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Complete Relativistic Description of the $N^*(1520)$

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1 Introduction

In the last years there has been a large interest in the description of hadronic properties in a strongly interacting medium. This interest is induced by the fact that various experiments give indications for modifications of the mass and the width of hadrons when put into the medium. The changes of hadron masses in a medium are probably related to the restoration of chiral symmetry as one of the fundamental symmetries of QCD which is spontaneously broken in the vacuum (for further discussion see [RW00]).

The ρ -meson is an ideal probe for in-medium studies because it decays into dileptons. They can travel nearly undisturbed through the medium making it possible to measure the in-medium properties. The in-medium effects are particularly strong for ρ -mesons at rest as could be shown by Post et al. [PLM01] and Würfel [Wür04]. Especially in a nucleon dominated medium (nuclear matter) scattering of such low energetic ρ -mesons with nucleons will create the $N^*(1520)$ resonance, making it especially important for the in-medium properties of the ρ -meson. A coupled channel in-medium calculation involving the $N^*(1520)$ and several other baryon resonances and in the meson sector ρ , π and η has been performed by Post et al. [PLM04]. Typically due to the complexity of the problem a number of approximations are involved in such calculations.

Especially the selfenergies of the considered particles are either calculated non-relativistically [PLM04] or at least a simplified spin structure is assumed [PLM01]. As a prelude to a more complete treatment it is important to understand the vacuum case in its full relativistic structure which can serve as a starting point for more involved in-medium computations.

In a previous relativistic calculation of Post et al. [PLM01] on the in-medium properties of the ρ -meson it could be shown that for a resonance with negative parity the mentioned approximation leads to a wrong sign for the imaginary part of the selfenergy of the resonance. This in turn leads to a negative cross section. By changing the propagator by hand it was possible to overcome this short coming. This shows that the relativistic effects can be non-trivial. A correct and fully relativistic description of the $N^*(1520)$ in the vacuum is therefore desirable.

A study of the correct description of higher spin particles is also of interest due to the non-trivial character of their interactions. Though field theoretical descriptions for higher spin particles were already introduced and discussed in the 40s of the last century by Fierz and Pauli [FP39] and Rarita and Schwinger [RS41], the question how to introduce couplings is still open. When electromagnetic interactions

are introduced via minimal coupling acausal propagations [VZ69] and non positive definite anticommutation relations [JS61] arise. In [Cox89] it was shown that both inconsistencies appear because the interaction violates the proper number of degrees of freedom (DOF) of the free theory.

A first analysis for hadronic interactions was done in the 70s. A still widely used coupling for $N\Delta\pi$ was proposed in [NEK71]. Though the authors claim in this paper that the above mentioned inconsistencies do not enter for a special choice of parameters, Hagen [Hag71] and Singh [Sin73] proved them wrong. Many general forms of interactions were ruled out on this ground. But still fully relativistic calculations were performed with these "inconsistent" couplings leading to reasonable description of experimental data [Kor97]. At the same time an idea was proposed how to introduce "consistent" couplings [PT99]. It is based on the finding that interactions which have the same symmetries as the massless free theory will not violate the DOF of the theory. The mass term will be introduced to break the symmetry and rises the number of DOF to the correct value. Because the interaction does not introduce further DOF the mass term which breaks the symmetry correctly in the free case will do it also in the interacting case. Such an approach always leads to a consistent interaction.

So far these calculations were performed only for the $\Delta(1232)$ isobar. The corresponding calculation for the $N^*(1520)$ is a non trivial extension for two reasons. First, it is a particle with negative parity leading to the above mentioned complications. Second, the $N^*(1520)$ decays also into unstable particles leading to a more complicated structure of the selfenergy.

In this work we will investigate the $\Delta(1232)$ and $N^*(1520)$ by calculating the full relativistic structure of the propagator. This will be done in a framework proposed by Pascalutsa and Timmermans [PT99]. The aim is to find out whether it is feasible to calculate fully relativistic propagators in the PASCALUTSA framework. One interesting finding will be that the selfenergies are actually simpler compared to the conventional approach.

For a further understanding of higher spin particles comparison with the experiment will be important. For a description of baryonic resonances formed e.g. in πN collisions a coupled-channel approach is needed including also background terms from t-channels etc. Typically such calculations do not consider the full selfenergy structure of the baryon resonances. To give an example, in K-matrix calculations [PM02] only the on-shell part of the involved two-particle propagators is taken into account. In this way analyticity is violated to some extent. The present work aims at a fully relativistic calculation of baryon resonance properties respecting all constraints of a local field theory like analyticity and unitarity. (Note that also a K-matrix calculation is unitary by construction.) On the other hand, as compared to coupled-channel calculations, the present work is more modest by concentrating mainly on the $N^*(1520)$ and the $\Delta(1232)$. Therefore a detailed description of scattering data is beyond the scope of the present work.

The work is structured in the following way:

Chapter 2 is an introduction to the field theoretical description of massive higher spin fields. Especially the PROCA [Pro36] and RARITA-SCHWINGER [RS41] equations will be derived. Possible free LAGRANGIANS leading to these equations of motion will be introduced. The free propagator will be given for spin 1 and spin 3/2 particles. Interactions of spin 3/2 resonances will be discussed and the PASCALUTSA formalism introduced as a possibility to overcome inconsistencies. This chapter gives a foundation for the calculation of the selfenergies which will be performed in chapter 3. First the selfenergy of the ρ -meson will be calculated and discussed giving a good example of the treatment of higher spin fields. Afterwards the calculation of the selfenergy for spin 3/2 particles in the PASCALUTSA framework will be introduced.

In chapter 4 the spectral function will be motivated using the KÄLLEN-LEHMANN representation. The dressed propagator is calculated by the DYSON-SCHWINGER equation. With the dressed propagator an analytical form for the spectral functions can be found. The spectral function of the ρ -meson is calculated explicitly and discussed as an easy example of a higher spin spectral function. The width and the mass of a resonance will be defined by comparing the spectral function to a BREIT-WIGNER form. Later the spectral functions for spin 1/2 and spin 3/2 particles are presented and the width and the mass of fermionic resonances will be discussed.

In chapter 5 the couplings for the three main channels of the $N^*(1520)$ are introduced and their selfenergies calculated. This will be presented in a somewhat more general way by including not only the $N^*(1520)$ resonance but also particles with positive parity and different isospin. To give a better understanding of the results the non-relativistic limit of the widths will be calculated and compared to non-relativistic calculations.

The specific results for the $\Delta(1232)$ and $N^*(1520)$ will be presented in chapter 6. In the beginning the parameters used for the calculations are presented. Next the selfenergies are depicted and discussed for different cut-off parameters. The widths are discussed next followed by the spectral functions. They will be discussed for different parameters and compared to simplified versions.

In chapter 7 a summary of the main findings will be given.

In the appendices we present some details about how to calculate the number of DOF for a theory with constraints.

2 Massive Higher Spin Fields

In this chapter we will introduce the fields and equations of motion for massive particles with arbitrary spin and especially for particles with spin 1 and spin 3/2, these are the massive vector and the RARITA-SCHWINGER fields. We will introduce suitable LAGRANGIANS for the free theory and check whether they include the correct number of degrees of freedom (DOF) using the method introduced in appendix A. Interactions for the vector mesons will be introduced in a way preserving the DOF of the free theory. In the case of the spin 3/2 particles we will discuss problems which occur when interactions are included. As an example of a consistent interaction for spin 3/2 particles the PASCALUSA formalism will be discussed. In this formalism the interaction is introduced without spoiling the number of DOF of the free theory. The correspondence between the PASCALUTSA and the *conventional* couplings will be investigated.

2.1 Fields and Equations of Motion

We will construct fields obeying the DIRAC equation for massive particles with higher spin. The starting point will be rather general but later it will be specified to the case of spin 1 and spin 3/2 particles. This derivation is based on [Gre87].

In the rest frame of the particle the solutions of the free DIRAC equation are given as

$$\begin{aligned}\psi^r &= \omega^r(0)e^{-i\epsilon_r mt} & r &= 1 \dots 4 \\ \epsilon_r &= \begin{cases} +1 & r = 1, 2 \\ -1 & r = 3, 4 \end{cases} \\ \omega_i^r(0) &= \delta_{ir} & i &= 1 \dots 4.\end{aligned}$$

Here r labels the different solutions and i the components of the spinor.

Without loss of generality we can take the positive energy solutions and call them $\omega^{(+)}$. It is possible to construct a tensor product

$$\begin{aligned}\tilde{\omega}_{a_1 a_2 \dots a_{2s}} &= \omega_{a_1}^{(+)}(0) \dots \omega_{a_{2s}}^{(+)}(0) \\ &= \delta_{a_1 r_1} \dots \delta_{a_{2s} r_{2s}}.\end{aligned}$$

We have constructed the product in such a way leading to $2s$ indices. Now we search for the totally symmetric part of the above tensor product. There are $2s + 1$ possibilities to do this

$$\begin{aligned}
 \omega_{a_1 \dots a_{2s}}(0, i = 0) &= \delta_{a_1 1} \delta_{a_2 1} \dots \delta_{a_{2s} 1} \\
 \omega_{a_1 \dots a_{2s}}(0, i = 1) &= \delta_{a_1 2} \delta_{a_2 1} \delta_{a_3 1} \dots \delta_{a_{2s} 1} + \delta_{a_1 1} \delta_{a_2 2} \delta_{a_3 1} \dots \delta_{a_{2s} 1} + \dots + \\
 &\quad + \delta_{a_1 1} \delta_{a_2 1} \delta_{a_3 1} \dots \delta_{a_{2s} 2} \\
 &\quad \vdots \\
 \omega_{a_1 \dots a_{2s}}(0, i = 2s) &= \delta_{a_1 2} \delta_{a_2 2} \dots \delta_{a_{2s} 2}.
 \end{aligned}$$

All $\psi^r(x)$ are also eigen-functions of the total spin operator

$$\Sigma_{a_1 a'_1}^3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

where σ_3 is the third Pauli matrix defined as

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then the totally symmetric spinor as constructed above is also an eigen-vector of the totally symmetric spin operator. It can be constructed in this way

$$\begin{aligned}
 \frac{1}{2} \hbar \Sigma_{a_1 a'_1 a_2 a'_2 \dots a_{2s} a'_{2s}}^3 &= \frac{1}{2} \hbar \Sigma_{a_1 a'_1}^3 \delta_{a_2 a'_2} \dots \delta_{a_{2s} a'_{2s}} + \frac{1}{2} \hbar \Sigma_{a_2 a'_2}^3 \delta_{a_1 a'_1} \dots \delta_{a_{2s} a'_{2s}} + \dots + \\
 &\quad + \dots + \frac{1}{2} \hbar \Sigma_{a_{2s} a'_{2s}}^3 \delta_{a_1 a'_1} \dots \delta_{a_{2s-1} a'_{2s-1}} \quad .
 \end{aligned}$$

When calculating the eigen-functions for each symmetric tensor product the solutions are

$$\frac{1}{2} \hbar \Sigma^3 \omega^{(+)}(0, j) = \hbar(s - j) \omega^{(+)}(0, j).$$

Because $j = 0, \dots, 2s$ the total number of eigen-vectors are $2s + 1$ and the eigen-solutions are $s, s - 1, \dots, -s + 1, -s$. This shows that the symmetric multispinors defined above are the correct fields to describe a particle with spin s where the z component can take $2s + 1$ different values.

Fields for particles with negative energy can be constructed analogously. Now it is possible to boost the spinor to get the general fields

$$\begin{aligned}
 \psi_{a_1 \dots a_s}^{(+)}(x, p, i) &= \omega_{a_1 \dots a_s}^{(+)}(\mathbf{p}, i) e^{-ip \cdot x}, \\
 \psi_{a_1 \dots a_s}^{(-)}(x, p, i) &= \omega_{a_1 \dots a_s}^{(-)}(\mathbf{p}, i) e^{+ip \cdot x}.
 \end{aligned}$$

The total field is the superposition of these fields. They are the solutions of the DIRAC equation for every index, the so called BARGMANN-WIGNER equations [BW48]:

$$\begin{aligned} (i\gamma \cdot \partial - m)_{a_1 a'_1} \psi_{a'_1 a_2 \dots a_{2s}}(x) &= 0, \\ &\vdots \\ (i\gamma \cdot \partial - m)_{a_{2s} a'_{2s}} \psi_{a_1 a_2 \dots a'_{2s}}(x) &= 0. \end{aligned} \tag{2.1}$$

Because each component is a solution of the DIRAC equation it is also a solution of the KLEIN-GORDON equation

$$(\partial_\mu \partial^\mu + m^2) \psi_{a_1 a_2 \dots a_{2s}}(x) = 0. \tag{2.2}$$

The $\psi_{a_1 a_2 \dots a_{2s}}(x)$ represent particles with mass m and spin s . This can be concluded because it has $(2s+1)$ linear independent components where each of them is an eigenfunction of the total spin operator. Additionally all components obey equation (2.1) and (2.2).

2.2 The Proca Equation

From the general consideration above we want to calculate the equations of motion for vector fields. Vector particles have spin 1, which means that the BARGMANN-WIGNER fields have two indices. The two DIRAC equations of the symmetric matrix ψ_{ab} are of the form of equation (2.1)

$$\begin{aligned} (i\gamma \cdot \partial - m)_{aa'} \psi_{a'b}(x) &= 0, \\ (i\gamma \cdot \partial - m)_{bb'} \psi_{ab'}(x) &= 0. \end{aligned}$$

Because ψ_{ab} is a symmetric 4×4 spinor it can be expanded in the CLIFFORD algebra \mathbb{D} of the form

$$\psi_{ab} = (\mathbb{1})_{ab} s + (\gamma^\mu)_{ab} v_\mu + (\sigma^{\mu\nu})_{ab} t_{\mu\nu} + (\gamma^5)_{ab} p + (\gamma_5 \gamma^\mu)_{ab} w_\mu,$$

where $\sigma^{\mu\nu}$ is defined as:

$$\sigma^{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]. \tag{2.3}$$

ψ is symmetric which means that $\psi^T = \psi$. This would transpose the elements of the CLIFFORD algebra, which will give constraints on the coefficients. But it is not an easy calculation. Because the charge conjugation operator $C = i\gamma_0 \gamma_2$ can be used to transpose DIRAC gamma matrices, $C\gamma^\mu C = (\gamma^\mu)^T$, it is more convenient to expand ψ in elements of $\mathbb{D}C$. This leads to

$$\psi_{ab} = (C)_{ab} s + (\gamma^\mu C)_{ab} v_\mu + (\sigma^{\mu\nu} C)_{ab} t_{\mu\nu} + (\gamma^5 C)_{ab} p + (\gamma_5 \gamma^\mu C)_{ab} w_\mu.$$

Some basic properties of C are

$$\begin{aligned} C &= i\gamma^2\gamma^0 = -C^{-1} = -C^\dagger = -C^T \\ &\Rightarrow C^2 = -C^{-1}C = -1 \\ &\Rightarrow C^T C = 1 \end{aligned}$$

Now we can calculate the symmetry property of each of the elements of $\mathbb{D}C$.

$$\begin{aligned} C^T &= -C \Rightarrow \text{antisymmetric} \\ (\gamma^\mu C)^T &= C^T (\gamma^\mu)^T = C^T C \gamma^\mu C = \gamma^\mu C \Rightarrow \text{symmetric} \\ (\sigma^{\mu\nu} C)^T &= \frac{1}{2} (\gamma^\mu \gamma^\nu C - \gamma^\nu \gamma^\mu C)^T \\ &= \frac{1}{2} (C^T C \gamma^\nu C C \gamma^\mu C - C^T C \gamma^\mu C C \gamma^\nu C) = -\sigma^{\nu\mu} C = \sigma^{\mu\nu} C \Rightarrow \text{symmetric} \\ (\gamma^5 C)^T &= C^T (\gamma^5)^T = -C \gamma^5 = -\gamma^5 C \Rightarrow \text{antisymmetric} \\ (\gamma^\mu \gamma^5 C)^T &= C^T (\gamma^5)^T (\gamma^\mu)^T = \gamma^5 \gamma^\mu C = -\gamma^\mu \gamma^5 C \Rightarrow \text{antisymmetric} \end{aligned}$$

Because ψ is totally symmetric, only the symmetric quantities of $\mathbb{D}C$ can have non vanishing values. ψ has then the form

$$\Psi_{ab} = (\gamma^\mu C)_{ab} v_\mu + (\sigma^{\mu\nu} C)_{ab} t_{\mu\nu}$$

with v_μ as a vector and $t_{\mu\nu}$ an antisymmetric covariant tensor. From the BARGMANN-WIGNER equation it follows that

$$\begin{aligned} (i\gamma \cdot \partial - m)_{aa'} ((\gamma^\mu C)_{a'b} v_\mu(x) + (\sigma^{\mu\nu} C)_{a'b} t_{\mu\nu}(x)) &= 0, \\ (i\gamma \cdot \partial - m)_{bb'} ((\gamma^\mu C)_{ab'} v_\mu(x) + (\sigma^{\mu\nu} C)_{ab'} t_{\mu\nu}(x)) &= 0. \end{aligned}$$

One sees that the coefficients v_μ and $t_{\mu\nu}$ do not take part in the matrix multiplication. As a consequence it is possible to take them out to the right of all γ matrices. Using the relation $C\gamma_\mu^T = -\gamma_\mu C$ also C can be taken to the right. This leads to the equations

$$\begin{aligned} (i\gamma^\alpha \gamma^\mu \partial_\alpha v_\mu - m\gamma^\mu v_\mu + i\partial_\alpha \gamma^\alpha \sigma^{\mu\nu} t_{\mu\nu} - m\sigma^{\mu\nu} t_{\mu\nu}) C &= 0, \\ (-i\gamma^\mu \gamma^\alpha \partial_\alpha v_\mu - m\gamma^\mu v_\mu - i\partial_\alpha \sigma^{\mu\nu} \gamma_\alpha t_{\mu\nu} - m\sigma^{\mu\nu} t_{\mu\nu}) C &= 0. \end{aligned}$$

Adding both equations leads to

$$\sigma^{\alpha\mu} C (\partial_\alpha v_\mu - \partial_\mu v_\alpha - 2mt_{\alpha\mu}) - 2\gamma^\mu C (2\partial^\alpha t_{\alpha\mu} + mv_\mu) = 0.$$

Because the matrices $\sigma^{\alpha\mu} C$ and $\gamma^\mu C$ are linearly independent each term has to be zero separately. Introducing $F^{\mu\nu} = 2t^{\mu\nu}$ and $A^\mu = \frac{1}{m}v^\mu$ the equation given above

leads to the PROCA equation [Pro36]:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad (2.4)$$

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0. \quad (2.5)$$

Contracting equation (2.5) with ∂_μ leads to a constraint

$$m^2 \partial_\mu A^\mu = 0.$$

Using this constraint the equation of motion can be written as

$$\begin{aligned} (\square + m^2) A^\mu &= 0, \\ \partial_\mu A^\mu &= 0 \end{aligned} \quad (2.6)$$

giving it a similar form as for a scalar field. Alternative derivation and further information on the vector fields can be found in standard textbooks e.g. [Gas66] [Wei95][Ryd96].

2.2.1 The Lagrangian

For the equation of motion a LAGRANGIAN can be constructed of the form

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu. \quad (2.7)$$

We use Dirac's formalism for counting degrees of freedom (DOF) explained in appendix A and especially theorem 7 presented there to derive the number of DOF for a massive and for a massless vector field.

The generalized momentum derived from the LAGRANGIAN (2.7) is

$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_\mu)} = F^{\mu 0}.$$

One sees immediately the first primary constraint arising due to the antisymmetric structure of the field strength $F^{\mu\nu}$. The primary constraint is

$$\Pi^0 \approx 0 = \phi_1.$$

We can write the HAMILTONIAN as stated in equation (A.1)

$$\begin{aligned} \mathcal{H} &= \Pi^\mu \partial_0 A_\mu - \mathcal{L} \\ &= F^{r0} \partial_0 A_r + \frac{1}{4} F^{rs} F_{rs} + \frac{1}{2} F^{r0} F_{r0} - \frac{1}{2} m^2 A_\mu A^\mu \\ &= \frac{1}{4} F^{rs} F_{rs} - \frac{1}{2} F^{r0} F_{r0} + F^{r0} \partial_r A_0 - \frac{1}{2} m^2 A_\mu A^\mu \\ &= \frac{1}{4} F^{rs} F_{rs} + \frac{1}{2} \Pi^r \Pi_r - A_0 \partial_r \Pi^r - \frac{1}{2} m^2 A^0 A_0 + \frac{1}{2} m^2 A^r A_r. \end{aligned}$$

Now we need to work out the consistency check explained in section A.2.1

$$0 \approx [\Pi^0, \mathcal{H}].$$

When expanding the Poisson bracket one gets

$$\begin{aligned} [\Pi^0, \mathcal{H}] &= \frac{\partial \Pi^0}{\partial A_\mu} \frac{\partial \mathcal{H}}{\partial \Pi^\mu} - \frac{\partial \Pi^0}{\partial \Pi^\mu} \frac{\partial \mathcal{H}}{\partial A_\mu} \\ &= \partial_r \Pi^r + m^2 A_0 = 0. \end{aligned}$$

The above equation is of the form of category 2 in section A.2.1. This leads to the secondary constraints

$$\phi_2 = \partial_r \Pi^r + m^2 A_0.$$

Now we have to perform a consistency check for this constraint again:

$$0 \approx [\phi_2, \mathcal{H}] + u_1 [\phi_2, \phi_1].$$

Calculating the first term one gets

$$[\phi_2, \mathcal{H}] = \frac{\partial \phi_2}{\partial A_\mu} \frac{\partial \mathcal{H}}{\partial \Pi^\mu} - \frac{\partial \phi_2}{\partial \Pi^\mu} \frac{\partial \mathcal{H}}{\partial A_\mu} = -m^2 \partial_r A^r.$$

The second term gives

$$\begin{aligned} [\phi_1, \phi_2] &= [\Pi^0, \partial_r \Pi^r + m^2 A_0] \\ &= \frac{\partial \Pi^0}{\partial A_\mu} \frac{\partial \phi_2}{\partial \Pi^\mu} - \frac{\partial \Pi^0}{\partial \Pi^\mu} \frac{\partial \phi_2}{\partial A_\mu} \\ &= -m^2. \end{aligned} \tag{2.8}$$

When comparing these solutions with definition 1, in Appendix A, one finds that ϕ_1 and ϕ_2 are *first class* constraints for $m = 0$ and *second class* constraints for $m \neq 0$. The mass plays a crucial role for the number of DOF in this theory.

Now we discuss further the case of massless particles. As long as category 1 or 3 is not fulfilled, one has to impose the consistency checks on each constraint. This has to be done until category 1 or 3 in section A.2.1 is fulfilled. It is trivially fulfilled for $m = 0$. For $m = 0$ the primary constraint ϕ_1 and secondary constraint ϕ_2 are *first class* constraints and the consistency conditions reduce to $0 = 0$. This means we have found all constraints and the extended HAMILTONIAN of equation A.15 is fully known as

$$H_E = H + v_1 \phi_1 + v_2 \phi_2.$$

The v 's are arbitrary functions leading to a gauge symmetry of the theory. Because we have found all constraints it is possible to count the degrees of freedom. The fields are real four-vectors which leads to eight DOF. We have two primary and secondary constraints reducing the DOF to six. Both constraints are first class constraints reducing the DOF further to four. Which leads to $2 + 2$ DOF as it should be for a massless spin 1 vector field e.g. the photon. (Note that we count separately field amplitude and its time derivative. This corresponds to position and velocity in a mechanical system.)

For the case of a massive vector field equation (2.8) does not reduce to zero leading to a consistency equation of category 3. This gives a solution for $u_1 = \partial_r A^r$. No further constraints arise so we can write the extended HAMILTONIAN as

$$H_E = H + u_1 \phi_1.$$

In contrast to the massless case u_1 is not an arbitrary function anymore. The massive vector field possesses no gauge symmetry. Because there are no first class constraints in the theory the number of DOF raises to $3 + 3$ as it should be for a massive spin 1 vector field.

2.2.2 The Propagator

Using the LAGRANGIAN of equation (2.7) the free propagator of the vector field [PLM04] can be derived. The mass m which appears in the LAGRANGIAN is the bare mass of the particle. The bare mass is in general not equal to the physical mass. To separate both masses the bare mass will be called m_0 and the physical mass m_R . Using this convention the undressed propagator has the form:

$$\begin{aligned}\mathcal{D}_0^{\mu\nu} &= \frac{1}{q^2 - m_0^2 + i\epsilon} P_T^{\mu\nu}(q) - \frac{1}{m_0^2} P_L^{\mu\nu}(q), \\ P_{\mu\nu}^T(q) &= g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \\ P_{\mu\nu}^L(q) &= \frac{q_\mu q_\nu}{q^2},\end{aligned}$$

with $P_{\mu\nu}^T(q)$ as the transverse and $P_{\mu\nu}^L(q)$ as the longitudinal projector. These projectors satisfy

$$\begin{aligned}P^L + P^T &= 1, \\ (P^L)^2 &= P^L, \\ (P^T)^2 &= P^T, \\ P^L P^T &= P^T P^L = 0, \\ q^\mu P_{\mu\nu}^T(q) &= q^\nu P_{\mu\nu}^T(q) = 0.\end{aligned}$$

2.2.3 Interactions

In section 2.2 a formalism for a massive vector field was introduced to describe a particle with spin 1 leading to an equation of motion and a constraint in (2.6). The constraint is needed because the field has 2×4 DOF whereas the physical massive and massless vector field have 2×3 and 2×2 internal DOF, respectively. In section 2.2.1 we showed that the free LAGRANGIAN (2.7) has the correct number of DOF for the massive and massless case. Furthermore it was shown that the massless case is invariant under a gauge transformation. The mass term breaks this gauge invariance changing all first class constraints to second class constraint. This rises the number of DOF to the right value.

For the free case this is fine but what will happen when interactions are introduced? These interactions can violate the constraints leading to unphysical DOF. It is desirable to have a formalism which starts from a correct free theory and introduces interactions correctly.

An interesting point to start with is the gauge invariance of the massless case. It is interesting because theorem 7 in appendix A says that gauge transformations with the same number of parameters and time-derivatives acting on these parameters will always lead to the same number of constraints. Thus by introducing an interaction term which possesses the same gauge transformations as the free massless theory one will have the same number of constraints.

For example couplings for photons can be introduced in this way. The field tensor $F^{\mu\nu}$ is invariant under the gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \epsilon$. A coupling of the form

$$igA_\mu j^\mu$$

with matter fields entering j will transform as

$$igA_\mu j^\mu \rightarrow ig(A_\mu j^\mu + \partial_\mu \epsilon j^\mu) = ig(A_\mu j^\mu - \epsilon \partial_\mu j^\mu).$$

Requiring that the interaction should also be gauge invariant under these transformation leads to the condition $\partial_\mu j^\mu = 0$ which is just current conservation.

Introducing a mass will break the gauge symmetry and change all first class constraints to second class constraints. But because the interaction has not introduced further constraints a mass term which rises the number of DOF to the correct number in the free case will also give the correct number of DOF in the interacting case. The presented scheme can be used generally for particles with spin ≥ 1 .

