Meson spectrum from functional methods beyond Rainbow-Ladder

Lunchclub Seminar 14. April 2021

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Theoretical Physics

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Overview

1. The meson spectrum
2. Functional methods
3. Beyond Rainbow-Ladder

What are mesons?

- $q\overline{q}$ bound states
- Mainly characterized by:
  - Quark content $(u, d, s, \ldots)$
  - Quantum numbers J^{PC}

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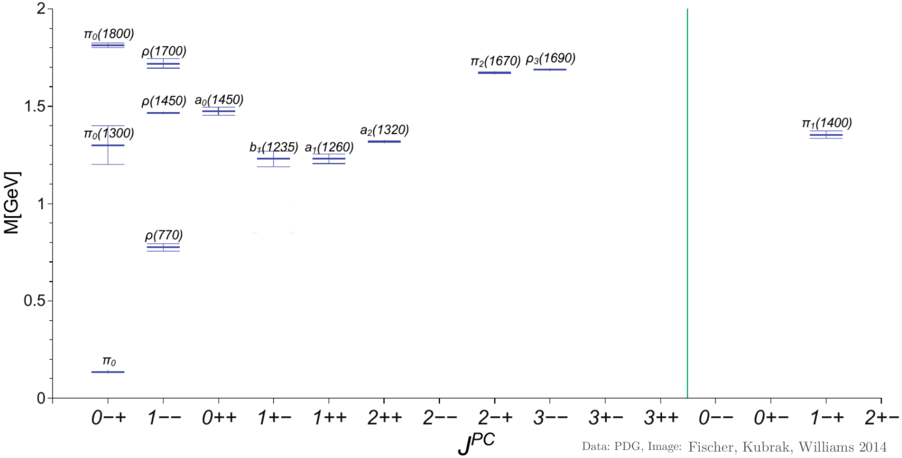
  - Quantum numbers J^{PC}

Example: The pion

- Consists of u- and d-quarks
- $J^{PC} = 0^{-+}$
- $m_\pi = 139.57$ MeV

Light meson spectrum



Data: PDG, Image: Fischer, Kubrak, Williams 2014

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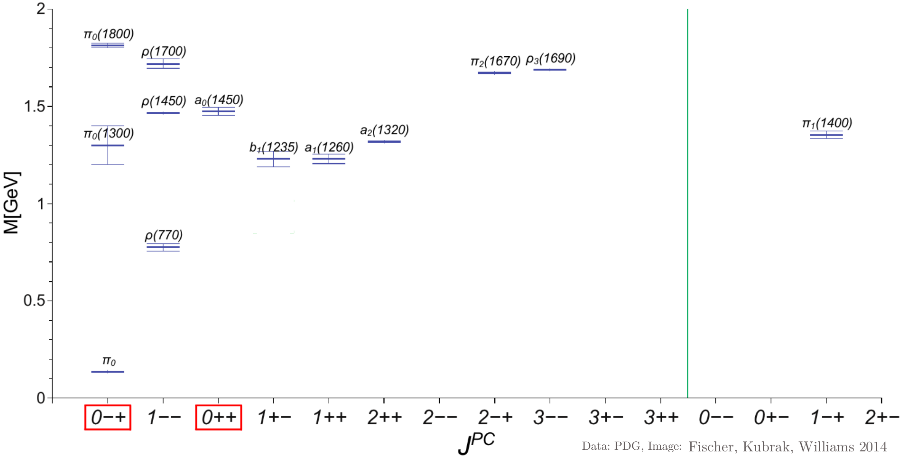

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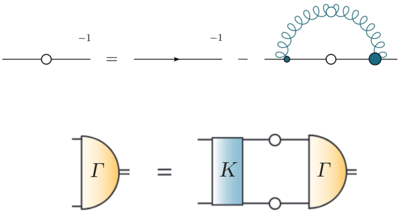
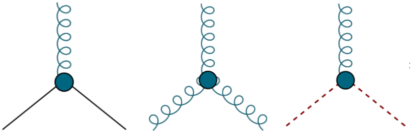
Our goals:

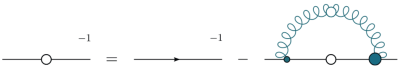
- Reproducing the experimental spectrum from fundamental equations of QCD

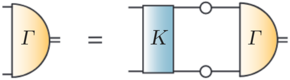


Our goals:

- Reproducing the experimental spectrum from fundamental equations of QCD
- Understanding the nature of QCD interactions

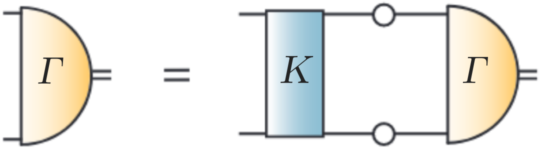






Overview

The Bethe-Salpeter equation



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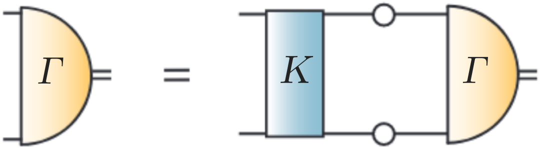

$$[\Gamma(p,P)]_{tu} = \int đ^4 q [S(q_+)\Gamma(q,P)S(q_-)]_{sr} K^{rs}_{tu}(q,p,P)$$

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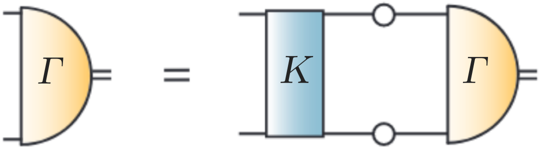

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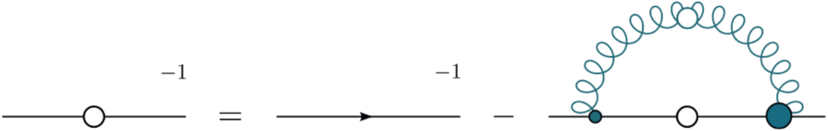
We need as input:

- Quark propagator $S(p)$
- Scattering kernel $K(p,q,P)$

Obtaining the quark propagator

The Dyson-Schwinger equation



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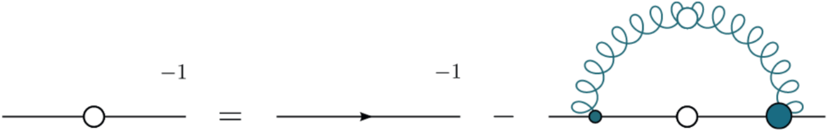
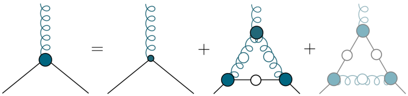


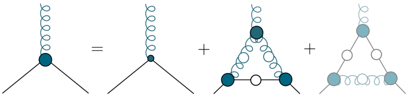
$$S^{-1}(p) = Z_2 S_0^{-1}(p) + Z_{1F} g^2 C_f \int đ^4 q \gamma_\mu S(q) \Gamma_\nu(q,p) D^{\mu\nu}(p-q)$$

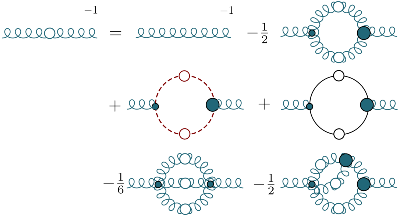
More DSEs...



Williams, Fischer, Heupel 2016

More DSEs. . .





