Thermodynamics from the quark condensate

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Based on:
PI, Fischer, Steinert, arXiv:2012.04991

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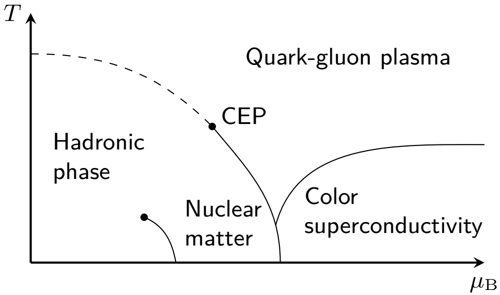
Phase transitions in QCD



Nonperturbative approaches:

- Lattice QCD ... limited to $\mu_B/T \lesssim 3$ due to sign problem
- Effective models ... generalizable?
- Functional methods ... all QCD degrees of freedom & no sign problem (but truncations are necessary)

Equation of state (EoS)

$p(T, \mu_B)$, $s(T, \mu_B)$, and $\varepsilon(T, \mu_B)$
All encoded in thermodynamic potential Ω

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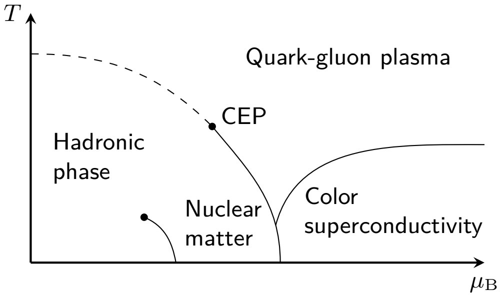

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Dyson-Schwinger equations

- Nonperturbative functional approach
- Correlation functions on quark-gluon level
- Bound states as composite objects of quarks and gluons (solve Bethe-Salpeter/Faddeev equations)

Working areas:

Hadron physics

- Meson and baryon spectra
- Scattering amplitudes
- Decays
- Form factors
- Exotics (tetraquarks, glueballs, and hybrids)
- In-medium properties of mesons

Phase diagram

- Phase structure of QCD
- Spectral functions
- Thermodynamics

Additionally

- Muon $g-2$ (HLbL)
- Higher n-point functions
- ...

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1
Fischer, PPNP 105 (2019) 1
Eichmann, Fischer, Heupel, Santowsky, Wallbott, FBS 61 (2020) 38

Setting the scene: Euclidean QCD

Generating functional

$$\mathcal{Z}_{\mathrm{QCD}} = \int \mathcal{D}[\bar{\psi}\psi A\tilde{c}c]\,\exp\Big\{ -\int_0^{1/T} \mathrm{d}x_4 \int \mathrm{d}^3\vec{x}\, \Big(\bar{\psi}\,(\not{D} + \hat{m} + \gamma_4\,\hat{\mu})\,\psi + \frac{1}{4} F^a_{\nu\sigma} F^a_{\nu\sigma} + \text{gauge fixing} + \text{sources} \Big) \Big\}$$

Propagators in Landau gauge and momentum space, $q = (\omega_q, \vec{q}\,)$:

$$S_f(q) = \Big[\mathrm{i}\,(\omega_q + \mathrm{i}\,\mu_f)\,\gamma_4\, C_f(q) + \mathrm{i}\,\vec{\not{q}}\, A_f(q) + B_f(q) \Big]^{-1}$$

$$D_{\nu\sigma}(q) = \frac{Z^{\mathrm{T}}(q)}{q^2}\, P^{\mathrm{T}}_{\nu\sigma}(q) + \frac{Z^{\mathrm{L}}(q)}{q^2}\, P^{\mathrm{L}}_{\nu\sigma}(q)$$

How to truncate?

