Der Wissenschaftsfonds.

Vector and Axial-Vector Mesons in Nuclear Matter

Ralf-Arno Tripolt

Lunch Club Seminar, Justus-Liebig-Universität Gießen

June 9, 2021

We work for **tomorrow**

Spectral Functions and Transport Coefficients from the Functional Renormalization Group

TECHNISCHE UNIVERSITÄT DARMSTADT

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HGS-HIRe for FAIR — Helmholtz Graduate School for Hadron and Ion Research

H-QM Helmholtz Research School Quark Matter Studies

October 14, 2015 | Ralf-Arno Tripolt | Spectral Functions and Transport Coefficients from the FRG | 1

The "Resonances Via Padé" (RVP) Method

Ralf-Arno Tripolt, ECT*, Trento, Italy

Based on arXiv: 1610.03252
Ralf-Arno Tripolt, Idan Haritan, Jochen Wambach, Nimrod Moiseyev

Gießen, December 7th, 2016

ECT* EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

1 / 64

Spectral functions with the FRG

Ralf-Arno Tripolt

In collaboration with:
Chris Jung, Fabian Rennecke, Naoto Tanji,
Lorenz von Smekal, Jochen Wambach, Johannes Weyrich

Lunch Club Seminar, Justus-Liebig-Universität Giessen, November 21, 2018

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CRC-TR 211

1 / 54

Fermionic spectral functions with the Functional Renormalization Group

Ralf-Arno Tripolt
(Goethe University Frankfurt)

Lunch Club Seminar, Justus-Liebig-Universität Giessen, December 11, 2019

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DFG Deutsche Forschungsgemeinschaft

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arXiv: 2105.00861

Vector and Axial-Vector Mesons in Nuclear Matter

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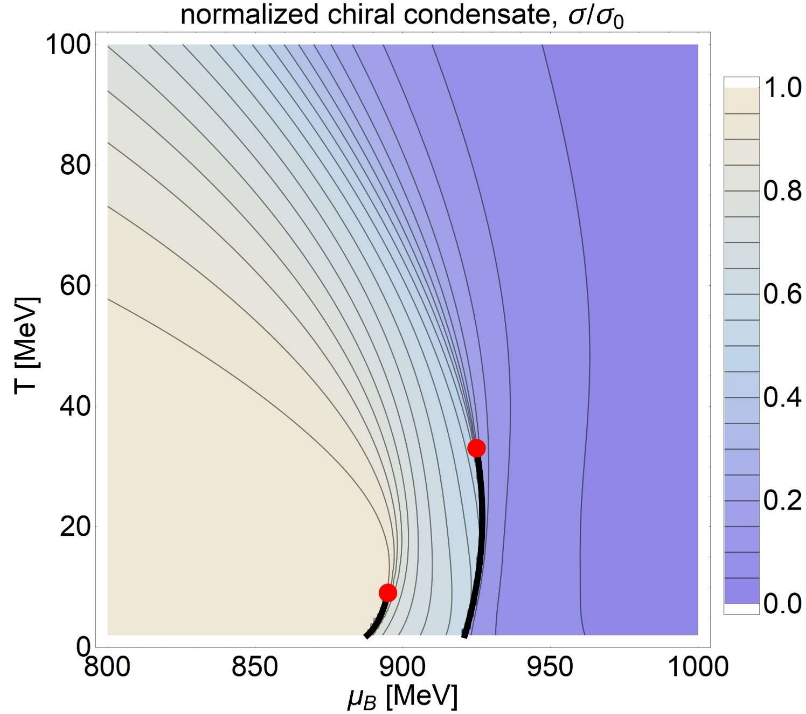
As a first step towards a realistic phenomenological description of vector and axial-vector mesons in nuclear matter, we calculate the spectral functions of the ρ and the a_1 meson in a chiral baryon-meson model as a low-energy effective realization of QCD, taking into account the effects of fluctuations from scalar mesons, nucleons, and vector mesons within the Functional Renormalization Group (FRG) approach. The phase diagram of the effective hadronic theory exhibits a nuclear liquid-gas phase transition as well as a chiral phase transition at a higher baryon-chemical potential. The in-medium ρ and a_1 spectral functions are calculated by using the previously introduced analytically-continued FRG (aFRG) method. Our results show strong modifications of the spectral functions in particular near the critical endpoints of both phase transitions which may well be of relevance for electromagnetic rates in heavy-ion collisions or neutrino emissivities in neutron-star merger events.

PACS numbers: 05.10.Cc, 12.38.Aw, 12.38.Lg, 11.30.Rd, 11.10.Wx
Keywords: vector mesons, nuclear matter, functional renormalization group, spectral function

Thermodynamics and equation of state of nuclear and neutron matter

Research aims and questions:

- develop a realistic **hadronic effective theory for nuclear and neutron matter** based on chiral symmetry
- what is the **phase structure of nuclear and neutron matter** at high densities and moderate temperatures and what is the **nature of the chiral phase transition?**
- what are the **thermodynamical properties** and the **equation of state (EOS)** of isospin symmetric and asymmetric strong-interaction matter?



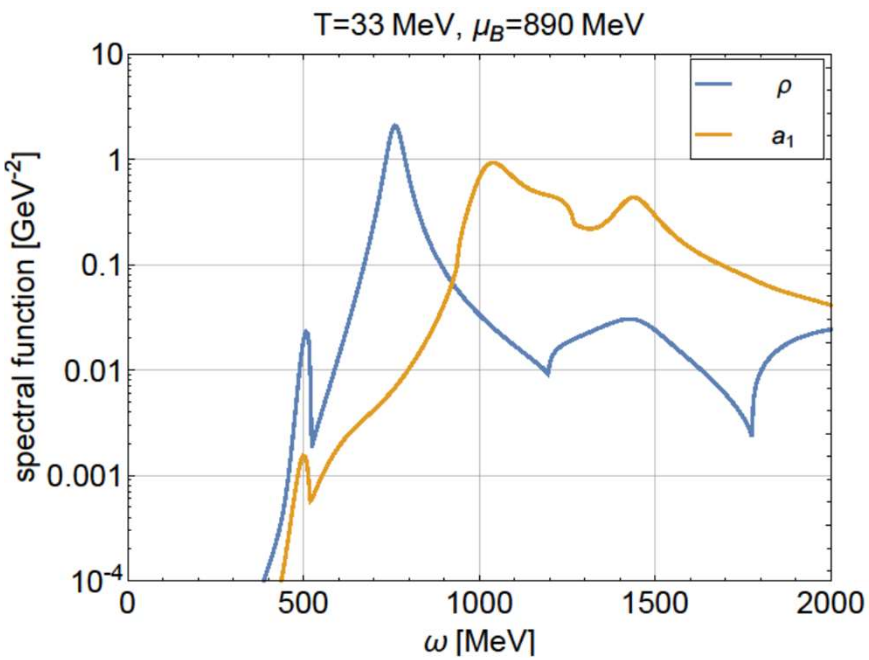
phase diagram of the parity-doublet model

[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

(Axial-)vector mesons, dileptons and neutrinos

Research aims and questions:

- compute **vector and axial-vector meson spectral functions** based on the effective theory **for nuclear and neutron matter**
- how do their **spectral properties** change at finite temperature and density, in particular near phase transitions, and how are the changes **related to chiral symmetry**?
- what are the expected **dilepton rates and spectra** in heavy-ion collision experiments?
- is it possible to make a robust prediction for **experimental signatures of phase transitions** and effects of chiral symmetry in dilepton spectra?
- what is the electroweak response of neutron-rich matter in terms of **thermal neutrino rates**?



ρ and a_1 spectral function in nuclear matter

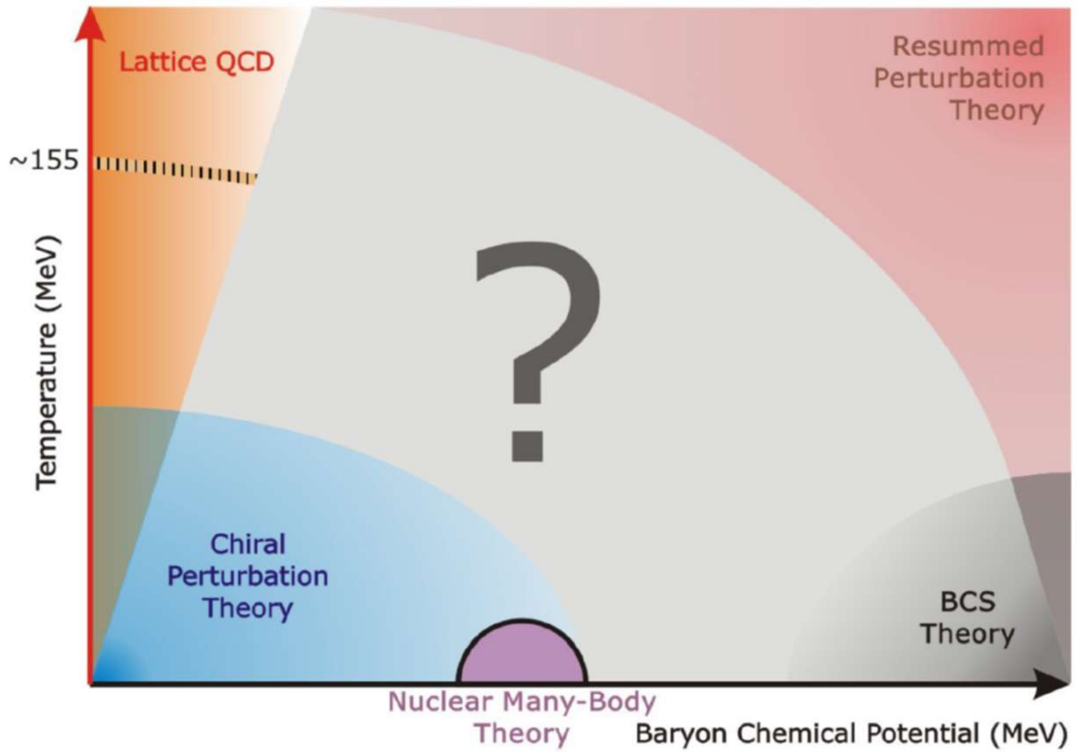
[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Functional Renormalization Group (FRG)

The FRG is a **non-perturbative** framework that is able to describe the effects of **quantum fluctuations** and can be applied at **finite temperature and density**.