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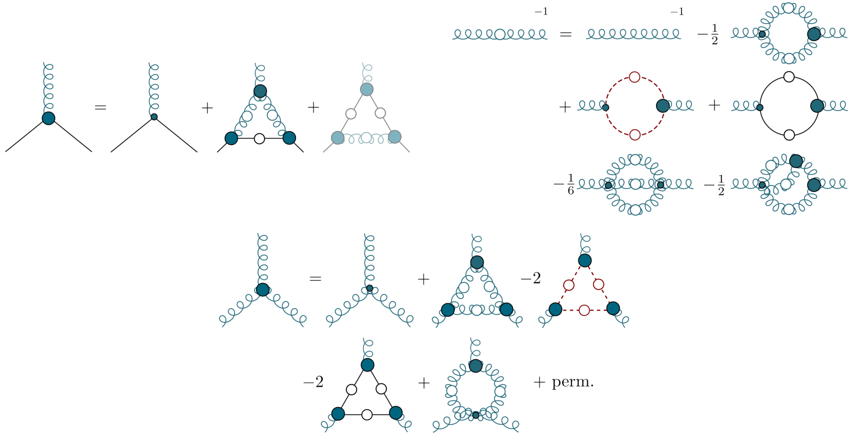

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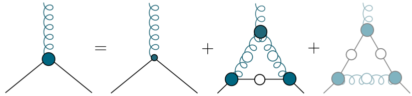

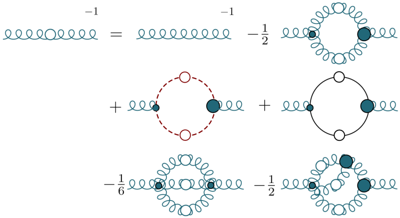

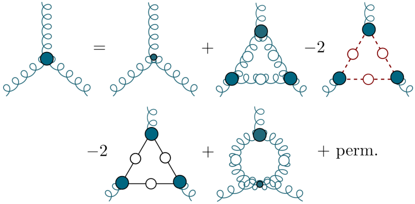

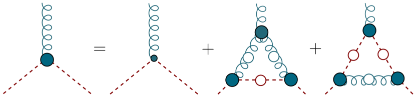

Williams, Fischer, Heupel 2016

More DSEs...

And so on...

More DSEs...

And so on...

...forever.

More DSEs. . .

Solution: Truncation!

The rainbow-ladder truncation

General idea

- Simplify the tensor structure of the quark-gluon vertex

General form of the vertex:

$$\Gamma^\mu(p,q) \in \{\gamma^\mu, p^\mu, q^\mu\} \otimes \left\{\mathbb{1}, \not{p}, \not{q}, \left[\not{p}, \not{q}\right]_-\right\}$$

Truncated form:

$$\Gamma^\mu(p,q) \propto \gamma^\mu$$

General idea

- Simplify the tensor structure of the quark-gluon vertex
- Replace the gluon propagator by an effective coupling

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Truncated form:

$$\Gamma^\mu(p,q) \propto \gamma^\mu$$

General form of the gluon propagator (in Landau gauge):

$$D^{\mu\nu}(k) = \frac{Z(k^2)}{k^2}\left(\delta^{\mu\nu} - k^\mu k^\nu / k^2\right)$$

