Heavy-quark diffusion coefficient in the quark-gluon plasma: Theoretical study

NAJMUL HAQUE

School of Physical Science (SPS),
National Institute of Science Education and Research (NISER),
India

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Outline

HQ diffusion coefficient

- When a heavy-quark passes through a thermal medium, it losses its kinetic energy due the collision as well as due to the radiation.
- Heavy quark diffusion coefficient is related with the collisional energy loss and momentum boardaning of the heavy-quark
- The momentum of the heavy quark evolves according to the Langevin equations as

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i, \quad \left\langle \xi_i(t)\xi_j\left(t'\right)\right\rangle = \kappa\delta_{ij}\delta\left(t-t'\right)$$

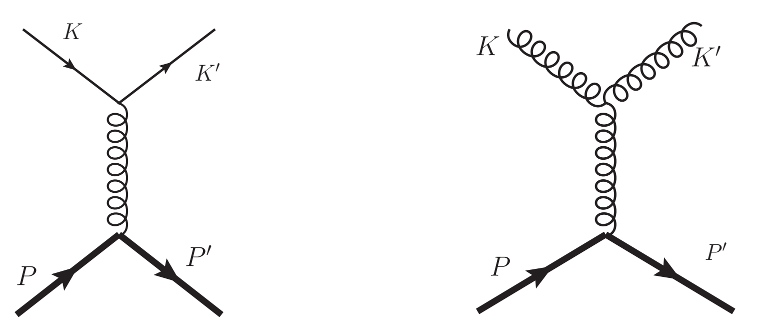
- The diffusion constant in space, D_s, can be found by starting a particle at $x=0$ at $t=0$ and finding the mean-squared position at a later time,

$$\langle x_i(t)x_j(t)\rangle = 2Dt\delta_{ij} \quad \rightarrow \quad 6D_s t = \left\langle x^2(t)\right\rangle .$$

The relation between position and momentum $x_i(t) = \int_0^t dt' \frac{p_i(t')}{M}$, we have

$$6D_s t = \int_0^t dt_1 \int_0^t dt_2 \frac{1}{M^2} \left\langle p\left(t_1\right) p\left(t_2\right)\right\rangle = \frac{6Tt}{M\eta_D}$$

$$\Rightarrow D_s = \frac{T}{M\eta_D} = \frac{2T^2}{\kappa}.$$

- Momentum diffusion κ from the t-channel diagram of $qH \to qH$ and $gH \to gH$ scattering.
- 

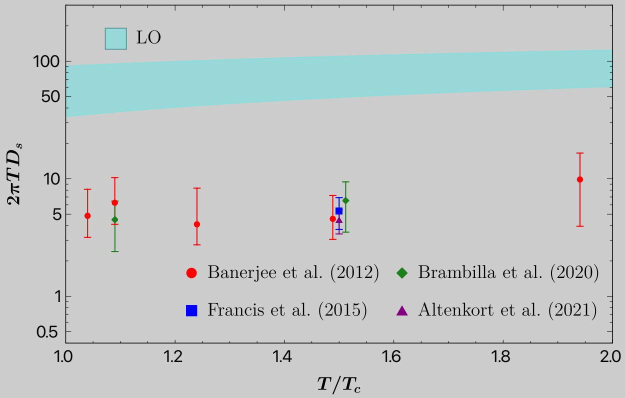
- $$3\kappa = \frac{1}{16M^2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^4 kk'}\int q^2 dq \int_{-1}^{1} d\cos\theta_{\boldsymbol{kq}}\,\delta\left(k'-k\right) q^2 \times\Bigg[|\mathcal{M}|^2_{\text{quark}}\, n_F(k)\left[1-n_F\left(k'\right)\right]+|\mathcal{M}|^2_{\text{gluon}}\, n_B(k)\left[1+n_B\left(k'\right)\right]\Bigg].$$

