RP & F. Rennecke: 2401.06130

Why is it SO hard to find the χPT light gbs $\bar{c}$ THREE flavors?

GR8 surprise from the lattice

No 1st order for m_χ^{crit}

$m_\chi^{crit} < 50$ Bazavov + ... 1701.03548

" < 100 Kuramashi + ... 2001.04398

" < 90 Dini + ... 2111.12599

$m_\chi^{crit} = 0$!! Cuteri, Philipsen & Sciarra 2107.12739

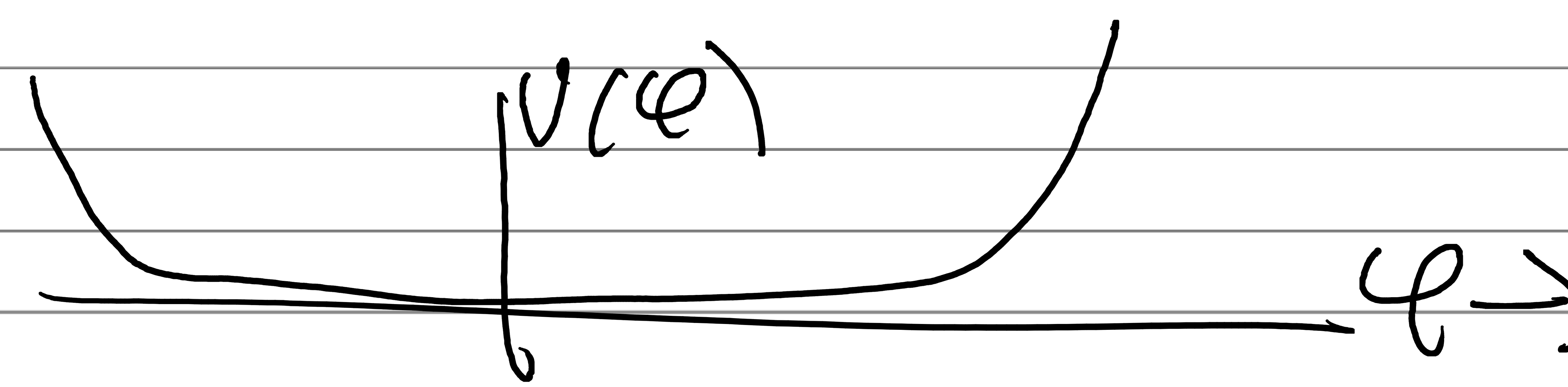
Also, why Parganlija + ... 1208.0585 is good 1511.05035, 1605.05154, 1612.09218

φ^3 & 1st order PT

Assume inv. $\varphi \to -\varphi$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2} m^2 \varphi^2 + \lambda \varphi^4$$

