

Identifying the Time Scales in Electron-Positron Production from Ultra-Strong Electric Fields

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¹based on work with
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- 1 Motivation: Time scales in the quantum world
- 2 Basic Aspects of the Sauter-Schwinger Pair Production
 - The Different Regimes of Pair Production
 - Pair Production in Crossed Laser Beams
- 3 Dirac-Heisenberg-Wigner Approach to Sauter-Schwinger Effect
 - Formalism
 - Selected Numerical Results
- 4 Physics of Adiabatic Particle Number
- 5 Time Scales of Particle Formation
- 6 Summary

Time scales in the quantum world

- Vastly different time scales for quantum phenomena, e.g., lifetime of
 - Z_0 resonance: $\tau \approx 3 \times 10^{-25}$ s
 - ^{124}Xe nucleus: $\tau \approx 6 \times 10^{+29}$ s

Long time scales due to tunnel effect, resp., evanescent waves

- “Textbook” Quantum Field Theory:
In S -matrix *only* asymptotic *in*- and *out*-states at $t_{i,f} \rightarrow \mp\infty$.
- Understanding of time scales in quantum systems? 😞
Atomic ionisation: Expressions for “tunneling times” experimentally falsified, see, e.g., A.S. Landsman and U. Keller, Phys. Reports **547** (2015) 1.
- Non-equilibrium Quantum Field Theory:
One short time scale in simple systems and/or pert. processes
vs.
several time scales in complex quantum systems and/or
non-perturbative processes???

What about quantum phenomena in time-dependent backgrounds?

As, e.g.,

- particle production in an expanding universe,
- Hawking radiation of Black Holes (grav. collapse!), or
- $e^+ e^-$ pair production in ultra-strong crossed laser beams
(\leftrightarrow Sauter-Schwinger effect).

Fundamental conundrum:

The time interval for extracting the particle number should be short enough to restrict the variation of the background field to a negligible amount. However, Heisenberg's uncertainty principle leads for short time intervals to a large spread in energy, and thus to the possibility of a large number of virtual pairs, and a correspondingly undetermined particle number.

Time scales in the quantum world

Is the question for time scales in such processes well-defined?

If so, are there several distinct time scales?

To which sub-processes can they be attributed?

How long do the sub-processes and the whole process take?

What are the consequences for our understanding of quantum systems?

... a long-standing prediction ...

Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs.

Von Fritz Sauter in München.

Mit 6 Abbildungen. (Eingegangen am 21. April 1931.)

Es werden die Lösungen der Diracgleichung mit dem Potential $V = uz$ angegeben und ihr Verhalten diskutiert. Zu dem Funktionsverlauf, der auch bei nichtrelativistischer Rechnung auftritt, kommt in der Diracschen Theorie noch ein Gebiet hinzu, in dem Elektronenimpuls und -geschwindigkeit entgegengesetztes Vorzeichen besitzen. Im Anschluß daran wird für ein Elektron die Wahrscheinlichkeit berechnet, aus dem Gebiet „positiven Impulses“ in das mit „negativem Impuls“ überzugehen. Es ergibt sich, daß die Durchgangswahrscheinlichkeit erst dann endliche Werte annimmt, wenn die Größe des Potentialanstieges auf einer Strecke gleich der Comptonwellenlänge vergleichbar wird mit der Ruheenergie des Elektrons. Die von O. Klein berechneten großen Werte für die Durchgangswahrscheinlichkeit durch einen Potentialsprung von der Größenordnung der doppelten Ruheenergie sind in dem Sinne als Grenzwerte im Falle unendlich steilen Potentialanstieges zu verstehen.

Vor einiger Zeit erschien eine interessante Arbeit von O. Klein* über

Schwinger's formula

1931: First calculation [F. Sauter, Z. Phys. **69** (1931) 742]

1932: Discovery of the positron [C.D. Anderson, Phys. Rev. **43** (1933) 491]

1936: Theoretical description of pair production from fields

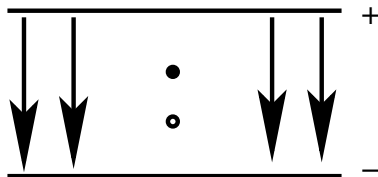
[W. Heisenberg and H. Euler, Z. Phys. **98** (1936) 714 [arXiv:physics/0605038]]

1950/51: quantum field theoretical calculation

[J. Schwinger, Phys. Rev. **82** (1951) 664.]

1971: alternating field

[E. Brezin, C. Itzykson, Phys. Rev. **D2** (1970) 1191.]



Static & spatially uniform electric field \implies “Vacuum” decays

Schwinger's formula

Full one-loop calculation in background of classical electric field for (boson/fermion) pair production provides **vacuum persistence probability / volume · time** (Schwinger's formula):

$$P_0 = \frac{e^2 E^2}{4\pi^3 c \hbar^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n\pi m_e^2 c^3 / \hbar e E}$$

- Due to tunneling of $n e^+ e^-$ pairs out of “vacuum”.
- $n = 1$ term dominates ...