Leading order HTL

PRC 71, 064904 (2005), Moore & Teaney

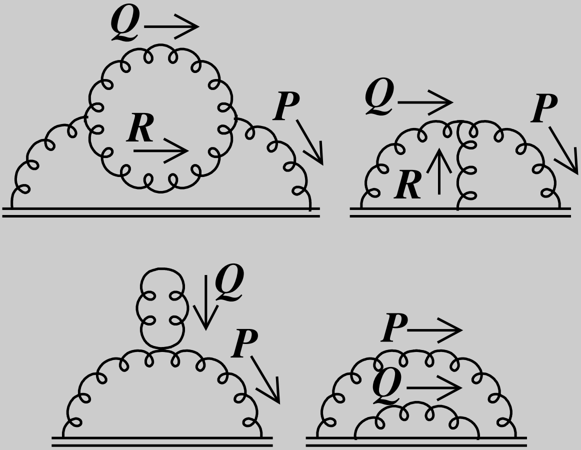
$$
\begin{aligned}
3\kappa &= \frac{C_F g^4}{4\pi^3}\int_0^\infty k^2 dk \int_0^{2k} q\ dq \frac{q^2}{(q^2+m_D^2)^2} \\
&\times \left[N_f n_F(k)[1-n_F(k)]\left(2-\frac{q^2}{2k^2}\right)+N_c n_B(k)[1+n_B(k)]\left(2-\frac{q^2}{k^2}+\frac{q^4}{4k^4}\right)\right] \\
&= \frac{C_F g^4}{18\pi}\left[\left(N_c+\frac{N_f}{2}\right)\left[\ln\frac{2T}{m_D}+\frac{1}{2}-\gamma_E+\frac{\zeta'(2)}{\zeta(2)}\right]+\frac{N_f}{2}\ln 2\right]
\end{aligned}
$$

PRC 71, 064904 (2005), Moore & Teaney



Next-to-Leading order HTLL

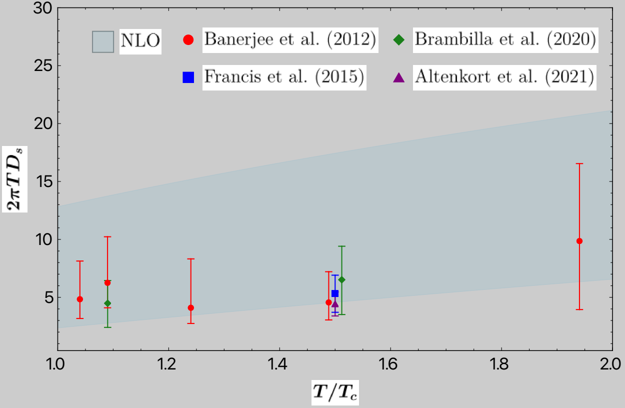
PRL100, 052301 (2008), Caron-Huot& Moore



Next-to-Leading order HTLL

PRL100, 052301 (2008), Caron-Huot& Moore

$$3\kappa = \frac{C_F g^4}{18\pi}\left[\left(N_c + \frac{N_f}{2}\right)\left[\ln\frac{2T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\right] + \frac{N_f}{2}\ln 2 + 2.3302\ \frac{N_c m_D}{T}\right)\right]$$



Motivation

- The gluon and ghost propagators with Gribov quantization show an IR suppression and enhancement, respectively, as compared to the Faddeev-Popov case.
- These results heuristically encompass desirable features of confinement: the IR-suppressed gluon propagator indicates gluon confinement at large distance, and the IR-enhanced ghost is responsible for confinement.
- It was shown by Zwanziger (PRL94, 182301 (2005)) in a phenomenological way that a free gas of Gribov quasiparticles qualitatively captures the nonperturbative features of the lattice equation of state.