2.3 The Rarita Schwinger Equation

Now we work out the fields for spin 3/2 particles. First we will calculate the equations of motion for spin 3/2 particles. In this case the BARGMANN-WIGNER multispinor has $2s = 3$ indices and equations. The BARGMANN-WIGNER equations

are:

$$\begin{aligned}(i\gamma \cdot \partial - m)_{aa'}\Psi_{a'bc}(x) &= 0, \\ (i\gamma \cdot \partial - m)_{bb'}\Psi_{ab'c}(x) &= 0, \\ (i\gamma \cdot \partial - m)_{cc'}\Psi_{abc'}(x) &= 0.\end{aligned}$$

$\Psi_{abc}(x)$ is a totally symmetric DIRAC quantity and can be expanded in the CLIFFORD algebra \mathbb{D} . As discussed for the case of the vector meson it is more convenient to expand Ψ in \mathbb{DC} . Because Ψ is totally symmetric, only the symmetric quantities of \mathbb{DC} will give nonvanishing values. Because Ψ has now three indices it is possible to expand it in two different ways. First

$$\Psi_{abc} = (\gamma^\mu C)_{ab}v_{\mu c} + (\sigma^{\mu\nu} C)_{ab}t_{\mu\nu c},$$

this guarantees symmetry of the indices a and b with v_μ as a vector spinor and $t_{\mu\nu}$ an antisymmetric covariant tensor spinor but also

$$\Psi'_{abc} = (\gamma^\mu C)_{ac}v_{\mu b} + (\sigma^{\mu\nu} C)_{ac}t_{\mu\nu b},$$

which guarantees symmetry of the indices a and c . When both equations are equal then Ψ is totally symmetric with respect to its indices. To explore further constraints we contract both equations with an element of the CLIFFORD algebra $\Gamma_{bc} \in \mathbb{D}$

$$\begin{aligned}\Psi_{abc}\Gamma_{bc} &= (\gamma^\mu C)_{ab}\Gamma_{bc}(v_\mu)_c + (\sigma^{\mu\nu} C)_{ab}\Gamma_{bc}(t_{\mu\nu})_c, \\ \Psi'_{abc}\Gamma_{bc} &= (\gamma^\mu C)_{ac}\Gamma_{bc}(v_\mu)_b + (\sigma^{\mu\nu} C)_{ac}\Gamma_{bc}(t_{\mu\nu})_b \\ &\quad \pm (\gamma^\mu C)_{ac}\Gamma_{cb}(v_\mu)_b \pm (\sigma^{\mu\nu} C)_{ac}\Gamma_{cb}(t_{\mu\nu})_b \\ &= \pm \Psi_{abc}\Gamma_{bc}.\end{aligned}$$

The plus sign occurs when Γ_{bc} is symmetric. Then everything is alright because due to the symmetric structure $\Psi_{abc} = \Psi'_{abc}$. But when Γ_{bc} is antisymmetric then

$$\Psi_{abc}\Gamma_{bc} = 0.$$

Because there are three linear independent antisymmetric quantities in the CLIFFORD algebra we get three constraints. For simplicity we choose $\Gamma_{bc} \in C^T\mathbb{D}$. Then the three constraints are

$$\gamma^\mu v_\mu + \sigma^{\mu\nu} t_{\mu\nu} = 0, \tag{2.9}$$

$$\gamma^\mu \gamma_5 v_\mu + \sigma^{\mu\nu} \gamma_5 t_{\mu\nu} = 0, \tag{2.10}$$

$$\gamma^\mu \gamma_5 \gamma^\alpha v_\mu + \sigma^{\mu\nu} \gamma_5 \gamma^\alpha t_{\mu\nu} = 0. \tag{2.11}$$

Multiplication of the first equation with γ_5 from the left and subtraction and addition with (2.10) leads to equivalent equations:

$$\gamma_\mu v^\mu = 0, \tag{2.12}$$

$$\sigma_{\mu\nu} t^{\mu\nu} = 0. \tag{2.13}$$

Using equations (2.12) and (2.13) we want to investigate equation (2.11). First we multiply this equation with γ_5 . To use equations (2.12) and (2.13) we need to commute the γ^μ matrices and the $\sigma^{\mu\nu}$ matrices. The (anti)commutation relations are

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad (2.14)$$

$$[\gamma^\alpha, \sigma^{\mu\nu}] = 2(g^{\alpha\mu}\gamma^\nu - g^{\alpha\nu}\gamma^\mu). \quad (2.15)$$

Then (2.11) reduces to

$$v^\mu - 2\gamma_\nu t^{\nu\mu} = 0.$$

This equation already contains (2.13) since when we contract it with γ_μ and use (2.12) we obtain (2.13). To summerize we find:

$$\gamma_\mu v^\mu = 0, \quad (2.16)$$

$$v^\mu = 2\gamma_\nu t^{\nu\mu}. \quad (2.17)$$

These are $4 + 16 = 20$ linear equation for the $16 + 24 = 40$ components of v^μ and $t^{\mu\nu}$, which is the correct number of components for a total symmetric tensor of rank three in a four-dimensional vector space.

With these representations for Ψ we can try to get some information out of the BARGMANN-WIGNER equations. The first BARGMANN-WIGNER equation contracted with the first component of Ψ leads to

$$[(i\gamma^\alpha\partial_\alpha - m)\gamma_\mu C]_{ab} v_c^\mu + [(i\gamma^\alpha\partial_\alpha - m)\sigma_{\mu\nu} C]_{ab} t_c^{\mu\nu} = 0.$$

Contracting the second BARGMANN-WIGNER equation the second component of Ψ will transpose the DIRAC matrices

$$\begin{aligned} & [(i(\gamma^\alpha\partial_\alpha - m)\gamma_\mu C)]_{ab}^T v_c^\mu + [(i\gamma^\alpha\partial_\alpha - m)\sigma_{\mu\nu} C]_{ab}^T t_c^{\mu\nu} = 0 \\ \Leftrightarrow & [(i\gamma_\mu(\gamma^\alpha\partial_\alpha - m)C)]_{ab} v_c^\mu + [\sigma^{\mu\nu}(i\gamma^\alpha\partial_\alpha - m)C]_{ab} t_c^{\mu\nu} = 0. \end{aligned}$$

Subtracting these two equations and using the commutation relations leads to two equation

$$\begin{aligned} -2it^{\mu\nu} &= \frac{1}{m} (\partial^\mu v^\nu - \partial^\nu v^\mu), \\ -2\partial_\mu it^{\mu\nu} &= -mv^\mu. \end{aligned}$$

We define $\psi_{\mu\nu} = -2it_{\mu\nu}$ and $\psi^\mu = \frac{1}{m}v^\mu$. The above equation can be written in a more convenient way

$$\psi^{\mu\nu} = \partial^\mu \psi^\nu - \partial^\nu \psi^\mu, \quad (2.18)$$

$$\partial_\mu \psi^{\mu\nu} = -m^2 \psi^{\mu\nu}. \quad (2.19)$$

Because of the antisymmetry of $\psi^{\mu\nu}$ we find $\partial_\mu \psi^\mu = 0$. This leads to the usual KLEIN-GORDON equation

$$\square \psi^\mu + m^2 \psi^\mu = 0.$$

Now it is possible to use equation (2.17) to get a first order equation of motion for ψ^μ :

$$m\psi^\mu = i\gamma_\nu \psi^{\nu\mu} = i\gamma_\nu (\partial^\nu \psi^\mu - \partial^\mu \psi^\nu) = i\gamma \cdot \partial \psi^\mu.$$

In the last two steps we used equation (2.18) and (2.16). This equation is just the DIRAC equation for every LORENTZ component.

So a spin 3/2 particle can be described as a vector spinor field where all LORENTZ components fulfill the DIRAC equation

$$(i\gamma \cdot \partial - m)\psi^\mu = 0$$

and four constraints

$$\gamma_\mu \psi^\mu = 0.$$

All other constraints can be derived from these two equations. These equations are the RARITA-SCHWINGER equations [RS41] for spin 3/2 particles.

2.3.1 The Langrangian

There are different ways to find a LAGRANGIAN for the RARITA-SCHWINGER fields. Already in [RS41] a possible LAGRANGIAN was proposed. The first LAGRANGIANS for massive particles with arbitrary higher non-integer spin were introduced in [SH74]. Later also proposals for massless fields arose [FF78].

Already in the first paper of Rarita and Schwinger [RS41] a gauge invariance of the massless fields was pointed out. Later in [Cur79] it was shown that starting with a DIRAC field with j LORENTZ indices where each component obeys the DIRAC equation, unique LAGRANGIANS can be found for massless higher spin particles when requiring invariance with respect to a local gauge transformation of the form

$$\delta \Psi_{\mu_1 \dots \mu_j} = (1/j) [\partial_{\mu_1} \epsilon_{\mu_2 \dots \mu_j} + \dots + \partial_{\mu_j} \epsilon_{\mu_1 \dots \mu_{j-1}}]. \quad (2.20)$$

When looking for the most general free LAGRANGIAN one arbitrary parameter shows up which is fixed when requiring above gauge transformation. This is meant when talking about "unique" LAGRANGIANS. These LAGRANGIANS describe particles of spin $s = j + 1/2$.

As an example we want to show how this local transformation leads to the correct DOF for the case of spin 3/2 fields.

The spin 3/2 RARITA-SCHWINGER field has 16 complex components. Four from the DIRAC structure on which a LORENTZ structure is introduced. For the case of spin 3/2 fields equation (2.20) can be written

$$\psi_\mu \rightarrow \psi_\mu + \partial_\mu \epsilon. \quad (2.21)$$

With theorem 7 in appendix A it is easy to calculate the DOF for this theory. For a massless field we expect 2 components. Each component is complex-valued and has therefore 2 DOF. The number of independent components can be calculated as

$$2 \times N_{comp} - 2 \times N_I - N_{II}$$

and should yield 2×2 . Here N_{comp} is the number of complex components of the field. N_I is the number of first class constraint and N_{II} the number of second class constraints.

From theorem 7 we can calculate N_I . The components of the parameters of the local transformation is equal to the components of ϵ . Because ϵ is a DIRAC quantity it has $n = 4$ components. The highest order in the time derivative of the local transformation is $d = 1$ so

$$N_I = n \times (d + 1) = 4 \times (1 + 1) = 8.$$

The total number of second class constraints is according to theorem 7:

$$N_{II} = N_{comp} - n = 16 - 4 = 12.$$

Keeping in mind that every first class constraint reduced two DOF and every second class constraints one DOF the total number of DOF is then

$$2 \times 16 - 2 \times 8 - 12 = 4.$$

Thus a LAGRANGIAN of the field ψ_μ invariant with respect to the gauge transformation (2.21) leads to the correct number of DOF.

A suitable Lagrangian leading to the correct equation of motion and constraints is

$$\mathcal{L} = \frac{1}{2} \bar{\psi}_\mu \{ \sigma^{\mu\nu}, (i \not{\partial}) \} \psi_\nu$$

where $\sigma^{\mu\nu}$ is defined in (2.3). The invariance is readily checked by the identity:

$$\{ \sigma_{\mu\nu}, \gamma_\alpha \} = \varepsilon_{\mu\nu\alpha\lambda} \gamma^\lambda \gamma_5. \quad (2.22)$$

The massive case can be introduced by breaking the gauge symmetry. This will be generated by the mass term. Such an explicit breaking of the symmetry turns all first class constraints into second class constraints. This has to be done in such

a way that the total number of DOF is $2 \times (2s + 1)$, in our case of spin 3/2 particles 8. Introducing the mass term by replacing the derivative in the LAGRANGIAN by

$$\partial^\mu \rightarrow \partial^\mu + \frac{1}{4}iM\gamma^\mu$$

leads to the correct number of DOF [PT99]. This leads to a LAGRANGIAN of the form

$$\mathcal{L} = \frac{1}{2}\bar{\psi}_\mu\{\sigma^{\mu\nu}, (i\cancel{\partial} - M)\}\psi_\nu. \quad (2.23)$$

When starting with the requirement that each component of the field obeys the DIRAC equation and that the massless case is invariant under a gauge transformation of the form (2.21) leads to the correct LAGRANGIAN with respect to the DOF of the theory.

2.3.2 The Propagator

From the LAGRANGIAN of the free RARITA-SCHWINGER field derived in section 2.3.1 we can extract the propagator as the GREEN's function of the equation [dJM92]

$$\{\sigma_{\mu\nu}, (\cancel{p} - M)\} \mathcal{G}_0^{\nu\tau} = g_\mu{}^\tau. \quad (2.24)$$

The free RARITA-SCHWINGER propagator can be expanded in the basis of the projection operators of the spin states. The definition used here is taken from [dJM92] apart from a misprint there. The projection operators are given by

$$\begin{aligned} P^{3/2} &= g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3p^2}(\cancel{p}\gamma^\mu p^\nu + p^\mu\gamma^\nu\cancel{p}), \\ P_{11}^{1/2} &= \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{p^\mu p^\nu}{p^2} + \frac{1}{3p^2}(\cancel{p}\gamma^\mu p^\nu + p^\mu\gamma^\nu\cancel{p}), \\ P_{22}^{1/2} &= \frac{p^\mu p^\nu}{p^2}. \end{aligned}$$

For a complete system one also needs

$$\begin{aligned} P_{12}^{1/2} &= \frac{1}{\sqrt{3}p^2}(p^\mu p^\nu - \cancel{p}\gamma^\mu p^\nu), \\ P_{21}^{1/2} &= \frac{1}{\sqrt{3}p^2}(\cancel{p}p^\mu\gamma^\nu - p^\mu p^\nu). \end{aligned}$$

This set of projection operators satisfies the orthonormality and completeness conditions [VN81]

$$(P_{ij}^I)_{\mu\nu}(P_{kl}^J)^{\nu\delta} = \delta^{IJ}\delta_{jk}(P_{il}^J)_\mu{}^\delta, \quad (2.25)$$

$$P^{3/2} + P_{11}^{1/2} + P_{22}^{1/2} = g^{\mu\nu}. \quad (2.26)$$

Expanding the propagator in these projection operators reads

$$\mathcal{G}_0^{\mu\nu} = A P^{3/2} + B P_{11}^{1/2} + C P_{22}^{1/2} + D P_{12}^{1/2} + E P_{21}^{1/2}$$

where the indices of the projection operators are omitted.

Also the operator in equation (2.24) can be written in this basis leading to an equation for the propagator of the form

$$\left[(\not{p} - M) P^{3/2} - 2(\not{p} - M) P_{11}^{1/2} + \sqrt{3} M P_{12}^{1/2} + \sqrt{3} M P_{21}^{1/2} \right] \mathcal{G}_0^{\mu\nu} = P^{3/2} + P_{11}^{1/2} + P_{22}^{1/2}.$$

On the right hand side we used equation (2.26) the completeness of these projection operators. Using also the orthonormality conditions (2.25) it is possible to extract the solutions

$$\begin{aligned} A &= \frac{\not{p} + M}{p^2 - M^2}, \\ B &= 0, \\ C &= -\frac{2}{3M^2}(\not{p} + M), \\ D &= \frac{1}{\sqrt{3}M}, \\ E &= D, \end{aligned}$$

leading to the free propagator of the form

$$\mathcal{G}_0^{\mu\nu} = \frac{\not{p} + M}{p^2 - M^2} P^{3/2} - \frac{2}{3M^2} (\not{p} + M) P_{22}^{1/2} + \frac{1}{\sqrt{3}M} (P_{12}^{1/2} + P_{21}^{1/2}). \quad (2.27)$$

When inserting the projection operators the propagator can be written in a form often found in the literature [Kor97][PT99]

$$\mathcal{G}_0^{\mu\nu} = \frac{\not{p} + M}{p^2 - M^2} \left[g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^{\nu\nu} - \frac{2}{3M^2} p^\mu p^\mu + \frac{1}{3M} p^\mu \gamma^\nu - p^\nu \gamma^\mu \right].$$

An interesting observation is that only the part of the propagator proportional to the spin 3/2 state has a pole structure (first term in equation (2.27)). Thus, on-shell the propagator only propagates spin 3/2 fields.

2.3.3 Interacting Spin 3/2 Particles

For the case of spin 3/2 particles problems arise when the RARITA-SCHWINGER fields are coupled to photons via minimal coupling. Johnson and Sudarshan [JS61] showed

that such a coupling leads to a non-positive definite anticommutation relation. A few years later Velo and Zwanziger [VZ69] showed that the propagation becomes acausal. Later it was shown that both problems have the same origin [KT87][Jen74]. They arise due to the above mentioned violation of the constraints by the interaction [Cox89]. This leads to unphysical extra DOF which manifest themselves as a lower spin background.

A coupling of spin 3/2 fields to a nucleon and a pion was introduced 1971 in [NEK71] as the most general LAGRANGIAN containing only a first-order derivative. With h.c. as the hermitian conjugate it reads

$$\begin{aligned}\mathcal{L}_{int} &= \frac{f_{RN\pi}}{m_\pi} \bar{\psi}_\mu \Theta^{\mu\nu} \Psi \partial_\mu \phi + h.c., \\ \Theta^{\mu\nu} &= g^{\mu\nu} + \left(-Z - \frac{1}{2}\right) \gamma^\mu \gamma^\nu.\end{aligned}\tag{2.28}$$

This coupling is still widely used for hadronic interactions and often referred to as the *conventional* coupling to spin 3/2 fields. Z in the LAGRANGIAN is a parameter. The authors claimed in the original paper that when requiring that the interaction obeys the correct (anti)commutation relations Z has to take the value $\frac{1}{2}$.

As a reaction to this paper Hagen [Hag71] showed that this claim is incorrect because the anticommutation relations are non-positive definite. Later Singh [Sin73] showed that this coupling also propagates particles in an acausal way.

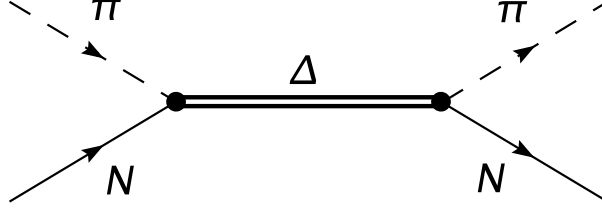
In 2.2.3 a possible solution was already discussed. Here we want to apply it to the case of spin 3/2 particles.

2.3.4 The Pascalutsa Formalism

In 2.2.3 a possible scheme to introduce consistent couplings was described. In section 2.3.1 it was shown that the free massless RARITA-SCHWINGER field is invariant under a gauge transformation of the form $\psi_\mu \rightarrow \psi_\mu + \partial_\mu \epsilon$. In the same section it was shown that the free massless field possesses the right number of DOF and a possible mass term was introduced that breaks the gauge invariance and rises the number of DOF to the correct value of $2 \times (2s + 1)$. An interaction LAGRANGIAN possessing the same gauge invariance as the free massless theory will not introduce further constraints. The mass term that breaks the symmetry in the correct way for the free case will do so also in the interacting case.

Note that the gauge invariance discussed here has nothing to do with the gauge invariance of the standard model. It is only needed as a tool to construct the correct interaction. Because it is explicitly broken by the mass term no physical symmetries arise.

As an example we will work out possible interactions for the easiest hadronic case, the $N\Delta\pi$ coupling. The lowest in derivative and explicitly gauge invariant $\pi N\Delta$


 Figure 2.1: FEYNMAN Δ -exchange graph.

interaction can be written as [Pas98]

$$\mathcal{L}_{int} = g\bar{\Psi}\sigma^{\mu\nu}(\partial_\mu\psi_\nu)\phi + h.c. \quad (2.29)$$

This LAGRANGIAN is gauge invariant under the transformation introduced in equation (2.21). The change after applying the gauge transformation is

$$\delta\mathcal{L}_{int} = g\bar{\Psi}\sigma_{\mu\nu}(\partial_\mu\partial_\nu\epsilon)\phi + h.c. = 0.$$

It is zero because $\partial_\mu\partial_\nu$ is symmetric under exchange of indices but $\sigma^{\mu\nu}$ is antisymmetric.

However this interaction is trivial because it couples to $\partial \cdot \psi$ and $\gamma \cdot \psi$, the spin 1/2 sector of the Δ . One sees it explicitly when writing $\sigma^{\mu\nu} = -g^{\mu\nu} + \gamma^\mu\gamma^\nu$. The LAGRANGIAN (2.29) can be transformed as:

$$\begin{aligned} \mathcal{L}_{int} &= g\bar{\Psi}(-g^{\mu\nu} + \gamma^\mu\gamma^\nu)(\partial_\mu\psi_\nu)\phi + h.c. \\ &= g\bar{\Psi}(-\partial \cdot \psi + \partial \cdot \gamma \gamma \cdot \psi)\phi + h.c. \end{aligned}$$

Calculating the scattering amplitude with the above LAGRANGIAN will lead to a vanishing contribution. The amplitude corresponding to a tree-level FEYNMAN graph, depicted in figure 2.1, for the πN scattering via a virtual Δ is

$$\mathcal{M}(p) = \Gamma_\alpha(p)\mathcal{G}^{\alpha\beta}(p)\Gamma_\beta(p) = 0.$$

The explicit calculation is straightforward: The vertex functions are given by

$$\Gamma_\alpha(p) \sim p^\mu\sigma_{\mu\alpha}.$$

Due to the product of symmetric and antisymmetric quantities the contraction of the vertex function with momentum is zero:

$$p^\alpha\Gamma_\alpha(p) = 0.$$

Then the only parts of the RARITA-SCHWINGER propagator which could have non-zero contributions are

$$\begin{aligned}\mathcal{G}_{\text{eff}}^{\alpha\beta} &= \frac{1}{\not{p} - M + i\epsilon} (g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta) \\ &= \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} (g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta).\end{aligned}$$

Now it is possible to calculate \mathcal{M}

$$\begin{aligned}\mathcal{M}(p) &= \Gamma_\alpha(p) \mathcal{G}_{\text{eff}}^{\alpha\beta}(p) \Gamma_\beta(p) \\ &= \Gamma_\alpha(p) \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} (g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta) \Gamma_\beta(p).\end{aligned}$$

Using equation (2.15) one can calculate

$$[\Gamma_\alpha, \not{p}] = -2(p^2\gamma_\alpha - p_\alpha\not{p}).$$

Commutating Γ_α with \not{p} in the numerator of $\mathcal{G}_{\text{eff}}^{\alpha\beta}$ gives

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$$

with

$$\begin{aligned}\mathcal{M}_1(p) &= \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} \Gamma_\alpha(p) (g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta) \Gamma_\beta(p) \\ &\sim p^\mu \sigma_{\mu\alpha} (g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta) \sigma_{\nu\beta} p^\nu \\ &= p^\mu \sigma_{\mu\alpha} g^{\alpha\beta} \sigma_{\nu\beta} p^\nu + p^\mu \sigma_{\mu\alpha} (-\frac{1}{3}\gamma^\alpha\gamma^\beta) \sigma_{\nu\beta} p^\nu \\ &= -8p^\mu \gamma_\mu \gamma_\nu p^\nu - 4p^\mu g_{\mu\nu} p^\nu + 12p^\mu \gamma_\mu \gamma_\nu p^\nu \\ &= -12p^2 + 12p^2 = 0\end{aligned}$$

and

$$\begin{aligned}\mathcal{M}_2(p) &= -2(p^2\gamma_\alpha - p_\alpha\not{p}) \left(g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta\right) \Gamma_\beta \\ &\sim p^2\gamma^\beta\Gamma_\beta - \not{p}p^\beta\Gamma_\beta - \frac{1}{3}p^2\gamma_\alpha\gamma^\alpha\gamma^\beta\Gamma_\beta + \frac{1}{3}p^2\gamma^\beta\Gamma_\beta.\end{aligned}$$

Because $p^\beta\Gamma_\beta = 0$ and $\gamma_\alpha\gamma^\alpha = 4$ above calculation reduces to

$$\mathcal{M}_2(p) \sim p^2\gamma^\beta\Gamma_\beta - \frac{4}{3}p^2\gamma^\beta\Gamma_\beta + \frac{1}{3}p^2\gamma^\beta\Gamma_\beta = 0.$$

Because the total scattering amplitude is the sum of both quantities it is also zero. This already shows that the basic idea is reasonable because the spin 1/2 parts of the RARITA-SCHWINGER fields are "invisible" and do not have contributions to physical quantities and processes.

The next lowest in derivatives gauge invariant interaction can be written as [Pas98]

$$\mathcal{L} = f \varepsilon^{\mu\nu\alpha\beta} \bar{\Psi} \gamma_5 \gamma_\alpha (\partial_\mu \psi_\nu) \partial_\beta \phi + h.c. \quad (2.30)$$

Let us check the gauge invariance: The change in the LAGRANGIAN is

$$\delta \mathcal{L} = f \varepsilon^{\mu\nu\alpha\beta} \bar{\Psi} \gamma_5 \gamma_\alpha (\partial_\mu \partial_\nu \epsilon) \partial_\beta \phi + h.c. = 0.$$

Also here the coupling of a symmetric quantity $\partial_\mu \partial_\nu$ to the antisymmetric $\varepsilon^{\mu\nu\alpha\beta}$ guarantees that this LAGRANGIAN is invariant under the gauge transformation. The LAGRANGIAN defined in equation (2.30) can be written in a manifestly gauge invariant way as known from the electromagnetic case

$$\begin{aligned} \mathcal{L} &= f \bar{\Psi} \gamma_5 \gamma_\mu \tilde{G}^{\mu\nu} \partial_\nu \phi + h.c. \quad , \\ G^{\mu\nu} &= \partial^\mu \psi^\nu - \partial^\nu \psi^\mu, \\ \tilde{G}^{\mu\nu} &= \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}. \end{aligned}$$

The expression for the vertex reads

$$\Gamma^\mu(k, p) = i g \varepsilon^{\mu\nu\alpha\beta} p_\nu \gamma_5 \gamma_\alpha k_\beta.$$

Now the FEYNMAN amplitude for figure 2.1 does not vanish leading to

$$\begin{aligned} \mathcal{M}(k', k; p) &= \Gamma^\alpha(k', p) \mathcal{G}_{\alpha\beta}(p) \Gamma^\beta(k, p) \\ &= \frac{f^2}{\not{p} - m} p^2 P_{\alpha\beta}^{3/2}(p) k'^\alpha k^\beta. \end{aligned}$$

In the second step the LEVI-CEVITA tensor is exchanged by DIRAC matrices using identity (2.22). $P_{\alpha\beta}^{3/2}(p)$ is the spin 3/2 projection operator defined above. This operator has the well-known property of projecting on the spin 3/2 states and is a clear signature of spin 3/2 components. This amplitude is independent of the spin 1/2 sector of the RARITA-SCHWINGER field.

Here the vertex function also vanishes when contracted with the momentum of the RARITA-SCHWINGER field. This is a general feature of particles with gauge symmetries and known in electrodynamics under the name WARD identity [PS95]. When looking at it on a technical basis the RARITA-SCHWINGER field ψ_μ always couples via a derivative. The change after a gauge transformation is then always a symmetric quantity $\partial_\alpha \psi_\beta \rightarrow \partial_\alpha \psi_\beta + \partial_\alpha \partial_\beta \epsilon$. Thus there will be an antisymmetric part in the LAGRANGIAN to make the gauge transformed part zero, let us call it

$A^{\mu\beta}$. Because on the FEYNMAN graph level all derivatives will become momenta of the particle $\partial_\alpha \psi_\beta \rightarrow p_\alpha \psi_\beta$ the vertex function will be

$$\Gamma^\mu \sim p_\beta A^{\mu\beta}.$$

Contraction with momentum will give a similar result as the gauge transformation

$$p_\mu \Gamma^\mu \sim p_\mu p_\beta A^{\mu\beta} = 0.$$

When looking at the projection operators for the spin states defined above one sees that only the projection operator on the spin 3/2 states have parts which are independent of p , so on the FEYNMAN graph level the spin 1/2 states will decouple and the effective propagator will read

$$\mathcal{G}_{0\text{ eff}}^{\mu\nu} = \frac{1}{\not{p} - M + i\epsilon} P^{3/2}.$$

With the same arguments as given above also couplings to other hadrons can be constructed as will be done in section 5.3 and 5.4.

2.3.5 Correspondence of Conventional and Pascualutsa Interactions

After establishing the PASCALUTSA formalism as a tool to get consistent interactions one needs to study the connections between conventional and PASCALUTSA couplings. It is possible to establish a correspondence theorem between two LAGRANGIANS, when taking the perspective that two LAGRANGIANS leading to the same observables are equivalent. In the language of scattering theory this is true when different LAGRANGIANS lead to the same on-shell S-matrix. Note that to some extent we have to change our point of view: So far we have first introduced the free field theory for a spin 3/2 state and then its interaction with other particles. In other words, the spin 3/2 state was the central object. However, discussing S-matrix theory one start from the asymptotic states. The spin 3/2 state, e.g. the Δ , is an intermediate state formed by the asymptotic ones, e.g. N and π . Therefore when talking about correspondence of different interactions, one means different interactions of the asymptotic states.

By taking this point of view Pascualutsa showed explicitly in [Pas01] that an "inconsistent" interaction of massive spin 3/2 fields can be related on the S-matrix level to a consistent one by a redefinition of the spin 3/2 fields.

Such redefinitions of fields can be done generally as was shown in [KOS61] and [CWZ69]. An interaction LAGRANGIAN with a set of variables ϕ can be written as

$$L[\phi] = L_0[\phi] + L_1[\phi]$$

where $L_0[\phi]$ is the LAGRANGIAN of the free field and $L_1[\phi]$ the interaction LAGRANGIAN. A further set of variables χ can be introduced which expresses ϕ as a nonlinear local function of the form

$$\phi = \chi F[\chi], \quad F[0] = 1. \quad (2.31)$$

It is possible to separate the resulting LAGRANGIAN also in a free and an interacting part of the form

$$L[\chi F[\chi]] = L_0[\chi] + L_2[\chi].$$

The on-mass-shell S matrix of $L[\phi]$ and

$$L'[\phi] = L_0[\phi] + L_2[\phi]$$

is identical if $F[\phi]$ is a local power series in the fields ϕ and $L[\phi]$ is a local power series in the fields ϕ and their derivatives.

As shown in [Pas01] a conventional interaction LAGRANGIAN of N , π and Δ is equivalent to a consistent interaction LAGRANGIAN plus an additional $\pi N \pi N$ contact interaction. On the S-matrix level both approaches can be seen as equivalent because they yield the same observables.

