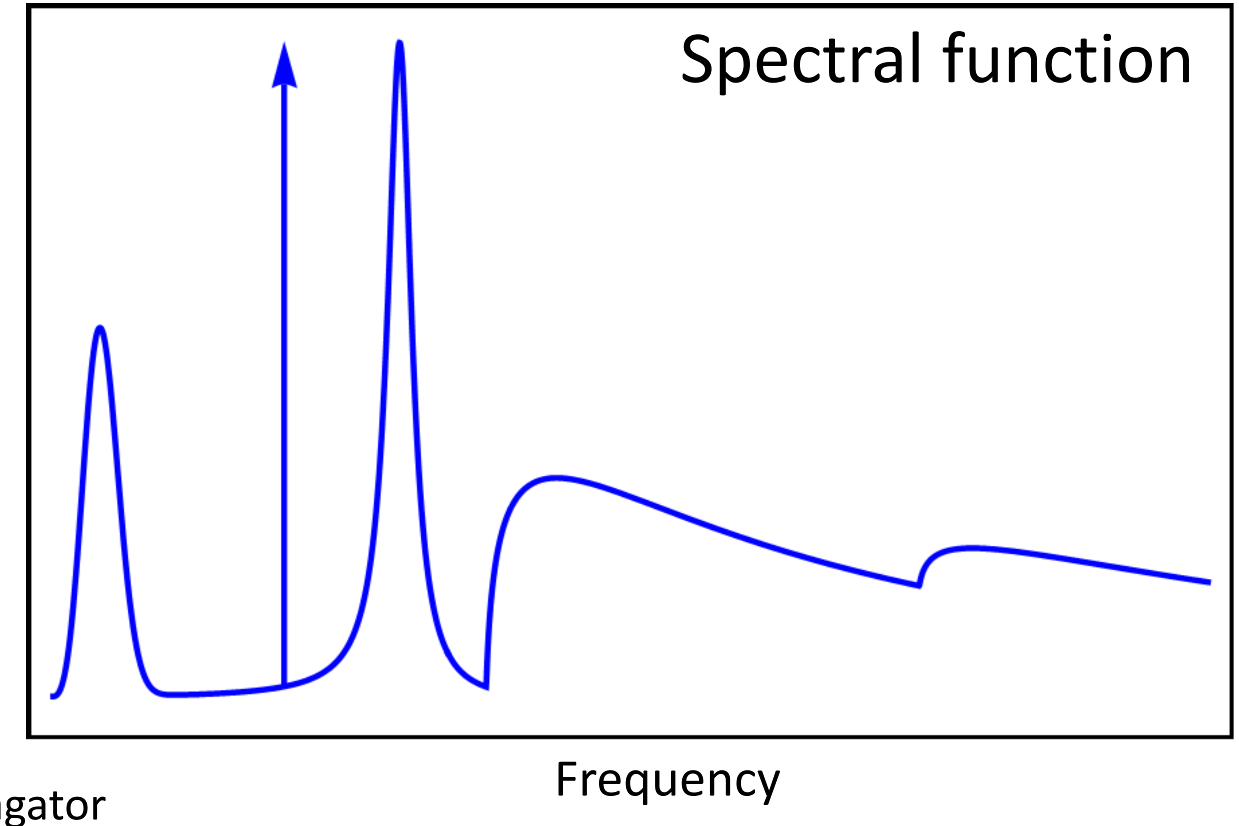
Physical picture :

- Encodes the spectrum of the theory
- Linear response functions

Pragmatic picture :

- $$G_E(p_0, \mathbf{p}) = \int \frac{\mathrm{d}\eta}{2\pi} \frac{\rho(\eta, \mathbf{p})}{\eta - \mathrm{i}p_0}$$

  Integral representation of the (Euclidean) propagator



- Statement about the analytic structure of the propagator

Axiomatic/Mathematical picture :

- Existence linked to a restriction of the functional space (enforces causality)

Higher order spectral representations

What about vertices?

→ **Vertices admit a spectral representation!**

Consider

$$\Gamma^{(n)}(p_1, p_2, \ldots, p_n)$$

Constrained by $\sum \varepsilon_i = 0$

Analytically continue with $p_i \to -\mathrm{i}(\omega_i + \mathrm{i}\varepsilon_i)$

Three-point function

	$\varepsilon_1/\varepsilon$	$\varepsilon_2/\varepsilon$	$\varepsilon_3/\varepsilon$
$\Gamma^{(2)}_{\mathrm{RAA}}$	$+2$	-1	-1
$\Gamma^{(2)}_{\mathrm{ARA}}$	-1	$+2$	-1
$\Gamma^{(2)}_{\mathrm{AAR}}$	-1	-1	$+2$
$\Gamma^{(2)}_{\mathrm{ARR}}$	-2	$+1$	$+1$
$\Gamma^{(2)}_{\mathrm{RAR}}$	$+1$	-2	$+1$
$\Gamma^{(2)}_{\mathrm{RRA}}$	$+1$	$+1$	-2

Spectral representation of three-point functions:

$$\Gamma^{(3)}(p_0, r_0) = \int \frac{d\eta_1}{2\pi}\frac{d\eta_2}{2\pi} \frac{-\mathrm{sgn}(p_0)\mathrm{sgn}(r_0)}{(\eta_1+\eta_2) - \mathrm{i}(p_0+r_0)} \left[\frac{\rho_1(\eta_1,\eta_2)}{\eta_1 - \mathrm{i}p_0} + \frac{\rho_2(\eta_1,\eta_2)}{\eta_2 - \mathrm{i}r_0}\right]$$

Identities:

$$\Gamma^{(3)}_{\alpha\alpha\alpha} = 0 \quad \text{and} \quad \Gamma^{(3)}_{\alpha\beta\gamma} = \left(\Gamma^{(3)}_{\bar\alpha\bar\beta\bar\gamma}\right)^*$$

preliminary

Spectral functions:

$$\rho_1 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{ARA} + \Gamma^{(3)}_{AAR}\right)$$

$$\rho_2 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{RAA} + \Gamma^{(3)}_{AAR}\right)$$

Degenerate for a identical fields:

$$\rho_1(\eta_1,\eta_2) = \rho_2(\eta_2,\eta_1)$$

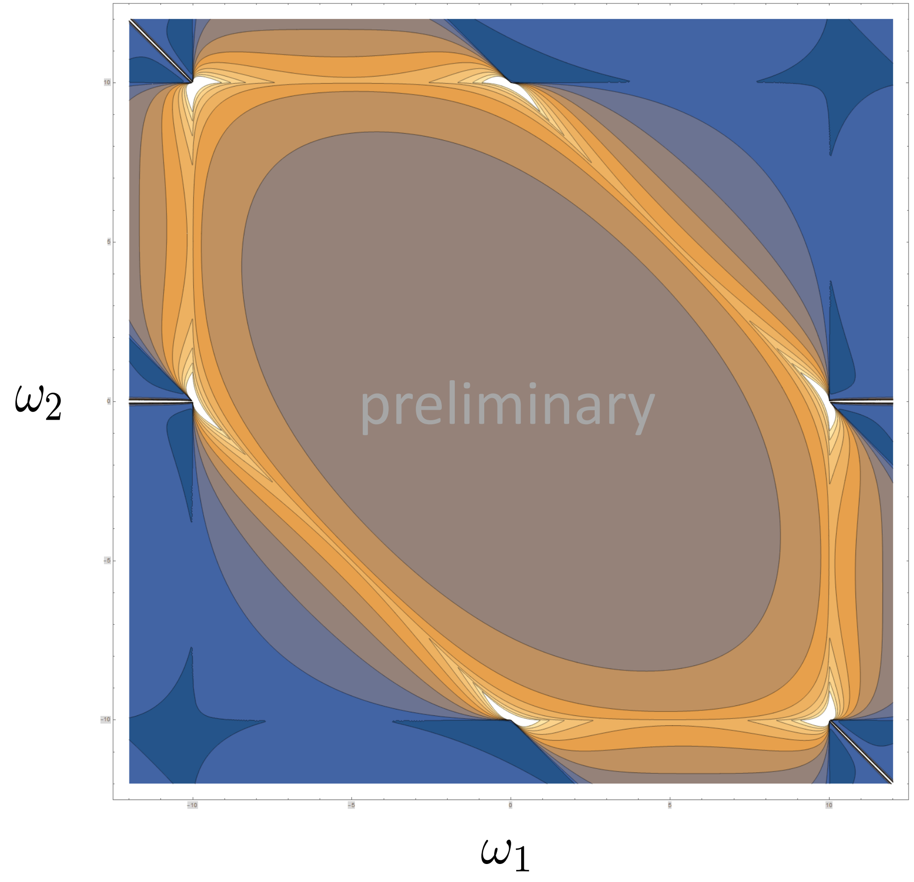
Generalizes to n-point functions

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
Bodeker, Sangel, JCAP 1706 (2017)
Pawlowski, NW, work in progress

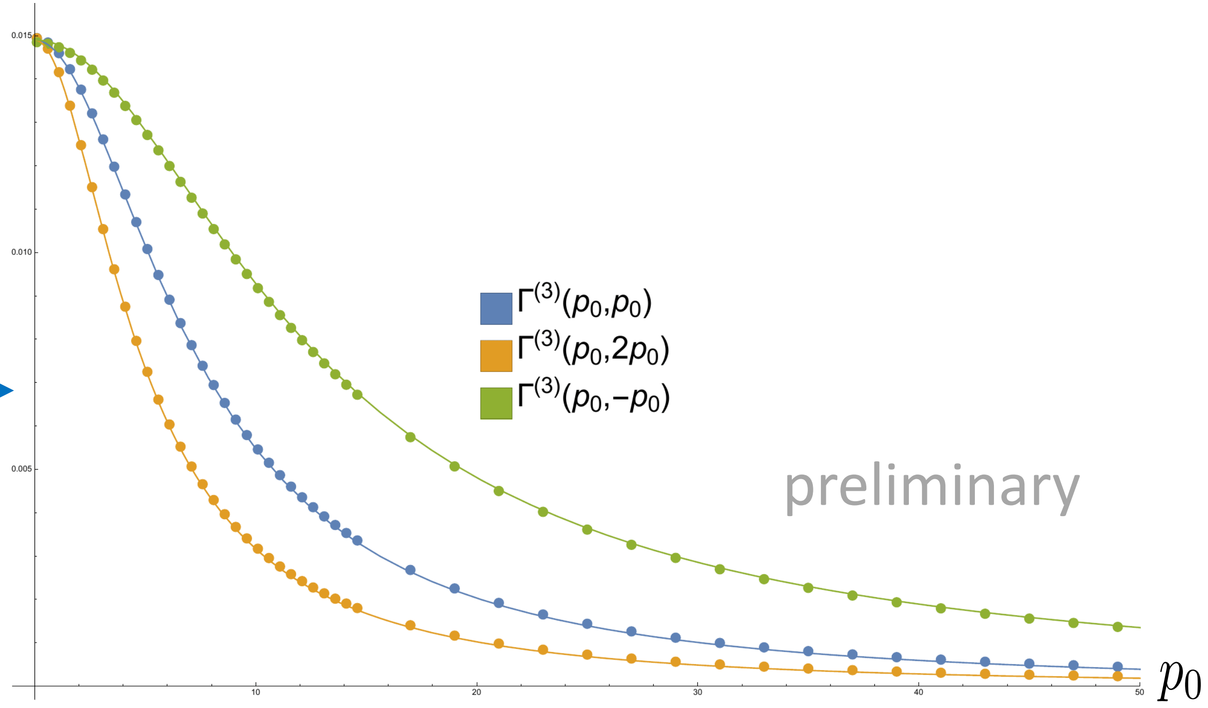
Application to scalar field

1st iteration for a scalar field

Spectral function



Euclidean Dressing



$$\Gamma^{(3)}(p_0, r_0) = \int \frac{d\eta_1}{2\pi} \frac{d\eta_2}{2\pi} \frac{-\text{sgn}(p_0)\text{sgn}(r_0)}{(\eta_1+\eta_2) - i(p_0+r_0)} \left[\frac{\rho_1(\eta_1,\eta_2)}{\eta_1 - ip_0} + \frac{\rho_2(\eta_1,\eta_2)}{\eta_2 - ir_0} \right]$$