Schwinger's formula

Estimate of scales:

electron Compton wavelength

$$\frac{\lambda_e}{2\pi} = \frac{\hbar}{m_e c} = 386 \text{ fm}$$

rest energy of $e^+ e^-$ pair

$$2m_e c^2 = 1.022 \text{ MeV}$$

work of field on charge e over Compton wavelength = rest energy

$$eE_c \frac{\lambda_e}{2\pi} = m_e c^2 \quad \Longrightarrow \quad E_c = \frac{m_e^2 c^3}{e\hbar} \approx 1.3 \cdot 10^{18} \frac{\text{V}}{\text{m}}$$

$E \ll E_c$: pair production is a quantum tunneling process

\Longrightarrow amplitude $\propto \exp(-\pi E_c/E) \propto \exp(-\frac{1}{e})$




is **NON-perturbative!**²

²Not strictly for pulses of finite duration and extension

Schwinger's formula

Can we test Schwinger's formula?

And how does light transform into matter at mesoscopic scales?

- **huge** electric field! 
- homogeneity over “mesoscopic” scale $\lambda_{Compton}^e \ll \mathbf{d} \ll \mu\text{m}$ 
- static for “mesoscopic” time scales? 

Produced pairs \rightarrow **time dependence** of electric field!!!

Laser pulses:

Mesoscopically-extended “long”-pulsed fields \leftrightarrow few photons.



The Different Regimes of Pair Production

Keldysh adiabaticity parameter (*cf.* ionisation by electric fields)

$$\gamma = \frac{\omega m_e c}{eE}$$

allows to distinguish

- Sauter-Schwinger ($\gamma \ll 1$),
- multiphoton ($\gamma = \mathcal{O}(1)$), and
- low-order perturbative ($\gamma \gg 1$)

regime.

NB: Non-Markovian!

From quasi-continuously varying field strengths
via many photons
to two-photon fusion.



Pair Production in Crossed Laser Beams?

H. M. Fried et al., Phys. Rev. **D63** (2001) 125001;

A. Ringwald, Phys. Lett. **B510** (2001) 107;

R. A. et al., Phys. Rev. Lett. **87** (2001) 193902.

Single laser beam: **No** pair production possible!

Due to

- (i) momentum and energy conservation and
- (ii) gauge and Lorentz invariance.

- (i) $nk^\mu \neq p_e^\mu + q_e^\mu$ for arbitrary n (easily seen in CMS of $e^+ e^-$ pair)
- (ii) gauge and Lorentz invariant description of general e.m. field:

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} (\vec{E}^2 - \vec{B}^2), \quad \mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \vec{E} \cdot \vec{B}.$$

e.m. plane waves, light-like fields: $\mathcal{F} = \mathcal{G} = 0!$

Energy of photons arbitrary small in specific frame!

Field strengths of the order of E_{crit} via crossed laser beams



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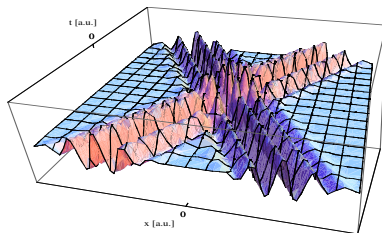
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- **HIBEF / XFEL (DESY):** $E \simeq 0.1 E_{\text{cr}}$ reachable \rightarrow **focusing?!**
- **Optical Laser (ELI):** Probably 'only' $E \simeq 0.01 E_{\text{cr}}$ reachable



Crossed laser beams: 2 counter-propagating wave packages

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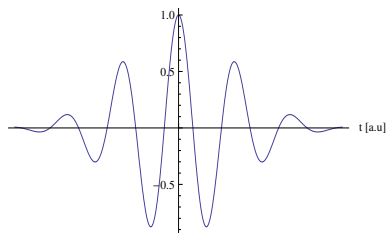
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Interaction region $x = 0$: **Oscillation with Gaussian Envelope**

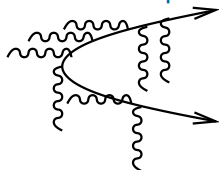
Pair Production in Crossed Laser Beams?

Distinguish Schwinger effect from multi-photon production:

Example: Two (IR) optical lasers, i.e. $\hbar\omega \approx 1\text{eV}$, cross at 90° with zero angle between polarization vectors.

Assume overlap volume $V = (10\mu\text{m})^3$ and intensities up to 10^{26} W/m^2 .

\implies At least $n = 2m_e c^2 / \hbar\omega \approx 10^6$ photons to be absorbed coherently:



i.e. a factor e^n in amplitude or α^n in cross section!!!

But:

Suppose in volume V are $N \gg n$ “available” photons.

\implies combinatorial factor $\frac{N!}{n!(N-n)!} \approx (N/n)^n$

compensates the miniscule α^n factor if $N/n > \alpha^{-1} \approx 137!!!$

Pair Production in Crossed Laser Beams?

Laser parameters:	ELI Ultra-High Field (... still in planing ...)	X-ray FEL
λ	800 - 900 nm	0.15 nm
$\hbar\omega$	~ 10 eV (HHG)	8.3 keV
peak power	200 PW	5 TW
spot radius	~ 900 nm	0.15 nm
coherence time	5 - 100 as	80 fs
intensity	$\sim 10^{27}$ W/m ²	$7 \cdot 10^{31}$ W/m ²
peak electric field	$\sim 10^{16}$ V/m	up to $2 \cdot 10^{17}$ V/m
E_0/E_c	~ 0.01	up to 0.1
$\hbar\omega/2m_e c^2$	10^{-5}	0.008

Pair Production in Crossed Laser Beams?