- In covariant gauge, the gluon propagator is

$$D_{\mu\nu}^{ab}(K) = -\frac{\delta_{ab}}{K^2}\left[g_{\mu\nu} - (1-\xi)\,\frac{K_\mu K_\nu}{K^2}\right]$$

- Faddeev-Popov action with ghost field

$$\begin{aligned} S &= S_{YM} + S_{GF} + S_{ghost} \\ &= S_{YM} + \int d^4x \left(\bar{c}^a \partial^\mu \left(D_\mu c\right)^a - \frac{1}{2\xi}\left(\partial_\mu A^{\mu a}\right)^2\right) \end{aligned}$$

- Gribov demonstrated for the first time in 1978 that the gauge condition proposed by Faddeev and Popov is not ideal.

- Gribov considered the question of, given a certain physical configuration, how many different gauge copies of this configuration obey the particular gauge condition.

- Consider two gauge fields A^a_μ and $A^{a\prime}_\mu$, and assume they both obey the Landau gauge condition. If $A^{a\prime}_\mu$ is a gauge copy of A^a_μ,

$$A^{a'}_\mu = A^a_\mu + \mathcal{D}^{ab}_\mu \omega^b$$

If both fields obey the Landau gauge condition

$$\partial_\mu \mathcal{D}^{ab}_\mu \omega^b = 0$$

$\rightarrow$ The Faddeev-Popov operator has at least one zero mode. If the gauge field is infinitesimally small, this operator will not have zero modes. The set of gauge has its first zero mode is called the "Gribov horizon".

- The set of all gauge fields where the Faddeev-Popov operator has no zero modes is called the “first Gribov region” Ω.
- If gauge fields have gauge copies, these fields will be overcounted in the path integral. In order to overcome that overcounting, Gribov argued we should limit the path integral to the first Gribov region.
- In order to do so, he considered the ghost propagator, which is the vacuum expectation value of the inverse of the Faddeev–Popov operator. If this operator is always positive definite, the ghost propagator cannot have poles — which is called the “no-pole condition”.
- In usual perturbation theory (using the usual Faddeev-Popov formalism), the propagator does have a pole, which means we left the first region and overcounted some configurations.

- In the Gribov quantization, the YM partition function in Euclidean space reads

$$Z = \int_{\Omega} \mathcal{D}A(x) V(\Omega) \delta(\partial \cdot A) \det[-\partial \cdot D(A)] e^{-S_{\rm YM}}$$

The restriction of the integration to the Gribov region is realized by inserting a function $V(\Omega)$ into the partition function, where

$$V(\Omega) = \theta[1 - \sigma(0)] = \int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{d\beta}{2\pi i\beta} e^{\beta[1-\sigma(0)]}$$

represents the no-pole condition. Here, $1 - \sigma(P)$ is the inverse of the ghost dressing function $Z_G(P)$.

- The integration variable β is identified as the Gribov mass parameter γ_G after some redefinition.

- Gribov's gluon propagator in the Landau gauge reads

$$D_A(P) = \delta^{ab}\frac{P^2}{P^4+\gamma_{\mathrm{G}}^4}\left(\delta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right)$$

- The ghost propagator in the Landau gauge

$$D_c(P) = \delta^{ab}\frac{1}{1-\sigma(P)}\cdot\frac{1}{P^2},$$

- The inverse of the ghost dressing function is

$$Z_G^{-1} \equiv [1-\sigma(P)] = \frac{N_{\mathrm{c}}g^2}{128\pi^2}\left[-5+\left(3-\frac{\gamma_G^4}{P^4}\right)\ln\left(1+\frac{P^4}{\gamma_G^4}\right)\right.$$
$$\left.+\frac{\pi P^2}{\gamma_G^2}+2\left(3-\frac{P^4}{\gamma_G^4}\right)\frac{\gamma_G^2}{P^2}\arctan\frac{P^2}{\gamma_G^2}\right]$$

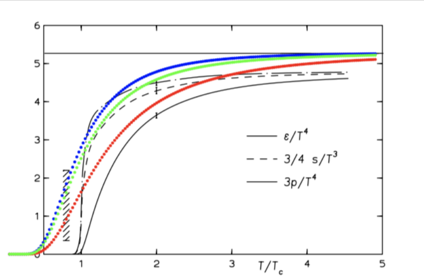
Applicability of Gribov confinement scenario