Quark DSE

Dressed quark-gluon vertex:

- No lattice results yet
- Explicit solutions at $T = 0$

  Fischer, Williams, PRL 103 (2009) 122001
  Mitter, Pawlowski, Strodthoff, PRD 91 (2015) 054035
  Williams, EPJA 51 (2015) 53
  Williams, Fischer, Heupel, PRD 93 (2016) 034026
  Sternbeck et al., PoS (LATTICE2016) 349
- $T \neq 0$: Ansatz based on STI and known perturbative behavior

  Preliminary results at $T \neq 0$ (solving vertex DSE):
  Contant, Huber, Fischer, Welzbacher, Williams,
  Acta Phys. Polon. B Proc. Supp. 11, 483 (2018)

Dressed gluon propagator:

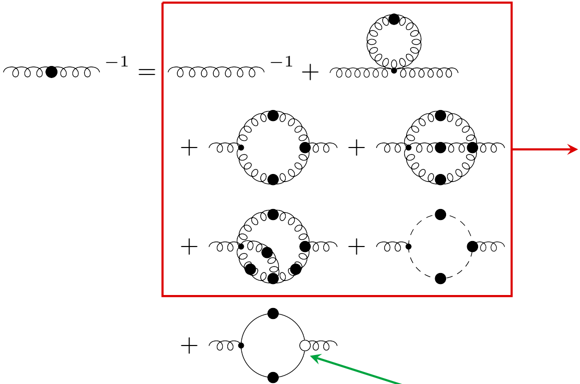
- Two strategies:
  - Model gluon propagator

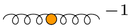
    Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301
    Gao, Liu, PRD 94 (2016) 076009
  - Explicit treatment of gluonic sector
- Here: Use the latter
  - Consistent flavor dependencies
  - Gluon becomes sensitive to chiral dynamics

Third way: expand about FRG vacuum data + treat medium as perturbation

Gao, Pawlowski, PRD 102 (2020) 034027
Gao, Pawlowski, arXiv:2010.13075

How to truncate?

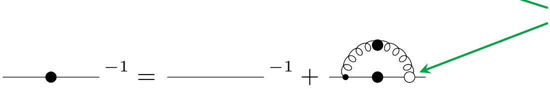




quenched, T-dependent
lattice gluon propagator

Fischer, Maas, Mueller, EPJC 68 (2010) 165

Maas, Pawlowski, von Smekal, Spielmann,
PRD 85 (2012) 034037

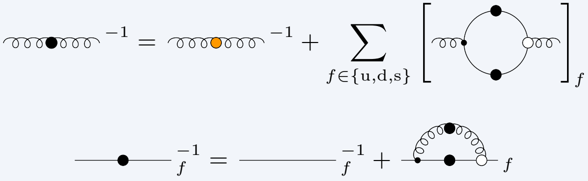


(T, μ)-dependent ansatz
for quark-gluon vertex

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022
(and references therein)

Three-flavor QCD with Dyson-Schwinger equations

Final set of truncated DSEs



- Quenched lattice gluon propagator as input & unquenching via quark loops
- Nontrivial coupling between different quark flavors
- Vertex ansatz built along STI and perturbation theory

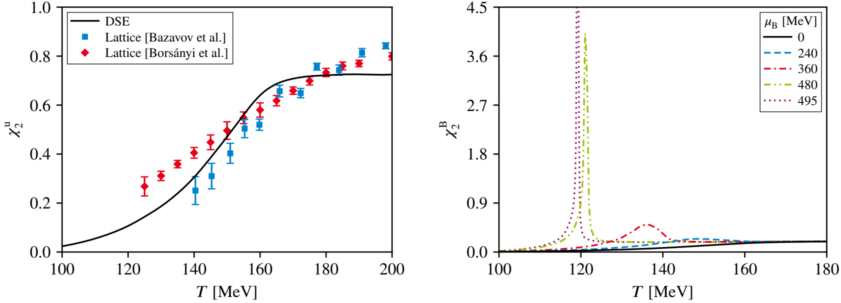
Result: Dressed (i.e., nonperturbative) quark and unquenched gluon propagators

Thermodynamics from DSEs: Status quo

- Our current status: Quark and baryon number fluctuations

  PI, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

$$\chi_{ijk}^{\mathrm{uds}} = -T^{(i+j+k)-4} \frac{\partial^{i+j+k}\Omega}{\partial\mu_{\mathrm{u}}^i \partial\mu_{\mathrm{d}}^j \partial\mu_{\mathrm{s}}^k}$$



- Starting point for fluctuations: density $n_X = \partial\Omega/\partial\mu_X$
- Full thermodynamics extremely difficult within DSEs
  $\Rightarrow$ Approach needed to access Ω

