Imaginary Chemical Potentials and Mesonic Contributions to the Columbia Plot

Julian Bernhardt

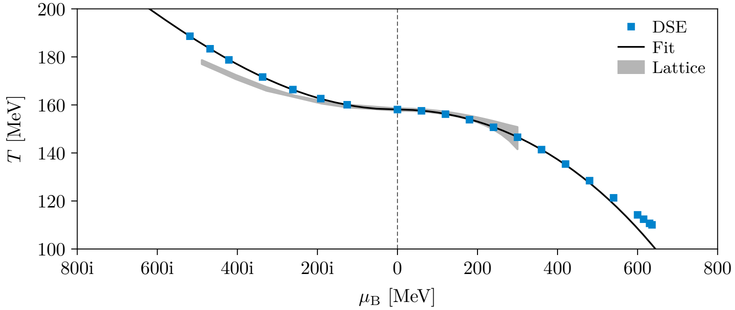
Institut für Theoretische Physik
Justus-Liebig-Universität Gießen

Based on:
JB, Fischer, arXiv:2305.01434
and
JB, Fischer (in preparation)

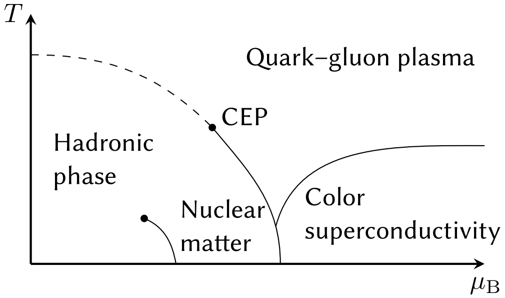
ITP Lunch Club Seminar, JLU Gießen
2023-05-03

Outline

First Objective: Imaginary Chemical Potentials



Motivation: Why Imaginary Chemical Potentials?



- Sign problem of lattice QCD prevents direct calculation of nonzero (real) chemical potentials
- Different methods to bypass/mitigate:
  - Reweighting, Taylor expansion around $\mu = 0$, ...
  - Calculation of imaginary chemical potentials and extrapolation/analytical continuation to real ones
- Functional methods can do real and imaginary $\mu \to$ possible to gauge quality of extrapolation

Framework: Dyson–Schwinger Equations

Master DSE

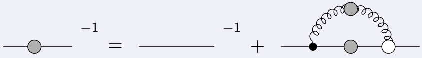
$$0 = \int \mathcal{D}\varphi \, \frac{\delta}{\delta\varphi} \exp\big(-\mathcal{S}[\varphi] + \langle \varphi, J \rangle\big) = \left\langle -\frac{\delta \mathcal{S}}{\delta \varphi} + J \right\rangle$$

- Quantum equations of motion of Euclidean n-point functions
- Non-perturbative, functional approach
- Obtained by taking appropriate number of functional derivatives of master DSE (and setting $J = 0$)
- Infinite tower of coupled, self-consistent equations $\rightarrow$ truncation needed
- Wide range of applications
  - Phase diagram, Columbia Plot, thermodynamics, ...
  - Together with BSEs: Hadron physics (spectroscopy, decays, ...)
  - Muon $g - 2$, QED3, ...
  - ...

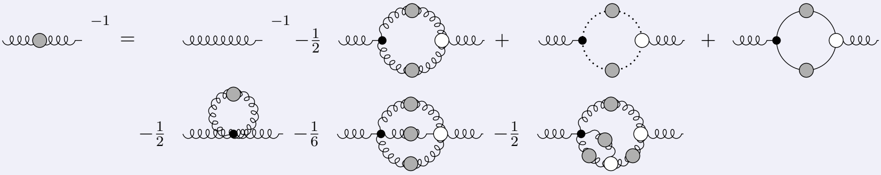
Reviews: Fischer, PPNP 105 (2019) 1
Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