Pawlowski, NW, work in progress

Analytic structure of the propagator

Consider three "different" propagators

- Retarded propagator $G_R(\omega, \mathbf{p})$
- Advanced propagator $G_A(\omega, \mathbf{p})$
- Euclidean propagator $G_E(p_0, \mathbf{p})$

(G_R and G_A:)

- Linked in equilibrium via complex conjugation
- Sum of two real (dependent) distributions
- Unique analytic continuation to a holomorphic half-plane

The Euclidean propagator are the two holomorphic half-planes "glued" together

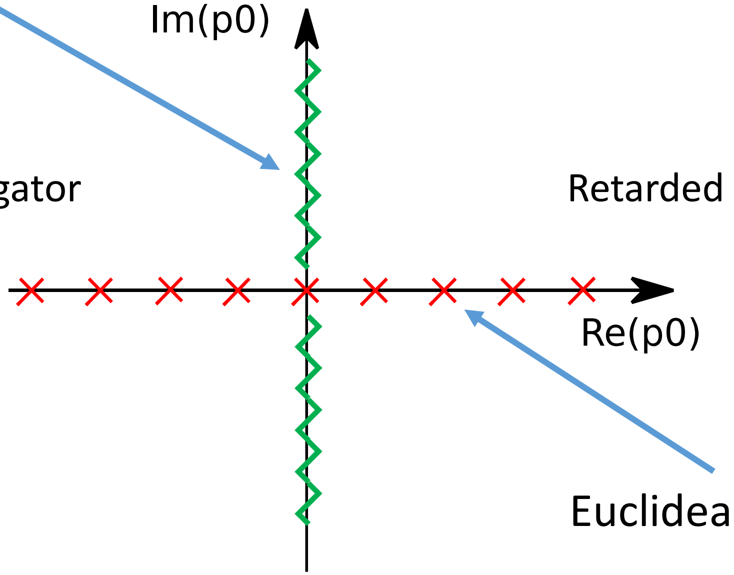
- Holomorphic up to the line $Re(p_0) = 0$
- Discontinuity defines a real distribution, the spectral function

$$G_E(p_0, \mathbf{p}) = \int \frac{\mathrm{d}\eta}{2\pi} \frac{\rho(\eta, \mathbf{p})}{\eta - \mathrm{i}p_0}$$

Spectral function (discontinuity)

Advanced propagator

Retarded propagator



Euclidean data

Propagator in the complex plane

Reconstruction

Spectral reconstruction

Invert: $$G_E(p_0,\mathbf{p}) = \int \frac{\mathrm{d}\eta}{2\pi}\frac{\rho(\eta,\mathbf{p})}{\eta - \mathrm{i}p_0}$$

→ more convenient $$G_E(p_0,\mathbf{p}) = \int_0^\infty \frac{\mathrm{d}\eta}{\pi}\eta\frac{\rho(\eta,\mathbf{p})}{\eta^2+p_0^2}$$

Consider finite temperature (includes vacuum as special case)

→ Reconstruct analytic function from equally spaced points in one half-plane

Matsubara modes

Mathematically:

→ Uniqueness by Carlson's theorem

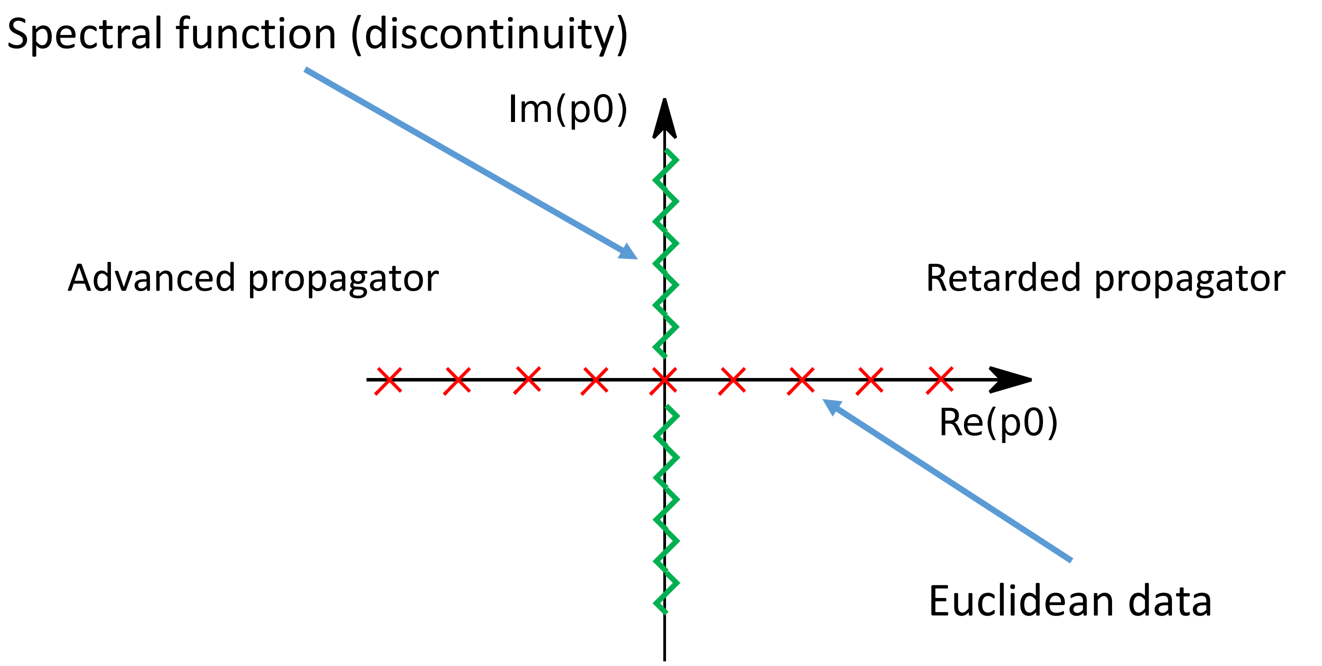
→ Explicit construction of spectral function possible, however the problem is ill-conditioned

Cuniberti, De Micheli, Viano, Commun.Math.Phys. 216 (2001)

Usual reconstructions:

→ Linked functional basis and determination of coefficients

Idea: Physically inspired basis that resprects analytic structure for the propagator



Propagator in the complex plane

Spectral reconstruction

$$\sim \left(\frac{1}{(p_0 + \Gamma)^2 + M^2} \right)^{\delta}$$

Guiding principles:

- Chose a suitable functional basis ⟷ Utilize structures with a physics picture
  - Start from generalized Breit-Wigners
- Utilize all prior knowledge ⟷ Include/Enforce known asymptotics

$$\lim_{p_0 \to 0^+} \partial_{p_0} G(p_0) = -\frac{1}{2} \lim_{\omega \to 0^+} \partial_\omega \rho(\omega)$$

c.f. talk by A. K. Cyrol last week

- Determine coefficients in a reliable way ⟷ Different levels of quality

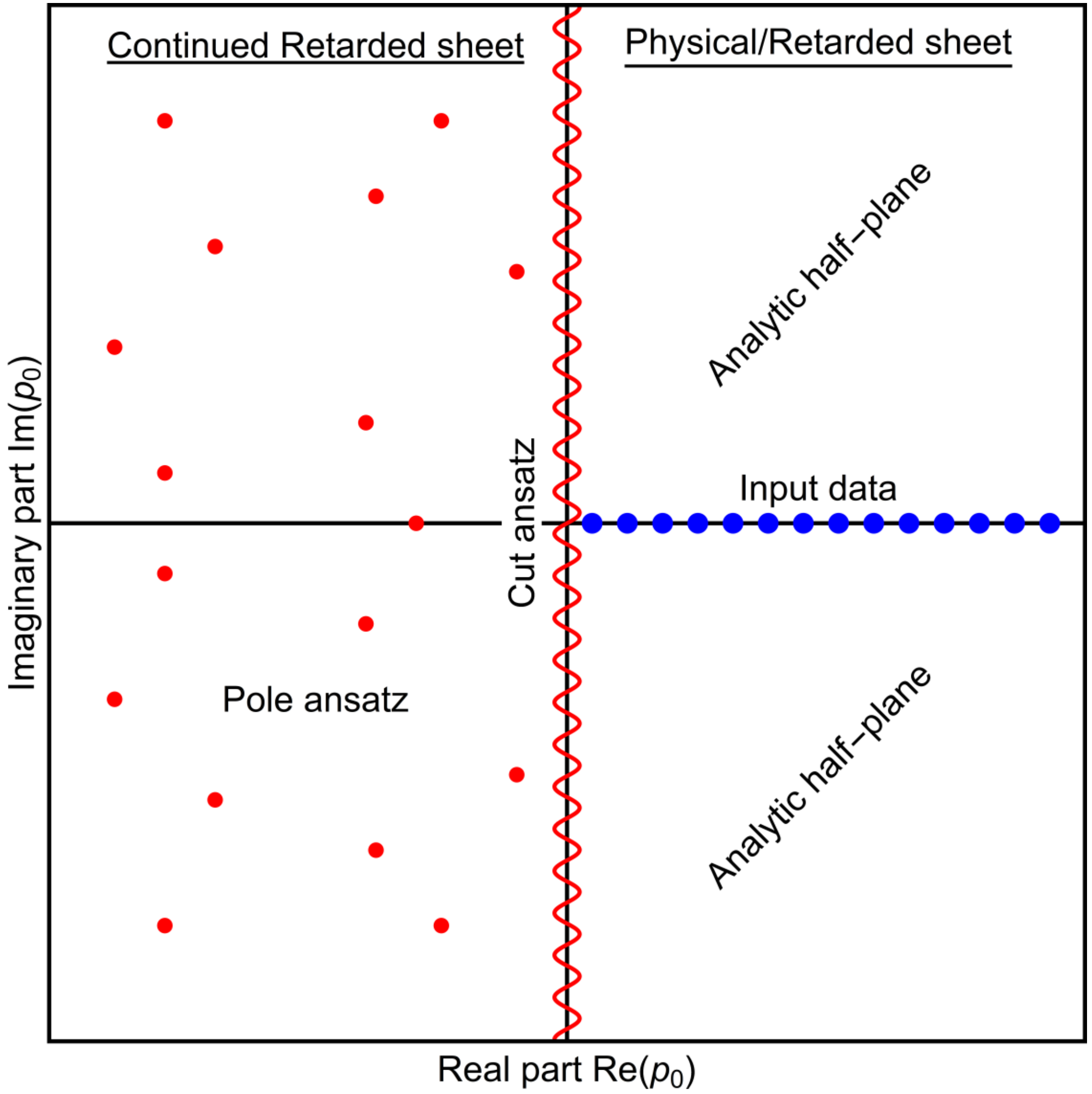
χ^2-fit ⇒ Bayesian Inference ⇒ Hamiltonian Monte-Carlo