Advantages:

- preserves **symmetries** and their breaking patterns
- properly describes **phase transitions** since it includes both thermal and quantum fluctuations
- allows for a well-defined and straightforward **analytic continuation** procedure
- thermodynamic consistency: describes **thermodynamical and spectral properties** on the same footing



[CRC-TR 211]

Functional Renormalization Group (FRG)

Euclidean partition function for a scalar field:

$$Z[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] + \int d^4x \, J(x)\varphi(x)\right)$$

Wilson‘s coarse-graining idea: split field into low- and high-frequency modes:

$$\varphi(x) = \varphi_{q\leq k}(x) + \varphi_{q>k}(x)$$

define scale-dependent partition function that only includes fluctuations with $q > k$:

$$Z[J] = \int \mathcal{D}\varphi_{q\leq k} \underbrace{\int \mathcal{D}\varphi_{q>k} \, \exp\left(-S[\varphi] + \int d^4x J(x)\varphi(x)\right)}_{Z_k[J]}$$

Functional Renormalization Group (FRG)

Scale-dependent partition function can also be defined as:

$$Z_k[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] - \Delta S_k[\varphi] + \int d^4x J(x)\varphi(x) \right)$$

with a regulator term that suppresses IR modes:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \, \varphi(-q) R_k(q) \varphi(q)$$

Switch to scale-dependent effective action, $\phi(x) = \langle \varphi(x) \rangle$:

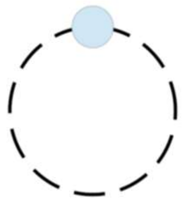
$$\Gamma_k[\phi] = \sup_J \left(\int d^4x \, J(x)\phi(x) - \log Z_k[J] \right) - \Delta S_k[\phi]$$

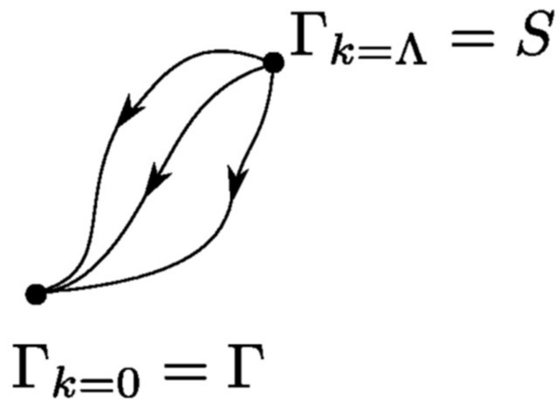
Functional Renormalization Group (FRG)

Flow equation for the effective average action:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

[C. Wetterich, Phys. Lett. B 301 (1993) 90]

$$\partial_k \Gamma_k = \frac{1}{2}$$ 



[wikipedia.org/wiki/Functional_renormalization_group]

- Γ_k **interpolates** between the **bare action** ***S*** **in the UV** and the **effective action** Γ **in the IR**
- **regulator** R_k acts as a **mass term** and suppresses fluctuations with momenta *p*<*k*

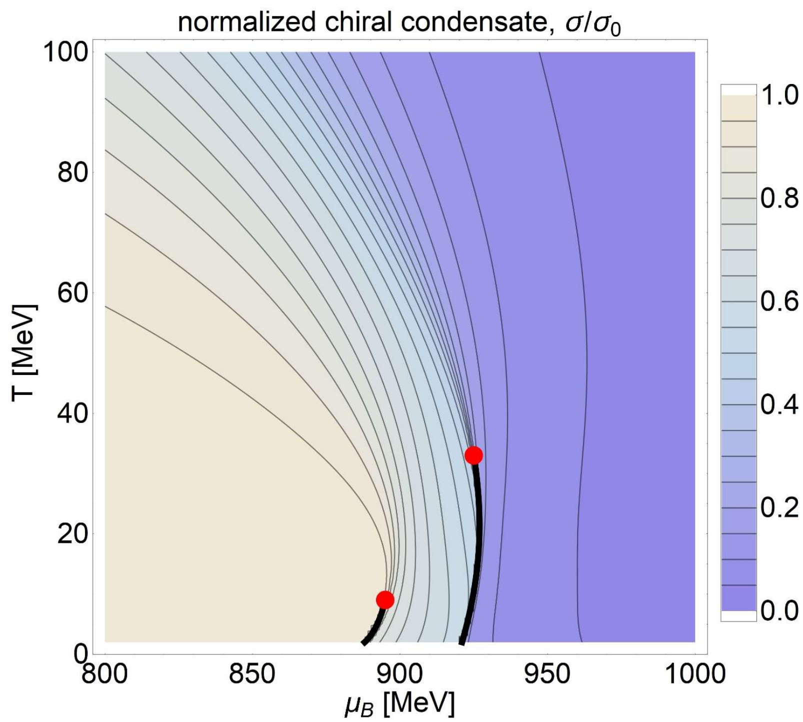
Effective hadronic theory for nuclear matter

The **parity-doublet** (or mirror baryon) **model** describes **nucleons** along with their **parity partners** and can account for a finite nucleon mass in a chirally-invariant way:

$$\Gamma_k = \int d^4x \left\{ \bar{N}_1 \left(\not{\partial} - \mu_B \gamma_0 + h_1 \left(\sigma + \mathrm{i}\vec{\tau}\vec{\pi}\gamma_5 \right) \right) N_1 + \right.$$
$$\bar{N}_2 \left(\not{\partial} - \mu_B \gamma_0 + h_2 \left(\sigma - \mathrm{i}\vec{\tau}\vec{\pi}\gamma_5 \right) \right) N_2 +$$
$$\left. m_{0,B} \left(\bar{N}_1 \gamma_5 N_2 - \bar{N}_2 \gamma_5 N_1 \right) + \frac{1}{2} (\partial_\mu \phi)^2 + U_k(\phi^2) \right\}$$

- $N_1 = (p, n)$, $N_2 = N(1535)$
- describes the **nuclear liquid-gas transition together with a chiral phase transition!**

phase diagram:



[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

[C. E. Detar, T. Kunihiro, Phys. Rev. D 39 (1989) 2805]
[D. Jido, M. Oka, A. Hosaka, Prog. Theor. Phys. 106 (2001) 873]
[J. Weyrich, N. Strodthoff, L. v. Smekal, Phys. Rev. C 92 (2015) 015214]

Parity-doublet model: Mirror assignment

Possible **representations for baryons** under chiral $SU(2)_L$ x $SU(2)_R$ transformations:

$$\left((\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})\right)\otimes\left((\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})\right)\otimes\left((\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})\right)$$
$$= 5\left(\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})\right)\oplus 3\left((1,\tfrac{1}{2})\oplus(\tfrac{1}{2},1)\right)\oplus\left((\tfrac{3}{2},0)\oplus(0,\tfrac{3}{2})\right)$$

Two possibilities for a two-baryon system with species ψ_1 and ψ_2 in $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation:

$$(\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})$$
$$\psi_{1,l}\ \psi_{1,r}\quad \psi_{2,l}\ \psi_{2,r}$$

$$\psi_{1,l}\to\Omega_L\psi_{1,l},\ \psi_{1,r}\to\Omega_R\psi_{1,r}$$
$$\psi_{2,l}\to\Omega_L\psi_{2,l},\ \psi_{2,r}\to\Omega_R\psi_{2,r}$$

naive assignment

$$\psi_{l/r}\equiv\frac{1\mp\gamma_5}{2}\psi$$

$$\Omega_{R,L}\in SU(2)_{R,L}$$

$$(\tfrac{1}{2},0)\oplus(0,\tfrac{1}{2})$$
$$\psi_{1,l}\ \psi_{1,r}\quad \psi_{2,r}\ \psi_{2,l}$$

$$\psi_{1,l}\to\Omega_L\psi_{1,l},\ \psi_{1,r}\to\Omega_R\psi_{1,r}$$
$$\psi_{2,r}\to\Omega_L\psi_{2,r},\ \psi_{2,l}\to\Omega_R\psi_{2,l}$$

mirror assignment

[C. E. Detar, T. Kunihiro, Phys. Rev. D 39 (1989) 2805]
[D. Jido, M. Oka, A. Hosaka, Prog. Theor. Phys. 106 (2001) 873]
[J. Weyrich, N. Strodthoff, L. v. Smekal, Phys. Rev. C 92 (2015) 015214]

Parity-doublet model: chirally invariant mass

Kinetic term is **invariant** under chiral transformations for both assignments:

$$\mathcal{L}_{\mathrm{kin}} = \sum_{i=1,2} \bar{\psi}_i \mathrm{i} \partial\!\!\!/ \psi_i$$

Standard **Dirac-mass term breaks chiral symmetry** for both assignments:

$$\mathcal{L}_m = -m \bar{\psi}_i \psi_i$$

Mass term for two Fermion species **in the mirror assignment is invariant** under chiral symmetry:

$$\begin{aligned}\mathcal{L}_{m,\mathrm{mirror}} &= m_0 (\bar{\psi}_2 \psi_1 + \bar{\psi}_1 \psi_2) \\ &= m_0 (\psi_{2r}^\dagger \psi_{1l} + \psi_{1l}^\dagger \psi_{2r} + \psi_{1r}^\dagger \psi_{2l} + \psi_{2l}^\dagger \psi_{1r})\end{aligned}$$

[C. E. Detar, T. Kunihiro, Phys. Rev. D 39 (1989) 2805]
[D. Jido, M. Oka, A. Hosaka, Prog. Theor. Phys. 106 (2001) 873]
[J. Weyrich, N. Strodthoff, L. v. Smekal, Phys. Rev. C 92 (2015) 015214]

Parity-doublet model: model construction and masses

We are using $N_1 \equiv \psi_1$ and $N_2 \equiv \gamma_5 \psi_2$ so that N_2 has the **opposite parity** of N_1 and the eigenvalues of the mass matrix in the chiral limit are both m_0, rather than $\pm m_0$.

The original baryon N_1 and the **mirror baryon** N_2 transform as $N_1 \rightarrow e^{i\theta^a \gamma^5 T^a} N_1$ and $N_2 \rightarrow e^{-i\theta^a \gamma^5 T^a} N_2$

under axial transformations → requires **opposite signs in their Yukawa couplings** to the pion!

The fermionic part of the Lagrangian is then given by $\mathcal{L} = \bar{\Psi} S_0^{-1} \Psi$ with $\Psi = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$ and

$$S_0^{-1} = \begin{pmatrix} -\mathrm{i}\not{p} + h_1(\sigma + \mathrm{i}\gamma_5 \vec{\pi}\vec{\tau}) - \mu_B \gamma^0 & m_0 \gamma_5 \\ -m_0 \gamma_5 & -\mathrm{i}\not{p} + h_2(\sigma - \mathrm{i}\gamma_5 \vec{\pi}\vec{\tau}) - \mu_B \gamma^0 \end{pmatrix}$$

which gives **masses**

$$m_{N_1}^2 = \frac{1}{2}\left(+ (h_1 - h_2)\sigma_0 + \sqrt{4m_0^2 + \sigma_0^2(h_1 + h_2)^2} \right)$$

$$m_{N_2}^2 = \frac{1}{2}\left(- (h_1 - h_2)\sigma_0 + \sqrt{4m_0^2 + \sigma_0^2(h_1 + h_2)^2} \right)$$

[J. Weyrich, N. Strodthoff, L. v. Smekal, Phys. Rev. C 92 (2015) 015214] [RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Parity-doublet model: Flow equation for the effective potential

Flow equation for the effective potential is solved numerically using the grid method:

$$\partial_k U_k = \frac{k^4}{12\pi^2}\Big\{\frac{1+2n_B(E_{\sigma,k})}{E_{\sigma,k}} + \frac{3(1+2n_B(E_{\pi,k}))}{E_{\pi,k}} + \frac{4N_f}{E_{N_1,k}E_{N_2,k}}\Big[-(E_{N_1,k}+E_{N_2,k}) + E_{N_2,k}n_F(E_{N_1,k}-\mu_B)$$
$$+ E_{N_1,k}n_F(E_{N_2,k}-\mu_B) + E_{N_2,k}n_F(E_{N_1,k}+\mu_B) + E_{N_1,k}n_F(E_{N_2,k}+\mu_B)\Big]\Big\}.$$

Effective energies and masses:

