Hadron physics with functional methods

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Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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universitätbonn

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF Helmholtz Forschungsakademie Hessen für FAIR

Topic

Blue

Important !

… please pay attention…

Topic

Red

Derivation

… you may take a nap
if you are not interested…

Further reading material

- R. Alkofer and L. von Smekal, ``The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states," Phys. Rept. 353 (2001) 281 [hep-ph/0007355].

- P. Maris and C. D. Roberts, ``Dyson-Schwinger equations: A Tool for hadron physics," Int. J. Mod. Phys. E 12 (2003) 297 [nucl-th/0301049]

- C. S. Fischer, ``Infrared properties of QCD from Dyson-Schwinger equations," J. Phys. G 32 (2006) R253 [hep-ph/0605173].

- A. Bashir, L. Chang, I. C. Cloet, B. El-Bennich, Y. X. Liu, C. D. Roberts and P. C. Tandy, ``Collective perspective on advances in Dyson-Schwinger Equation QCD," Commun. Theor. Phys. 58 (2012) 79 [arXiv:1201.3366].

- G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C.S. Fischer, Progress in Particle and Nuclear Physics 91, 1-100 [arXiv:1606.09602]

- C.S.Fischer, ``QCD at finite temperature and chemical potential from DysonSchwinger equations," Prog. Part. Nucl. Phys. 105 (2019), 1-60 [arXiv:1810.12938 [hep-ph]].

Why hadron physics ??

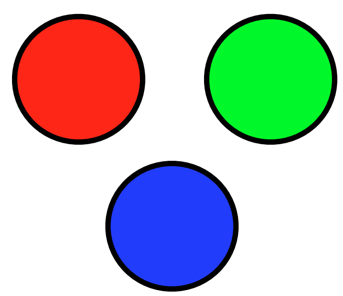
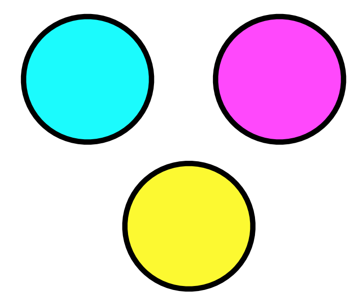
- Bridge to particle physics (standard model and beyond)

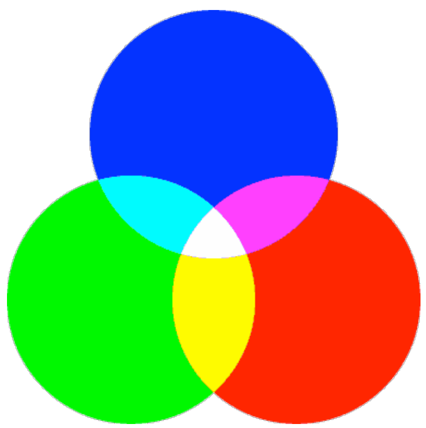
  see later (maybe)

  - Confinement
  - Dynamical mass generation
  - Properties of protons, neutrons…

- Bridge from fundamental physics to effective nuclear forces

Electromagnetic charge vs. color-charge

	Charge	Anti-charge
Electromagnetism	−	+
Strong force		

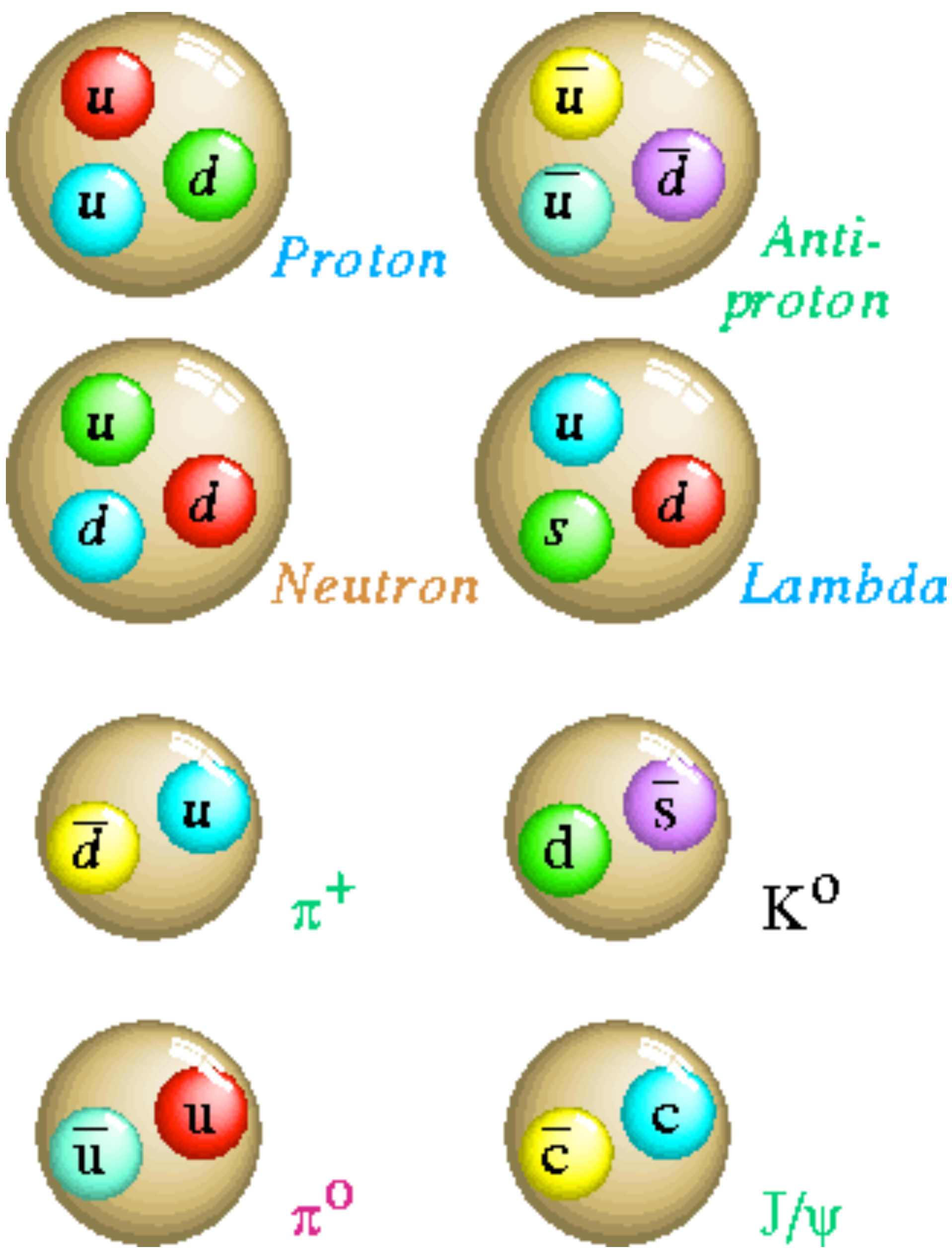


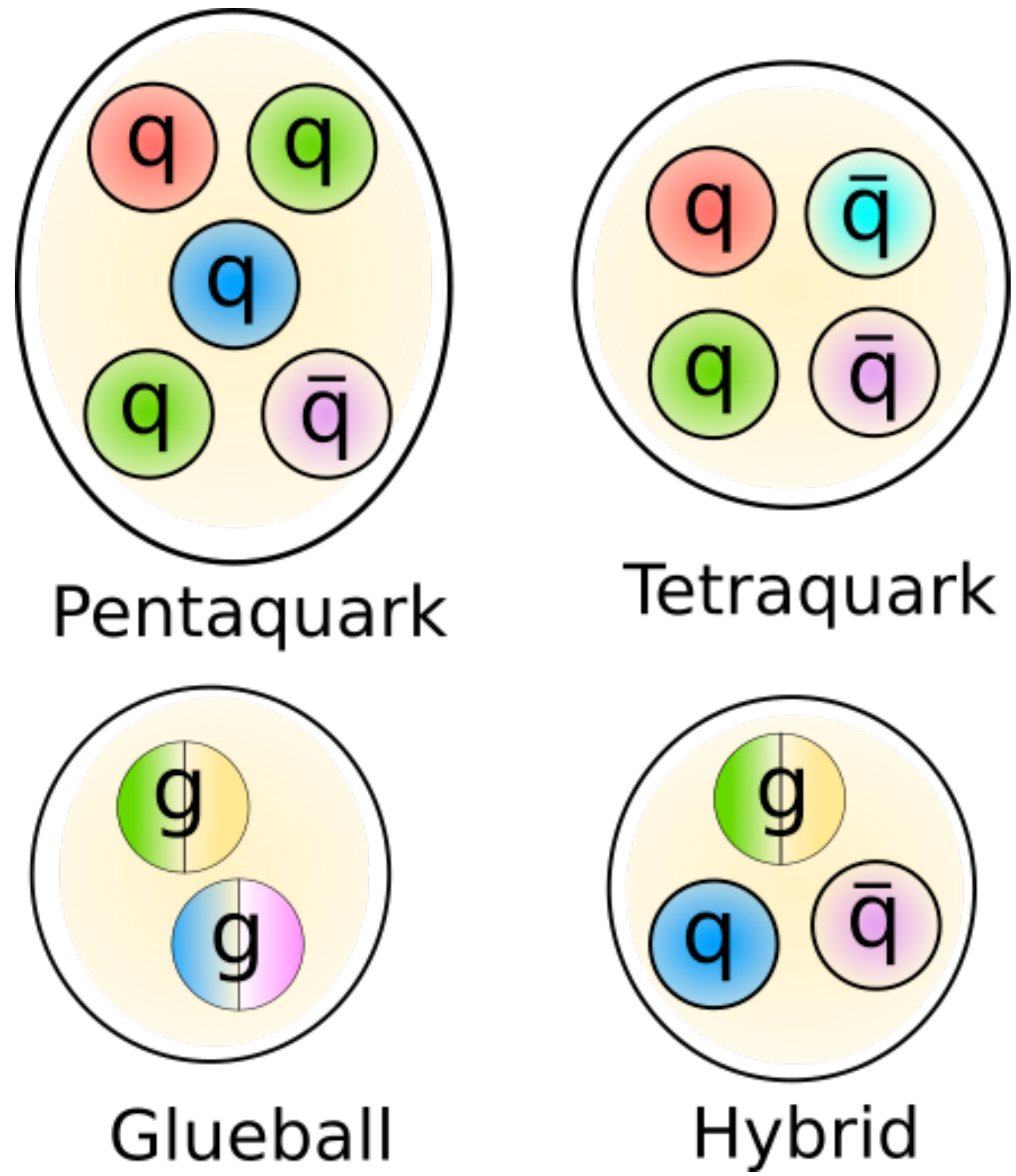
- **Confinement:** All observable particles are colorless

Hadrons: baryons, mesons and ... exotics !

Baryons

Mesons





Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better

2. Properties of QCD and correlation functions
   - Confinement and dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…

3. Bound states and applications (rainbow-ladder approximation)
   - Mesons
   - Baryons
   - Tetraquarks

4. Form factors and decays (rainbow-ladder approximation)
   - Meson and baryon form factors
   - The anomalous magnetic moment of the muon

5. Beyond rainbow ladder
   - Confinement and glueballs
   - Hadron results beyond rainbow-ladder

Nonpert. QCD: Complementary approaches

Quarks and gluons

- Lattice simulations
  - Ab initio
  - Gauge invariant

Quarks, gluons or… Hadrons

- Effective theories (NRQCD, χPT, ...)
  - Dof integrated out
  - —>Physical dof

- Functional approaches (DSE, FRG,Hamilton):
  - Space-time continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible
  - Chemical potential: no sign problem
  - Access to structural information

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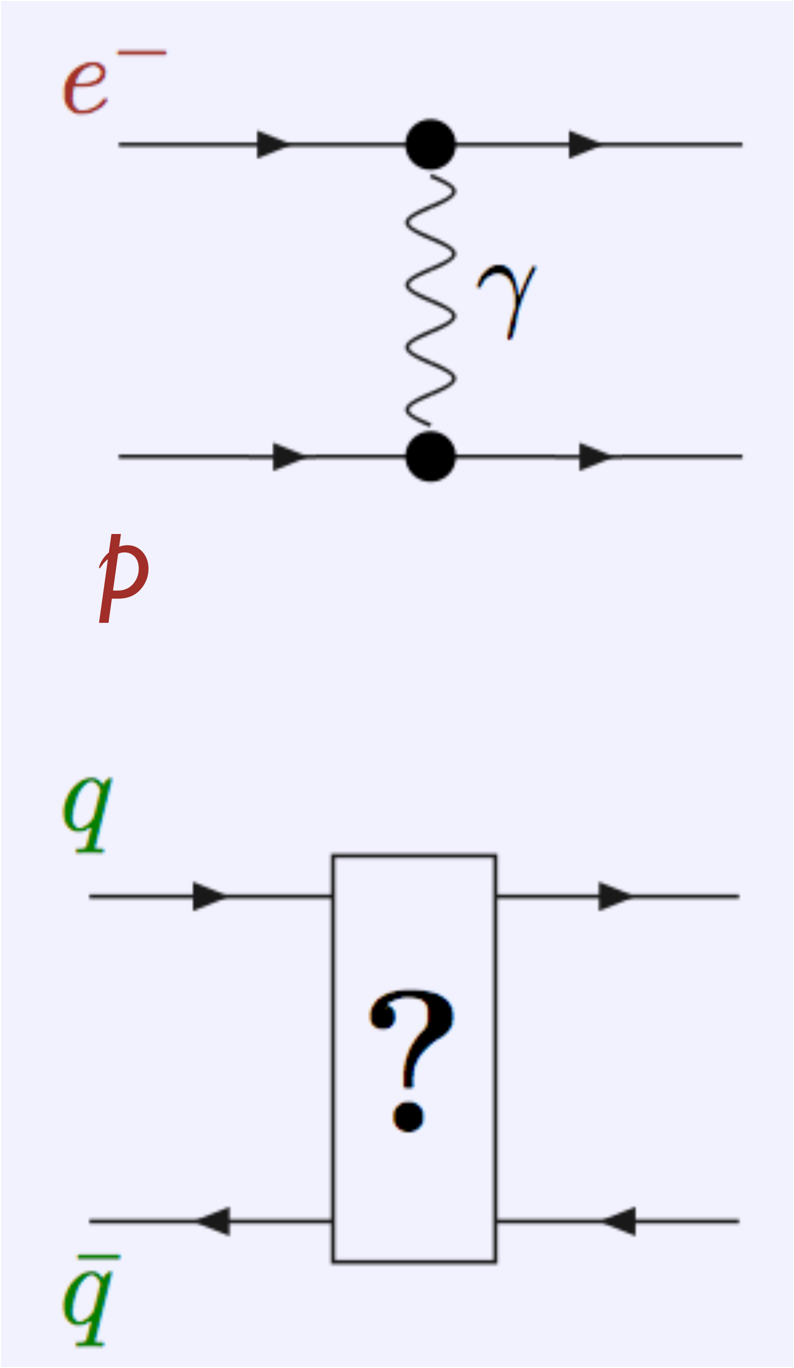
Phenomenological tool: Quark-model

Bound states in QED and QCD

Proton-electron-system:

- $F(r) \sim \frac{1}{r^2}$
- Hydrogen can be ionised...



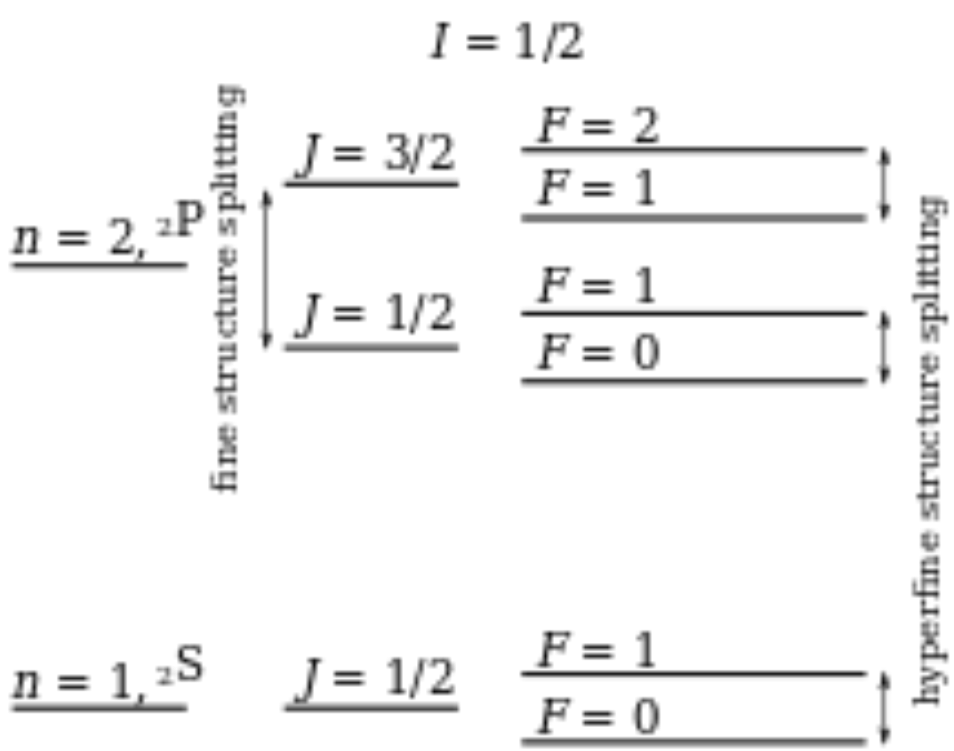
Quark-Antiquark-System

- $F(r) \sim const.$
- Confinement

Similarities: bound states of two spin 1/2 particles...