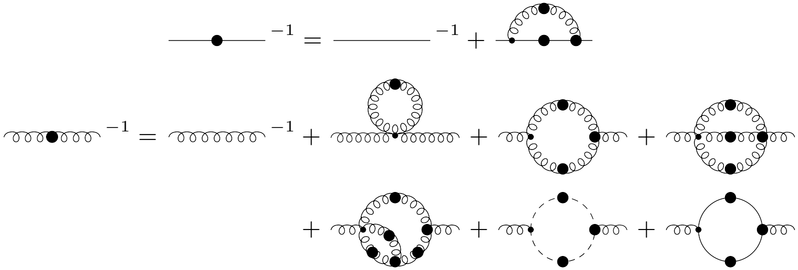
Thermodynamics from DSEs: The 'problem' with Ω

Master equation

$$0 = \int \mathcal{D}[\vec{\varphi}]\, \frac{\delta}{\delta\varphi_k} \exp\left(-\mathcal{S}[\vec{\varphi}] + \int \mathrm{d}^4x\, \vec{J}\cdot\vec{\varphi}\right)$$

$$\Downarrow$$

$$\frac{\delta\Gamma_{1\mathrm{PI}}}{\delta\widetilde{\varphi}_k} = \frac{\delta\mathcal{S}}{\delta\varphi_k}\left[\varphi_\ell \to \pm\left(\frac{\delta}{\delta J_\ell} + \widetilde{\varphi}_\ell\right)\right]1$$



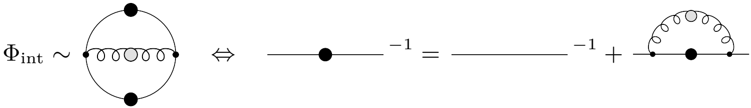
$\Gamma_{1\mathrm{PI}} \sim -\Omega$ at physical point $\Rightarrow$ 'integration' neccessary

Thermodynamics from DSEs: The ‘problem’ with Ω

- 2PI formalism (quark-only):
  Cornwall, Jackiw, Tomboulis, PRD 10 (1974) 2428

$$\Omega[S] = -\frac{T}{V}\left(\mathrm{Tr}\log\frac{S^{-1}}{T} - \mathrm{Tr}\left[\mathbb{1} - S_0^{-1}S\right] + \Phi_{\mathrm{int}}[S]\right)$$

- Physical propagator from stationary condition: $\delta\Omega/\delta S = 0$
  $\Rightarrow S^{-1} = S_0 + \Sigma$ with $\Sigma \sim \delta\Phi_{\mathrm{int}}/\delta S$
- Closed form for Φ_{int}: quark-gluon vertex must not depend on quark



- So far: thermodynamics from DSEs only in rainbow-ladder truncation
  Blaschke, Roberts, Schmidt, PLB 425 (1998) 232
  Xu, Yan, Cui, Zong, IJMPA 30 (2015) 1550217
  Gao et. al., PRD 93 (2016) 094019
- Needed: Ω from a **truncation-independent** method

Thermodynamics from DSEs: Maxwell method

- Consider: $\Omega = \Omega(T, \mu; m)$
- Current-quark mass: external source for bilinear $\bar{\psi}\psi$
  $\Rightarrow \langle\bar{\psi}\psi\rangle(T, \mu; m) = \partial\Omega(T, \mu; m)/\partial m$
- Integrate:

$$\Omega(T, \mu; m_2) - \Omega(T, \mu; m_1) = \int_{m_1}^{m_2} \mathrm{d}m' \, \langle\bar{\psi}\psi\rangle(T, \mu; m')$$

- Ω and $\langle\bar{\psi}\psi\rangle$ are divergent but **suitable derivatives are finite!**
- Derivative w.r.t. T:

$$s(T, \mu; m_2) - s(T, \mu; m_1) = -\int_{m_1}^{m_2} \mathrm{d}m' \, \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T}(T, \mu; m')$$

- Integral limits: let $m_1 = m$ and $m_2 \to \infty$

General relation between s and $\langle\bar{\psi}\psi\rangle$

$$s(T, \mu; m) = s_{\mathrm{YM}}(T) + \int_{m}^{\infty} \mathrm{d}m' \, \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T}(T, \mu; m')$$