Flat regulator: $r(y) = \left(\frac{1}{y} - 1\right)\theta(1-y)$

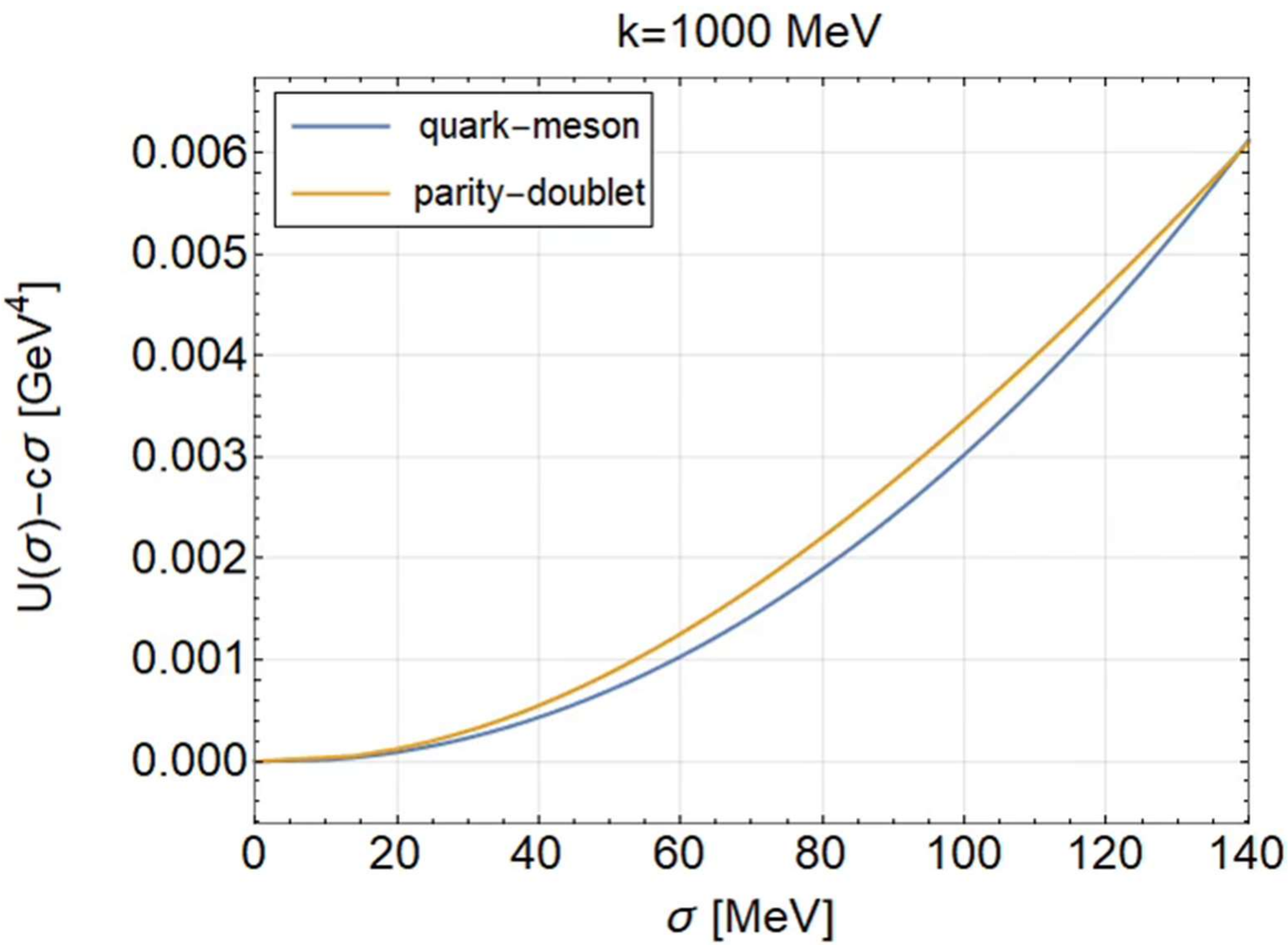
$$E_{\alpha,k} \equiv \sqrt{k^2 + m_{\alpha,k}^2}, \qquad \alpha \in \{\pi, \sigma, N_1, N_2\}, \qquad m_{\pi,k}^2 = 2U_k', \qquad m_{\sigma,k}^2 = 2U_k' + 4U_k''\phi_0^2 \qquad \phi^2 \equiv \sigma^2 + \vec{\pi}^2$$

Parameters at UV scale of Λ = 1 GeV: $U_\Lambda(\phi^2) = b_1\phi^2 + b_2(\phi^2)^2 + b_3(\phi^2)^3$

b_1 $[\Lambda^2]$	b_2	b_3 $[\Lambda^{-2}]$	c $[\Lambda^3]$	$m_{0,N}$ [MeV]	$h_{s,1}$ $= h_{v,1}$	$h_{s,2}$ $= h_{v,2}$	$f_\pi \equiv \sigma_0$ [MeV]	m_π [MeV]	m_σ [MeV]	m_{N_1} [MeV]	m_{N_2} [MeV]
0.395189	-4.66855	52.3117	$1.74303\cdot10^{-3}$	800	6.94073	13.3493	92.8	137	474	938	1533

[J. Weyrich, N. Strodthoff, L. v. Smekal, Phys. Rev. C 92 (2015) 015214] [RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

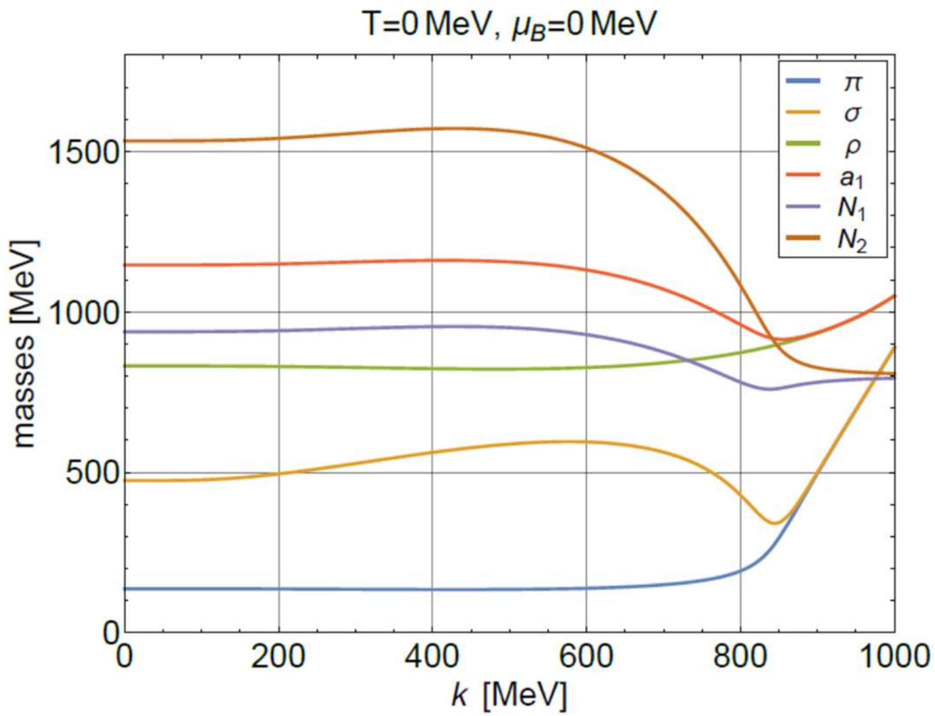
Flow of the effective potential



Flow of (Euclidean) masses

Dynamical chiral symmetry breaking is due to baryonic fluctutations:

- chiral partners have same mass in UV
- masses split up near chiral symmetry breaking scale of $k_\chi \approx 850$ MeV
- mesonic fluctuations dominate below $k \approx 600$ MeV and tend to restore chiral symmetry

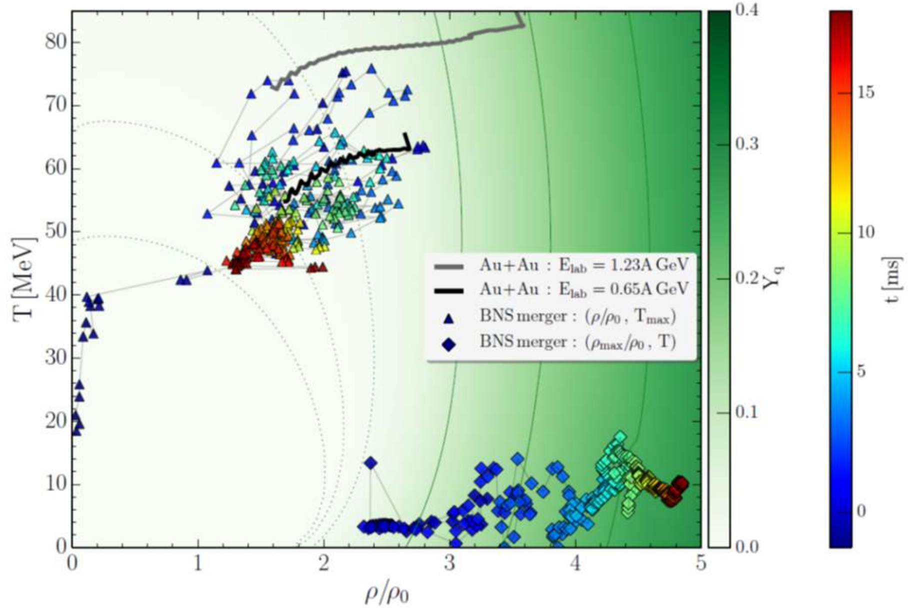


[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Connection to heavy-ion collisions and neutron star mergers

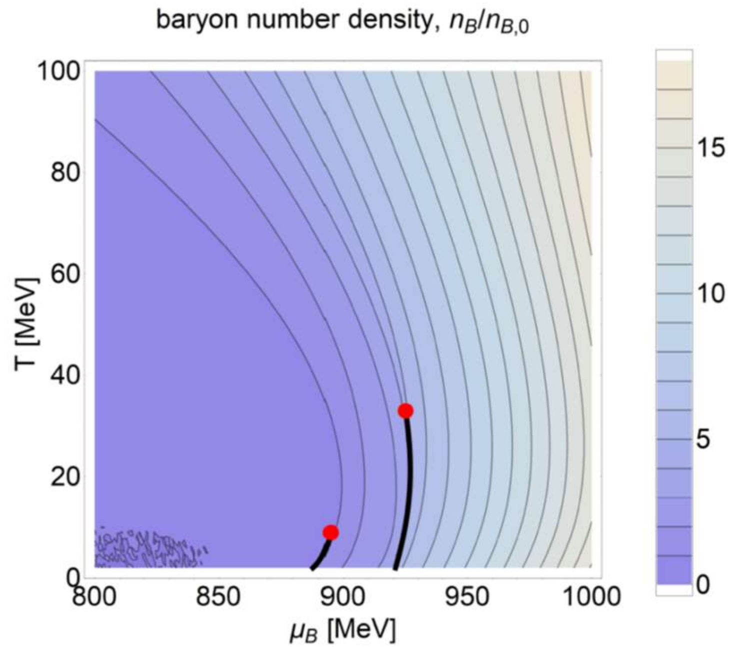
We aim at describing nuclear matter in a regime relevant for **heavy-ion collisions (HICs)** and **binary neutron star (BNS) mergers**:

Chiral phase transition occurs at density of 3-4 ρ_0!