But from a physical point of view where it is desirable to understand the reactions taking place it is more appropriate to use the PASCALUTSA coupling to separate between resonances which are formed in the reactions and 4-point interactions.

We have demonstrated that the PASCALUTSA framework is sufficient to provide consistent interactions for higher spin states. We note, however, that the gauge invariance (2.21) might not be a necessary requirement but only a sufficient one. Here further answers will be given by ongoing researches in effective field theories (see e.g. [HWGS05]).

In the following we use the PASCALUTSA framework to construct consistent hadronic interactions.

3 Selfenergies

The selfenergy is introduced in quantum field theory textbooks as part of the renormalization procedure [PS95]. More generally it is defined as the one-particle irreducible function.

The importance of the selfenergy in hadronic field theories is that it is used in the DYSON-SCHWINGER formalism to dress the propagator as will be introduced in section 4.2.

In this chapter we will introduce the selfenergy of the ρ -meson. It is needed for further calculations. Later we will give a short introduction how to calculate the selfenergy for spin 3/2 states in the PASCALUTSA framework. At the end we will discuss how to calculate selfenergies for a baryon-baryon-meson system.

3.1 Selfenergy of the ρ -Meson

To calculate the selfenergy of the ρ -meson a coupling to pions is needed. A possible coupling is [HFN93]:

$$\begin{aligned}\mathcal{L}_{\rho\pi} &= (D_\mu\pi)^*(D^\mu\pi) - m_\pi^2\pi^*\pi - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_V^2\rho_\mu\rho^\mu, \\ D_\mu &= \partial_\mu + ig_\rho\rho_\mu.\end{aligned}\tag{3.1}$$

With this LAGRANGIAN and the propagator introduced in section 2.2.2 one can calculate the selfenergy for a vector meson. The selfenergy is depicted as FEYNMAN graphs in figure 3.1 to one-loop accuracy [HFN93]. Both graphs are needed in the

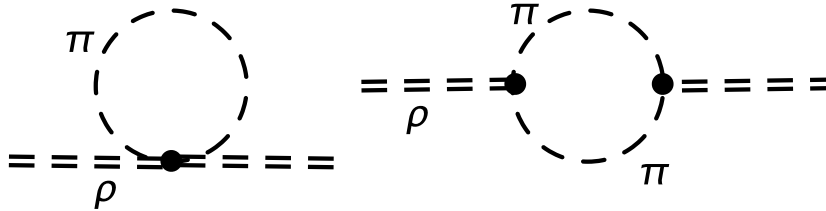


Figure 3.1: The two contributing graphs for the lowest order calculation of the self-energy of the ρ -meson.

lowest order calculation to preserve transversality, i.e. $q_\mu \Sigma^{\mu\nu} = 0$ (for further discussion see [PLM04]). Note that this is the gauge invariance requirement discussed in section 2.2.3. The selfenergy is:

$$\begin{aligned} -i\tilde{\Sigma}^{\mu\nu}(q) = & -g_\rho^2 \int \frac{d^4k}{(2\pi)^4} \frac{2g^{\mu\nu}}{k^2 - m_\pi^2 + i\epsilon} \\ & + g_\rho^2 \int \frac{d^4k}{(2\pi)^4} \frac{(2k - q)^\mu (2k - q)^\nu}{(k^2 - m_\pi^2 + i\epsilon)((k - q)^2 - m_\pi^2 + i\epsilon)} \quad . \end{aligned} \quad (3.2)$$

The first term is independent of the energy and real. The second term is energy dependent and complex giving the ρ -meson its width.

This selfenergy $\tilde{\Sigma}^{\mu\nu}$ is divergent. The denominator of the second integrand is a polynomial of rank 4, while the numerator has rank 2. This means that for high energies the theory does not describe the physics anymore. This is expected because at high energies the ρ -meson is not a pointlike particle but consist of quarks which will be "seen". But when restricting ourselves to low energies the LAGRANGIAN will be sufficient to describe the physics.

If one wants to preserve the transversality of the selfenergy a regularization via a cut-off on the vertices is not possible. Similar to [HFN93] we choose a PAULI-VILLARS regularization scheme [PS95]. An alternative method by subtracted dispersion relations is proposed by [KKW96]. The latter, however, does not yield a normalized spectral function.

Before we regularize the selfenergy it is possible to learn more about its structure. As stated above it is transverse which means

$$q_\mu \Sigma^{\mu\nu}(k) = q_\nu \Sigma^{\mu\nu}(k) = 0.$$

As shown in [BD67] this means that $\Sigma^{\mu\nu}$ is a tensor of rank two which always can be written as

$$\begin{aligned} \Sigma^{\mu\nu}(q) &= \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right] \Sigma(q) \\ &= -P_T^{\mu\nu} \Sigma(q) \end{aligned}$$

with

$$\Sigma = -\frac{1}{3} g_{\mu\nu} \Sigma^{\mu\nu}$$

and P_T defined in section 2.2.2.

Using this information the dressed propagator can be derived before calculating the selfenergy explicitly. This is done by calculating the SCHWINGER-DYSON equation:

$$\mathcal{D}_\rho^{\mu\nu} = \mathcal{D}_\rho^{0\mu\nu} + \mathcal{D}_\rho^{0\mu\alpha} \Sigma_{\alpha\beta} \mathcal{D}_\rho^{\beta\nu}. \quad (3.3)$$

Using the tensor structure of Σ and inserting it into the SCHWINGER-DYSON equation the result for the dressed propagator is:

$$\mathcal{D}_\rho^{\mu\nu}(q) = \frac{1}{q^2 - m_0^2 - \Sigma} P_T^{\mu\nu}(q) + \frac{1}{m_0^2} P_L^{\mu\nu}(q).$$

Since the selfenergy is transverse it is not surprising that the selfenergy appears only in the transverse part of the propagator and $q_\mu D_\rho^{\mu\nu} = q_\mu D_0^{\mu\nu}$ due to the properties of the projection operators.

The idea of the PAULI-VILLARS regularization scheme is that the divergences can be eliminated when adding to the original $\pi\rho$ Lagrangian (3.1) additional interaction terms where fictitious particles with higher masses Λ_i are coupled to the ρ -meson. The description for regularization of the selfenergy will be

$$\Sigma^{\mu\nu}(q) = \tilde{\Sigma}^{\mu\nu}(q, m_\pi) - \sum_i B_i \tilde{\Sigma}^{\mu\nu}(q, \Lambda_i)$$

where the number of subtractions depends on the rank of the divergences.

The coefficients B_i will be determined by requiring that the selfenergy is finite. Λ_i will be fixed later. Since Λ_i are large the integrands in (3.2) are unaffected for small q but cut off smoothly when $q \geq \Lambda_i$.

Since the first term of equation (3.2) is quadratically divergent two subtractions are needed. The finite result for the selfenergy's first term is:

$$\Sigma_1 = 2ig_\rho^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - m_\pi^2 + i\epsilon} - \sum_{i=1,2} \frac{B_i}{k^2 - \Lambda_i^2 + i\epsilon} \right].$$

The integral converges for

$$B_1 = \frac{\Lambda_2^2 - m_\pi^2}{\Lambda_2^2 - \Lambda_1^2}, \quad B_2 = \frac{\Lambda_1^2 - m_\pi^2}{\Lambda_1^2 - \Lambda_2^2},$$

with the solution [PS95]

$$\Sigma_1 = -\frac{g_\rho}{(2\pi)^2} \sum_{i=1,2} B_i \Lambda_i^2 \ln \left(\frac{\Lambda_i}{m_\pi} \right).$$

Working this out also for the second term one gets for the real and imaginary part

of the selfenergy a fully analytical form. Taking $s = q^2$ it can be written as:

$$\begin{aligned} \text{Re } \Sigma_2(s) &= -\frac{g_\rho^2}{24\pi^2} s \left[\mathcal{G}(s, m_\pi) - \sum_{i=1,2} B_i \mathcal{G}(s, \Lambda_i) + \ln \left(\frac{\Lambda_1^{B_1} \Lambda_2^{B_2}}{m_\pi} \right) \right] \\ &\quad + \frac{g_\rho^2}{(2\pi)^2} \sum_{i=1,2} B_i \Lambda_i^2 \ln \left(\frac{\Lambda_i}{m_\pi} \right), \\ \text{Im } \Sigma_2(s) &= -\frac{g_\rho^2}{48\pi} s \left[\Theta(s - 4m_\pi^2) \left(1 - \frac{4m_\pi^2}{s} \right)^{3/2} \right. \\ &\quad \left. - \sum_{i=1,2} B_i \Theta(s - 4\Lambda_i^2) \left(1 - \frac{4\Lambda_i^2}{s} \right)^{3/2} \right]. \end{aligned}$$

The function \mathcal{G} is defined as

$$\mathcal{G}(s, m) := \begin{cases} \left(\frac{4m^2}{s} - 1 \right)^{3/2} \arctan \left(\left(\sqrt{\frac{4m^2}{s} - 1} \right)^{-1} \right) & 0 < s < 4m^2, \\ -\frac{1}{2} \left(1 - \frac{4m^2}{s} \right)^{3/2} \ln \left| \frac{\sqrt{1 - \frac{4m^2}{s}} + 1}{\sqrt{1 - \frac{4m^2}{s}} - 1} \right| & 4m^2 < s, s < 0. \end{cases}$$

The total selfenergy is the sum of both parts Σ_1 and Σ_2 . Because the most divergent parts cancel it is possible to take the limit $\Lambda_2 \rightarrow \infty$. Only one cut-off parameter remains $\Lambda = \Lambda_1$. The limit yields:

$$\begin{aligned} \text{Re } \Sigma(s) &= -\frac{g_\rho^2}{24\pi^2} s \left[\mathcal{G}(s, m_\pi) - \mathcal{G}(s, \Lambda) \right. \\ &\quad \left. + 4 \frac{\Lambda^2 - m_\pi^2}{s} + \ln \left(\frac{\Lambda}{m_\pi} \right) \right], \\ \text{Im } \Sigma(s) &= -\frac{g_\rho^2}{48\pi} s \left[\Theta(s - 4m_\pi^2) \left(1 - \frac{4m_\pi^2}{s} \right)^{3/2} \right. \\ &\quad \left. - \Theta(s - 4\Lambda^2) \left(1 - \frac{4\Lambda^2}{s} \right)^{3/2} \right]. \end{aligned}$$

The effect of the PAULI-VILLARS regularization scheme can be observed in the imaginary part of the selfenergy. There two identical functions with different mass terms are subtracted. Taking $s \rightarrow \infty$ leads to a constant term which can be seen when Taylor expanding the brackets:

$$\text{Im } \Sigma(s) = -\frac{g_\rho^2}{8\pi} (\Lambda^2 - m_\pi^2) \quad \text{for } s \rightarrow \infty.$$

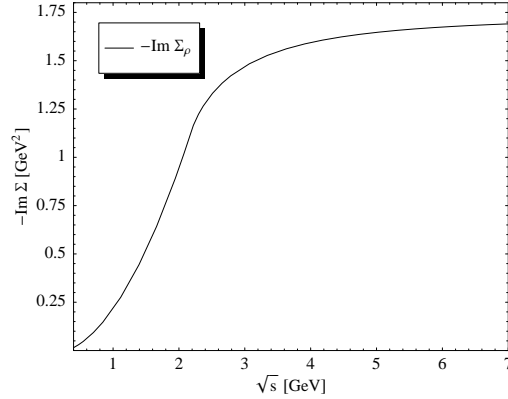


Figure 3.2: Imaginary part of the selfenergy for the ρ -meson in vacuum.

In contrast, without the PAULI-VILLARS term $\text{Im } \Sigma$ would rise linearly with s for large s .

The effect of the regularization can be seen in figure 3.2 where the imaginary part of the selfenergy is depicted. The selfenergy rises for larger \sqrt{s} until a value of approx. 4 GeV where it starts to reach asymptotically a constant value.

The parameter Λ is fixed to 1 GeV. This cut-off region is motivated by the fact that for hadronic models the quark and gluon degrees freeze out, which is guaranteed at soft processes much below 1 GeV.

There are two free parameters in the theory. The bare mass of the ρ -meson m_0 entering the propagator and the coupling constant g_ρ . The parameters are fitted in [HFN93] to the values $g_\rho = 6.05$ and $m_\rho = 875 \text{ MeV}$. The fit was done by adjusting the position and the height of the peak in the pion electromagnetic form factor.

3.2 Selfenergy of a Spin 3/2 Particle in the Pascualutsa Framework

The selfenergy has the same structure as the propagator leading to a DIRAC structure for fermionic selfenergies and for the case of the spin 3/2 particle also to a LORENTZ structure.

Due to the completeness of the spin 3/2 projection operators defined in section 2.3.2, the selfenergy can be written as

$$\Sigma_{\mu\nu} = P^{3/2}a + P_{11}^{1/2}b + P_{22}^{1/2}c + P_{12}^{1/2}d + P_{21}^{1/2}e$$

with the DIRAC matrices

$$a = a_s(p^2) + \not{p}a_v(p^2)$$

and the corresponding decomposition for b-e.

The gauge invariant structure of the interaction leads to a transverse selfenergy

$$p_\mu \Sigma^{\mu\nu}(p) = p_\nu \Sigma^{\mu\nu}(p) = 0.$$

This can be written as

$$(P_{22}^{1/2})_{\mu\nu} \Sigma^{\mu\nu} = \Sigma^{\mu\nu} (P_{22}^{1/2})_{\mu\nu} = 0.$$

Then some of the terms given above drop out. Since

$$(P_{22}^{1/2})(P_{21}^{1/2}) = (P_{21}^{1/2})$$

and

$$(P_{12}^{1/2})(P_{22}^{1/2}) = (P_{12}^{1/2})$$

we find

$$P_{22}^{1/2} c + P_{21}^{1/2} e = 0,$$

$$P_{22}^{1/2} c + P_{12}^{1/2} d = 0.$$

Contracting the upper (lower) equation with $P_{11}^{1/2}$ from right (left) leads to $e = 0$ and $d = 0$ since

$$(P_{21}^{1/2})(P_{11}^{1/2}) = (P_{21}^{1/2})$$

and

$$(P_{11}^{1/2})(P_{12}^{1/2}) = (P_{12}^{1/2}).$$

Then also $c = 0$ and the number of coefficients reduces to two

$$\Sigma_{\mu\nu} = P^{3/2}(a_1 + \not{p}a_2) + P_{11}^{1/2}(a_3 + \not{p}a_4). \quad (3.4)$$

We define for later purposes

$$\begin{aligned} \Sigma &= \frac{1}{2} P_{\mu\nu}^{3/2} \Sigma^{\mu\nu} \\ &= a \\ &= \Sigma_s + \not{p} \Sigma_v. \end{aligned} \quad (3.5)$$



Figure 3.3: FEYNMAN graph for a resonance selfenergy. The double line denotes the resonance, the full line the nucleon and the dashed line the pion.

If $\Sigma_{\mu\nu}$ is known the coefficients a_i can be determined via traces over $\Sigma_{\mu\nu}$:

$$\begin{aligned} a_1 &= \frac{1}{8} \text{Tr} \left(\Sigma^\mu{}_\mu - \frac{1}{3} \gamma_\nu \gamma_\mu \Sigma^{\mu\nu} \right), \\ a_2 &= \frac{1}{8p^2} \text{Tr} \left(\not{p} \Sigma^\mu{}_\mu - \frac{1}{3} \not{p} \gamma_\nu \gamma_\mu \Sigma^{\mu\nu} \right), \\ a_3 &= \frac{1}{12} \text{Tr} \left(\gamma_\nu \gamma_\mu \Sigma^{\mu\nu} \right), \\ a_4 &= \frac{1}{12} \text{Tr} \left(\not{p} \gamma_\nu \gamma_\mu \Sigma^{\mu\nu} \right). \end{aligned}$$

This is a striking result because in the conventional approach all ten coefficients are needed [Kor97]. We can conclude already here that it is not only feasible to work in the PASCALUTSA framework but much easier.

3.3 Calculation of the Selfenergy for a Resonance-Baryon-Meson System

Different approaches are possible when calculating the imaginary part of the self-energy. The most common way is via the optical theorem [PS95]. We will use an equivalent approach [PLM04][SSW⁺89]. Indices and derivatives will be omitted for simplicity and it is understood that derivatives in the LAGRANGIAN will be substituted by momenta in the selfenergy.

For a given LAGRANGIAN of the form

$$\mathcal{L}_{RB\phi} = g \bar{\Psi}_R \Gamma_a \Psi_B \Gamma_b \phi + c \phi^* \Gamma_{b'} \bar{\Psi}_B \Gamma_{a'} \Psi_R$$

the selfenergy in one-loop accuracy can be calculated using the FEYNMAN graph depicted in figure 3.3:

$$\Sigma(p) = g^2 i \int \frac{d^4 q}{(2\pi)^4} \Gamma_{b'} \Gamma_{a'} D(p-q) G(q) \Gamma_a \Gamma_b$$

with $D(k)$ and $G(k)$ as the propagator of the meson and the baryon, respectively. The imaginary part is then given as [PLM04][SSW⁺89]:

$$Im \Sigma(p) = -g^2 \int \frac{d^4 q}{8\pi^2} \Gamma_{b'} \Gamma_{a'} \mathcal{A}(p-q) \rho(q) \Gamma_a \Gamma_b \Theta(p_0 - q_0) \Theta(q_0) \quad (3.6)$$

with $\mathcal{A}(k)$ and $\rho(k)$ as the spectral function of the meson and baryon, respectively. The spectral function will be defined and discussed in the next chapter. For a stable scalar particle one gets

$$\mathcal{A}(k) \Theta(k_0) = \delta(k^2 - m^2) \Theta(k_0) = \begin{cases} \frac{1}{2k_0} \delta(k_0 - \sqrt{\mathbf{k}^2 + m^2}) \\ \frac{1}{2|\mathbf{k}|} \delta(|\mathbf{k}| - \sqrt{k_0^2 - m^2}) \end{cases} \quad (3.7)$$

For the calculation (3.6) it is convenient to use $\mathbf{p} = 0$ and $p_0 = \sqrt{s}$. The presence of two stable particles allows a complete analytical evaluation of the integral.

The real part $Re \Sigma$ is calculated via a dispersion relation [PLM04][SSW⁺89].

4 The Propagator and its Representations

Two major components of field theory are the fields and their propagators. In this chapter we will discuss the propagators of a vector meson and a spin 3/2 baryon. We start with the spectral representation which defines the spectral function. Then we will introduce the dressed propagator from which the analytical form of the spectral function can be deduced. The bare mass and the width of the particles will be defined by comparing the spectral function to a BREIT-WIGNER form. This will be done first for the bosonic case as a simple example and later for the case of spin 3/2 baryons.

4.1 The Källen-Lehmann Representation

The propagator, or more precisely the FEYNMAN propagator of an arbitrary field ϕ is defined as the two-point correlation function or two-point GREEN's function [PS95]

$$\mathcal{D}(x, y) = -i \langle \Omega | T \phi(x) \phi(y) | \Omega \rangle. \quad (4.1)$$

Here Ω is understood as the vacuum of the theory which in general will differ for interacting and non-interacting theories.

For a non-interacting scalar field of mass m there are analytic solutions of the fields $\phi(x)$. Then it is possible to calculate the two-point function in (4.1) and its Fourier transform:

$$\begin{aligned} \mathcal{D}_0(q^2) &= -i \int d^4x e^{iqx} \langle 0 | T \phi(x) \phi(0) | 0 \rangle \\ &= \frac{1}{q^2 - m^2 + i\epsilon} \quad . \end{aligned}$$

In an interacting theory there are in general no analytical solutions of ϕ . The propagator will not only have a single pole with strength one but several poles can arise due to bound states as well as branch cuts starting at the threshold energy of the formation of multi-particle states. But some general features can be deduced.

Assuming that the theory has asymptotic free states $|n\rangle$ and is invariant under translation, the momentum and coordinate dependence can be extracted:

$$\begin{aligned}\langle n|\phi(x)|m\rangle &= \langle n|e^{iQ\cdot x}\phi(0)e^{-iQ\cdot x}|m\rangle \\ &= e^{i(q_n - q_m)\cdot x}\langle n|\phi(0)|m\rangle.\end{aligned}$$

When inserting a total set of eigenstates into the FEYNMAN propagator, it can be written as

$$\begin{aligned}\mathcal{D}(x, y) &= -i \sum_n \langle \Omega|\phi(0)|n\rangle \langle n|\phi(0)|\Omega\rangle \\ &\quad \times (\Theta(y_0 - x_0)e^{-iq_n(x-y)} - \Theta(x_0 - y_0)e^{iq_n(x-y)}) \\ &= \mathcal{D}(x - y).\end{aligned}$$

Introducing

$$1 = \int d^4k \delta^4(q_n - k),$$

one can define a spectral amplitude

$$\varrho(q) = (2\pi)^3 \sum_n \delta^4(q_n - k) |\langle \Omega|\phi(0)|n\rangle|^2$$

with which the propagator will be

$$\mathcal{D}(x, y) = \frac{-i}{(2\pi)^3} \int d^4k \varrho(k) (\Theta(y_0 - x_0)e^{-ik(x-y)} - \Theta(x_0 - y_0)e^{ik(x-y)}).$$

$\varrho(k)$ is positive semidefinite and only non-zero for $k^2 \geq 0$ and for $k_0 \geq 0$ since all eigenstates satisfy $q_n \geq 0$ and $q_n^0 \geq 0$. Because $\varrho(k)$ is invariant under LORENTZ transformations one can write

$$\varrho(k) = \rho(k^2)\Theta(k_0).$$

The propagator is given as [BD67]

$$\begin{aligned}\mathcal{D}(x - y) &= \frac{-i}{(2\pi)^3} \int d^4k \varrho(k) (\Theta(y_0 - x_0)e^{-ik(x-y)} - \Theta(x_0 - y_0)e^{ik(x-y)}) \\ &= \int_0^\infty d\sigma^2 \rho(\sigma^2) \mathcal{D}_0(x - y)\end{aligned}$$

where $\mathcal{D}_0(x - y)$ takes the mass σ . In momentum space the propagator can be written as

$$\mathcal{D}(q^2) = \int_0^\infty d\sigma^2 \rho(\sigma^2) \frac{1}{q^2 - \sigma^2 + i\epsilon} \quad . \quad (4.2)$$

To fulfill particle number conservation a normalization condition can be derived from the quantization condition of the fields [Pos03]

$$\int_{q_{\min}^2}^{\infty} dq^2 \rho(q^2) = 1 \quad (4.3)$$

where $\sqrt{q_{\min}^2}$ is the minimal energy to create the boson.

This representation was introduced by Källen and Lehmann [Käl52][Leh54] and can be found in textbooks as the KÄLLEN-LEHMANN representation [PS95] or spectral representation [BD67] for fields. The function $\rho(q^2)$ is usually called the spectral function of the field ϕ .

For the case of a non-interacting spin 1/2 field the FEYNMAN propagator has the form [PS95]

$$\mathcal{G}_0(p) = \frac{1}{\not{p} - M + i\epsilon} \quad (4.4)$$

The KÄLLEN-LEHMANN representation of the fermionic propagator must be a sum of elements of the CLIFFORD algebra:

$$\rho_{\alpha\beta}(p) = \rho(p)g_{\alpha\beta} + \rho_\mu(p)\gamma_{\alpha\beta}^\mu + \rho_{\mu\nu}(p)\sigma_{\alpha\beta}^{\mu\nu} + \tilde{\rho}(p)\gamma_{\alpha\beta}^5 + \tilde{\rho}_\mu(\gamma^\mu\gamma^5)_{\alpha\beta}.$$

After incorporating invariance under LORENTZ transformations and invariance under parity only a two terms survive [BD67]

$$\rho_{\alpha\beta}(p) = \rho_v(p^2)\not{p}_{\alpha\beta} + \rho_s(p^2)\delta_{\alpha\beta}.$$

The spectral representation of the FEYNMAN propagator of fermionic particles can be written as

$$\mathcal{G}(p^2) = \int_0^\infty dk^2 [\rho_v(k^2)\not{k} + \rho_s(k^2)] \frac{1}{p^2 - k^2 + i\epsilon} \quad (4.5)$$

ρ_v and ρ_s are scalar functions and have some fundamental properties:

$$\rho_v(p^2) \text{ and } \rho_s(p^2) \text{ are both real,} \quad (4.6)$$

$$\rho_v(p^2) \geq 0, \quad (4.7)$$

$$\sqrt{p^2}\rho_v(p^2) - \rho_s(p^2) \geq 0. \quad (4.8)$$

As in the scalar case it is possible to derive a normalization condition for ρ_v [Pos03]

$$\int_{p_{\min}^2}^{\infty} dp^2 \rho_v(p^2) = 1. \quad (4.9)$$

For the case of spin 3/2 fields it is a delicate task to perform a general spectral representation [Kor97][dJM92]. But in the PASCALUTSA framework the effective

propagator of the spin 3/2 fields has a similar structure as the spin 1/2 fields (see section 2.3.4). The effective spectral representation of spin 3/2 fields in the PASCALUTSA formalism can then be written as

$$\mathcal{G}_{\text{eff}}^{\mu\nu}(p^2) = \int_0^\infty dk^2 [\not{p}\rho_v(k^2) + \rho_s(k^2)] \frac{1}{p^2 - k^2 + i\epsilon} P^{3/2}(p^2).$$

This means that in the PASCALUTSA framework the spectral functions for spin 1/2 and spin 3/2 particles basically have the same structure which is a great simplification.

4.2 The Dressed Propagator

To calculate a propagator for an interacting theory a resummation is appropriate. This can be done consistently using the SCHWINGER-DYSON equation (3.3):

$$\mathcal{D}(q^2) = \mathcal{D}_0(q^2) + \mathcal{D}_0(q^2)\Sigma(q^2)\mathcal{D}(q^2). \quad (4.10)$$

where \mathcal{D}_0 is the bare or non-interacting propagator.

$\Sigma(q^2)$ is the selfenergy of the particle including all possible decay channels. In the case of a scalar field the geometric series of (4.10) can be summed up. To highlight the character of the mass as a bare mass we rewrite $m \rightarrow m_0$. Then the result of the resummation reads:

$$\begin{aligned} \mathcal{D}(q^2) &= \frac{\mathcal{D}_0(q^2)}{1 - \mathcal{D}_0(q^2)\Sigma(q^2)} \\ &= \frac{1}{q^2 - m_0^2 - \Sigma(q^2)} \quad . \end{aligned} \quad (4.11)$$

For a fermionic field (4.4) the resummation leads to

$$\mathcal{G}(p) = \frac{1}{\not{p} - M_0 - \Sigma(p)}, \quad (4.12)$$

where M is substituted with M_0 . The structure of this propagator has the form

$$\mathcal{G}(p) = \frac{F_s + \not{p}F_v}{(\tilde{s} - \tilde{M}^2)^2 + Z^2} \quad (4.13)$$

with the quantities ($s = p^2$):

$$\tilde{s} = s [(1 - \text{Re } \Sigma_v)^2 - (\text{Im } \Sigma_v)^2], \quad (4.14)$$

$$\tilde{M}^2 = (M_0 + \text{Re } \Sigma_s)^2 - (\text{Im } \Sigma_s)^2, \quad (4.15)$$

$$Z = 2 [s (1 - \text{Re } \Sigma_v) \text{Im } \Sigma_v + (M_0 + \text{Re } \Sigma_s) \text{Im } \Sigma_s], \quad (4.16)$$

$$\text{Re} F_s = (M_0 + \text{Re } \Sigma_s) (\tilde{s} - \tilde{M}^2) - \text{Im } \Sigma_s Z, \quad (4.17)$$

$$\text{Im} F_s = \text{Im } \Sigma_s (\tilde{s} - \tilde{M}^2) + (M_0 + \text{Re } \Sigma_s) Z, \quad (4.18)$$

$$\text{Re} F_v = (1 - \text{Re } \Sigma_v) (\tilde{s} - \tilde{M}^2) + \text{Im } \Sigma_v Z, \quad (4.19)$$

$$\text{Im} F_v = -\text{Im } \Sigma_v (\tilde{s} - \tilde{M}^2) + (1 - \text{Re } \Sigma_v) Z. \quad (4.20)$$

Resummation of the propagator for the RARITA-SCHWINGER field is also possible. The free propagator is derived in section 2.3.2:

$$\mathcal{G}_0^{\mu\nu}(p) = \frac{1}{\not{p} - M_0 + i\epsilon} P^{3/2} - \frac{2}{3M_0^2} (\not{p} + M_0) P_{22}^{1/2} + \frac{1}{\sqrt{3}M_0} (P_{12}^{1/2} + P_{21}^{1/2}).$$

After resummation by the SCHWINGER-DYSON equation the full propagator reads

$$\mathcal{G}^{\mu\nu}(p) = \frac{1}{\not{p} - M_0 - \Sigma} P^{3/2} + \frac{1}{3M_0^2} (-2(\not{p} + M_0) - \tilde{b}) P_{22}^{1/2} + \frac{1}{\sqrt{3}M_0} (P_{12}^{1/2} + P_{21}^{1/2})$$

with Σ and $\tilde{b} = a_3 - \not{p}a_4$ taken from equation (3.5) and equation (3.4).