The model interaction

$$\alpha(k^2) = \alpha_{\mathsf{IR}}(k^2) + \alpha_{\mathsf{UV}}(k^2)$$

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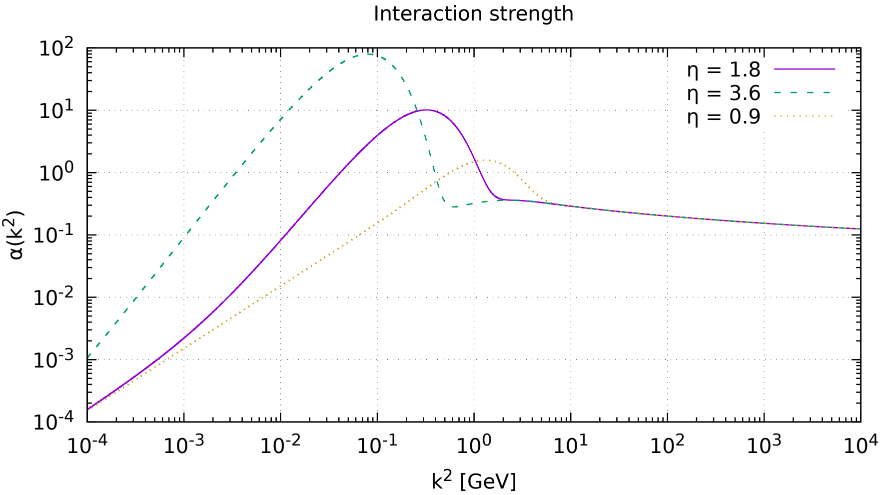
$$\alpha_{\mathrm{UV}}(k^2) = \frac{\pi\gamma_m \left(1 - e^{-k^2/\Lambda_0^2}\right)}{\ln\sqrt{e^2 - 1 + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2}}$$

The model interaction



Results

Solving the DSE

General form of the quark propagator:

$$S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$$

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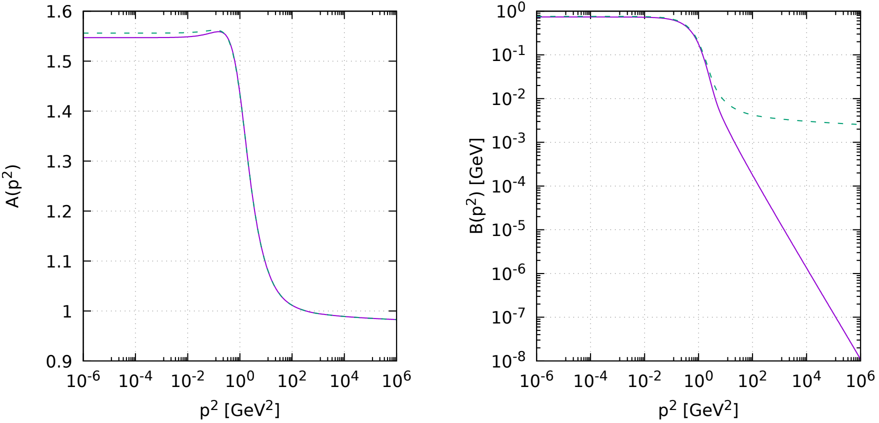
Coupled integral equations for $A(p^2)$ and $B(p^2)$

$$A(p^2) = Z_2 + \Sigma_A(p^2)$$
$$B(p^2) = Z_2 Z_m m_c + \Sigma_B(p^2)$$

Numerical results



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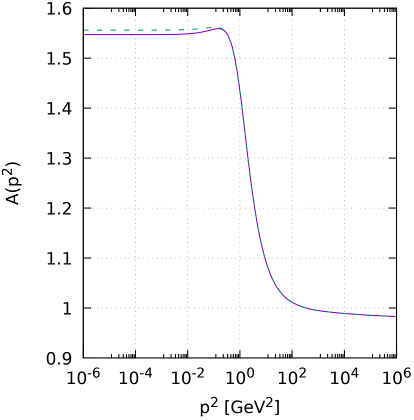

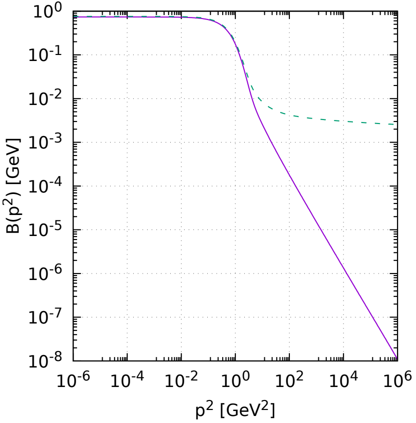

$$M_{\text{eff}} = 488 \text{ MeV}$$

Back to the Bethe-Salpeter equation

Specifying a scattering kernel

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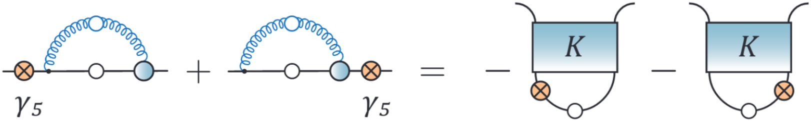

