Meson spectrum from functional methods beyond Rainbow-Ladder

Lunchclub Seminar 14. April 2021





Theoretical Physics



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Overview



2 Functional methods



Beyond Rainbow-Ladder

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What are mesons?

- $q\overline{q}$ bound states
- Mainly characterized by:
 - Quark content (u, d, s, ...)
 Quantum numbers J^{PC}

What are mesons?

- qq bound states
- Mainly characterized by:
 - Quark content (*u*, *d*, *s*, ...)
 - Quantum numbers J^{PC}

Example: The pion

• Consists of *u*- and *d*-quarks

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$$J^{PC} = 0^{-+}$$

•
$$m_{\pi}=139.57$$
 MeV

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Light meson spectrum



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Light meson spectrum



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Our goals:

 Reproducing the experimental spectrum from fundamental equations of QCD



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Our goals:

- Reproducing the experimental spectrum from fundamental equations of QCD
- Understanding the nature of QCD interactions





Overview



2 Functional methods



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The Bethe-Salpeter equation



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The Bethe-Salpeter equation



$$[\Gamma(p,P)]_{tu} = \int \mathrm{d}^4 q [S(q_+)\Gamma(q,P)S(q_-)]_{sr} \mathcal{K}_{tu}^{rs}(q,p,P)$$

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The Bethe-Salpeter equation



$$[\Gamma(p,P)]_{tu} = \int \mathrm{d}^4 q [S(q_+)\Gamma(q,P)S(q_-)]_{sr} \mathcal{K}_{tu}^{rs}(q,p,P)$$

We need as input:

- Quark propagator S(p)
- Scattering kernel K(p, q, P)

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Obtaining the quark propagator

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The Dyson-Schwinger equation



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Image: A math a math

The Dyson-Schwinger equation



$$S^{-1}(p) = Z_2 S_0^{-1}(p) + Z_{1F} g^2 C_f \int d^4 q \gamma_\mu S(q) \Gamma_
u(q,p) D^{\mu
u}(p-q)$$

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Williams, Fischer, Heupel 2016

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Williams, Fischer, Heupel 2016

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Williams, Fischer, Heupel 2016

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Williams, Fischer, Heupel 2016

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And so on...

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And so on. forever.

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Solution: Truncation!

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The rainbow-ladder truncation

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General idea

General form of the vertex:

• Simplify the tensor structure of the quark-gluon vertex $\Gamma^{\mu}(p,q) \in \{\gamma^{\mu}, p^{\mu}, q^{\mu}\} \otimes \{\mathbb{1}, p, q, [p, q]_{-}\}$

Truncated form:

 $\Gamma^{\mu}(\pmb{p},\pmb{q})\propto\gamma^{\mu}$

General idea

General form of the vertex:

- Simplify the tensor structure of the quark-gluon vertex
- Replace the gluon propagator by an effective coupling

 ${\sf \Gamma}^{\mu}({\sf p},{\it q})\in\{\gamma^{\mu},{\it p}^{\mu},{\it q}^{\mu}\}\otimes\left\{1,{\it p},{\it q},\left[{\it p},{\it q}
ight]_{-}
ight\}$

Truncated form:

 $\Gamma^\mu(\pmb{p},\pmb{q})\propto\gamma^\mu$

General form of the gluon propagator (in Landau gauge):

$$D^{\mu
u}(k) = rac{Z(k^2)}{k^2} \left(\delta^{\mu
u} - k^{\mu} k^{
u} / k^2
ight)$$

$$\alpha(k^2) = \alpha_{\rm IR}(k^2) + \alpha_{\rm UV}(k^2)$$

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$$\alpha(k^2) = \alpha_{\rm IR}(k^2) + \alpha_{\rm UV}(k^2)$$

$$\alpha_{\rm IR}(k^2) = \pi \eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 e^{-\eta^2 k^2/\Lambda^2}$$

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$$\alpha_{\rm IR}(k^2) = \pi \eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 e^{-\eta^2 k^2/\Lambda^2}$$

$$lpha_{\mathrm{UV}}(k^2) = rac{\pi \gamma_m \left(1 - e^{-k^2/\Lambda_0^2}
ight)}{\ln \sqrt{e^2 - 1 + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2
ight)^2}}$$

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Results

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Solving the DSE

General form of the quark propagator:

$$S^{-1}(p) = i \not p A(p^2) + B(p^2)$$

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Solving the DSE

General form of the quark propagator:

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Coupled integral equations for $A(p^2)$ and $B(p^2)$

$$A(p^2) = Z_2 + \Sigma_A(p^2)$$
$$B(p^2) = Z_2 Z_m m_c + \Sigma_B(p^2)$$

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Numerical results



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Numerical results



$M_{\rm eff} = 488 \; { m MeV}$

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Back to the Bethe-Salpeter equation

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Specifying a scattering kernel

Kernel cannot be chosen arbitrarily

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Specifying a scattering kernel

Kernel cannot be chosen arbitrarily Symmetries must be conserved

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Specifying a scattering kernel

Kernel cannot be chosen arbitrarily Symmetries must be conserved



Specifying a scattering kernel

Kernel cannot be chosen arbitrarily Symmetries must be conserved



Gell-Mann-Oakes-Renner relation:

$$f_\pi^2 m_\pi^2 = -2m_c \left< \overline{\psi} \psi \right> / \mathit{N} + \mathit{O}(m_c^2)$$

