Chiral Symmetry Breaking with Dynamical Hadronization

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Lunch Club, JLU Giessen, 23.06.2021

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Outline

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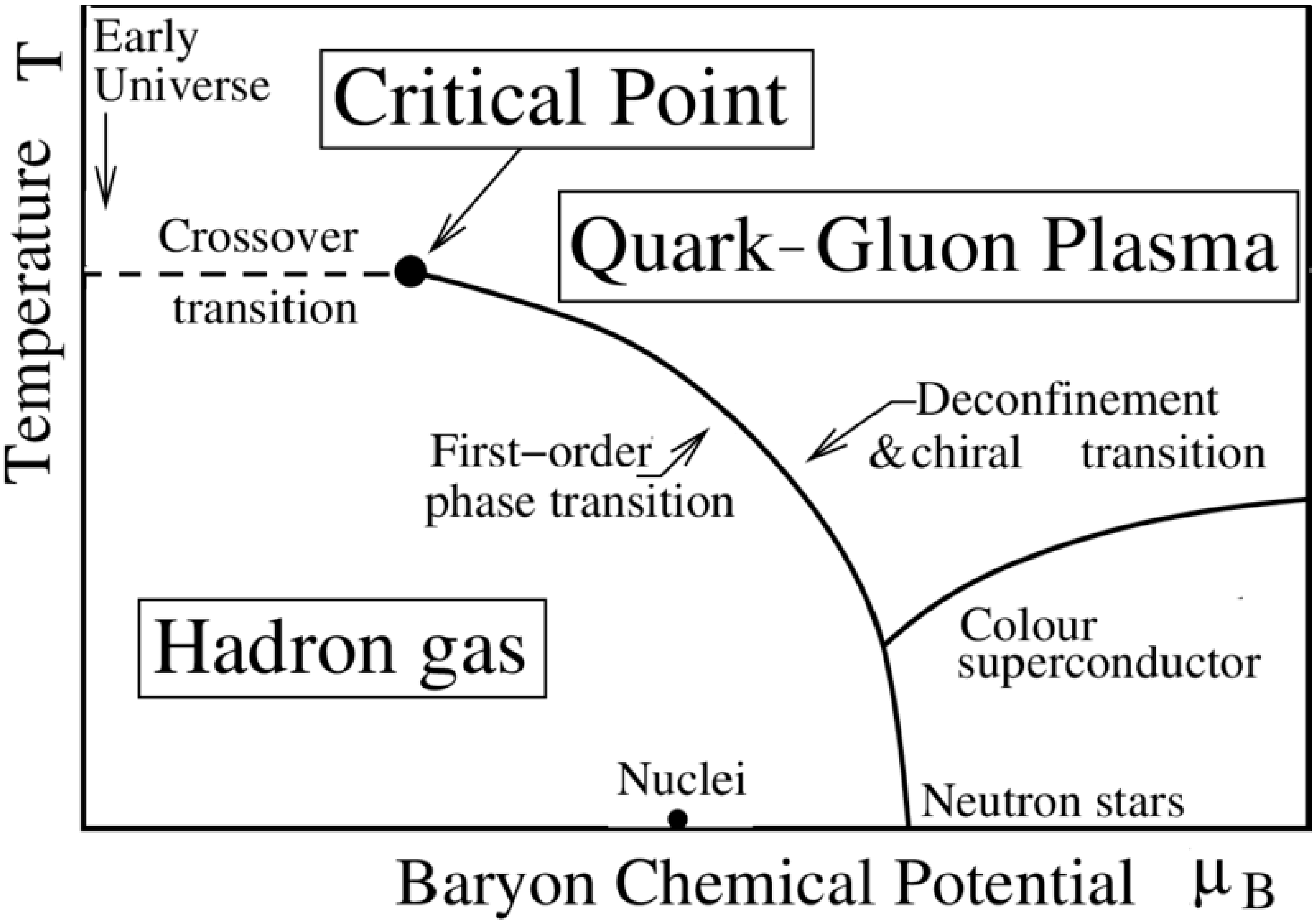
2) Quark-Meson Model