In the last two years:

Multi-Petawatt Physics Prioritisation (MP3) Workshops
to discuss physics opportunities for future ultra-strong lasers,
cf. arXiv:2211.13187.

⇒ NSF design award for the EP-OPAL laser at the Laboratory for
Laser Energetics (LLE), University of Rochester

October this year:

White papers for the CNRS Apollon Research Infrastructure and
Apollon Laser Facility at CEA Saclay



Phase-Space formulation of Schwinger effect: $\{\vec{x}, \vec{p}, t\}$

- Generalisation of Quantum Kinetic Theory needed for inhomogeneous electric and/or magnetic fields.
- Momentum \vec{k} conjugate variable of $\vec{x} \rightarrow$ **No direct generalization!**

Approach: Dirac-Heisenberg-Wigner (DHW) function

see, e.g., F. Hebenstreit, PhD thesis, April 2011

F. Hebenstreit, R. A., H. Gies, Phys.Rev.D **82** (2010) 105026;

Phys.Rev.Lett. **107** (2011) 180403

- Based on gauge-invariant density operator for Dirac fields
$$\hat{C}_{\alpha\beta}(r, s) = \mathcal{U}(A, r, s) [\bar{\psi}_{\beta}(r - s/2), \psi_{\alpha}(r + s/2)]$$

Dirac-Heisenberg-Wigner formalism

- Gauge-invariant density operator for Dirac fields
 $\hat{C}_{\alpha\beta}(r, s) = \mathcal{U}(A, r, s) [\bar{\psi}_\beta(r - s/2), \psi_\alpha(r + s/2)]$
- Wilson line factor $\mathcal{U}(A, r, s) = \exp\left(i e s \int_{-1/2}^{1/2} d\xi A(r + \xi s)\right)$
- **Hartree approximation:** Mean field $F^{\mu\nu}(x) \approx \langle \hat{F}^{\mu\nu}(x) \rangle$
- DHW function $\mathcal{W}_{\alpha\beta}(r, p) = \langle \Phi | \frac{1}{2} \int d^4s e^{ips} \hat{C}_{\alpha\beta}(r, s) | \Phi \rangle$.
- E.g., equation for vanishing magnetic field ($\vec{B} = 0$):

$$D_t \mathcal{W}_{\alpha\beta} = -\frac{1}{2} \nabla \left[\gamma^0 \vec{\gamma}, \mathcal{W} \right]_{\alpha\beta} - i \left[m \gamma^0, \mathcal{W} \right]_{\alpha\beta} - i \left\{ \gamma^0 \vec{\gamma} \vec{p}, \mathcal{W} \right\}_{\alpha\beta}$$

$$\text{with } D_t = \partial_t + e \int_{-1/2}^{1/2} d\lambda \vec{E}(\vec{x} + i\lambda \partial_p, t) \partial_p$$

$E(\mathbf{x}, t)$ and $B(\mathbf{x}, t)$: DHW Formalism

C. Kohlfürst, PhD thesis, 2015, [arXiv:1512.06082](https://arxiv.org/abs/1512.06082)

- full equations including electric and magnetic fields
- 3+1, 2+1 and 1+1 dimensions
- selected symmetries as *e.g.* cylindrically symmetric fields
- Quantum Kinetic Theory in homogeneous limit
- most efficient numerical solution by pseudo-spectral methods (check for convergence at late time)
- calculations of observables from Wigner components straightforward
- onset of (semi-)classical propagation



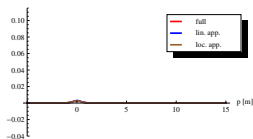
Numerical Results: Single Pulse in 1+1 Dimension

Electric Field: $E(x, t) = E_0 \operatorname{sech}^2(t/\tau) \exp(-x^2/2\lambda^2)$

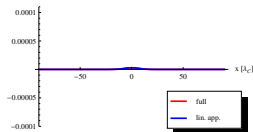
- **Spatial**: $\sim \exp(-x^2/2\lambda^2)$
- **Temporal**: $\sim \operatorname{sech}^2(t/\tau)$
- 3 relevant scales: E_0, τ, λ
- Simplification: Consider **QED₁₊₁** instead of **QED₃₊₁**
- Particle **momentum** density: $f_p(\mathbf{p}, t) = \int [d\mathbf{x}] f(\mathbf{x}, \mathbf{p}, t)$
- Particle **space** density: $f_x(\mathbf{x}, t) = \int [d\mathbf{p}] f(\mathbf{x}, \mathbf{p}, t)$
- Charge **momentum** density: $q_p(\mathbf{p}, t) = \int [d\mathbf{x}] q(\mathbf{x}, \mathbf{p}, t)$
- Charge **space** density: $q_x(\mathbf{x}, t) = \int [d\mathbf{p}] q(\mathbf{x}, \mathbf{p}, t)$