QED: electron-proton interaction

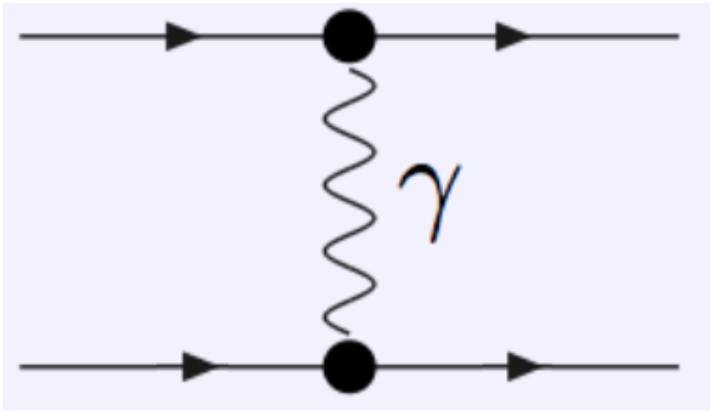
- hydrogen:



Coulomb potential
spin-orbit coupling (LS): fine splitting
spin-spin coupling (SS): hyperfine splitting

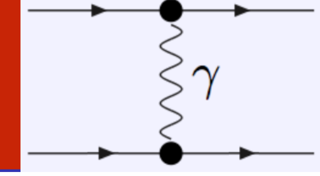
Calculation e.g. via Schrödinger equation and perturbation theory

- field theory:



$$
\begin{aligned}
V_{Fermi-Breit} = &-\frac{\alpha}{r} + \frac{\alpha\pi}{2}\delta(\mathbf{r})\left[\frac{1}{m_e^2}+\frac{1}{m_p^2}\right] + \frac{8\pi\alpha}{3m_e m_p}\mathbf{S}_e\mathbf{S}_p\delta(\mathbf{r}) \\
&+ \frac{\alpha}{m_e m_p r^3}\left[3(\mathbf{S}_e\mathbf{r})(\mathbf{S}_p\mathbf{r}) - \mathbf{S}_e\mathbf{S}_p\right] \\
&+ \frac{\alpha}{r^3}\left[\frac{\mathbf{S}_e\mathbf{L}_e}{2m_e^2} - \frac{\mathbf{S}_p\mathbf{L}_p}{2m_p^2} + \frac{\mathbf{S}_p\mathbf{L}_e - \mathbf{S}_e\mathbf{L}_p}{2m_p m_e}\right] \\
&+ \frac{\alpha}{2m_e m_p r}\left(\mathbf{p}_e\mathbf{p}_p + \frac{(\mathbf{r}\mathbf{p}_p)(\mathbf{r}\mathbf{p}_e)}{r^2}\right)
\end{aligned}
$$

Derivation: Fermi-Breit force



We start with the formula for the scattering amplitude in momentum space

$$M = e^2 \bar{u}_e(\mathbf{p}_e') \gamma_\mu u_e(\mathbf{p}_e) \frac{1}{\mathbf{q}^2} \bar{u}_p(\mathbf{p}_p') \gamma_\mu u_p(\mathbf{p}_p) \qquad (1)$$

with generic spinors in non-relativistic approximation

$$u(\mathbf{p}) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m}\chi \end{pmatrix} \quad \rightarrow \quad u(\mathbf{p}) = \left(1 - \frac{\mathbf{p}^2}{8m^2}\right) \begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{2m}\chi \end{pmatrix} \qquad (2)$$

Using this and $\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$ we obtain

$$\begin{aligned} M = & -\frac{e^2}{\mathbf{q}^2} \left(1 - \frac{\mathbf{p}_e^2 + \mathbf{p}_e'^2}{8m_e^2}\right) \left(1 - \frac{\mathbf{p}_p^2 + \mathbf{p}_p'^2}{8m_p^2}\right) \times \\ & \left[\chi_p^\dagger \left(1 + \frac{\mathbf{p}_p\mathbf{p}_p' + i\boldsymbol{\sigma}(\mathbf{p}_p' \times \mathbf{p}_p)}{4m_p^2}\right) \chi_p \chi_e^\dagger \left(1 + \frac{\mathbf{p}_e\mathbf{p}_e' + i\boldsymbol{\sigma}(\mathbf{p}_e' \times \mathbf{p}_e)}{4m_e^2}\right) \chi_e \right. \\ & \left. - \chi_p^\dagger \frac{\mathbf{p}_p + \mathbf{p}_p' - i\boldsymbol{\sigma}\mathbf{q}}{2m_p} \chi_p \chi_e^\dagger \frac{\mathbf{p}_e + \mathbf{p}_e' - i\boldsymbol{\sigma}\mathbf{q}}{2m_e} \chi_e \right] \end{aligned} \qquad (3)$$

Derivation: Fermi-Breit force

Reminding ourselves that scattering amplitude and potential are connected via a Fourier-transformation

$$V(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{qr}} M \tag{1}$$

we obtain the familiar Coulomb potential from the leading term

$$V_{Coulomb}(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{qr}} \frac{-e^2}{\mathbf{q}} = -\frac{\alpha}{r} \tag{2}$$

and with $\mathbf{S} = \boldsymbol{\sigma}/2$ as well as $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ we obtain the **LS**-coupling term

$$V_{LS} = -\int \frac{d^3q}{(2\pi)^3} \frac{e^2}{\mathbf{q}^2} \frac{i\boldsymbol{\sigma}(\mathbf{p}_e' \times \mathbf{p}_e)}{4m_e^2} e^{-i\mathbf{qr}} \tag{3}$$

$$= \frac{e^2}{em_e^2} \frac{\boldsymbol{\sigma}(\mathbf{r} \times \mathbf{p}_e)}{4\pi r^3} = \frac{\alpha}{2m_e^2 r^3} \mathbf{LS} \tag{4}$$

Derivation: Fermi-Breit force

The other terms of order p^2/m_e^2 combine to the Darwin-term

$$V_D = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{qr}} \frac{\mathbf{p}_e^2 + \mathbf{p}_e'^2 - 2\mathbf{p}_e\mathbf{p}_e'}{8m_e^2} = \frac{\alpha\pi}{2m_e^2}\delta(\mathbf{r})\,, \tag{1}$$

whereas terms of order p/m_e can be interpreted as spin-spin interactions between proton and electron

$$V_{SS} = V_{hyp} + V_{tensor} \tag{2}$$

$$V_{hyp} = \frac{8\pi\alpha}{3m_e m_p}\mathbf{S}_e\mathbf{S}_p\delta(\mathbf{r}) \tag{3}$$

$$V_{tensor} = \frac{\alpha}{m_e m_p r^3}\left(3(\mathbf{S}_e\mathbf{r})(\mathbf{S}_p\mathbf{r}) - \mathbf{S}_e\mathbf{S}_p\right)\,. \tag{4}$$

This is what makes the hyperfine structure of the hydrogen atom !
Finally the remaining term denotes the **LL** interaction and can be written as

$$V_{LL} = \frac{\alpha}{2m_e m_p r}\left(\mathbf{p}_e\mathbf{p}_p + \frac{(\mathbf{r}\mathbf{p}_p)(\mathbf{r}\mathbf{p}_e)}{r^2}\right) \tag{5}$$

(Non-relativistic) Quark model

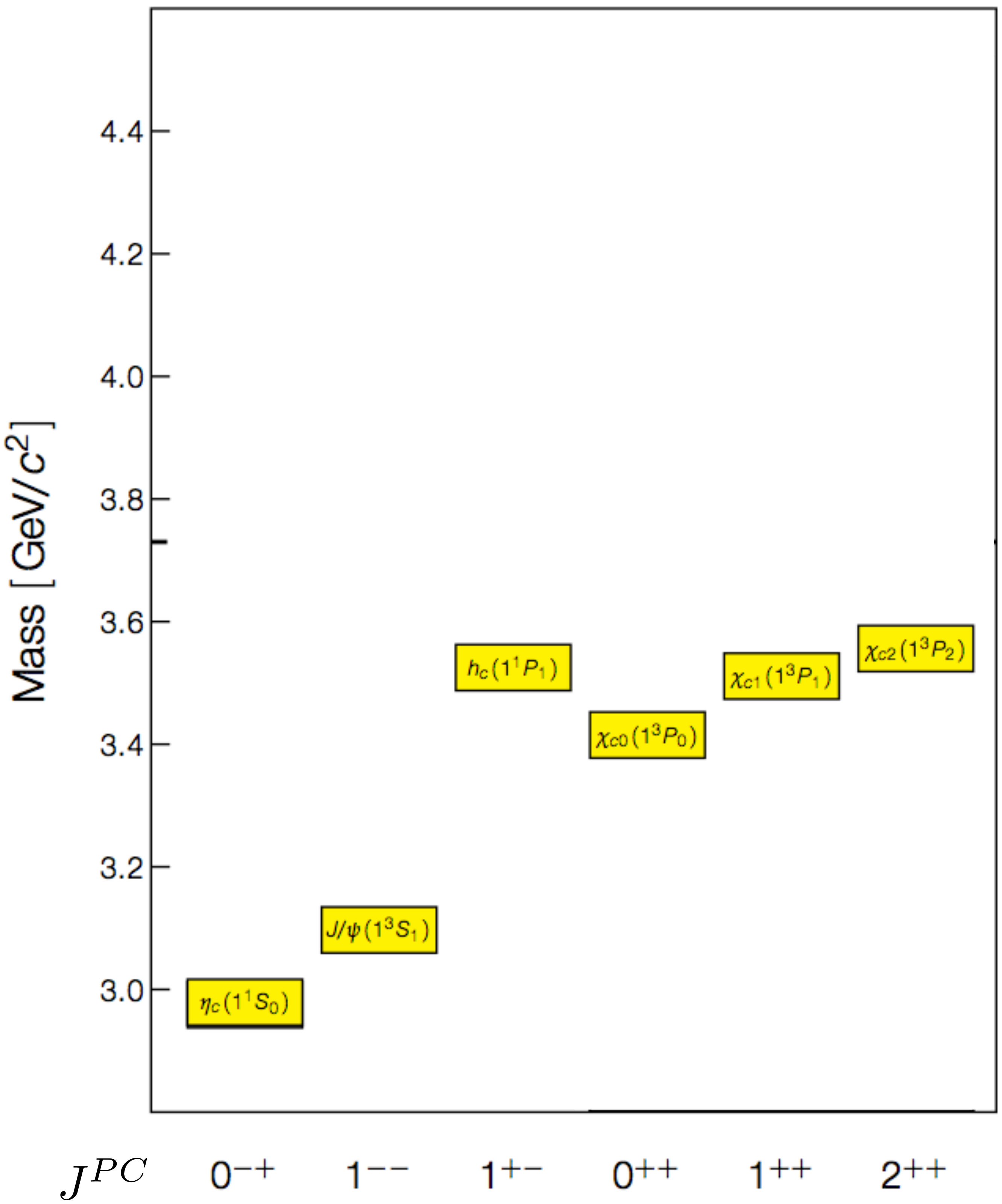
Basic ideas:

- Consider heavy quarks (charm, bottom): non-relativistic
- Bound states of two spin 1/2 particles:
  similar forces than QED ?
- Quarks are pointlike (=contituents) with mass m
- simplest assumption: interaction dominated by one-gluon exchange (vector-vector type of interaction) —> Fermi-Breit
- replace α_{QED} with α_S and Coulomb- by Cornell-potential

$$V_{Coulomb} = -\frac{\alpha}{r} \qquad \rightarrow \qquad V_{Cornell} = br - \frac{\alpha_S}{r}$$

- introduce parameters to play with strength of different contributions

Spectrum of ground state charmonia



Do we understand level ordering ?

Quantum numbers (quark model)

Coupling a quark and an antiquark:

$$S : 1/2 \otimes 1/2 \rightarrow 0 \oplus 1$$

$$P : (-1)^{L+1}$$

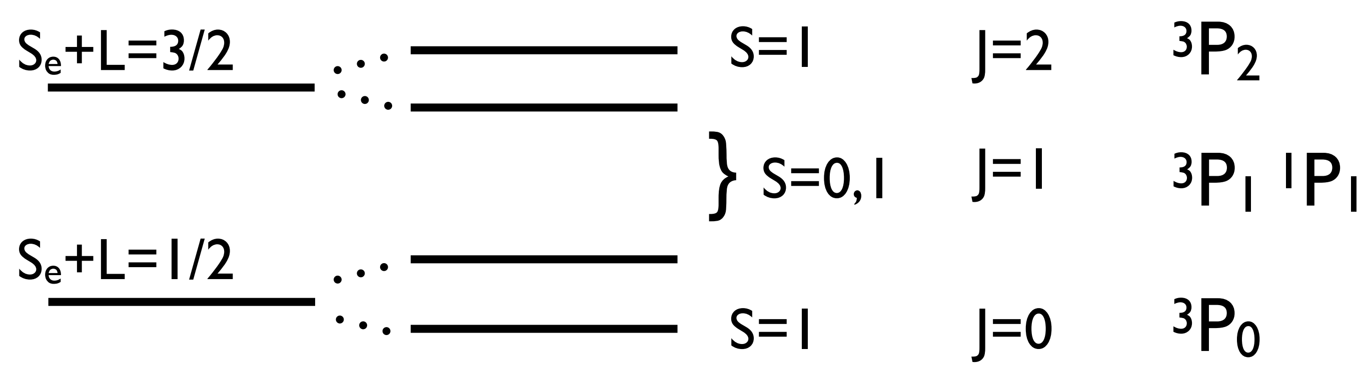
$$C : (-1)^{L+S}$$

S	L	J^{PC}	
0	0	0^{-+}	
1	0	1^{--}	
0	1	1^{+-}	**$^1P_1$**
1	1	0^{++}	$^3P_0$
		1^{++}	$^3P_1$
		2^{++}	$^3P_2$

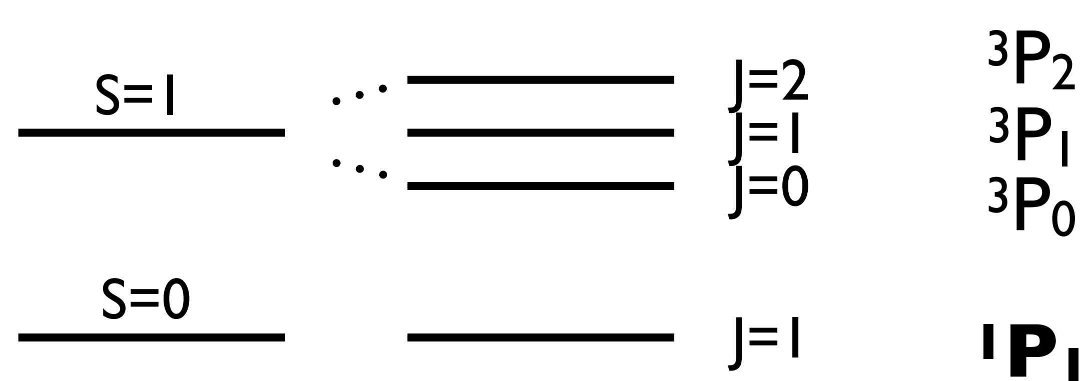
J^{PC} or $^{2S+1}L_J$

Spectrum for L=1 states

dominant LS-coupling:

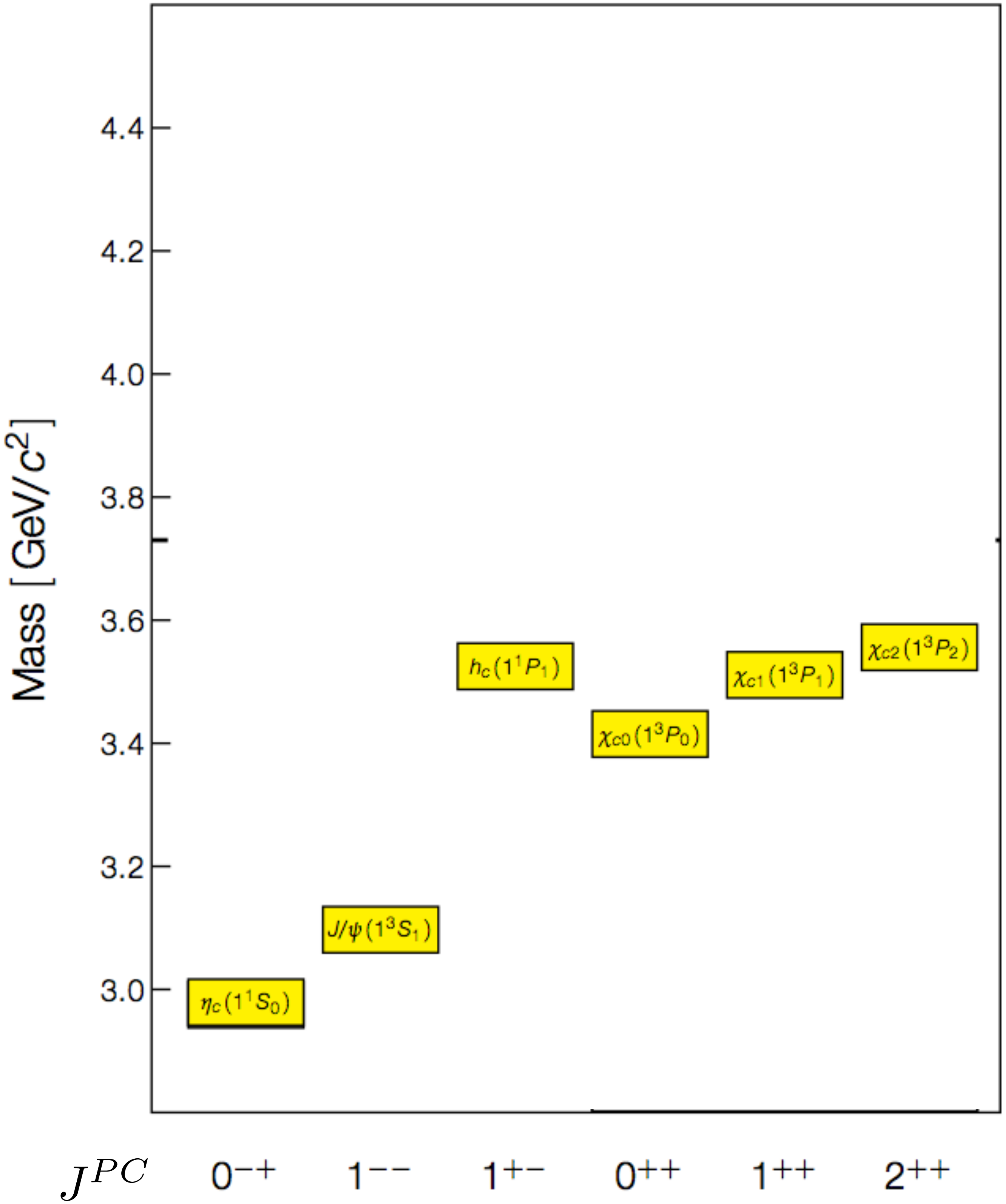


dominant SS-coupling:



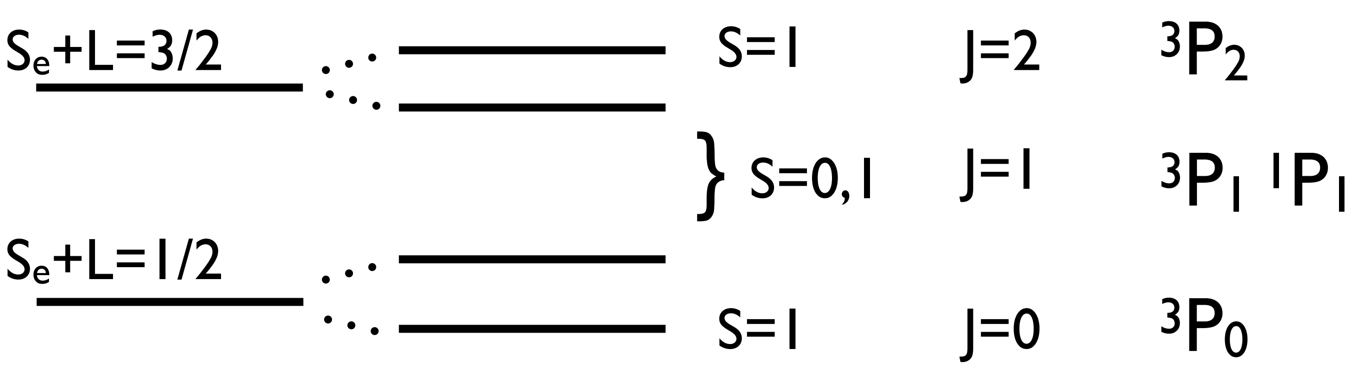
Note: 'exotic' quantum numbers such as $0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$ etc. not possible !