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Spectral reconstruction

Connection to the analytic structure

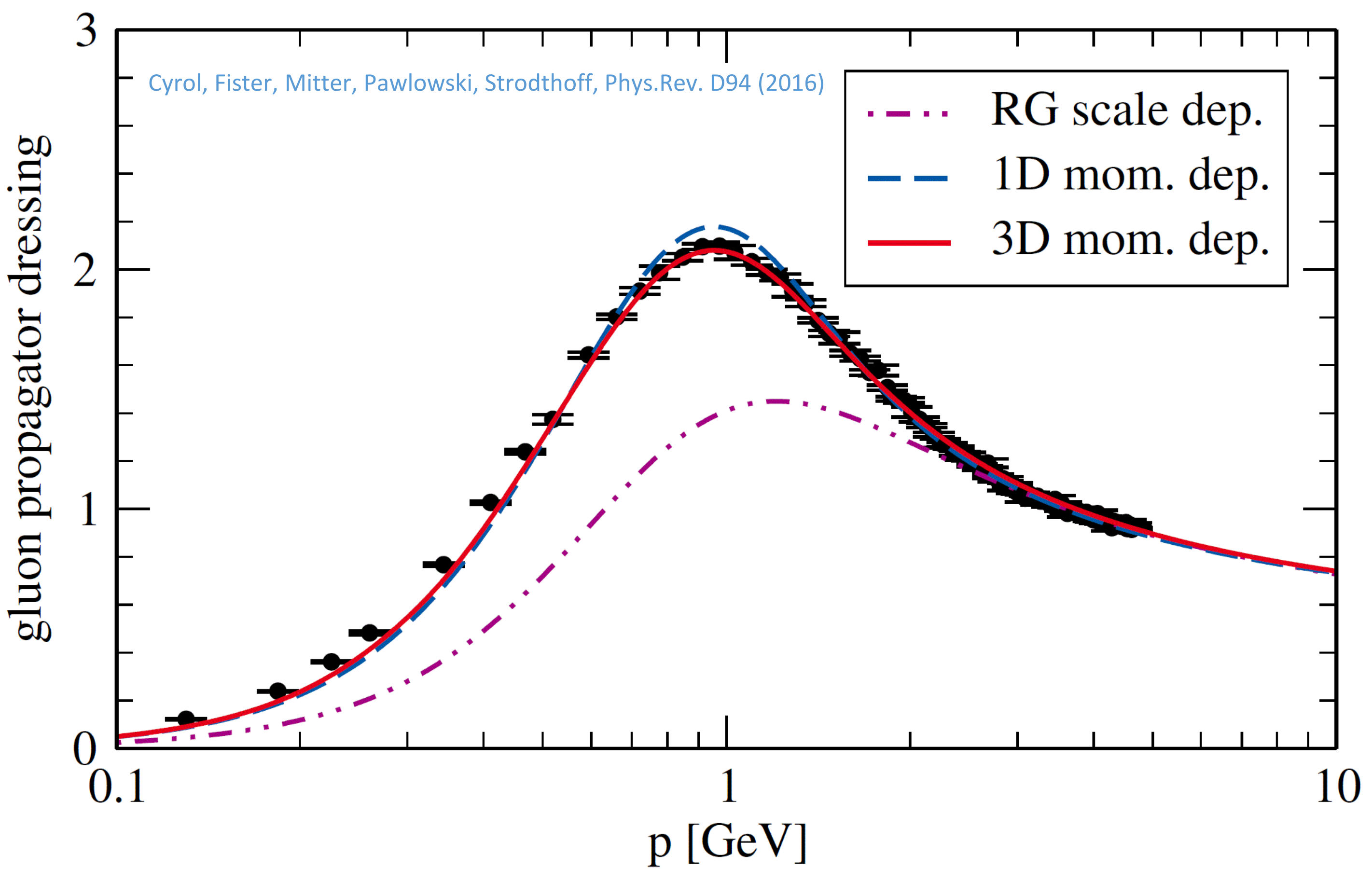
- Consider the analytically continued retarded propagator
- The other half-plane is necessarily meromorphic
- Ansatz for the complex structure of the retarded propagator
- Previous knowledge easily included



Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Reconstructing the gluon

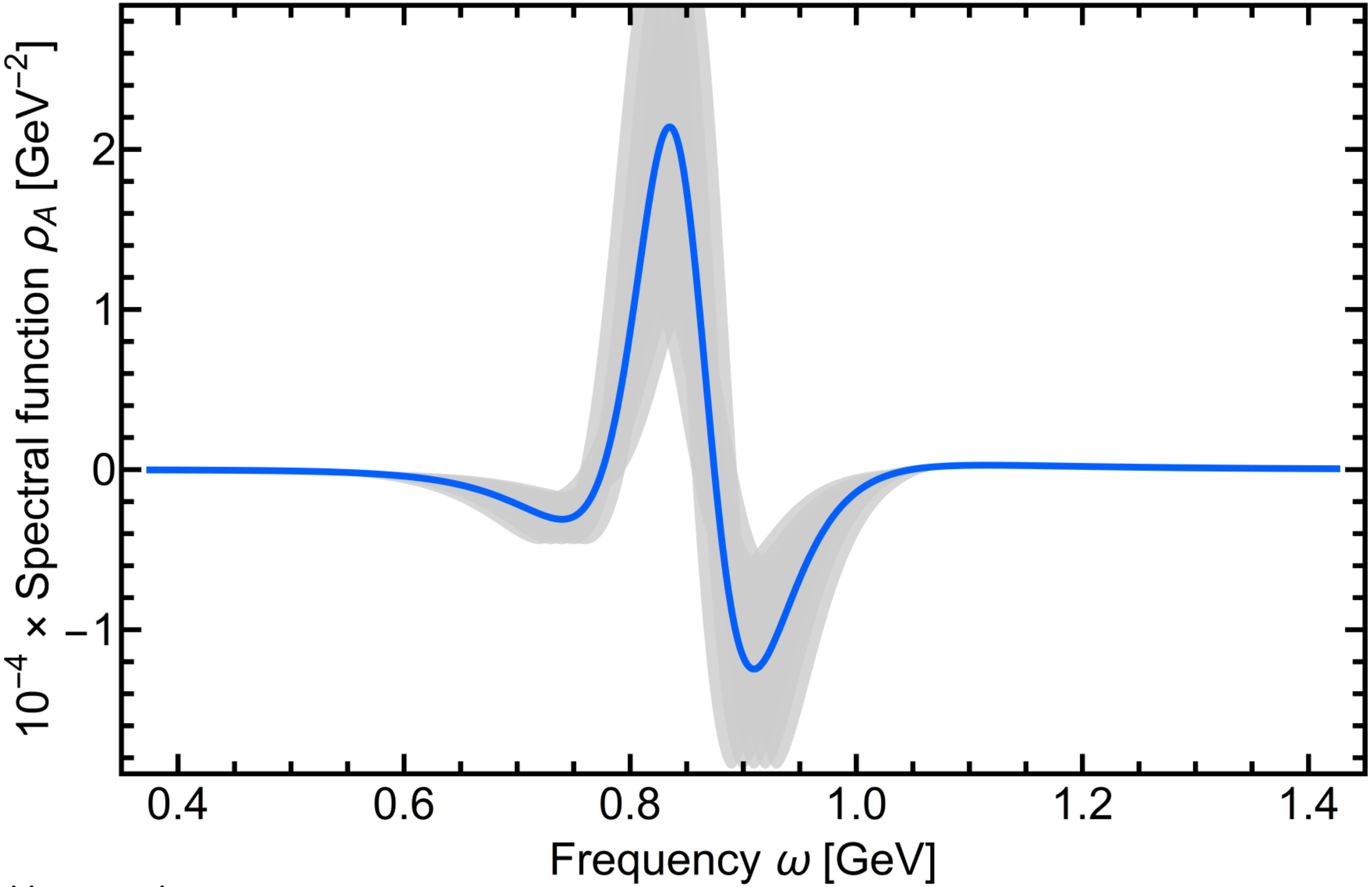
- Gluon admits positivity violation
  - Most reconstruction methods fail (miserably)
- Ansatz includes
  - Generalized Breit-Wigners
  - Polynomials
  - IR & UV asymptotic cuts (negative IR!)
- Determine coefficients via χ^2-fit
  - First start for improvement, but HMC requires uniqueness of the coefficients
- Shape reliable, quantitative details are not