The effective propagator of the spin 3/2 field in the PASCALUTSA formalism is then given as

$$\mathcal{G}_{\text{eff}}^{\mu\nu} = \frac{1}{\not{p} - M_0 - \Sigma} P^{3/2} = \mathcal{G} P^{3/2} \quad (4.21)$$

where in the last step we used equation (4.12). This means that the effective propagator of the spin 3/2 fields can be written as the propagator of the spin 1/2 field multiplied by the projection operator $P^{3/2}$.

4.3 The Spectral Function

Having calculated the dressed propagator the analytic structure of the spectral function can be easily derived. This will be done for the bosonic and the fermionic case separately and a summary of the constraints derived in section 4.1 will be given. As an easy example for a spectral function we calculate it explicitly for the ρ -meson and discuss its properties.

4.3.1 Bosonic Properties

For stable bosons one sees immediately from equation (4.2) that the spectral function is given as

$$\rho(q^2) = \delta(q^2 - m^2).$$

For unstable particles the spectral function can be calculated using the property

$$\text{Im} \frac{1}{q^2 - \sigma^2 + i\epsilon} = -\pi \delta(q^2 - \sigma^2). \quad (4.22)$$

Taking the imaginary part of the FEYNMAN propagator leads to

$$\text{Im} \mathcal{D}(q^2) = \int_0^\infty d\sigma^2 \rho(\sigma^2) \text{Im} \frac{1}{q^2 - \sigma^2 + i\epsilon}$$

since $\rho(q^2)$ is a real quantity. Using equation (4.22) one gets

$$\rho(q^2) = -\frac{1}{\pi} \text{Im} \mathcal{D}_F(q^2).$$

With the representation (4.11) of the dressed propagator the bosonic spectral function can be obtained as:

$$\rho(q^2) = -\frac{1}{\pi} \text{Im} \mathcal{D}(q) = -\frac{1}{\pi} \frac{\text{Im} \Sigma}{(q^2 - m_0^2 - \text{Re} \Sigma)^2 + \text{Im} \Sigma^2} \quad (4.23)$$

ρ is normalized as stated in equation (4.3)

$$\int_{q_{\min}^2}^\infty dq^2 \rho(q^2) = 1.$$

4.3.2 The Spectral Function of the ρ -Meson

Using the selfenergy calculated in section 3.1 the propagator is fully determined. The spectral function for bosons was introduced in the last section. Due to the transversity of the coupling the effective propagator of the ρ -meson can be written as for the scalar case multiplied by the transverse projection operator $P_{\mu\nu}^T$ defined in section 2.2.2. This means that the important structure of the spectral function is given by equation (4.23).

The result is plotted in figure 4.1 with $q^2 = s$. On the right hand side of the figure one can see, that the KÄLLEN-LEHMANN normalization condition introduced in equation (4.3) is fulfilled. There we have plotted

$$\text{norm}_\rho(s) = \int_{4m^2}^s ds' \rho(s'). \quad (4.24)$$

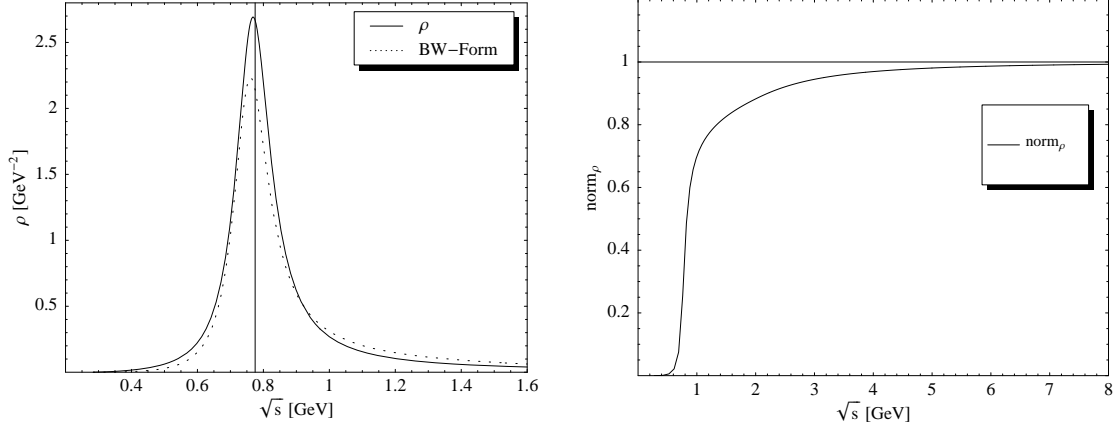


Figure 4.1: Left: Spectral function of the ρ -meson in vacuum (full line) as compared to a simpler BREIT-WIGNER form (dotted). Right: Normalization function of the spectral function as defined in (4.24).

One sees that $norm(s_\rho) \rightarrow 1$ for large s .

The calculated selfenergy or the spectral function fully describes the resonance. From them all measurable quantities can be calculated, e.g. phase shifts. On the other hand, selfenergies and spectral functions are not directly measurable quantities. For example, with a reparametrization (2.31) the spectral function of a resonance might change without changing any observable. Therefore one introduces simpler quantities to characterize a resonance which are closer to theory than phase shifts and closer to experiment than spectral functions. These quantities, the mass m_R of the resonance and its (on-shell) width Γ , are only two numbers instead of a full spectral shape. The price to pay for such an oversimplification is an ambiguity how to define mass and width. In the following we will define these quantities by comparing our spectral function (4.23) to a relativistic BREIT-WIGNER form [Eea04]

$$S = \frac{1}{\pi} \frac{\sqrt{s} \Gamma}{(s - m_R^2)^2 + s \Gamma^2} \quad (4.25)$$

where Γ is the width and m_R the physical mass of the resonance. Note that this is not the only possibility. We could have taken as well e.g. the peak position of ρ to define the mass. In turn this means that for complicated selfenergies it should not be too surprising if the mass as we defined it deviates to some extent from the peak position.

A direct comparison of the BREIT-WIGNER form and the spectral function in equation (4.23) is not possible due to the more complicated structure of the spectral function. On the other hand, it is convenient to define the particle's mass and width at the point where the real part of the inverse propagator, i.e. $s - m_0^2 - Re \Sigma(s)$

vanishes. Then we can expand the denominator of ρ around this point, which is the physical mass $m_R = m_\rho$ of the ρ -meson:

$$s - m_0^2 - \text{Re } \Sigma \sim \frac{1}{c}(s - m_\rho^2) + O((s - m_\rho^2)^2). \quad (4.26)$$

The coefficient c can then be extracted as the first derivative of $s - m_0^2 - \text{Re } \Sigma$ on the mass-shell:

$$\frac{1}{c} = \left. \frac{d}{ds}(s - m_0^2 - \text{Re } \Sigma) \right|_{s=m_\rho} = 1 - \left. \frac{d}{ds} \text{Re } \Sigma \right|_{s=m_\rho}.$$

Then the spectral function has the form

$$\begin{aligned} \rho(s) &= -\frac{1}{\pi} \frac{\text{Im } \Sigma}{(s - m_0^2 - \text{Re } \Sigma)^2 + \text{Im } \Sigma^2} \\ &\sim -\frac{1}{\pi} \frac{c^2 \text{Im } \Sigma}{(s - m_\rho^2)^2 + c^2 \text{Im } \Sigma^2} \\ &= -\frac{c}{\pi} \frac{c \text{Im } \Sigma}{(s - m_\rho^2)^2 + (c \text{Im } \Sigma)^2}. \end{aligned} \quad (4.27)$$

Now the physical mass m_ρ and the width Γ_ρ can be extracted. This is achieved for the mass by putting (4.26) on the mass-shell

$$m_\rho^2 - (m_0)^2 - \text{Re } \Sigma(m_\rho^2) = 0.$$

One can interpret this relation such that the bare mass of the ρ -meson is shifted by the real part of the selfenergy leading to the physical mass m_ρ .

Also the width can be read off by comparing (4.27) to the BREIT-WIGNER form (4.25). One can read off the width of the ρ -meson from the denominator or numerator leading to the same quantity

$$\Gamma_\rho = - \left. \frac{c(s)}{\sqrt{s}} \text{Im } \Sigma(s) \right|_{s=m_\rho^2}. \quad (4.28)$$

Note that without the additional c in front of the BREIT-WIGNER type form in (4.27) the spectral function would be normalized incorrectly. The appearance of c in (4.28) leads to a squeezed or an enlarged width, depending whether c is larger or smaller than one. Or looking at the definition of c the spectral function will be squeezed (enlarged) if the derivative of the real part of the selfenergy is larger (smaller) than zero. This is clear because then the real part of the selfenergy will increase (decrease).

A further interesting observation is the fact that the width is proportional to $\text{Im } \Sigma / \sqrt{s}$. As shown in section 3.1 $\text{Im } \Sigma$ reaches a constant value for large s . Because the width is proportional to $\text{Im } \Sigma / \sqrt{s}$ it will reach zero for large s . This

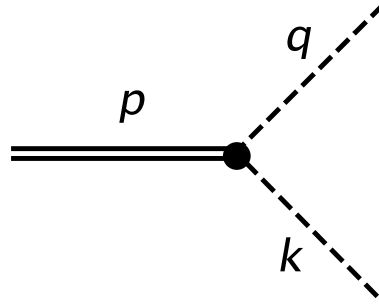


Figure 4.2: ρ -meson (double line) decaying into two pions (dashed lines).

is an effect of the PAULI-VILLARS regularization scheme applied to $Im \Sigma$. Without regularization $Im \Sigma$ would rise linearly with s and the width with \sqrt{s} for large s . Such a width would spoil the normalization of the spectral function.

Now we can compare the width of the ρ -meson with the general property that close to threshold the energy dependence of the width is determined by the orbital angular momentum l

$$\Gamma(\sqrt{s} \approx \sqrt{s_{thr}}) \sim \mathbf{q}^{2l+1}$$

with \mathbf{q} as the center of mass momentum of the decay products. Away from threshold further powers of the momenta can occur due to relativistic effects. Around threshold the kinetic energies are small compared to the mass of the particles because most of the energy was used to create the particles. Because the ρ -meson has negative parity and the pion also has negative parity the angular momentum has to be at least $l = 1$. Around threshold the width has the form

$$\Gamma_\rho(\sqrt{s} \approx \sqrt{s_{thr}}) \sim \mathbf{q}^3. \quad (4.29)$$

Because the width is proportional to the imaginary part of the selfenergy calculated in section 3.1 we can write its analytical form for small kinetic energies as

$$\begin{aligned} \Gamma_\rho &= -\frac{c}{\sqrt{s}} Im \Sigma \sim \frac{s}{\sqrt{s}} \left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} \\ &= \frac{1}{s} (s - 4m_\pi^2)^{3/2} \\ &\sim \frac{1}{4m_\pi^2} (s - 4m_\pi^2)^{3/2}. \end{aligned} \quad (4.30)$$

This equation can be understood when looking at the decay process of the ρ -meson.

It decays into two pions as shown in figure 4.2. In the rest frame of the ρ -meson the kinematic quantities are

$$\begin{aligned} p &= (\sqrt{s}, 0), & p^2 &= s, \\ q &= (q_0, \mathbf{q}), & q_0^2 &= \mathbf{q}^2 + m_\pi^2, \\ k &= (k_0, -\mathbf{q}), & k_0^2 &= \mathbf{q}^2 + m_\pi^2 = q_0^2. \end{aligned}$$

The momenta of the decay products can be calculated as

$$\begin{aligned} s &= p^2 = (q + k)^2 \\ &= (q_0 + k_0)^2 = 4q_0^2 = 4(\mathbf{q}^2 + m_\pi^2) \\ &\Rightarrow \mathbf{q}^2 = \frac{1}{4}(s - 4m_\pi^2). \end{aligned}$$

Comparing this result with (4.30) one finds the expected result in (4.29).

By taking the structure of the spectral function and the width one could proceed the other way and introduce a "pseudo-relativistic" spectral function in a BREIT-WIGNER form

$$\mathcal{A}_{pseudo} = \frac{1}{N} \frac{\sqrt{s} \Gamma_{pseudo}}{(s - m_\rho^2)^2 + s \Gamma_{pseudo}^2} \quad . \quad (4.31)$$

with N as a normalization factor requiring

$$1 = \int_{4m_\pi^2}^{\infty} ds \mathcal{A}_{pseudo}(s).$$

The width is motivated by equation (4.29)

$$\Gamma_{pseudo} = a \mathbf{q}^3 = \frac{a}{8} (s - 4m_\pi^2)^{3/2} \quad (4.32)$$

and a is determined by the requirement that on the mass-shell this width is equal to the value given in [Eea04]. The result is plotted in figure 4.1. The shape of the spectral function is reproduced but the peak differs. One can understand this when comparing the guessed width (4.32) and the microscopically determined width (4.30). The latter width has an extra factor of $\frac{1}{s}$ which suppresses the spectral function for higher \sqrt{s} . Because both functions are normalized this suppression will shift spectral strength to lower \sqrt{s} leading to a higher peak. This means that imposing a normalization condition and a width which behaves correctly near threshold is in general not sufficient to generate a spectral function that agrees with the full calculation.

4.3.3 Fermionic Properties

For stable spin 1/2 particles the spectral functions can be read off immediately from equation (4.5) as

$$\begin{aligned}\rho_v(p^2) &= \delta(p^2 - M^2), \\ \rho_s(p^2) &= \sqrt{p^2} \delta(p^2 - M^2).\end{aligned}$$

Generally the spectral functions can be calculated using equation (4.22). Because the spectral function is defined as the imaginary part of the propagator one needs to clarify what the imaginary part of a DIRAC quantity is. The imaginary part is defined via the hermitian rather than the complex conjugate [BD67][PLM04][Frö01]:

$$\text{Re } \mathcal{G}(p) = \frac{1}{2}(\mathcal{G}(p) + \gamma_0 \mathcal{G}^\dagger \gamma_0), \quad \text{Im } \mathcal{G}(p) = \frac{1}{2i}(\mathcal{G}(p) - \gamma_0 \mathcal{G}^\dagger \gamma_0).$$

This definition treats \not{p} as a real quantity.

Now it is possible to calculate the imaginary part of a propagator in the KÄLLEN-LEHMANN representation

$$\begin{aligned}\text{Im } \mathcal{G}(p^2) &= \int_0^\infty dk^2 [\not{p} \rho_v(k^2) + \rho_s(k^2)] \text{Im} \frac{1}{p^2 - k^2 - i\epsilon} \\ &= \int_0^\infty dk^2 [\not{p} \rho_v(k^2) + \rho_s(k^2)] (-\pi \delta(p^2 - k^2)) \\ &= -\pi [\not{p} \rho_v(p^2) + \rho_s(p^2)].\end{aligned}$$

The spectral functions can be extracted as specific traces over the imaginary part of the FEYNMAN propagator (4.5) or (4.13):

$$\begin{aligned}\rho_v(p^2) &= -\frac{1}{4\pi p^2} \text{Tr}[\not{p} \text{Im } \mathcal{G}(p^2)], \\ \rho_s(p^2) &= -\frac{1}{4\pi} \text{Tr}[\text{Im } \mathcal{G}(p^2)].\end{aligned}$$

Finally we repeat equations (4.6)-(4.9):

$$\begin{aligned}\rho_v(p^2) \text{ and } \rho_s(p^2) &\text{ are both real,} \\ \rho_v(p^2) &\geq 0, \\ \sqrt{p^2} \rho_v(p^2) - \rho_s(p^2) &\geq 0, \\ \int_{p_{min}^2}^\infty dp^2 \rho_v(p^2) &= 1.\end{aligned}\tag{4.33}$$

These properties also hold for spin 3/2 particles where the spectral functions are given as:

$$\begin{aligned}\rho_v(p^2) &= -\frac{1}{8\pi p^2} \text{Tr}[\not{p} \text{Im } \mathcal{G}_{\text{eff}}^{\mu\nu}(p^2) P_{\mu\nu}^{3/2}] \\ &= -\frac{1}{4\pi p^2} \text{Tr}[\not{p} \text{Im } \mathcal{G}(p^2)], \\ \rho_s(p^2) &= -\frac{1}{8\pi} \text{Tr}[\text{Im } \mathcal{G}_{\text{eff}}^{\mu\nu}(p^2) P_{\mu\nu}^{3/2}] \\ &= -\frac{1}{4\pi} \text{Tr}[\text{Im } \mathcal{G}(p^2)]\end{aligned}$$

where $\mathcal{G}_{\text{eff}}^{\mu\nu}$ is the dressed effective spin 3/2 propagator (4.21) and \mathcal{G} is the dressed spin 1/2 propagator (4.12). Which means that in the PASCALUTSA framework the spectral functions of a spin 1/2 and a spin 3/2 particle are equal.

4.3.4 The Spectral Functions of Spin 1/2 and Spin 3/2 Particles

As we have seen in section 4.2 the propagator of a spin 1/2 and spin 3/2 particle can be calculated via the selfenergy of the particle. In this way, one obtains the spectral function in equation (4.13) directly in an analytical form

$$\begin{aligned}\text{Im } \mathcal{G} &= \frac{\text{Im } F_s + \not{p} \text{Im } F_v}{(\tilde{s} - \tilde{M}^2)^2 + Z^2} \\ &= -\pi(\not{p}\rho_v(s) + \rho_s(s)).\end{aligned}$$

The analytical form of the ρ s are

$$\rho_v(s) = -\frac{1}{\pi} \frac{\text{Im } F_v}{(\tilde{s} - \tilde{M}^2)^2 + Z^2}, \quad (4.34)$$

$$\rho_s(s) = -\frac{1}{\pi} \frac{\text{Im } F_s}{(\tilde{s} - \tilde{M}^2)^2 + Z^2}. \quad (4.35)$$

Because ρ_v (ρ_s) and $\text{Im } F_v$ ($\text{Im } F_s$) are proportional to each other the relations (4.6)-(4.8) also hold for $-\text{Im } F_s$ and $-\text{Im } F_v$:

$$\begin{aligned}\text{Im } F_v(s) \text{ and } \text{Im } F_s(s) &\text{ are both real,} \\ -\text{Im } F_v(s) &\geq 0, \\ \text{Im } F_s(s) - \sqrt{s} \text{Im } F_v(s) &\geq 0.\end{aligned} \quad (4.36)$$

For later use we define the BJORKEN-DRELL function as:

$$BD(s) = \text{Im } F_v(s) - \sqrt{s} \text{Im } F_s(s). \quad (4.37)$$

Note that this function must not get negative.

As for the bosonic case an extraction of the mass and the width of the fermionic resonance is desirable. This is not needed to describe the resonance, because the resonance is fully characterized by selfenergy or spectral functions, but to compare the results of this work with other results. We recall the discussion in section 4.3.2 that there are several possibilities to define mass and width. Because the ansatz with the BREIT-WIGNER form (4.25) worked well for the bosonic case we also want to use it for the fermionic case. This will be more complicated because the denominator of the spectral function is much more involved and not only one spectral function exists but two.

To put the spectral functions in a more convenient form for comparing to the BREIT-WIGNER form we expand the first term of the denominator in (4.34) and (4.35) around the physical mass M_R of the resonance:

$$\tilde{s} - \tilde{M}^2 \sim \frac{1}{c}(s - M_R^2) + O((s - M_R^2)^2).$$

The physical mass M_R is defined such that for $\sqrt{s} = M_R$:

$$\tilde{s}(M_R^2) - \tilde{M}^2(M_R^2) = 0. \quad (4.38)$$

We recall that using this definition the physical mass M_R in general cannot be read off as the peak of a spectral function.

The bare mass can be extracted when inserting the definitions for \tilde{M} from equation (4.15):

$$M_0 = \sqrt{\tilde{s}(M_R^2) + Im \Sigma_s(M_R^2)^2 - Re \Sigma_s(M_R^2)}. \quad (4.39)$$

The coefficient c of the TAYLOR expansion is given by the first derivative of $\tilde{s} - \tilde{M}^2$:

$$c = \left[\frac{d}{ds} \left(\tilde{s}(s, Im \Sigma_v(s), Re \Sigma_v(s)) - \tilde{M}^2(s, Im \Sigma_s(s), Re \Sigma_s(s)) \right) \right]_{s=M_R^2}^{-1}.$$

The spectral functions ρ_v (4.34) and ρ_s (4.35) can be approximated around $\sqrt{s} \approx M_R$ by

$$\begin{aligned} \rho_v &\approx -\frac{c}{\pi} \frac{c Im F_v}{(s - M_R^2)^2 + c^2 Z^2} \quad , \\ \rho_s &\approx -\frac{c}{\pi} \frac{c Im F_s}{(s - M_R^2)^2 + c^2 Z^2} \quad . \end{aligned}$$

As in the bosonic case the width can be read off by comparison with the BREIT-WIGNER form defined in equation (4.25). Due to the complicated structure of the

spectral functions not only one possible definition of a width exists but three:

$$\begin{aligned}\Gamma_Z(s) &= -\frac{c}{\sqrt{s}}Z(s), \\ \Gamma_V(s) &= -\frac{c}{\sqrt{s}}\text{Im } F_v(s), \\ \Gamma_S(s) &= -\frac{c}{\sqrt{s}M_0}\text{Im } F_s(s).\end{aligned}$$

Γ_Z is read off from the denominator, whereas Γ_V and Γ_S are read off from the numerator of ρ_v and ρ_s , respectively. In the bosonic case discussed in section 4.3.2 such an ambiguity was not present.

The factor $\frac{1}{M_0}$ in the case of Γ_S is motivated by the fact that a width has energy as the proper unit. When neglecting the real parts of the selfenergy and quadratic terms one can write these widths as:

$$\begin{aligned}\Gamma_Z &= -\frac{c}{\sqrt{s}}2[s(1 - \text{Re } \Sigma_v) \text{Im } \Sigma_v + (M_0 + \text{Re } \Sigma_s) \text{Im } \Sigma_s] \\ &\approx -\frac{2c}{\sqrt{s}}[s\text{Im } \Sigma_v + M_0\text{Im } \Sigma_s], \\ \Gamma_V &= -\frac{c}{\sqrt{s}}\left[-\text{Im } \Sigma_v(\tilde{s} - \tilde{M}^2) + (1 - \text{Re } \Sigma_v) Z\right] \\ &\approx -\frac{c}{\sqrt{s}}\left[-\text{Im } \Sigma_v(\tilde{s} - \tilde{M}^2) + Z\right] \\ &= \frac{c}{\sqrt{s}}\text{Im } \Sigma_v(\tilde{s} - \tilde{M}^2) + \Gamma_Z, \\ \Gamma_S &= -\frac{c}{\sqrt{s}M_0}\left[-\text{Im } \Sigma_s(\tilde{s} - \tilde{M}^2) + (M_0 - \text{Re } \Sigma_s) Z\right] \\ &\approx -\frac{c}{\sqrt{s}M_0}\left[-\text{Im } \Sigma_s(\tilde{s} - \tilde{M}^2) + M_0Z\right] \\ &= \frac{c}{\sqrt{s}M_0}\text{Im } \Sigma_s(\tilde{s} - \tilde{M}^2) + \Gamma_Z.\end{aligned}$$

For $\text{Re } \Sigma \rightarrow 0$ and $\sqrt{s} = M_R$

$$\Gamma_Z = \Gamma_V = \Gamma_S$$

because on the mass-shell $\tilde{s} - \tilde{M}^2 = 0$. But for large real parts of the selfenergy and away from the mass-shell deviations occur. Examples for all three widths will be given in chapter 6.

For the plots and fits of this work Γ_V has been chosen as a reasonable definition for the width. This choice is motivated from the fact that it is a positive definite quantity which follows from the fact that $-\text{Im } F_v$ is positive definite according to (4.36).

Further difficulties arise when considering partial widths of a particle. When a particle has more than one decay channel the total selfenergy will be the sum of the selfenergies of each channel. Then the total width of the particle can be extracted by calculating the propagator with the total selfenergy. Because the selfenergies enter the width non-linearly a partial width cannot be defined as the width where one channel is calculated independently neglecting all the other channels. Such a definition leads to a different bare mass for each channel and the sum of the partial widths would differ from the total width calculated using the full selfenergy. Therefore the partial widths are calculated using the total selfenergy for the real parts of the selfenergy and nonlinear terms in ImF_v and Z (see section 4.2). The partial selfenergies are used for the linear terms only. The sum of all partial widths will be equal to the total width. M_0 is calculated using the total selfenergy.

5 Couplings and Selfenergies of a Spin 3/2 Resonance to $N\pi$, $N\rho$ and $\Delta\pi$

In this chapter interaction LAGRANGIANS for a spin 3/2 resonance to $N\pi$, $N\rho$ and $\Delta\pi$ are introduced. All three interactions are constructed in the PASCALUTSA framework. Using these interactions the selfenergy for each channel is calculated using the method described in section 3.3. For the case of the $N\rho$ and $\Delta\pi$ channel also the selfenergies for the unstable ρ and Δ are calculated.

This full calculation is compared to commonly used simplifications where the widths of the particles are approximated by phase space considerations. For the case of the $N\pi$ coupling the width is also compared to a calculation using conventional coupling.

5.1 Form Factor

Due to the non-renormalizable character of the LAGRANGIANS used in this work a form factor has to be imposed. This is a reasonable approach because we are only interested in the kinematical regime of small energies. The form factor chosen throughout this work is [Kor97]:

$$FF(s) = \exp\left[-\frac{s - s_{threshold}}{\Lambda^2}\right]. \quad (5.1)$$

It is chosen in such a way to be 1 at threshold energy and suppresses the selfenergies by one over e for $\sqrt{s} = \sqrt{s_{threshold} + \Lambda^2}$.

5.2 Selfenergy of a Spin 3/2 Baryon $N\pi$ System

We present here the results for a spin 3/2 baryon decaying into $N\pi$. The LAGRANGIAN for such a state is given by the following formula, the top (bottom) line for particles with positive (negative) parity:

$$\mathcal{L}_{RN\pi} = g_{RN\pi} \bar{\psi}_R^{\mu\nu} \gamma_\mu \begin{Bmatrix} i \\ \gamma_5 \end{Bmatrix} \Psi_N \partial_\nu \pi + h.c.$$

where $\psi^{\mu\nu}$ is the field strength tensor of the spin 3/2 baryon field and is defined in analogy to the electromagnetic case as

$$\psi^{\mu\nu} = \partial^\mu \psi^\nu - \partial^\nu \psi^\mu$$

and its dual

$$\tilde{\psi}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \psi_{\rho\sigma}.$$

The imaginary part of the selfenergy can be determined by four coefficients a_i . They were defined in section 3.2 and are only nonzero for $s > (M_N + m_\pi)^2$. All have the following form:

$$\text{Im } a_i(s) = \frac{N_I}{8\pi} g_{RN\pi}^2 F F(s)^2 \frac{q_*}{\sqrt{s}} b_i(s).$$

N_I is 1 (3) for a resonance of isospin 3/2 (1/2). Furthermore we introduce the notation:

$$q_*^2 = \frac{1}{4s} [(M_N^2 - m_\pi^2 - s)^2 - 4sm_\pi^2], \quad (5.2)$$

$$k_{0*} = s + M_N^2 - m_\pi^2. \quad (5.3)$$

For the functions $b_i(s)$ we obtain

$$b_1(s) = -\frac{1}{3} P q_*^2 s M_N,$$

$$b_2(s) = -\frac{1}{6} q_*^2 k_{0*},$$

$$b_3(s) = 4 b_1(s),$$

$$b_4(s) = 4 b_2(s).$$

P is +1 (−1) for particles with positive (negative) parity. The real part is calculated numerically using the dispersion relation

$$\text{Re } a_i(s) = \frac{\mathcal{P}}{\pi} \int_{(M_N+m_\pi)^2}^{\infty} d\sigma \frac{\text{Im } a_i(\sigma)}{\sigma - s}.$$

The physical meaning of equation (5.2) and (5.3) can be seen when exploring the kinematics of the decaying process depicted in figure 5.1. p , k and q are the four-momenta of the resonance, nucleon and pion, respectively. In the rest frame of the resonance these quantities can be written as

$$p = (\sqrt{s}, 0), \quad p^2 = s, \quad (5.4)$$

$$q = (q_0, \mathbf{q}), \quad q_0^2 = \mathbf{q}^2 + m_\pi^2, \quad (5.5)$$

$$k = (k_0, -\mathbf{q}), \quad k_0^2 = \mathbf{q}^2 + M_N^2.$$

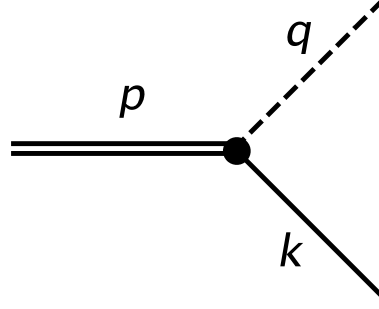


Figure 5.1: Two-body decay.