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Rainbow-ladder kernel

Choose K analogous to the quark self-energy - single gluon exchange



Rainbow-ladder kernel

Choose K analogous to the quark self-energy - single gluon exchange



$$K_{abcd}(p,q,P) = -C_f Z_2^2 4\pi T_{\mu\nu}(k) \gamma^{\mu}_{ab} \gamma^{\nu}_{cd} \frac{\alpha(k^2)}{k^2}$$

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Rainbow-ladder kernel

Choose K analogous to the quark self-energy - single gluon exchange



$$K_{abcd}(p,q,P) = -C_f Z_2^2 4\pi T_{\mu\nu}(k) \gamma^{\mu}_{ab} \gamma^{\nu}_{cd} \frac{\alpha(k^2)}{k^2}$$

Eigenvalue equation of the form:

$$(KG_0)(m) \cdot \Gamma(m) = \lambda(m)\Gamma(m)$$

 $\lambda(m) = 1$ at the physical meson mass.

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Solving the BSE



Eigenvalue curve

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Goldstone's theorem



$$f_{\pi}^2 m_{\pi}^2 = -2m_c \left\langle \overline{\psi}\psi \right\rangle / N + O(m_c^2)$$

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Comparing to experimental results



Comparing to experimental results



Overview



2 Functional methods



Beyond Rainbow-Ladder

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• Start with classical action $S[\Phi]$

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- Start with classical action $S[\Phi]$
- \bullet Transform to effective action $\Gamma[\Phi]$

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- Start with classical action $S[\Phi]$
- \bullet Transform to effective action $\Gamma[\Phi]$
- Take first order derivative and evaluate at $\Phi=\Phi_0$ to obtain self-energy

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- Start with classical action S[Φ]
- \bullet Transform to effective action $\Gamma[\Phi]$
- Take first order derivative and evaluate at $\Phi=\Phi_0$ to obtain self-energy
- Take second order derivative and evaluate at $\Phi=\Phi_0$ to obtain the scattering kernel

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But...

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- Start with classical action S[Φ]
- \bullet Transform to effective action $\Gamma[\Phi]$
- Take first order derivative and evaluate at $\Phi=\Phi_0$ to obtain self-energy
- Take second order derivative and evaluate at $\Phi=\Phi_0$ to obtain the scattering kernel
- But... we can go the other way around!

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• Specify a kernel

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- Specify a kernel
- Use the AVWTI to construct a self-energy

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- Specify a kernel
- Use the AVWTI to construct a self-energy

Starting with K(p, q, P), we get

$$B(p^2) = Z_2 Z_m m_c - \frac{1}{4} \operatorname{tr} \left[\gamma_{ab}^5 \int d^4 q \mathcal{K}_{bcde}(p,q,P) \gamma_{cd}^5 B(q^2) d(q^2) \right]$$

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- Specify a kernel
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Reminder: $S^{-1}(p) = i p A(p^2) + B(p^2)$

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From a 3PI effective action:

$$K_{abcd}(p,q,P) = -g^2 C_F D_{\mu\nu}(k) [\Gamma^{\mu}(p_-,q_-)]_{ab} [\Gamma^{\nu}(p_+,q_+)]_{cd}$$

Williams, Fischer, Heupel 2016

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Williams, Fischer, Heupel 2016

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General quark-gluon vertex:

$$\Gamma^{\mu} = ig\left(\sum_{i=1}^{4} \lambda_i L_i^{\mu} + \sum_{i=1}^{8} \tau_i T_i^{\mu}\right)$$

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Rainbow-Ladder:

$$L_1^\mu(p,q) = \gamma^\mu$$

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Rainbow-Ladder:

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Adding a second structure:

$$L_3^{\mu}(p,q) = il^{\mu} \equiv i(p+q)^{\mu}$$

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The generalized kernel



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Results

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Quark propagator



Quark propagator



 $M_{\rm eff} = 275 \; {\rm MeV}$

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Goldstone's theorem



$$f_{\pi}^2 m_{\pi}^2 = -2m_c \left\langle \overline{\psi}\psi \right\rangle / N + O(m_c^2)$$

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Meson spectrum



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Meson spectrum



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Meson spectrum



 \Rightarrow The additional structure increases the scalar meson mass!

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Conclusion & outlook

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Conclusion & Outlook

• We derived a self energy from a general kernel

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Conclusion & Outlook

- We derived a self energy from a general kernel
- We added more structures to the quark-gluon vertex

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- We derived a self energy from a general kernel
- We added more structures to the quark-gluon vertex
- We improved the predicted scalar meson mass

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- We derived a self energy from a general kernel
- We added more structures to the quark-gluon vertex
- We improved the predicted scalar meson mass
- It only took a few seconds of CPU time

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• Calculate more channels ((axial-)vector, tensor mesons)

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- Calculate more channels ((axial-)vector, tensor mesons)
- Add even more structures to the vertex

- We derived a self energy from a general kernel
- We added more structures to the quark-gluon vertex
- We improved the predicted scalar meson mass
- It only took a few seconds of CPU time

- Calculate more channels ((axial-)vector, tensor mesons)
- Add even more structures to the vertex
- Apply to a wider range of problems
 - Heavy mesons
 - Baryons
 - Tetraquarks
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