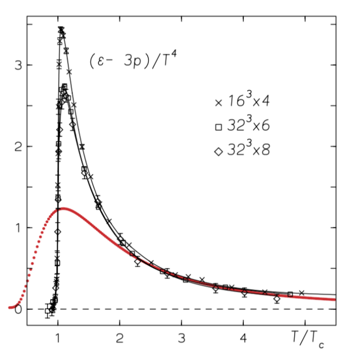
Ref: D. Zwanziger,PRD76, 125014 (2007)

- Long-distance behavior of the color-Coulomb potential $V_{\rm coul}(R) \sim \sigma_{\rm coul}\, R$, $\sigma_{\rm coul} \sim 3\sigma$ and σ being the physical string tension between a pair of external quarks.
- It was also found numerically that the long-distance behavior of $V_{\rm coul}\,(R)$ is consistent with a linear increase, $\sigma_{\rm coul}\ > 0$, above the phase transition temperature, $T > T_c$, where σ vanishes.
- Investigation of the temperature dependence of $\sigma_{\rm coul}$ revealed that in the deconfined phase, the Coulomb string tension increases with T, which is consistent with a magnetic mass $\sigma_{\rm coul}^{1/2}(T) \sim g_s^2(T)\, T$.
- Thus, from the numerical evidence one can say that the Gribov parameter is nonzero in the deconfined phase also.

Gluon Thermodynamics

- Gluon thermodynamics with Gribov term was calculated for the first time (in our knowledge) in 2005 in PRL 94, 182301 (2005) by D Zwanziger considering Coulomb gauge.
- The unknown Gribov parameter was determined by fitting the lattice trace anomaly at high temperature





Gluon Thermodynamics

- In 2013, K Fukushima & N Su used the Gribov modified gluon and ghost propagator and calculated the gluon thermodynamics in Landau gauge.
- The Gribov mass parameter was determined by the variational principle, leading to the following gap equation:

$$\sum\!\!\!\!\!\!\int_P \frac{1}{P^4+\gamma_{\mathrm{G}}^4} = \frac{d}{(d-1)N_{\mathrm{c}}g^2}$$

- After isolating and subtracting UV divergence, gap equation becomes

$$\begin{aligned} 1 \;\; &= \;\; \frac{3N_{\mathrm{c}}g^2}{64\pi^2}\left[\frac{5}{6}-\ln\left(\frac{\gamma_{\mathrm{G}}^2}{\Lambda^2}\right)+\frac{4}{i\gamma_{\mathrm{G}}^2}\right. \\ &\qquad \left.\times\int_0^\infty dp p^2\left(\frac{n_B\left(E_-\right)}{E_-}-\frac{n_B\left(E_+\right)}{E_+}\right)\right] \end{aligned}$$

where $E_\pm = \sqrt{\mathbf{p}^2 \pm i\gamma_{\mathrm{G}}^2}$.

Ref: K Fukushima, N Su, PRD 88, 076008 (2013)

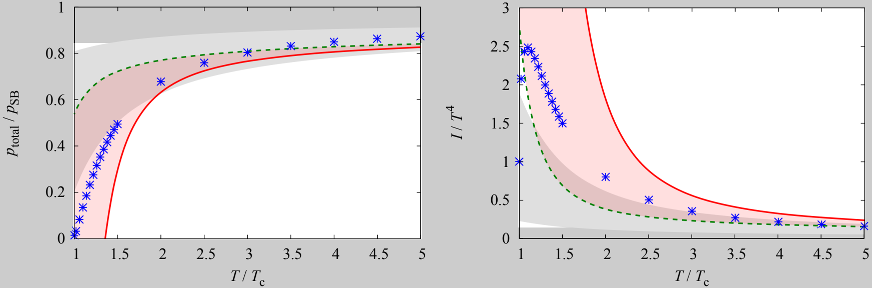
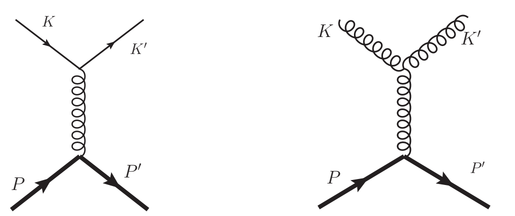