Spectrum of states in the charmonia region

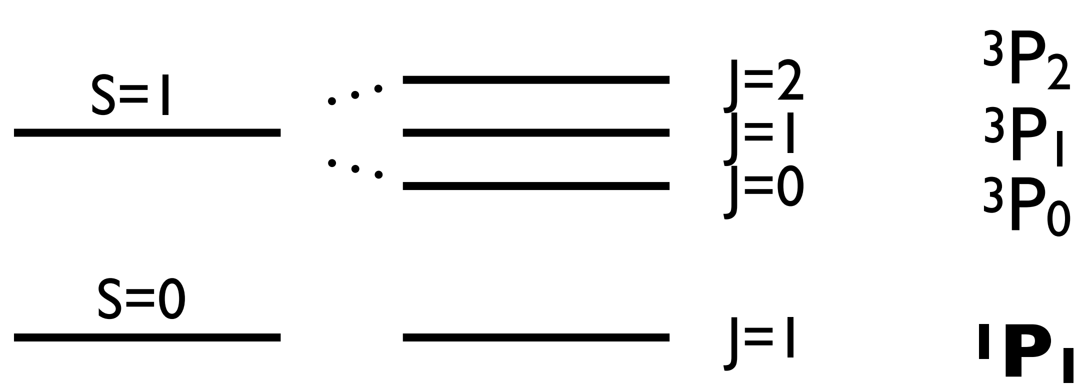


Wolfgang Gradl, BESIII, St Goar 2015

dominant LS-coupling:



dominant SS-coupling:



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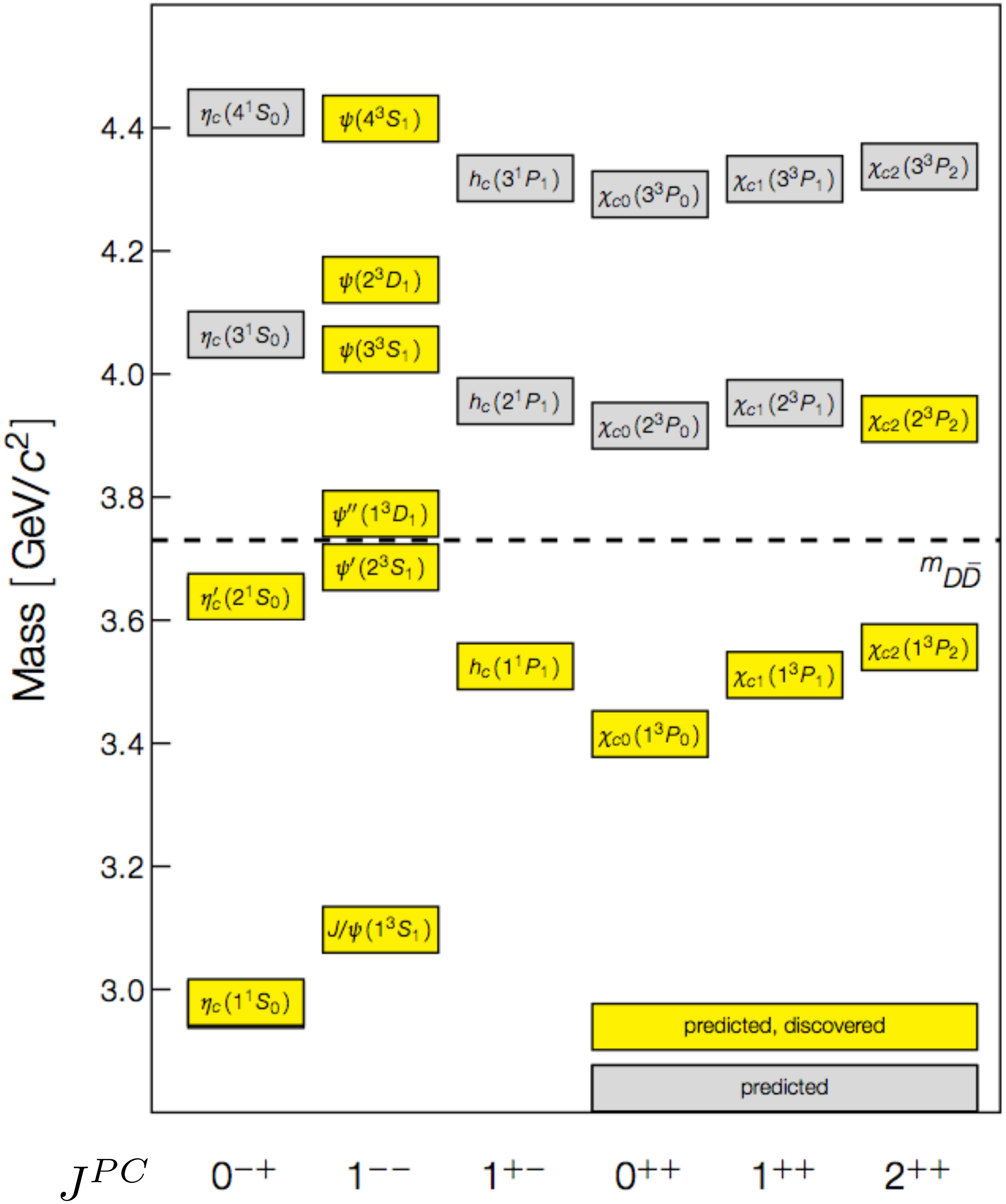

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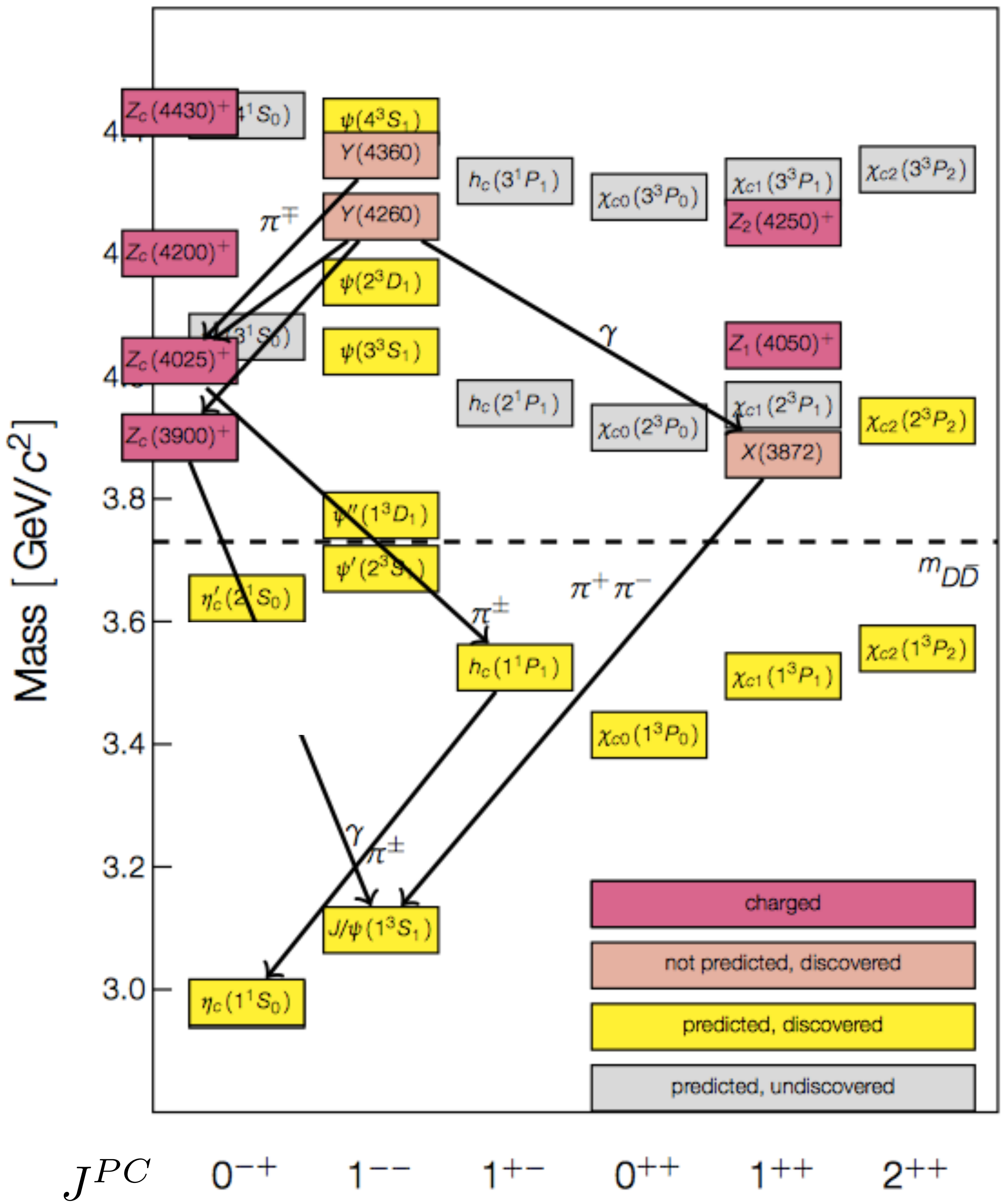
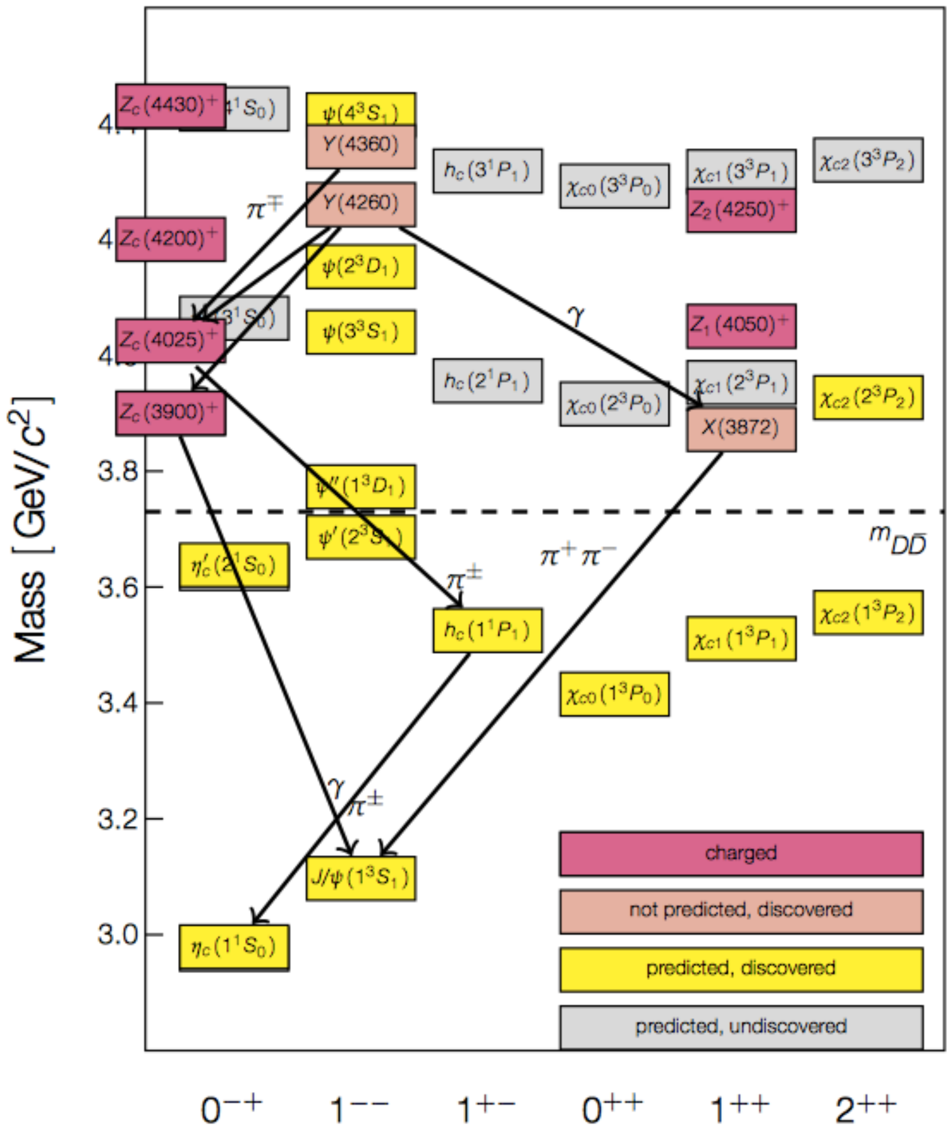


Wolfgang Gradl, BESIII, St Goar 2015

- many new states, not predicted by quark model
- some of these are charged… : candidates for tetraquarks
- but also: hybrids ? glueballs ?

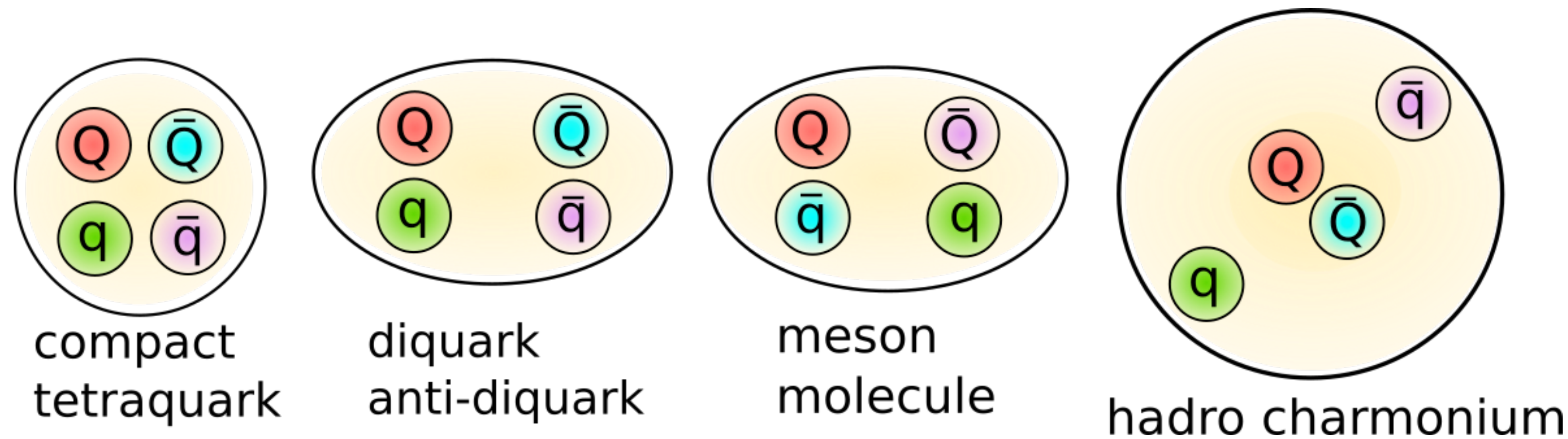
Experiments: Belle (II), BaBAR, BES III, LHCb, GlueX/JLAB, PANDA/FAIR

Tetraquark candidates in charmonium region



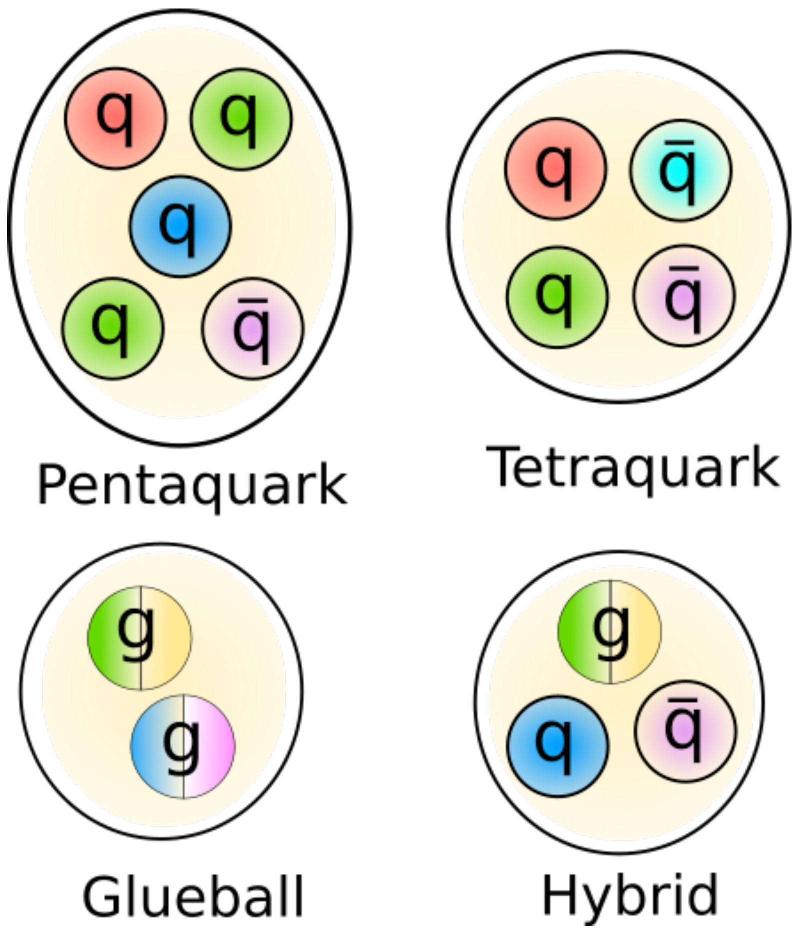
Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



Related to details of underlying QCD forces between quarks

Tetraquarks in the light meson sector



K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)

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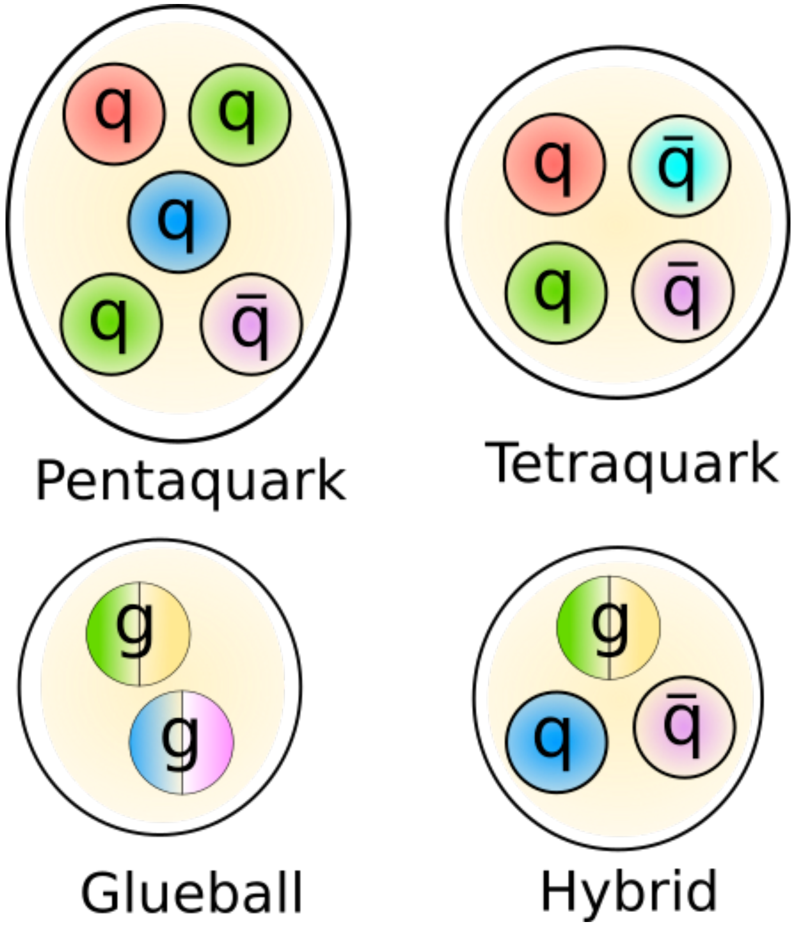
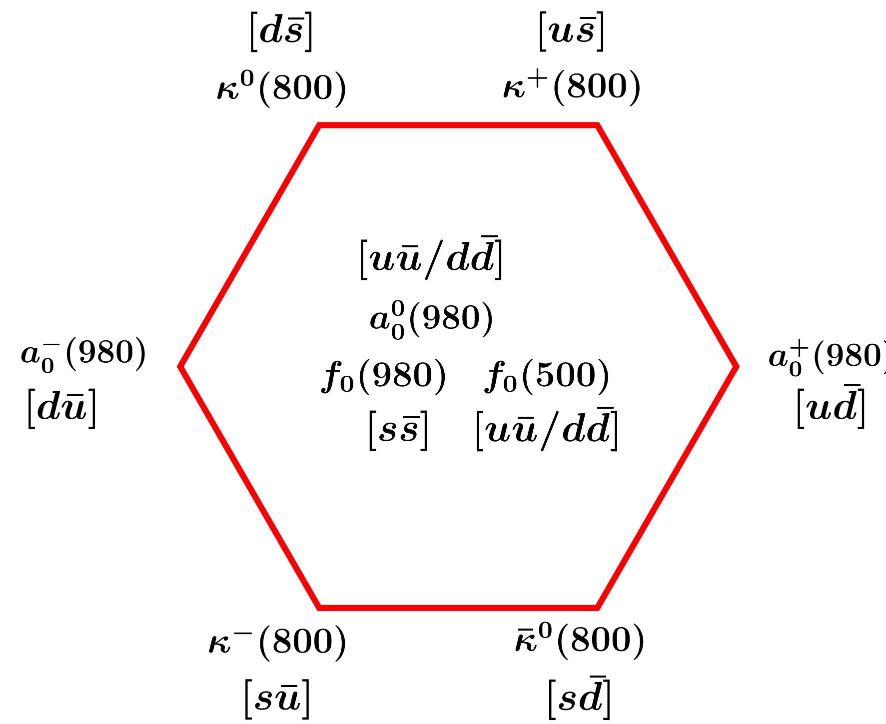


Light meson sector: scalars!