Time evolution: Small $\lambda = 10\lambda_C$

$$f_p(p, t)$$

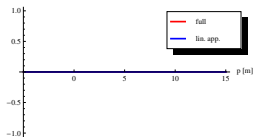


$$f_x(x, t)$$

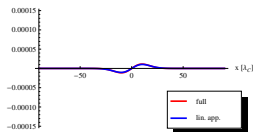


$$t = -2\tau$$

$$q_p(p, t)$$

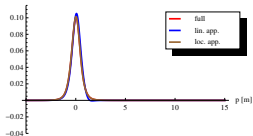


$$q_x(x, t)$$

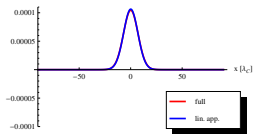


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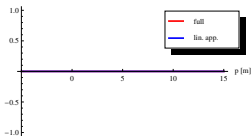


$$f_x(x, t)$$

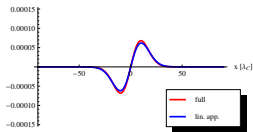


$$t = -\tau$$

$$q_p(p, t)$$

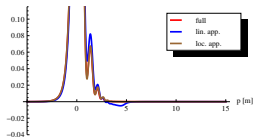


$$q_x(x, t)$$

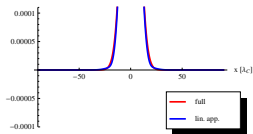


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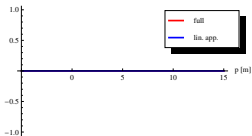
$$f_p(p, t)$$



$$f_x(x, t)$$

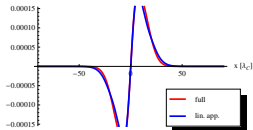


$$q_p(p, t)$$



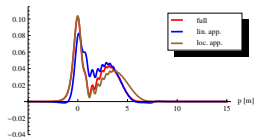
$$t = 0$$

$$q_x(x, t)$$

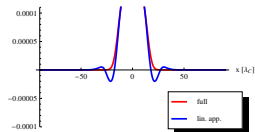


Time evolution: Small $\lambda = 10\lambda_C$

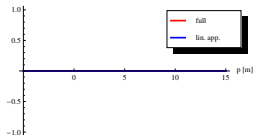
$$f_p(p, t)$$



$$f_x(x, t)$$

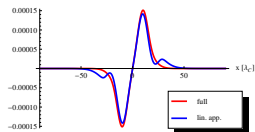


$$q_p(p, t)$$



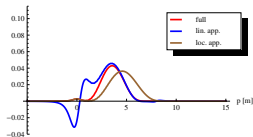
$$t = \tau$$

$$q_x(x, t)$$

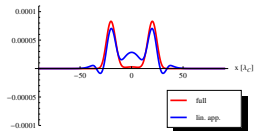


Time evolution: Small $\lambda = 10\lambda_C$

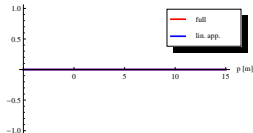
$$f_p(p, t)$$



$$f_x(x, t)$$

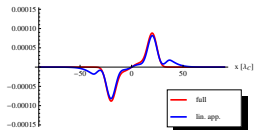


$$q_p(p, t)$$



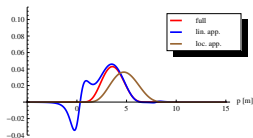
$$t = 2\tau$$

$$q_x(x, t)$$

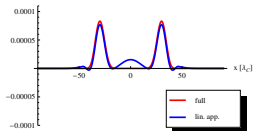


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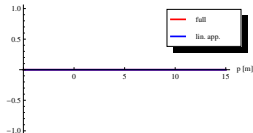
$$f_p(p, t)$$



$$f_x(x, t)$$

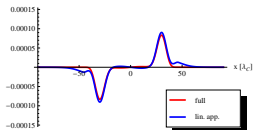


$$q_p(p, t)$$



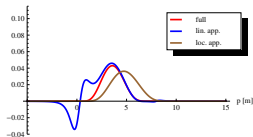
$$t = 3\tau$$

$$q_x(x, t)$$

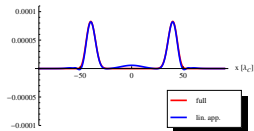


Time evolution: Small $\lambda = 10\lambda_C$

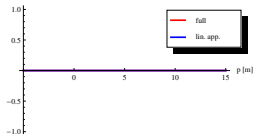
$$f_p(p, t)$$



$$f_x(x, t)$$

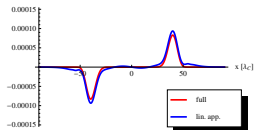


$$q_p(p, t)$$



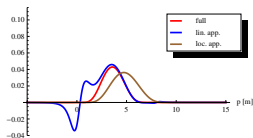
$$t = 4\tau$$

$$q_x(x, t)$$

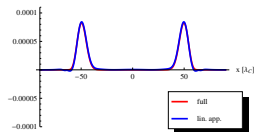


Time evolution: Small $\lambda = 10\lambda_C$

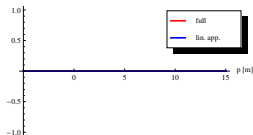
$$f_p(p, t)$$



$$f_x(x, t)$$

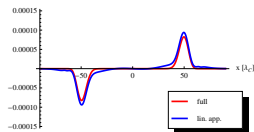


$$q_p(p, t)$$



$$t = 5\tau$$

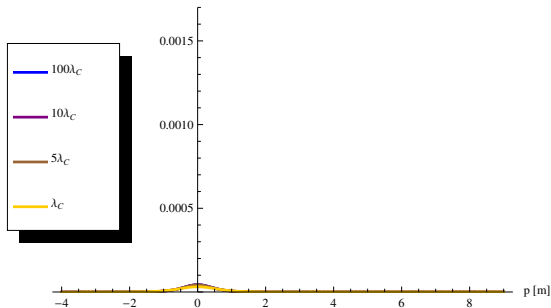
$$q_x(x, t)$$



Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p, t) \equiv \frac{f_p(p, t)}{\lambda}$