c.f. talk by A. K. Cyrol last week

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Reconstructing the gluon



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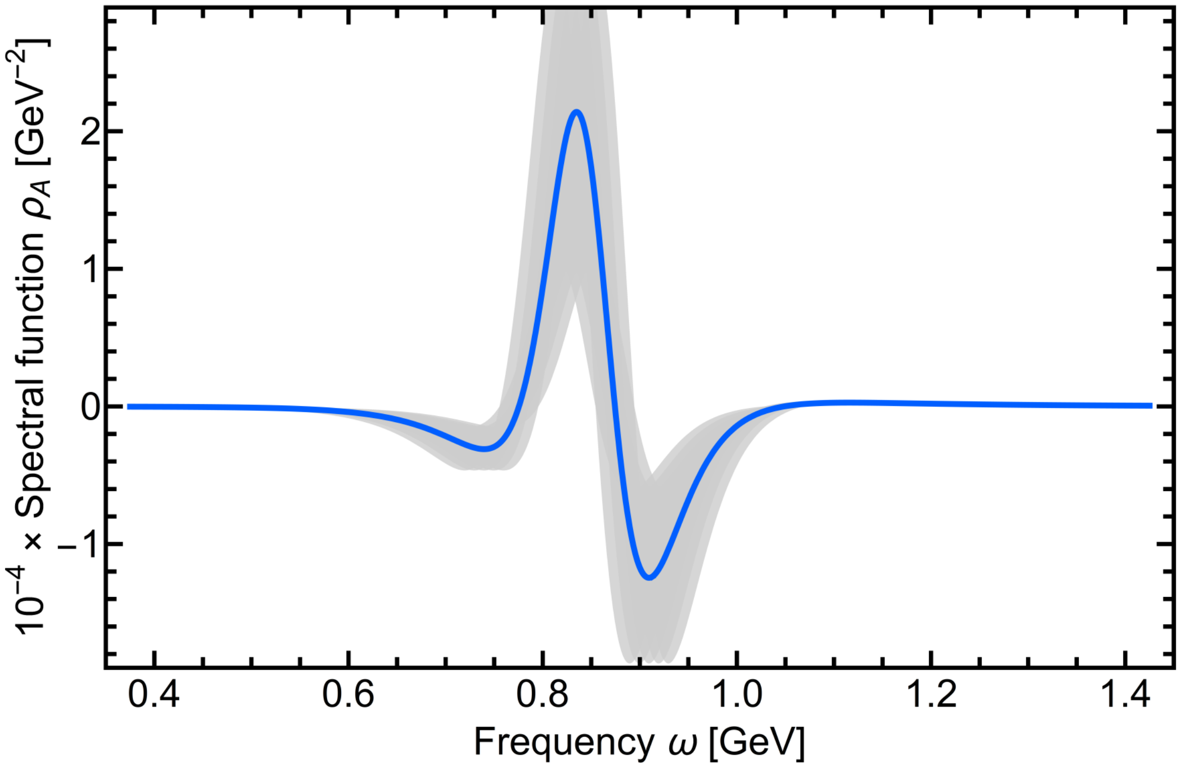
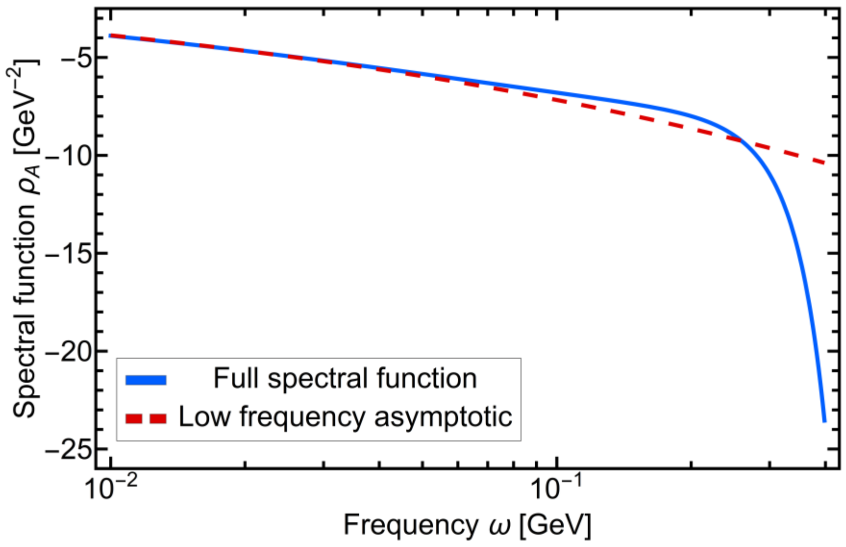
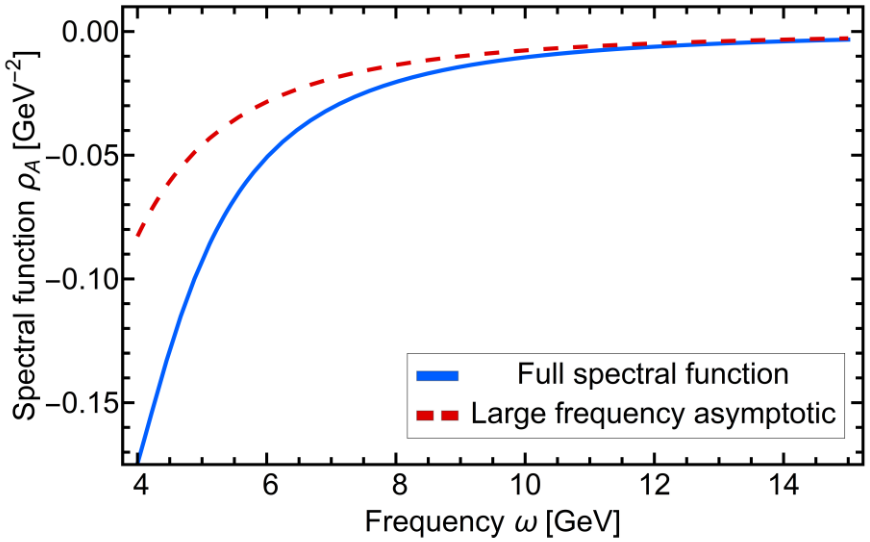
Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Reconstructing the gluon







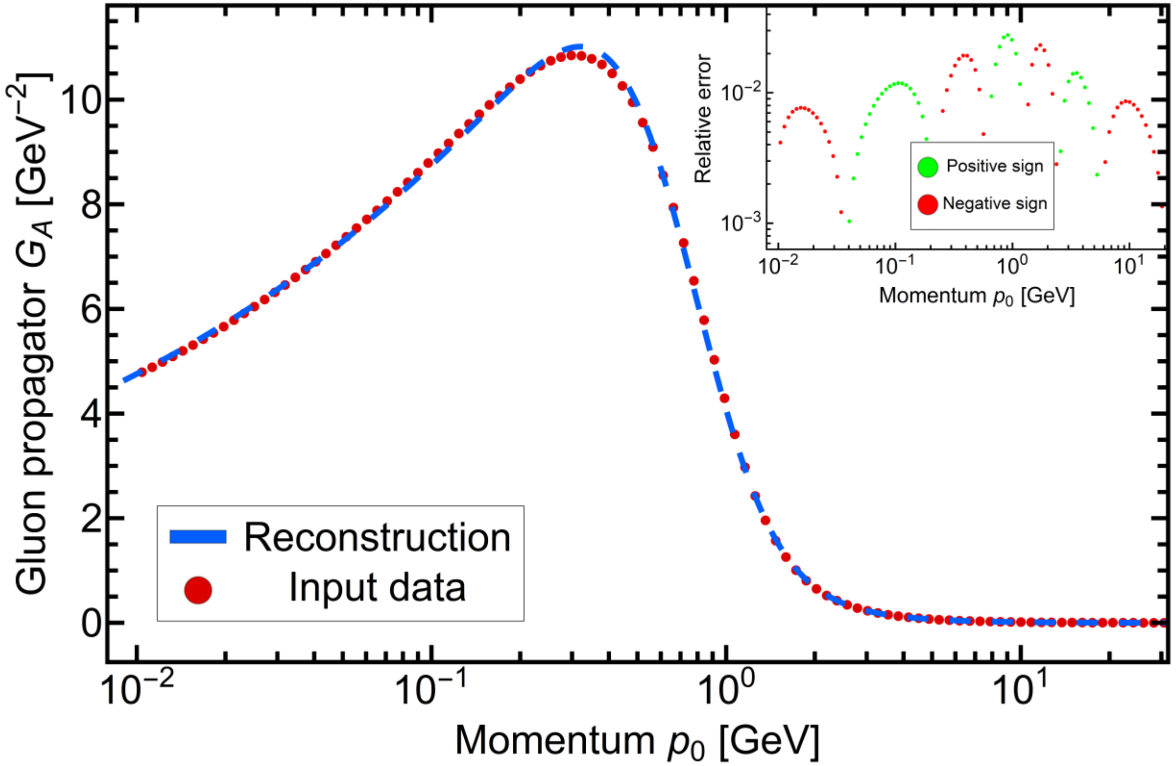
➡ In qualitative agreement with direct DSE calculation and other reconstructions

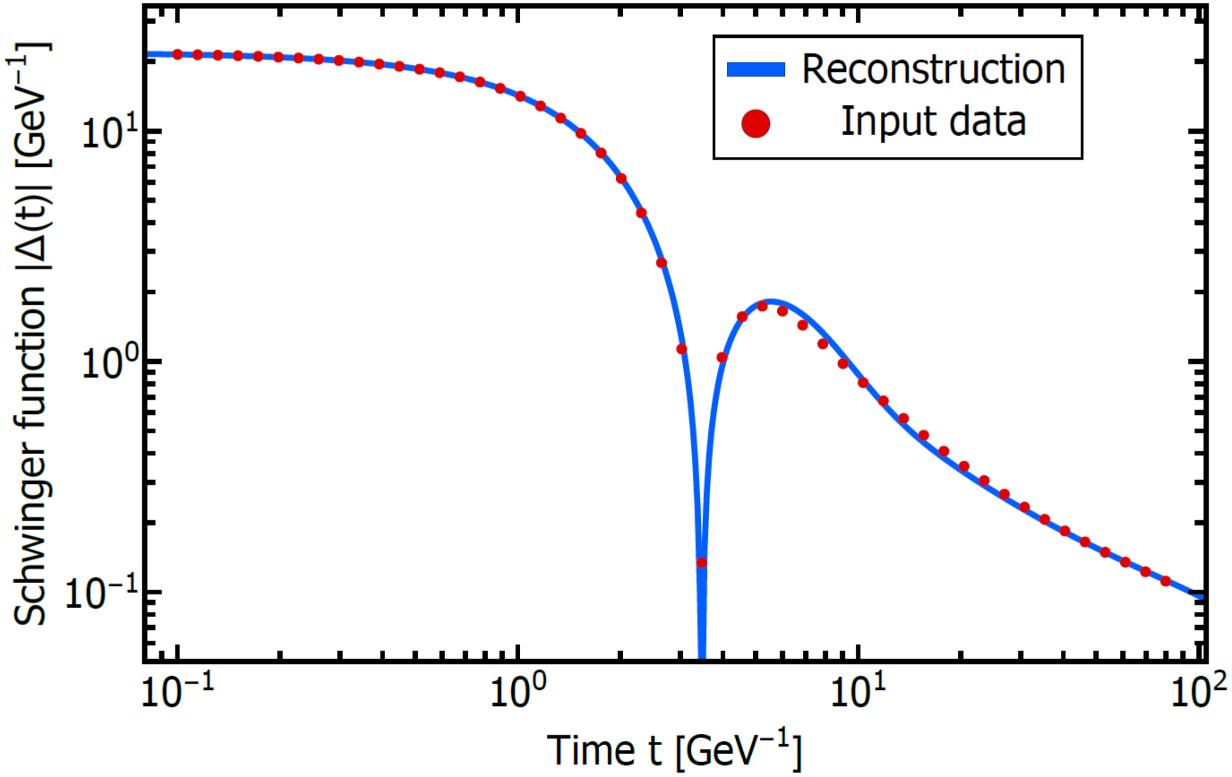
see e.g. Strauss, Fischer, Kellermann, Phys.Rev.Lett. 109 (2012)
Dudal, Oliveira, Silva, Phys.Rev. D89 (2014)

c.f. talk by A. K. Cyrol last week

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Reconstructing the gluon





c.f. talk by A. K. Cyrol last week

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

~~Our~~ Applications: gluon spectral function

unfortunately nothing new yet

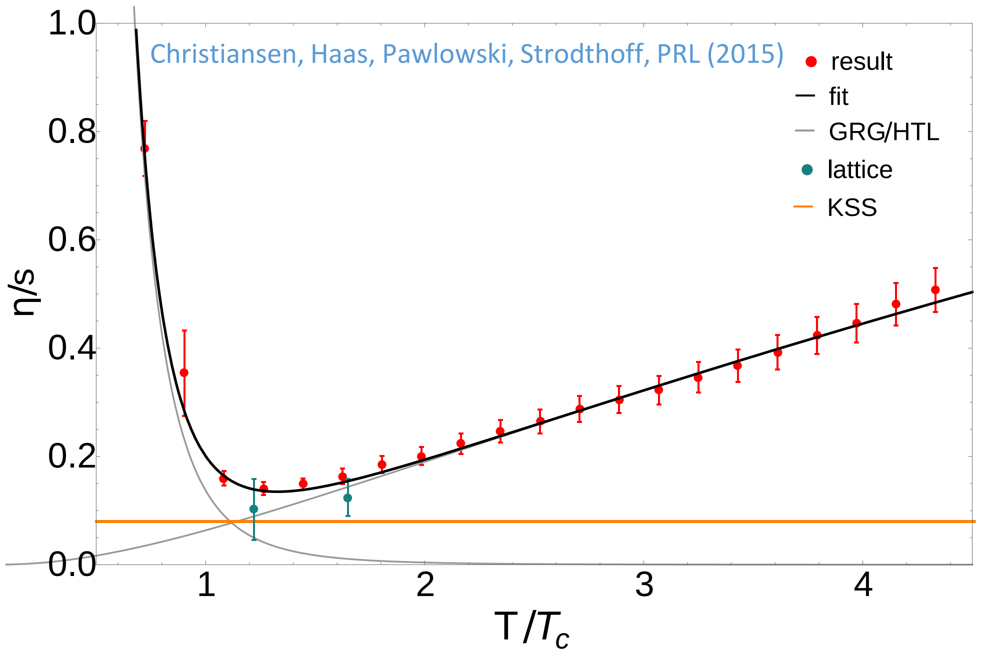
Transport coefficients

Shear viscosity:

$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{\rho_{\pi\pi}(\omega, 0)}{\omega}$$

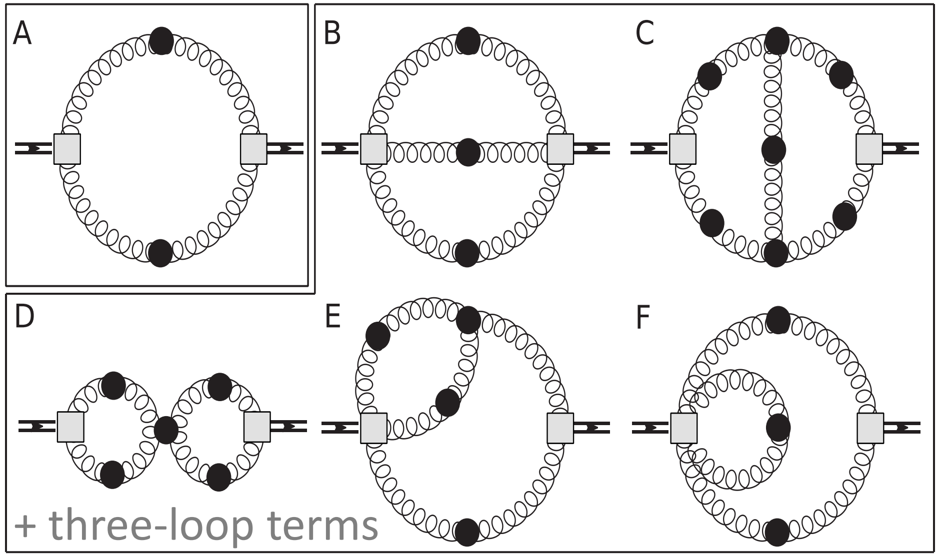
Bulk viscosity:

$$\zeta = \frac{1}{2} \lim_{\omega \to 0} \frac{\rho_{\mathcal{PP}}(\omega, 0)}{\omega}$$



Composite Dyson-Schwinger equation

➡ Exact representation with a finite number of loops



Christiansen, Haas, Pawlowski, Strodthoff, PRL (2015)
Pawlowski, NW, work in progress

Calculation

Implications from the analytic structure

Applies to all functional methods (e.g. pert. theory, FRG, DSE, 2PI, …)

Euclidean result is unique

- Analytic continuation to Minkowski spacetime is unique
- Deformation of integration contours necessarily required
- Unique integration prescription can be obtained by a smooth deformation of the Euclidean path (keep all residues)
- Analytic continuation problem at finite temperature resolved by demanding preservation of this structure

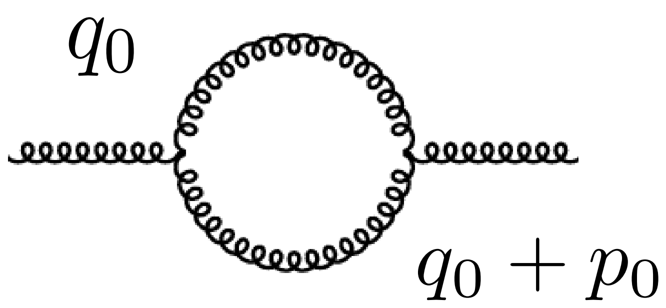
$$G_E(p_0, \mathbf{p}) = \int_0^\infty \frac{\mathrm{d}\eta}{\pi} \eta \frac{\rho(\eta, \mathbf{p})}{\eta^2 + p_0^2}$$

Map cuts to poles via their spectral representations

Baym, Mermin, Journal of Mathematical Physics 2, 232 (1961)

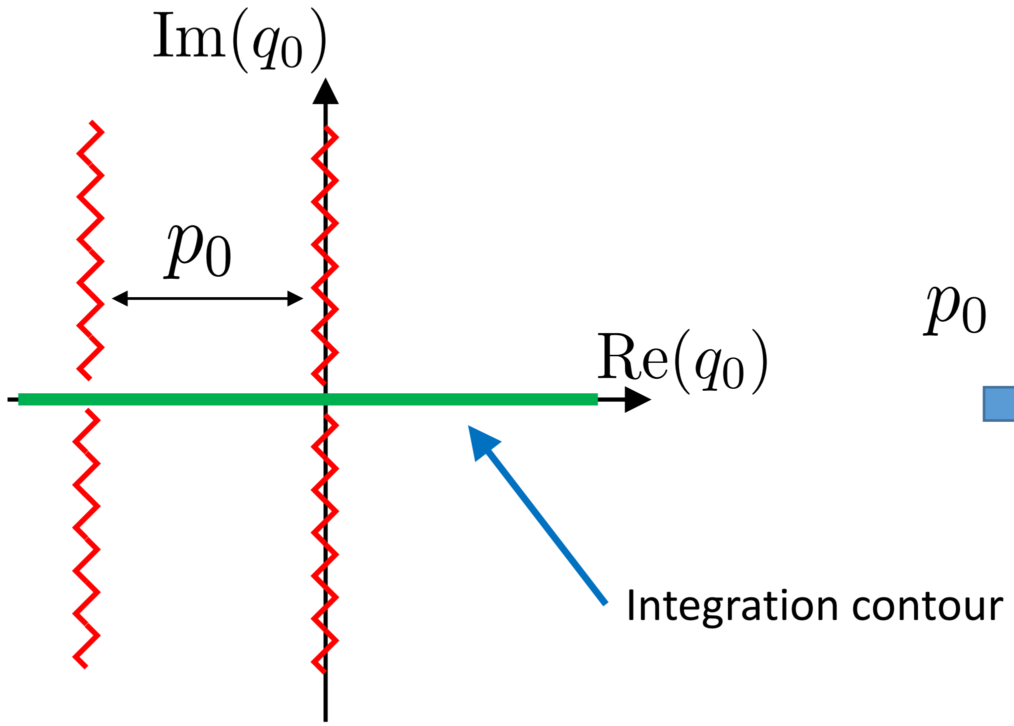
Evans, Nucl.Phys. B374 (1992)

A simple example



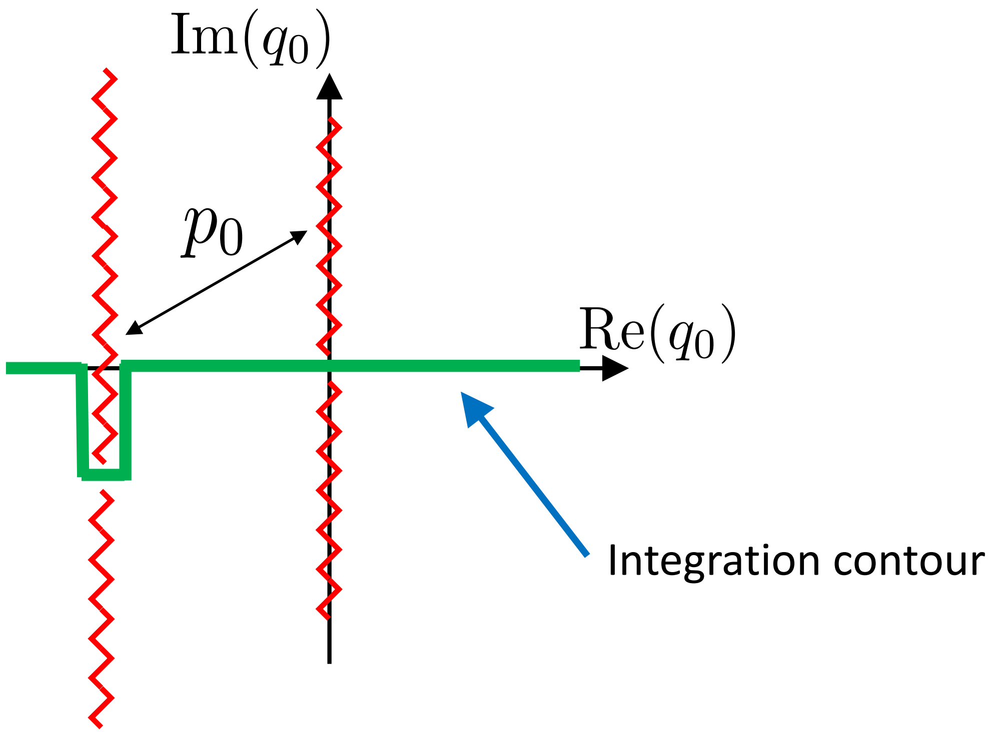
Gluon vacuum polarization

- For illustration consider the diagram with full propagator and trivial vertices



Analytic structure gluon polarization diagram

$p_0 \in \mathbb{C}$



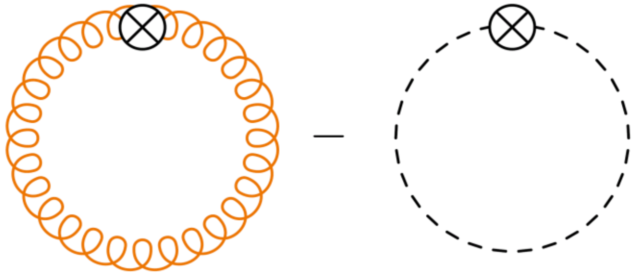
→ **Generalizes to vertices**

Application to the FRG

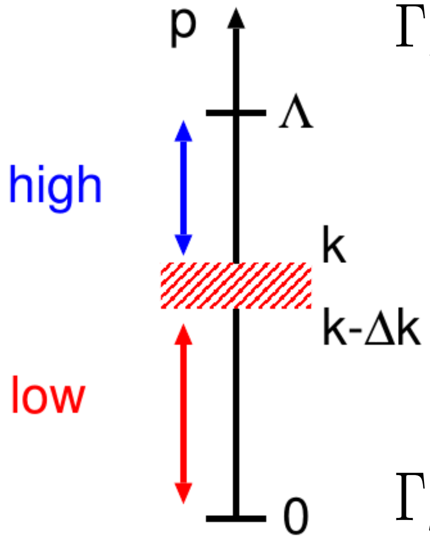
Functional Renormalization Group

Example equation: Yang-Mills

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$



Equations for n-point functions obtained via functional derivatives



$$\Gamma_{k=\Lambda} = S$$

- Non-perturbative first principle method
- Access to physical mechanisms
- No sign problem
  - Chemical potential
  - Real time

$$\Gamma_{k=0} = \Gamma$$

c.f. talk by A. K. Cyrol last week

➡ No new (major) conceptual problems

Lorentz invariant regulator introduce artificial poles

Take them explicitly into account

see e.g. Foerchinger, JHEP 1205 (2012) 021
Pawlowski, Strodthoff, NW, arxiv:1711.07444