$f_0(980)$ [j] $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 40$ to 100 MeV

$f_0(980)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	476
$K\overline{K}$	seen	36
$\gamma\gamma$	seen	495

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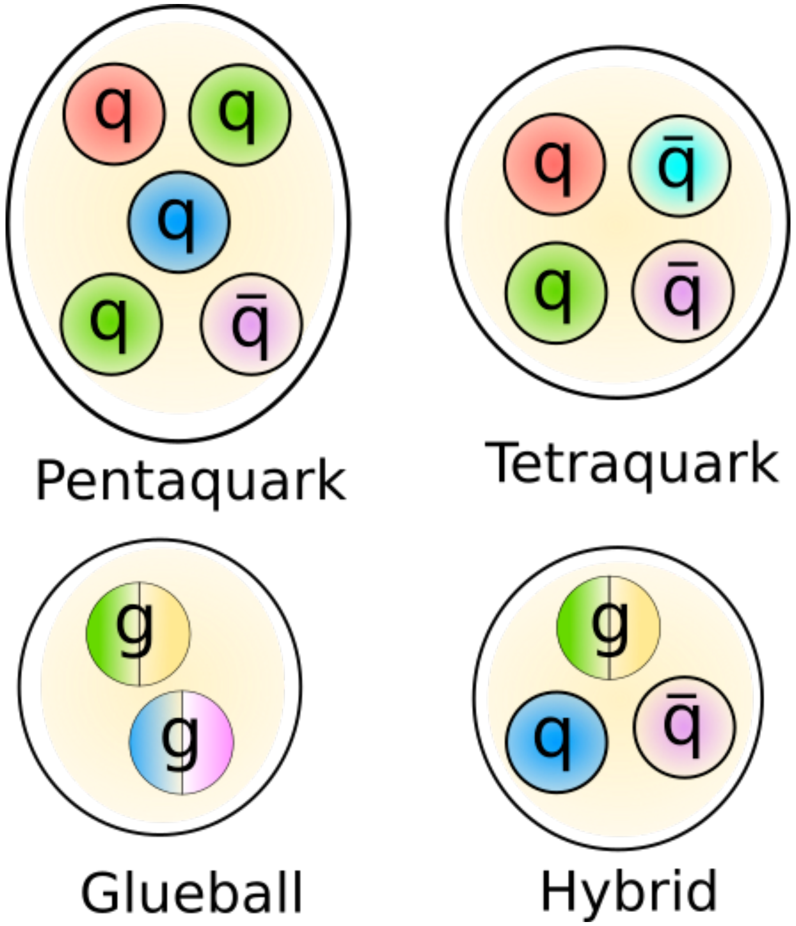

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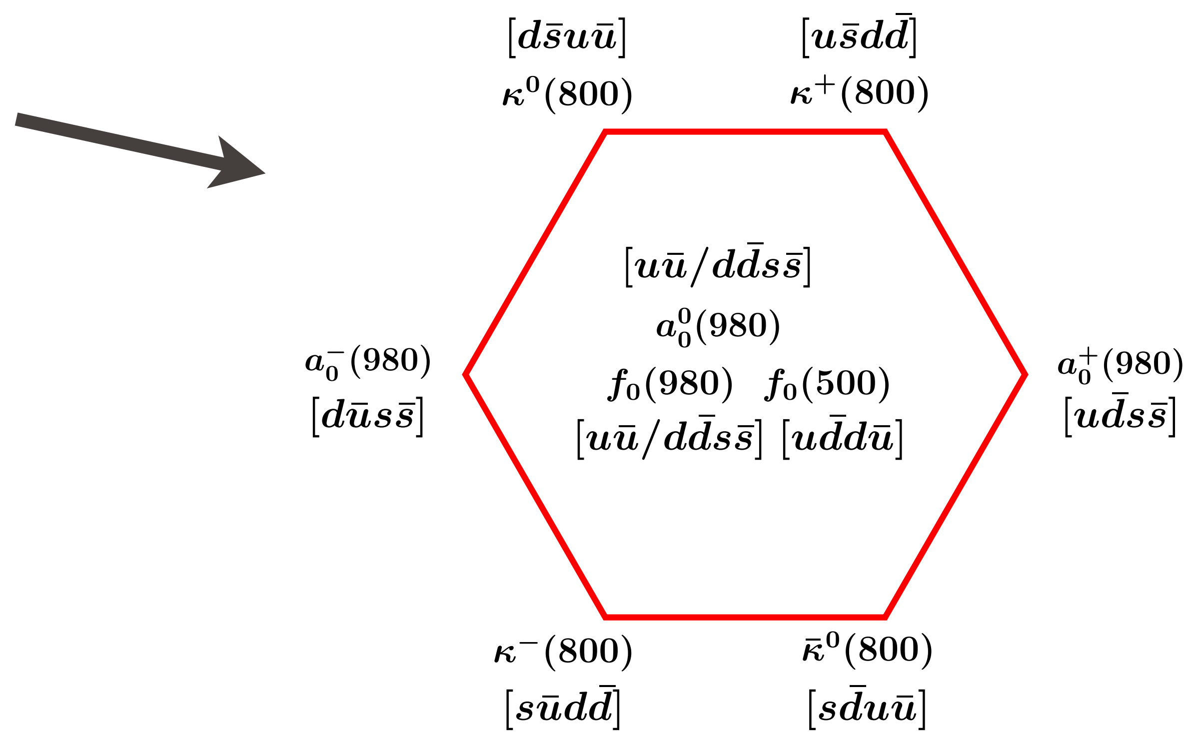

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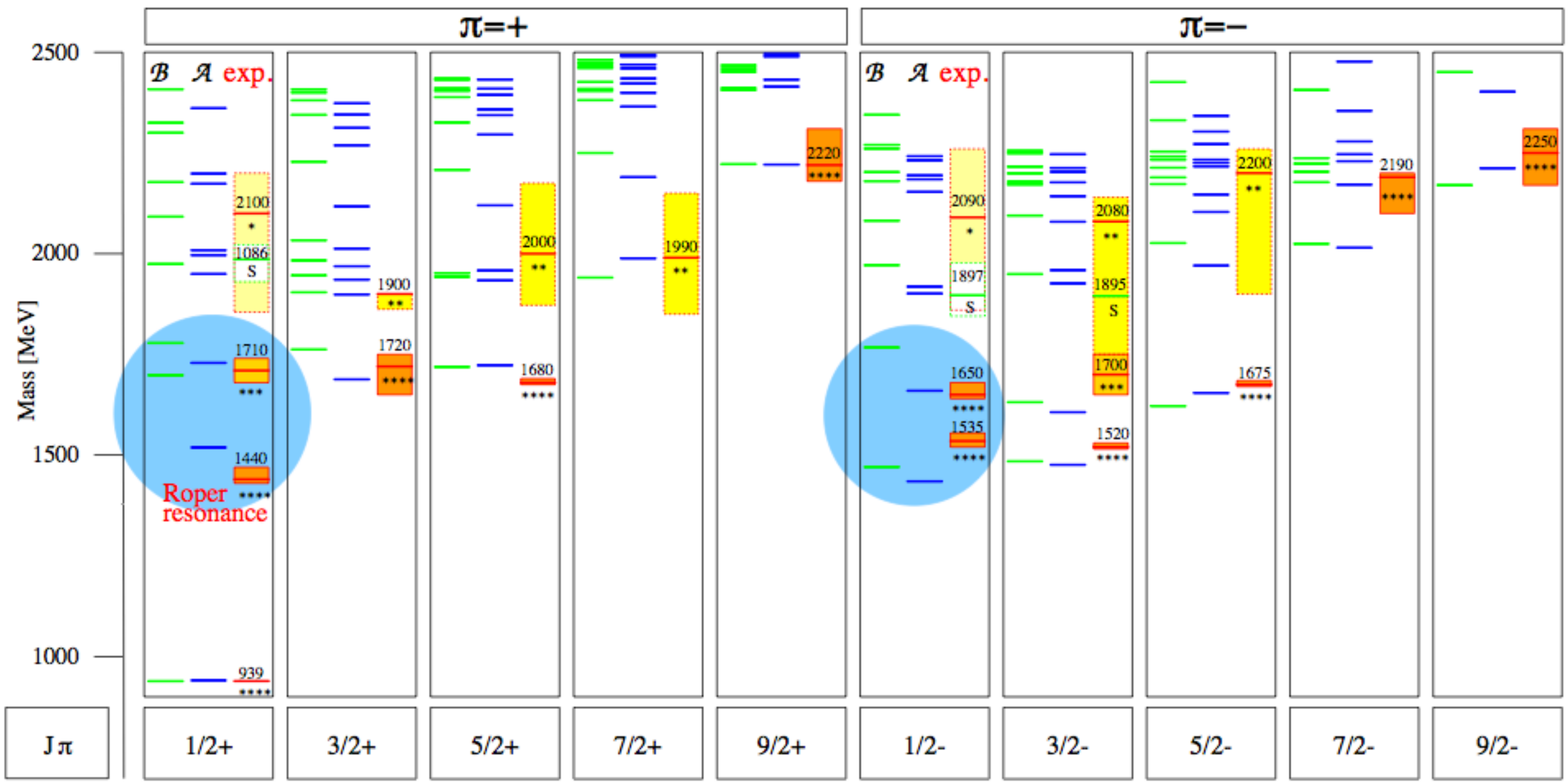
Baryons: quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- ‘missing resonances’ ?! diquarks ??
- level ordering between channels with opposite parity ??

Shortcomings of quark model

- Concept of constituent quarks ? see later…
- Use of potentials justified for light quarks ? No !
- Use of potentials justified even for charm ? NRQCD
- Relation of (phenomenological) potential to QCD ? unclear…
- Different parameters for different problems (mesons-baryons)
- Exotic states ? (tetraquarks, hybrids…) not well developed
- Many unsolved problems: Roper …

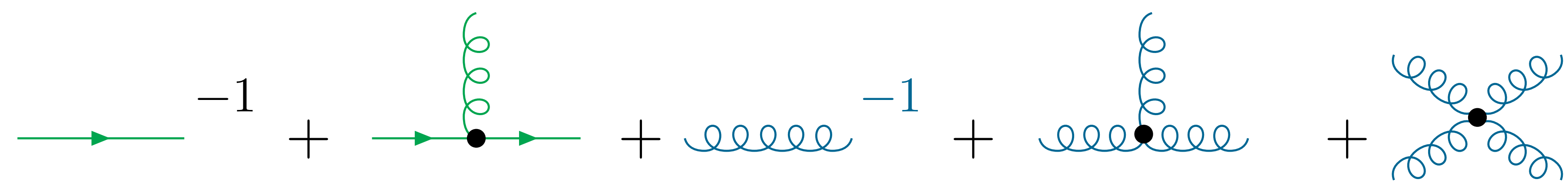
Still: quark model provides base line calculation
which allows us to formulate many useful questions !

S. Capstick and W. Roberts,
Quark models of baryon masses and decays,
Prog. Part. Nucl. Phys. 45 (2000) S241

The QCD generating functional

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A]\ \exp\left\{ - \int d^4x \Big(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \Big) \right\}$$

$S_{QCD} = \int d^4x \,\Big($



$\Big)$

- Euclidean space
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$
- $D_\mu = \partial_\mu + i g t^a A^a_\mu$
- Landau gauge: $\partial_\mu A^a_\mu = 0$

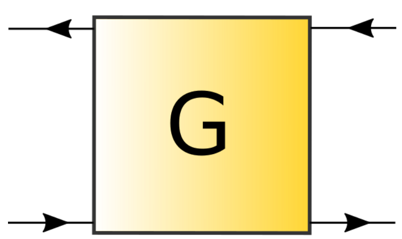
QCD correlation functions

$$\mathcal{Z} = \int \mathcal{D}[A,\Psi,\bar{\Psi}] \quad e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

$$\rightarrow \langle \mathcal{O} \rangle = \int \mathcal{D}[A,\Psi,\bar{\Psi}]\ \mathcal{O}\ e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

Examples:

$$\langle \Psi\bar{\Psi} \rangle = \int \mathcal{D}[A,\Psi,\bar{\Psi}]\ \Psi\bar{\Psi}\ e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

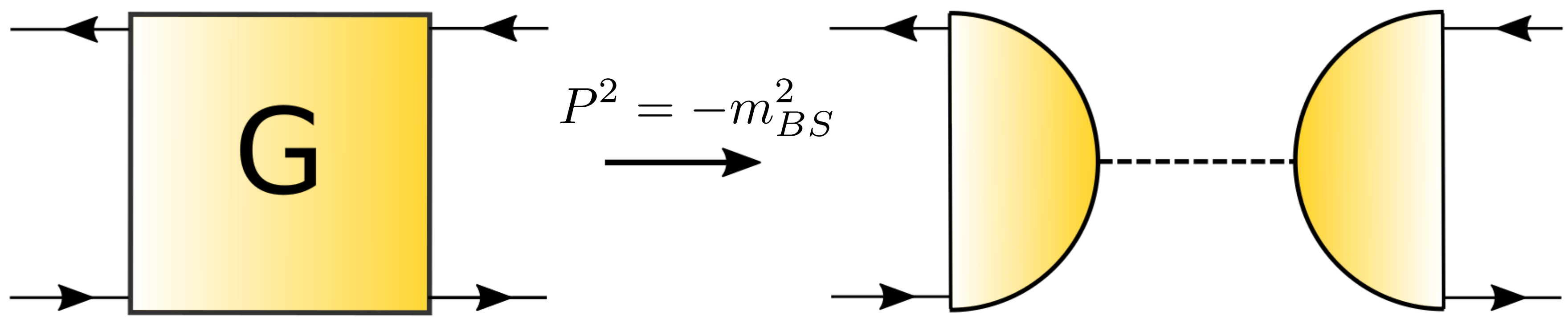


$$\langle \Psi\bar{\Psi}\Psi\bar{\Psi} \rangle = \int \mathcal{D}[A,\Psi,\bar{\Psi}]\ \Psi\bar{\Psi}\Psi\bar{\Psi}\ e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

- can be gauge invariant or gauge dependent

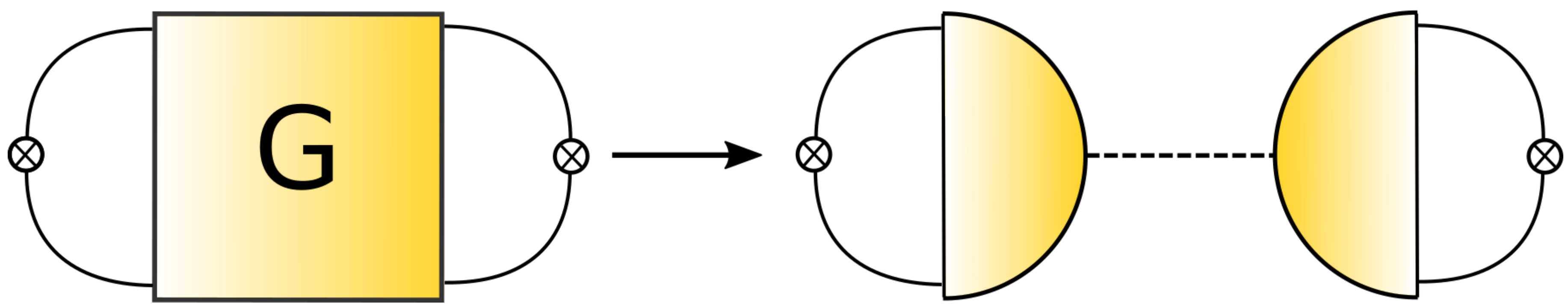
Extracting spectra from QCD-correlators

functional:



Lattice:

see lecture
of Daniel Mohler



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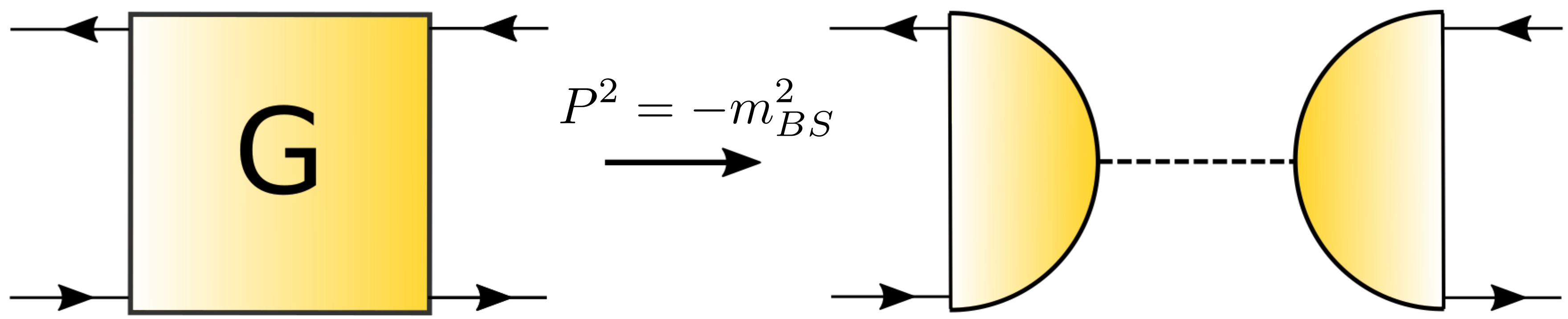

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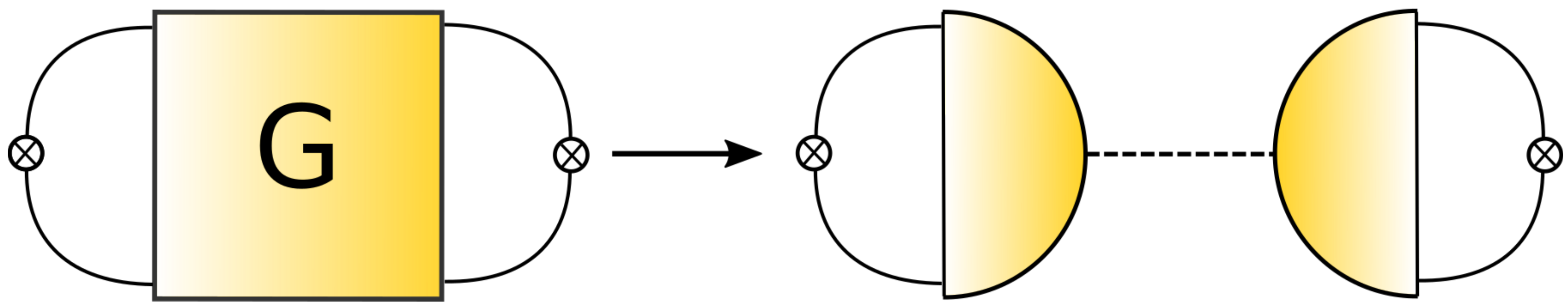
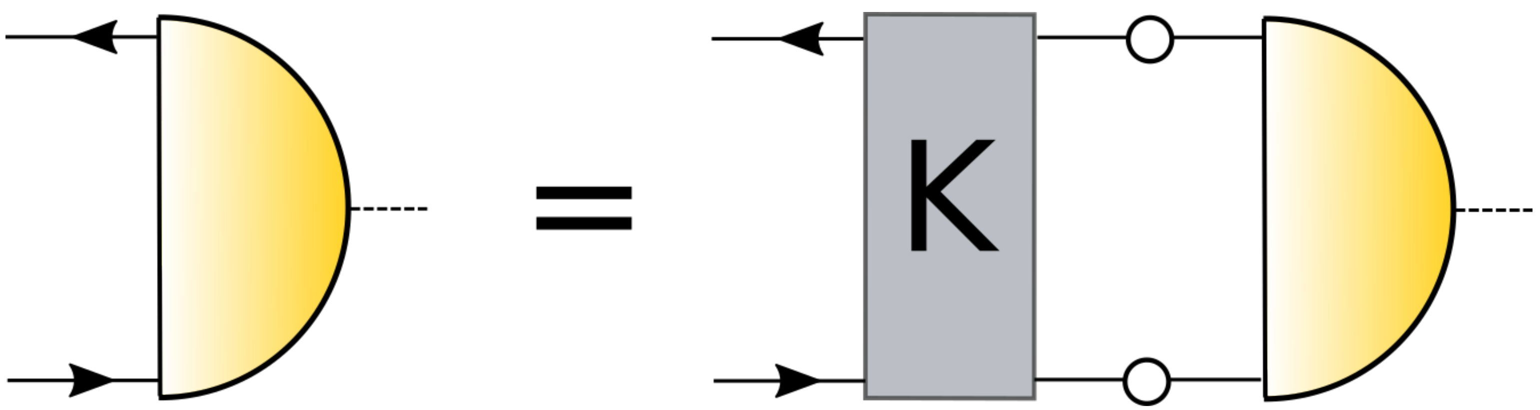
see lecture
of Daniel Mohler



exact BSE:

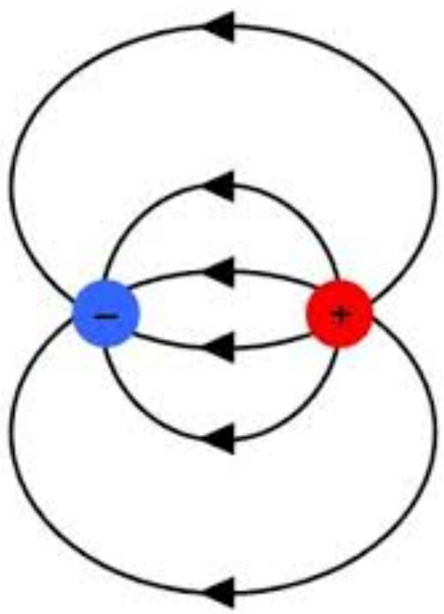


Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Confinement and dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
3. Bound states and applications (rainbow-ladder approximation)
   - Mesons
   - Baryons
   - Tetraquarks
4. Form factors and decays (rainbow-ladder approximation)
   - Meson and baryon form factors
   - The anomalous magnetic moment of the muon
5. Beyond rainbow ladder
   - Confinement and glueballs
   - Hadron results beyond rainbow-ladder

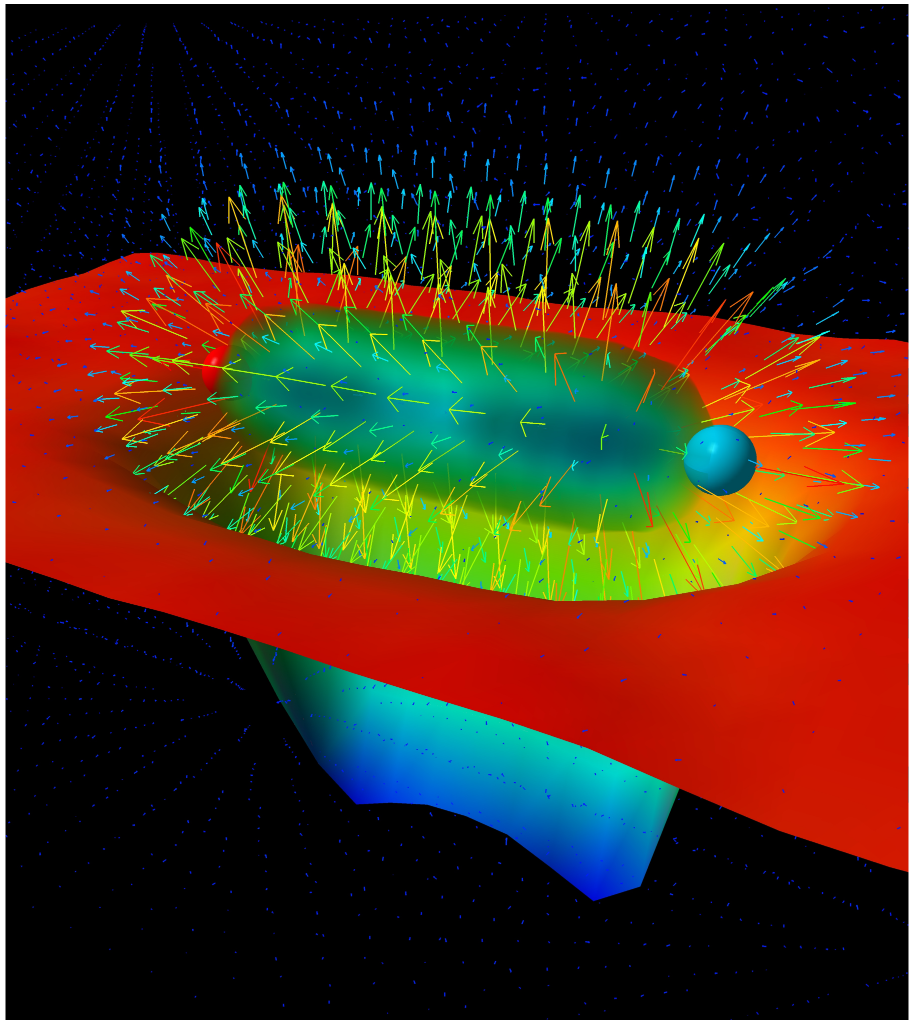
Confinement: flux tube

Electromagnetic force



$$F(r) \sim \frac{1}{r^2}$$

Strong force



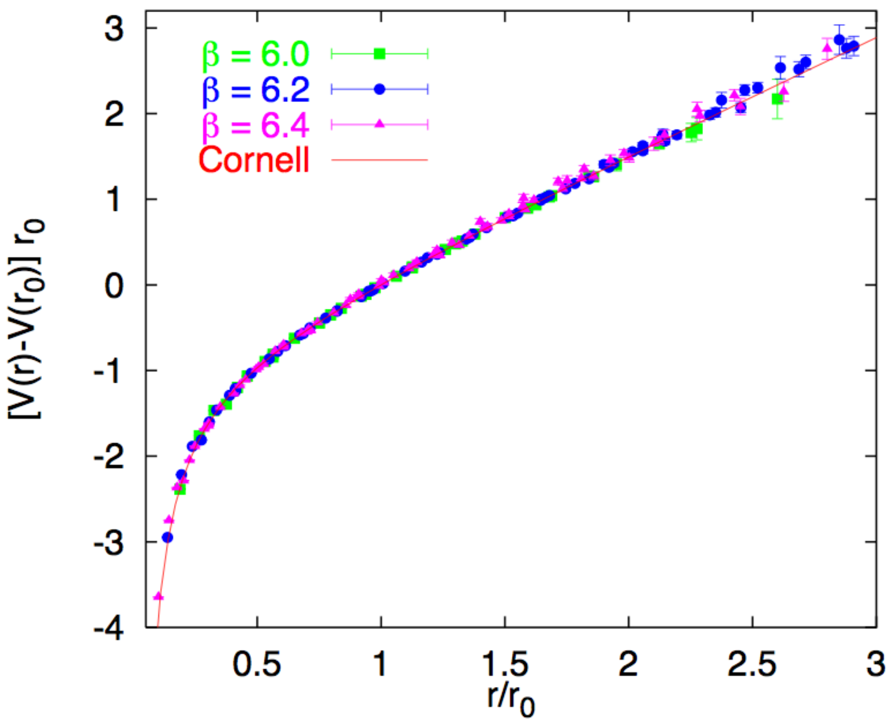
picture: D. Leinweber, Adelaide University

$$F(r) \sim const.$$

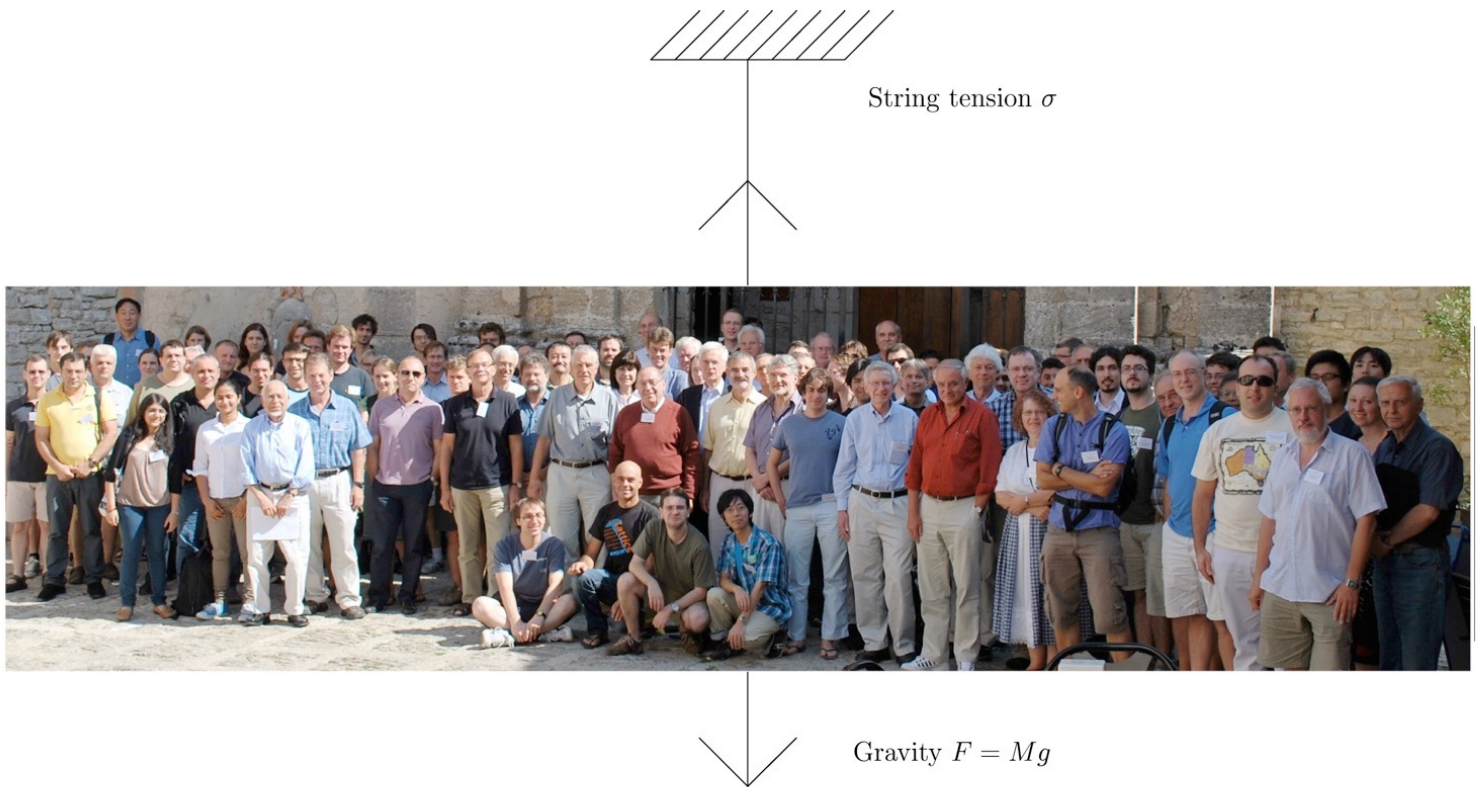
- Flux tube of force lines between two static quarks

Confinement: string tension

Yang-Mills theory with infinitely heavy test quarks:



Bali, Phys. Rept 343 (2001)



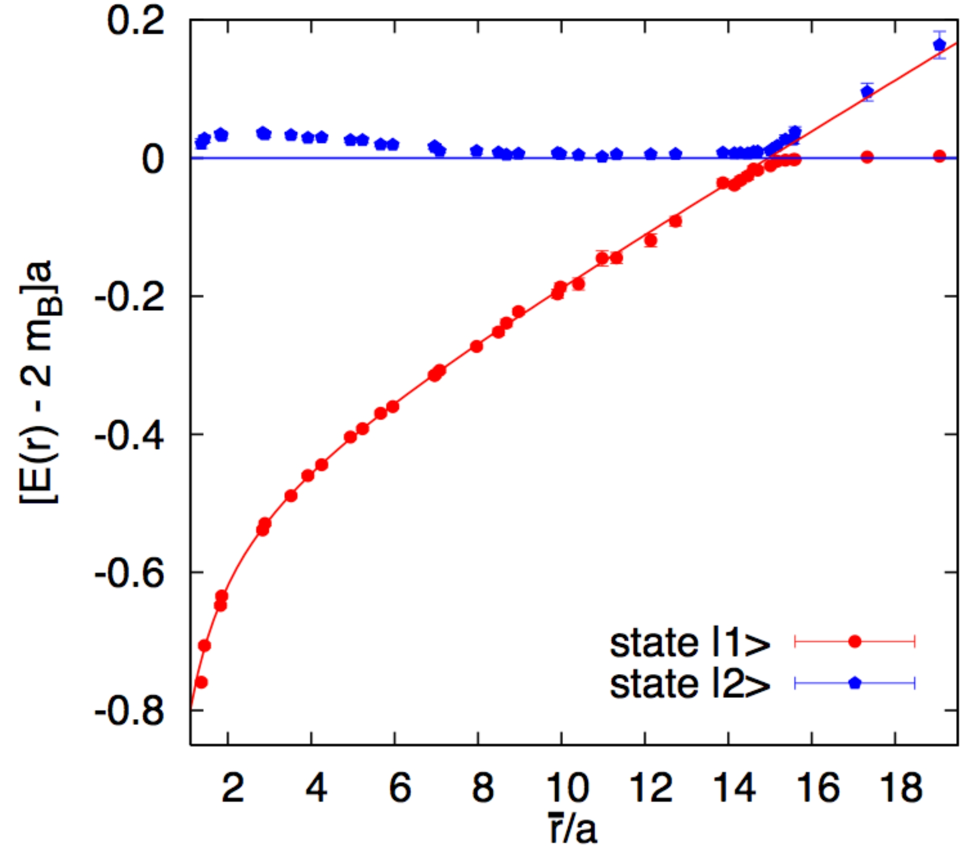
U.J.Wiese

$$E = L \int d^2 x_\perp \frac{1}{2} E_k^a(x) E_k^a(x) = L\sigma$$

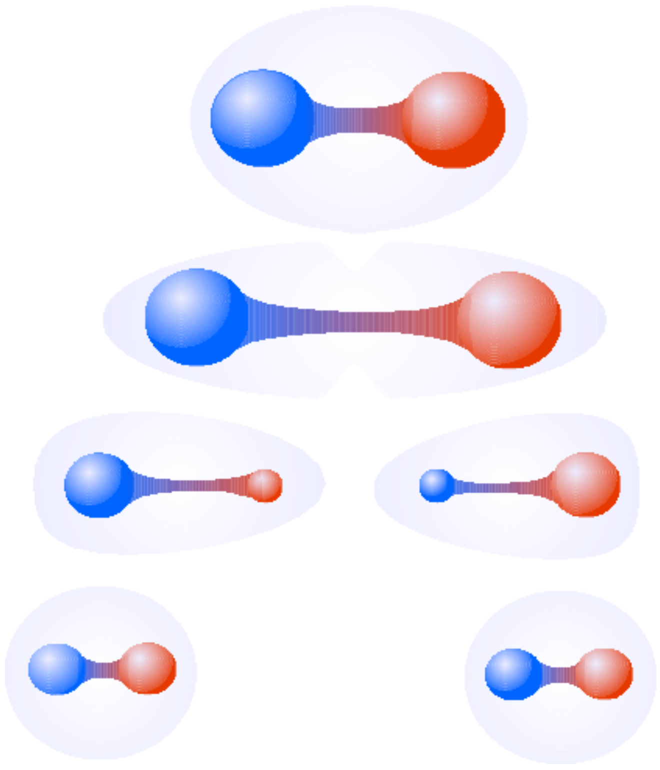
- String tension σ
- σ≈1cm thick steel cable
- isolate quark has infinite energy

Confinement: string breaking

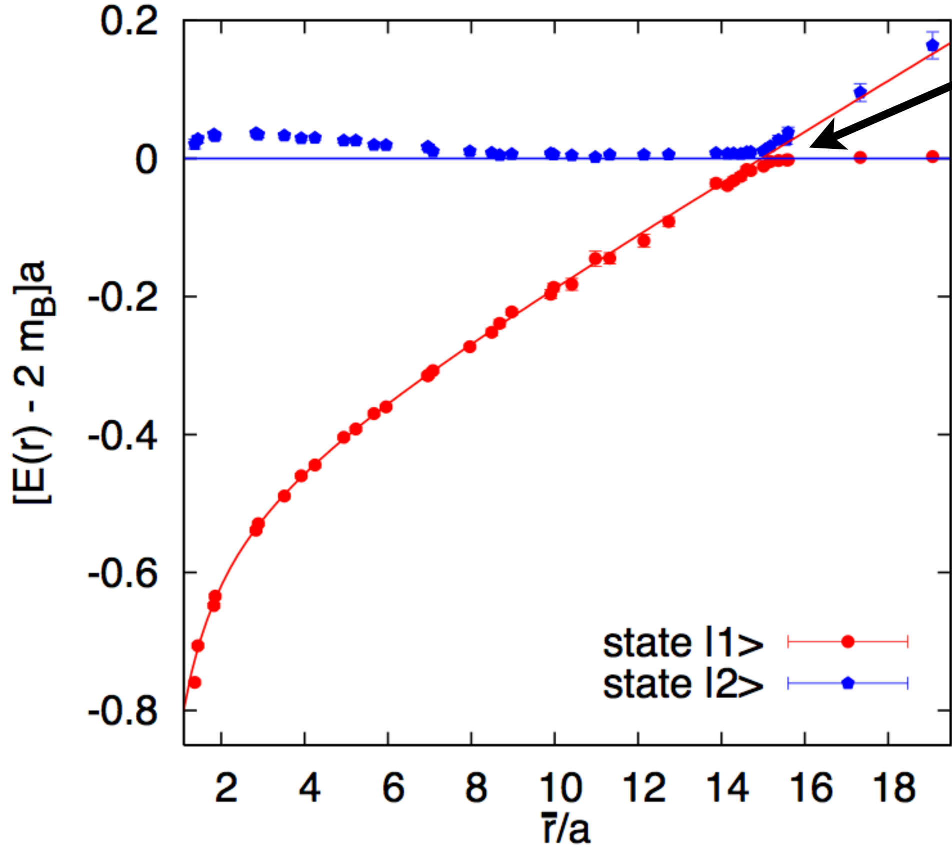
QCD:



Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513

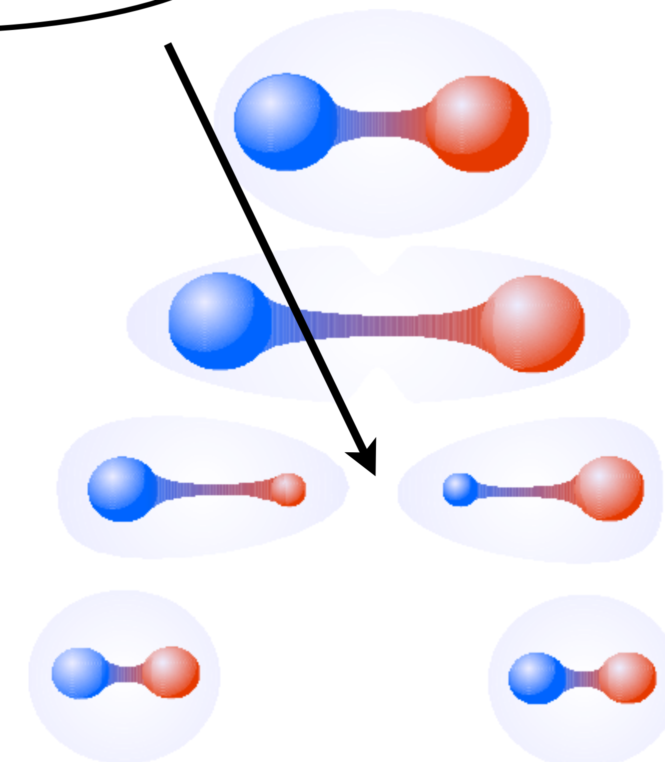


QCD:



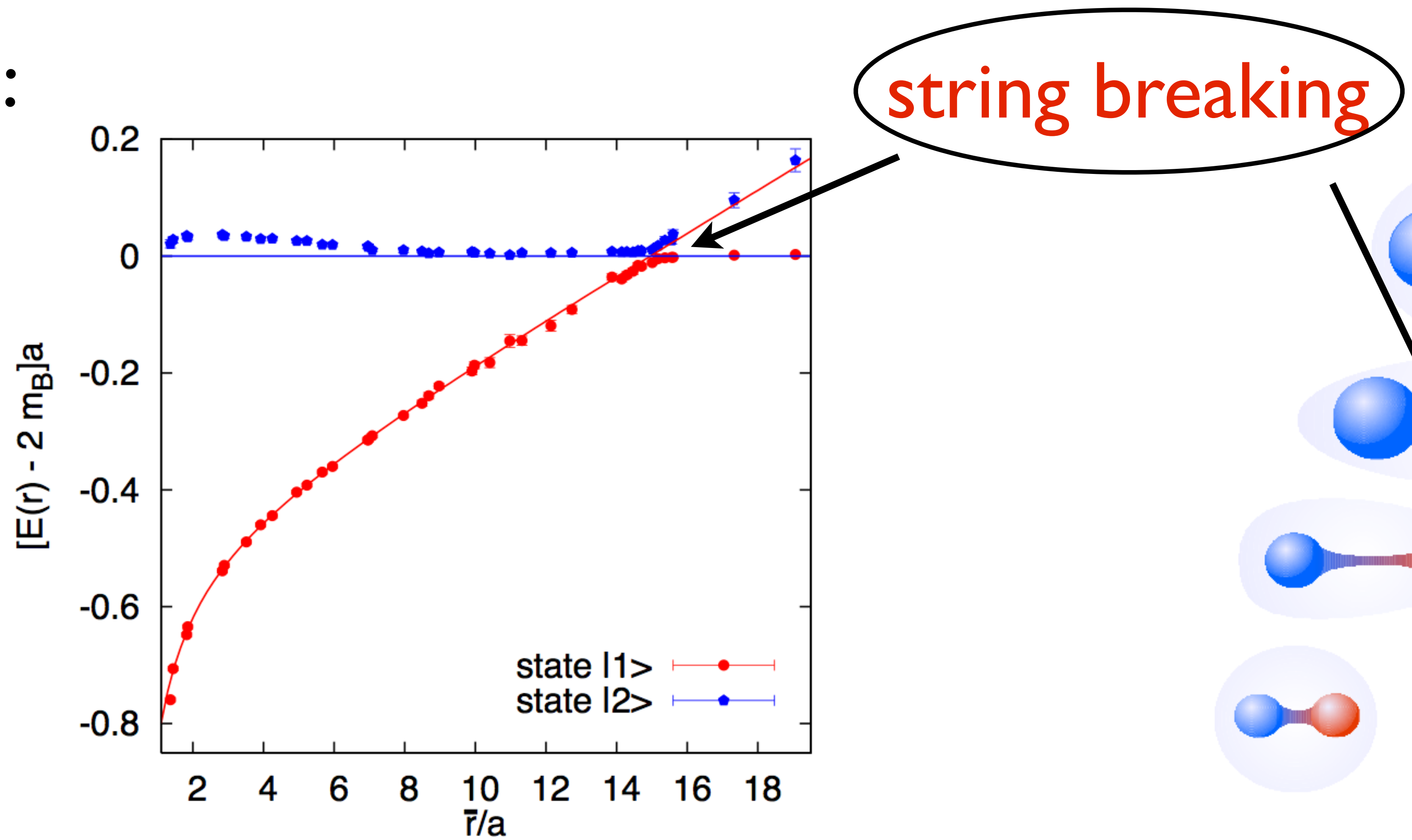
Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513

string breaking

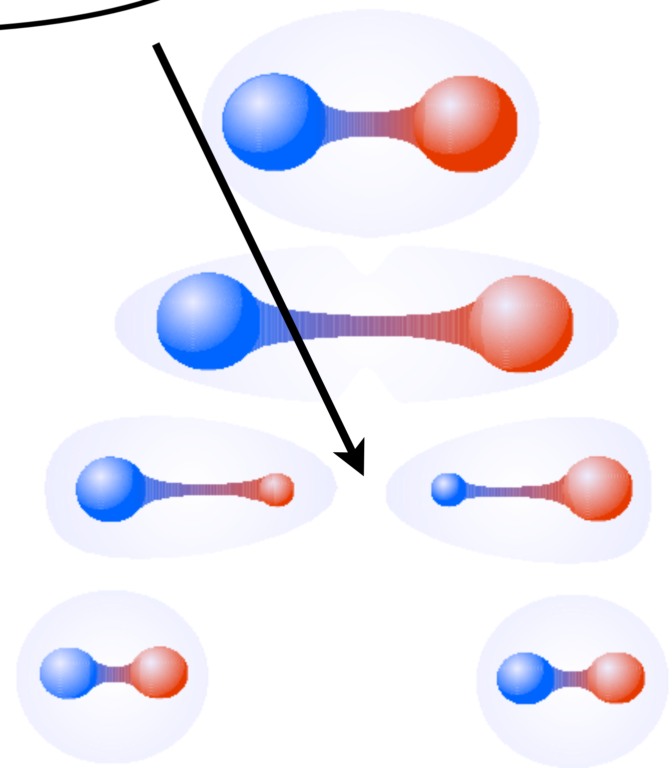


Confinement: string breaking

QCD:



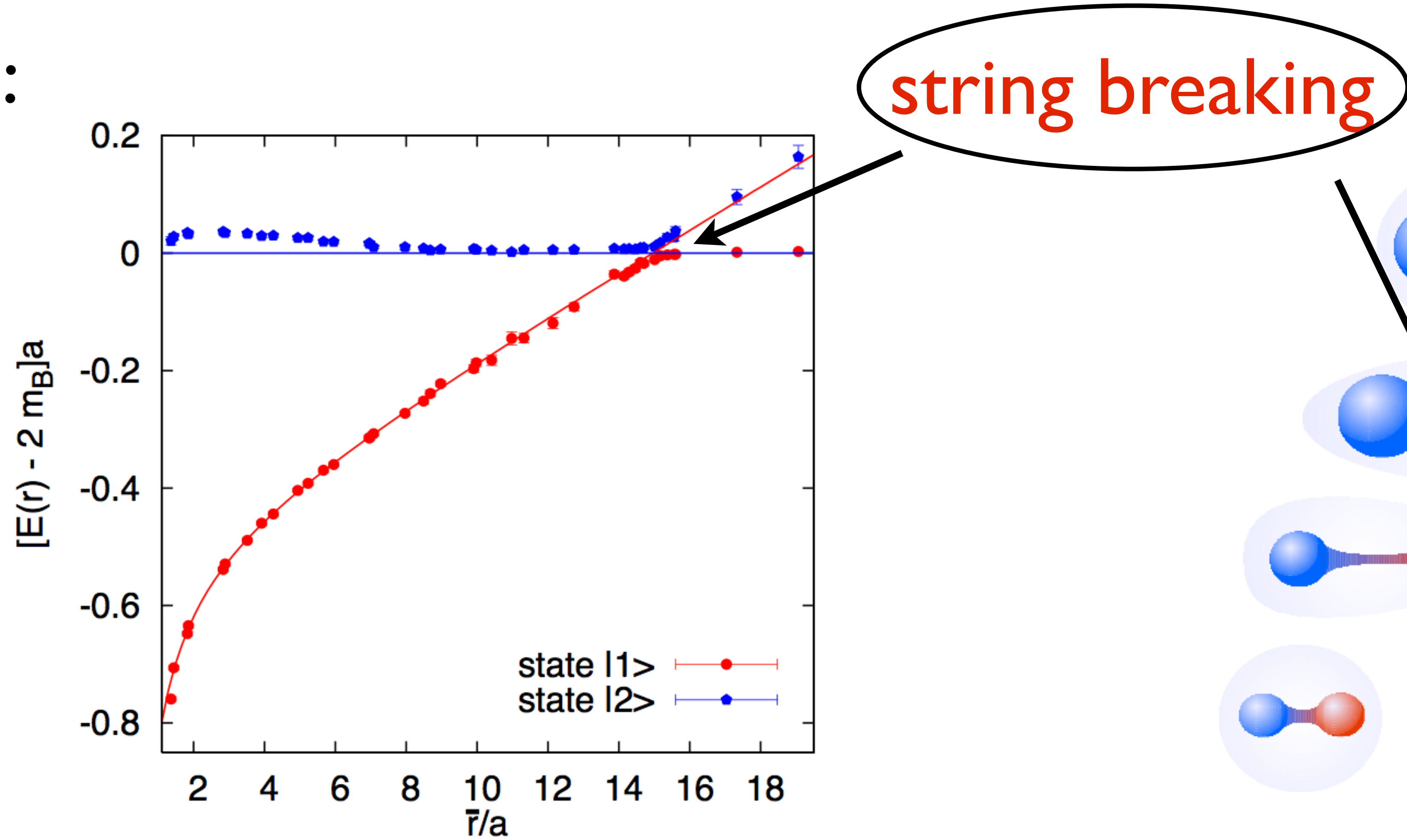
Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513



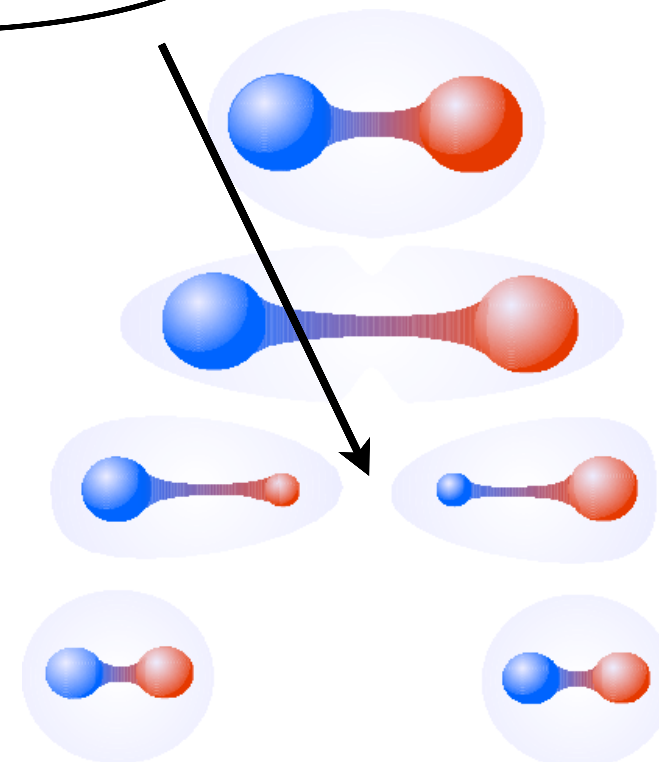
- String breaking by dynamical charges in fundamental representation of SU(N_c)
- Bound states do not see string breaking scale

Confinement: string breaking

QCD:



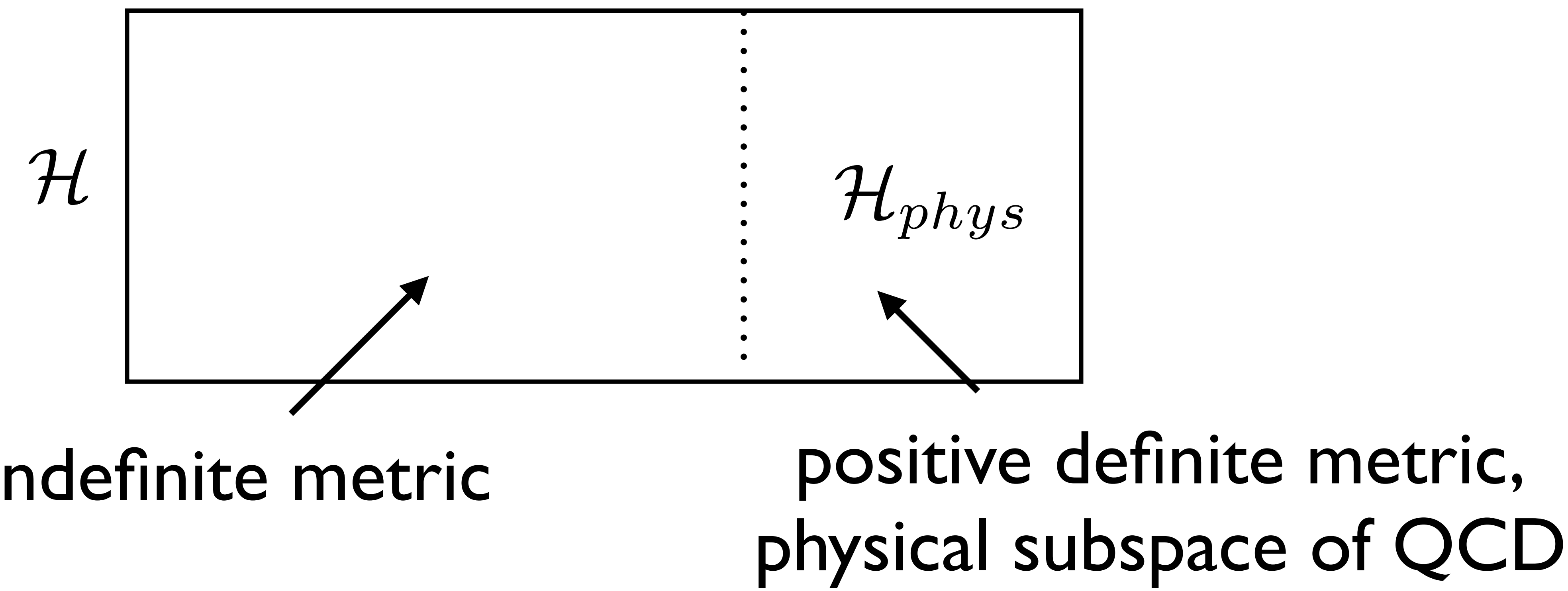
Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513



- String breaking by dynamical charges in fundamental representation of SU(N_c)
- Bound states do not see string breaking scale

provides some justification for quark model potential

Confinement and positivity violation



- State space of (gauge fixed) QCD: indefinite metric
- Physical particles need to live in $\mathcal{H}_{phys}$
- Particle that live not in $\mathcal{H}_{phys}$ are confined.

Axiomatic QFT (Osterwalder-Schrader):

physical particle $\longrightarrow$ $D(t, \mathbf{p}) \geq 0$

Motivation to look at propagators !

Properties of QCD: Dynamical mass generation

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
Nobel prize 2008

	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
M_{strong} $[MeV/c^2]$	350	350	350	350	350	350
M_{total} $[MeV/c^2]$	350	350	450	1500	4800	176000

$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

Motivation to look at propagators !

Properties of QCD: Dynamical mass generation

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
Nobel prize 2008

Input parameters in N_f=2+1 QCD

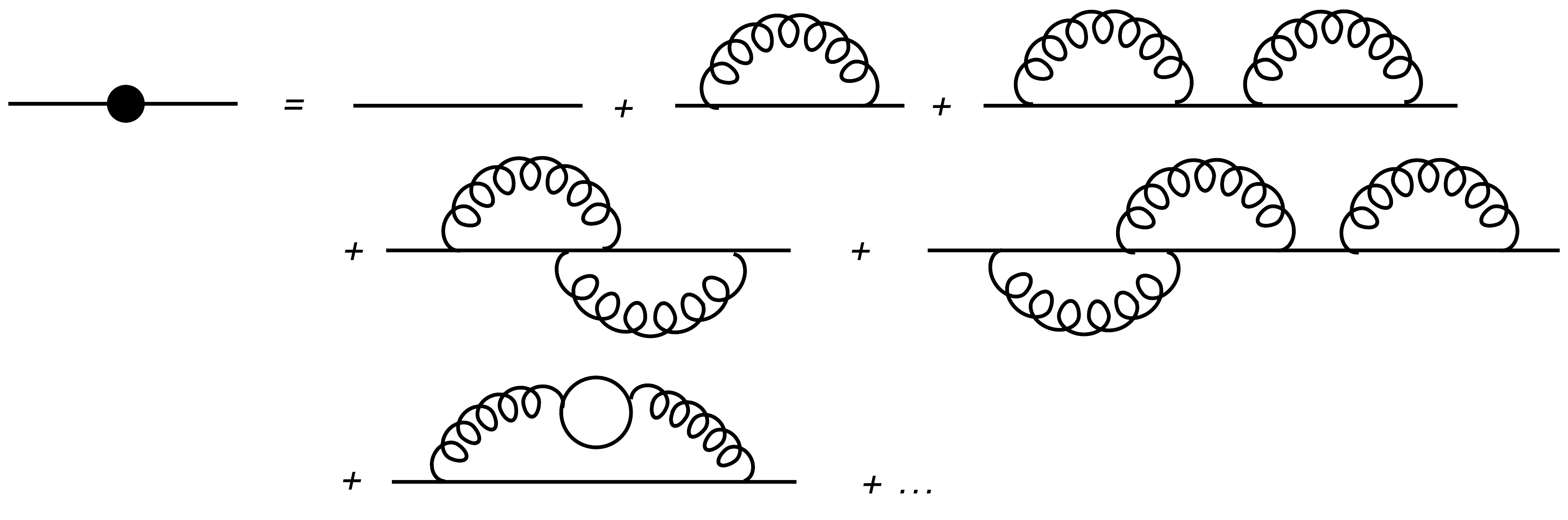
	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
M_{strong} $[MeV/c^2]$	350	350	350	350	350	350
M_{total} $[MeV/c^2]$	350	350	450	1500	4800	176000

$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

Motivation to look at propagators !

Derivation of DSEs I

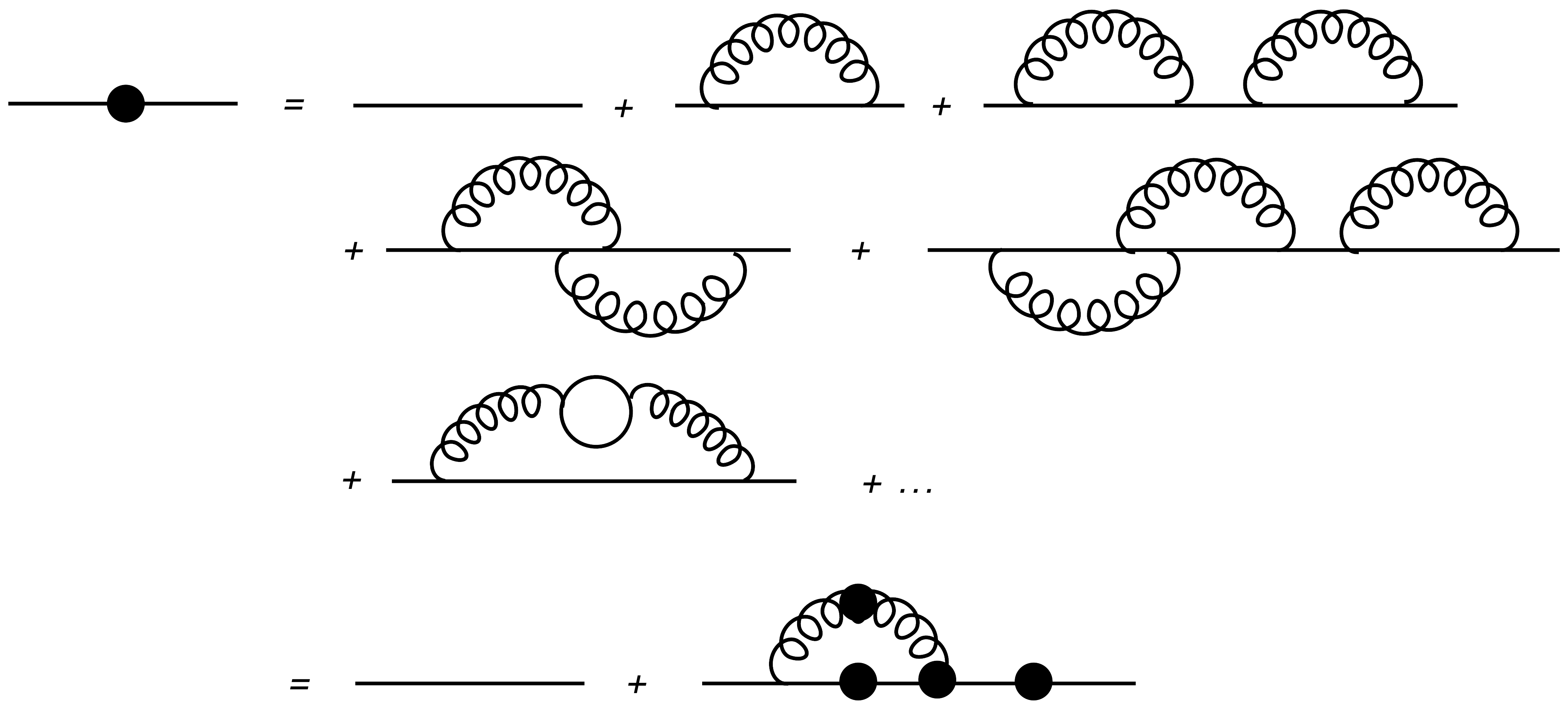
Graphical: start with perturbation theory and resum



$$S_0(p) = \frac{-i\not{p} + m}{p^2 + m^2} \quad \rightarrow \quad S(p) = \frac{-i\not{p}\,A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$