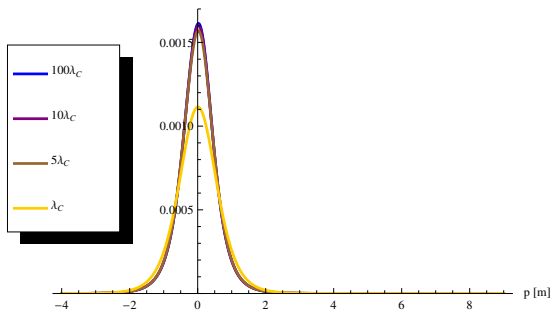
$$t = -2\tau$$



Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p, t) \equiv \frac{f_p(p, t)}{\lambda}$

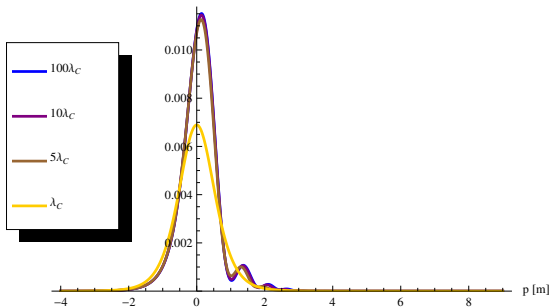
$$t = -\tau$$



Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p, t) \equiv \frac{f_p(p, t)}{\lambda}$

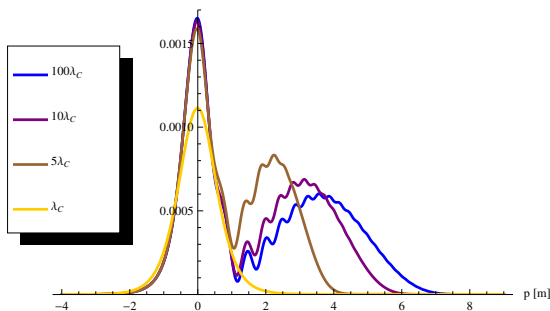
$t = 0$



Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p, t) \equiv \frac{f_p(p, t)}{\lambda}$

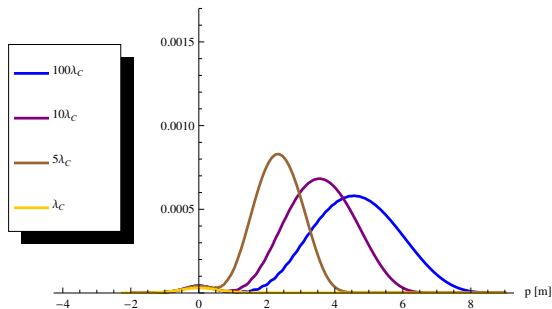
$$t = \tau$$



Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p, t) \equiv \frac{f_p(p, t)}{\lambda}$

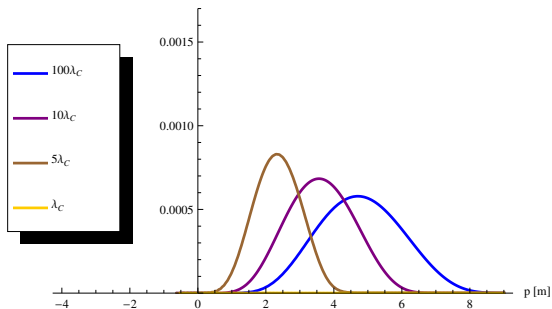
$$t = 2\tau$$



Reduced particle number density

Reduced particle momentum distribution: $\bar{f}_p(p, t) \equiv \frac{f_p(p, t)}{\lambda}$

$$t = 3\tau$$

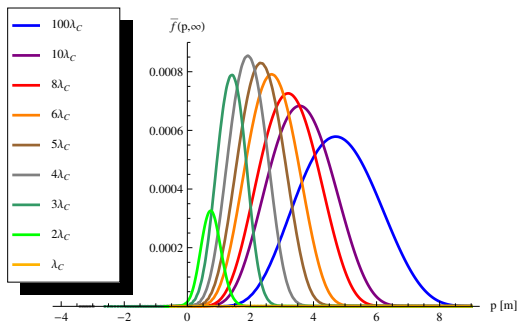


- Peak momentum shifted to smaller values: self-focussing effect
- Sharp drop for small $\lambda \rightarrow$ particle creation terminates

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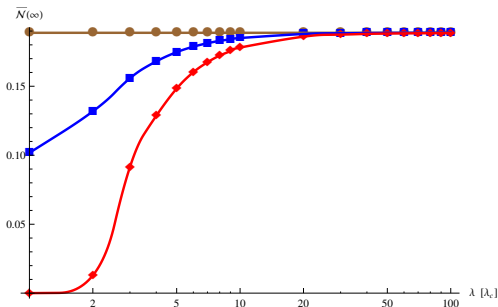


- Peak momentum shifted to smaller values: self-focussing effect
- Sharp drop for small $\lambda \rightarrow$ particle creation terminates

Total number of created particles

- Trivial scaling effect \rightarrow **reduced** number of created particles

$$\bar{N}(\infty) = \frac{N(\infty)}{\lambda}$$



local density apprx.
linear apprx.
full solution

- Need to include **higher derivatives** for small λ !
- Sharp drop for small $\lambda \rightarrow$ particle creation **terminates** because field energy becomes less than $2m$ — **only** seen in full solution!