Take them explicitly into account

see e.g. Kamikado, Strodthoff, von Smekal, Wambach, Eur.Phys.J. C74, 2806 (2014)
Tripolt, Strodthoff , von Smekal, Wambach, Phys.Rev. D89, 034010 (2014)

Application

Transport approach to QCD

- Describe non-equilibrium QCD in the linear response regime around an equilibrium state

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

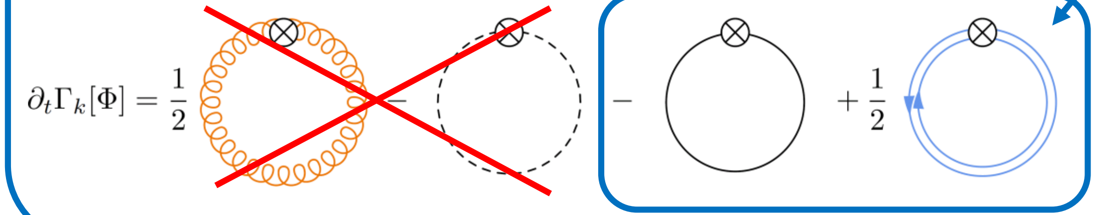
Quantum equation of motion

Noise field (Dissipation-Fluctuation)

- Evolution of critical mode via a transport equation
- Utilize 2+1 flavor low energy effective description of QCD
- FRG for equilibrium calculations

Flow equation for QCD

Bound states efficiently taken into account via Dynamical Hadronization



Low-energy effective theory of QCD

Describes QCD at small chemical potentials and medium temperatures

Capture all dynamically relevant phenomena

→ 2+1 Quark Flavors

Remaining phase structure qualitatively similar to the conjectured QCD phase diagram

$$\Gamma_k = \int_x \left\{ i\bar{q} Z_{q,k} \left(\gamma_\mu \partial_\mu + \gamma_0 \mu\right) q + i\bar{q}\, h_{q,k} \cdot \Sigma_5 q + \mathrm{tr}\left(Z_{\Sigma,k} \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger\right) + \tilde{U}_k(\Sigma, \Sigma^\dagger) \right\}$$

with $q = (u, d, s)$

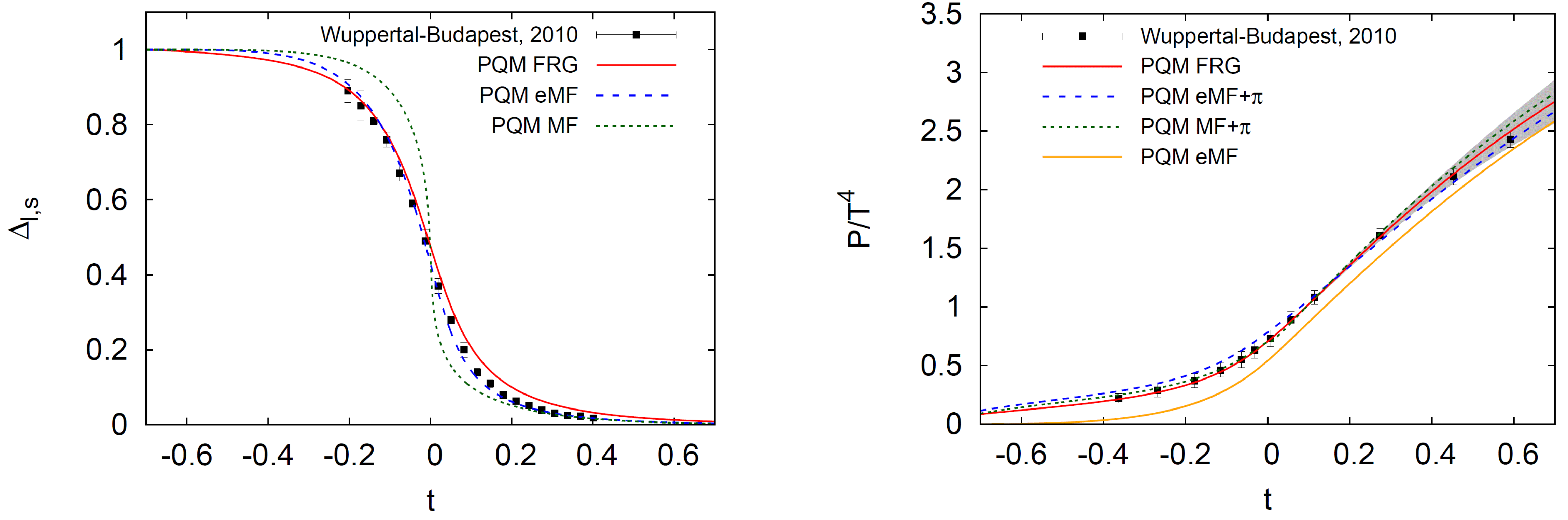
(Pseudo)scalar nonet field

$$\Sigma = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\sigma_L + a_0^0 + i\eta_L + i\pi^0\right) & a_0^- + i\pi^- & \kappa^- + iK^- \\ a_0^+ + i\pi^+ & \frac{1}{\sqrt{2}}\left(\sigma_L - a_0^0 + i\eta_L - i\pi^0\right) & \kappa^0 + iK^0 \\ \kappa^+ + iK^+ & \bar{\kappa}^0 + i\bar{K}^0 & \frac{1}{\sqrt{2}}\left(\sigma_S + i\eta_S\right) \end{pmatrix}$$

Schäfer, Rennecke, PRD96 (2017)

Low-energy effective theory of QCD

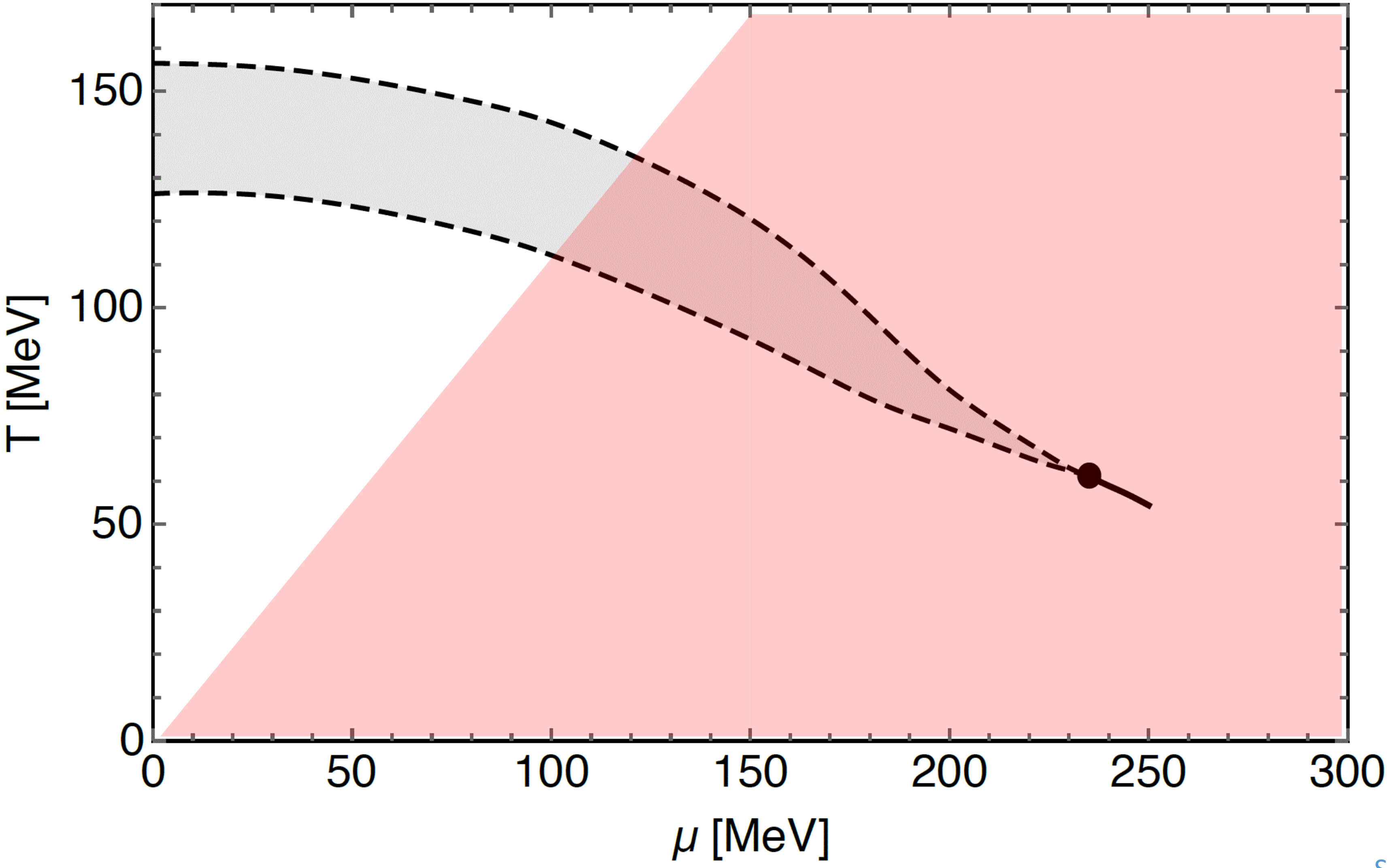
Excellent description of phase structure at vanishing chemical potential



Herbst, Mitter, Pawlowski, Schäfer, Stiele, PLB731 (2014)

Low-energy effective theory of QCD

Phase structure contains a critical endpoint



Schäfer, Rennecke, PRD96 (2017)

Towards non-equilibrium

Interested in the time evolution of the critical mode ➡ Sigma meson

Look at time evolution around the linear response regime of the equilibrium result

➡ Calculate spectral functions to leading order in LPA'+Y expansion ➡ no back coupling of momentum dependences