Figure: Gray band: three-loop HTL, Blue star: LQCD, Red band: for Gribov plasma

t-channel heavy quark scattering



$$|\mathcal{M}|^2_{\text{gluon}} = 16\mathrm{N_c} C_F g^4 M^2 k^2 \times (1+\cos^2\theta_{kk'})\frac{q^4}{(q^4+\gamma_G^4}$$

$$|\mathcal{M}|^2_{\text{quark}} = 16 N_f C_F g^4 M^2 k^2 \times (1+\cos\theta_{kk'})\frac{q^4}{(q^4+\gamma_G^4)^2}$$

$$\cos\theta_{kk'} = 1 - \frac{(QM)^2}{2(K\cdot P)^2} = 1 - \frac{q^2}{2k^2}\; viz\;((K\cdot P)\sim Mk, \mathrm{Q}-\text{purely spatial}) \tag{1}$$

$$3\kappa = \frac{C_F g^4}{4\pi^3} \int\limits_0^\infty k^2 dk \int\limits_0^{2k} q dq \frac{q^6}{(q^4+\gamma_G^4)^2} \times \Bigg[N_f\, n_F(k)[1-n_F(k)] \left(2 - \frac{q^2}{2k^2}\right) + N_c\, n_B(k)[1+n_B(k)] \left(2 - \frac{q^2}{k^2} + \frac{q^4}{4k^4}\right) \Bigg]$$

Fixing γ_G perturbatively

Analytic form of γ_G, in the limit $T \to \infty$, Phys. Rev. D 88, 076008

$$\gamma_G(T) = \frac{d-1}{d}\frac{\mathrm{N_c}}{4\sqrt{2}\pi}g^2(T)T \ , \tag{2}$$

d is dimension of space time(here, 4), $g(T)$ is the running coupling, in perturbative limit(one loop):

$$\frac{g^2(T)}{4\pi} = \frac{6\pi}{(11\mathrm{N_c} - 2\mathrm{N_f})\ln\left(\frac{\Lambda}{\Lambda_{\overline{\mathrm{MS}}}}\right)} \ , \tag{3}$$

where

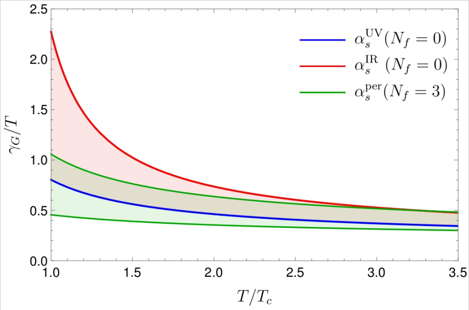
- $\pi T \leq \Lambda \leq 4\pi T$, and
- $\Lambda_{\overline{\mathrm{MS}}} = 0.176$ GeV, (lattice result) .

Fixing γ_G from LQCD fitted coupling

PRD 88, 076008

$$\alpha_S(T/T_c) \equiv \frac{g^2(T/T_c)}{4\pi} = \frac{6\pi}{11\ \mathrm{N_c}\ \ln[c(T/T_c)]}\ ,$$

$c = 1.43$ for IR and $c = 2.97$ for UV $\rightarrow \alpha^{IR}_{T=T_c} = 1.59$ and $\alpha^{UV}_{T=T_c} = 0.524$. The fitted parameter values corresponding to the coupling data extracted from the large distance (IR) and the short distance (UV) behaviour of the heavy quark free energy .