DSEs of QCD Propagators

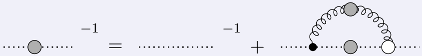
Quark Propagator



Gluon Propagator

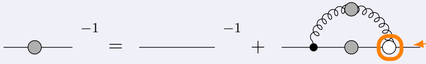


Ghost Propagator



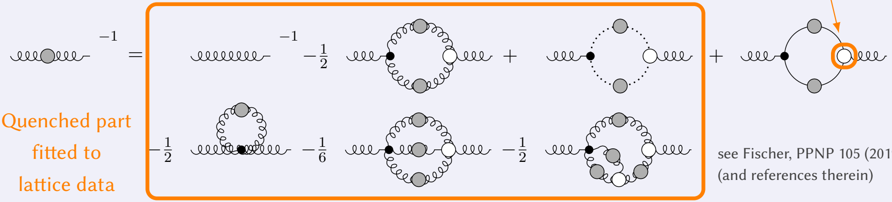
Truncation Scheme

Quark Propagator



Quark–gluon vertex ansatz

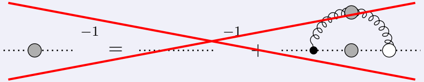
Gluon Propagator



Quenched part fitted to lattice data

see Fischer, PPNP 105 (2019) 1 (and references therein)

Ghost Propagator

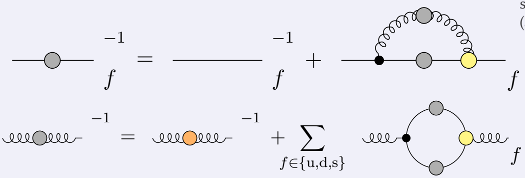


Not needed

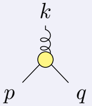
Truncated Set of DSEs

Truncated DSEs for Quarks and Gluons

see Fischer, PPNP 105 (2019) 1
(and references therein)



Quark–Gluon Vertex Ansatz



$$\Gamma^{f}_{\mu}(k,p,q) = \Gamma(k,p,q)\Gamma^{f,\mathrm{BC}}_{\mu}(p,q)$$ (Information about quarks)

Quenched Gluon Propagator

$$D^{\mathrm{que}}_{\mu\nu}(k) = D^{\mathrm{que}}_{\mu\nu}(k;T)$$ (Temperature-dependent fit to lattice data)

reference for lattice data: Fischer, Maas, Müller, EPJC 68 (2010) 165-181
Maas, Pawlowski, von Smekal, Spielmann, PRD 85 (2012) 034037

Review: Matsubara Formalism

- Finite temperature ($T = \beta^{-1}$) $\rightarrow$ bounded imaginary time ($\tau := \mathrm{i}t$) integration

$$\int_{-\infty}^{\infty} \mathrm{d}\tau\, L \;\rightarrow\; \int_{0}^{\beta} \mathrm{d}\tau\, L$$

- Spin–statistics theorem dictates boundary conditions:

$$\begin{aligned} \varphi(\tau+\beta) &= +\varphi(\tau) \quad \text{for bosons,} \\ \varphi(\tau+\beta) &= -\varphi(\tau) \quad \text{for fermions} \end{aligned}$$

- Together: bounded integral + (anti)periodicity $\rightarrow$ discrete Fourier transform

  - Only discrete energies possible

Inclusion of Temperature and Chemical Potential

Matsubara Frequencies

$$\omega_n = \begin{cases} 2n\pi T & \text{for bosons}\,, \\ (2n+1)\pi T & \text{for fermions}\,, \end{cases} \qquad n \in \mathbb{Z}$$

- At finite T, energy integral becomes sum over Matsubara frequencies

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q_4}{2\pi} K(q_4) \to T \sum_{n=-\infty}^{\infty} K(\omega_n)$$

- Chemical potential corresponds to imaginary shift of energy

$$\omega_n \to \tilde{\omega}_n := \omega_n + \mathrm{i}\mu$$

  - Valid for $\mu \in \mathbb{C}$

Quark Condensate and Susceptibility

- Quantities of interest:

$$\langle\overline{\psi}\psi\rangle(T) \sim T \sum_{\omega_n} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \operatorname{Tr}\big[S_\mathrm{u}(q)\big]$$

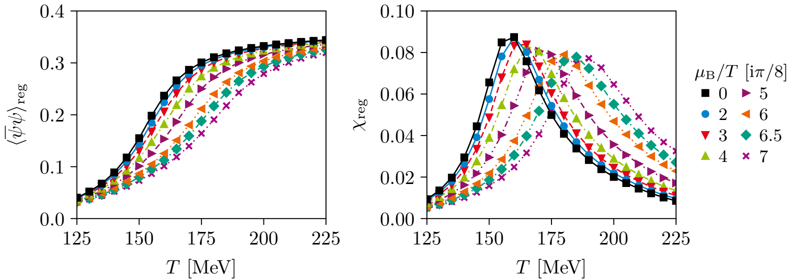
$$\chi(T) = \frac{\partial\langle\overline{\psi}\psi\rangle(T)}{\partial m_\mathrm{u}}$$