➡ Expand frequency dependence in low order polynomials ➡ transport regime

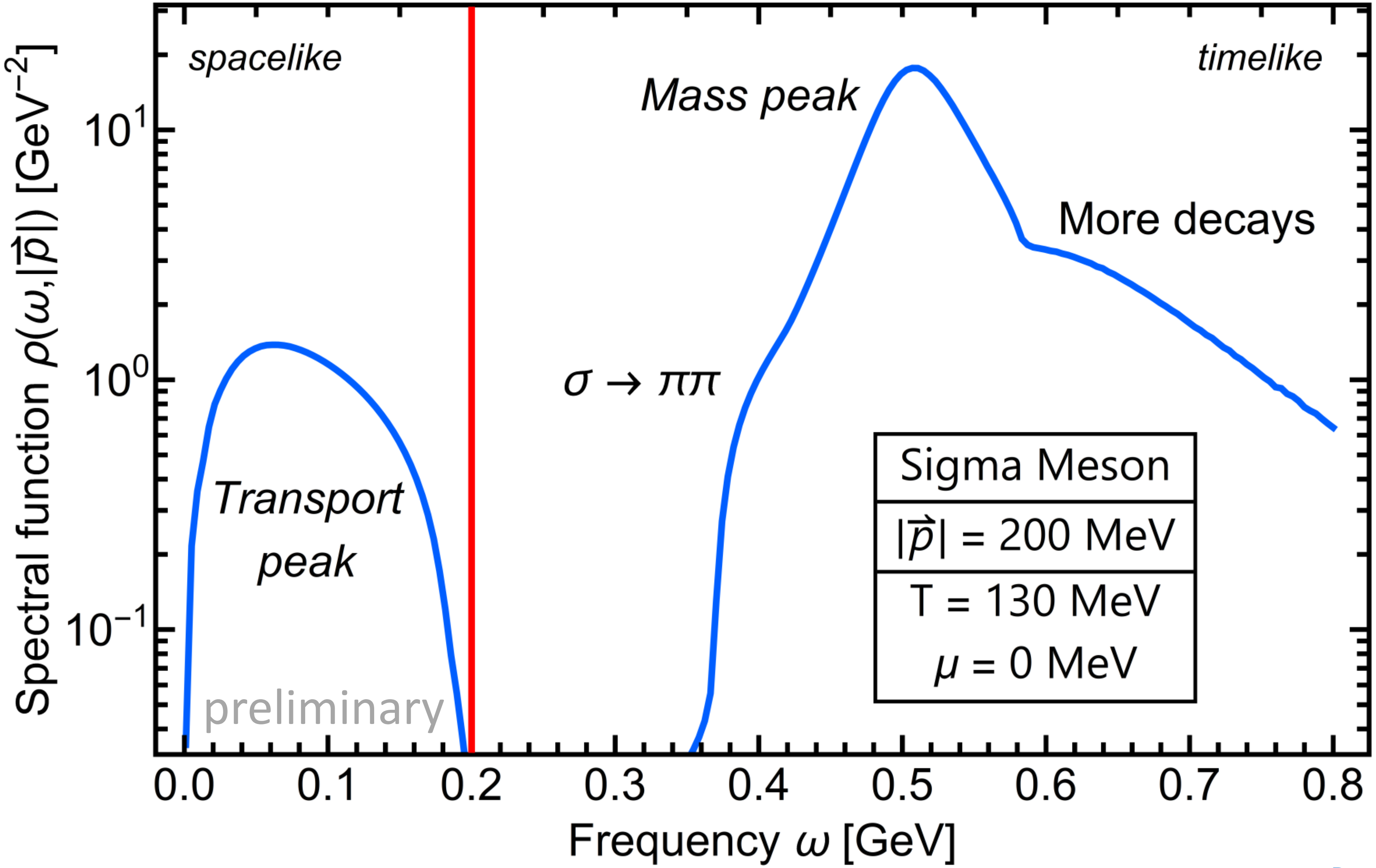
➡ spatial momentum dependence fully taken into account

Use Effective Potential and Spectral Functions of the sigma meson as input

Pawlowski, Rennecke, NW, in prep.
Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Linear response function

Sigma meson spectral function at T = 130 MeV and vanishing chemical potential



Pawlowski, Rennecke, NW, in prep.

Linear response function

Sigma meson spectral function at T = 130 MeV
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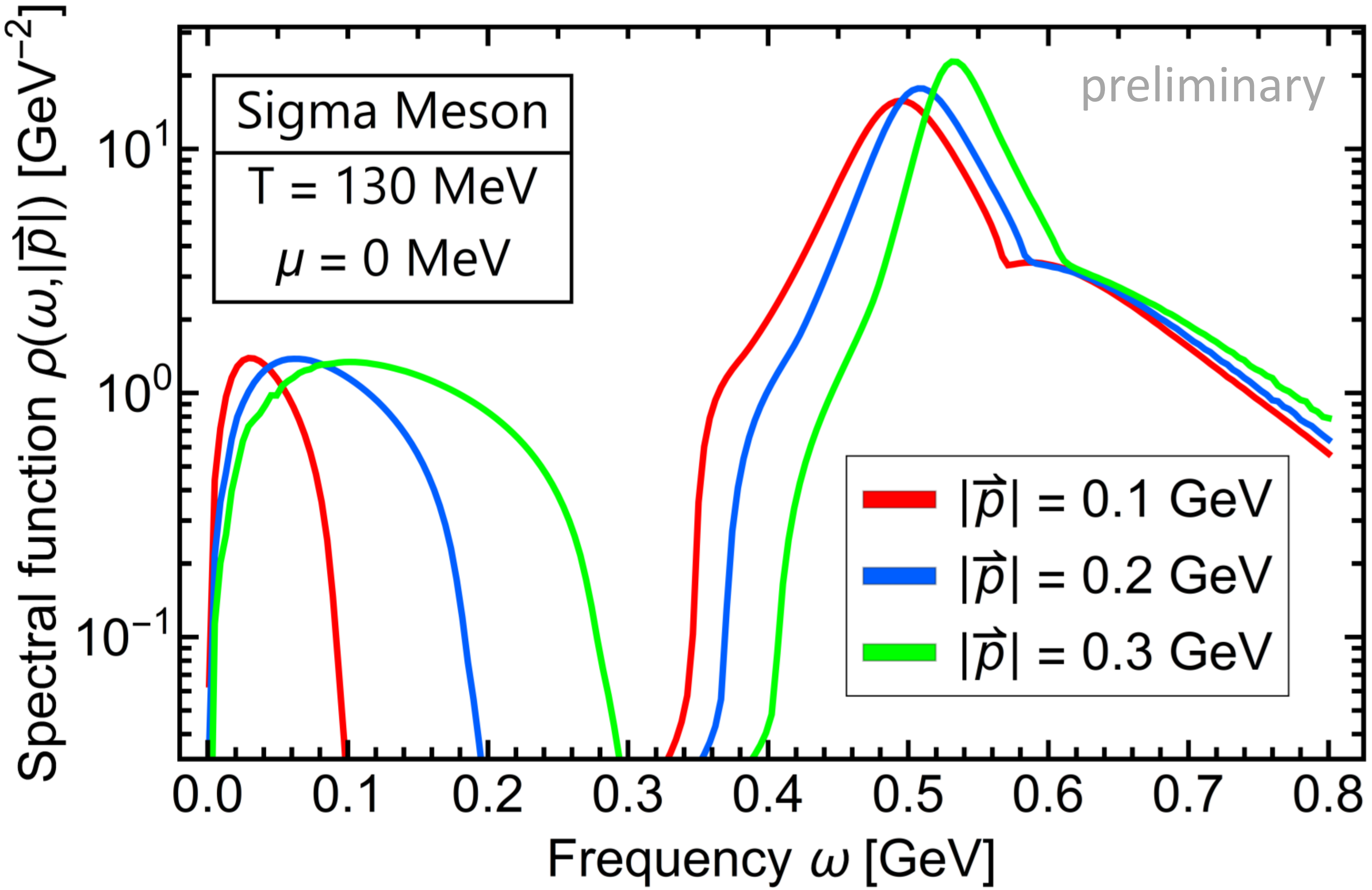


Pawlowski, Rennecke, NW, in prep.

Transport equation

Evolution governed by transport equation:

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

with

$$\left\{\mathrm{Re}\,\Gamma^{(2)}_\sigma(\omega,\vec{p}),\ \mathrm{Im}\,\Gamma^{(2)}_\sigma(\omega,\vec{p}),\ U(\sigma)\right\} \in \Gamma$$

$$\sigma(r,t) = \sigma_0 + \delta\sigma(r,t)$$

Split into equilibrium and fluctuation part

White noise approximation:

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(t') \rangle = \frac{1}{V}\delta(t-t')m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$

Spatial isotropy approximation:

$$\sigma(\vec{x}) = \sigma(r)$$

Initial conditions:

Quench from „high temperature“

$$\sigma(r) = 0 = \partial_t \sigma(r)$$

Transport equation

Evolution governed by transport equation:

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

with

$$\left\{ \mathrm{Re}\,\Gamma_\sigma^{(2)}(\omega,\vec{p}),\ \mathrm{Im}\,\Gamma_\sigma^{(2)}(\omega,\vec{p}),\ U(\sigma) \right\} \in \Gamma$$

$$\sigma(r,t) = \sigma_0 + \delta\sigma(r,t)$$

Split into equilibrium and fluctuation part

Calculate (standardized) cumulants

χ_n : nth central moment of the sigma field

Mean : $\sigma_0 = \langle\sigma\rangle$

Variance : $\sigma^2 = \left\langle(\sigma-\sigma_0)^2\right\rangle$

Skewness : $S = \frac{\chi_3}{\chi_2^{3/2}}$

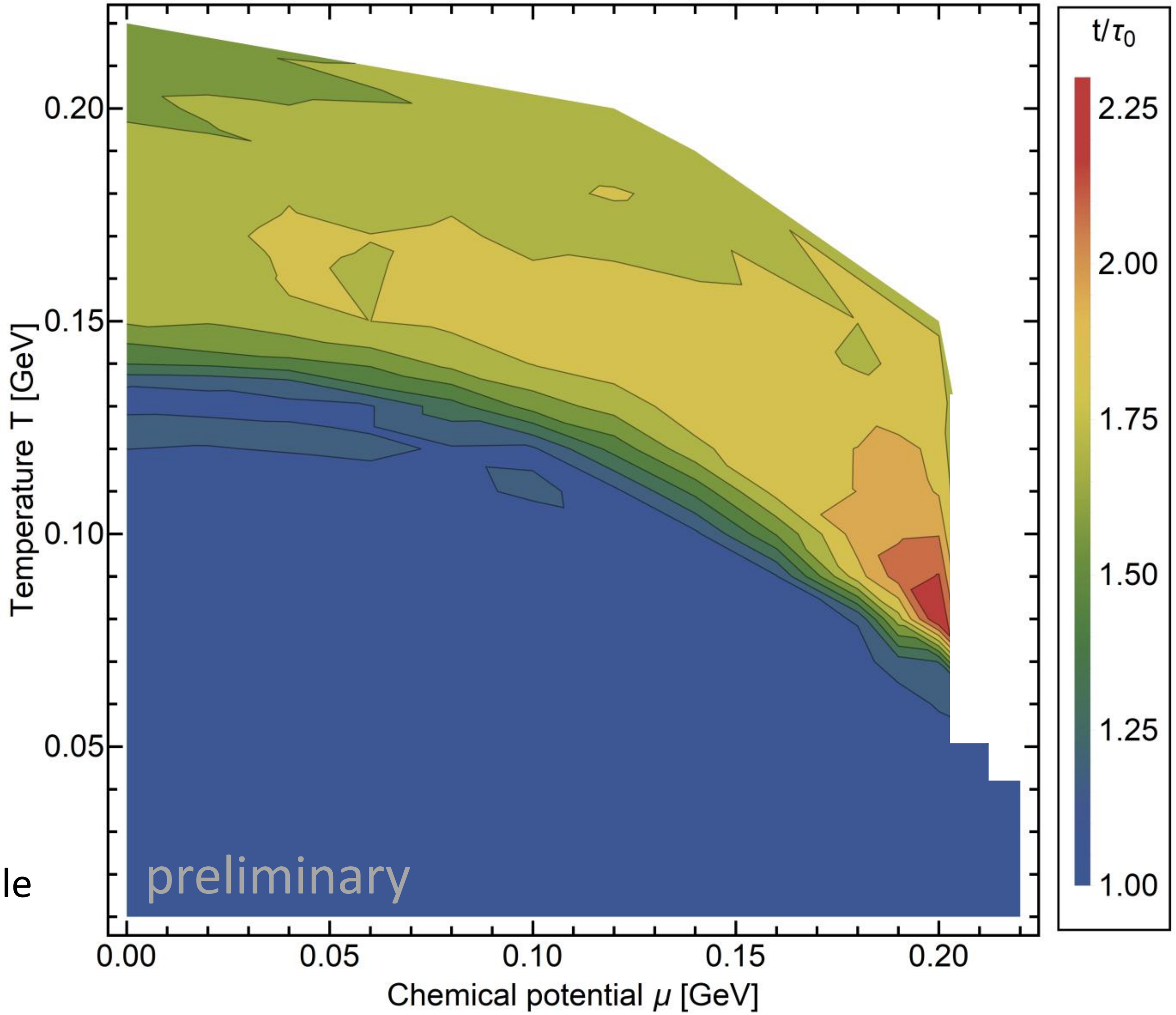
Kurtosis : $\kappa = \frac{\chi_4}{\chi_2^2} - 3$