Using momentum conservation it is possible to calculate \mathbf{q} because

$$\begin{aligned}
 s &= p^2 = (q + k)^2 = (q_0 + k_0)^2 \\
 &= q_0^2 + k_0^2 + 2q_0k_0 \\
 &= 2\mathbf{q}^2 + m_\pi^2 + M_N^2 + 2\sqrt{\mathbf{q}^2 + m_\pi^2}\sqrt{\mathbf{q}^2 + M_N^2} \\
 &\Leftrightarrow (s - 2\mathbf{q}^2 - m_\pi^2 - M_N^2)^2 = 4(\mathbf{q}^2 + m_\pi^2)(\mathbf{q}^2 + M_N^2) \\
 &\Leftrightarrow 4\mathbf{q}^2s = (s - m_\pi^2 - M_N^2)^2 - 4m_\pi^2M_N^2 \\
 \Rightarrow \mathbf{q}^2 &= \frac{1}{4s} [(s - m_\pi^2 - M_N^2)^2 - 4m_\pi^2M_N^2] \\
 &= \frac{1}{4s} [(s + m_\pi^2 - M_N^2)^2 - 4sm_\pi^2] \tag{5.6}
 \end{aligned}$$

$$= \frac{1}{4s} [(s - m_\pi^2 + M_N^2)^2 - 4sM_N^2] . \tag{5.7}$$

Comparing equation (5.2) with (5.6) shows that $q_* = |\mathbf{q}|$ is the momentum of the pion in the rest frame of the resonance. Equation (5.3) becomes clear when calculating the energy of the pion and nucleon using equation (5.4), (5.5) and (5.6),(5.7):

$$\begin{aligned}
 k_0^2 &= \mathbf{q}^2 + M_N^2 = \frac{1}{4s} [(s - m_\pi^2 + M_N^2)^2] , \\
 q_0^2 &= \mathbf{q}^2 + m_\pi^2 = \frac{1}{4s} [(s + m_\pi^2 - M_N^2)^2] .
 \end{aligned}$$

The quantity in (5.3) is proportional to the energy of the nucleon in the rest frame of the resonance.

On the mass shell the conventional and the PASCALUTSA coupling are the same [Pas01]. The relation between the PASCALUTSA coupling $g_{RN\pi}$ and conventional coupling $f_{RN\pi}$ defined in (2.28) is given by

$$g_{RN\pi} = \frac{f_{RN\pi}}{m_\pi M_R} .$$

Next we analyse the phase space of the obtained width. As shown in section 4.3.4, when neglecting the real parts and quadratic terms the width near the on-shell point is:

$$\Gamma = -\frac{2c}{\sqrt{s}} (s \text{Im } \Sigma_v + M_0 \text{Im } \Sigma_s).$$

When neglecting the real parts the bare mass M_0 is equal to the physical mass M_R . Inserting the selfenergy calculated in this section the width can be written as

$$\begin{aligned} \Gamma &= -\frac{2c}{\sqrt{s}} (s \text{Im } a_2 + M_0 \text{Im } a_1) \\ &= -\frac{2c}{\sqrt{s}} \frac{N_I}{8\pi} g_{RN\pi}^2 \frac{q_*}{\sqrt{s}} (s b_2 + M_R b_1) \\ &= c \frac{N_I}{12\pi} g_{RN\pi}^2 q_*^3 \frac{s}{\sqrt{s}} \left(\frac{1}{2\sqrt{s}} k_{0*} + P M_N \frac{M_R}{\sqrt{s}} \right). \end{aligned}$$

As shown above $k_0 = \frac{1}{2\sqrt{s}} k_{0*}$ is the energy of the nucleon and q_* is the momentum of the pion. Inserting the conventional coupling the width reads

$$\Gamma = c \frac{N_I}{12\pi m_\pi^2} f_{RN\pi}^2 q_*^3 \frac{s}{M_R^2} \frac{k_0 + P M_N \frac{M_R}{\sqrt{s}}}{\sqrt{s}}. \quad (5.8)$$

For $c=1$ and on the mass-shell this width agrees with the width calculated in [PM02].

Now we can compare this result with the fact [PLM04] that around the threshold the energy dependence of the width is determined by the orbital angular momentum l :

$$\Gamma(\sqrt{s} \approx \sqrt{s_{thr}}) \sim \mathbf{k}^{2l+1} \quad (5.9)$$

with \mathbf{k} as the center of mass momentum of the decay products. For small kinetic energies we can expand the energy in the non-relativistic limit with

$$k_0 = \sqrt{\mathbf{q}^2 + M_N^2} \sim M_N + \frac{\mathbf{q}^2}{2M_N}.$$

The width (5.8) is proportional to

$$\begin{aligned} \Gamma(s) &\sim \mathbf{q}^3 (k_0 + P M_N) = \mathbf{q}^3 \left(M_N (1 + P) + \frac{\mathbf{q}^2}{2M_N} \right) \\ &\sim \begin{cases} \mathbf{q}^3 & \text{for positive parity } P = +1 \\ \mathbf{q}^5 & \text{for negative parity } P = -1 \end{cases}. \end{aligned}$$

The angular momentum can be read off using equation (5.9): $l = 1$ for a particle with positive parity and $l = 2$ for negative parity. The LAGRANGIAN in this section will describe for positive parity a P-wave and for negative parity a D-wave resonance decaying into $N\pi$.

5.3 Selfenergy of a Spin 3/2 Baryon $N\rho$ System

Because the ρ -meson is not a stable particle the spectral function is not trivially given as a δ -function. A lot of work have been devoted to the ρ -meson and its properties (e.g. [KKW96] [HFN93]). In this work we took the selfenergies calculated in [HFN93] and presented in section 3.1. They come in a full analytical form and preserve unitarity. Using this selfenergies the spectral function ρ of the ρ -meson can be derived directly as shown in section 4.3.2.

We construct a relativistic gauge invariant LAGRANGIAN of the form

$$\mathcal{L} = \frac{g_{RN\rho}}{2} \bar{\psi}^{\mu\nu} \begin{Bmatrix} i\gamma_5 \\ 1 \end{Bmatrix} \Psi \rho_{\mu\nu} + h.c.$$

with top (bottom) line for particles with positive (negative) parity. $\psi^{\mu\nu}$ and $\rho_{\mu\nu}$ are the field strength tensors of the spin 3/2 baryon and ρ -meson, respectively. The imaginary parts of the coefficients a_i are only nonzero for $s > (M_N + 2m_\pi)^2$ and all have the form:

$$\begin{aligned} \text{Im } a_i(s) = & -\frac{N_I}{4\pi^2} g_{RN\rho}^2 F F(s)^2 \int_{M_N}^{z_*(s)} dk_0 b_i(s, k_0) \\ & \times \sqrt{k_0^2 - M_N^2} \rho \left(\sqrt{s - 2\sqrt{s}k_0 + M_N^2} \right). \end{aligned} \quad (5.10)$$

The lower limit of the integral is introduced by one of the Θ -functions in 3.6. The upper limit comes because the spectral function of the ρ -meson is zero ($\rho(q^2) = 0$) for $q^2 < 4m_\pi^2$ with

$$z_*^2 = \frac{1}{4s} (s + M_N^2 - 4m_\pi^2)^2.$$

The integral is evaluated numerically. The functions $b_i(s, k_0)$ are obtained as:

$$\begin{aligned} b_1(s, k_0) &= -\frac{s}{3M_N} P (M_N^4 + 2(s - 3k_0\sqrt{s} + k_0^2)M_N^2 + sk_0^2), \\ b_2(s, k_0) &= -\frac{\sqrt{s}k_0}{3M_N^2} (M_N^4 + 2(s - 3k_0\sqrt{s} + k_0^2)M_N^2 + sk_0^2), \\ b_3(s, k_0) &= b_1(s, k_0), \\ b_4(s, k_0) &= b_2(s, k_0). \end{aligned}$$

The real part is given by the numerical integration of

$$\text{Re } a_i(s) = \frac{\mathcal{P}}{\pi} \int_{(M_N+2m_\pi)^2}^{\infty} d\sigma \frac{\text{Im } a_i(\sigma)}{\sigma - s}.$$

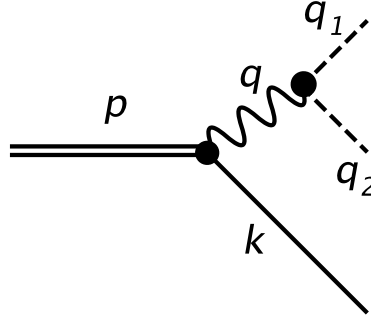


Figure 5.2: Three-body decay of a resonance into $N\pi\pi$ via a virtual ρ -meson.

The quantity z_*^2 can be understood when exploring the three-body kinematics of this process in figure 5.2. In the rest frame of the resonance the two pions have the same momentum when the nucleon has maximum energy. The kinematical quantities are then:

$$\begin{aligned} p &= (\sqrt{s}, 0), & p^2 &= s, \\ q_1 &= q_2 = (q_0, \mathbf{q}), & q_0^2 &= \frac{\mathbf{k}^2}{4} + m_\pi^2, \\ k &= (k_0, \mathbf{k}), & k_0^2 &= \mathbf{k}^2 + M_N^2. \end{aligned}$$

Using momentum conservation the energy of the nucleon can be calculated

$$\begin{aligned} s &= p^2 = (q_1 + q_2 + k)^2 \\ &= (2q_0 + k_0)^2 \\ &= 2k_0^2 - M_N^2 + 4m_\pi^2 + 4q_0k_0 \\ &\Leftrightarrow (s - 4m_\pi^2 + M_N^2 - 2k_0^2)^2 = 16q_0^2k_0^2 \\ &\Rightarrow 4sk_0^2 = (s - 4m_\pi^2 + M_N^2)^2 \\ &\Rightarrow k_0^2 = \frac{1}{4s}(s + M_N^2 - 4m_\pi^2)^2 = z_*^2. \end{aligned}$$

Now the limits of the integration become clear. The lower limit is the energy where the nucleon is at rest and the upper limit is the energy where the nucleon has maximum energy and the ρ -meson is at rest. Because the ρ -meson is not a stable particle k_0 can take all possible values between these two limits. In this picture ρ serves as the probability to find a ρ -meson with energy k_0 .

Taking the ρ -meson as a stable particle with mass m_ρ the integration in equation (5.10) can be carried out. Then the coefficients a_i are nonzero for $s > (M_N + m_\rho)^2$ and one obtains

$$\text{Im } a_i(s) = -\frac{N_I}{8\pi^2} g_{RN\rho}^2 F(s)^2 \frac{k_*}{\sqrt{s}} b_i(s, k_{0*}).$$

With the notations

$$k_{0*}^2 = \frac{1}{4s}(s + M_N^2 - m_\rho^2)^2,$$

$$k_*^2 = k_{0*}^2 - M_N^2.$$

Equivalent to the last section $k_0 = k_{0*}$ is the energy and $\mathbf{k}^2 = k_*^2$ the momentum of the nucleon. Inserting this notation the coefficients $b_i(s, k_{0*})$ can be calculated further leading to

$$b_1(s) = -P \frac{s}{3M_N} B(s),$$

$$b_2(s) = \frac{\sqrt{s}k_{0*}}{3M_N^2} B(s),$$

$$B(s) = ((2M_N^2 + s)k_*^2 + 3m_\rho^2 M_N^2).$$

The nonrelativistic limit of the width is calculated only for stable ρ -mesons. The approach is equal to the previous section:

$$\Gamma_{N\rho} \sim \sqrt{\mathbf{k}^2}(s b_2 + M_R b_1)$$

$$\sim \sqrt{\mathbf{k}^2} B(s)(k_0 - P M_N).$$

Expanding k_0 and $B(s)$ in the non-relativistic limit will give in leading order:

$$k_0 = \sqrt{\mathbf{k}^2 + M_N^2} \sim M_N + \frac{\mathbf{k}^2}{2M_N},$$

$$B(s) \sim 3m_\rho^2 M_N^2.$$

The width is proportional to

$$\Gamma_{N\rho} \sim \sqrt{\mathbf{k}^2}(\mathbf{k}^2 + M_N - P M_N)$$

leading to a P-wave for a resonance with positive parity and an S-wave for a resonance with negative parity.

5.4 Selfenergy of a Spin 3/2 Baryon $\Delta\pi$ System

Due to the unstable character of the Δ its spectral function ρ is not trivial anymore.

The Δ on its own is a spin 3/2 particle, so the LAGRANGIAN must be invariant under a simultaneous gauge transformation of both spin 3/2 particles. A LAGRANGIAN satisfying allsymmetries has the form

$$\mathcal{L} = \frac{g_{R\Delta\pi}}{2} \bar{\psi}^{\mu\nu} \gamma_\alpha \left\{ \begin{matrix} i\gamma_5 \\ 1 \end{matrix} \right\} \Delta_{\mu\nu} \partial^\alpha \pi + h.c.$$

The imaginary parts of the coefficients a_i are only non-zero for $s > (M_N + 2m_\pi)^2$ and have the form:

$$Im\ a_i(s) = N_I \frac{3}{2\pi^3} g_{R\Delta\pi}^2 F F^2 \int_{ka_0(s)}^{\sqrt{s}-m_\pi} dk_0 q_{vec} \rho(m_\pi^2 + 2\sqrt{s}k_0 - s) b_i(s, k_0) \quad (5.11)$$

with $\rho = \rho_s$ for $i = 1, 3$ and $\rho = \rho_v$ for $i = 2, 4$. ρ_s and ρ_v are the scalar and vectorial parts of the Δ spectral function (section 4.3).

The upper limit of the integral comes from (3.7). If the Δ would get more energy there would be no energy left to create a pion. The lower limit is introduced when demanding that $\rho(k^2) = 0$ for $k^2 < (M_N + m_\pi)^2$. This is the minimal energy needed to create a unstable Δ . The integral is evaluated numerically.

We introduce the notation

$$ka_0(s) = \frac{1}{2\sqrt{s}}(M_N^2 + 2M_N m_\pi + s),$$

$$q_{vec}^2(s, k_0) = (k_0 - \sqrt{s})^2 - m_\pi^2,$$

where $ka_0(s)$ is the least energy needed to create a Δ in the rest frame of the decaying resonance. q_{vec} is the momentum of the pion in the same frame.

For the functions b_i we obtain:

$$b_1(s, k_0) = -\frac{1}{9} P s [2M_N^4 + (7k_0^2 - 8\sqrt{s}k_0 + 2s)M_N^2 + k_0^2(7s - 10k_0\sqrt{s})],$$

$$b_2(s, k_0) = \frac{\sqrt{s}}{9} [4(k_0 - \sqrt{s})M_N^4 + k_0(5k_0^2 - 14\sqrt{s}k_0 + 4s)M_N^2 + 5k_0^3s],$$

$$b_3(s, k_0) = -\frac{2}{9} P s [M_N^4 + (k_0^2 + 2\sqrt{s}k_0 + s)M_N^2 - k_0^2(2\sqrt{s}k_0 + s)],$$

$$b_4(s, k_0) = -\frac{2\sqrt{s}}{9} [(k_0 + 2\sqrt{s})M_N^4 + 2k_0(k_0^2 + 2\sqrt{s}k_0 - s)M_N^2 + k_0^3s].$$

The real part is given by the numerical integration of

$$Re\ a_i(s) = \frac{\mathcal{P}}{\pi} \int_{(M_N+2m_\pi)^2}^{\infty} d\sigma \frac{Im\ a_i(\sigma)}{\sigma - s}.$$

Taking the Δ -resonance as a stable particle with mass M_Δ one can carry out the integration in equation (5.11) leading to

$$Im\ a_i(s) = -N_I \frac{1}{4\pi} g_{R\Delta\pi}^2 F F(s)^2 k_{vec} b_i(s)$$

with k_{vec} as the momentum of the Δ which is, in the rest frame of the resonance, given by

$$k_{vec}^2(s) = \frac{1}{4s} ((s - m_\pi^2 + M_\Delta^2)^2 - 4sM_\Delta^2).$$

The functions b_i are similar to the unstable case. b_1 has to be multiplied by M_Δ , $M_N \rightarrow M_\Delta$ and k_0 is not a free parameter anymore because the Δ can only be on the mass-shell with the condition $k_0^2 = k_{vec}^2 + M_\Delta^2$. In the rest frame of the resonance it is given by

$$k_0^2(s) = \frac{1}{4s}(s - m_\pi^2 + M_\Delta^2)^2.$$

Before we calculate the non-relativistic limit for the width as was done in the previous sections we want to rearrange b_1 and b_2 in a more convenient form using the on-shell condition leading to:

$$\begin{aligned} b_1 &= -\frac{s}{9}M_\Delta P \left[(M_\Delta^2 + s)(9M_\Delta^2 + 7k_{vec}^2) - 2\sqrt{s}k_0(9M_\Delta^2 - 5k_{vec}^2) \right], \\ b_2 &= \frac{1}{9} \left[\sqrt{s}k_0(M_\Delta^2 + s)(9M_\Delta^2 + 5k_{vec}^2) - 2sM_\Delta^2(9M_\Delta^2 + 7k_{vec}^2) \right]. \end{aligned}$$

The on-shell width, in first order of k_{vec} , is proportional to

$$\begin{aligned} \Gamma_{\Delta\pi} &\sim k_{vec}(M_R b_2 + b_1) \\ &= k_{vec} \left(\frac{M_R^2}{9} \left[(M_\Delta^2 + M_R^2)(k_0 - P M_\Delta)9M_\Delta^2 - 2M_R M_\Delta (M_\Delta - P k_0)9M_\Delta^2 \right] \right) \\ &= k_{vec} (M_R^2 M_\Delta^2 (k_0 - P M_\Delta)(M_\Delta + P M_R)^2). \end{aligned}$$

In the non-relativistic limit $k_0 \rightarrow M_\Delta + \frac{k_{vec}^2}{2M_\Delta}$ the width is proportional to

$$\Gamma_{\Delta\pi} \sim k_{vec} (M_\Delta(1 - P) + k_{vec}^2).$$

This channel is an S-wave (P-wave) for a resonance with negative (positive) parity which is the correct description.

6 Results for $P_{33}(1232)$ and $D_{13}(1520)$

In this chapter the results for the $P_{33}(1232)$ and $D_{13}(1520)$ resonances will be presented. We will introduce the parameters and discuss the off-shell width of the particles, their selfenergies and spectral functions.

The calculations were done using the following parameters: bare mass M_0 and for each decay channel i a coupling g_i and a cut-off Λ_i . For simplicity all Λ_i are chosen to be the same. We will study, however, how the results change when Λ is varied. The best way to obtain the values of the parameters would be a fit to experimental data (e.g. phase shifts). But this approach would go far beyond the scope of this work. The emphasis of this work is to find out whether it is feasible to calculate propagators for spin 3/2 particles in a fully relativistic framework for different channels. The priority is set to the implementation of the full relativistic structure of the propagator. Calculating experimental data, as for example cross sections and phase shifts, from the derived selfenergies and spectral functions is not a trivial task involving also background terms etc. (cf. the corresponding discussion in the introduction chapter). Therefore the parameters were fitted only to the partial widths and the mass of the resonance taken from [Eea04]. This leads to some complications because mass and width are not directly measurable observables leaving an ambiguity how to define them as discussed in section 4.3.

Further problems arise because the parameters M_0 and g are coupled to each other. To calculate M_0 from equation (4.39) the full knowledge of the selfenergy is needed which will only be possible when the couplings are known. They can be extracted by demanding that the respective partial width of the resonance on the mass-shell is equal to the partial width published in [Eea04]. But for the calculation of the widths M_0 is needed. In the case where only one channel exists this problem can be solved by inserting equation (4.39) into the propagator. In the case of different channels M_0 needs to be calculated with the sum of all selfenergies as discussed in section 4.3.4. Variation of the coupling in each channel will influence every other channel making fine tuning a difficult task.

Obviously It is impossible to fit the three parameters M_R , g and Λ , using only two input parameters. This leaves one parameter, the cut-off Λ , open which leads to an ambiguity on the results because the selfenergy depends largely on Λ . Although it is not possible to pin down Λ exactly it will be possible to give arguments for a

reasonable range of values which resolves the ambiguity. The reasonable range of Λ is around $\Lambda = 1$ GeV. When not stated differently all plots depicted in this chapter are calculated using such a value for Λ .

The form factor (5.1) used in this work is 1 at threshold and decreases exponentially afterwards. This means that the value of the coupling g is given at threshold energy and not on the mass-shell. In the literature (e.g. [PLM04]) the form factors are often chosen in such a way to be 1 on the mass-shell of the particles. The given values of the coupling constants are hence the values on the mass-shell. To make it easier to compare the values in this work with other works also the effective coupling constant on the mass-shell g^{eff} will be listed. The effective coupling constant is defined as

$$g^{\text{eff}} = g * FF(M_R^2).$$

6.1 Parameters of the $\Delta P_{33}(1232)$

Much work has already been devoted to the complete relativistic structure of the Δ in conventional [Kor97] and PASCALUTSA [AKS02] coupling. We present these results for completeness and as a good example for the structure of a relativistic spin 3/2 propagator. In addition, the spectral functions of the Δ are needed as an input for the decay channel $N^*(1520) \rightarrow \Delta\pi$.

The calculations are done using three parameters, bare mass M_0 , coupling $g_{\Delta N\pi}$ and cut-off Λ . For given Λ the parameters M_0 and $g_{\Delta N\pi}$ are chosen such that M_R in equation (4.39) takes the physical value 1.232 GeV and the width on the mass-shell is

$$\Gamma_V(M_R) = \Gamma_{\text{exp}} = 120 \text{ MeV}.$$

The Δ is a particle with isospin 3/2 so $N_I = 1$, and positive parity $P = +1$. The remaining parameters for different cut-offs and the effective coupling are listed in table 6.1.

In view of these parameters a motivation for the above mentioned value of $\Lambda = 1$ GeV can be given. For small Λ the effective coupling deviates largely from the coupling at threshold energy. This is not desirable because in the physical meaningful region from threshold energy to approximately 1 GeV above threshold the coupling should not deviate too much. Consistently small values of Λ can be excluded. For large values of Λ the bare mass is shifted to a very high value. It is not likely that the selfenergy of the P_{33} changes its own mass so dramatically. There will be a further argument why large values of Λ lead to undesired features when discussing the normalization function of the P_{33} . All this arguments raise strong indications for a reasonable cut-off parameter at around 1 GeV.

Λ [GeV]	M_0 [GeV]	$g_{\Delta N\pi}$ [GeV $^{-2}$]	$g_{\Delta N\pi}^{\text{eff}}$ [GeV $^{-2}$]
0.6	1.237	34.8	12.80
1.0	1.334	22.5	13.05
2.0	2.303	17.0	15.08

Table 6.1: Parameter for the P_{33} resonance

6.2 Parameters of the $N^* D_{13}(1520)$

There are three major decay channels for the $D_{13}(1520)$, which are $N\pi$, $N\rho$ and $\Delta\pi$. We calculate the selfenergy of each channel separately. The total selfenergy is then given as the sum of each individual channel.

The $D_{13}(1520)$ has negative parity and is an isospin 1/2 hadron. This fixes $P = -1$. $N_I = 3$ for the channels $N\pi$ and $N\rho$ and $N_I = 12$ for the channel $\Delta\pi$.

The coupling constants are fitted via the partial width of each channel in such a way that the contribution to the total width on the mass-shell of the $N\pi$ channel is 55% as proposed in [Eea04]. The contribution of the $N\rho$ channel is chosen to be 10 MeV or 26 MeV. The results in this work are calculated with this two values because it is hard to extract the coupling of the ρ -meson to the D_{13} resonance. Information on the coupling of baryon resonances to the $N\rho$ channel originates mainly from an analysis of the reaction $\pi N \rightarrow \pi\pi N$. This is a formidable task because the $N^+(1520)$ is nominally subthreshold to a ρN final state. In other words, only the low-energy tail of the ρ -meson contributes at the N^* mass shell. Analysis of the experimental data by Manley [MAGT84][MS92] lead to the value of $\Gamma_{N\rho} = 26$ MeV. This value is also similar to the partial width given by the particle data group (PDG) [Eea04]. A somewhat different approach was taken by Leupold and Post [LP05] where the coupling was extracted in a QCD sum rule analysis of in-medium modifications of the ρ -meson. This analysis leads to the smaller value of $\Gamma_{N\rho} = 10$ MeV. A similar value $\Gamma_{N\rho} = 12$ MeV was deduced by Vrana [VDL00] in an analysis of the experimental data similar to Manley. In the present work both partial widths will be used to see how much the spectral function of the D_{13} depends on the value of the coupling to $N\rho$. The $\Delta\pi$ channel is adjusted such that one always gets a total width of $\Gamma = 120$ MeV.

Because the definition for the partial width discussed in section 4.3.4 leads to coupled equations the couplings are fitted such that the above requirements are fulfilled as good as possible.

In table 6.2 the parameters for different Λ 's and $\Gamma_{N\rho} = 10$ MeV can be found. In table 6.3 the couplings are fitted to give a contribution for the $N\rho$ channel of 26 MeV.

Λ [GeV]	M_0 [GeV]	$g_{N^*N\pi}$	$g_{N^*N\pi}^{\text{eff}}$	$g_{N^*N\rho}$	$g_{N^*N\rho}^{\text{eff}}$	$g_{N^*\Delta\pi}$	$g_{N^*\Delta\pi}^{\text{eff}}$
0.6	1.27	153.6	6.3	40.3	3.9	8.85	0.86
1.0	1.55	22.2	7.0	9.6	4.1	2.1	0.91
2.0	3.98	9.7	7.3	5.2	4.2	1.16	0.94

Table 6.2: Parameter for the D_{13} resonance with a partial width of $\Gamma_{\rho N} = 10$ MeV. The sum of all partial widths is 120 MeV and the cut-off parameter for the Δ resonance is set to 1 GeV. The units of the couplings are $[\text{GeV}^{-2}]$ for the $N\pi$ and $N\rho$ channels and $[\text{GeV}^{-3}]$ for the $\Delta\pi$ channel.

Λ [GeV]	M_0 [GeV]	$g_{N^*N\pi}$	$g_{N^*N\pi}^{\text{eff}}$	$g_{N^*N\rho}$	$g_{N^*N\rho}^{\text{eff}}$	$g_{N^*\Delta\pi}$	$g_{N^*\Delta\pi}^{\text{eff}}$
0.6	1.35	154.6	6.3	65.6	6.4	7.1	0.65
1.0	1.65	22.3	7.0	15.9	6.9	1.68	0.72
2.0	5.77	9.8	7.3	8.4	6.9	0.98	0.79

Table 6.3: As figure 6.2 but using $\Gamma_{N\rho} = 26$ MeV.

6.3 Selfenergies

The selfenergies of the P_{33} are depicted in figure 6.1. The imaginary part does not change sign and goes to zero for large \sqrt{s} due to the form factor applied to it. For the P_{33} both imaginary parts of the selfenergy are negative which is characteristic for a particle with positive parity in this channel.

The real parts are small compared to the mass of the P_{33} but compared to the imaginary parts equally large on the mass-shell. This is only the case for the cut-off parameter chosen for the plots which is $\Lambda = 1$ GeV. Choosing different values for the cut-off leads to a large variation of the real part, depicted in figure 6.2. One sees that for large values of the cut-off Λ the real parts get very large and do not change the sign anymore. As discussed above such high values of Λ are not reasonable. When taking smaller values of Λ the real parts become small on the mass-shell. But for too low values of Λ the variation of the coupling is strong and also the suppression in the physical meaningful energy region is high.

For the case of the D_{13} the selfenergies are depicted in figure 6.3. The imaginary parts do not change sign, as for the case of the P_{33} , but the sign is opposite for Σ_v and Σ_s . The energy dependence of the selfenergy is much stronger for the case of the D_{13} leading to a larger shoulder in the spectral function which will be discussed later.