- Divergent for $m_\mathrm{u} > 0$, need to be regularized:

$$\langle\overline{\psi}\psi\rangle_\mathrm{reg}(T) = \big[\langle\overline{\psi}\psi\rangle(T) - \langle\overline{\psi}\psi\rangle(0)\big]\frac{m_\mathrm{u}}{f_\pi^4}$$

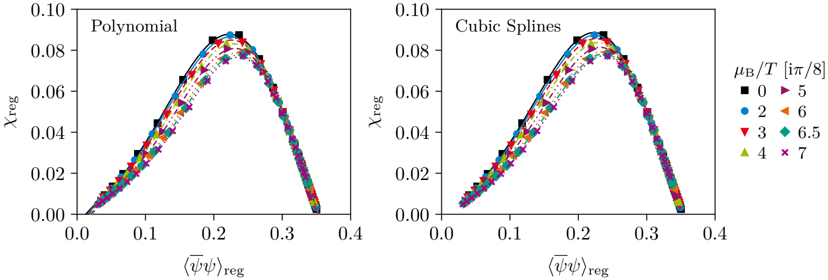
$$\chi_\mathrm{reg}(T) = \big[\chi(T) - \chi(0)\big]\frac{m_\mathrm{u}^2}{f_\pi^4}$$

Results for Condensate and Susceptibility at Imaginary μ_B



- Inflection point of $\langle\overline{\psi}\psi\rangle$ and maximum of χ move towards higher T for increasing $\mathrm{Im}(\mu_B)$
  - Qualitative agreement with the lattice

Determination of Pseudocritical Temperature



Procedure analogous to PRL 125 (2020) 052001:

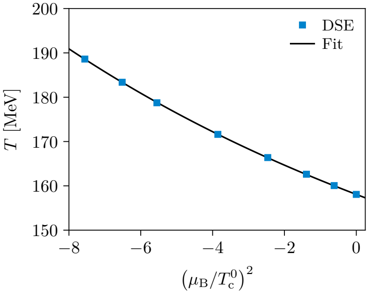
1. Convert $\langle\bar{\psi}\psi\rangle(T)$ and $\chi(T)$ data to dependence $\chi\big(\langle\bar{\psi}\psi\rangle\big)$
2. Use either a fit to fifth-order polynomial or cubic spline interpolation to determine peak position $\rightarrow$ defines $\langle\bar{\psi}\psi\rangle(T_{\mathrm{c}})$
3. Interpolate $\langle\bar{\psi}\psi\rangle(T)$ to extract T_{c} from $\langle\bar{\psi}\psi\rangle(T_{\mathrm{c}})$

JB, Fischer, arXiv:2305.01434

Function to Extrapolate

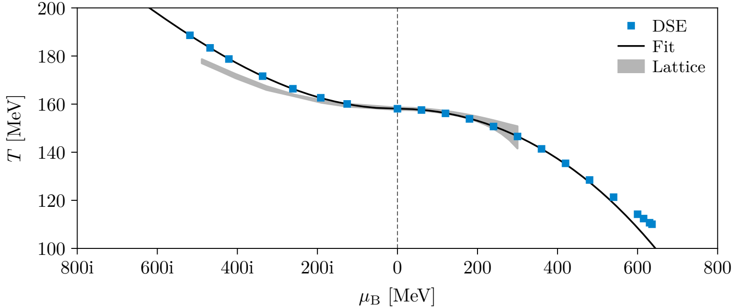
Parametrization of the Crossover Line

$$\frac{T_\mathrm{c}(\mu_\mathrm{B})}{T_\mathrm{c}^0} = 1 - \kappa_2 \left(\frac{\mu_\mathrm{B}}{T_\mathrm{c}^0}\right)^2 - \kappa_4 \left(\frac{\mu_\mathrm{B}}{T_\mathrm{c}^0}\right)^4 , \quad T_\mathrm{c}^0 = T_\mathrm{c}(\mu_\mathrm{B} = 0)$$