....

Standardized cumulants → 0 for a Gaussian

Equilibration time τ obtained from the Kurtosis

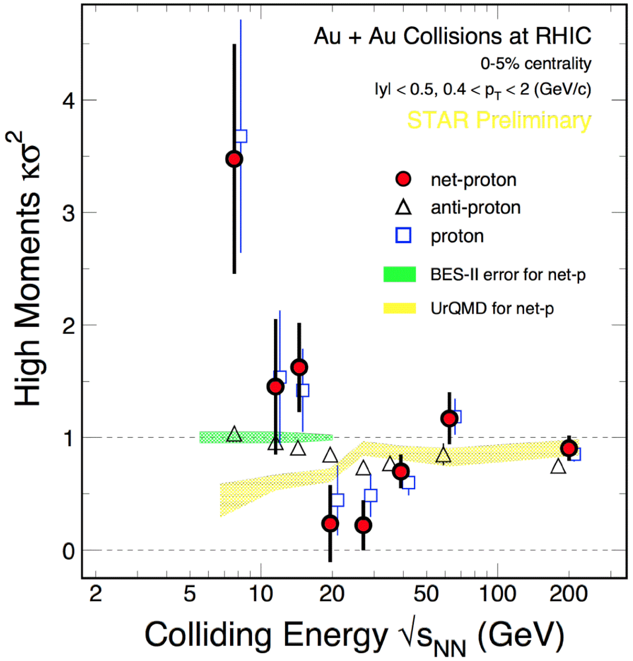
Equilibration time



Reference time scale
$\tau_0 \approx 0.4$ fm/c

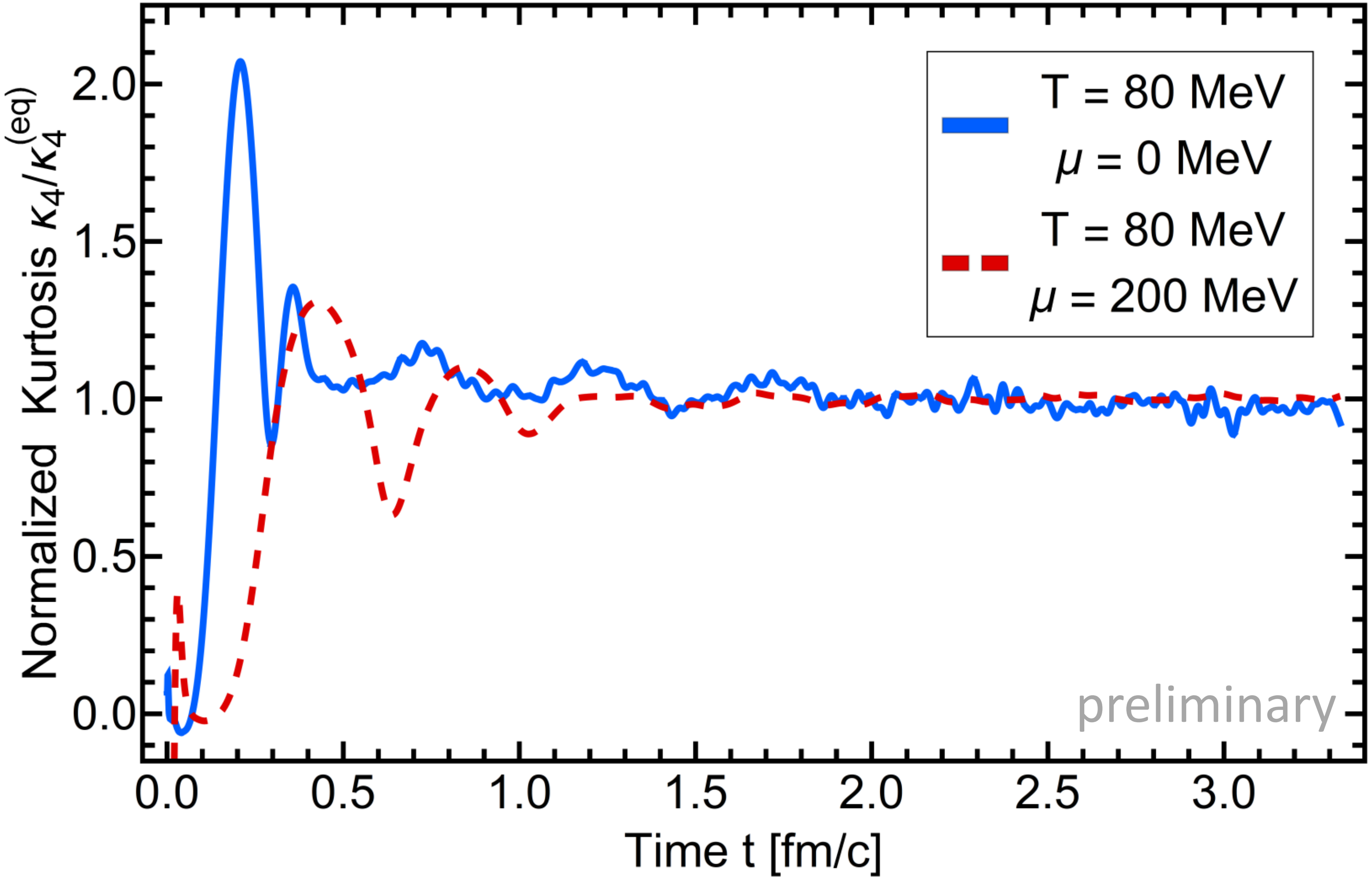
Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

- Critical endpoint and phase boundary clearly identifiable
- Critical slowing down at the critical endpoint
- Impact on observables?



Luo, Xu, Nucl.Sci.Tech. 28 (2017)

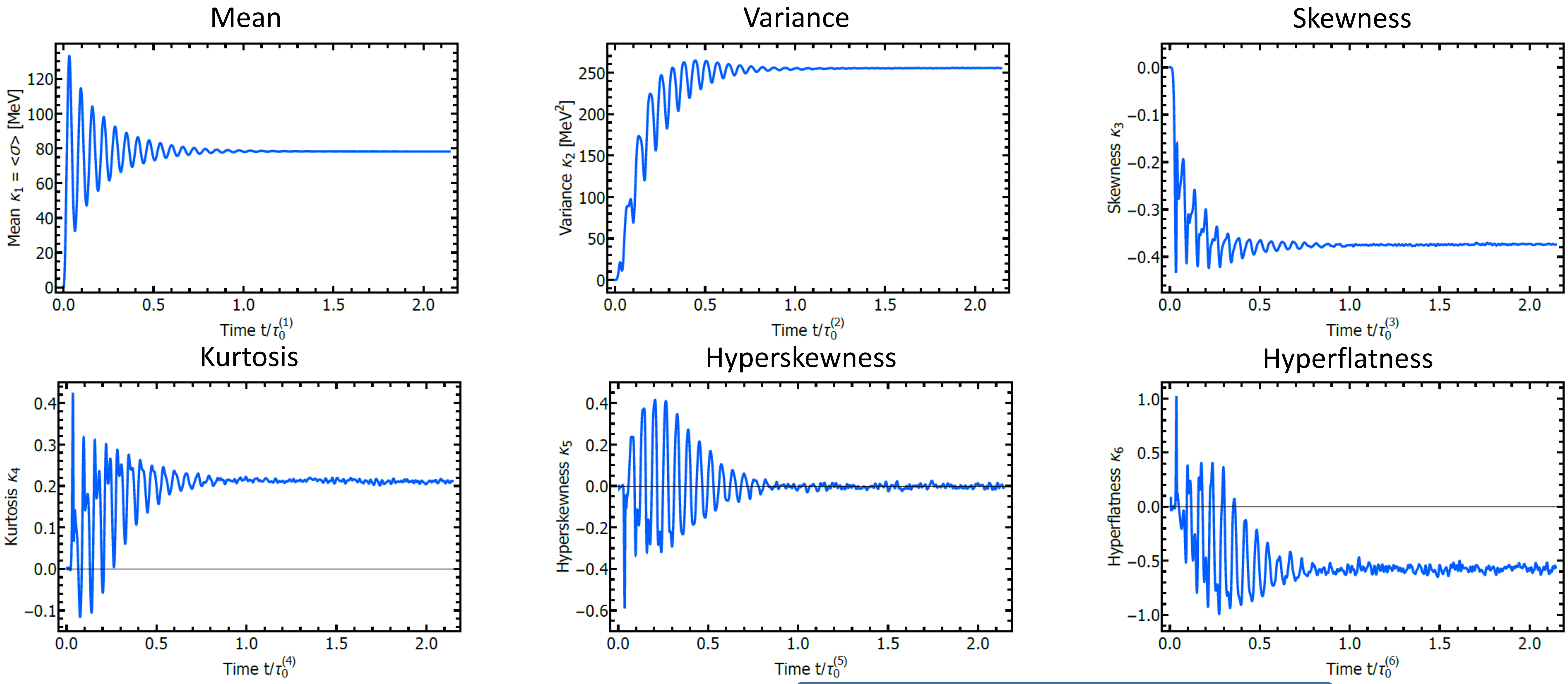
Time evolution of cumulants



Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Time evolution of cumulants

Disclaimer: For the O(N)-model Qualitatively the same → Way more statistics



Recovers correct equilibrium cumulants

preliminary

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Summary & Outlook

- Spectral representations and their implications
- Spectral functions from reconstruction
- Spectral functions from direct computation
- QCD-assisted transport approach

Thank you for your attention!

- Higher order spectral representations
- Transport coefficients
- Direct calculation of spectral functions in QCD