The real parts of the D_{13} are small around the mass-shell of the resonance and change its sign. This is, as for the case of the P_{33} , only true for the cut-off parameter around $\Lambda = 1$ GeV. For lower Λ the on-shell value of the real parts can become large

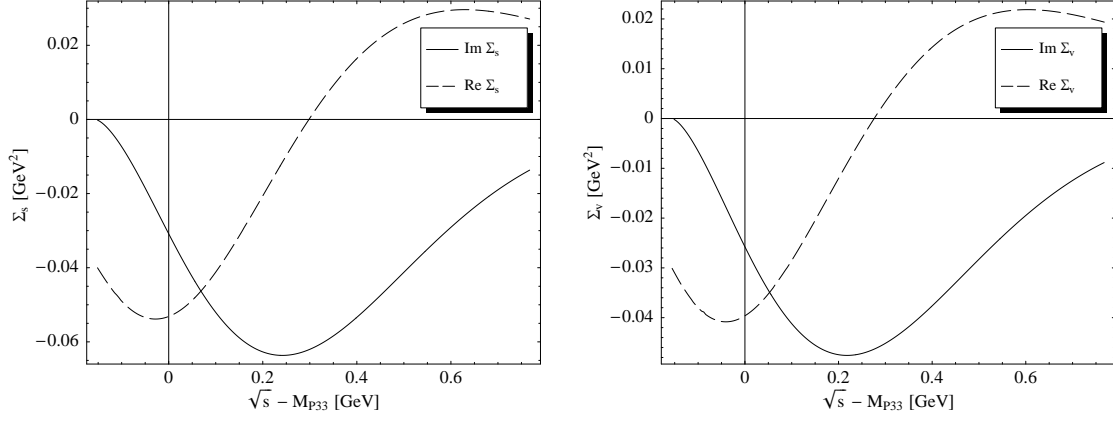


Figure 6.1: The selfenergy of the $P_{33}(1232)$ resonance as defined in equation (3.5). Left: Scalar part. Right: Vector part.

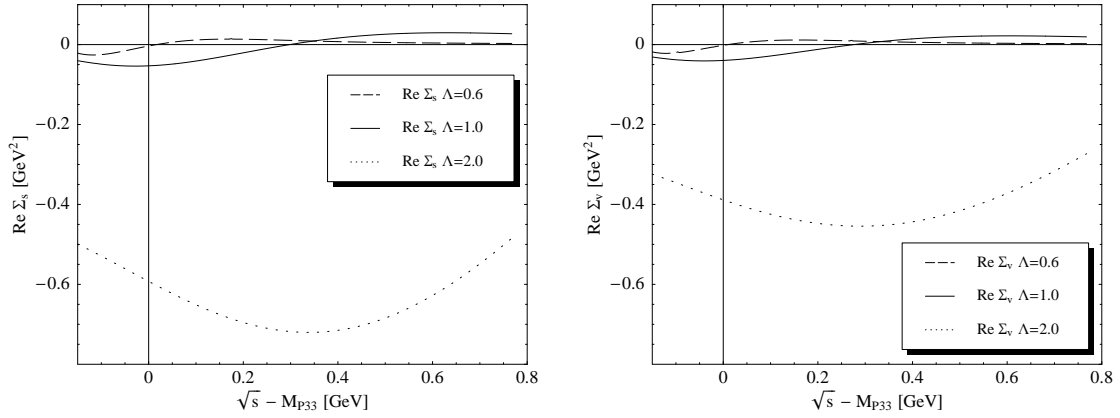


Figure 6.2: Real part of the $P_{33}(1232)$ resonance selfenergies with different form factor Λ as defined in equation (5.1). Left: Scalar part. Right: Vector part.

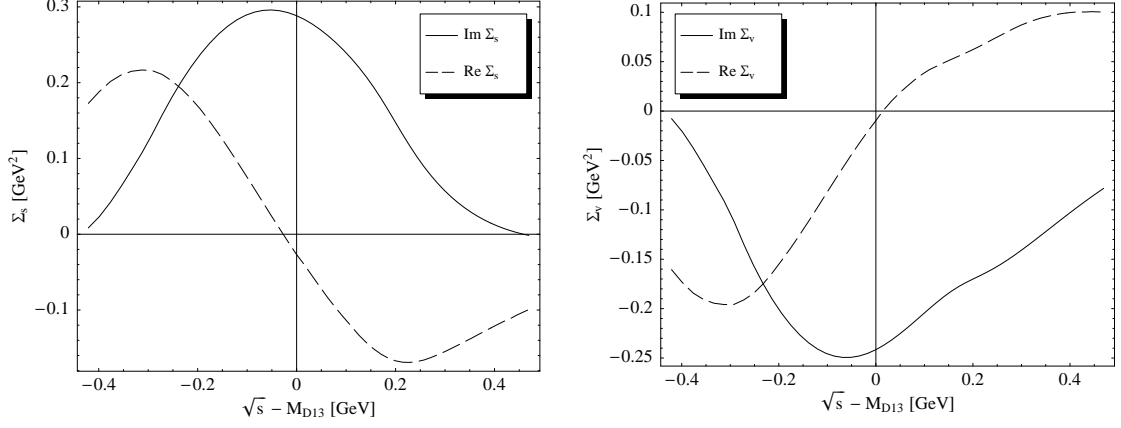


Figure 6.3: The selfenergy of the $D_{13}(1520)$ resonance as defined in equation 3.5. Left: Scalar part. Right: Vector Part.

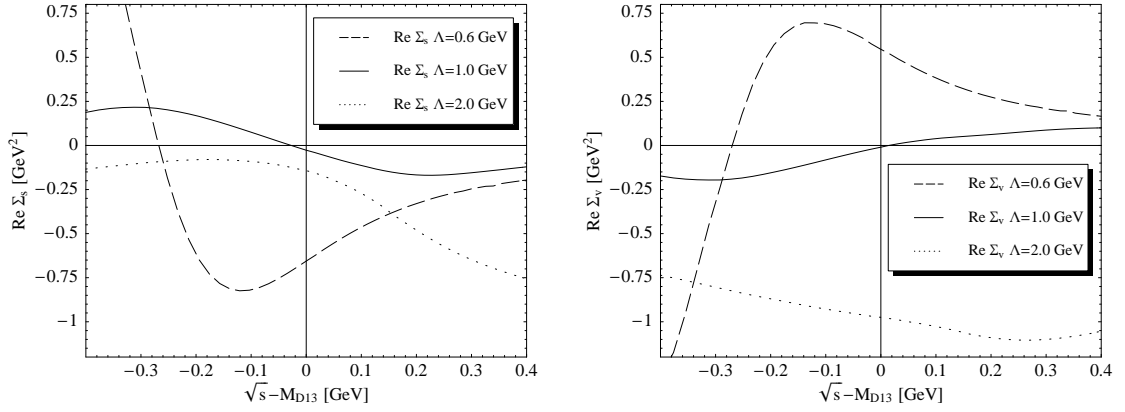


Figure 6.4: Real parts of the D_{13} resonance selfenergies plotted for different cut-off parameter Λ . The corresponding form factor is defined in section 5.1. Left: Scalar part. Right: Vector part.

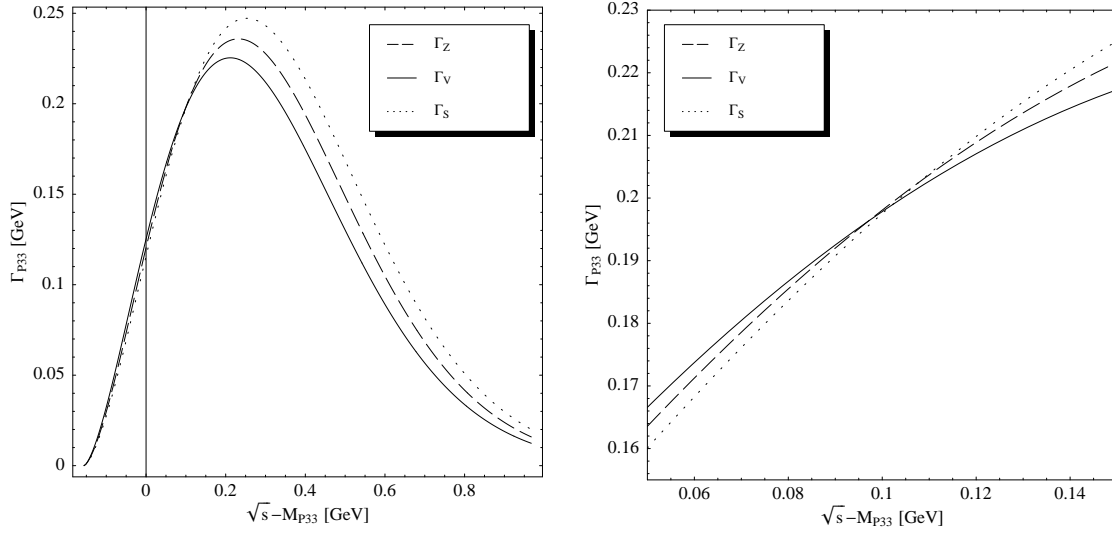


Figure 6.5: Left: Γ_Z , Γ_V and Γ_S . Right: Γ_Z , Γ_V and Γ_S zoomed.

as depicted in figure 6.4. A difference to the P_{33} case can be seen because there for lower Λ the real parts became smaller which means that this is not generally true but differ from case to case. When Λ is large the real parts do not change sign anymore and become large on the mass-shell.

6.4 The Width

As discussed in section 4.3.4 there is an ambiguity concerning which function is to call the width when going off-shell. In figure 6.5 all three candidates are depicted for the case of the P_{33} . They have large energy dependences with a maximum above the mass-shell energy. This means that spectral strength will be concentrated above mass-shell energies.

In the case of the P_{33} all three widths are similar over the whole energy range. All three widths become equal and cross each other slightly above the mass-shell energy. This is the case because the real parts of the selfenergies become zero and change sign also above the mass-shell energy as can be seen in figure 6.1. Because the values of the real parts are small, compared to the mass, the deviations are small. Γ_Z , Γ_V and Γ_S are positive definite which is not in general the case because only for Γ_V such a constraint exists (cf. the discussion in section 4.3.4). But it is not surprising that all widths are positive definite in the case of the P_{33} resonance because the width is approximately proportional to

$$\Gamma \sim -(s \text{Im} \Sigma_v + M_0 \text{Im} \Sigma_s)$$

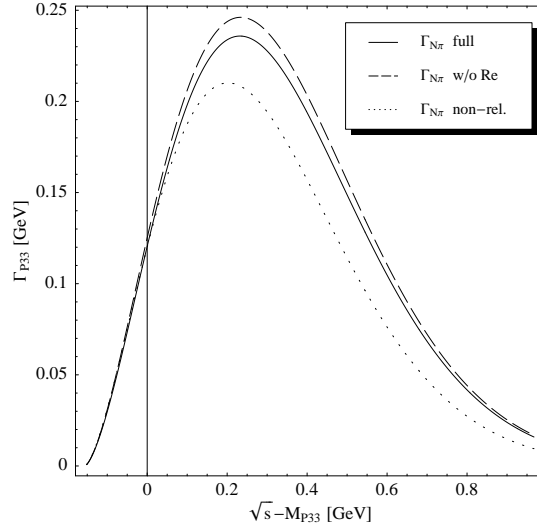


Figure 6.6: Width of the $P_{33}(1232)$. The dashed line is the width calculated by Penner [PM02]. The dotted line is a simple non-relativistic width.

as was shown in section 4.3.4. The imaginary parts of the selfenergy are both negative when the resonance has positive parity as was discussed in the last section. Then Γ will not become negative. This is in general not the case as was shown for the D_{13} where the imaginary parts of the selfenergy have opposite signs. There, Γ_Z and Γ_S will become negative leaving Γ_V as the only good candidate for a definition of the width. In the case of the P_{33} all three candidates are good choices.

It is possible to compare the width of the P_{33} with a calculation where the real parts of the selfenergy are neglected and conventional coupling is used, published in [PM02]. We have discussed this width in section 5.2 equation (5.8). Both widths agree well from threshold to slightly above the mass-shell region as depicted in figure 6.6. The similarity near the threshold region is induced by the fact that both widths have to reach the same non-relativistic limit. This can be seen when comparing both widths with a simple non-relativistic width of the form

$$\Gamma_{\text{non-rel.}}(s) = f \mathbf{q}^3 F F(s)^2 = f \left[\frac{1}{4s} [(s - m_\pi^2 - M_N^2)^2 - 4m_\pi^2 M_N^2] \right]^{3/2} F F(s)^2 \quad (6.1)$$

which is discussed in section 5.2 and also depicted in figure 6.6. The quantity \mathbf{q} is the center of mass momentum of the decay products. The constant f is fitted such that on the mass-shell the width is equal to 120 MeV.

Near the mass-shell the full width and the approximation by Penner [PM02] are equal when the real parts of the selfenergy proceeds zero as was shown in section 5.2. Because the real parts are small for the cut-off parameter chosen both widths

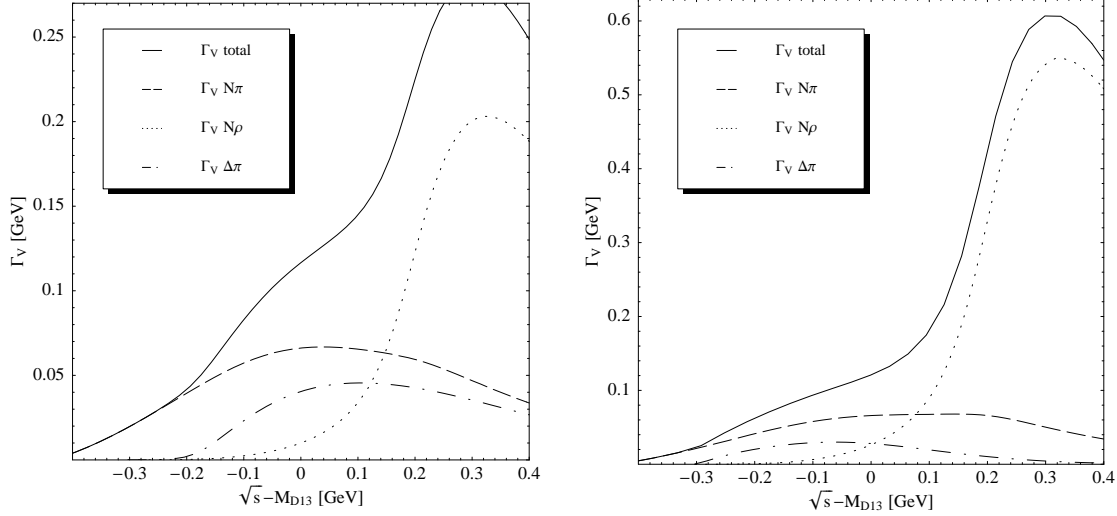


Figure 6.7: Left: Solid line is the width of the $D_{13}(1520)$ with $\Gamma_{N\rho} = 10$ MeV. The dashed lines are the partial widths of the various channels. Right: Same as left but with $\Gamma_{N\rho} = 26$ MeV.

agree well on the mass-shell. This two constraint, at threshold energy and at the mass-shell region, keeps both widths close together. Above this energy region no further constraint occurs and both functions start to differ. The difference is small, leading to the conclusion that the approximations are reasonable.

The non-relativistic width gives a good description of the width for the P_{33} until the region of its on-shell mass. Similar as above this can be understood because the full width will give the correct non-relativistic limit for small \sqrt{s} and on the mass-shell both widths are fitted to be equal. Above the on-shell mass region the description is not reasonable indicating that the more involved character of the width plays a role in this region where relativistic effects cannot be neglected.

The width of the D_{13} is depicted in figure 6.7. On the left the width and the partial widths are depicted for a partial width of the $N\rho$ channel $\Gamma_{N\rho} = 10$ MeV and on the right for $\Gamma_{N\rho} = 26$ MeV. One sees that the widths in both cases are highly energy dependent and have their maximum above the mass-shell region.

The dashed curves in the plot of the D_{13} are the partial widths of each channel. For high energies the coupling to $N\rho$ dominates. This can be understood because the nominal threshold region of this channel lays at $M_N + m_\rho \approx 1.7$ GeV, so only the tail of the mass spectrum of the ρ -meson contributes at the D_{13} mass shell. Even though the partial width of this channel is comparably small, due to the subthreshold nature, the coupling must be large to gain such a width. This can be seen when the phase space opens up and the $N\rho$ channel dominates the total width.

For the larger value of the partial width of the $N\rho$ channel the energy dependence

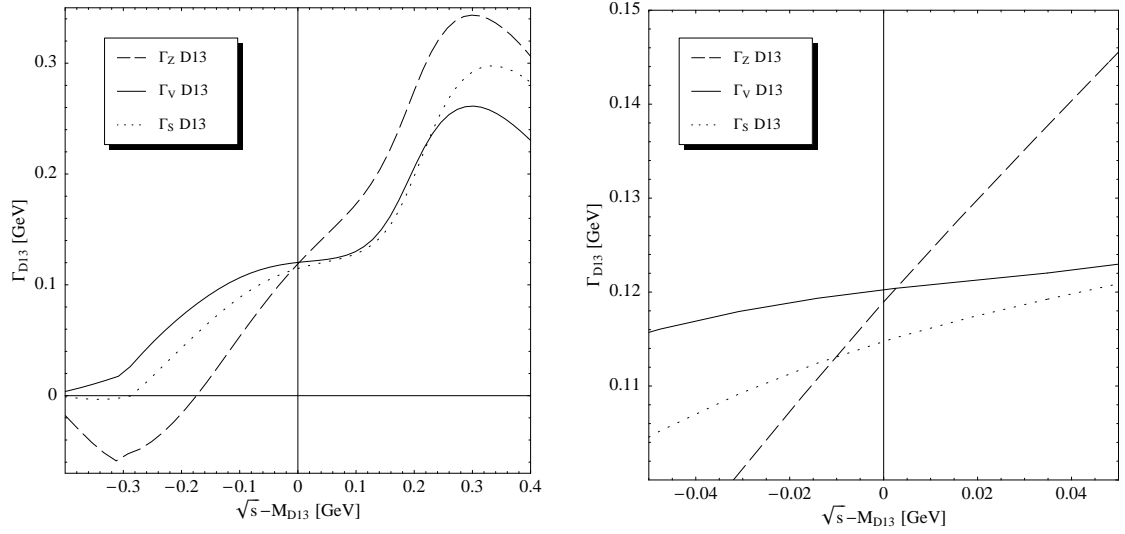


Figure 6.8: Left: Different definitions for the width Γ_Z , Γ_V and Γ_S , Right: Γ_Z , Γ_V and Γ_S zoomed at on-shell energy.

is even stronger leading to a three times higher maximum value of the width. In the spectral function a transfer of spectral strength to higher energies is expected.

The alternative widths are depicted in figure 6.8. They clearly differ as expected. For small \sqrt{s} the quantities Γ_Z and Γ_S become negative making them a bad choice for a width. When the real parts of the selfenergies are small, which is the case on the mass-shell, all quantities are about the same. Because for the cut-off parameter chosen the real parts are small it is not surprising that all three widths are nearly equal on the mass-shell as shown in figure 6.8.

Because the real parts are small on the mass-shell there is no ambiguity choosing a width in this energy region. Going off-shell it is preferable to take Γ_V because it is always a positive definite quantity as shown in section 4.3.4.

In the case of the D_{13} an easy comparison to a non-relativistic width is not possible due to the unstable character of the decay products. To calculate the momentum of the stable particles, a complicated analysis of three-body kinematics would be needed. Even taking the decay products of the D_{13} resonance as stable particles does not solve the problem because the threshold energy of the $N\rho$ channel would be above the on-shell mass region of the D_{13} making it impossible to fit the non-relativistic partial width.

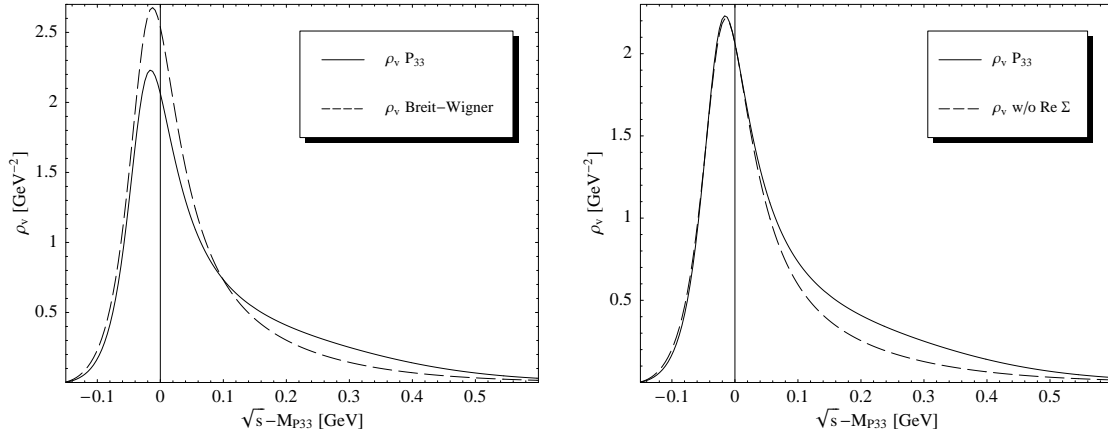


Figure 6.9: Left: Spectral function of the P_{33} resonance compared with a simple BREIT-WIGNER approximation. Right: Spectral function of the P_{33} resonance with and without real parts.

6.5 The Spectral Function

Examining the properties of the fermionic spectral representation in section 4.3.3 one sees that the resonance is not described by one but two spectral functions. But only one of them ρ_v is normalized and positive definite making it the proper choice for the spectral information of a particle. In the following only the results for ρ_v will be discussed and depicted.

The spectral function of the P_{33} shown in figure 6.9 has the expected asymmetric form. It rises quickly and decreases with a rather large tail. The large tail is induced because the width of the P_{33} has its maximum in this region. The form of the spectral function is typical for BREIT-WIGNER type quantities. Comparing the spectral function with a "pseudo-relativistic" spectralfunction, deduced from a BREIT-WIGNER form, as discussed in section 4.3.2 and especially equation (4.31), makes its similarity clear. For the width in (4.31) equation (6.1) is used and fitted to the correct width of 120 MeV on the mass-shell. N of equation (4.31) is deduced by demanding that the "pseudo-relativistic" spectral function is normalized to one. Comparing the full spectral function of the P_{33} with this very simple BREIT-WIGNER approximation, depicted in figure 6.9, shows that only for small \sqrt{s} both quantities agree. The "pseudo-relativistic" spectral function shifts too much spectral strength to lower energies. This is the case because at higher \sqrt{s} relativistic effects take place which are not accounted for in the approximation. This means that such a simple approximation already fails for the easiest spin 3/2 resonance.

As already discussed in section 4.3.4 the mass of the resonance cannot be read off as the energy at the peak of the spectral function. This is possible when the width

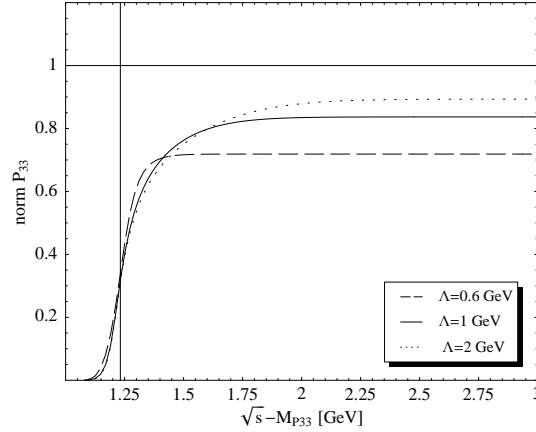


Figure 6.10: Right: Normalization of the P_{33} for different cut-off parameters, without real part of the selfenergy.

is a constant then

$$\left. \frac{d}{ds} \rho_v(s) \right|_{s=M_R^2} = 0.$$

Taking the width as an s dependent function will shift the peak away from the mass-shell. This is already true for the simple BREIT-WIGNER form depicted at the right hand side of figure 6.9. The mass of the resonance is defined as the solution of equation (4.38).

Beside the BREIT-WIGNER form it is possible to approximate the spectral function by neglecting the real parts. Because the real parts are calculated through a dispersion relation their calculation includes some numerical effort. The real parts are small indicating that the changes will not be dominant. Neglecting the real parts gives a better approximation than the "pseudo-relativistic" spectral function. It underpredicts the values for large \sqrt{s} but around the mass-shell both agree well. The real parts play a dominant role in rising the tail as can be seen in figure 6.9, right hand side. This is needed because by neglecting the real part of the selfenergies the normalization of the spectral function is violated as depicted in figure 6.10. There a similar normalization function as in equation (4.24) is depicted. It is defined as

$$norm(s) = \int_{(M_N+m_\pi)^2}^s ds' \rho_v(s'). \quad (6.2)$$

The violation depends on the cut-off parameter and is approx 20% in the case of $\Lambda = 1$ GeV.

A third approximation was proposed by Post et al. in [PLM01] using a simplified propagator. The full relativistic structure of the fermionic propagator as introduced

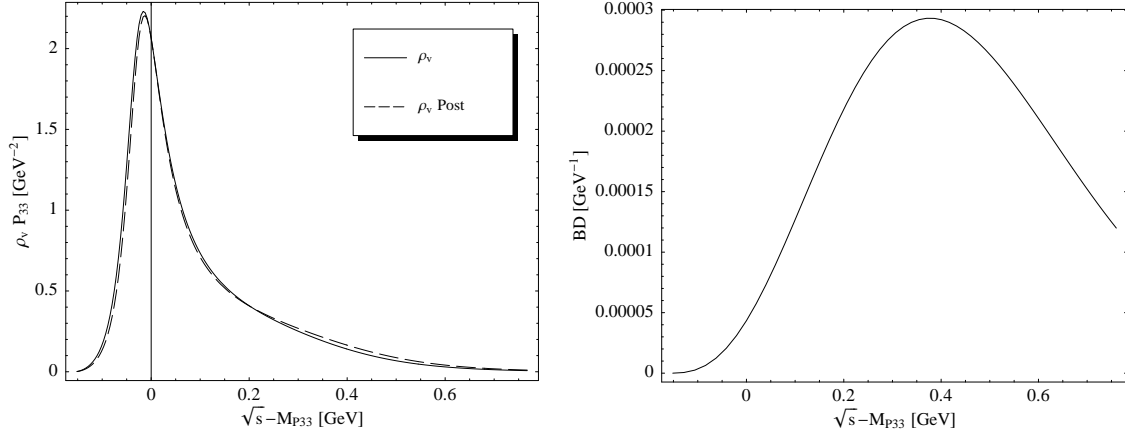


Figure 6.11: Left: Comparison of the full spectral function of the P_{33} resonance with simplified version proposed by Post et al. [PLM01]. Right: BJORKEN-DRELL function (4.37) for the P_{33} resonance.

in section 4.2 is quite involved. The complication arises due to the DIRAC structure of the selfenergy appearing in the denominator of the dressed propagator. When taking an averaged scalar selfenergy motivated as an averaging over the spins

$$\begin{aligned}\langle \Sigma(p) \rangle &= \frac{1}{2} \sum_s \bar{u}_s(p) \Sigma(p) u_s(p) \\ &= \frac{1}{2} \text{Tr} \left[(\not{p} + \sqrt{k^2} \Sigma(k)) \right]\end{aligned}$$

one can define a simplified propagator:

$$\mathcal{G}(p) = \frac{\not{p} + M}{\not{p} - M - \langle \Sigma \rangle} \quad .$$

But as shown in [PLM01] inconsistencies with the BJORKEN-DRELL relation (4.8) arise. In turn, this even leads to negative cross sections, see [PLM01] for details. Therefore it is an important consistency requirement to insist on the validity of (4.8). When computing ρ_s and ρ_v one finds

$$M\rho_v(p) - \rho_s(p) = 0.$$

For $\sqrt{p^2} > M$ equation (4.8) is violated. To solve this problem it was suggested in [PLM01] to change $M^2 \rightarrow \sqrt{p^2}$ in the numerator. This leads to an equation for the ρ 's of the form

$$\sqrt{p^2} \rho_v(p) - \rho_s(p) = 0$$

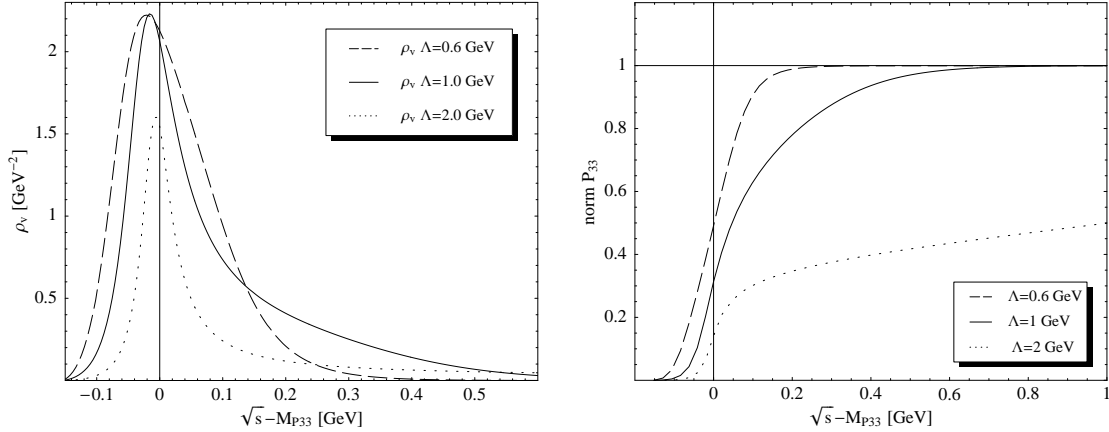


Figure 6.12: Left: Spectral function of the P_{33} resonance with different values of the cut-off parameter Λ . Right: Normalization function for P_{33} for different cut-off parameters Λ .

which is in agreement with the BJORKEN-DRELL relation (4.8). When comparing this to the full results one sees that in the full calculation the BJORKEN-DRELL relation is fulfilled but the BJORKEN-DRELL function (4.37) is not necessarily zero. Deviations in the spectral functions are expected when for given p the BJORKEN-DRELL function (4.37) is larger than zero. This can be seen for the spectral function of the P_{33} (1232) resonance depicted on the left hand side of figure 6.11 where the deviations are very small. This was expected because the BJORKEN-DRELL function is always small for all p , depicted on the right hand side of figure 6.11.