Evolution of maximum temperature (triangles) and maximum density (diamonds) regions in BNS merger simulation, and HIC trajectories (lines)

[M. Hanauske et al., Particles 2 (2019) no. 1]

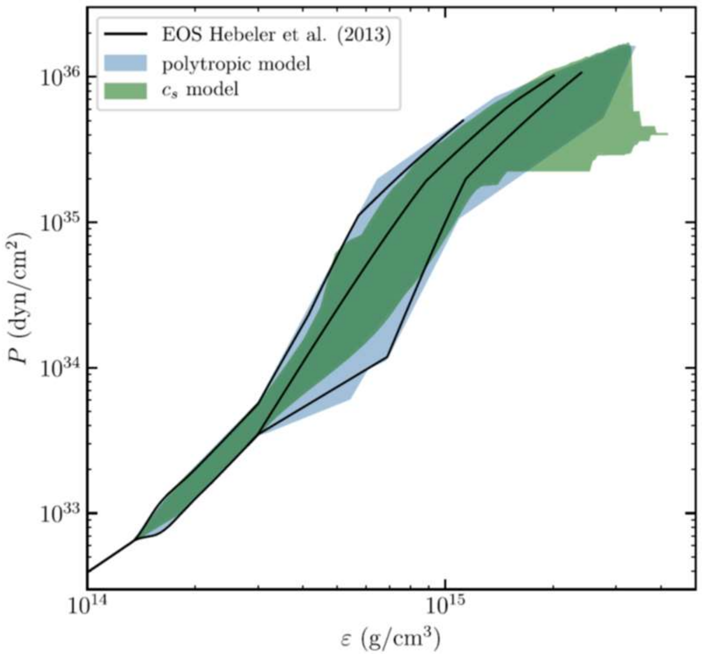


$$n(T,\mu) = \frac{\partial p(T,\mu)}{\partial \mu}$$

[RAT, L. v. Smekal, J. Wambach, preliminary]

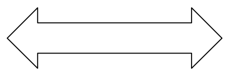
Parity-doublet model: Equation of state for nuclear matter

Knowledge of the **EOS of neutron-rich matter** is key to understand and model **r-process** sites as **supernovae** and **neutron stars** and their **mergers,** and the **gravitational wave signal** produced:

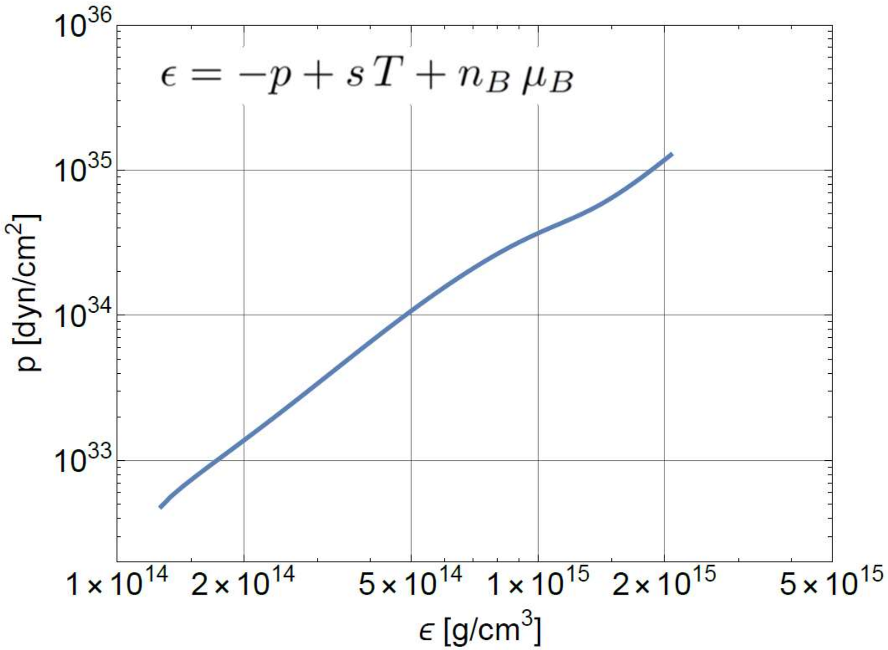


Allowed ranges for the pressure and energy density when using chiral EFT constraints at low densities

[S. K. Greif, G. Raaijmakers, K. Hebeler, A. Schwenk, A. L. Watts, Mon. Not. R. Astron. Soc. 485 (2019) 5363-5376]



EOS from the parity-doublet model at μ_B = 897 MeV:



[preliminary]

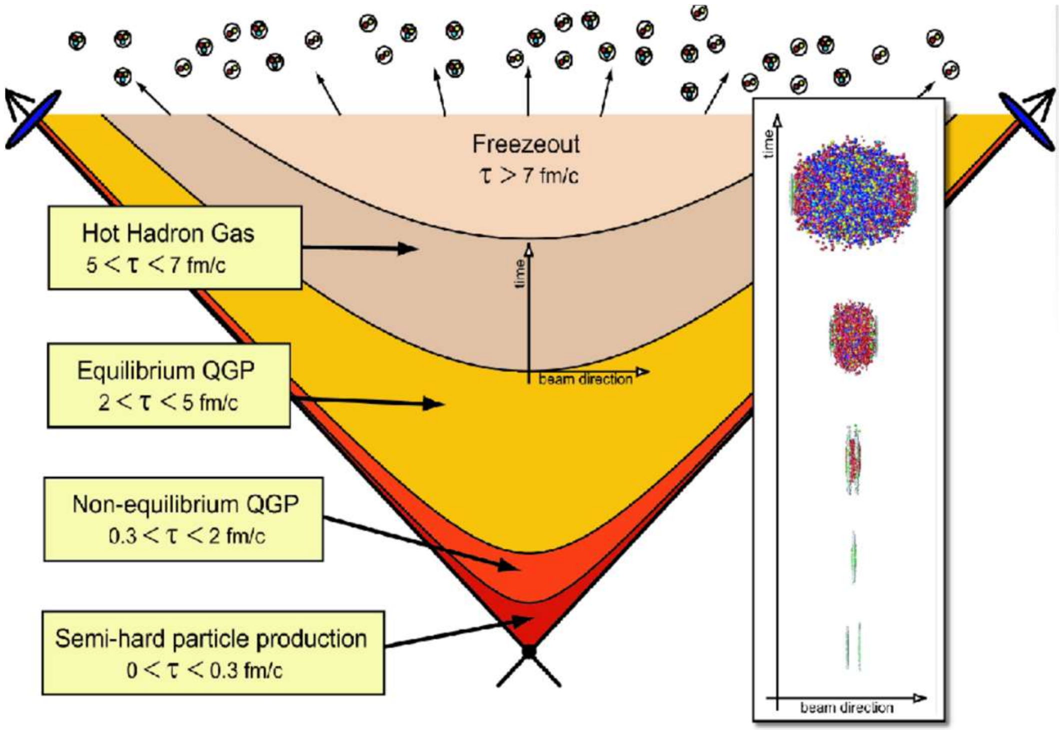
Heavy-ion collisions and electromagnetic (EM) probes

Compared to the fireball, EM probes, i.e. **photons and dileptons**, have a long mean free path and can therefore **leave the interaction zone undisturbed!**

They are therefore **ideal probes** and can be used as:

- thermometer
- chronometer
- spectrometer
- multimeter
- ...

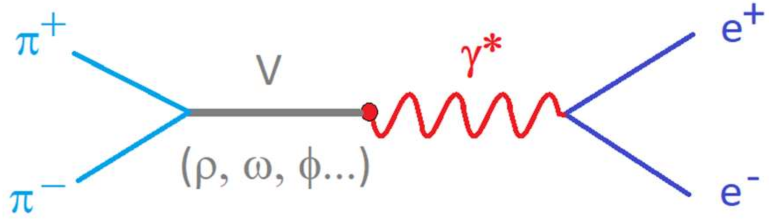
Sketch of the space-time evolution of a heavy-ion collision:



[M. Strickland, Acta Phys. Polon. B 45 (2014) 2355-2394]

Vector mesons and dileptons in heavy-ion collisions

Vector mesons have the same quantum numbers as photons and **can decay directly into dileptons**:

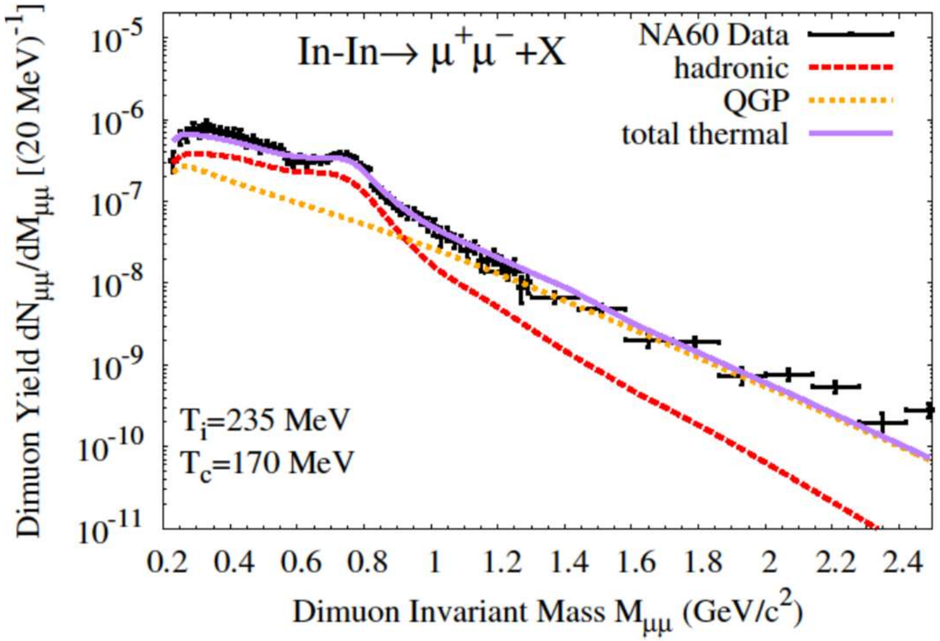


Dilepton rate can be expressed in terms of the **EM spectral function**:

$$\frac{dN_{ll}}{d^4x d^4q} \sim \mathrm{Im}\Pi^{\mu\nu}_{\mathrm{em}}(M, q; \mu, T)$$

In the low-energy regime ($M \leq 1$ GeV) the EM spectral function can be expressed in terms of the **spectral functions of the light vector mesons** (Vector Meson Dominance, VMD):

$$\mathrm{Im}\Pi^{\mu\nu}_{\mathrm{em}} \sim \mathrm{Im}D^{\mu\nu}_{\rho} + \frac{1}{9}\mathrm{Im}D^{\mu\nu}_{\omega} + \frac{2}{9}\mathrm{Im}D^{\mu\nu}_{\phi}$$



[R. Rapp, H. van Hees, Phys. Lett. B 753 (2016) 586-590]

Parity-doublet model with (axial-)vector mesons

The $\boldsymbol{\rho}$ **vector meson** and the $\boldsymbol{a_1}$ **axial-vector meson** are introduced to the **parity-doublet model** using a **novel formulation in terms of field strengths** $\rho_{\mu\nu}$:

$$\Gamma_k = \int d^4x \Big\{ \bar{N}_1 \left(\partial\!\!\!/ - \mu_B\gamma_0 + h_{s,1}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,1}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu + \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu}) \right) N_1$$
$$+ \bar{N}_2 \left(\partial\!\!\!/ - \mu_B\gamma_0 + h_{s,2}(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,2}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu - \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu} \right) N_2 + m_{0,N}\left(\bar{N}_1\gamma^5 N_2 - \bar{N}_2\gamma^5 N_1 \right)$$
$$+ U_k(\phi^2) - c\sigma + \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{4}\operatorname{tr}\partial_\mu\rho_{\mu\nu}\partial_\sigma\rho_{\sigma\nu} + \frac{m_v^2}{8}\operatorname{tr}\rho_{\mu\nu}\rho_{\mu\nu} \Big\} + \Delta\Gamma_{\pi a_1}.$$

The **interactions** of the (axial-)vector fields **with** the **(pseudo-)scalar pion and sigma** fields are determined from minimal coupling:

$$D_\mu = \partial_\mu + igV_\mu$$

[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]
[C. Jung, L. v. Smekal, Phys. Rev. D 100 (2019) 116009]
[J. Gasser, H. Leutwyler, Annals. Phys. 158 (1984) 142]