$m^2 = 0$: 2nd order

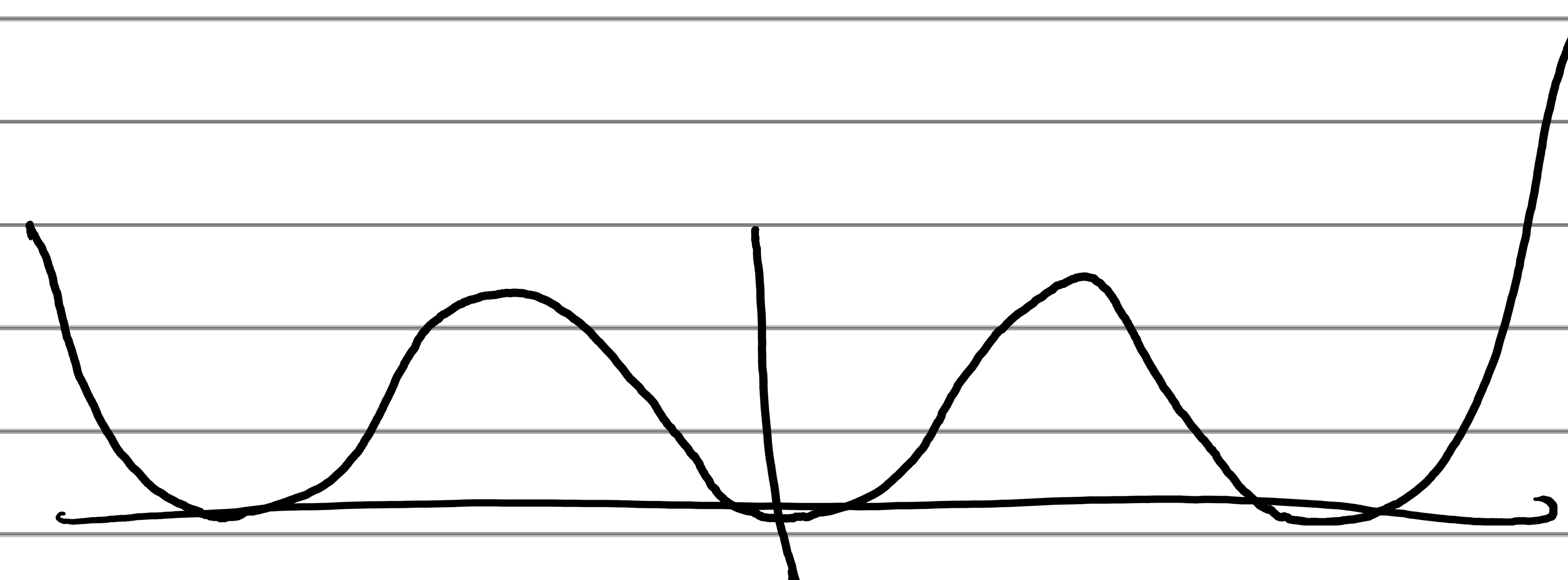


Not $\varphi \to -\varphi$. Shift so $\langle \varphi \rangle = 0$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{m^2}{2} \varphi^2 - \kappa \varphi^3 + \lambda \varphi^4$$

$m^2 = 0$:

$m^2 > 0$:



$\Rightarrow$ $\underline{\underline{1}}$st order

Two examples

SU(3), pure gauge: 1-form sym. Gaiotto+... 1412.5148

$$\ell = \mathrm{tr}\,\mathbb{P}\, e^{ig\int_0^{1/T} A_0\, dx} \longrightarrow e^{2\pi i/3}\,\ell$$