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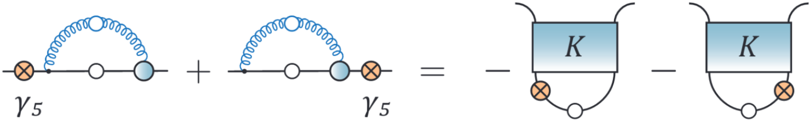
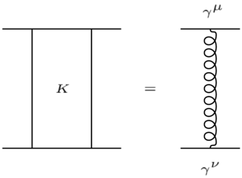


Gell-Mann-Oakes-Renner relation:

$$f_\pi^2 m_\pi^2 = -2m_c \langle \overline{\psi}\psi \rangle / N + O(m_c^2)$$

Rainbow-ladder kernel

Choose K analogous to the quark self-energy - single gluon exchange



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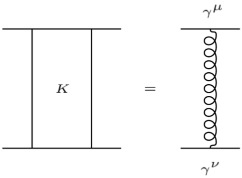

$$K_{abcd}(p,q,P) = -C_f Z_2^2 4\pi T_{\mu\nu}(k)\gamma^\mu_{ab}\gamma^\nu_{cd}\frac{\alpha(k^2)}{k^2}$$

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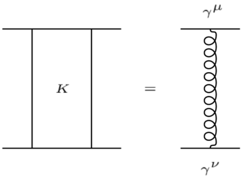

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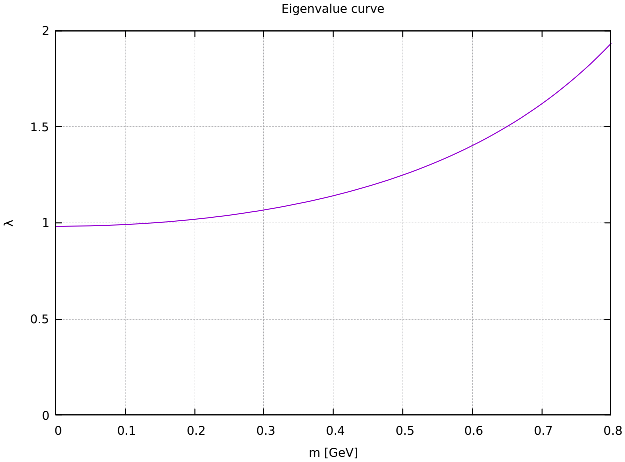
Eigenvalue equation of the form:

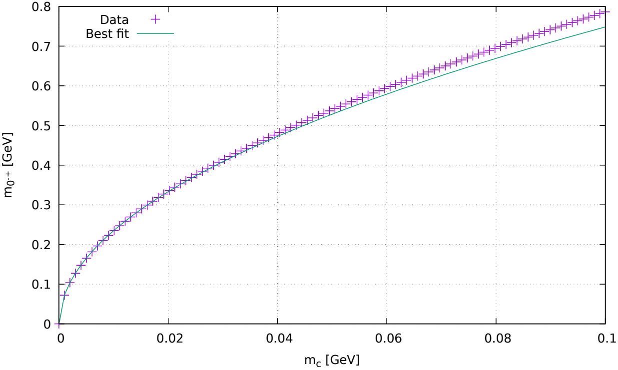
$$(KG_0)(m)\cdot\Gamma(m) = \lambda(m)\Gamma(m)$$

$\lambda(m) = 1$ at the physical meson mass.