Fluctuating vector mesons with the FRG

Describing massive spin-1 (axial-)vector mesons by fundamental fields in an effective theory is known to be problematic:

- in the **Proca** formalism the transversality of the corresponding Green functions is maintained only on-shell which, e.g., leads to a **pathological UV behavior** $\Gamma^{(2)}_{V,k} = (p^2 + m^2_{V,k})\,\Pi^T_{\mu\nu}(p)$
- in the **Stueckelberg** formalism **spurious massless single-particle contributions** to the vector Green functions arise when restoring transversality $\Gamma^{(2),\mathrm{E}}_{\mu\nu,k}(p) = (p^2 + m^2_{V,k})\,\Pi^T_{\mu\nu}(p) + (\lambda_k\,p^2 + m^2_{V,k})\,\Pi^L_{\mu\nu}(p)$

⇨ we introduce right and left-handed **vector mesons in terms of (anti-)selfdual field strengths**

$$\tilde{\rho}^{\pm}_{\mu\nu} = \pm\rho^{\pm}_{\mu\nu} \qquad \rho_{\mu\nu} = \rho^{+}_{\mu\nu} + \rho^{-}_{\mu\nu} = \vec{\rho}^{\,+}_{\mu\nu}\cdot\vec{T}_R + \vec{\rho}^{\,-}_{\mu\nu}\cdot\vec{T}_L$$

which transform according to the (1,0) and (0,1) representations of the Euclidean O(4) (T_R and T_L are so(4) Lie algebra generators in the adjoint representation)

The iso-triplet **vector and axial-vector fields** are then obtained in terms of **derivatives**:

$$\vec{\rho}_\mu = \frac{1}{2m_v}\,\mathrm{tr}\big(\partial_\sigma\rho_{\sigma\mu}\vec{T}_V\big) \qquad \vec{a}_{1\mu} = \frac{1}{2m_v}\,\mathrm{tr}\big(\partial_\sigma\rho_{\sigma\mu}\vec{T}_A\big)$$

where $\vec{T}_V = \vec{T}_R + \vec{T}_L$ and $\vec{T}_A = \vec{T}_R - \vec{T}_L$

[C. Jung, L. v. Smekal, Phys. Rev. D 100 (2019) 116009]
[J. Gasser, H. Leutwyler, Annals. Phys. 158 (1984) 142]
[N. Nakanishi, I. Ojima, World Scientific Lecture Notes in Physics 27, 1990]
[C. Itzykson, J.-B. Zuber, Quantum Field Theory, Dover, 1980]

Fluctuating vector mesons with the FRG

The **vector-meson part of the effective action** is then given by:

$$\mathcal{L}_0^\rho = -\frac{1}{4}\,\mathrm{tr}\,(\partial_\mu \rho_{\mu\nu})\partial_\sigma \rho_{\sigma\nu} + \frac{m_v^2}{8}\,\mathrm{tr}\,\rho_{\mu\nu}\rho_{\mu\nu}$$

The corresponding tree-level **two-point functions** for free (axial-)vector mesons is then:

$$\Gamma^{(2)}_{\mu\nu}(p) = -\frac{m_v^2}{p^4}(p^2 + m_v^2)\left(p^2\delta_{\mu\nu} - p_\mu p_\nu\right)$$

(Axial-)vector fluctutations are included **within the FRG** by temporarily adding longitudinal terms in order to be able to invert the correlation functions:

$$D_{\mu\nu,k}(p) \equiv \left(\Gamma_k^{(2)}(p) + R_k(p)\right)^{-1}_{\mu\nu} = \frac{-p^2}{m_{0,k}^2}\frac{1}{(p^2(1+r(y)) + m_{v,k}^2)}\Pi^T_{\mu\nu}(p) + \frac{-p^2}{m_{0,k}^2}\frac{1}{(p^2(1+r(y)) + \xi\frac{\Lambda^2}{k^2}m_{v,k}^2)}\Pi^L_{\mu\nu}(p)$$

with a regulator shape function $r(y)$, $y = p^2/k^2$ and the 4D projection operators

$$\Pi^T_{\mu\nu}(p) = \delta_{\mu\nu} - p_\mu p_\nu/p^2,$$
$$\Pi^L_{\mu\nu}(p) = \delta_{\mu\nu} - \Pi^T_{\mu\nu}(p) = p_\mu p_\nu/p^2$$

[J. Gasser, H. Leutwyler, Annals. Phys. 158 (1984) 142]
[C. Jung, L. v. Smekal, Phys. Rev. D 100 (2019) 116009]

Fluctuating vector mesons with the FRG

In the limit $k \to 0$ the **propagators become purely transverse** and fulfill the Ward identity:

$$p_\mu D^{\mu\nu}_{k\to 0} = 0$$

The **scale-dependent mass parameter** is defined as

$$m^2_{0,k} \equiv Z_k^{-1} m^2_{v,k}$$

with the running vector meson mass $m_{v,k}$ and the wave-function renormalization factor Z_k. The flow of the mass parameter $m_{0,k}$ is obtained by projecting the **transverse part of the vector-meson two-point function**:

$$\Gamma^{(2),T}_{\rho,k}(p) = \frac{-m^2_{0,k}}{p^2} \left(p^2 + m^2_{v,k}\right)$$

where the **flow equation for $m_{0,k}$** is obtained as

$$\partial_k m^2_{0,k} = -\frac{1}{3(N_f^2-1)} \lim_{p\to 0} \frac{\partial}{\partial \vec{p}^2} \left(p^2\, \Pi^T_{\mu\nu}(p)\, \mathrm{tr}\left(\partial_k \Gamma^{(2)}_{\rho,k}(p)\right)_{\mu\nu} \right)$$

[C. Jung, L. v. Smekal, Phys. Rev. D 100 (2019) 116009]

Fluctuating vector mesons with the FRG

The **flow of $m_{v,k}$** is then obtained from

$$\partial_k \left(m_{v,k}^2\, m_{0,k}^2\right) = -\frac{1}{3(N_f^2-1)} \lim_{p\to 0} p^2\, \Pi^T_{\mu\nu}(p)\, \mathrm{tr}\left(\partial_k \Gamma^{(2)}_{\rho,k}(p)\right)_{\mu\nu} = 0$$

which gives

$$\partial_k m_{v,k}^2 = -\frac{m_{v,k}^2}{m_{0,k}^2}\,\partial_k m_{0,k}^2$$

The **(axial-)vector meson masses** are then given by

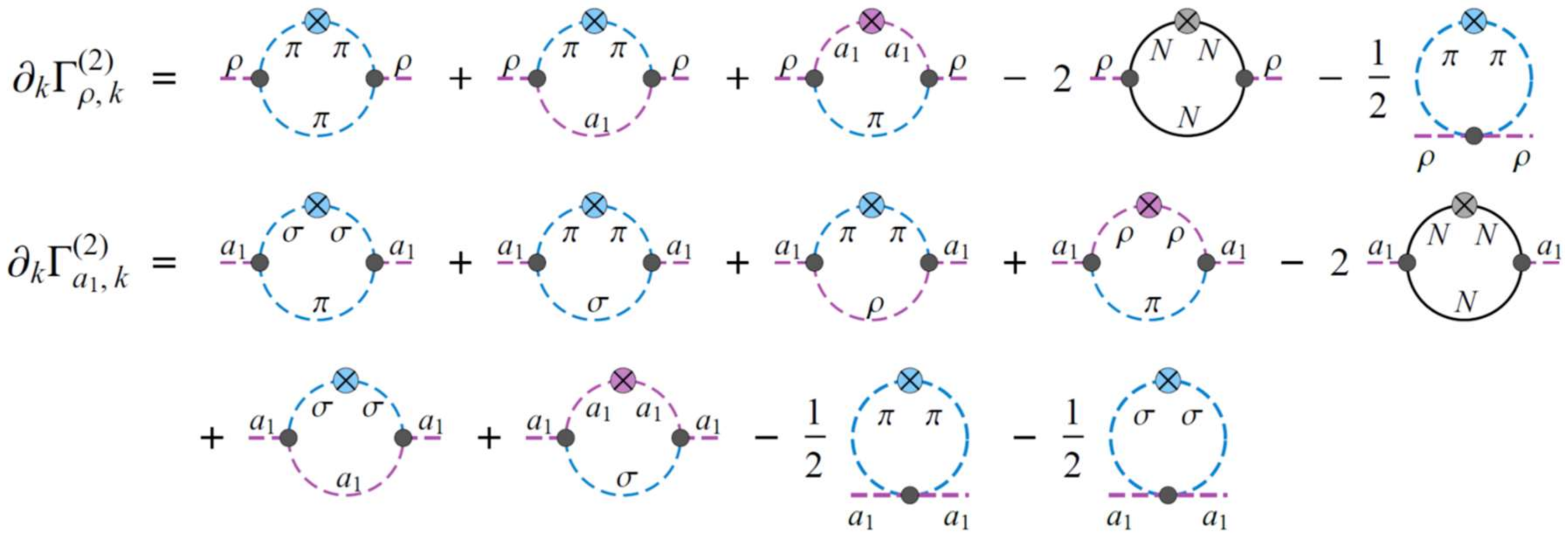
$$m_{\rho,k}^2 = m_{k,v}^2,$$
$$m_{a_1,k}^2 = m_{k,v}^2 + g^2\phi_0^2$$

The scale-dependent mass $m_{v,k}$ is used as input in the **flow equations for the two-point functions**:

$$\partial_k \Gamma^{(2)}_k(p) = \mathrm{STr}\Big\{(\partial_k R_k) D_k(q) \Gamma^{(3)}_k D_k(q+p) \Gamma^{(3)}_k D_k(q)\Big\} - \frac{1}{2}\mathrm{STr}\Big\{(\partial_k R_k) D_k(q) \Gamma^{(4)}_k D_k(q)\Big\}$$

[C. Jung, L. v. Smekal, Phys. Rev. D 100 (2019) 116009]

Flow equations for ρ and a_1 2-point functions



- loops with **dynamical (axial-)vector mesons** are **included now**!
- **vertices** are extracted **from** the ansatz for the **effective average action**
- **analytic continuation** is done using the **aFRG method**

[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

The aFRG method

The **analytic continuation** from imaginary to real-time energies is done via the **aFRG (analytically continued FRG) method** on the level of the flow equations in two steps:

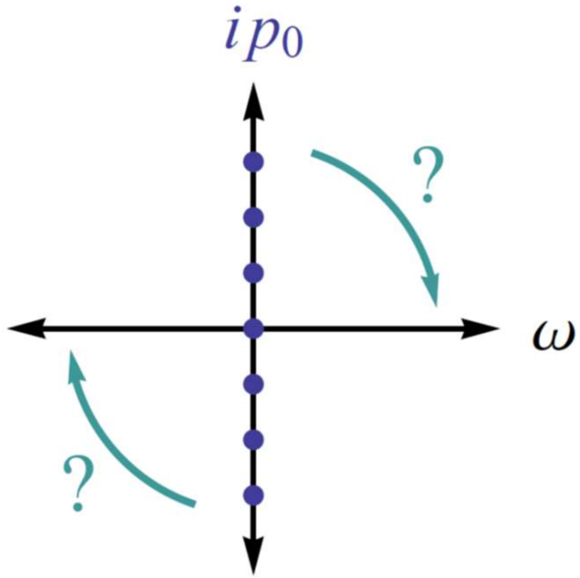
1. Periodicity of the occupation number factors w.r.t. the **imaginary Euclidean energy $\boldsymbol{ip_0 = i2n\pi T}$** is used:

$$n_{B,F}(E + ip_0) \rightarrow n_{B,F}(E)$$

2. Remaining ip_0 are substituted by **continuous real frequency $\boldsymbol{\omega}$**:

$$\Gamma^{(2),R}(\omega,\vec{p}) = -\lim_{\epsilon\to 0}\Gamma^{(2),E}(ip_0 \to -\omega - i\epsilon, \vec{p})$$

$$\rho(\omega,\vec{p}) = -\frac{1}{\pi}\mathrm{Im}\frac{1}{\Gamma^{(2),R}(\omega,\vec{p})} = \frac{1}{\pi}\frac{\mathrm{Im}\,\Gamma^{(2),R}(\omega,\vec{p})}{\left(\mathrm{Re}\,\Gamma^{(2),R}(\omega,\vec{p})\right)^2 + \left(\mathrm{Im}\,\Gamma^{(2),R}(\omega,\vec{p})\right)^2}$$