$Z(3)$

$\Rightarrow \ell^3 + \text{c.c.} = Z(3)$ inv. $\Rightarrow$ 1st order

χPT: $\underline{m_{qk} = 0}$

$$\Phi = \bar{q}_L q_R \longrightarrow e^{2\pi i/N_f}\,\Phi$$

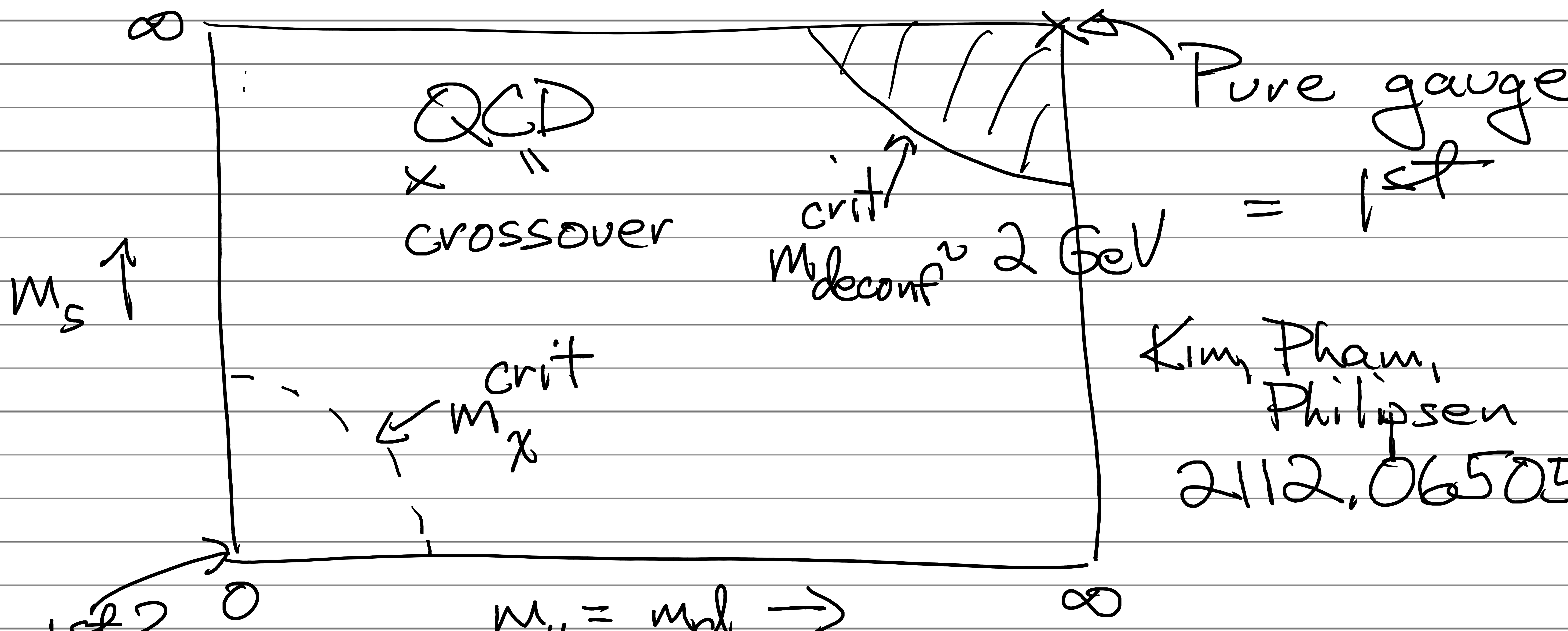
$2\pi i/N_f \longrightarrow$ # flavors

$\det \Phi \Rightarrow$ <u>same</u> = <u>cubic</u> for $N_f = 3$

= 1st order

But an<u>omalous</u> int. $\Rightarrow$ makes η' heavy

Columbia phase diagram



Kim, Pham, Philipsen 2112.06505

$\chi = 1^{st}$?

$\eta' =$ <u>heavy</u> $\Rightarrow m_\chi^{crit} =$ <u>large</u>

$m_\chi^{crit} = 150$ MeV, Schaefer & Wagner 0808.1491

86 Resch, Rennecke & Schaefer 1712.07961

110 (RDP) & Skokov, unpub'd

Lattice - <u>NO</u> m_χ^{crit} seen

χ sym

$\underline{m_q = 0}$: $\quad q_{L,R} \to e^{i\theta_V \mp i\theta_A/2}\, U_{L,R}\, q_{L,R}$

θ_V : $U(1)$ q # $= L+R$

θ_A : $\underline{\text{axial}}$ q # $= L-R$

$U_{L,R} = SU(N_f)_{L,R}$

$\Phi = \bar{q}_L q_R \to e^{i\theta_A}\, U_L^\dagger\, \Phi\, U_R$ $\quad$ ind. θ_V

$\Rightarrow \Phi^\dagger \Phi \to U_R^\dagger\, \Phi^\dagger \Phi\, U_R$ $\quad$ $\underline{\text{ind. } \theta_A}$

Eff. Lag.'s

$U_A(1)$ sym: $\quad \mathrm{tr}\, \Phi^\dagger \Phi$

$$(\mathrm{tr}\, \Phi^\dagger \Phi)^2, \quad \mathrm{tr}\, (\Phi^\dagger \Phi)^2 + \ldots,$$

$\cancel{U_A(1)}$: $\quad \det \Phi \to \det\left(e^{i\theta_A} U_L^\dagger \Phi U_R\right)$

$$= e^{i N_f \theta_A} \det \Phi$$

$\Rightarrow$ inv. if $\theta_A = \frac{2\pi}{N_f}$ $\Rightarrow$ $Z(N_f)$ inv.