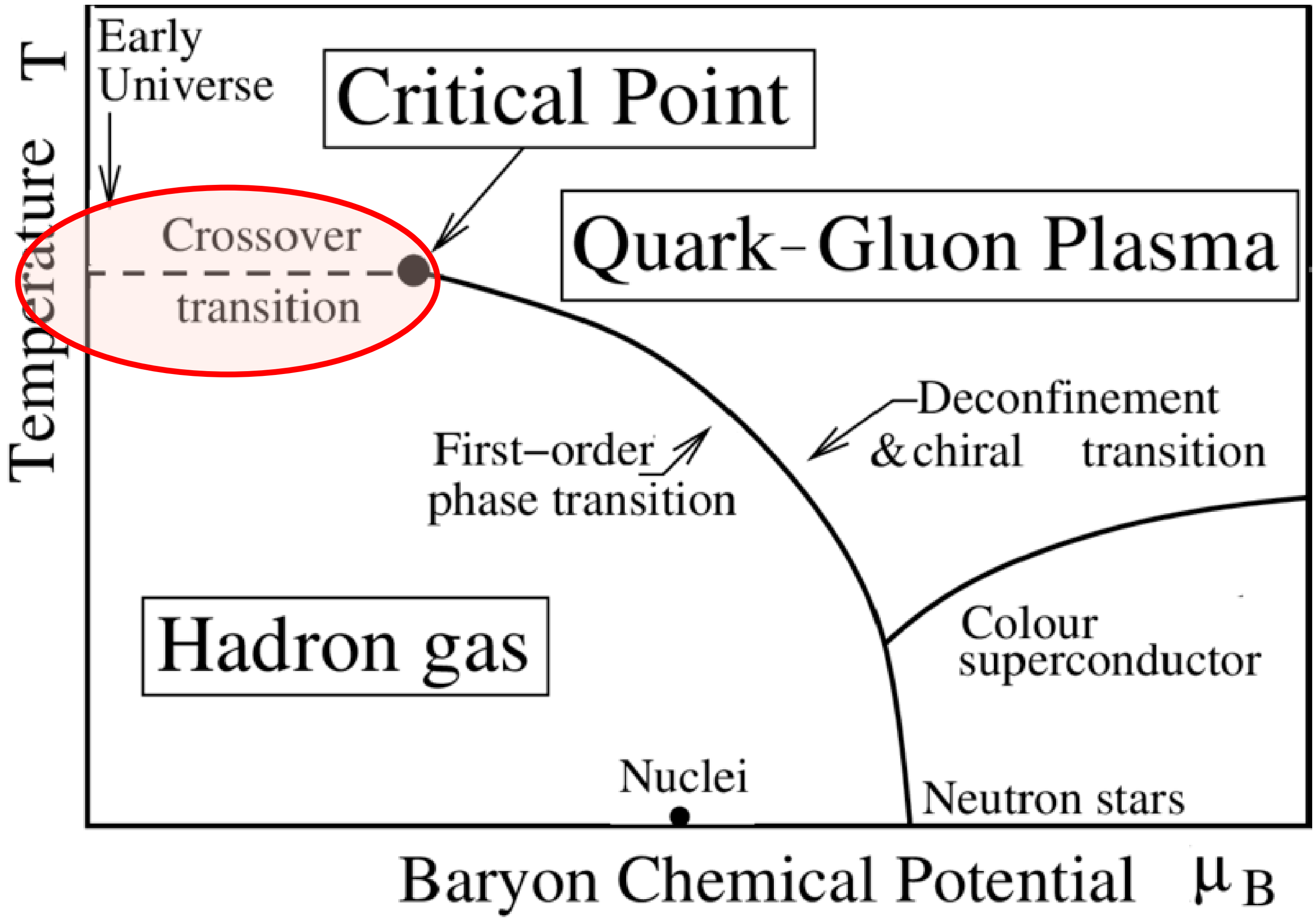
3) Dynamical Hadronization

4) Summary & Outlook

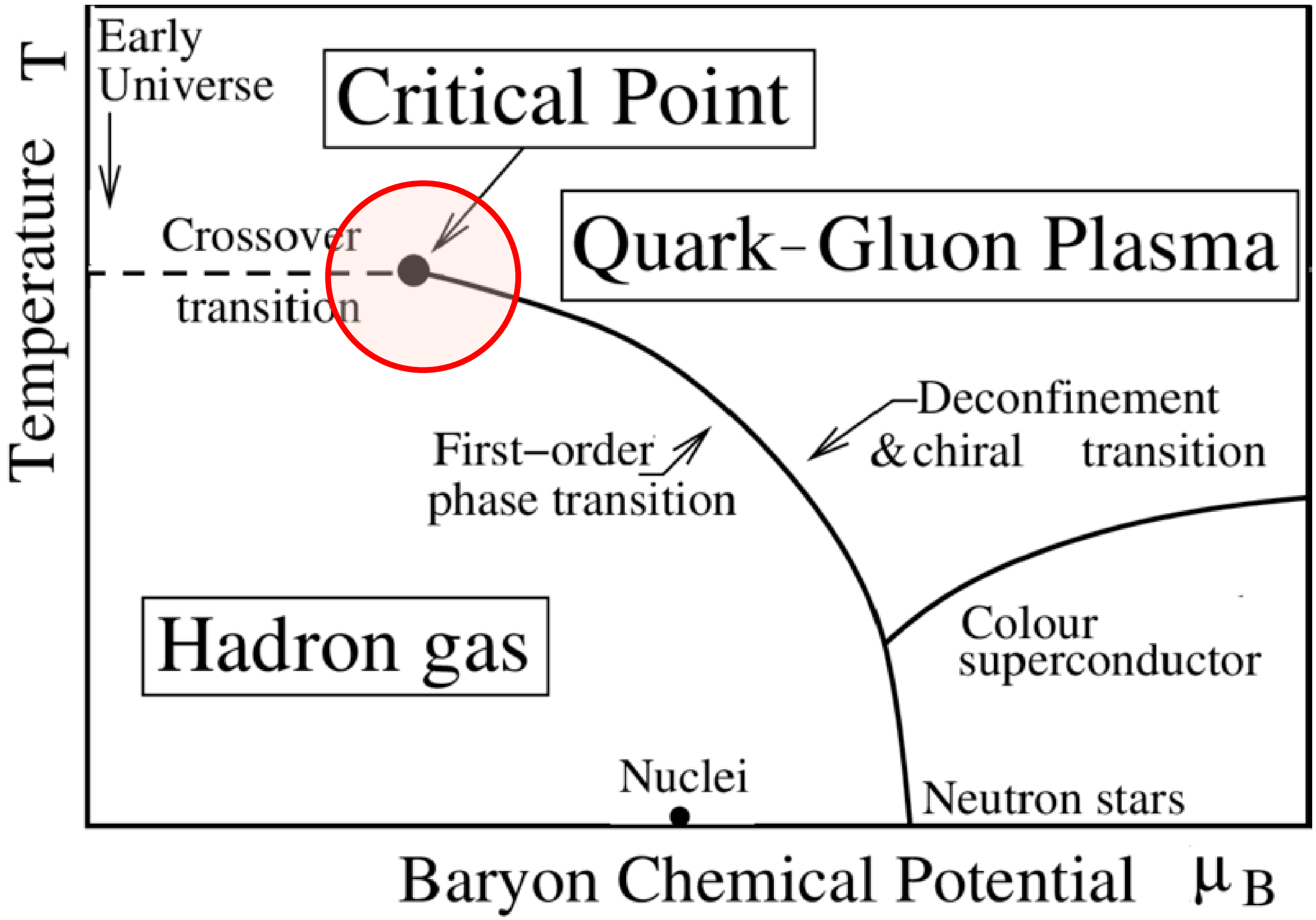
Introduction: QCD Phases



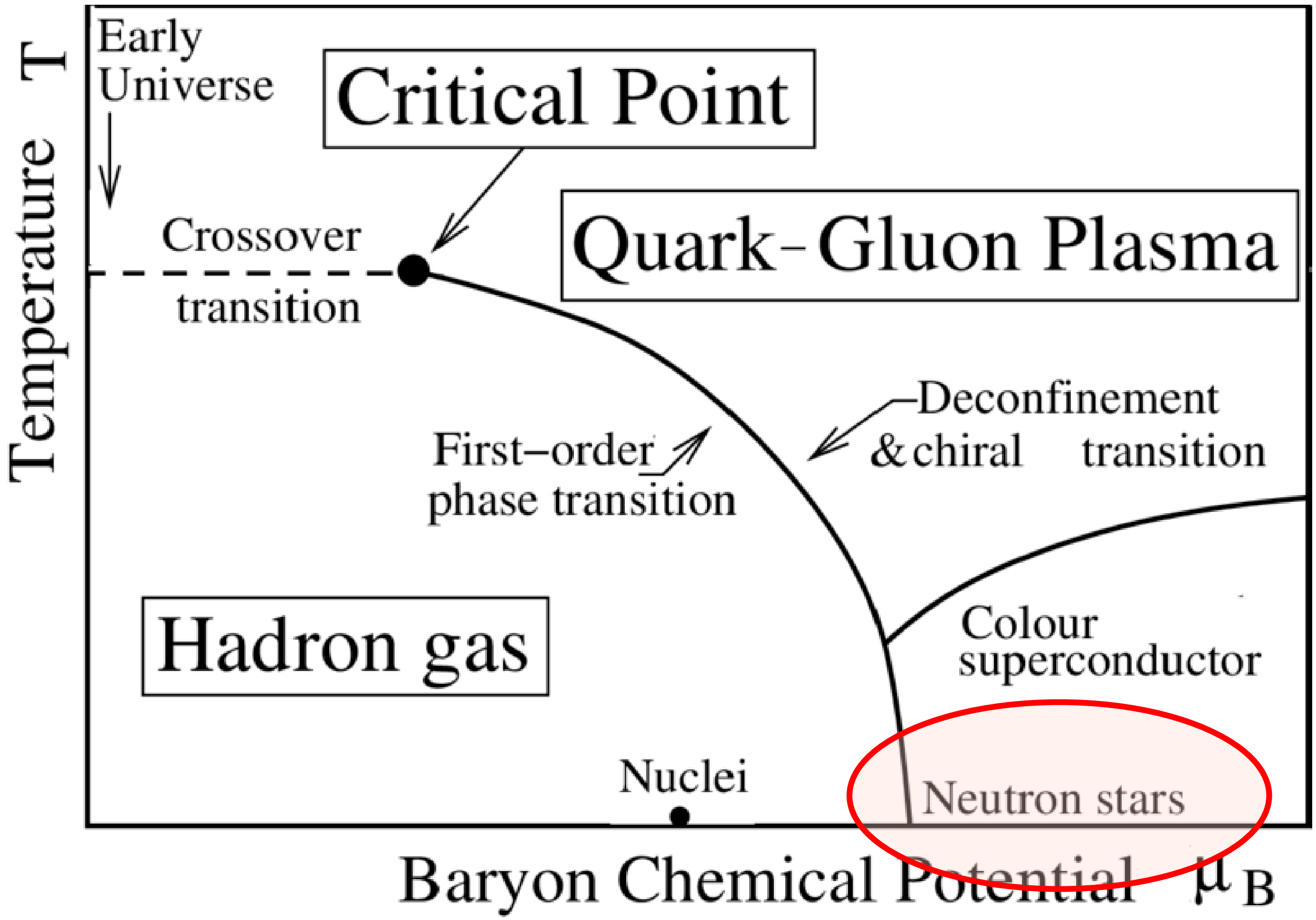
Introduction: QCD Phases



Introduction: QCD Phases



Introduction: QCD Phases



Functional Renormalization Group

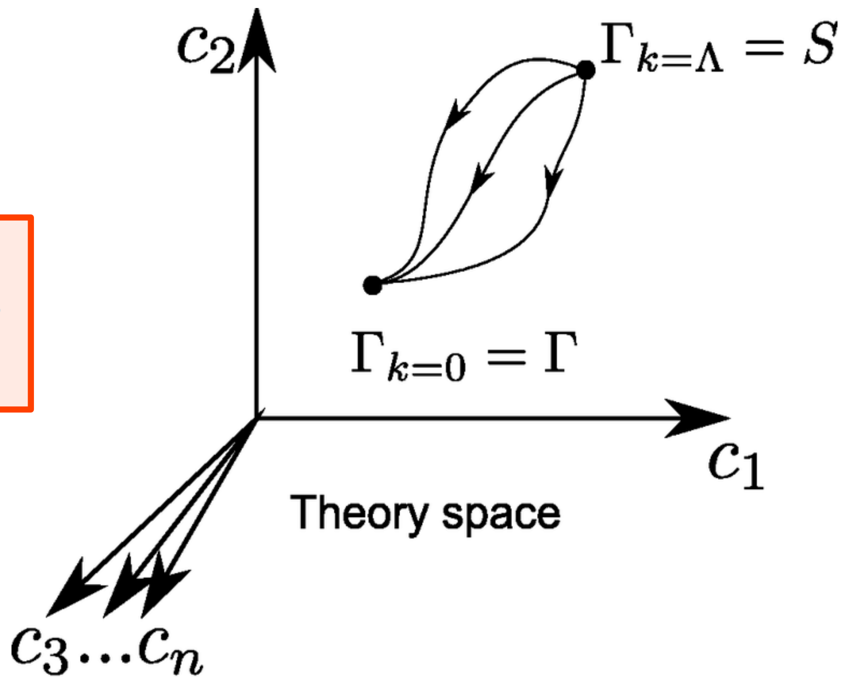
- Introduce an UV- and IR-regulation to our theory

  ➔ scale dependent effective action Γ_k

- Scale dependence of Γ_k (Wetterich equation):

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ (\partial_t \boldsymbol{R}_k) \left(\boldsymbol{\Gamma}_k^{(1,1)} + \boldsymbol{R}_k \right)^{-1} \right\}$$

$$(\partial_t = k \partial_k)$$



Objectives

- Phenomena of low energy QCD:
  - Confinement/Deconfinement
  - Chiral symmetry breaking
- Validate our model/truncation:
  - Appropriate to describe the correct physics? Necessary extensions?
- Research into chiral symmetry breaking and thermodynamics:
  - Chiral phase structure: Condensate, Critical End Point
  - Dynamical mass generation
  - Equation of State at high chem. potentials

3) Quark-Meson Model

Two Flavor Model

- **Effective Action:**

$$\phi = \begin{pmatrix} \sigma \\ \vec{\pi} \end{pmatrix}$$

$$\Gamma_k\left[\bar{\psi}, \psi, \phi\right] = \int_x \left\{ \bar{\psi}\left[Z_{\psi,k}\left(\not{\partial} - \mu\gamma_0\right) + g_k\left(\sigma + i\gamma_5\vec{\tau}\vec{\pi}\right)\right]\psi + \frac{Z_{\phi,k}}{2}\left(\partial_\mu\phi\right)^2 + \Omega_k\left(\phi^2\right) - c\sigma \right\}$$