Thermodynamics from DSEs: Maxwell method

Maxwell relation

$$\frac{\partial^2 \Omega}{\partial m \partial T} = \frac{\partial^2 \Omega}{\partial T \partial m} \quad \Rightarrow \quad \frac{\partial s}{\partial m} = -\frac{\partial \langle \bar{\psi}\psi \rangle}{\partial T}$$

- Pressure at vanishing chemical potential:

$$p(T,0) = p(T_0,0) + \int_{T_0}^{T} \mathrm{d}T' \, s(T',0)$$

- Nonzero chemical potential:

$$p(T,\mu) = p(T_0,0) + \int_{T_0}^{T} \mathrm{d}T' \, s(T',0) + \int_0^{\mu} \mathrm{d}\mu' \, n(T,\mu')$$

- Applicable as soon as the quark condensate is available!

Quark condensate from the propagator

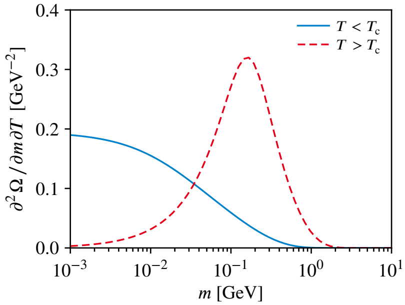
$$\langle \bar{\psi}\psi \rangle \sim \sum_{\omega_q} \int \frac{\mathrm{d}^3 \vec{q}}{(2\pi)^3} \, \mathrm{Tr}\big[S(\omega_q + \mathrm{i}\mu, \vec{q}) \big]$$

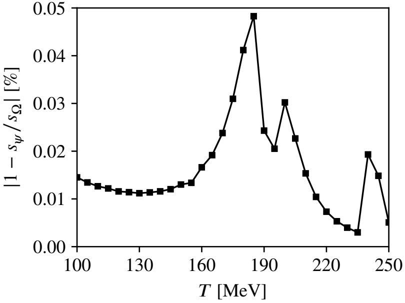
Detour: NJL model

- Mean-field NJL Lagrangian:

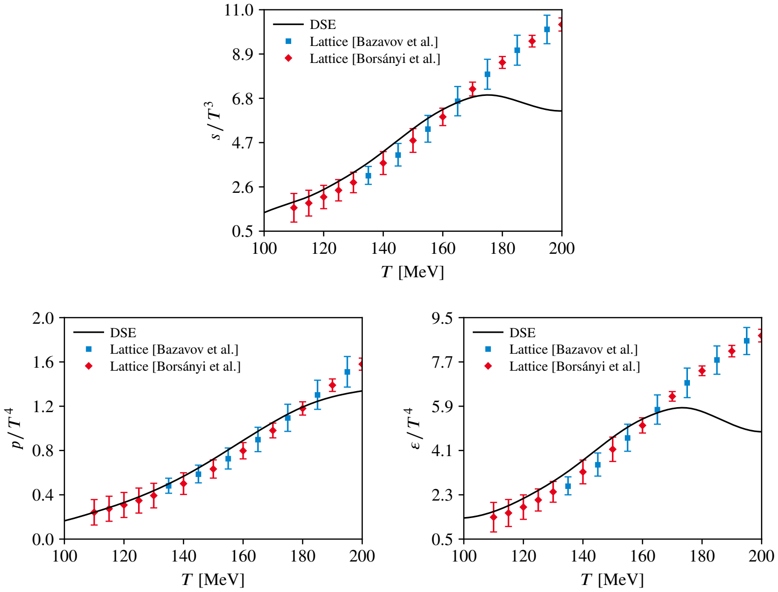
$$\mathcal{L}_{\mathrm{NJL}}^{(\mathrm{mf})} = \bar{\psi}(\mathrm{i}\not{\partial} - M)\psi - \frac{(M-m)^2}{4G}$$