- Fit describes the data perfectly

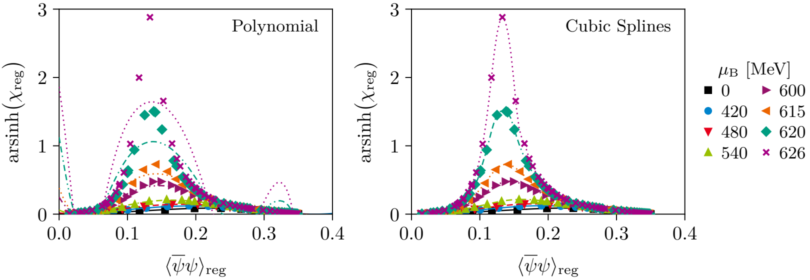
Quality of Extrapolation



- Extrapolation also works flawlessly up to $\mu_B \simeq 510$ MeV
  - Deviations in vicinity of CEP

JB, Fischer, arXiv:2305.01434

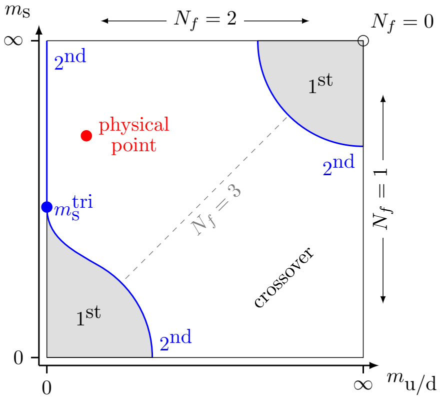
Behaviour in Vicinity of CEP



- Approximation of fifth-order polynomial breaks down
  - Expected since susceptibility has singularity at CEP

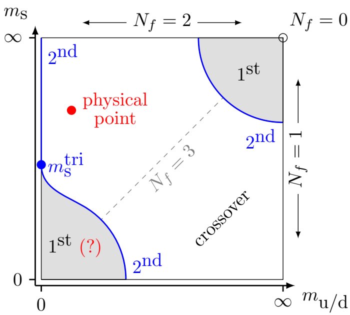
JB, Fischer, arXiv:2305.01434

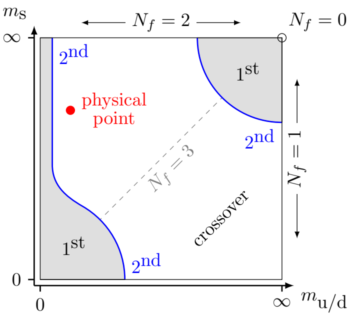
Second Objective: Mesonic Contributions to the Columbia Plot



Motivation: Columbia Plot(s)

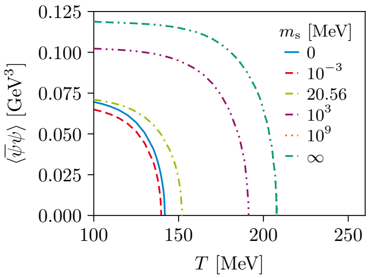
for reference on upper right corner in DSE framework, see Fischer, Luecker, Pawlowski, PRD 91 (2015) 014024

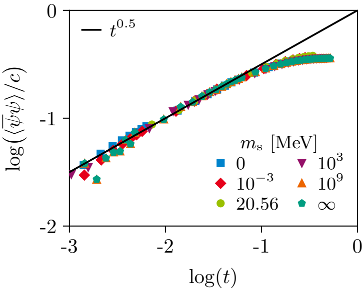


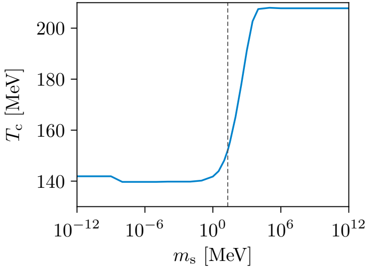


- Two different scenarios for Columbia Plot: anomalously broken (left) or restored (right) $\mathrm{U_A}(1)$-symmetry
- Existence of first order region in lower left corner (of left scenario) is not yet clear see Cuteri, Philipsen, Sciarra, JHEP 11 (2021) 141
- Chiral limit is difficult for lattice QCD but no conceptual problem for our framework

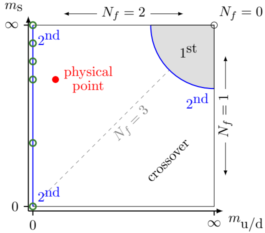
Results: Condensate and Critical Scaling in Chiral Limit







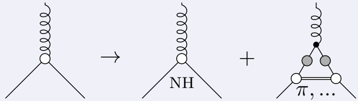
Resulting Columbia Plot:



Long-range Correlations in Vertex

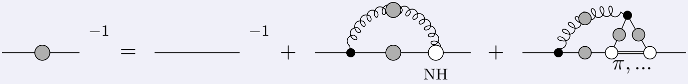
- In vicinity of a second-order phase transition, long-range correlations become important

Vertex Ansatz (from Skeleton Expansion of DSE)



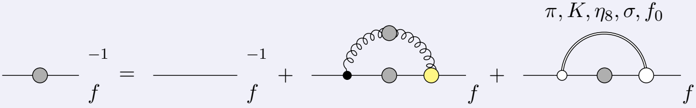
- Leads to modification of the quark self-energy → additional diagram

Resulting Quark DSE

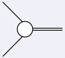


Meson-Backcoupling Setup

Quark DSE with Backcoupling Diagrams

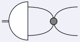


Bethe–Salpeter Amplitudes



(Goldberger–Treiman-like relations: quarks and decay constants)

Meson Decay Constants



(Generalized Pagels–Stokar relation)

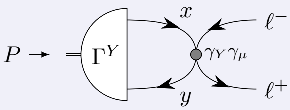
Free Meson Propagator

(Mass from Gell-Mann–Oakes–Renner fit)

details on meson backcoupling: Fischer, Müller, PRD 84 (2011) 054013; JB, Fischer (in preparation)

Essential: Meson Decay Constants

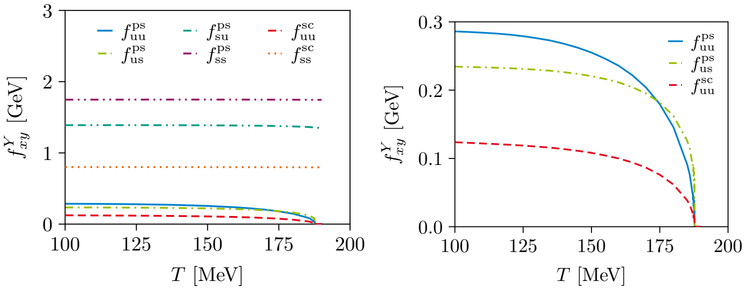
Generalized Pagels–Stokar Relation



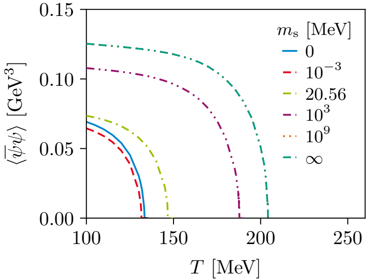
$= \mathrm{i}\tilde{P}_\mu \, f^Y_{xy}$

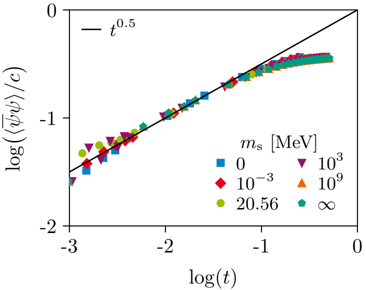
Decay constants in diagrams:

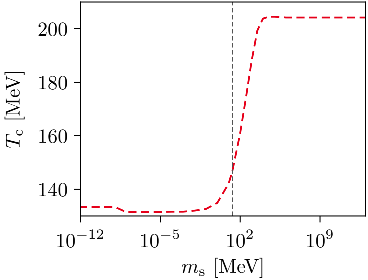
up	strange
π: $f^{\mathrm{ps}}_{\mathrm{uu}}$	
K: $f^{\mathrm{ps}}_{\mathrm{us}}$	K: $f^{\mathrm{ps}}_{\mathrm{su}}$
η_8: $f^{\mathrm{ps}}_{\mathrm{uu}}$	η_8: $f^{\mathrm{ps}}_{\mathrm{ss}}$
σ: $f^{\mathrm{sc}}_{\mathrm{uu}}$	f_0: $f^{\mathrm{sc}}_{\mathrm{ss}}$



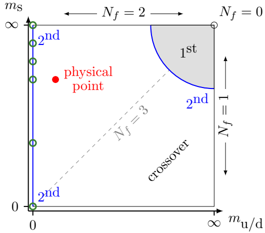
Results for Meson Backcoupling in Chiral Limit







Resulting Columbia Plot:



JB, Fischer (in preparation)