Label	Scale Dep.
LPA	Ω_k
LPA+Y	Ω_k, g_k
LPA'	$\Omega_k, Z_{\psi,k}, Z_{\phi,k}$
LPA'+Y	$\Omega_k, g_k, Z_{\psi,k}, Z_{\phi,k}$

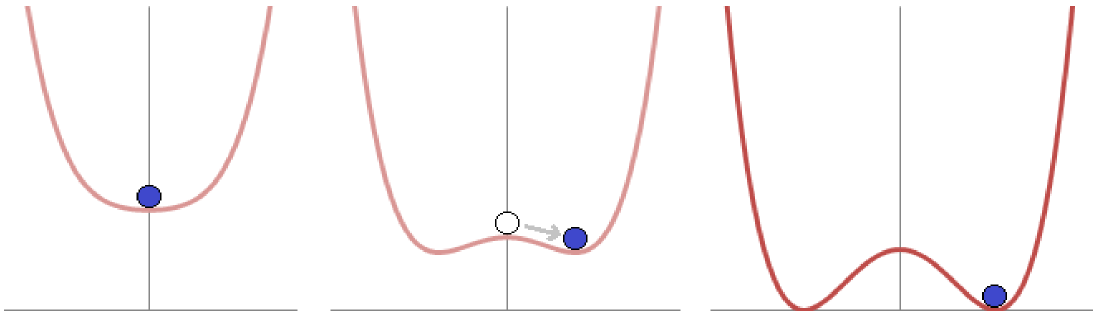
- Local Potential Approximation (LPA):
  - Only effective Potential $\Omega(\phi^2)$ scale dependent
  - $g_k \equiv g$ constant and $Z_{\phi,k} = Z_{\psi,k} = 1$

$\Longrightarrow$ classical propagators, e.g.

$$G_{\pi,k} = \frac{1}{p^2 + m_{\pi,k}^2} \quad \text{for } R_k \to 0$$

Initial Action

- UV and IR scale: $\Lambda = 900\,\mathrm{MeV},\ k_{IR} = 80\,\mathrm{MeV}$
- UV-Ansatz for the effective potential: $\Omega_\Lambda = a_1\phi^2 + \frac{a_2}{2}\phi^4$



- Parameter fixing (a_1,a_2,c and $g_{k=\Lambda}$) to get vacuum values

$$
\begin{aligned}
\bar{m}_\sigma &= 550\,\mathrm{MeV}\\
\bar{m}_\pi &= 138\,\mathrm{MeV}\\
\bar{m}_\psi &= 300\,\mathrm{MeV}\\
f_\pi &= 93\,\mathrm{MeV}
\end{aligned}
$$

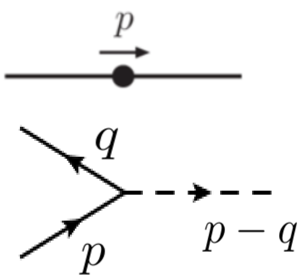
$$m^2_{\pi,k} = 2\Omega'_k$$

$$m^2_{\sigma,k} = 2\Omega'_k + 4\sigma^2\Omega''_k$$

$$m^2_{\psi,k} = g_k^2\sigma^2$$

Going “Beyond LPA”

- In general flow eq’s depend on fields and momenta



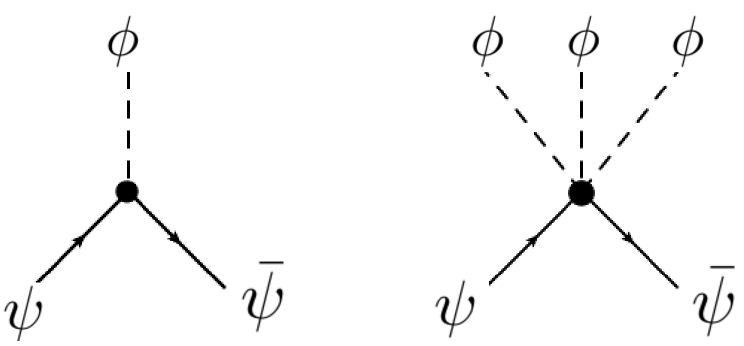
$$\Rightarrow Z_{\phi,k}\left(p^2,\phi^2\right), Z_{\psi,k}\left(p^2,\phi^2\right), g_k\left(p^2,q^2,\phi^2\right)$$

- Consequences:

$Z_k(p^2)$ $\Longrightarrow$ Non-trivial (higher order) momentum dependencies in propagators:

$$G_k(p^2) = \frac{1}{Z_k(p^2)\cdot p^2 + m_k^2}$$

$g_k(\phi^2)$ $\Longrightarrow$ Higher order vertices:

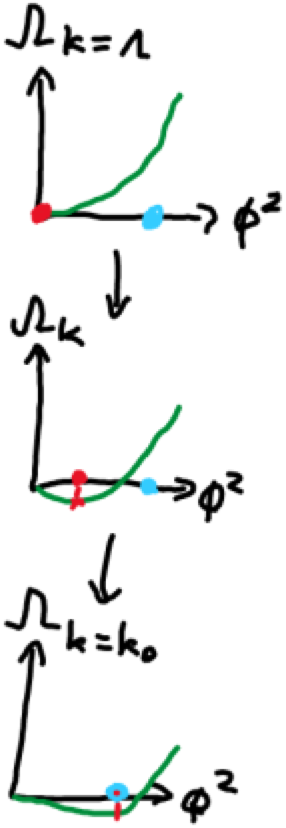


Simplified Flow Equations

- Approximate Z_k and g_k as field and momentum independent:

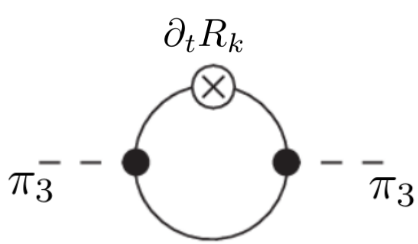
  ➔ Static bare/renormalized or "co-moving" evaluation point ϕ_0 (sketched with colored dots)

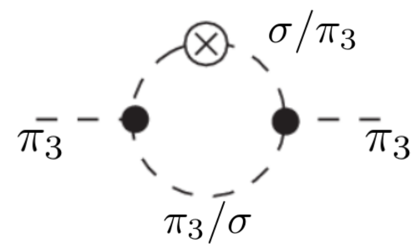
  ➔ Flow equations at fixed external momentum $p=p_{min}$



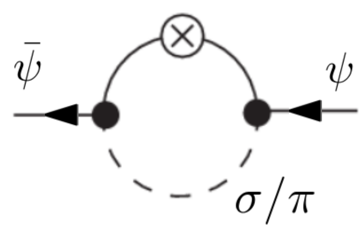
- Contributions:

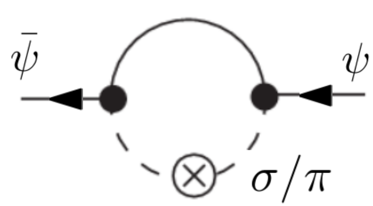
  ➔ For Z_ϕ:



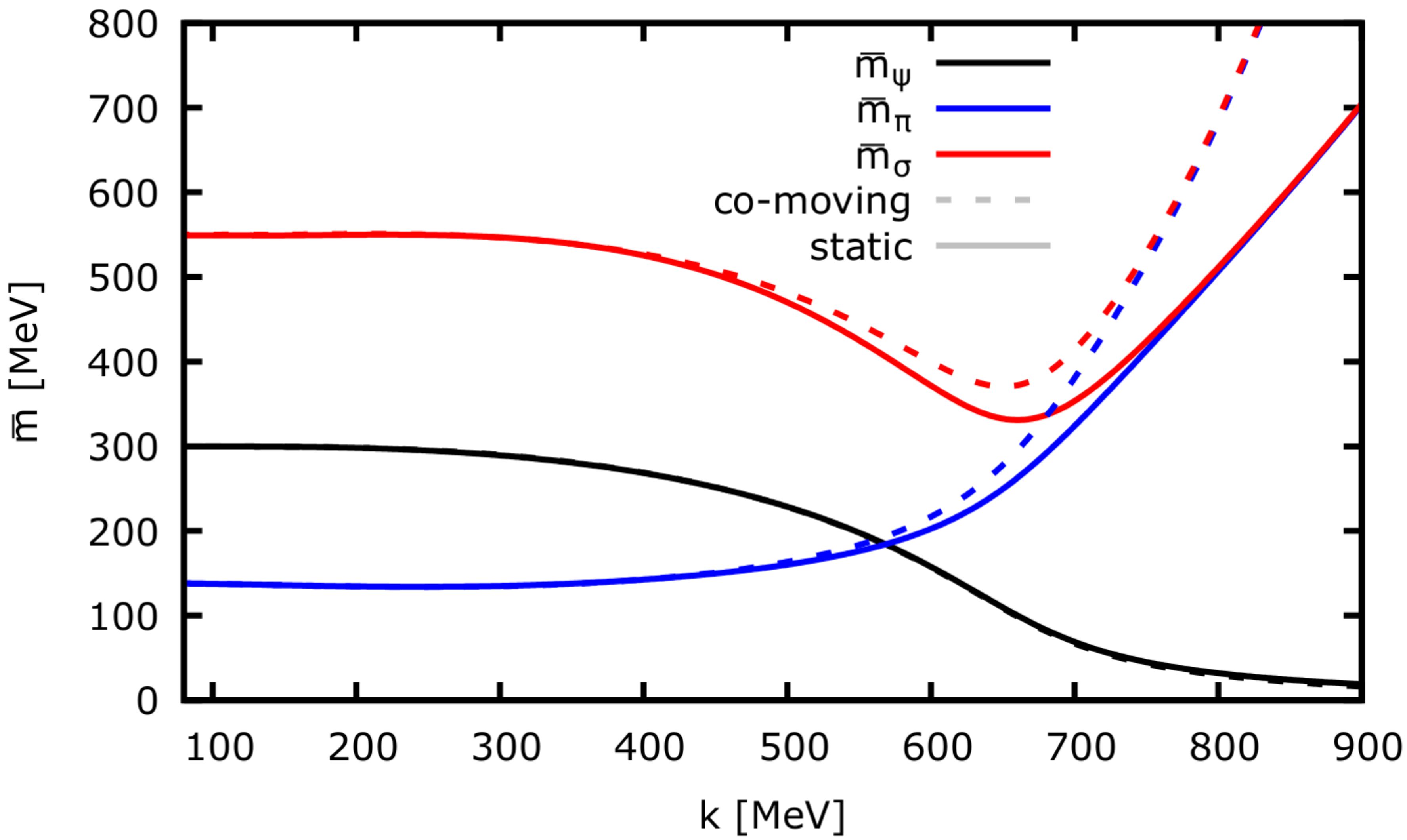


  ➔ For Z_ψ:

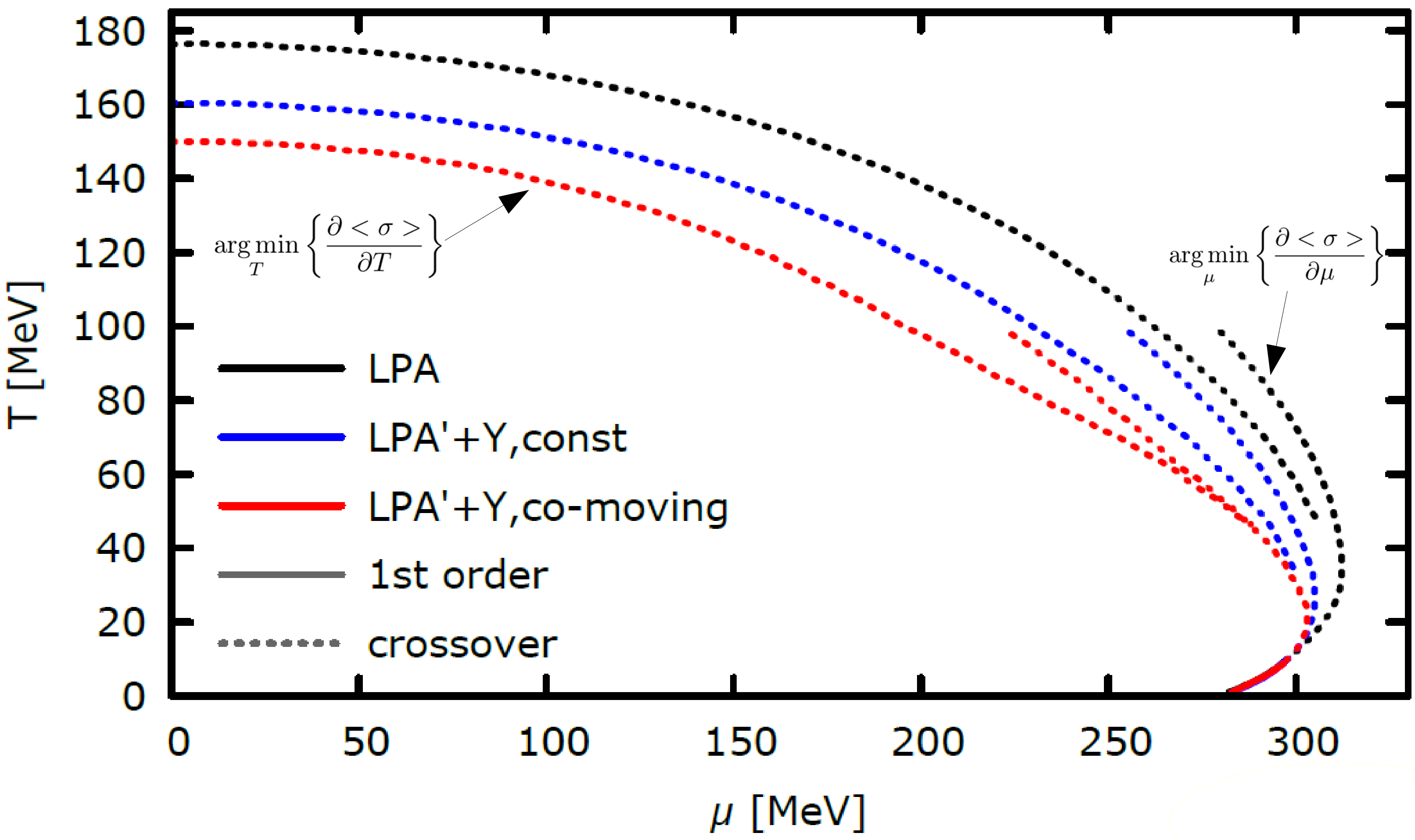




Scale Dependent Masses



Phase Diagram



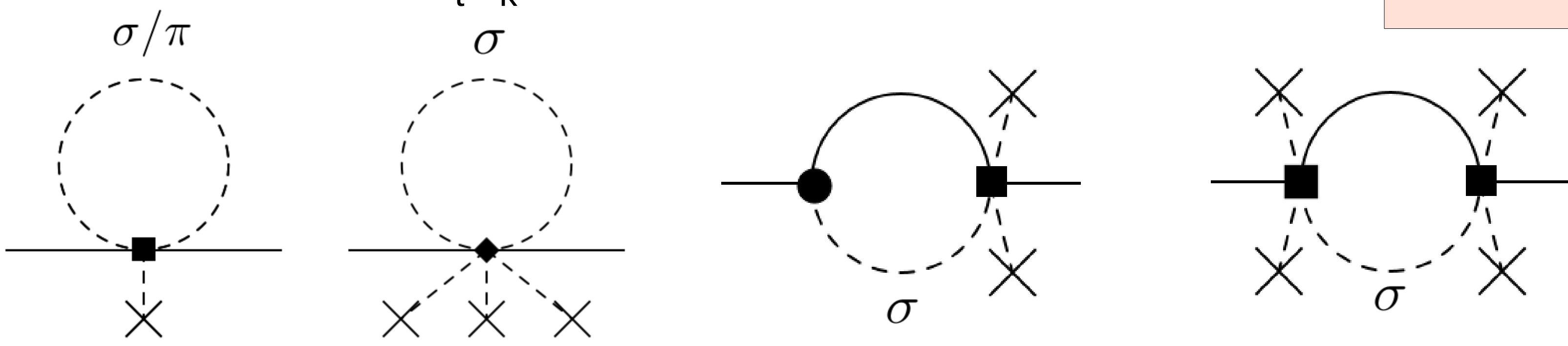
Field Dependent Yukawa Coupling

- <u>Now:</u> g_k field dependent

$g_k = g_k(\phi^2)$

- new contributions to $\partial_t g_k$:

$\times = \langle \sigma \rangle$

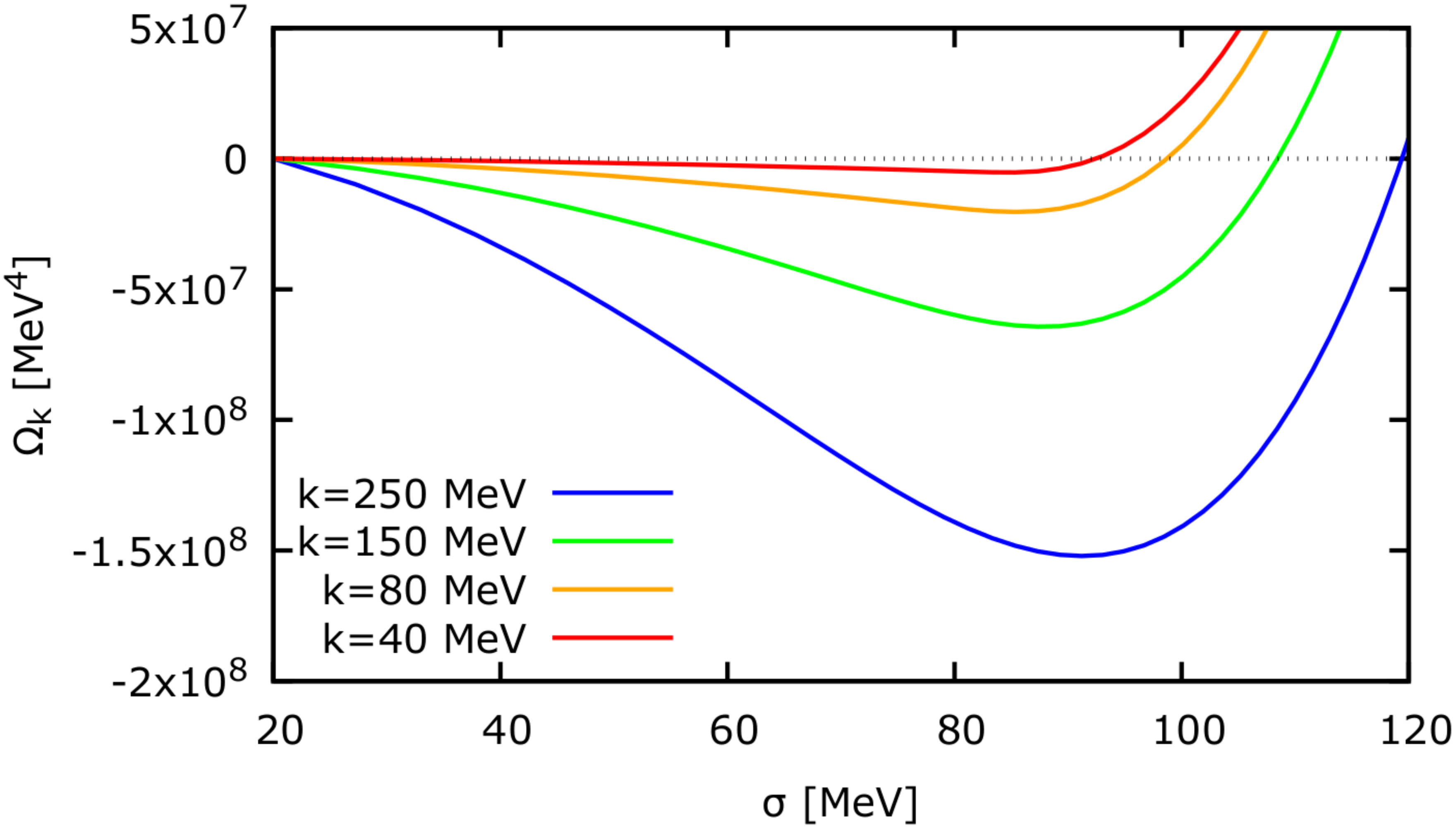


- In LPA+Y: g_k diverges for σ=0, leading term:

$$\partial_t g_k\left(\sigma^2 = 0\right) \sim -\frac{g_k^3(0)}{\left(1 + \underbrace{m_\pi^2(0)/k^2}_{\to -1 \text{ for } k \to 0}\right)^{3/2}}$$