[K. Kamikado, N. Strodthoff, L. v. Smekal, J. Wambach, Eur. Phys. J. C 74 (2014) 2806]
[RAT, N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. D 89 (2014) 034010]
[N. Landsman and C. v. Weert, Physics Reports 145, 3&4 (1987) 141]
[G. Baym, N. D. Mermin, J. Math. Phys. 2 (1961) 232]

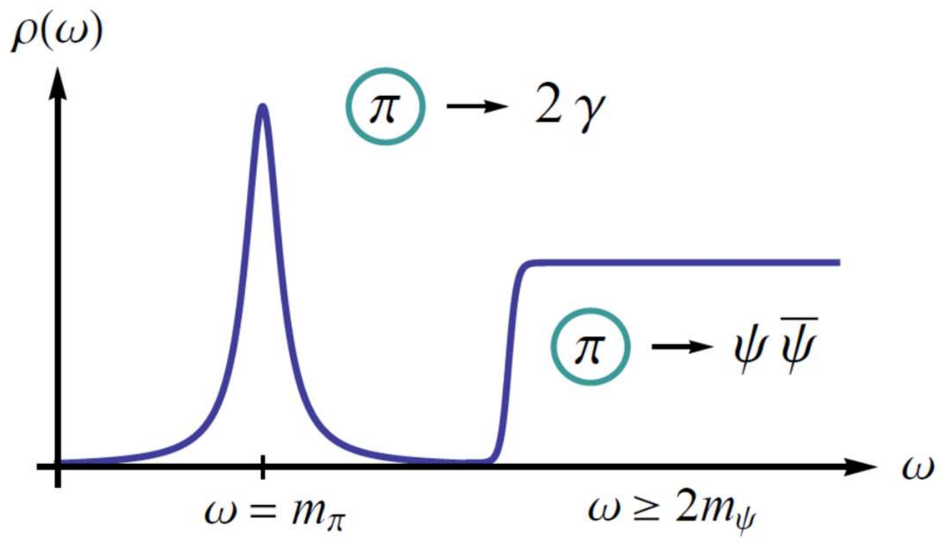
Introduction: spectral functions

The **spectral function** is defined as the imaginary part of the real-time propagator:

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \mathrm{Im}\, D^R(\omega, \vec{p})$$

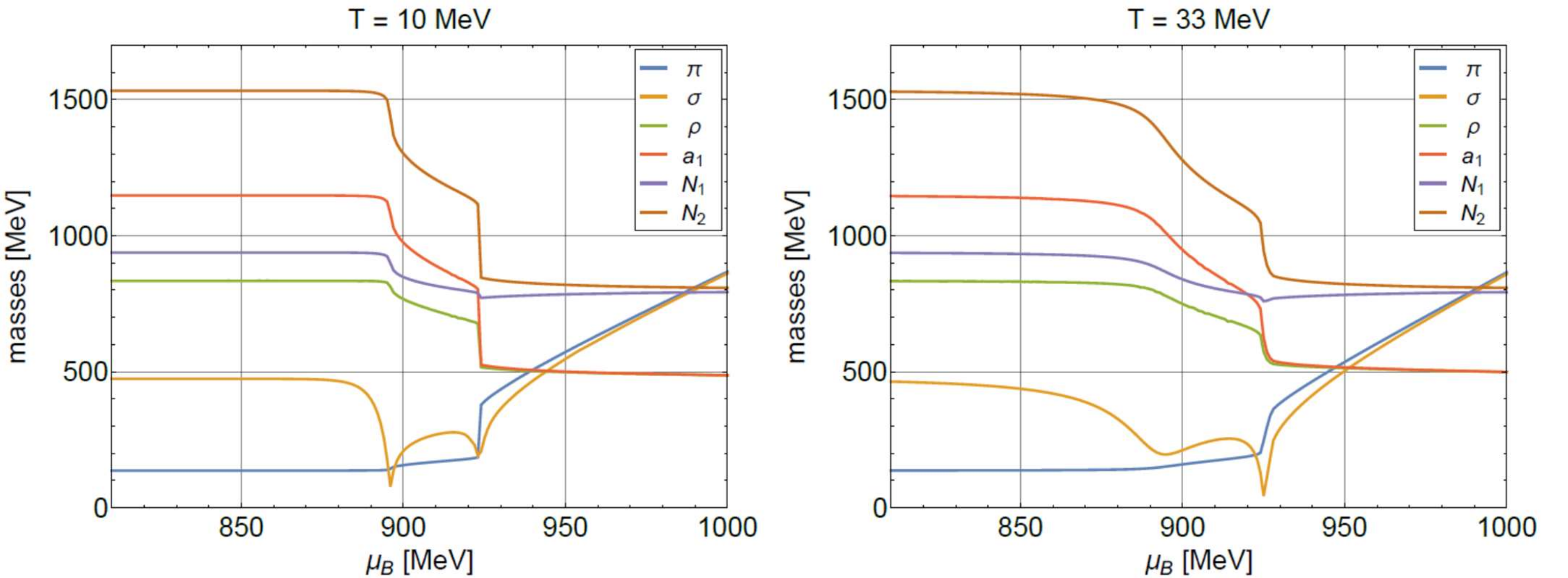
It contains **information on**:

- the (pole) mass of a particle
- the lifetime
- decay channels
- resonances
- collective excitations
- dispersion relations, ...



sketch of the pion spectral function

Euclidean masses



The **scale-dependent masses** serve as input for the flow equations of the two-point functions and **determine the locations of decay thresholds**!

[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Real-time two-point functions

In the UV, the **two-point functions** are given by

$$\Gamma^{(2),R}_{\rho,\Lambda}(\omega) = m^2_{0,\Lambda}\left(1 + \frac{m^2_{\rho,\Lambda}}{(\epsilon - i\omega)^2}\right)$$

$$\Gamma^{(2),R}_{a_1,\Lambda}(\omega) = m^2_{0,\Lambda}\left(1 + \frac{m^2_{a_1,\Lambda}}{(\epsilon - i\omega)^2}\right)$$

The **parameters** are chosen as

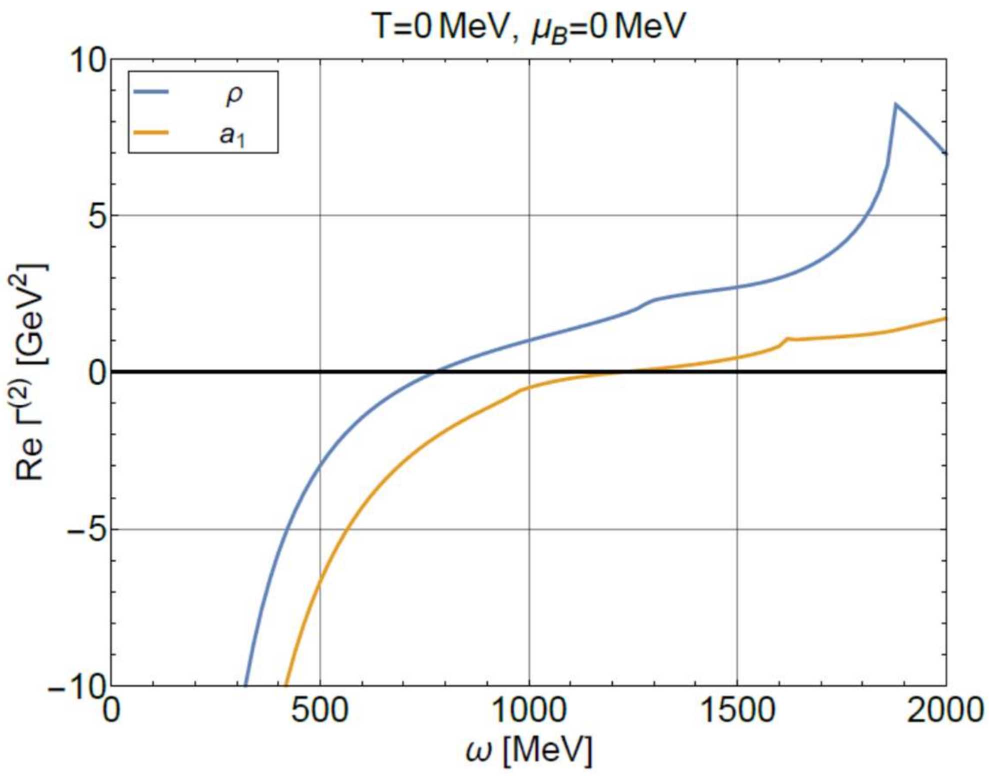
$$m_{v,\Lambda} = m_{0,\Lambda} = 1050 \ \text{MeV}$$

$$g = 8.5$$

in order to obtain for the **pole masses**:

$$m^p_\rho \approx 780 \ \text{MeV} \qquad m^p_{a_1} \approx 1240 \ \text{MeV}$$

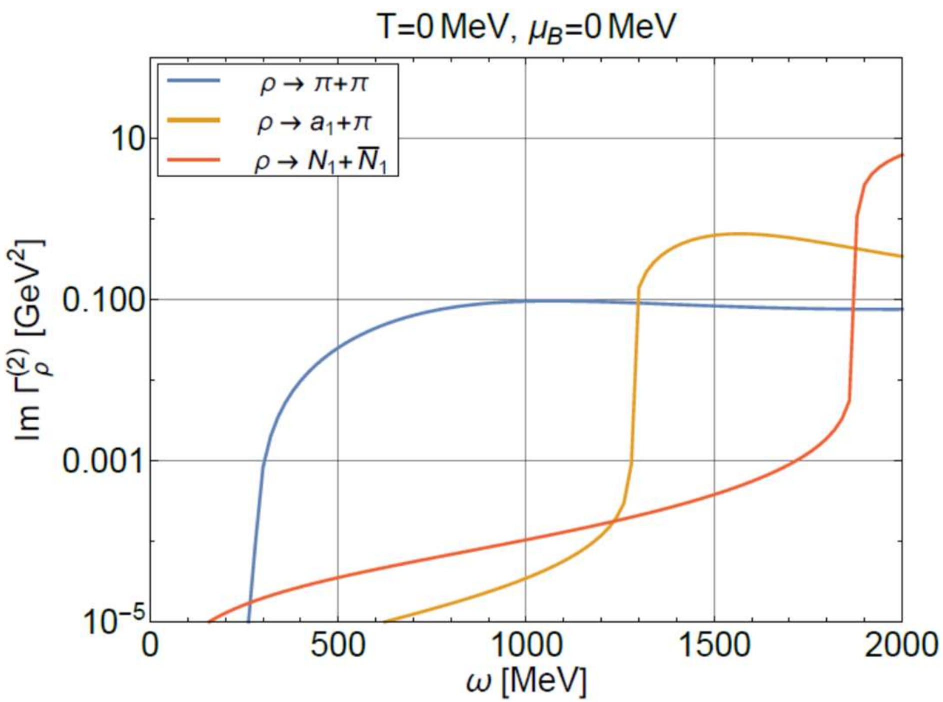
Real part of two-point functions determines pole masses:

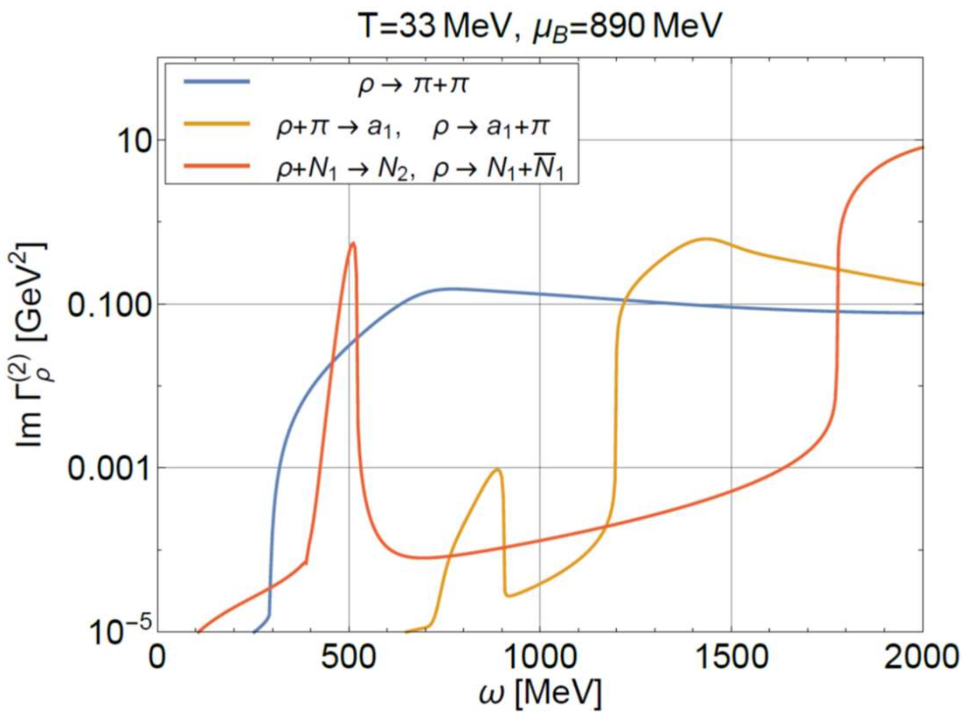


[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Imaginary part of ρ 2-point function

The **2-point function** is affected by several processes that give rise to **complicated in-medium modifications**:

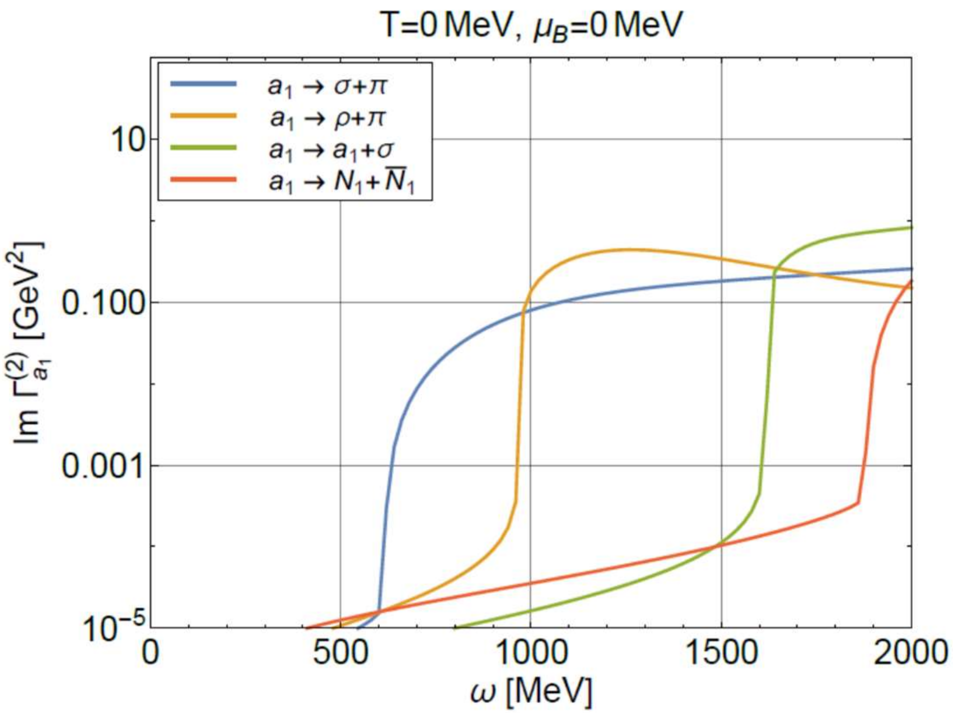


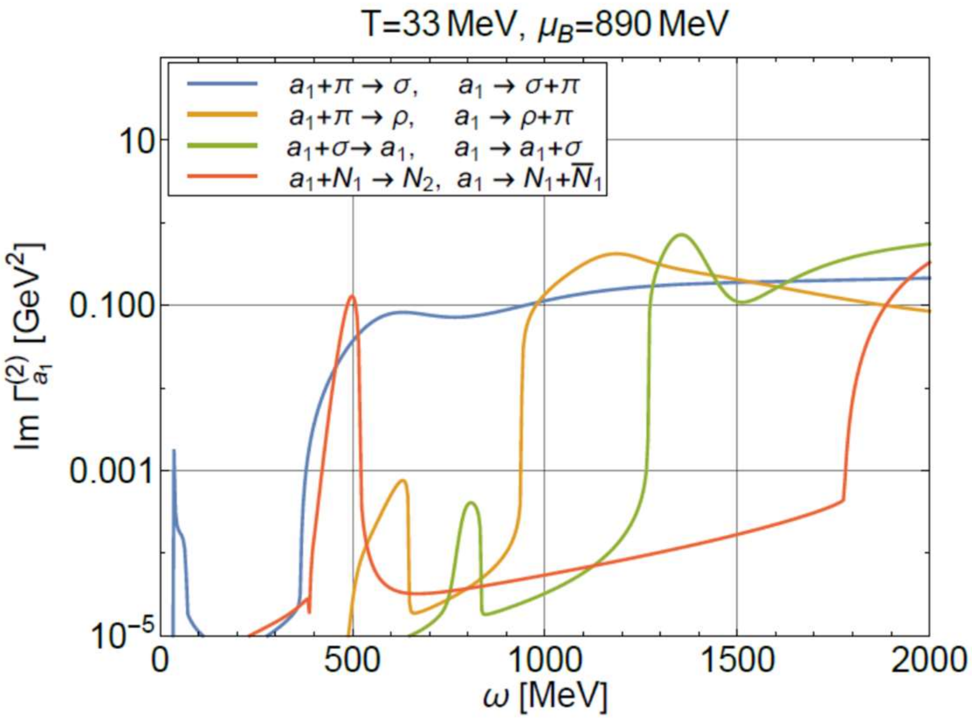


[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Imaginary part of a_1 2-point function

The **2-point function** is affected by several processes that give rise to **complicated in-medium modifications**:

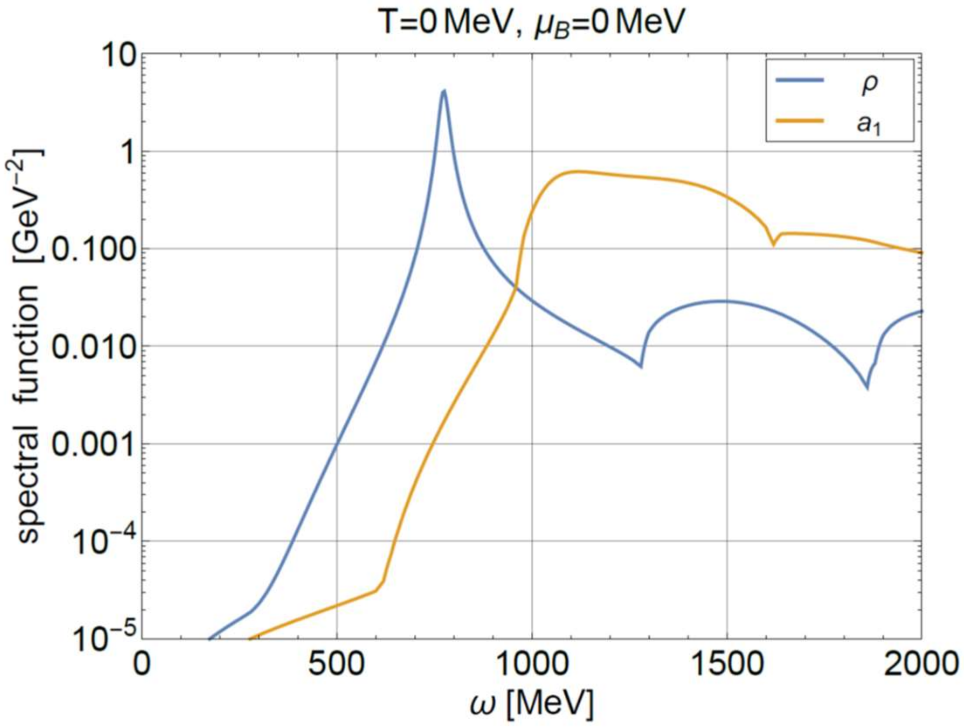


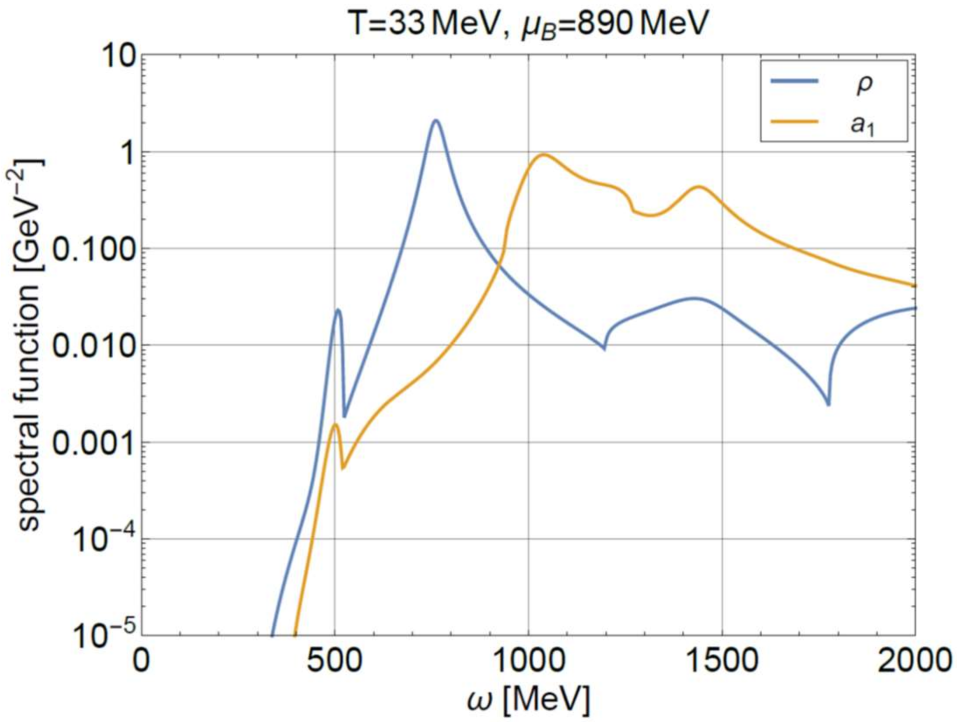


[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

ρ and a_1 spectral functions

The **medium modifications** of the the 2-point functions directly translate to the shape of the **spectral functions**:





[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

ρ and a_1 spectral functions near the chiral CEP

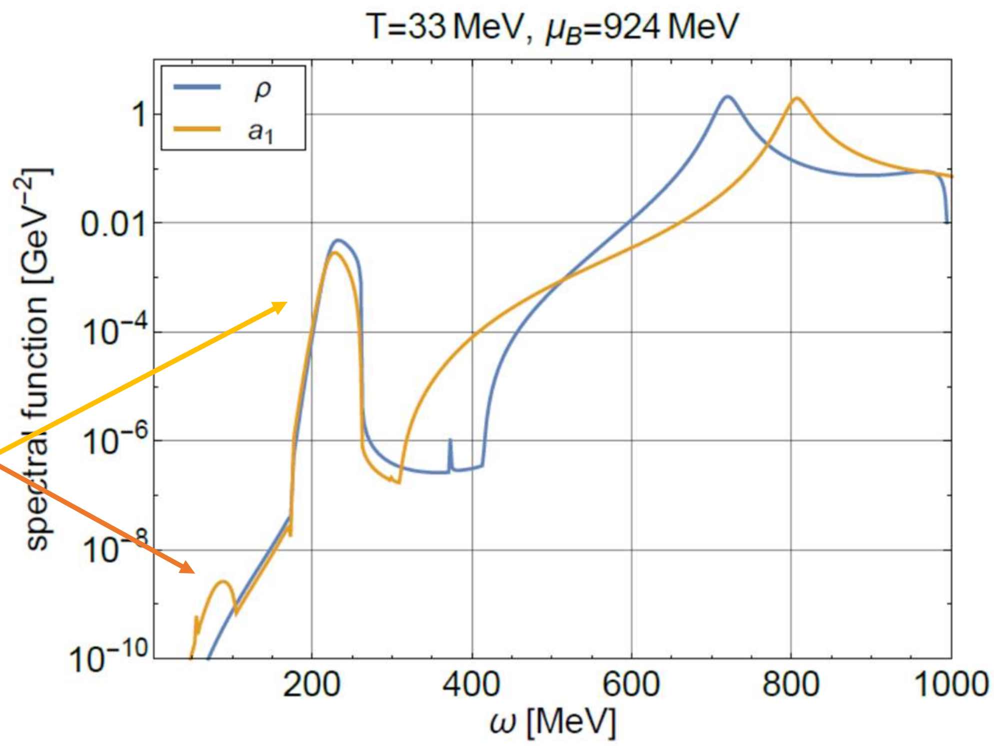
Near the chiral CEP, the **sigma meson** becomes **almost massless** and can combine with a virtual a_1 to form a pion:

$$a_1^* + \sigma \to \pi$$

Also, the **mass difference between N_1 and N_2** becomes **very small** which shifts the corresponding **resonance-production peak** to lower energies:

$$\rho^* + N_1 \to N_2$$

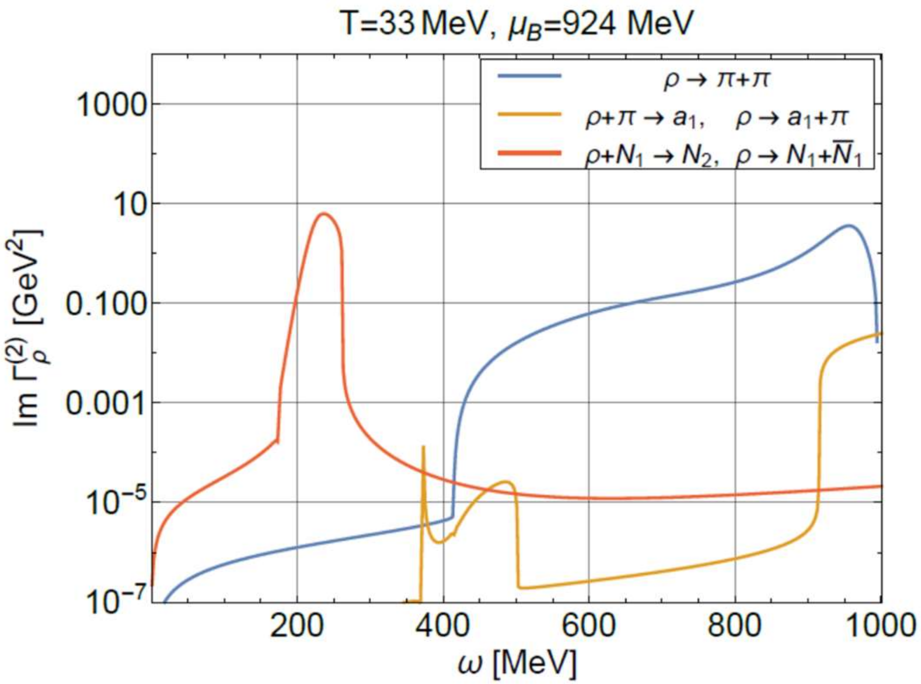
$$a_1^* + N_1 \to N_2$$

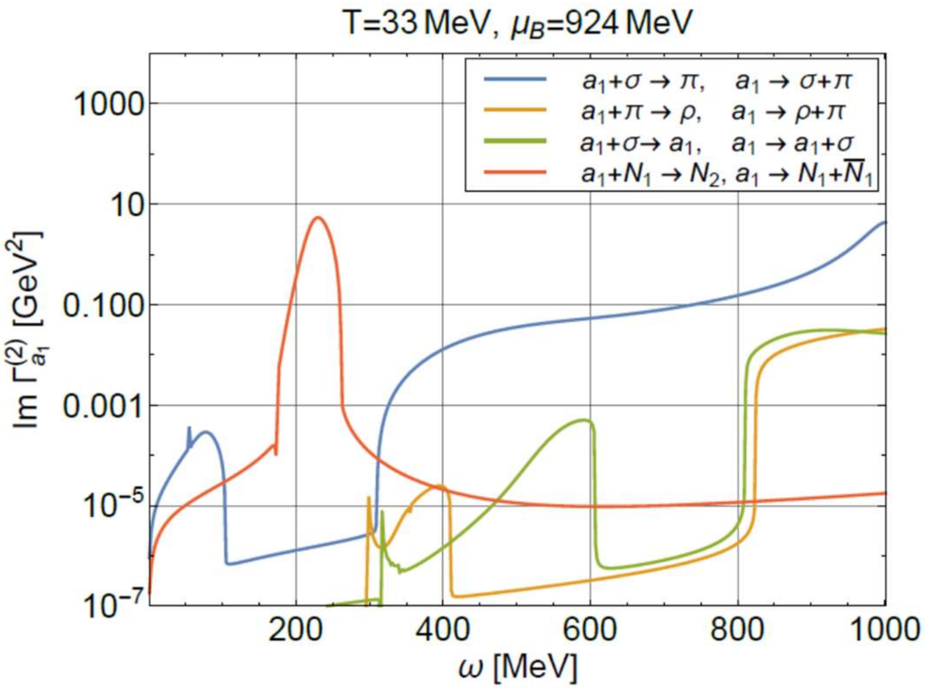


[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Imaginary part of ρ and a_1 2-point function

Near the chiral CEP, the $\boldsymbol{N_2}$ resonance production peak moves to lower energies:





[RAT, C. Jung, L. v. Smekal, J. Wambach, arXiv: 2105.00861]

Preliminary results on dilepton rate in nuclear matter

The **resonance-production** peak in the ρ spectral function due to the process

$$\rho^* + N_1 \to N_2$$

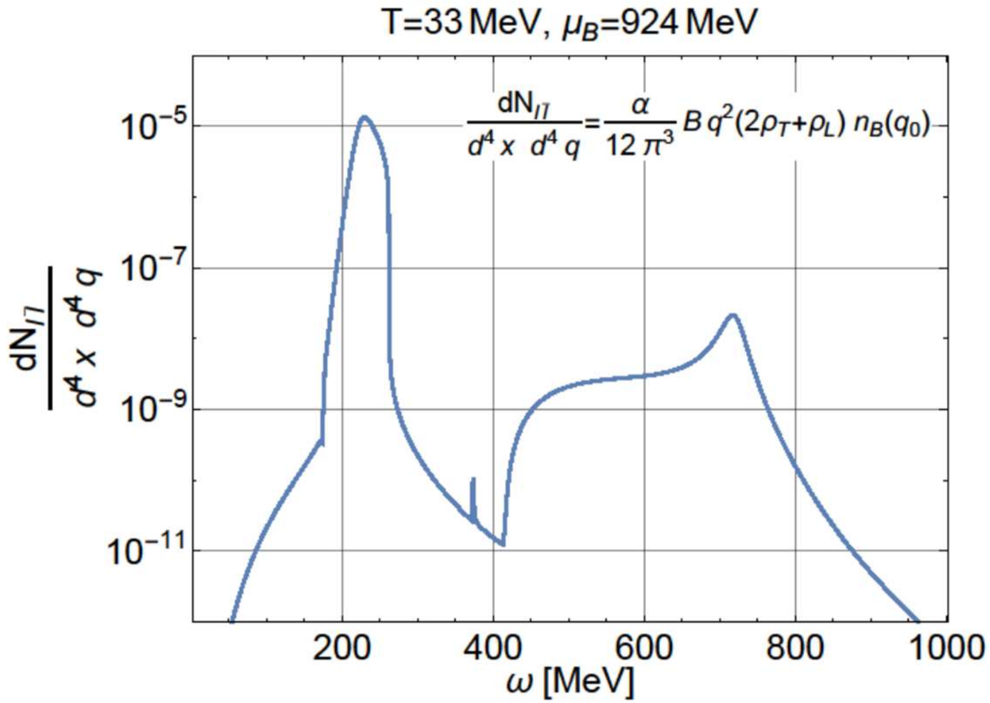
gives rise to a **strong increase in the dilepton rate**!

- **unique prediction** of the parity-doublet model
- detection would yield strong evidence in support of the **parity-doubling** scenario as providing the **mechanism for chiral symmetry restoration** inside dense nuclear matter!

An **overpopulation of N(1535)** states could be **measured by an increased η yield**:

N(1535) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	32–52 %	464
$N\eta$	30–55 %	176

[PDG, Particle Physics Booklet, 2020]



[RAT, L. v. Smekal, J. Wambach, preliminary]

Summary

- we computed **vector and axial-vector meson spectral functions in nuclear matter** based on the parity-doublet model and the aFRG method
- (axial-)vector mesons are introduced using a formulation **in terms of field strengths**
- phase diagram exhibits a **nuclear liquid-gas transition together with a chiral transition**
- effects of chiral symmetry restoration **near the chiral CEP** give rise to a promiment **low-energy peak** around 250 MeV due to $N(1535)$ resonance production
- preliminary estimates indicate that this effect **might be observed experimentally through an increased dilepton yield**!

Outlook

- include a **fluctuating $\boldsymbol{\omega}$ meson** within the FRG approach to the **parity-doublet model** on the level of the effective potential in oder to improve description of thermodynamics
- describe neutron-rich matter, i.e. **isospin-asymmetric matter**, by including a **fluctuating $\boldsymbol{\rho}$ meson**
- study the (chiral) phase structure and compute the **equation of state for nuclear and neutron matter** at low temperatures
- compute **$\boldsymbol{\rho}$ and $\boldsymbol{a_1}$ spectral function** in nuclear matter using such an extended parity-doublet model
- calculate the corresponding **dilepton rates,** identify measurable **signatures for phase transitions and for chiral symmetry restoration**

Thank you for your attention!