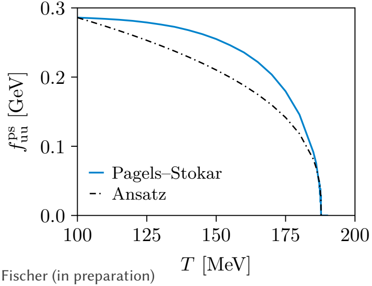
QCD Scaling

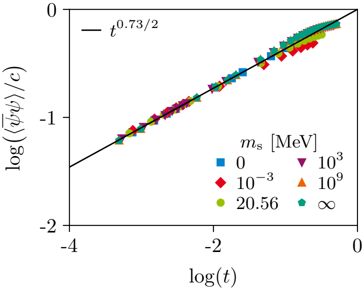
- We see mean-field critical exponents instead of $\mathrm{O}(4) \to$ can be bypassed: condensate inherits scaling behaviour from decay constants see Fischer, Müller, PRD 84 (2011) 054013

Scaling Ansatz

$$\tilde{f}^{Y}_{\mathrm{uu}}(T) = f^{Y}_{\mathrm{uu}}(T_0)\left(\frac{T_\mathrm{c}-T}{T_\mathrm{c}-T_0}\right)^{\beta}, \quad \tilde{f}^{\mathrm{ps}}_{\mathrm{us}}(T) = f^{\mathrm{ps}}_{\mathrm{us}}(T_0)\left(\frac{T_\mathrm{c}-T}{T_\mathrm{c}-T_0}\right)^{\beta/2},$$

$$\tilde{f}^{Y}_{xy}(T) = f^{Y}_{xy}(T_0)\,, \quad T_0 = 100\text{ MeV}\,, \quad \beta = 0.73/2$$

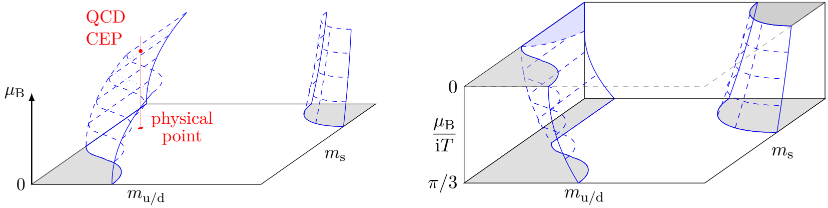




JB, Fischer (in preparation)

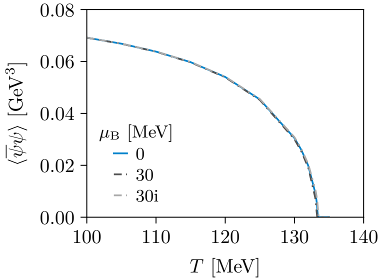
Three-dimensional Extension of Columbia Plot

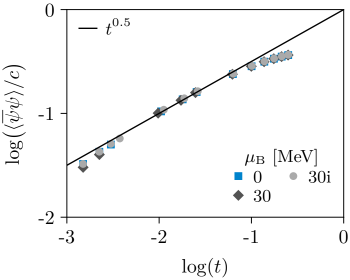
- Common extension of Columbia plot is to include (real and/or imaginary) chemical potential as third axis:



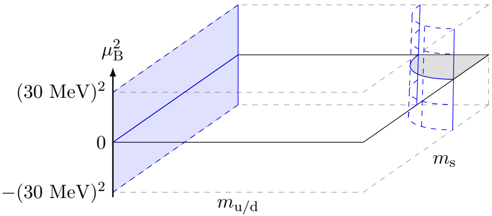
- Have only rough qualitative indications on behaviour of critical surface

Results for Small Nonzero Chemical Potentials





- Resulting three-dimensional Columbia plot:



JB, Fischer (in preparation)

Conclusion and Outlook

Conclusion:

1. Studied QCD phase diagram at both real and imaginary chemical potentials using DSEs and gauged quality of extrapolation
   - Qualitative aggreement with lattice
   - Extrapolation works almost perfectly across a very large portion of crossover line ($\sim 85\ \%$)
2. Investigated impact of mesonic degrees of freedom on Columbia plot
   - See second order phase transition across whole left edge, with both zero and small nonzero chemical potentials
   - Can recover correct scaling behaviour with appropriate ansatz of decay constants

Outlook:

- Investigate complex chemical potentials $\rightarrow$ Lee–Yang zeros
- Study finite-volume effects with meson backcoupling