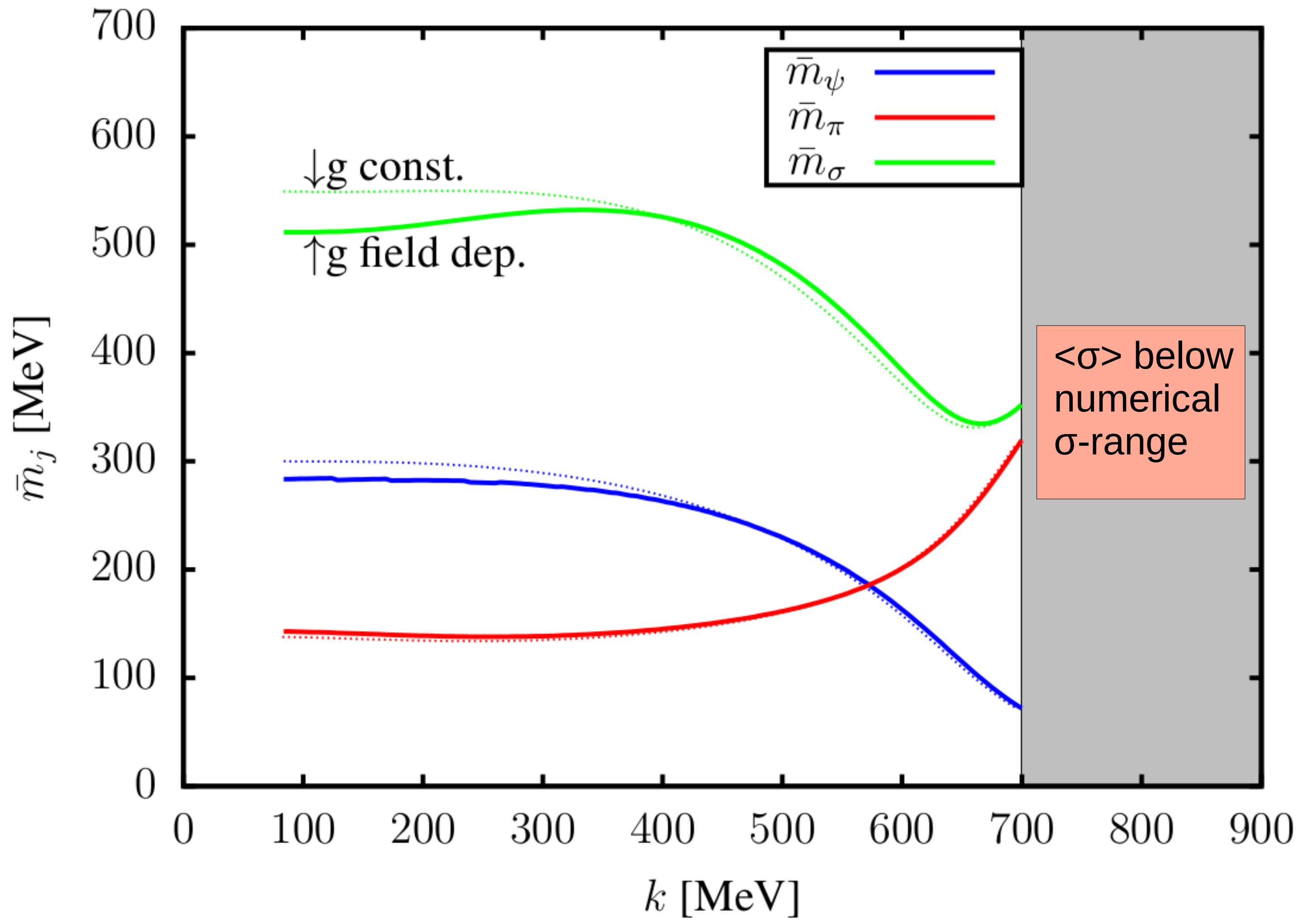
Field Dependent Yukawa Coupling

Effective Potential



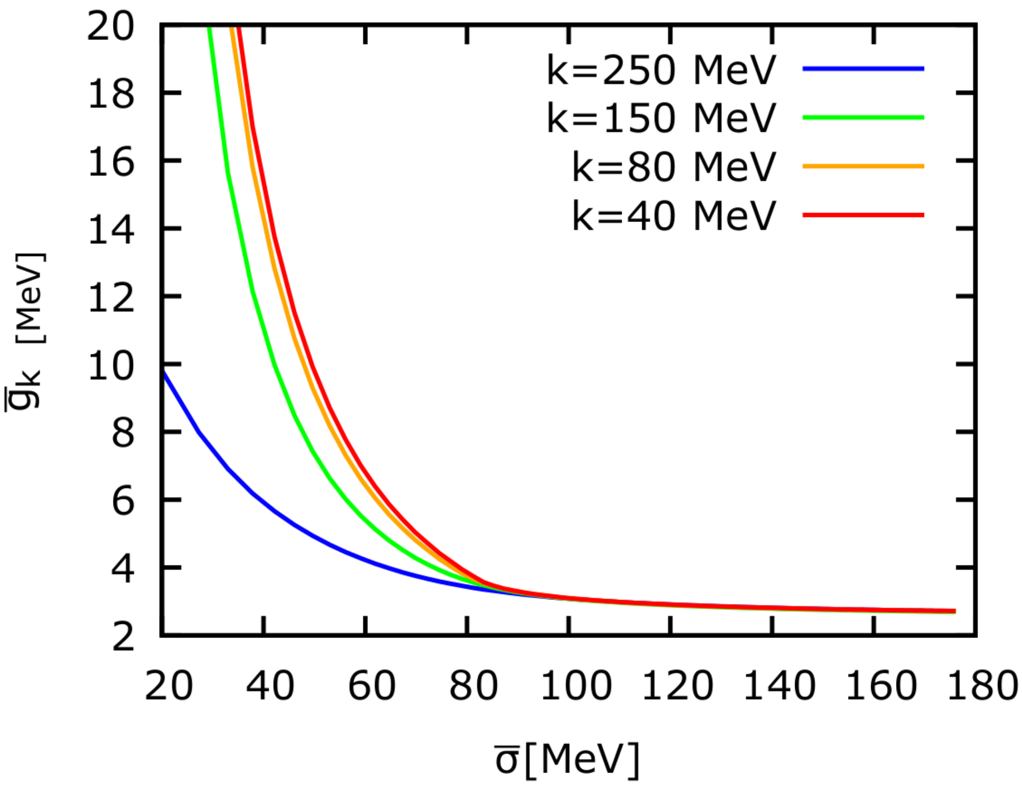
Field Dependent Yukawa Coupling

Vacuum flow of the masses

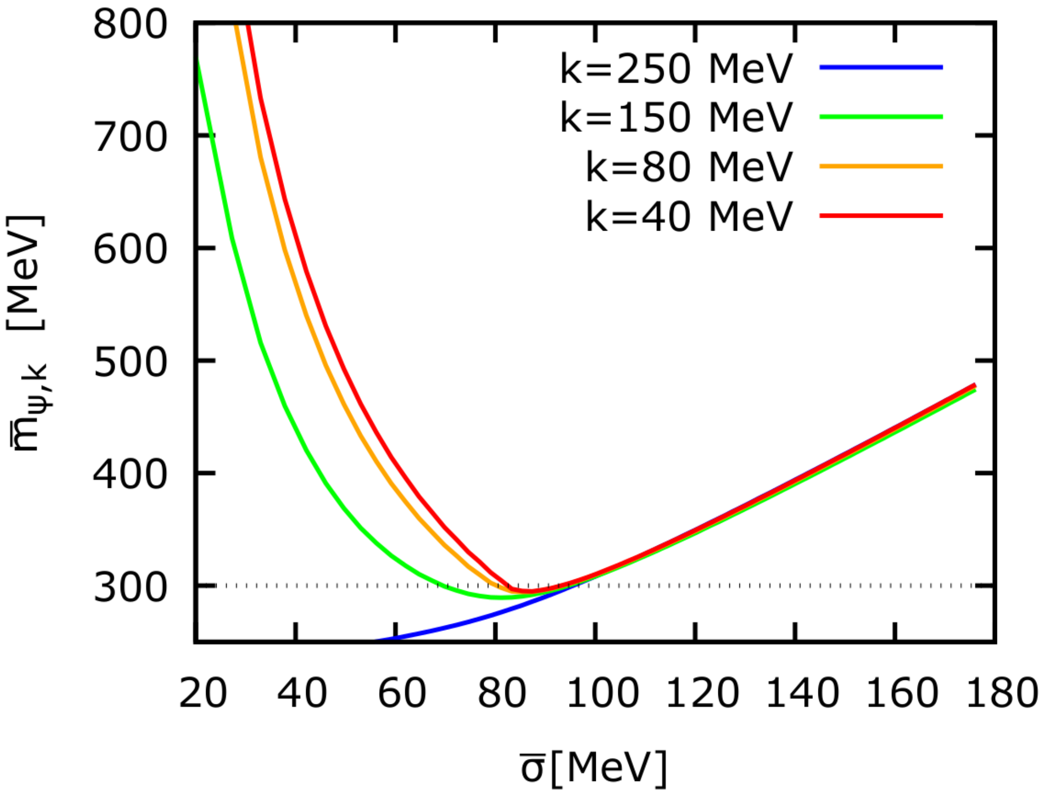


Field Dependent Yukawa Coupling

Yukawa Coupling $\bar{g}_k$



Quark Mass $\bar{m}_\psi = \bar{g}_k \bar{\sigma}$

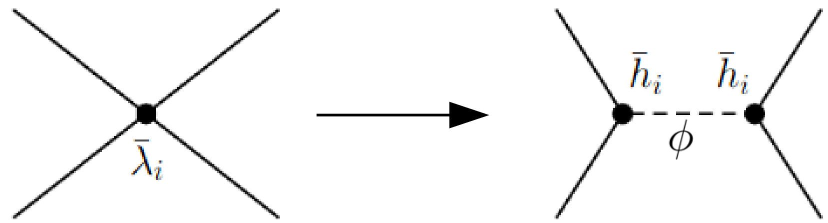


3) Dynamical Hadronization

Introducing Bosons

- Goal: Replace fermion interactions with effective d.o.f.

➔ For four-fermion interactions: use Hubbard-Stratonovich (HS) transformation:



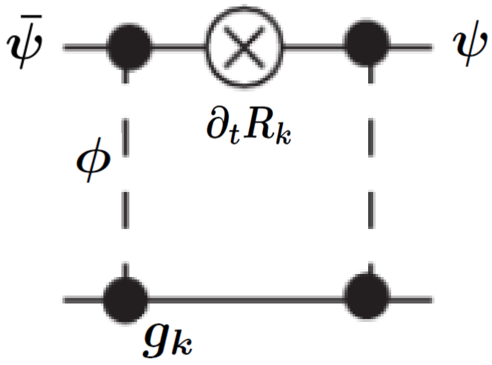
- At fixed scale k=Λ

  →Introduces UV-cutoff of partially bosonized theory
- Exact mathematical identity

Dynamical Hadronization: Why?

- Allows “smooth” transition between different d.o.f.
- Fermionic interactions regenerated at scales k<Λ via box diagrams:

$$\partial_t \lambda_k \sim g_k^4$$



  - Possible back-coupling into other flows
  - Effects usually neglected in low-energy models
- Goal: λ_k=0 for <u>all</u> scales k<Λ

➔Idea: “Re-bosonize” effective action at every step of the flow

Fierz Transformations

- Consider interaction term

$$\mathscr{L}_{\text{int}} = g_I \left(\bar{\psi} \hat{\Gamma}^{(I)} \psi \right)^2 = g_I \Gamma^{(I)}_{ij} \Gamma^{(I)}_{kl} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l$$

- For a complete basis $\left\{ \hat{\Gamma}^{(M)} \right\}$ one finds identities

$$\Gamma^{(I)}_{ij} \Gamma^{(I)}_{kl} = \sum_M c^I_M \Gamma^{(M)}_{il} \Gamma^{(M)}_{kj}$$

allowing to rewrite the interaction as

$$\mathscr{L}_{\text{ex}} = -g_I \sum_M c^I_M \left(\bar{\psi} \hat{\Gamma}^{(M)} \psi \right)^2$$

Example: Flavor/Color space

$$\begin{bmatrix} \mathbb{I}_{ij} \mathbb{I}_{kl} \\ (\tau_a)_{ij} (\tau^a)_{kl} \end{bmatrix} = \begin{bmatrix} \frac{1}{N} & \frac{1}{2} \\ 2\frac{N^2-1}{N^2} & -\frac{1}{N} \end{bmatrix} \begin{bmatrix} \mathbb{I}_{ij} \mathbb{I}_{kl} \\ (\tau_a)_{ij} (\tau^a)_{kl} \end{bmatrix}$$