Solving the BSE



Goldstone's theorem



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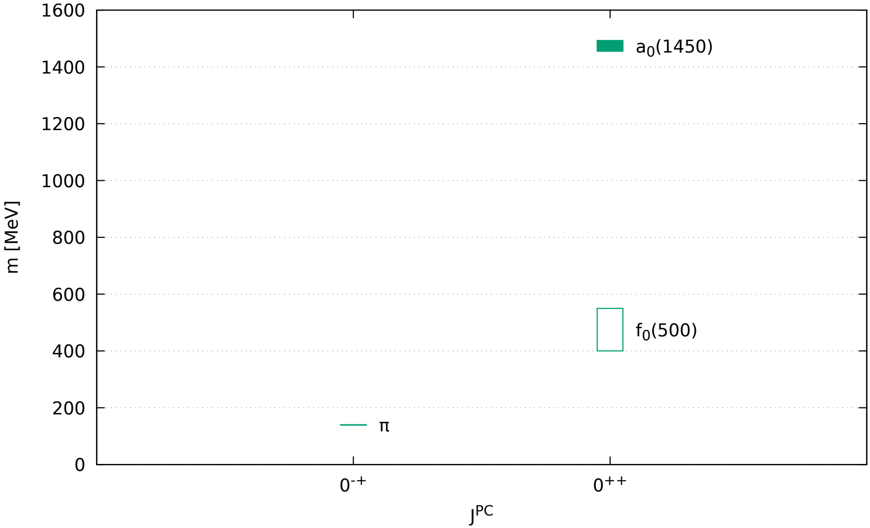
Comparing to experimental results



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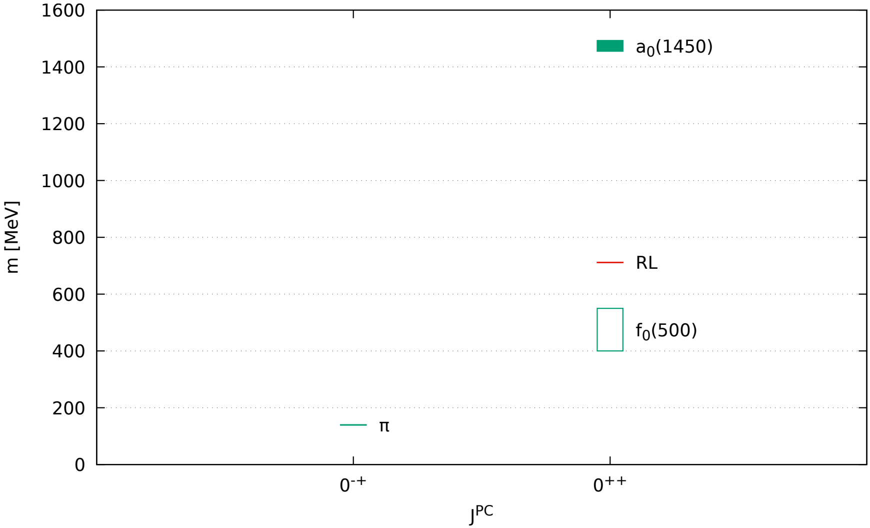

Overview

Usual approach

- Start with classical action $S[\Phi]$

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But...

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- Transform to effective action $\Gamma[\Phi]$
- Take first order derivative and evaluate at $\Phi = \Phi_0$ to obtain self-energy
- Take second order derivative and evaluate at $\Phi = \Phi_0$ to obtain the scattering kernel

But... we can go the other way around!