Result and conclusion

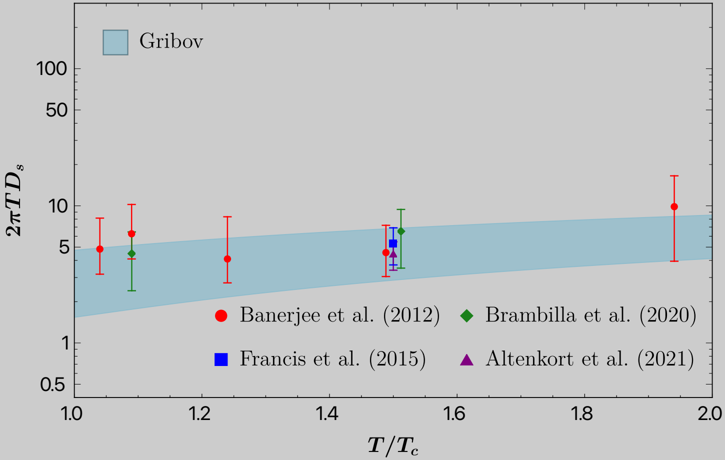


Figure: Plot of $(2\pi T)D_S$ vs (T/T_c). For LO and NLO 2 loop coupling has been taken with $T_c/\Lambda_{\overline{MS}} = 1.15$.

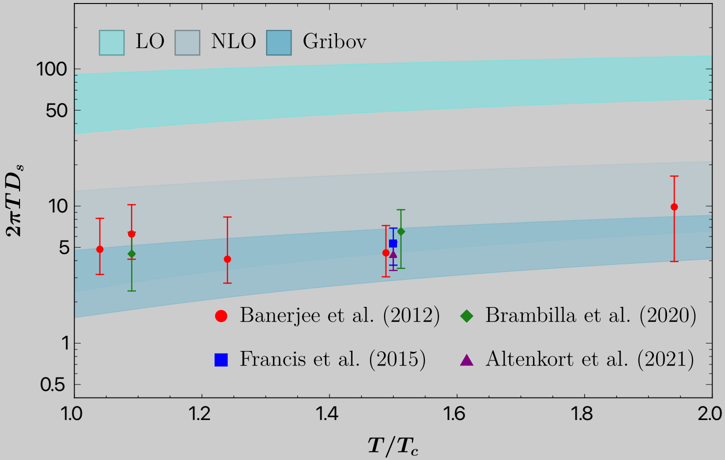


Figure: Plot of $(2\pi T)D_S$ vs (T/T_c). For LO and NLO 2 loop coupling has been taken with $T_c/\Lambda_{\overline{MS}} = 1.15$.

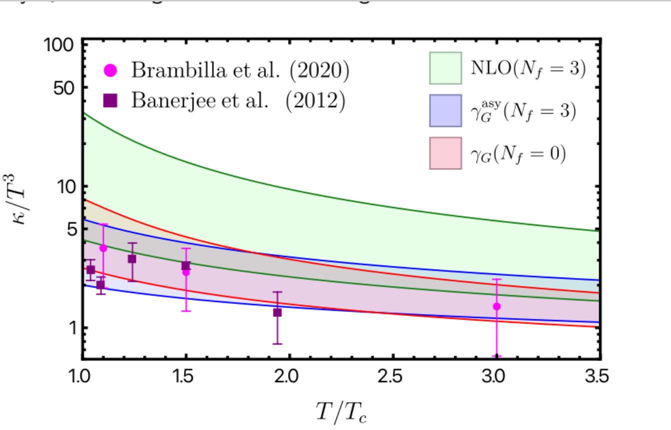


Figure: Plot κ/T^3 vs (T/T_c). For NLO, one loop coupling has been taken with $\Lambda_{\overline{\text{MS}}} = 0.176$ GeV.

Conclusion

- We have discussed existing LO and NLO HTLpt results for heavy quark diffusion coefficient.
- We have discussed the motivation to include Gribov quantization to the estimation of heavy-quark diffusion coefficients.
- We have also discussed our recent results for the heavy-quark diffusion rate in Gribov Plasma.

Thank you for your attention.