Dyn. Hadronization: How?

- Take into account that relevance of d.o.f. changes with scale

➔ FRG with scale dependent fields:

$$\partial_t \varphi_k = \mathcal{F}_k \left[\varphi, \psi, \bar{\psi}, \ldots\right]$$

[Gies et al. (2002)]

- $\mathcal{F}_k$ a priori arbitrary functional

➔ For suitable F_k: 4-fermion interactions completely translated into Yukawa interactions at all scales

Effective Action

- Implementation of DH into the flow equation:
  - Start with generating functional (only one field for sake of simplicity)

$$Z_k[j] = e^{W_k[j]} = \int \mathcal{D}\varphi \exp\left\{-S[\varphi] - \underline{\Delta S_k[\varphi_k] + \int j\varphi_k}\right\}$$

  - For the effective action this results in the flow equation (analogously to the “standard” Wetterich eq.)

$$\partial_t \Gamma_k|_{\phi_k} = \frac{1}{2}\mathrm{STr}\left[(\partial_t R_k)\, G_k\right] + \int \left(G_k \frac{\delta}{\delta \phi_k}\right) R_k\,(\partial_t \phi_k) - \int \frac{\delta \Gamma_k}{\delta \phi_k}(\partial_t \phi_k)$$

standard flow — field dep. of $\partial_t\phi_k$ — k-dep. of ϕ_k

Flow Equations

$$\partial_t \Gamma_k|_{\phi_k} = \frac{1}{2}\mathrm{STr}\left[\left(\partial_t R_k\right) G_k\right] + \int \left(G_k \frac{\delta}{\delta \phi_k}\right) R_k \left(\partial_t \phi_k\right) - \int \frac{\delta \Gamma_k}{\delta \phi_k}\left(\partial_t \phi_k\right)$$

- Flow equations can be splitted into an original part (without dynamical hadronization) and additional contributions due to the scale dependent fields!

➔ Generic flow equation:

$$\partial_t c_k = \partial_t^0 c_k + \Delta \partial_t c_k$$

without dyn. had. new contributions

➔ Both parts are derived as usual by projecting onto the corresponding terms of the flowing action