$\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{B}(\mathbf{x}, t)$: Model for electromagnetic field

C. Kohlfürst and R.A., Phys.Lett. **B756** (2016) 371 [arXiv:1512.06668]

- Superposition of left- and right-running pulses in 2+1 dim.:

$$\vec{A}(z, t) = \varepsilon \tau \left(\tanh\left(\frac{t}{\tau} + 1\right) - \tanh\left(\frac{t}{\tau} - 1\right) \right) \exp\left(-\frac{z^2}{2\lambda^2}\right) \vec{e}_x.$$

- ε maximal electric field strength
- τ temporal extent (difference of Sauter pulses)
- λ spatial extent (Gaussian)
- homogeneous Maxwell eqs. fulfilled by construction
- electric field: double-peak structure, antisymmetric in time
- magnetic field: maximal strength $\varepsilon\tau/\lambda^2$
- Field energy in
 - electric field for $\tau/\lambda \ll 1$
 - magnetic field for $\tau/\lambda \gg 1$

for more details and other model fields see:

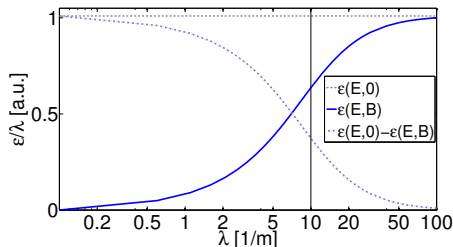
Chapter 7 of C. Kohlfürst, PhD thesis

Numerical Results for Model Field

- pseudoscalar Lorentz invariant $\tilde{F}_{\mu\nu}F^{\mu\nu} \propto \vec{E}\vec{B} = 0$
- scalar Lorentz invariant $F_{\mu\nu}F^{\mu\nu} \propto \vec{E}^2 - \vec{B}^2 \begin{matrix} (>) \\ (<) \end{matrix} 0$
- pair production only for regions in which $E^2(t, z) - B^2(t, z) > 0$

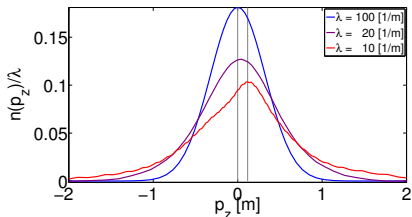
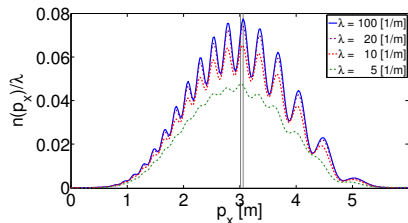
- Effectively available field energy

$$\mathcal{E}[\vec{E}, \vec{B}] = \int dtdz (E^2(t, z) - B^2(t, z)) \Theta(E^2(t, z) - B^2(t, z))$$



Numerical Results for Model Field

Reduced particle density as function of p_x and p_z
($\varepsilon = 0.707 E_C$, $\tau = 5/m$):



$\lambda \gg \tau, 1/m$: homogeneous limit

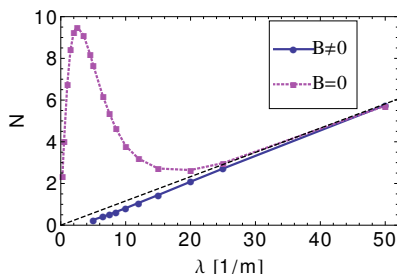
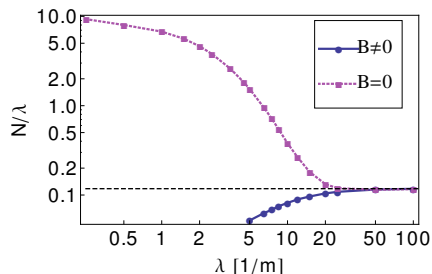
$\lambda \approx \tau$: deflection due to magnetic field

Suppression of pair production?

Numerical Results for Model Field

Comparison to calculation with magnetic field neglected

Reduced total particle number, resp., total particle number
($\varepsilon = 0.707 E_C$, $\tau = 10/m$):



Significantly overestimated particle number for small λ !

NB: Regions with negative “particle distribution”!

Physics of Adiabatic Particle Number

If there is no well-defined particle number at finite times:
How can extracted time scales have physical meaning?

A. Ilderton, Phys.Rev. **D 105** (2022) 016021 [arXiv:2108.13885];

M. Diez, RA and C. Kohlfürst, Phys. Lett. **B 844** (2023) 138063 [arXiv:2211.07510]

- Time-dependent background \Rightarrow time-dependent “Dirac vacuum”
- Adiabatic (i.e., instantaneous) eigenstates of the Hamiltonian:
Preferred basis
- At $t < \tau$ (pulse length):
Many more “adiabatic particles” than asymptotic particles
- “Adiabatic particles” unphysical?
- Gedankenexperiment: Shut off the background field rapidly
e.g., $E(t, x) \rightarrow E(t, x)\Theta_{reg}(t_0 - t)$
 \Rightarrow calculated spectrum for $t = t_0$ accurately represents the then
measured spectrum.