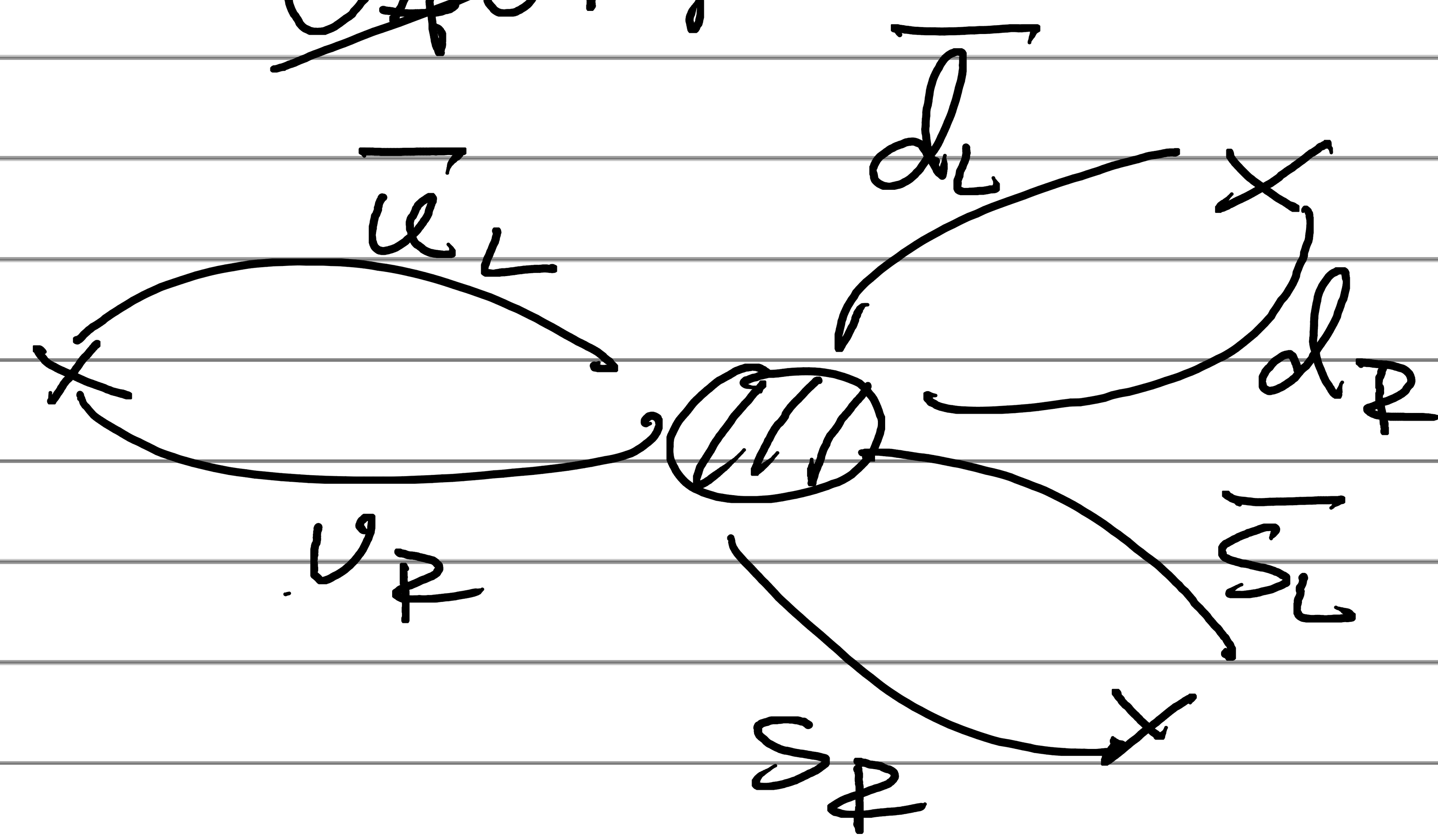
part of $SU(N_f)_{L,R}$

not $U(1)_A$

$\cancel{U_A(1)}$

$\det \Phi$;