To summarize, the P_{33} resonance has only one major decay channel giving it a less involved structure than for the D_{13} . Even for such a simple spectral function a "pseudo-relativistic" approach will lead to bad agreements already in the mass-shell region. Neglecting the real parts of the selfenergy gives a good approximation from threshold energy to the mass-shell region but spoils the normalization condition. A very good approximation is given by the simplified propagator of Post et al. [PLM01] leading to good agreements for all \sqrt{s} without spoiling normalization.

As discussed above, the cut-off parameter is not fitted to experimental data. The spectral function of the P_{33} resonance depends largely on this parameter as can be seen in figure 6.12. The shape of all three functions is different. Small values of Λ lead to a broadening of the spectral function around the peak by decreasing the tail. This can be understood when looking at the normalization functions for different Λ as depicted in figure 6.12. For small Λ unity is reached earlier. For large Λ the normalization function reaches unity at $\sqrt{s} - M_{P_{33}} = 4$ GeV leading to a strongly compressed spectral function with a long tail. Because the energy region where unity is reached is far beyond the physical energy region modeled here such large Λ s should be excluded. From the discussion in the beginning also small Λ s are physically

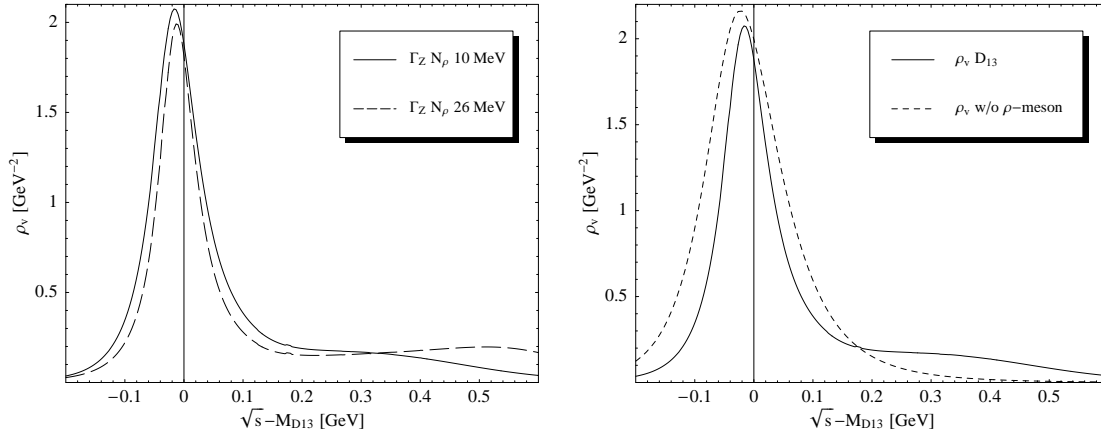


Figure 6.13: Left: Spectral function of the D_{13} resonance with both widths of the $N\rho$ channel. Right: Spectral function of the D_{13} resonance when the decay into the ρ -meson is ignored.

unreasonable. From this consideration it can be concluded that reasonable cut-off parameters will be found in the region of 1 GeV.

The spectral function of the D_{13} compared to the case of the P_{33} is more symmetric around the peak with a shoulder arising in the region of the invariant mass of the $N\rho$ channel as can be seen in figure 6.13. This shoulder is larger when taking a larger partial width for the $N\rho$ channel. When using this value for the width of the $N\rho$ channel structures arise in the region of the $N\rho$ threshold due to the opening phase space for this reaction. In the last section it was shown that the width becomes heavily larger in this energy region when taking the higher partial width for the $N\rho$ channel. This means that spectral strength is transferred to higher energies leading to the higher shoulder.

The shoulder vanishes when the $N\rho$ channel is ignored as depicted in figure 6.13. Spectral strength is transferred to lower energies leading to a broadening of the spectral function. These changes indicate the importance of the ρ -meson in this energy region.

In the discussion of the selfenergy the strong energy dependence of the real parts indicated a shoulder for the spectral function. This claim can be approved when neglecting the real parts which are depicted in figure 6.14. There one sees that the shoulder decreases when the real parts are switched off, showing that the shoulder is induced by the real parts of the selfenergy. This is needed to fulfill the normalization condition given in section 4.1. Without the real parts of the selfenergy the normalization is spoiled as shown in figure 6.15.

Because the normalization condition is spoiled when neglecting the real parts of the selfenergy this is not a good approximation for the spectral function. A "pseudo-relativistic" approximation was already unsatisfactory in the simple case

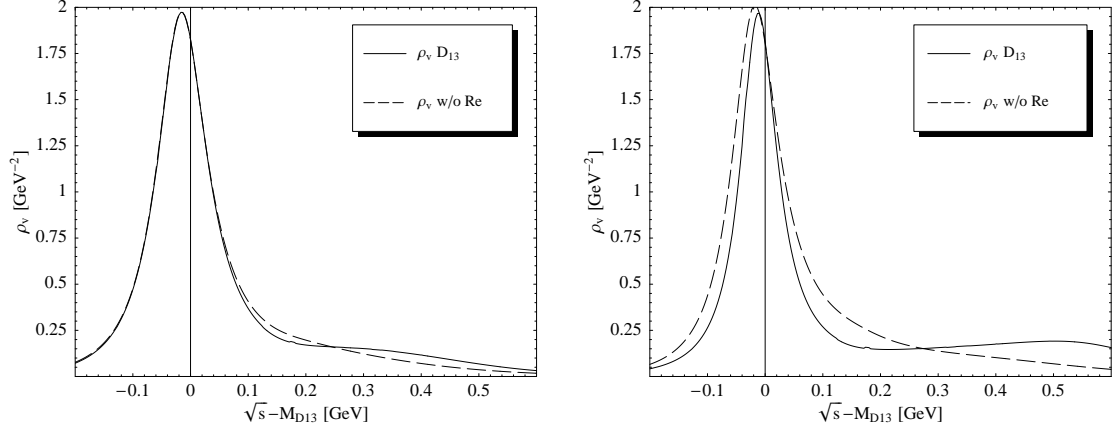


Figure 6.14: Spectral function of the D_{13} resonance with and without real parts. Left: Width of the $N\rho$ channel is 10 MeV. Right: Width of the $N\rho$ channel is 26 MeV.

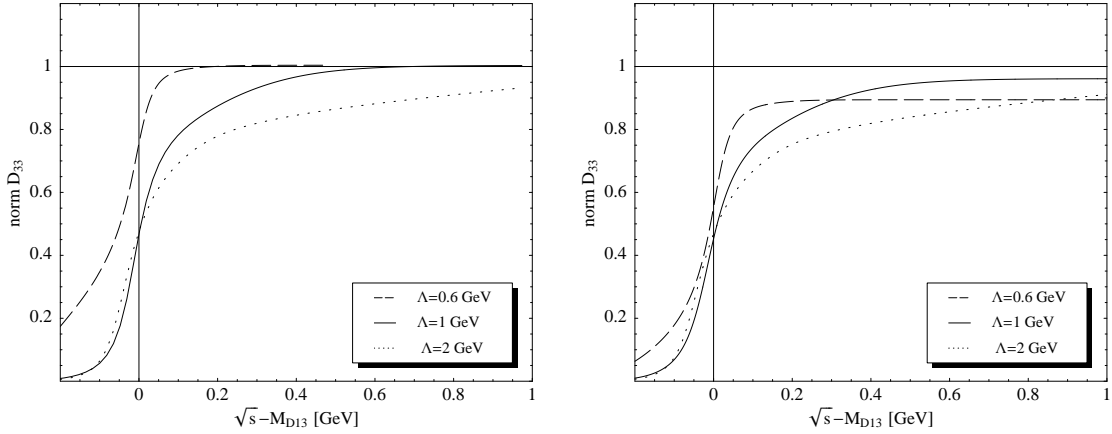


Figure 6.15: Normalization function for D_{13} . Left: For different cut-off parameter. Right: For different cut-off parameters without the real part of the selfenergy.

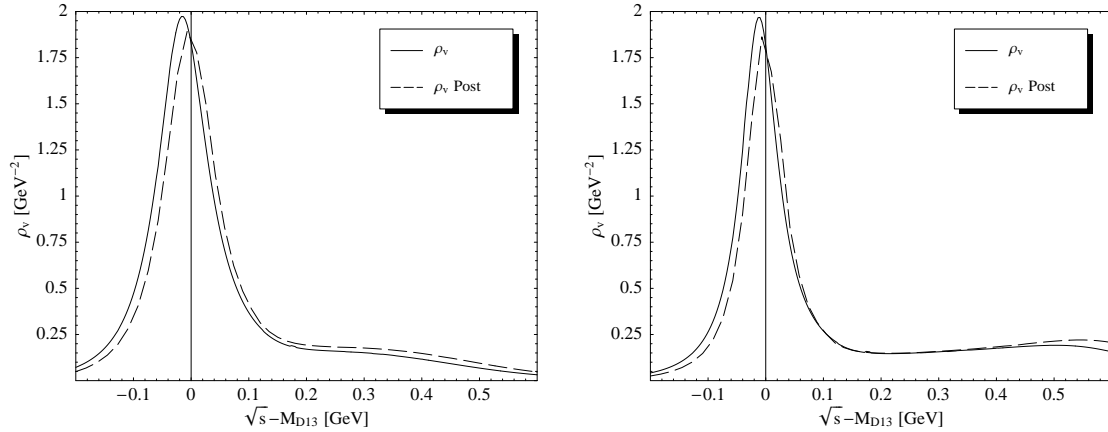


Figure 6.16: Spectral function of the full calculation for the D_{13} resonance compared with the simplified calculation of [PLM01]. Left: Width of the $N\rho$ channel is 10 MeV. Right: Width of the $N\rho$ channel is 26 MeV.

of the P_{33} . In addition it is difficult to construct non-relativistic widths due to the off threshold behavior of the $N\rho$ channel. This means that a comparison with a "pseudo-relativistic" spectral function is not possible. But the spectral function of the D_{13} can be simplified using the method of Post et al. as discussed in the case of the P_{33} . The results are depicted in figure 6.16. Spectral strength is transferred to higher energies by the approximation and the peak is shifted towards the mass-shell region. This shift takes place because the denominator in the approximation is much simpler than in the full case. But still the results are in good agreement for both widths of the $N\rho$ channel. The deviations for the D_{13} case are larger than for the P_{33} case because also the BJORKEN-DRELL function defined in equation (4.37) has larger values than in the P_{33} case as depicted in figure 6.17. In the simplified version the BJORKEN-DRELL function is always zero. The BJORKEN-DRELL function (4.37) can be seen as a measure of the quality for such an approximation for a given particle.

The spectral function of the D_{13} depends strongly on the cut-off parameter Λ as depicted in figure 6.18. Similar to the P_{33} case this comes due too different distribution of the spectral strength over different energies. The distribution can be seen when comparing the normalization functions for different cut-off parameters, depicted in figure 6.15. Larger cut-off parameters distribute the strength in a larger region and into the high energy parts of the spectrum. As in the case of the P_{33} such a broad distribution of spectral strength outside the physical meaningful energy region is disturbing. This excludes large values of Λ as a reasonable choice.

A smaller cut-off parameter surpresses strongly already in the physical meaningful region and large portions of spectral strength can be found at energies much below the mass-shell leading to a large tail for small energies. This tail is induced by the larg energy dependence of the real parts in this energy region. Already in the begin-

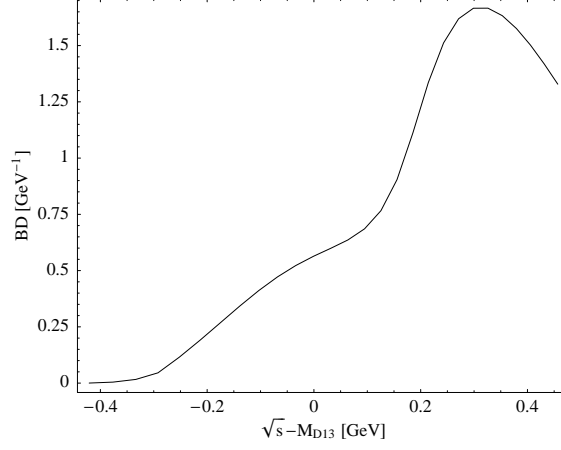


Figure 6.17: Same as figure 6.11, r.h.s., but for D_{13} .

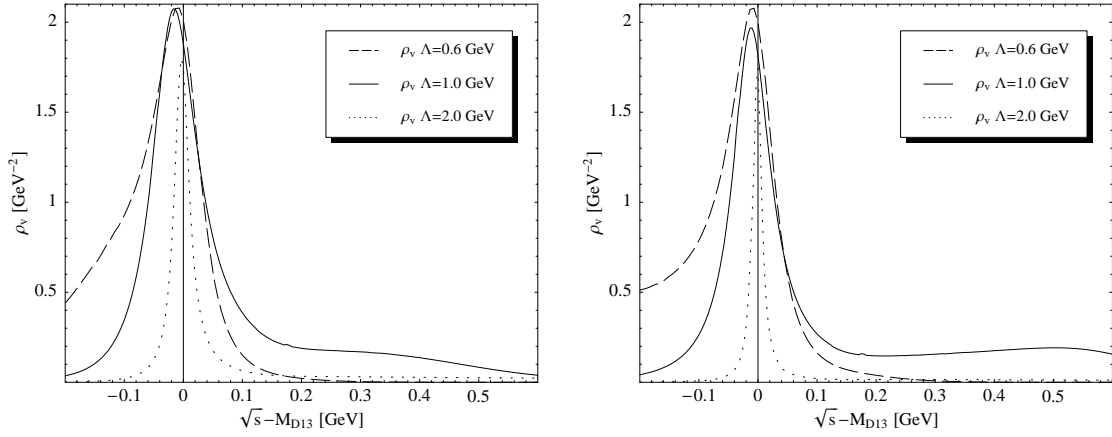


Figure 6.18: Spectral function of the D_{13} resonance with different values of the cut-off parameter Λ . Left: Partial width $\Gamma_{N\rho} = 10$ MeV. Right: Partial width $\Gamma_{N\rho} = 26$ MeV.

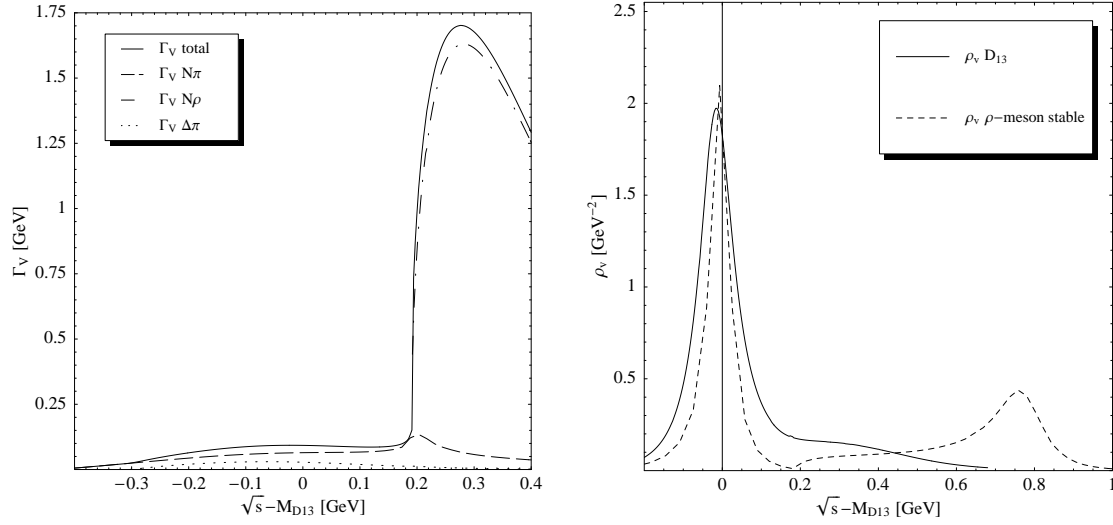


Figure 6.19: Left: Partial and total width of D_{13} when considering the ρ -meson as a stable particle. Right: Spectral function of D_{13} for the case of an unstable or stable ρ -meson.

ning of this chapter we have argued that small values of Λ are unreasonable when looking at the variation of the coupling in the physically meaningful region. Taking all arguments together supplies strong indications that small cut-off parameters will not give a correct physical description of the resonance. We conclude that also for the D_{13} resonance a physically meaningful value for the cut-off parameter will be around $\Lambda = 1$ GeV.

6.6 Influence of Unstable Particle

One of the major difficulties when calculating the selfenergy and the spectral function of the D_{13} resonance compared to the P_{33} resonance is the fact that the D_{13} decays into unstable particles. The complications arise because when going from a stable to an unstable particle a δ -function in the selfenergy has to be exchanged by a spectral function (see section 3.3). This leads to a higher numerical effort because the integrations cannot be solved analytically anymore. If the unstable particle is a baryon as in the case of the $\Delta\pi$ channel the spectral function will even have a more complicated structure leading again to much higher numerical effort.

Assuming the particles to be stable would be a large simplification especially for the $\Delta\pi$ channel. Because in this work the full propagator is available it is interesting to compare the results of the full spectral function to a simplified one where one particle is assumed to be stable.

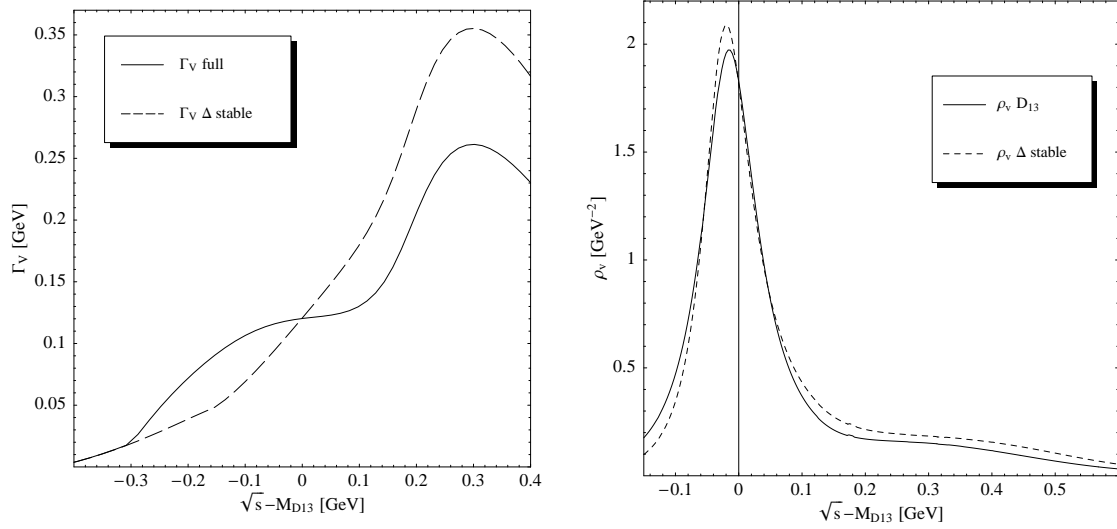


Figure 6.20: Left: Total width of D_{13} when considering the Δ resonance as an unstable (full line) and stable (dashed line) particle. Right: Spectral function of D_{13} for the case of an unstable or stable Δ .

When inserting the ρ -meson as a stable particle its threshold value will be higher than the mass shell of the D_{13} . So it is not possible anymore to fit the coupling constant via the partial width. In figure 6.19 the partial widths and the spectral function are depicted taking the same input as for the unstable case. The partial width of the ρ -meson opens up dramatically reaching nearly seven times the value than in the unstable case. Due to this dramatic increase of the width around the threshold energy of $N\rho$, large amount of spectral strength is transferred above this threshold creating a new peak. The total shape of the spectral function changes leading to the conclusion that this simplification is not a good approximation.

When considering the Δ as a stable particle it is possible to refit the coupling constants of the $\Delta\pi$ channel. Taking the value of the width of the $N\rho$ channel to $\Gamma_{N\rho} = 10$ MeV the coupling is $g_{N^*\Delta\pi} = 2.05 \text{ GeV}^{-3}$ as compared to 2.1 GeV^{-3} in the unstable case. This means that the coupling strength remains the same.

The differences between a calculation using a stable or an unstable Δ will be mainly induced by the shift of the threshold for the width of this channel. Because the spectral function is normalized spectral strength has to be transferred to higher energies leading to an increased width for energies higher than the threshold region of $\Delta\pi$. This can be seen in the left plot in figure 6.20 where the width of the D_{13} is plotted for the full case and taking the Δ as a stable particle. On the right hand side of figure 6.20 the spectral function is compared with the result of the full calculation. One sees the shift of spectral strength to higher energies leading to a lowered start, higher peak and a slightly larger shoulder. The effect can be seen but is still

considerably small.

Taking the Δ as a stable particle is a huge simplification for the calculation due to the complicated structure of its spectral function. Comparing the effect of this simplification on the spectral function of the D_{13} and the possible simplification of the calculation leads to the conclusion that it is a good approximation when computing power is restricted.

It was possible to make the simplification where the Δ is taken as a stable particle because the mass-shell of the D_{13} is above the energy threshold to create a Δ . The effect by taking the Δ as a stable particle only shifts some spectral strength due to the shifted threshold of the partial width which is comparably small. Additionally the coupling of the D_{13} to the Δ is not too strong so only a small amount of spectral strength is transferred to higher energies. This leads to rather small changes in the spectral function making it a considerable simplification.

In the case of the ρ such a simplification changes the whole structure of the spectral function. This can be understood because the on-shell energy of the D_{13} is less than the threshold energy to create $N\rho$. Due to the strong coupling of the ρ -meson to the D_{13} a considerably large partial width has to be taken into account even at these energies. Additionally the mass of the ρ -meson is quite high leading to a large energy gap between the threshold energy of the unstable ρ -meson and the stable one. Taking the ρ -meson as a stable particle pushes all this spectral strength above threshold where it has a predominant impact on the shape of the spectral function.

7 Summary and Outlook

In this work interactions of spin $3/2$ resonances in hadron physics are investigated. Such interactions are not trivial and lead to many problems concerning the consistencies of the couplings. It was shown that it is feasible to calculate fully relativistic propagators for spin $3/2$ resonances in the PASCALUTSA framework. These considerations can serve as a prelude to more complicated treatments for in-medium calculations of hadronic properties.

Further specific calculations were performed for the case of the P_{33} and D_{13} resonances which are needed to overcome complications that occurred in simplifications. Such complications (violation of the BJORKEN-DRELL relation (4.33) accompanied by negative cross sections) vanish when the full relativistic structure is incorporated.

First we started in chapter 2 introducing the RARITA-SCHWINGER fields as a field theoretical description of spin $3/2$ particles. A free Lagrangian was introduced and checked for giving the correct number of DOF. Using this Lagrangian the free propagator was calculated.

In the same chapter the introduction of interactions was discussed. The conventional coupling of spin $3/2$ resonances to $N\pi$ introduced by Nath et al. [NEK71] is inconsistent because it leads to acausal propagation [Sin73] and non positive definite anti-commutation relations [Hag71]. These inconsistencies arise because the interaction violates the number of DOF of the free theory [Cox89]. To overcome these inconsistencies the PASCALUTSA framework was introduced. A scheme was given how to introduce consistent interactions by exploring the gauge invariance of the free massless LAGRANGIAN. The interacting LAGRANGIAN is introduced with the same gauge invariance as the free massless theory. This does not lead to further DOF. The mass term is introduced like in the free case to break the symmetry and raises the number of DOF to the correct value for massive higher spin fields. In an example for the case of the $\Delta N\pi$ coupling this was shown explicitly. There one could see that in the PASCALUTSA framework the spin $1/2$ and spin $3/2$ parts of the interaction are clearly separated. In chapter 5 we derived for the first time consistent couplings of spin $3/2$ particles to ρN and $\Delta\pi$ using the PASCALUTSA method.

At the end of chapter 2 we discussed the correspondence between conventional and PASCALUTSA coupling showing that on the S-matrix level they lead to the same observables because both approaches are equivalent up to some contact term. On the level of understanding the underlying physics the PASCALUTSA method is preferable because it separates spin $1/2$ and spin $3/2$ components of the interaction. Although it could be shown that the PASCALUTSA framework is sufficient to provide

consistent interactions for higher spin states it might not be a necessary requirement as ongoing research in effective field theories indicates [HWGS05].

On the basis of the PASCALUTSA framework the selfenergies of spin 3/2 resonances are introduced in chapter 3.

In chapter 4 we calculated the full relativistic structure of the dressed propagator for spin 3/2 resonances in an analytical form. With this starting point it was possible to calculate the analytical form of the spectral function for spin 3/2 resonances. It could be shown that in the PASCALUTSA framework the spectral function for spin 3/2 resonances has basically the same structure as spin 1/2 states. This is a major simplification and means that it is not only feasible to calculate spin 3/2 resonances in the PASCALUTSA framework but also easier than in the conventional approach.

The selfenergy and the spectral function describes fully a resonance. But because they are not measurable quantities the width and the mass of a resonance were introduced. They are closer to theory than experimental data and closer to experiment than spectral functions. But as pure numbers they can not represent the whole spectral shape. This leaves an ambiguity how to define them. In this work the width and the mass are defined by comparing the spectral function to a relativistic BREIT-WIGNER form. In the case of a spin 3/2 resonance there are three possible candidates for a width from which only one is a generally positive definite quantity.

To achieve the goal to calculate the spectral function of the D_{13} resonance the selfenergies of the major decay channels are needed. These are $N\pi$, $N\rho$ and $\Delta\pi$ and calculated in chapter 5. For $N\pi$ PASCALUTSA proposed a consistent coupling which was used. For the $N\rho$ and $\Delta\pi$ channel no couplings were previously available. Using the method proposed by PASCALUTSA they were derived. To check the results they are expanded for the non-relativistic limit by assuming stable particles, leading to the correct phase space behavior of the width. The $N\pi$ channel leads to a P-wave and D-wave for a particle with positive and negative parity, respectively. The $N\rho$ and $\Delta\pi$ channels lead to P-wave (S-wave) for particles with positive (negative) parity. The selfenergies were calculated in a general way making it possible to use them for all spin 3/2 resonances decaying into these three channels. Because the P_{33} resonance decays into $N\pi$ it was possible to calculate the properties of this resonance. It was also needed as a part of the D_{13} selfenergy in the $\Delta\pi$ channel because the unstable character of the Δ must be taken into account. The selfenergy of this channel contains an integration over the spectral function of the Δ making it a complicated and numerically involved quantity.

For the $N\rho$ and $\Delta\pi$ channel also the selfenergies for stable ρ and Δ were calculated. The results were compared to the non-relativistic limit yielding the correct behaviour as expected by phase space considerations.

The results for the P_{33} and D_{13} resonances were discussed in chapter 6. The spectral functions automatically fulfill all BJORKEN-DRELL conditions summarized in section 4.3.3. In detail this means that ρ_s and ρ_v are real quantities. ρ_v is positive definite and normalized. $\sqrt{s}\rho_v - \rho_s \geq 0$ for all \sqrt{s} . These conditions were

derived generally for spin 1/2 and spin 3/2 resonances. All spectral functions of these resonances have to fulfill them. In some simplifications as proposed by Post et al. [PLM01] the propagator had to be changed by hand to fulfill these conditions. It could be shown in this work, that by taking the full propagator such problems do not arise.