This Gedankenexperiment

- relates *virtual quantum fluctuations* to *real particles* (= localized wave-packets build from asymptotic states),
- explains why the intermediate adiabatic particle number generically exceeds strongly asymptotic one, (shutoff field contains high-frequency modes \rightarrow multiphoton pair production)
- disproves semi-classical expectations (“intuitive” picture of gradually forming and then accelerated particles),
- points towards multi-structure self-interfering particle phase-space distributions, and
- allows to identify sub-processes and corresponding multiple well-defined time scales.

Time Scales of Particle Formation

Matthias Diez, RA and Christian Kohlfürst, Phys. Lett. **B 844** (2023) 138063

[arXiv:2211.07510]

Model field

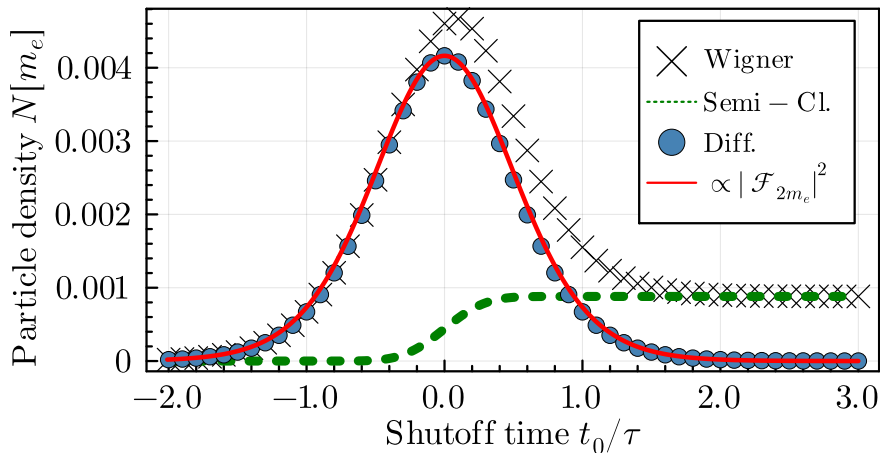
$$|e|\mathbf{E}(t, x) = \varepsilon m_e^2 \operatorname{sech}^2(t/\tau) \exp\left(-\frac{x^2}{2\lambda^2}\right) \mathbf{e}_x$$

in Schwinger regime: $0.1 \leq \varepsilon \leq 0.5$, $\tau \geq 10/m_e$ and $\lambda \geq 10/m_e$

- as in previous numerical studies,
- minimal quantum interference, and
- separation of charge carriers easily recognised.



Time Scales of Particle Formation



DHW particle number $N(t = t_0) =$ semi-classical Schwinger
+ perturbative “shut-off”
particle numbers

Time Scales of Particle Formation

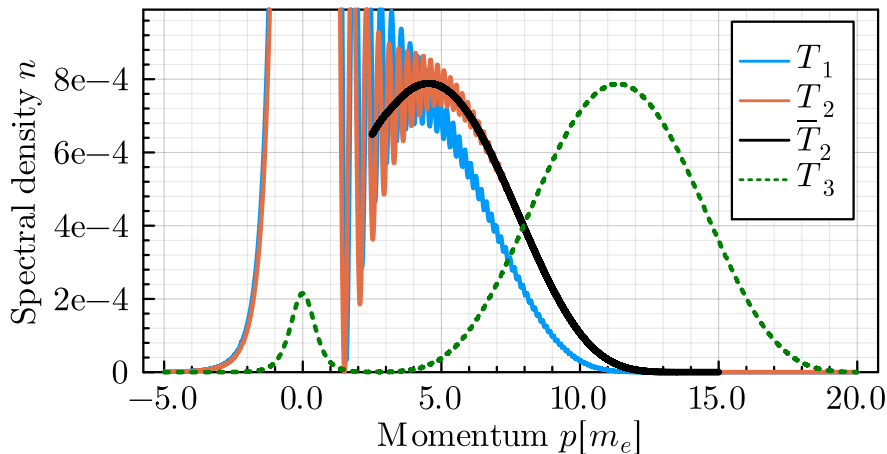
- Semi-classical Sauter-Schwinger particle creation at all times (green dashed curve)
- Perturbative particle creation due to “shut-off” high-frequency component scales with Fourier transform of $E(t)\Theta(t_0 - t)$ (full red curve)
- Full numerical DHW solution for adiabatic particle number (black crosses)
coincides with sum of both

Expectation based on Gedankenexperiment verified 😊

Shut-off & measurement \Leftrightarrow quantum interferences w.o. measurement

Time Scales of Particle Formation

Analysis of spectrum as function of kinetic momentum $p(t)$ reveals four subprocesses and **three time scales**:

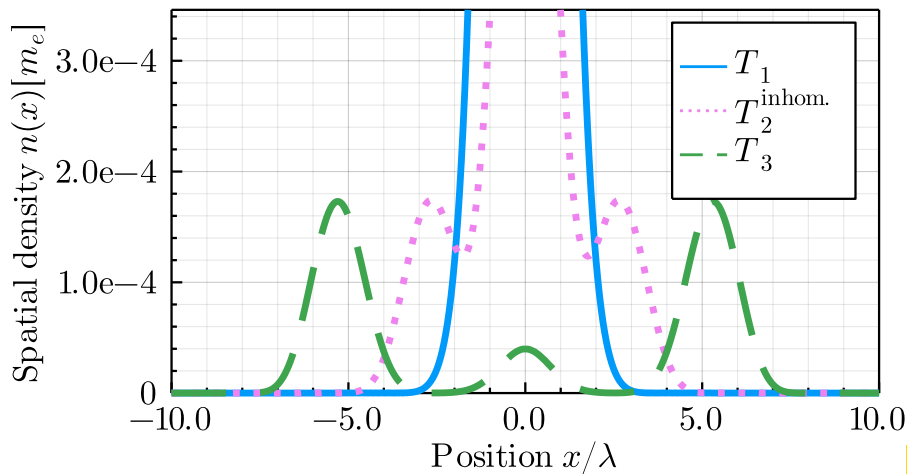


Time Scales of Particle Formation

- Early build-up of narrow perturbative peak at $p = 0$
- Self-interference \Rightarrow “side” peak at $p \neq 0$ appears at $T_1 \approx 0.65 \tau^{1/4} t_c^{3/4} / \varepsilon^2$,
in central peak destructive interference starts to dominate
- Partial wave packet starts to follow classical trajectory at $T_2 = T_1 + 0.06 \tau^{3/4} t_c^{1/4} / \varepsilon^{3/2}$
(\rightarrow pre-particle)
- Perturbative peak fades away (below 1% of maximal value) at $T_3 \approx 1.8 \tau$.
- ▶ Pre-particle number according to semi-class. expectation, accelerated by electric field.
- ▶ Perturbative peak height scales with ε^2 (power-like!), **not** accelerated by electric field.

Time Scales of Particle Formation

Creation of pre-electron and pre-positron at a mutual distance $d \approx 5\lambda$:



- ▶ X-ray FELs and multi-petawatt lasers (ELI, EP-OPAL, Apollon, ...) may test for the first time **strong-field non-perturbative QED** under controlled conditions.
- ▶ Strong interferences: Pair creation and *annihilation* will happen!
- ▶ Asymptotic particle production:
 $-1 \text{ MeV} \lesssim p_{\parallel} \lesssim 1 \text{ MeV}$ and $k_{\perp} \lesssim 1 \text{ MeV}$
- ▶ Solution within DHW formalism for **spatially inhomogeneous electric fields!** Shown here: Results in 1+1 dimensions!
- ▶ Solution within DHW formalism for **inhomog. time-dep. electromagnetic fields!** (Model field: eff. 2+1 dimensional)

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- ▶ First results for **particle formation time scales!**
- ▶ For the employed model field in the sub-critical Schwinger regime there is
 - 1 a time scale $T_1 \approx 0.65 \tau^{1/4} t_c^{3/4} / \varepsilon^2$ for the appearance of a peak in the particle distribution related to a pre-particle;
 - 2 a time scale $T_2 = T_1 + 0.06 \tau^{3/4} t_c^{1/4} / \varepsilon^{3/2}$ at which the pre-particle identifiable follows a classical trajectory; and
 - 3 a time scale $T_3 = 1.8 \tau$ at which quantum fluctuations (quantum interference and the 'perturbative' peak related to a would-be shutoff) fade out.

▶ Include

- first classical and
- then quantum

back-reaction and study Sauter-Schwinger effect at critical and super-critical field strengths:

- Sub-processes of particle production?
- Generic patterns for the time scales?
- Formation of a QED cascade and fundamental limitation for physically achievable field strengths?

▶ Apply real-time Quantum Field Theory to non-equilibrium physics in cosmology, astro-, molecular & atomic, particle physics,

To appear soon:

Spinning Pairs: Supporting 3P_0 Quark-Pair Creation from Landau Gauge Green's Functions

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Dept. Física Teórica and IPARCOS, Univ. Complutense de Madrid, Plaza de las Ciencias 1, 28040 Madrid, Spain*

(Dated: December 17, 2023)

Abundant phenomenology suggests that strong decays from relatively low-excitation hadrons into other hadrons proceed by the creation of a light quark-antiquark pair with zero total angular momentum, the so called 3P_0 mechanism originating from a scalar bilinear. Yet the Chromodynamics interaction is perturbatively mediated by gluons of spin one, and QCD presents a chirally symmetric Lagrangian. Such scalar decay term must be spontaneously generated upon breaking chiral symmetry. We attempt to reproduce this with the help of the quark-gluon vertex in Landau gauge, whose nonperturbative structure has been reasonably elucidated in the last years, and insertions of a uniform, constant chromoelectric field. This is akin to Schwinger pair production in Quantum Electrodynamics, and we provide a comparison with its two field-insertions diagram. We find that, the symmetry being cylindrical, the adequate quantum numbers to discuss the production are rather ${}^3\Sigma_0$, ${}^3\Sigma_1$ and ${}^3\Pi_0$ as in diatomic molecules, and we indeed find a sizeable contribution of the third decay mechanism, which may give a rationale for the 3P_0 phenomenology, as long as the momentum of the produced pair is at or below the scale of the bare or dynamically generated fermion mass. On the other hand, ultrarelativistic fermions are rather ejected with ${}^3\Sigma_1$ quantum numbers. In QED our results suggest that ${}^3\Sigma_0$ dominates whereas the constraint of producing a color singlet in QCD leads to ${}^3\Pi_0$ at sub-GeV momenta.