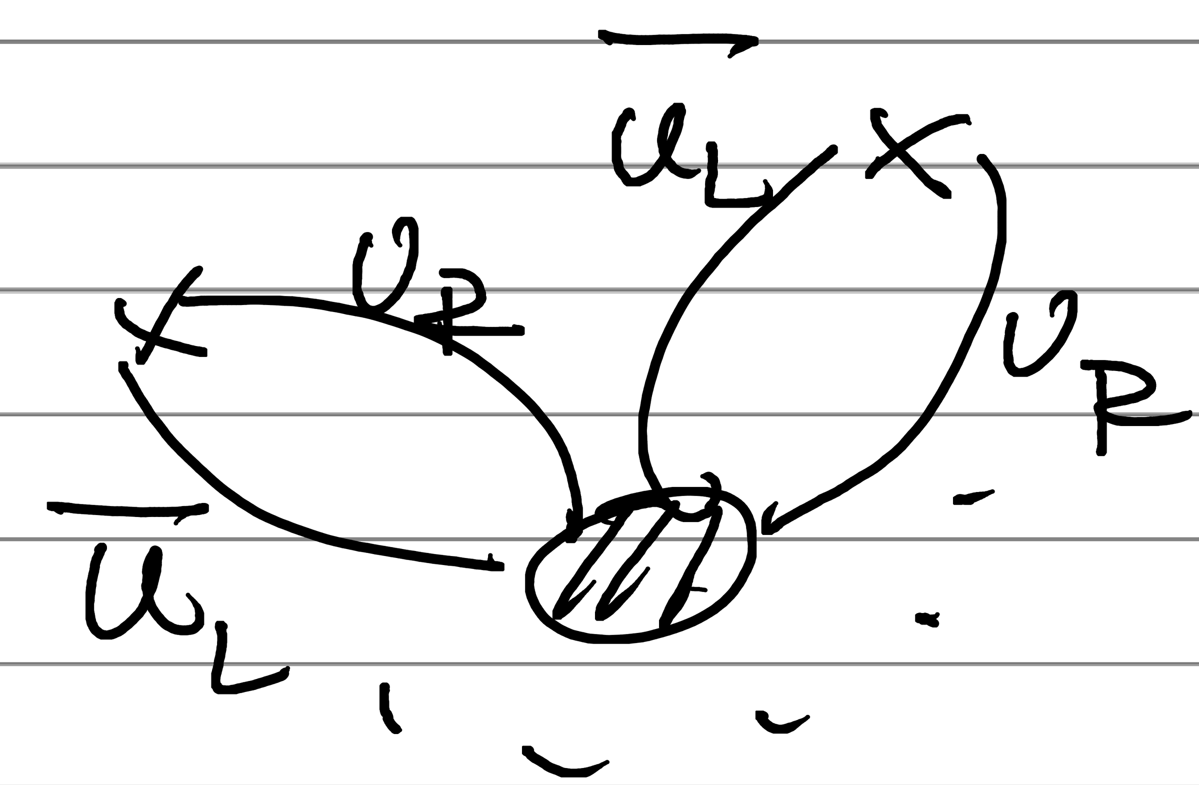
$Q_{top} = \underline{1}$



zero modes
instanton
$Q = \pm 1$

$(\det \Phi)^2$;

$Q_{top} = 2$



$Q = \pm 2$

RP & Rennecke, 1910.14052
" , 2003.13876

CP inv. :

$$(\det \Phi)^2 + (\det \Phi^\dagger)^2$$

Parganlija +, .., :

$$(\det \Phi - \det \Phi^\dagger)^2$$

$$= (\det \Phi)^2 + (\det \Phi^\dagger)^2 - 2 \det \Phi^\dagger \Phi$$

$$\parallel$$

poly. in $(\Phi^\dagger \Phi)^{N_f}$

Mean Field:

$$V = m^2 \operatorname{tr} \Phi^\dagger \Phi + \lambda_1 (\operatorname{tr} \Phi^\dagger \Phi)^2 + \lambda_2 \operatorname{tr} (\Phi^\dagger \Phi)^2$$

$$\xi_1 \det \Phi + \xi_1' \operatorname{tr} \Phi^\dagger \Phi \det \Phi + \xi_2 (\det \Phi)^2 + \ldots$$

Usual fit to $T = 0$ spectra:

$m^2, \lambda_1, \lambda_2, \xi_1 \neq 0 \quad \xi_1', \xi_2 = 0$

" " " $, \xi_2 \neq 0 \quad \xi_1, \xi_1' = 0$!

Parganlija + ...

<u>Need</u> ξ_1 <u>or</u> $\xi_2 \neq 0$ so η' heavy

SO: m_χ^{crit} is large - ?

Usual Wilson: operators det'd just by mass dim:

$$\mathrm{tr}\,\phi^\dagger\phi > (\mathrm{tr}\,\phi^\dagger\phi)^2 > (\mathrm{tr}\,\phi^\dagger\phi)^3 \text{ etc}$$

If true for $\cancel{U_A(1)}$,

$$\det\phi > (\det\phi)^2$$

Right? RP & FR – <u>no</u>.

Determine $\mathcal{L}_{eff}$ from size $\sim r_\chi$

Dominant op. from $\langle Q\rangle$ in r_χ.