The basic idea

- Specify a kernel

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Starting with $K(p, q, P)$, we get

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Reminder: $S^{-1}(p) = i\not{p} A(p^2) + B(p^2)$

Specifying a kernel

From a 3PI effective action:

$$K_{abcd}(p, q, P) = -g^2 C_F D_{\mu\nu}(k)[\Gamma^\mu(p_-, q_-)]_{ab}[\Gamma^\nu(p_+, q_+)]_{cd}$$

Williams, Fischer, Heupel 2016

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General quark-gluon vertex:

$$\Gamma^{\mu} = ig \left(\sum_{i=1}^{4} \lambda_i L_i^{\mu} + \sum_{i=1}^{8} \tau_i T_i^{\mu} \right)$$

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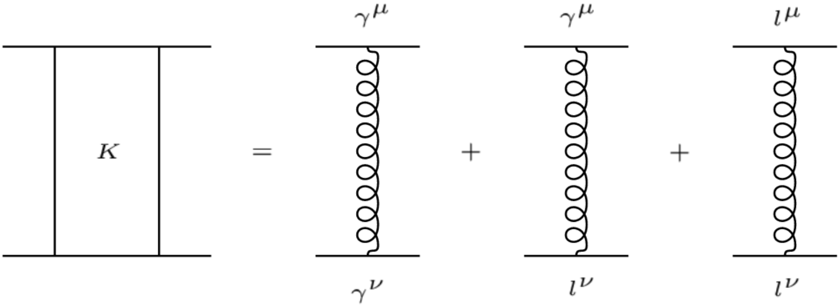
Adding a second structure:

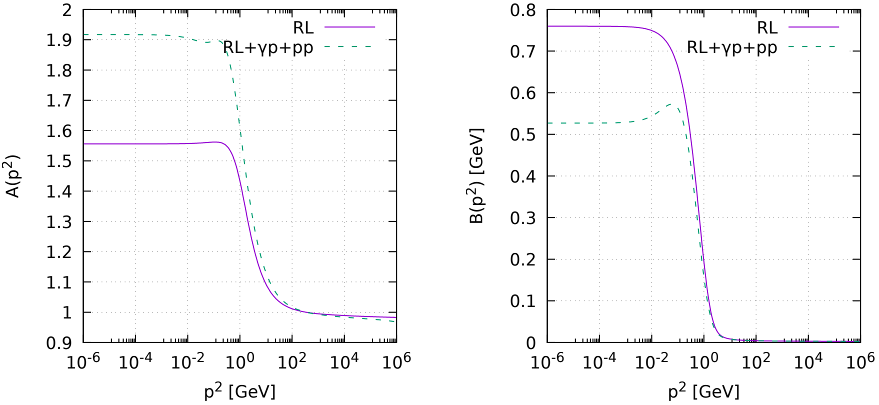
$$L_3^\mu(p, q) = il^\mu \equiv i(p+q)^\mu$$

The generalized kernel



Results

Quark propagator



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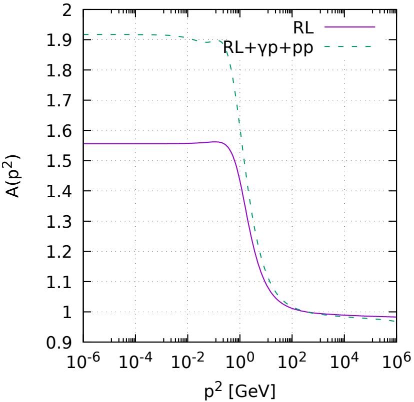

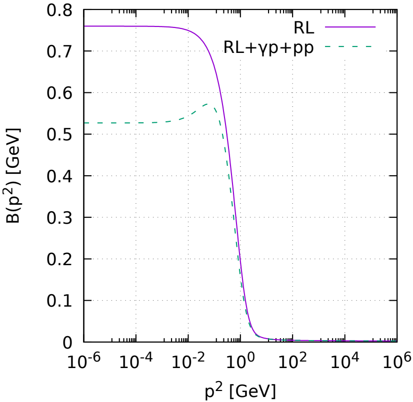

$$M_{\text{eff}} = 275 \text{ MeV}$$

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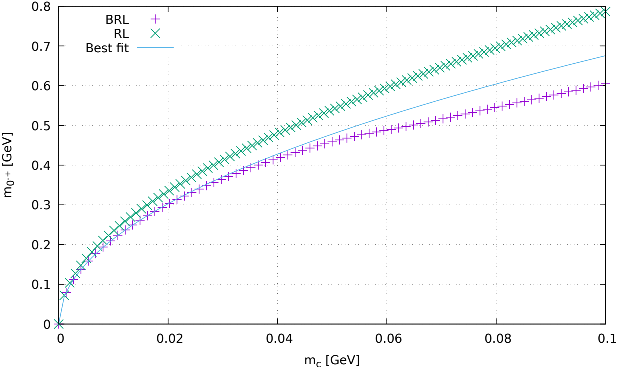