- Quark condensate: $\langle\bar{\psi}\psi\rangle = (m - M)/2G$

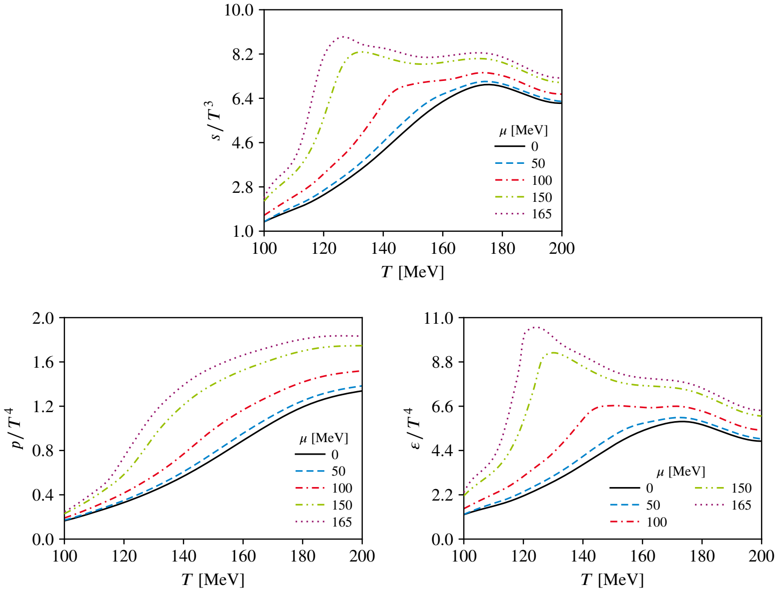




Thermodynamics from DSEs: Results at $\mu = 0$

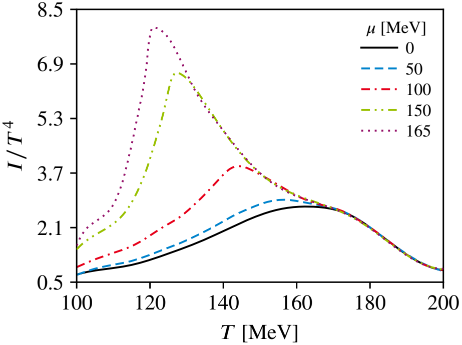


Thermodynamics from DSEs: Results at $\mu \neq 0$



Thermodynamics from DSEs: Results at $\mu \neq 0$

Interaction measure: $I(T,\mu) = \varepsilon(T,\mu) - 3p(T,\mu)$



Summary

EoS from DSEs:

- Proposed a truncation-independent method to compute thermodynamic quantities within the DSE approach
- Based on a general relation between $\langle\bar{\psi}\psi\rangle$ and s
- EoS in $(2+1)$-flavor QCD at $\mu = 0$ and $\mu \neq 0$:
  - Very good agreement with lattice results below and around T_c
  - Deviations at high temperatures related to the truncation of the quark-gluon vertex

Outlook / things to do:

- Systematic control over error budget
- Off-diagonal fluctuations
- Strangeness neutrality
- Need high-quality quark-gluon vertex for thermodynamics at large temperatures and/or densities

Backup slides

Ansatz for quark-gluon vertex

$$S_f^{-1}(p) = \mathrm{i}(\omega_p + \mathrm{i}\mu_f)\gamma_4 C_f(p) + \mathrm{i}\vec{\not p} A_f(p) + B_f(p)$$

Vertex ansatz:

$$\Gamma_\nu^f(q,p,k) = \widetilde{Z}_3\,\Gamma(k^2)\,\gamma_\nu \left(\delta_{4\nu}\,\frac{C_f(q)+C_f(p)}{2} + (1-\delta_{4\nu})\,\frac{A_f(q)+A_f(p)}{2} \right)$$

Phenomenological dressing function:

$$\Gamma(k^2) = \frac{d_1}{d_2 + k^2} + \frac{1}{1+\Lambda^2/k^2}\left(\frac{\alpha_\mathrm{s}\beta_0}{4\pi}\log\left(1+k^2/\Lambda^2\right)\right)^{2\delta}$$

- Abelian STI (leading term of Ball-Chiu vertex)
  Ball, Chiu, PRD 22, 2542 (1980)
- Perturbative running in the ultraviolet (quantitative)
- Ansatz for IR (qualitative)
  - d_1 fixed via T_c
  - d_2 fixed to match scale of quenched lattice gluon