<u>Conjecture</u>: $\langle Q\rangle_{r_\chi} > 1$

$\Rightarrow$ not $\det\Phi$

but $(\det\Phi)^{\mathrm{int.}\langle Q\rangle}$

What if $m_\chi^{cut} = 0$?

Unnatural for $\xi_1(T_\chi) = 0$.

Speculate

Perhaps ALL $\xi_j^{k,l,\dots}(T_\chi) = 0$

$\Rightarrow U_A(1)$ restored - dynamically - @ T_χ

Not 1 miracle, but an ∞.

$U_A(1)$ restored only @ T_χ.

If $m_\chi^{crit} = 0$: 2^{nd} order trans.?

$G_{cl} = SU(N_f)_L \times SU(N_f)_R \times U(1)_A$

To $\sim \varepsilon$ in $4-\varepsilon$ dim's : 1^{st} order

RP & Wilczek '84

True $\sim \varepsilon^5$!

Calabrese & Parruccini ph/0403140

Also MC in 3-dim's

Sorokin 2105.00072
2205.07199

Conformal bootstrap:
NEW IR stable fixed point for G_{cl}

Nakayama & Ohtsuki 1407.6195

Henriksson, Kousvos & Stergiou 2004.14388
" " 2209.02837

FRG: Fejos 2201.07909

$\Rightarrow$ 2^{nd} order PT possible for G_{cl}!

If $(\det \Phi)^2$ dominates : $\xi_2(0) \gg \xi_1(0)$

$T = 0$: $\sim \Phi_0^3 \det \Phi$

$T = T_\chi$: $\sim \xi_1(T_\chi) \det \Phi$

$N_f = 3$: weakly 1^{st} order OR 2^{nd} ?

$N_f = 2$: 2^{nd} order : $SU(2)_L \times SU(2)_R$

$\Rightarrow$ $\quad \shortparallel \quad \shortparallel \quad \times\, U(1)_A$

$\Rightarrow$ $m_\eta(T_\chi)$ small OR 0 ?

Brandt + ... 1608.06882 :

$m_\eta(T_\chi) \sim \frac{1}{10} m_\eta(0)$

$N_f = 1$

$\Phi = \phi + i\eta \qquad \det \Phi + \text{c.c.} = 2\phi$

$$V_{\text{gen}} = \xi_1 \phi + m^2(\phi^2 + \eta^2) + \xi_2(\phi^2 - \eta^2)$$
$$+ \xi_3 \phi(\phi^2 - \eta^2) + \xi_1^{(1,1)} \phi(\phi^2 + \eta^2)$$
$$+ \lambda(\phi^2 + \eta^2)^2 + \ldots$$

Naively: NO trans. $\langle\phi\rangle \sim \xi_1 \xrightarrow[T\to\infty]{} 0$

as instantons evaporate

If ALL $\xi_i^{\cdots}(T_\chi) = 0$, $U_A(1)$ restored @ T_χ

$\Rightarrow$ <u>2nd</u> order χPT.

Real test. <u>Not</u> easy on lattice

T & μ

Before: <u>un</u>natural for m^2 and $\xi_1 = 0$ @ T_χ

With $T \,\underline{\&}\, \mu$, $m^2 \,\underline{\&}\, \xi_1 = 0$ at <u>one</u> (T^A, μ^A)

Near (T^A, μ^A): light $\eta' \Rightarrow$ light η

HADES: # η's $\sim 2 *$ stat. dist.
Nature 15, 1040 (2019)

Decays of η & η' also change

Isospin, CP violation?

If ξ_1^{eff} small, Goldstone Bosons eigenstates of flavor:

$\pi^0 \sim \bar{u} u \qquad \eta \sim \bar{d} d \qquad \eta' \sim \bar{s} s$

$\Rightarrow$ isospin violation   RDP + Wilczek '84

Seen by HADES: 2312.06572 ?
2312.07176

$\cancel{CP}$: if $\xi_1(T,\mu)$ flips sign,

$\langle \Phi \rangle$: $\langle \phi \rangle \neq 0 \Rightarrow \underbrace{\langle \eta' \rangle \neq 0}_{\text{CP-\underline{odd} condensate}}$

for <u>some</u> region in plane of T + μ