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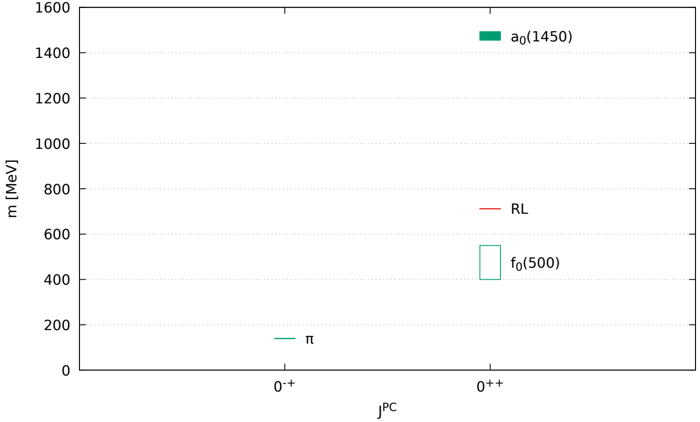
Meson spectrum



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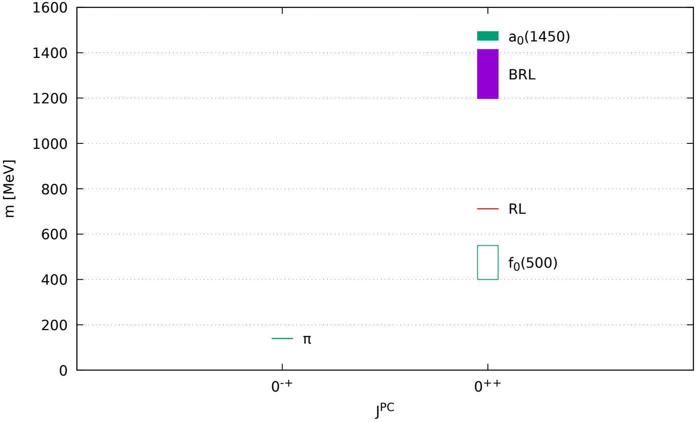

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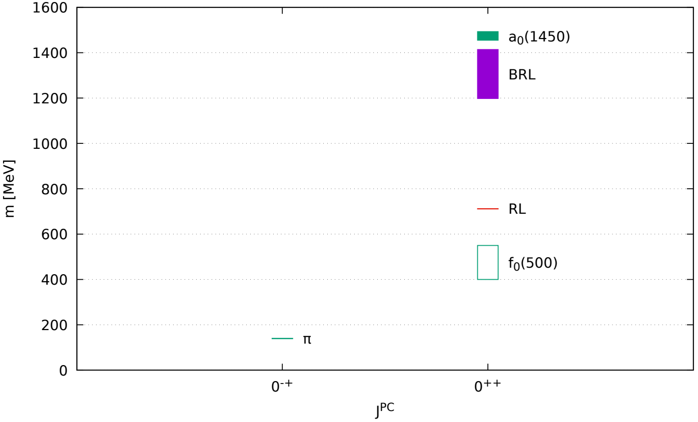

⇒ The additional structure increases the scalar meson mass!

Conclusion & outlook

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- Calculate more channels ((axial-)vector, tensor mesons)
- Add even more structures to the vertex
- Apply to a wider range of problems
  - Heavy mesons
  - Baryons
  - Tetraquarks
  - ...