The spectral function of the P_{33} resonance has a typical BREIT-WIGNER form. It is asymmetric with a quick rise and a long tail. The D_{13} resonance on the other hand has a long shoulder for high energies due to the strong coupling of the ρ -meson. This shoulder vanishes when the ρ -meson is neglected and more structure arise when the partial width is increased from $\Gamma_{N\rho} = 10$ MeV to $\Gamma_{N\rho} = 26$ MeV. These changes indicate the importance of the $N\rho$ channel for the whole structure of the D_{13} spectral function.

For the widths of the resonances it could be shown that for the P_{33} all three candidates for a width are proper choices. All three are positive definite and are similar over the whole energy range. In the case of the D_{13} the three candidates for a width differ largely and only one is positive definite making it the only choice for an off-shell width.

To investigate the dependence of the spectral function on the cut-off parameter, the latter was varied. The shape of the spectral function changes dramatically for both resonances. Because the parameters were only fitted to two quantities it was not possible to fit Λ precisely leaving an ambiguity which value is the correct one. Although it was not possible to pin down the number exactly it could be shown that only values of around 1 GeV give physically meaningful results. This resolves the ambiguity and gives some constraints for further fits when all three couplings are fitted to experimental data.

The full spectral function was compared with various simplifications to rate the quality of these simplifications. First, we compared the results of the P_{33} spectral function with a simple BREIT-WIGNER approach where the width is taken in a non-relativistic form. It could be shown that such a simple approach does not give a reasonable approximation for energies higher than the mass-shell. It is not an unexpected result because such a "pseudo-relativistic" approach will not work for higher energies where relativistic effects are not negligible.

Next it could be shown that neglecting the real parts spoils the normalization conditions for ρ_v . It has mainly an effect on the tails and shoulders of the spectral functions which are reduced.

Comparison of the full calculation with the approximation proposed by Post et al. [PLM01] shows that this is a reasonable approximation. The difference is mainly a slightly shifted peak and some spectral strength is transferred to higher energies. This effect is much smaller for the P_{33} than for the D_{13} case. Looking at the BJORKEN-DRELL function (4.37) one sees that for the P_{33} case it is nearly zero. Because in the Post et al. approach it is always zero, deviations from zero in the BJORKEN-DRELL function will also lead to deviations in the spectral functions. For the D_{13} the

deviation from zero are larger leading to larger deviations.

Taking the Δ and the ρ as stable particles lead to different conclusions. Taking the ρ as a stable particle has a major impact on the structure of the spectral function. Such an approximation is not reasonable. On the other hand, assuming the Δ as a stable particle has only marginal influence on the structure of the spectral function. Due to the large numerical effort needed when implementing the Δ as an unstable particle such a simplification is reasonable.

Using the results for the selfenergies it is possible to calculate the widths and spectral functions of all spin 3/2 resonances decaying into $N\pi$, $N\rho$, $\Delta\pi$. As an outlook we print out that with the explicit results for the D_{13} resonance it is possible to calculate reactions going over the D_{13} into dilepton or two pions as an end product, e.g.

$$\begin{aligned} N\pi &\rightarrow D_{13} \rightarrow N\rho \rightarrow Ne^+e^-, \\ N\pi &\rightarrow D_{13} \rightarrow N\rho \rightarrow N\pi\pi, \\ N\pi &\rightarrow D_{13} \rightarrow N\Delta \rightarrow N\pi\pi. \end{aligned}$$

Furthermore it is possible to rate the quality of approximations of spectral functions by comparing them to the results of the full calculation as was done in this work for some approximations. To pin down the input parameters more precise it is desirable to fit them to experimental data.

A Degrees of Freedom

To count the degrees of freedom (DOF) for a theory can be trivial when no constraints are imposed on the variables of the theory. But in general this is not the case.

For example the RARITA-SCHWINGER fields of spin $s = j + 1/2$ are introduced as a symmetric LORENTZ tensor-spinor $\psi_{\mu_1 \dots \mu_j}^\alpha$ of rank j , which is a convenient way to introduce higher spin fields in a LORENTZ covariant form. But such a tensor-spinor has $C_j = 4(j+1)(j+2)(j+3)/6$ complex components. On the other hand a physical massive particle with spin s has $2 \times (2s+1)$ DOF and clearly $2 \times (2s+1) = 2 \times (2(j+1)) < 2 \times C_j$ for all j s. One cannot avoid to introduce constraints eliminating the extra DOF. To understand the connection between constraints and DOF a detailed discussion is needed. The method used here is due to Dirac [Dir64]. It was introduced to quantize systems with constraints and is based on a HAMILTONIAN formalism.

A.1 Introduction to the Hamiltonian Formalism

The HAMILTONIAN formalism is a standard textbook formalism for classical mechanics [Kuy97]. Its advantage compared to the LAGRANGIAN formalism is the fact that an easy scheme for quantization exists [Mes99]. Nowadays this advantage has decreased because FEYNMAN introduced with the path integrals a scheme to quantize a system given in the LAGRANGIAN formalism [Mos04]. But it is not an easy task to introduce constraints in this formalism.

The main emphasis of this introduction is to go straight forward to the most relevant theorems for this work and not to give a detailed analysis of the HAMILTONIAN formalism. There will also be no discussion about how to quantize a system with constraints.

A.1.1 The Lagrangian

The *action* is defined as

$$\mathcal{S} = \int L dt$$

with $L = L(q_n, \dot{q}_n)$ as the LAGRANGIAN. This *action* has $2N$ degrees of freedom:

$$\begin{aligned} q_n & \text{ with } n = 1 \dots N, \\ \dot{q}_n & \text{ with } n = 1 \dots N. \end{aligned}$$

The equations of motion are derived via the EULER-LAGRANGE equation

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_n} \right) = \frac{\partial L}{\partial q_n} \quad .$$

A.1.2 The Hamiltonian

The HAMILTONIAN is introduced by defining the generalized momenta p_n as the partial derivatives of the LAGRANGIAN with respect to the velocities

$$p_n = \frac{\partial L}{\partial \dot{q}_n} \quad .$$

Until now we have assumed that the momenta and the coordinates are independent variables which form the phase space of the particles leading to $2N$ degrees of freedom. But this will not be the general case because it is possible that the momenta and the coordinates are not independent variables but have a dependence of the form

$$\phi_m(q_n, p_n) = 0, \quad m = 1 \dots M.$$

These functions are the M *primary constraints* of the HAMILTONIAN formalism. The HAMILTONIAN is introduced via a LEGENDRE transformation

$$L(q_n, \dot{q}_n) \longrightarrow H(q_n, p_n) = p_n \dot{q}_n - L.$$

But this will not lead to a unique HAMILTONIAN. An equally valid HAMILTONIAN can be introduced by adding a linear combination of the constraints:

$$H^* = H + u_m \phi_m \tag{A.1}$$

with until now arbitrary coefficients u_m .

Such a sum does not change the HAMILTONIAN because the constraints ϕ_m are zero for all p 's and q 's. So far this is trivial but variations of H^* lead to different equations of motion:

$$\begin{aligned} \dot{q}_n &= \frac{\partial H}{\partial p_n} + u_m \frac{\partial \phi_m}{\partial p_n}, \\ \dot{p}_n &= -\frac{\partial H}{\partial q_n} - u_m \frac{\partial \phi_m}{\partial q_n} \quad . \end{aligned}$$

with M coefficients u_m .

A.1.3 The Poisson Brackets

To explore these coefficients u_m we introduce the POISSON brackets

$$[f, g] = \frac{\partial f}{\partial q_n} \frac{\partial g}{\partial p_n} - \frac{\partial f}{\partial p_n} \frac{\partial g}{\partial q_n} \quad (\text{A.2})$$

with the properties

$$[f, g] = -[g, f] \quad \text{antisymmetric} \quad (\text{A.3})$$

$$[f_1 + f_2, g] = [f_1, g] + [f_2, g] \quad \text{linear} \quad (\text{A.4})$$

$$[f_1 f_2, g] = f_1 [f_2, g] + [f_1, g] f_2 \quad \text{product rule} \quad (\text{A.5})$$

$$[f, [g, h]] + [g, [h, f]] + [h, [f, g]] = 0 \quad \text{JACOBIAN identity} \quad (\text{A.6})$$

The equation of motion in this formalism is given by

$$\dot{g} = [g, H] + u_m [g, \phi_m] \quad (\text{A.7})$$

The POISSON bracket is only defined for quantities that are functions of q and p . But it is possible to generalize the definition.

Theorem 1. *When the POISSON brackets are generalized in such a way using only the equations (A.3) - (A.6) one can write equation (A.7) as*

$$\dot{g} = [g, H + u_m \phi_m]$$

Proof. $\dot{g} = [g, H + u_m \phi_m] = [g, H] + [g, u_m \phi_m]$ using equation (A.4).

From (A.5) one conclude $\dot{g} = [g, H] + [g, u_m] \phi_m + u_m [g, \phi_m]$

While $[g, u_m]$ is not defined this is not disturbing because it is multiplied by a zero, the constraints. \square

It is important to notice that the constraints ϕ_m must not be used *before* working out a POISSON bracket. The equations are written as *weak* equations, using " \approx " instead of " $=$ ". Then the equations of motion for $g(q, p)$ can be written as

$$\dot{g} \approx [g, H^*] \quad (\text{A.8})$$

A.2 General Equations of Motion

An equation of motion in the form of equation (A.8) has not much practical use because the u_m s are unknown coefficients. It is desirable to derive general equations of motion for a system with constraints. Then it would be possible to quantize such a system by demanding that the POISSON brackets become the commutation relation of the theory. To derive such general equations of motion we need information about the values of u_m . This will be achieved by classifying the constraints.

A.2.1 Consistency Conditions

The quantities ϕ_m are zero for all times. Hence their derivative must vanish. Inserting ϕ_m in (A.7) leads to M consistency conditions

$$0 \approx [\phi_m, H] + u_{m'}[\phi_m, \phi_{m'}]. \quad (\text{A.9})$$

These conditions have to be examined. It is possible that they lead to inconsistencies e.g. $1=0$. If this happens the original LAGRANGIAN has to be changed until all inconsistencies are eliminated. Then it is possible to sort these M equations into three categories:

1. Equation (A.9) reduces to $0 = 0$. Then these conditions are satisfied by the *primary constraints* ϕ_m .
2. Equation (A.9) reduces to another equation independent of the u_m . This equation is then independent of the first one.

This leads to new constraints, the *secondary constraints*

$$\chi_n(p, q) \approx 0$$

. They also raise some consistency conditions of the form

$$0 \approx [\chi_m, H] + u_{m'}[\chi_m, \chi_{m'}]$$

which again can be sorted into one of the three conditions listed here.

Then the theory has K *secondary constraints* and the total number of constraints adds up to

$$\phi_j \approx 0 \quad \text{with} \quad j = 1 \dots M, M+1 \dots M+K = J.$$

3. Equation (A.9) does not reduce to either way. This imposes a condition on the u_m s. They give a number of non-homogenous linear equations

$$u_m = U_m(p, q).$$

Because we have ruled out an inconsistent LAGRANGIAN, these equations will be solvable. But the solutions will be in general not unique because if we know one solution we can add any solution $V_m(q, p)$ of the equation with

$$V_m[\phi_j, \phi_m] = 0 \quad (\text{A.10})$$

which will lead to another solution because

$$[\phi_j, H] + (u_m + V_m)[\phi_j, \phi_m] \approx 0.$$

The most general solution can be written as a linear combination of all independent solutions

$$V_{am}(q, p) \quad \text{with} \quad a = 1 \dots A$$

as

$$u_m = U_m + v_a V_{am}$$

with arbitrary coefficients v_a .

Then the total HAMILTONIAN reads

$$H_T = H' + v_a \phi_a, \tag{A.11}$$

$$H' = H + U_m \phi_m, \tag{A.12}$$

$$\phi_a = V_{am} \phi_m. \tag{A.13}$$

This introduces some arbitrariness into the HAMILTONIAN. The v_a s can be chosen as functions of time without changing the physics of the theory. These arbitrary functions must mean that we are using a mathematical framework containing arbitrary elements, for example a coordinate system or gauges. To study the behavior of these functions we introduce some new definitions.

A.2.2 First and Second Class Constraints

Definition 1. We call ϕ_j a first class constraint if for any dynamical variable $R(q, p)$ the equation

$$[R, \phi_j] \approx 0 \quad \text{with} \quad j = 1 \dots J$$

holds. Otherwise we call it second class.

Now we can introduce a useful theorem

Theorem 2. The Poisson bracket of two first class quantities is also a first class quantity.

With this theorem one can deduce two other theorems:

Theorem 3. The HAMILTONIAN H' given in (A.12) is a first class quantity.

Proof. Interchanging u_m with U_m in equation (A.9) gives the proof. \square

Theorem 4. The ϕ_a s are also first class quantities.

Proof. When using the Poisson bracket with ϕ_a and ϕ_j one gets

$$[\phi_a, \phi_j] \stackrel{(A.13)}{=} V_{am}[\phi_m, \phi_j] \stackrel{(A.5)}{=} V_{am}[\phi_m, \phi_j] + [V_{am}, \phi_j]\phi_m \stackrel{(A.10)}{=} 0$$

because the second term is vanishing weakly. Due to the arbitrariness of the j the proposition is shown. \square

So the total HAMILTONIAN can be written by *first class* quantities namely the H' and some *first class* ϕ s.

Conclusion 1. *The number of independent arbitrary functions occurring in the general solution of the equations of motion is equal to the number of values which takes on an "a" as an index.*

They are equal to the number of independent primary first class constraints, because all the independent primary first class constraints are included in the sum of equation (A.11).

The *initial physical state* of a system is determined only by the p s and q s and not by the v_a s. How does then a state evolves in time for diverent v_a s when for given initial p s, q s at $t = 0$ it is g_0 ?

Theorem 5. *The difference between g s of two different v_a s after the time δt is equal to*

$$\Delta g(\delta t) = \varepsilon_a [g, \phi_a].$$

Proof.

$$\begin{aligned} g(\delta t) &= g_0 + \dot{g}\delta t \\ &= g_0 + [g, H_T]\delta t \\ &= g_0 + \delta t ([g, H'] + v_a [g, \phi_a]). \end{aligned}$$

The v_a s are compleatly arbitrary. Suppose we take different value $v_{a'}$ that gives a different $g(\delta t)$. Then

$$\Delta g(\delta t) = \delta t (v_a - v_{a'} [g, \phi_a]) = \varepsilon_a [g, \phi_a].$$

\square

Theorem 6. *When theorem 5 is applied twice with two different infinitesimal coefficients ε_a and $\gamma_{a'}$ the difference of the solutions depending on the sequence of applying theorem 5 is given as*

$$\Delta g(\delta t) = \varepsilon_a \gamma_{a'} [g, [\phi_a, \phi_{a'}]]. \quad (A.14)$$

The ϕ_a s are *primary first class* constraints. $\phi_b = [\phi_a, \phi_{a'}]$ can also be a *secondary first class* constraint, but due to theorem 2 it is still a *first class* constraint.

Conclusion 2. *The ϕ_a s and the ϕ_b s, as the primary and secondary first class constraints, act as a generator of infinitesimal contact transformations (A.14). They lead to changes in the qs and ps that do not affect the physical state.*

So it is convenient to introduce a more *general equation of motion*

$$\dot{g} = [g, H_E] \quad \text{with} \quad H_E = H_T + v'_a \phi'_a \quad (\text{A.15})$$

with the v'_a s as *secondary first class* constraints which were not introduced yet in H_T . Now that we have derived this *general equation of motion* we can count the *DOF* of the theory. Which are:

Conclusion 3. • *2N from the qs and ps.*

- *Every primary and secondary constraint reduces one DOF.*
- *Every primary and secondary first class constraint reduces an additional DOF due to the arbitrary functions v_a (or as a generator of the contact transformations).*

The connection to a local transformation is given by

Theorem 7 ([PT99]). *A LAGRANGIAN theory invariant under a local transformation with n independent parameters has n primary first class constraints [GT90]. The total number of first class constraints is $(d+1) \times n$, where d is the highest order of the time-derivative operator acting on the parameters of the transformation. Tho total number of second-class constraints is equal to the number of components of the system reduced by n .*

B Deutsche Zusammenfassung

In letzter Zeit hat sich ein großes Interesse daran entwickelt, wie sich hadronische Eigenschaften in einem stark wechselwirkenden Medium verändern. Bei den entsprechenden theoretischen Beschreibungen wird entweder nicht-relativistisch gerechnet [PLM04] oder zumindest eine vereinfachte Struktur der Selbstenergie [PLM01] angenommen. Bevor ausführlichere Rechnungen im Medium vorgenommen werden können, ist es nötig, die Struktur von Spin-3/2-Teilchen im Vakuum zu verstehen. Dafür ist eine vollständig relativistische Behandlung nötig. Die Einführung von Wechselwirkungen mit Spin-3/2-Resonanzen ist allerdings nicht trivial und kann zu Inkonsistenzen im Formalismus führen. Dies ist beispielsweise bei der konventionellen Kopplung von Spin-3/2-Resonanzen an $N\pi$ der Fall, die von Nath et al. [NEK71] eingeführt wurde. Eine Möglichkeit, konsistente Kopplungen einzuführen, wurde von Pascalutsa und Timmermans [PT99] vorgeschlagen. Es wurde allerdings noch nicht untersucht, wie aufwändig eine vollständig relativistische Beschreibung von beliebigen Spin-3/2-Resonanzen im PASCALUTSA-Formalismus ist. In dieser Arbeit wird eine relativistische Beschreibung am Beispiel der $N^*(1520)$ -Resonanz vorgestellt. Es zeigt sich, dass eine Beschreibung der Wechselwirkung von Spin-3/2-Resonanzen im PASCALUTSA-Formalismus nicht nur machbar sondern wesentlich einfacher als im konventionellen Zugang ist.

Im vereinfachten Ansatz von Post et al. [PLM01] musste der Propagator von Hand korrigiert werden, um Komplikationen zu vermeiden. Probleme treten insbesondere bei Teilchen mit negativer Parität auf. Das macht eine Untersuchung des $N^*(1520)$ als Resonanz mit negativer Parität interessant. In der vorliegenden Arbeit wird gezeigt, dass diese Komplikationen in einer vollständig relativistischen Rechnung nicht mehr auftreten.

In Kapitel 2 werden die RARITA-SCHWINGER-Felder als feldtheoretische Grundlage für die weitere Diskussion eingeführt. Die LAGRANGE-Funktion für freie Teilchen wird eingeführt und auf die korrekte Anzahl der Freiheitsgrade hin überprüft. Anhand dieser LAGRANGE-Funktion wird der freie Propagator berechnet.

Anschließend wird diskutiert, wie Wechselwirkungen eingeführt werden können. Die von Nath et al. [NEK71] eingeführte Kopplung ist inkonsistent, da sie zu einer nicht-kausalen Propagation [Sin73] und zu nicht-positiv definiten (Anti)Kommurationsrelationen [Hag71] führt. Diese Inkonsistenzen entstehen, da die Anzahl der Freiheitsgrade der freien Theorie durch die Wechselwirkung verändert wird. Der PASCALUTSA-Formalismus umgeht das Problem, indem die Wechselwirkung in einer eichinvarianten Form eingeführt wird. Zwei LAGRANGE-Funktionen mit der gle-

ichen Eichbinvarianz besitzen dieselbe Anzahl von Freiheitsgraden. Die freie LAGRANGE-Funktion für masselose Spin-3/2-Resonanzen besitzt diese Eichbedingung. Eine Wechselwirkung mit derselben Eichinvarianz führt keine neuen Freiheitsgrade ein. Ein Massenterm bricht die Eichinvarianz und erhöht die Anzahl der Freiheitsgrade auf den richtigen Wert. Das geschieht analog zur freien Theorie. An der $\Delta N\pi$ -Kopplung wird dieser Vorgang beispielhaft demonstriert. Es wird gezeigt, dass die Spin-1/2- und Spin-3/2-Anteile der Wechselwirkung tatsächlich voneinander separiert werden. In Kapitel 5 werden mittels dieser Methode neue Kopplungen für Spin-3/2-Resonanzen an ρN und $\Delta\pi$ entwickelt.

Am Ende des 2. Kapitels wird der Zusammenhang zwischen PASCALUTSA-Formalismus und konventioneller Kopplung diskutiert. Wenn nur die Observablen der asymptotischen Zustände von Interesse sind, führt die PASCALUTSA-Kopplung plus Kontaktterme einerseits und die konventionelle Kopplung andererseits zu den gleichen Ergebnissen. Diese Betrachtungsweise entspricht allerdings nicht dem Ansatz dieser Arbeit. Hier werden die Resonanz selbst und nicht die asymptotischen Zustände untersucht. Um die Resonanz zu verstehen, ist es angemessener die PASCALUTSA-Kopplung zu verwenden, da sich dann die Spin-1/2- und Spin-3/2-Anteile trennen lassen. Trotz der Vorteile ist noch nicht endgültig geklärt, ob der PASCALUTSA-Formalismus nur hinreichend oder auch notwendig ist, um wechselwirkende Spin-3/2-Resonanzen konsistent zu beschreiben. Diese Frage kann nur weitere Forschung im Bereich der effektiven Feldtheorien beantworten. Erste Ansätze finden sich in [HWGS05].

In Kapitel 3 werden Selbstenergien im PASCALUTSA-Formalismus diskutiert. Dabei ergibt sich das interessante Ergebnis, dass die vollständig relativistische Selbstenergie mit Hilfe von zwei Koeffizienten bestimmt werden kann. Im konventionellem Ansatz sind zehn Koeffizienten nötig.

Im 4. Kapitel wird der volle relativistische Propagator von Spin-3/2-Resonanzen in analytischer Form hergeleitet. Es wird gezeigt, dass die Struktur der Spektralfunktion von Spin-3/2-Resonanzen derjenigen von Spin-1/2-Resonanzen entspricht. Dies erleichtert die folgenden Untersuchungen. Die relativistische Struktur von Spin-1/2-Resonanzen wurde bereits untersucht, z.B. in [Frö01].

Selbstenergie und Spektralfunktion beschreiben die Eigenschaften eines Teilchens vollständig. Diese Größen sind aber nicht messbar und lassen sich nur schwer mit experimentellen Daten vergleichen. Es ist daher üblich, zwei Größen einzuführen, die experimentelle Daten und Rechnungen verbinden. Es handelt sich dabei um Masse und Breite einer Resonanz. Die vollständige Struktur der Selbstenergie oder der Spektralfunktion können diese Werte allerdings nicht ersetzen. Dies führt dazu, dass die Definition von Masse und Breite einer gewissen Mehrdeutigkeit unterliegen. In dieser Arbeit werden beide Größen aus einer relativistischen BREIT-WIGNER-Form abgeleitet. Die Spektralfunktion für Spin-3/2-Teilchen ist allerdings wesentlich komplizierter als die BREIT-WIGNER-Form. Daher ergeben sich drei Möglichkeiten, eine Breite zu definieren.

Im Falle der P_{33} -Resonanz sind alle drei Möglichkeiten Kandidaten für eine physikalisch sinnvolle Breite. Alle drei sind positiv definit und fast identisch. Beim D_{13} trifft das nicht zu. Die Unterschiede der möglichen Definitionen sind recht groß und nur eine davon ist positiv definit. Deshalb gibt es nur eine einzige sinnvolle Wahl einer s -abhängige Breite.

Um die D_{13} -Resonanz beschreiben zu können, müssen die Selbstenergien aller relevanten Zerfallskanäle ($N\pi$, $N\rho$ und $\Delta\pi$) berechnet werden. In Kapitel 5 werden die Kopplungen für diese Kanäle eingeführt und die Ergebnisse der Selbstenergien vorgestellt. Diese können anhand der daraus abgeleiteten Breite überprüft werden. Der nicht-relativistische Limes dieser Breite zeigt das korrekte Phasenraumverhalten.

Die Selbstenergien und Spektralfunktionen der P_{33} - und D_{13} -Resonanzen werden in Kapitel 6 diskutiert. Im Gegensatz zu vielen Vereinfachungen erfüllen die vollständigen Spektralfunktionen (ρ_s , ρ_v) automatisch alle BJORKEN-DRELL-Bedingungen, die in Kapitel 4.3.3 zusammengefasst sind. Im Einzelnen heißt das, dass ρ_s und ρ_v reelle Größen sind, ρ_v positiv definit und normiert ist und dass $\sqrt{s}\rho_v - \rho_s \geq 0$ für alle \sqrt{s} erfüllt ist. Alle Spektralfunktionen von Spin-1/2- und Spin-3/2-Teilchen müssen diese Bedingungen erfüllen.

Die P_{33} -Resonanz weist eine typische BREIT-WIGNER-Form auf. Sie ist asymmetrisch, steigt schnell an und fällt mit einem langen Ausläufer ab. Die D_{13} -Resonanz dagegen besitzt eine symmetrischere Struktur die bei hohen Energien in eine lange Schulter übergeht. Diese entsteht durch die starke Kopplung an das ρ -Meson. Die Schulter verschwindet, wenn das ρ -Meson vernachlässigt wird. Das Umgekehrte passiert, wenn die Kopplung an das ρ -Meson verstärkt wird. Daraus lässt sich auf den wichtigen Beitrag des ρ -Mesons zur Gesamtstruktur des D_{13} schließen.

Damit die Selbstenergien bei großen Energien endlich bleiben, wurde ein Dämpfungsparameter Λ eingeführt. Eine genaue Bestimmung dieses Parameters ist im Rahmen dieser Arbeit nicht möglich. Es wird jedoch gezeigt, dass Λ bei etwa 1 GeV liegen muss, um physikalisch sinnvolle Werte zu erhalten.

Die vollständig relativistische Spektralfunktion wird mit verschiedenen Vereinfachungen verglichen, um deren Qualität zu beurteilen. Zuerst wird das Ergebnis für die P_{33} -Resonanz mit einem einfacheren BREIT-WIGNER-Ansatz verglichen. Es zeigt sich, dass dieser einfache Ansatz für Energien über der Massenschale keine gute Näherung für die volle Spektralfunktion darstellt.

Als nächstes wird untersucht, inwieweit die Spektralfunktion ρ_v durch Vernachlässigung der Realteile der Selbstenergie vereinfacht werden kann. Dieser Ansatz scheitert daran, dass ρ_v nicht mehr normiert ist und damit seine Bedeutung verliert.

Die von Post et al. [PLM01] vorgeschlagene Näherung liefert eine gute Übereinstimmung mit der vollen Rechnung. Im Falle der P_{33} -Resonanz ist die Übereinstimmung fast perfekt. Bei der D_{13} -Resonanz ist das Maximum leicht verrückt und es wird zuviel spektrale Stärke hin zu höheren Energien verschoben. Diese

Abweichungen können anhand der BJORKEN-DRELL-Funktion (4.37) charakterisiert werden.

Unter der Annahme, dass Δ - und ρ -Resonanz stabile Teilchen sind, verändern sich Breite und Spektralfunktion der Teilchen. Diese Annahme ist reizvoll, da sie den numerischen Aufwand erheblich verringert. Für die Berechnung der Selbstenergie können die Spektralfunktionen der Teilchen dann durch δ -Funktionen ersetzt werden. Besonders im Falle des Δ ist der eingesparte Aufwand enorm, da die Spektralfunktion des Δ eine komplizierte Struktur besitzt. Nimmt man das ρ -Meson als stabil an, verändert sich die Spektralfunktion drastisch. Sie wird wesentlich schmaler und bei sehr hohen Energien entstehen neue Strukturen. Diese Näherung ist daher nicht physikalisch sinnvoll. Nimmt man dagegen die Δ -Resonanz als stabil an, verändert sich die Spektralfunktion nur marginal. Im Vergleich zum numerischen Aufwand, der dadurch eingespart wird, ist das im Bedarfsfall eine akzeptable Näherung.

Mit den in der vorliegenden Arbeit präsentierten Selbstenergien lassen sich alle Spektralfunktionen von Spin-3/2-Teilchen berechnen, die in $N\pi$, $N\rho$ oder $\Delta\pi$ zerfallen.

Die expliziten Ergebnisse für die D_{13} -Resonanz können benutzt werden, um Reaktionen von D_{13} -Resonanzen in Dileptonen oder zwei Pionen zu bestimmen, zum Beispiel:

$$\begin{aligned} N\pi &\rightarrow D_{13} \rightarrow N\rho \rightarrow Ne^+e^-, \\ N\pi &\rightarrow D_{13} \rightarrow N\rho \rightarrow N\pi\pi, \\ N\pi &\rightarrow D_{13} \rightarrow N\Delta \rightarrow N\pi\pi. \end{aligned}$$

Zusätzlich besteht die Möglichkeit, Vereinfachungen der Spektralfunktion qualitativ zu beurteilen, indem sie mit der vollständigen Rechnung verglichen werden. Das wird in dieser Arbeit für einige Vereinfachungen am Beispiel der D_{13} -Resonanz vorgeführt.

Um alle Parameter festzulegen, wäre es wünschenswert, diese direkt aus experimentellen Daten zu bestimmen.

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