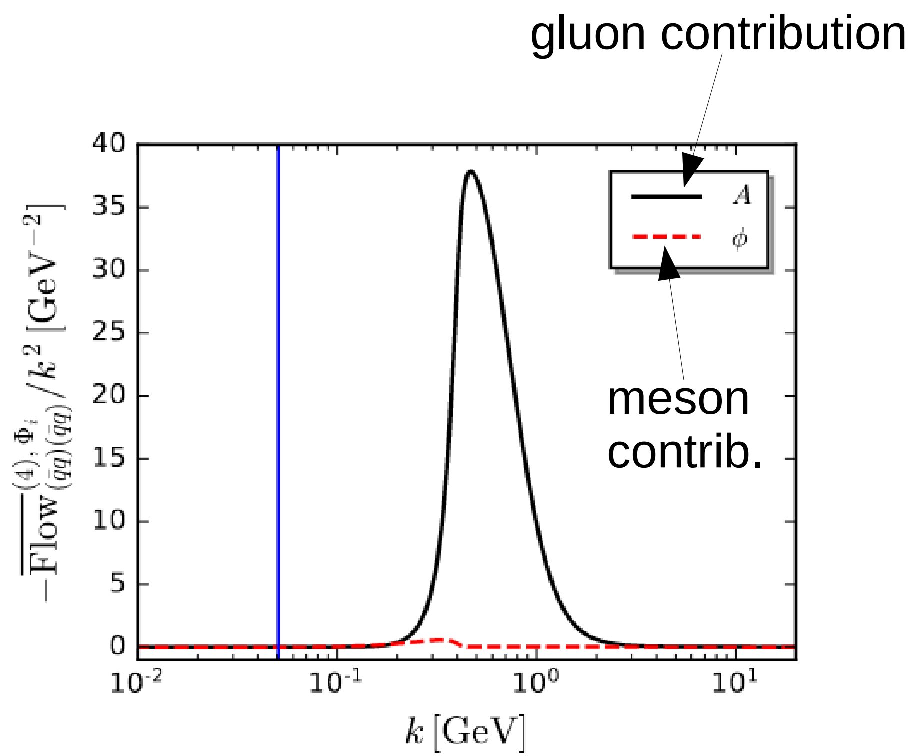
Relevance for low energy models

- Necessary for QM model? How large is the effect from box diagrams really?
  - In vacuum:

$f_\pi/\tilde{f}_\pi$	$M_q/\tilde{M}_q$	$M_\pi/\tilde{M}_\pi$	$M_\sigma/\tilde{M}_\sigma$
0.995	0.997	1.003	0.990

Table I: Effect of dynamical hadronisation on a quark-meson model: The quantities with/without a tilde are the results obtain from a solution of the flow equations of the quark-meson model with/without dynamical hadronisation techniques.

[Braun et al. (2016)]



[Fu et al. (2019)]

- comparison away from vacuum?

DH in the QM2 model

- Choose the following scaling for bosonic fields:

$$\partial_t \sigma_k = (\partial_t A_k) \int_q \bar{\psi}(q-p)\psi(q)$$

  - Hadronization function $\partial_t A_k$ fixed by condition

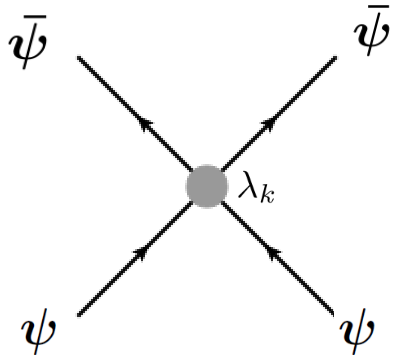
$$\partial_t \lambda_k \left[\partial_t A_k\right] \equiv 0$$

  - $\partial_t \sigma_k$ independent of bosonic fields
    - → one of the contributions to $\partial_t \Gamma_k$ vanishes

Four Fermion Interaction

- Term corresponding to (σ-π)-channel:

$$\mathcal{L}_{SP} = \frac{\lambda_k}{2}[(\underbrace{\bar{\psi}\psi}_{\hat{=}\boldsymbol{\sigma}})^2 + (\underbrace{\bar{\psi}i\gamma_5\tau_a\psi}_{\hat{=}\boldsymbol{\pi_a}})^2]$$



- But: Also different types of four fermion interactions are generated in $\partial_t\Gamma_k$, e.g. terms $(\bar{\psi}\not{\partial}\psi)^2$

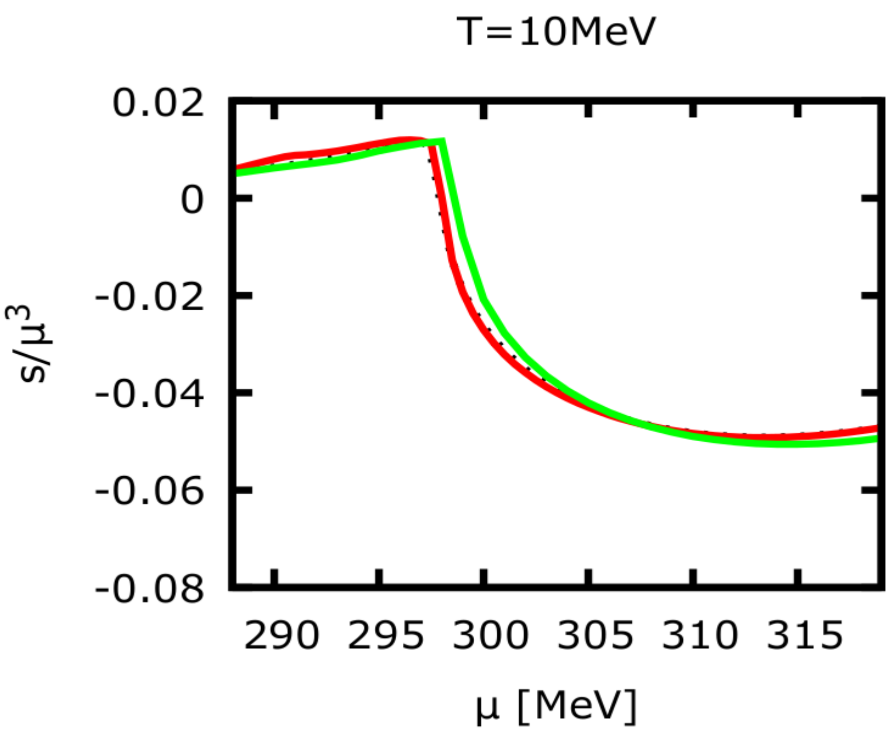
➔Flow strongly depends on Fierz transformations

Dynamical Hadronization: Results

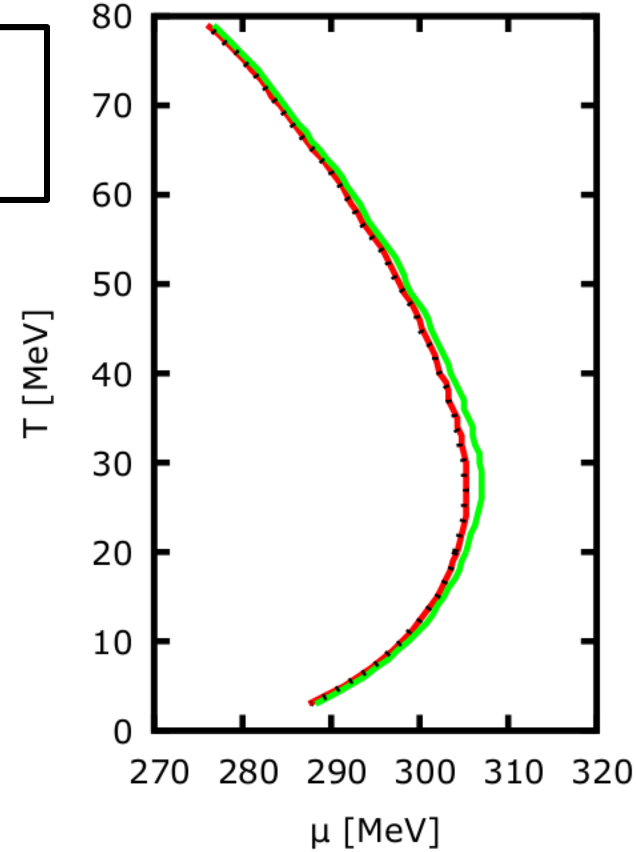
- Effects are very small! Examples:

approx.: DH with leading order $\partial_t \lambda_k$
full: DH with all contributions

a) Entropy density:



b) Phase Diagram:



[preliminary]

4) Summary

- Studied chiral symmetry breaking using a quark meson model
- Introduced different steps “beyond” leading truncation (LPA):
  - ➔ Additional scale dependencies
  - ➔ Allowed higher order quark-meson vertices
- Introduced dynamical hadronization and discussed it’s relevance for low energy models

4) Outlook

- Field dependent wave function renormalizations
- Equation of State at high densities/ temperature close to zero (neutron stars)
- Include higher order momentum dependencies to propagators

Some References

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