Derivation of DSEs I

Graphical: start with perturbation theory and resum



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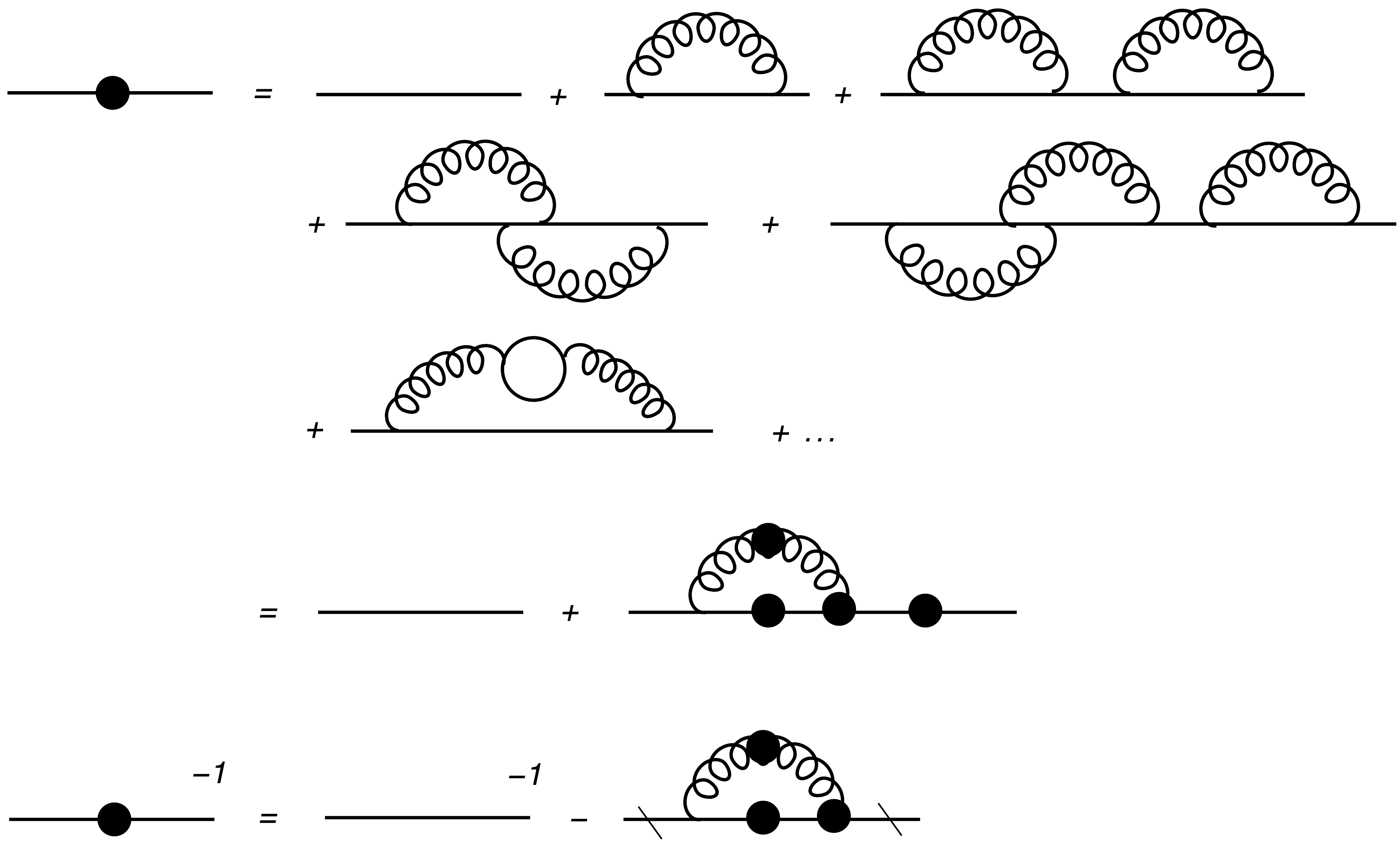
Graphical: start with perturbation theory and resum



$$S_0(p) = \frac{-i\not{p} + m}{p^2 + m^2} \quad \rightarrow \quad S(p) = \frac{-i\not{p}\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$

Derivation of DSEs II

Start from generating functional:

$$\mathcal{Z}[j] = \int \mathcal{D}[\Phi] \;\exp\left\{-S(\Phi) + j\Phi\right\} \qquad \text{with} \quad j\Phi = \int d^4x j(x)\Phi(x)$$

The integral of a total derivative vanishes:

$$0 = \frac{\delta}{\delta\Phi(y)}\mathcal{Z}[j] = \int \mathcal{D}[\Phi] \;\left(-\frac{\delta S(\Phi)}{\delta\Phi(y)} + j(y)\right)\; \exp\left\{-S(\Phi) + j\Phi\right\}$$

$$= \left\langle -\frac{\delta S(\Phi)}{\delta\Phi(y)} + j(y) \right\rangle$$

After a further derivative we set j=0 and obtain the DSE for the propagator:

$$0 = \frac{\delta^2}{\delta j(z)\delta\Phi(y)}\mathcal{Z}[j] = \left\langle -\frac{\delta S(\Phi)}{\delta\Phi(y)}\Phi(z) \right\rangle + \delta(y-z)\mathcal{Z}[0]$$

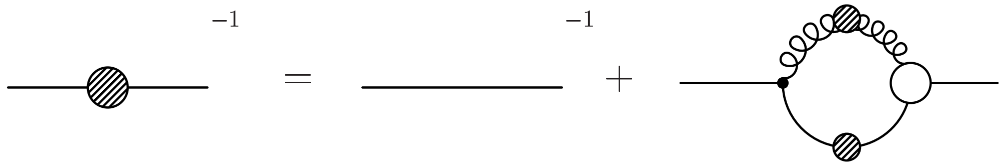
The quark DSE

For the DSE of the quark propagator we obtain:

$$S^{-1}(p) = S_0^{-1}(p) + g^2 \int \frac{d^4q}{(2\pi)^4}\, t^a \gamma_\mu \, S(q)\, D^{ab}_{\mu\nu}(q-p)\, \Gamma^b_\nu(q,p)$$

- Tower of DSEs for Euclidean n-point functions
- Similar tower from functional renormalization group (FRG): different structure but similar content !

FRG: H. Gies,``Introduction to the functional RG and applications to gauge theories,'' hep-ph/0611146.
J.M.Pawlowski,``Aspects of the functional renormalisation group,'' Annals Phys. 322 (2007) 2831 [hep-th/0512261].



Simple example:

Take bare gluon propagator: $D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}$

and bare quark-gluon-vertex: $\Gamma_\mu(p,q) = i\gamma_\mu$

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F \int \frac{d^4q}{(2\pi)^4}\, \gamma_\mu D_{\mu\nu}(k)\, S(q)\, \gamma_\nu$$

with $C_F = \frac{N_c^2 - 1}{2N_c} \rightarrow \frac{4}{3}$

$$S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$$

$$S_0^{-1}(p) = i\not{p} + m \qquad \rightarrow \text{project onto Dirac structures}$$

$$B(p^2) = m + g^2 \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k^2} \frac{3B(q^2)}{q^2A^2(q^2) + B^2(q^2)}$$

$$A(p^2) = 1 + g^2 \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k^2} \frac{A(q^2)}{q^2A^2(q^2) + B^2(q^2)} \left[-\frac{k^2}{p^2} + \frac{p^2+q^2}{2p^2} + \frac{(p^2-q^2)^2}{2p^2k^2} \right]$$

Dynamical chiral symmetry breaking II

In our simple example $A \approx 1$, then:

$$B(p^2) = m + g^2 \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k^2} \frac{3B(q^2)}{q^2 + B^2(q^2)}$$

Transform $\int d^4q$ in hyperspherical coordinates and perform angular integrals analytically ($\alpha = g^2/4\pi$):

$$B(p^2) = m + \alpha \int_0^{p^2} dq^2 \frac{q^2}{p^2} \frac{B(q^2)}{q^2 + B^2(q^2)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \frac{B(q^2)}{q^2 + B^2(q^2)}$$

This equation for the quark mass function $M(p^2) = B(p^2)/A(p^2)$ has a typical structure.

$\longrightarrow$

Dynamical chiral symmetry breaking III

Consider chiral limit $m=0$:

$$B(p) = \alpha \int_0^{p^2} dq^2 \, \frac{q^2}{p^2} \, \frac{B(q)}{q^2+B^2(q)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \, \frac{B(q)}{q^2+B^2(q)}$$

Dynamical chiral symmetry breaking III

Consider chiral limit $m=0$:

$$B(p) = \alpha \int_0^{p^2} dq^2 \, \frac{q^2}{p^2} \, \frac{B(q)}{q^2+B^2(q)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \, \frac{B(q)}{q^2+B^2(q)}$$

Three solutions:

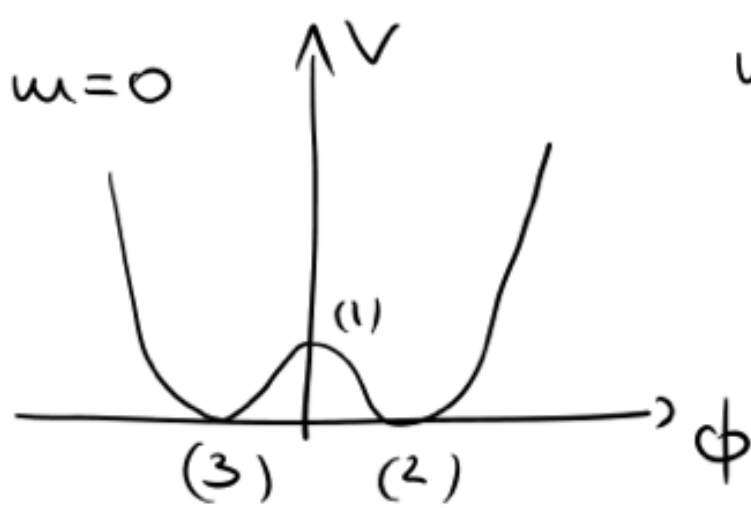
(1) $B(p) \equiv 0$ $\longrightarrow$ chiral symmetric: Wigner-Weyl

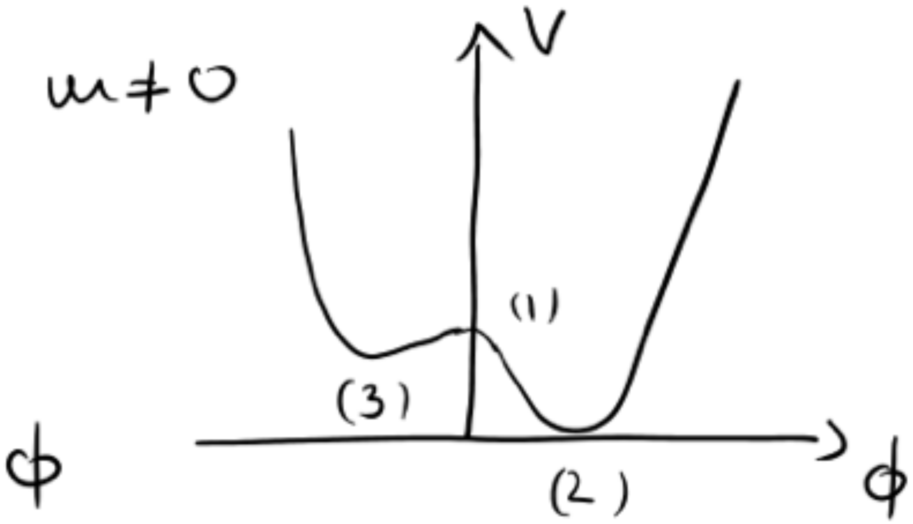
(2,3) $\pm B(p) \neq 0$ $\longrightarrow$ chiral symmetry broken: Nambu-Goldstone

cp. to effective potential in scalar models: see lecture of Pelaez

(1) metastable

(2,3) stable





Hadron physics with functional methods

Christian S. Fischer

Justus Liebig Universität Gießen

Lecture 2

Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better

2. Properties of QCD and correlation functions
   - Confinement and dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…

3. Bound states and applications (rainbow-ladder approximation)
   - Mesons
   - Baryons
   - Tetraquarks

4. Form factors and decays (rainbow-ladder approximation)
   - Meson and baryon form factors
   - The anomalous magnetic moment of the muon

5. Beyond rainbow ladder
   - Confinement and glueballs
   - Hadron results beyond rainbow-ladder

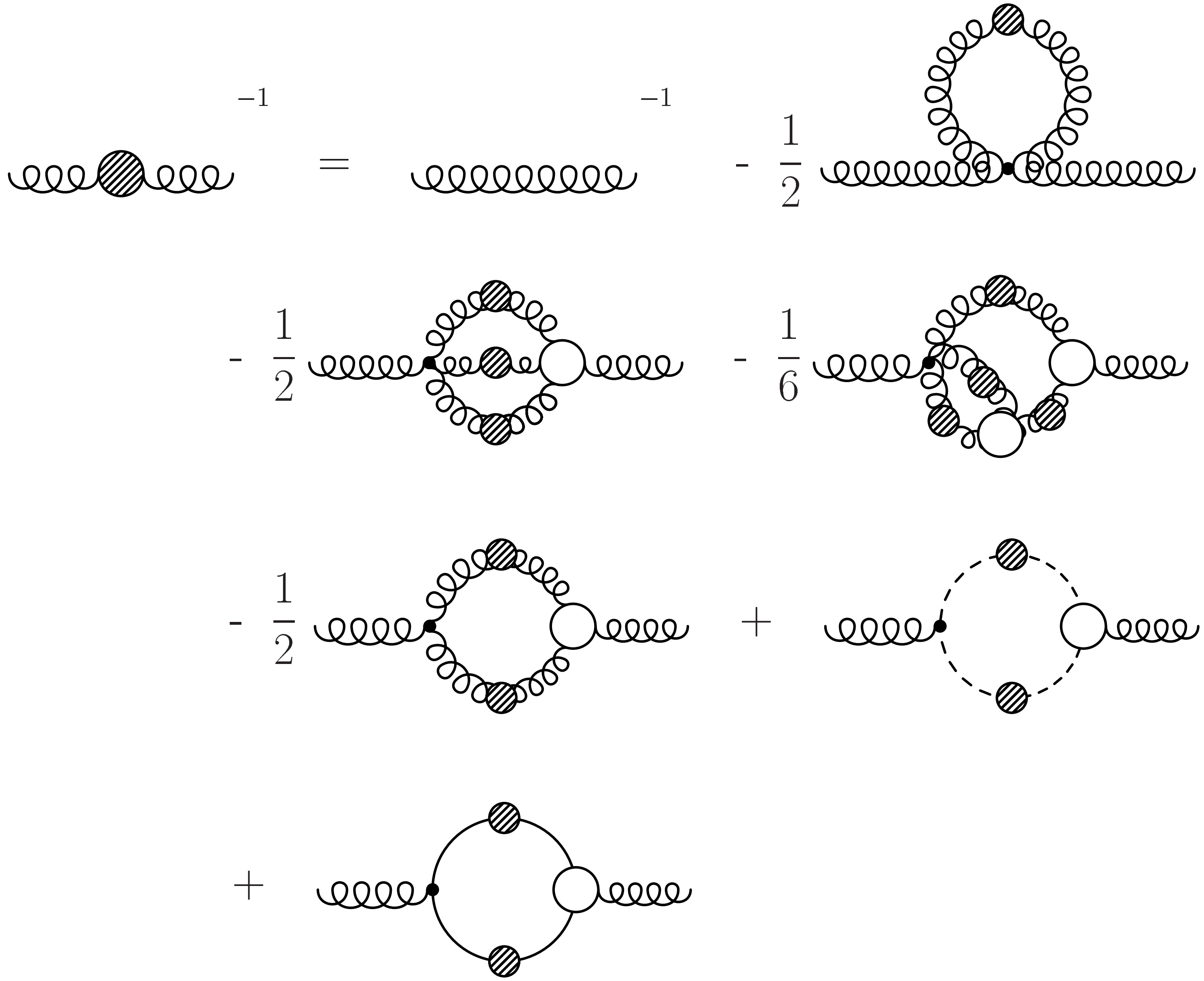
Bits and pieces to remember from Lecture 1

- non-relativistic quark model
  - source for classification of 'exotic quantum numbers'
    —> absent in quark-model but possible in relativistic theory
  - works with non-relativistic structure for forces (+rel. corr.)
    —> cp with exp. spectrum: LS dominates over SS

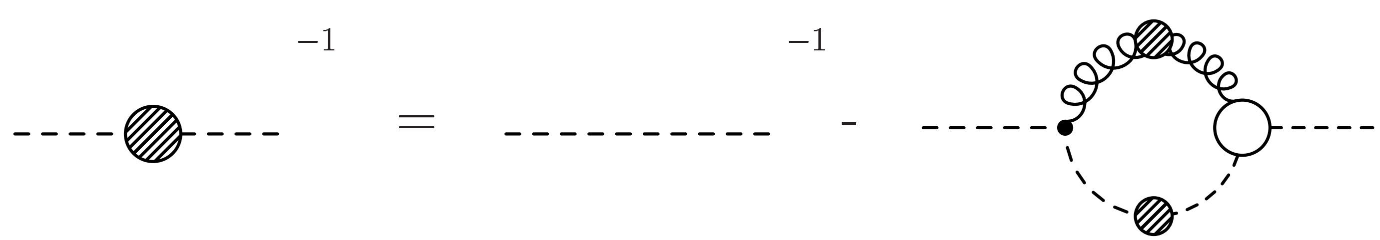
- functional methods: DSEs and BSEs (and FRGs)
  - derived exactly from QCD path integral
  - quark-DSE displays mechanism for dynamical mass generation
    —> already visible at simplest possible approximation
    —> not present in perturbation theory
    —> important part of dynamical chiral symmetry breaking

DSEs: Quark and gluon propagators

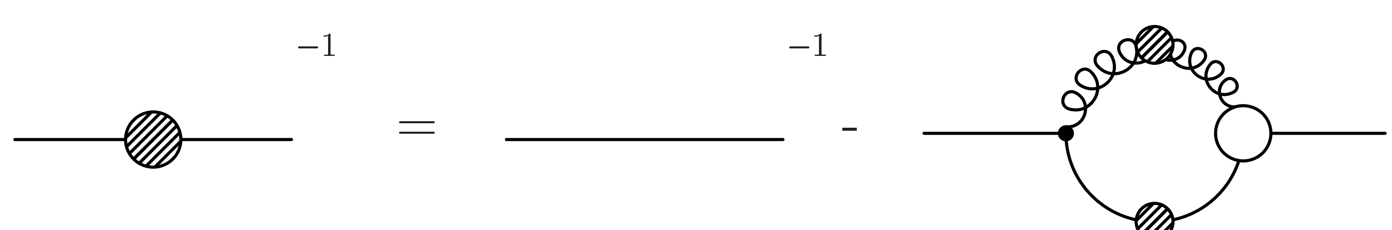
gluon:



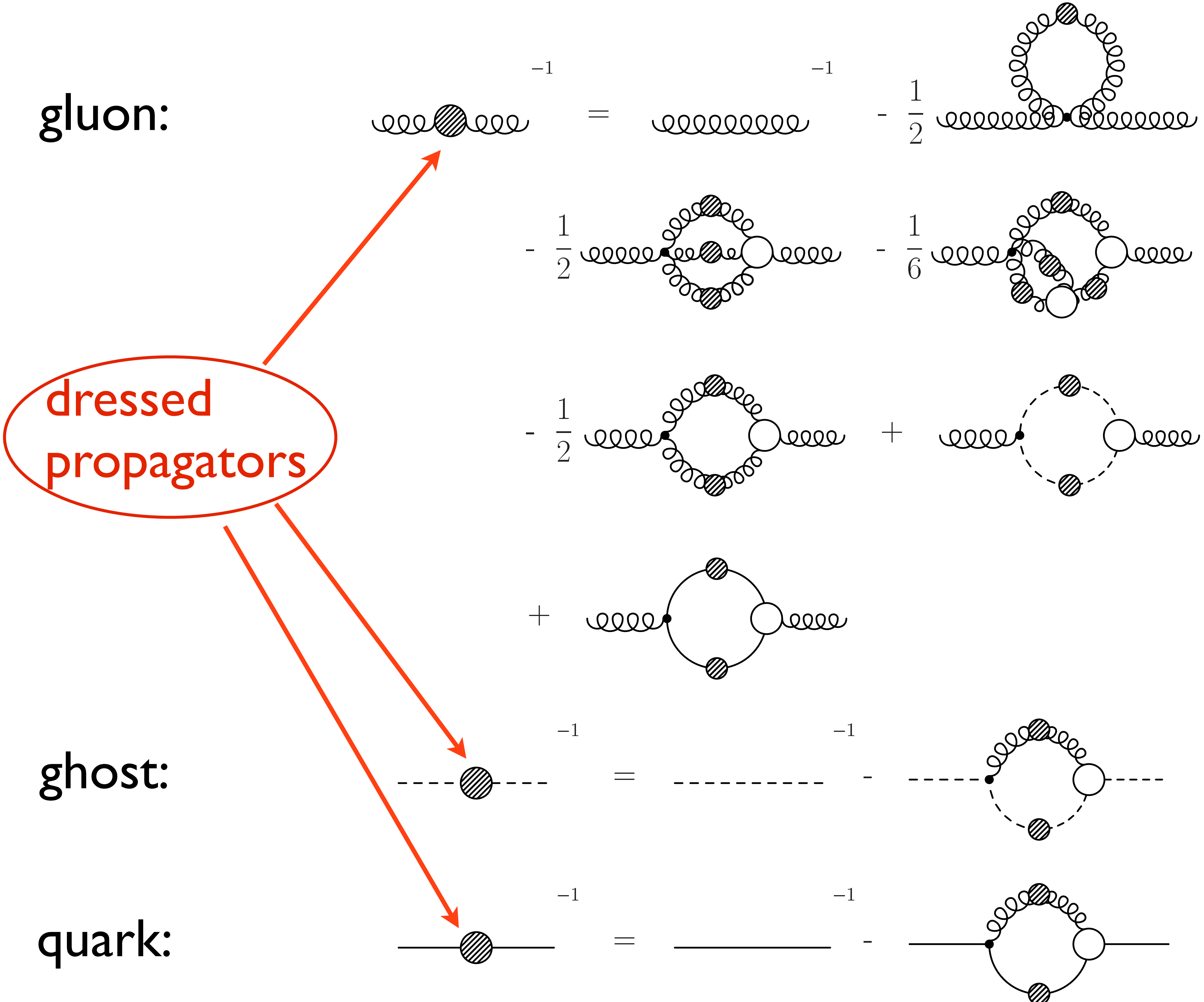
ghost:



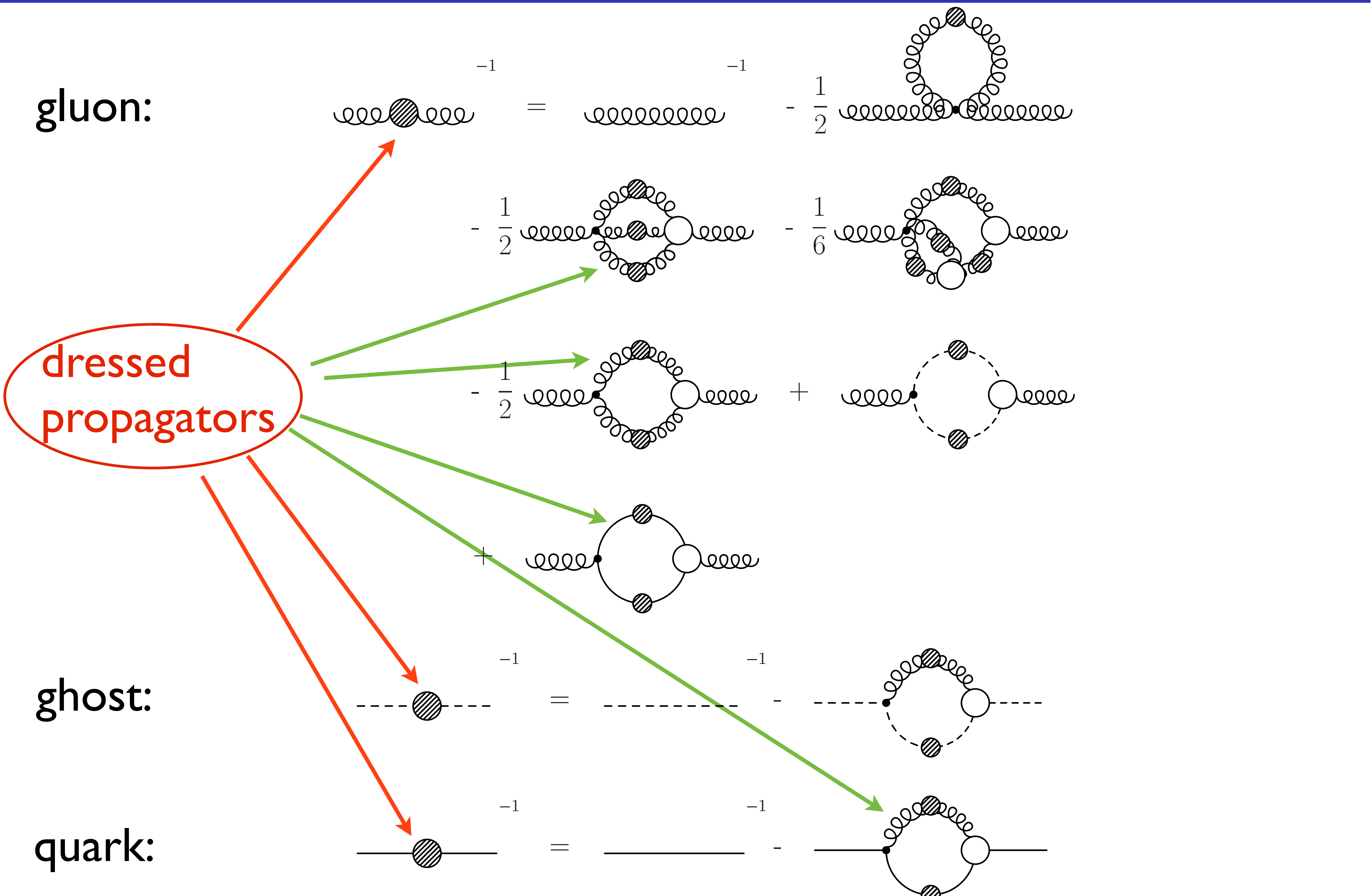
quark:



DSEs: Quark and gluon propagators



DSEs: Quark and gluon propagators

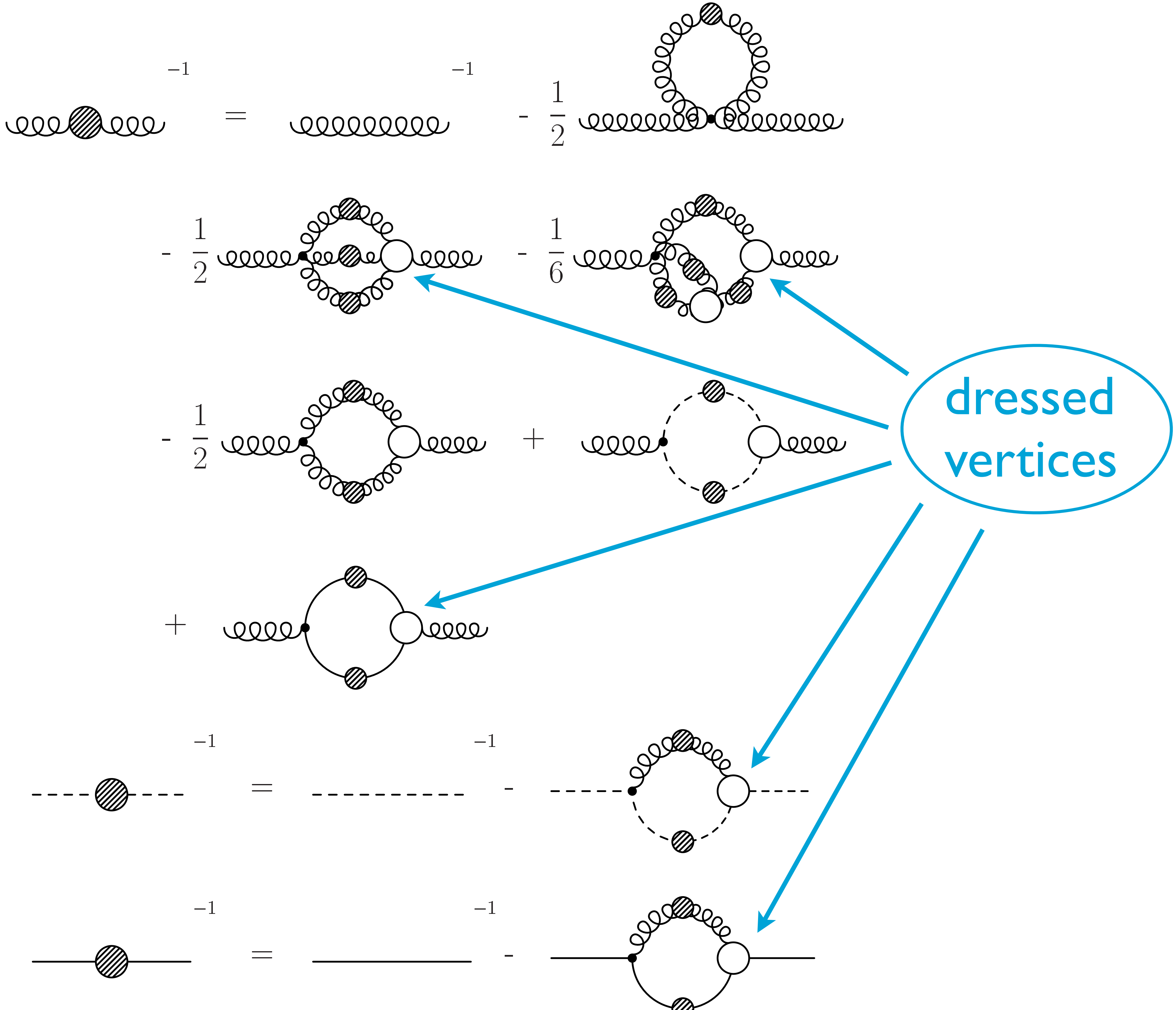


DSEs: Quark and gluon propagators

gluon:

ghost:

quark:



The DSE for the quark propagator

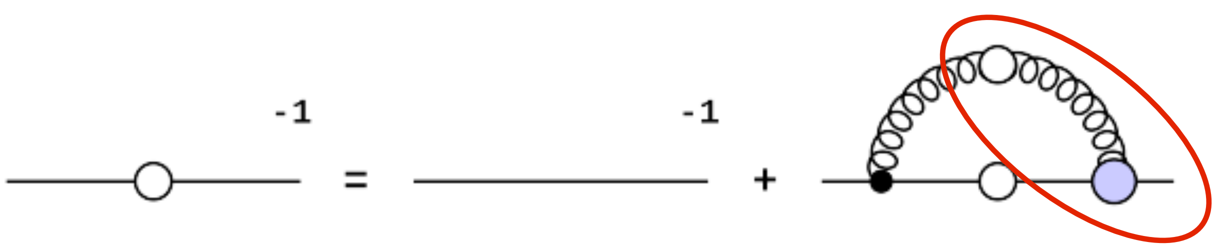
Approximations:

I) NJL/contact model:

II) Quark-diquark model:

Ansatz for quark prop
(and diquark wave function)

III) Rainbow-ladder:

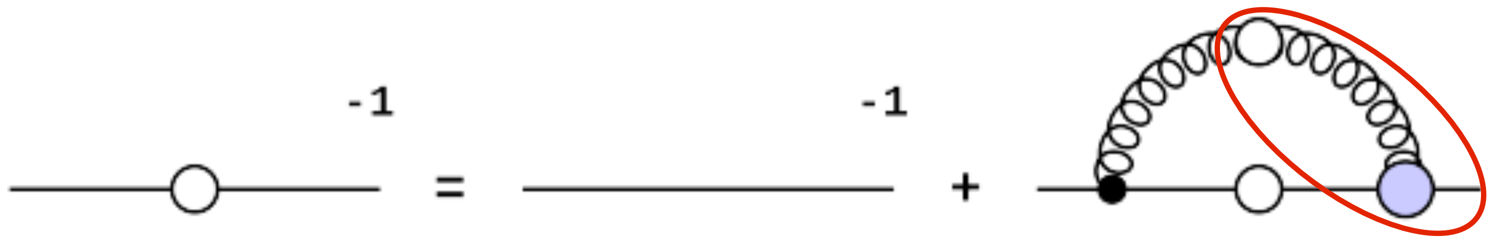


IV) Beyond rainbow-ladder:

CF and Alkofer, PRD 67 (2003) 094020
Williams, EPJA 51 (2015) 5, 57.
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD 93 (2016) 034026, and refs. therein

- solve DSEs for quarks, gluons and quark-gluon vertex

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

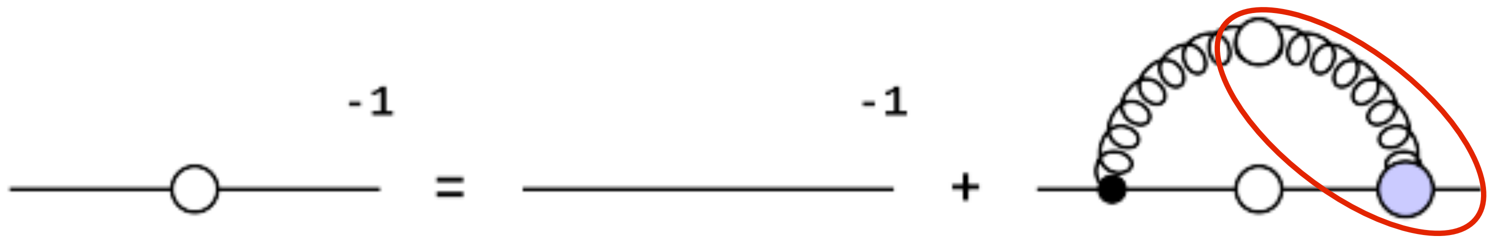
$$\sim \gamma^\mu \tau(k^2)$$

"approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction



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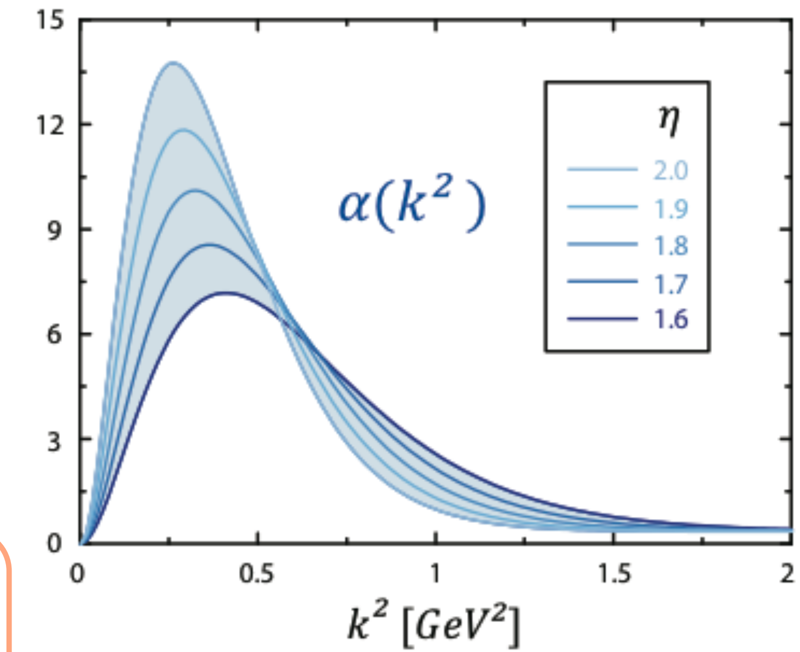


Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

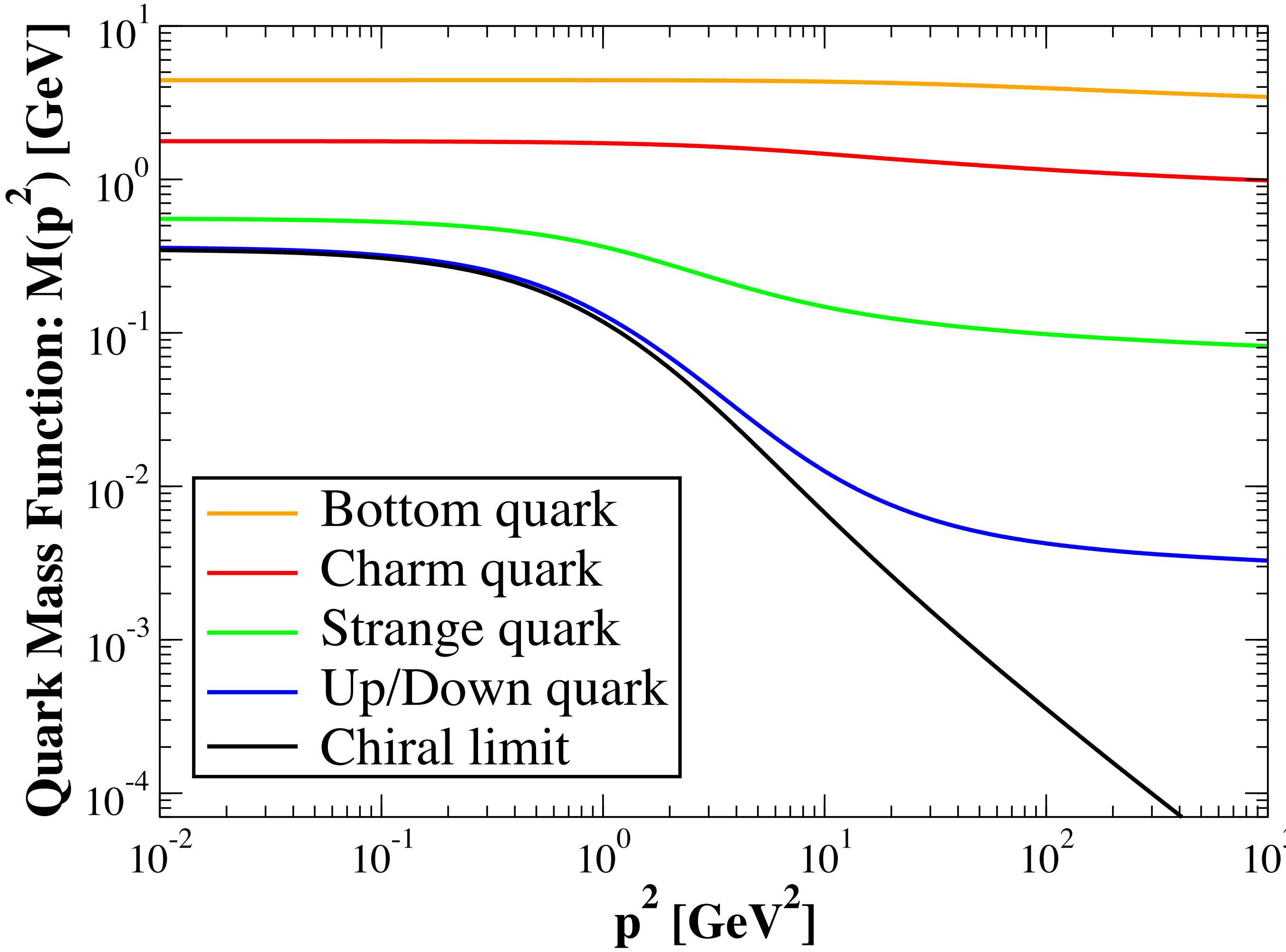
- scale Λ from f_π, masses $m_u = m_d$, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Quark mass: flavor dependence

$$S(p) = \frac{-i\not{p}\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$
$$= Z_f(p^2)\, \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

Typical solution:



- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: - $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

$$-\langle \bar{\Psi}\Psi \rangle = Z_2 Z_m N_c \int_p Tr S(p)$$

Quark mass: flavor dependence

Typical solution:

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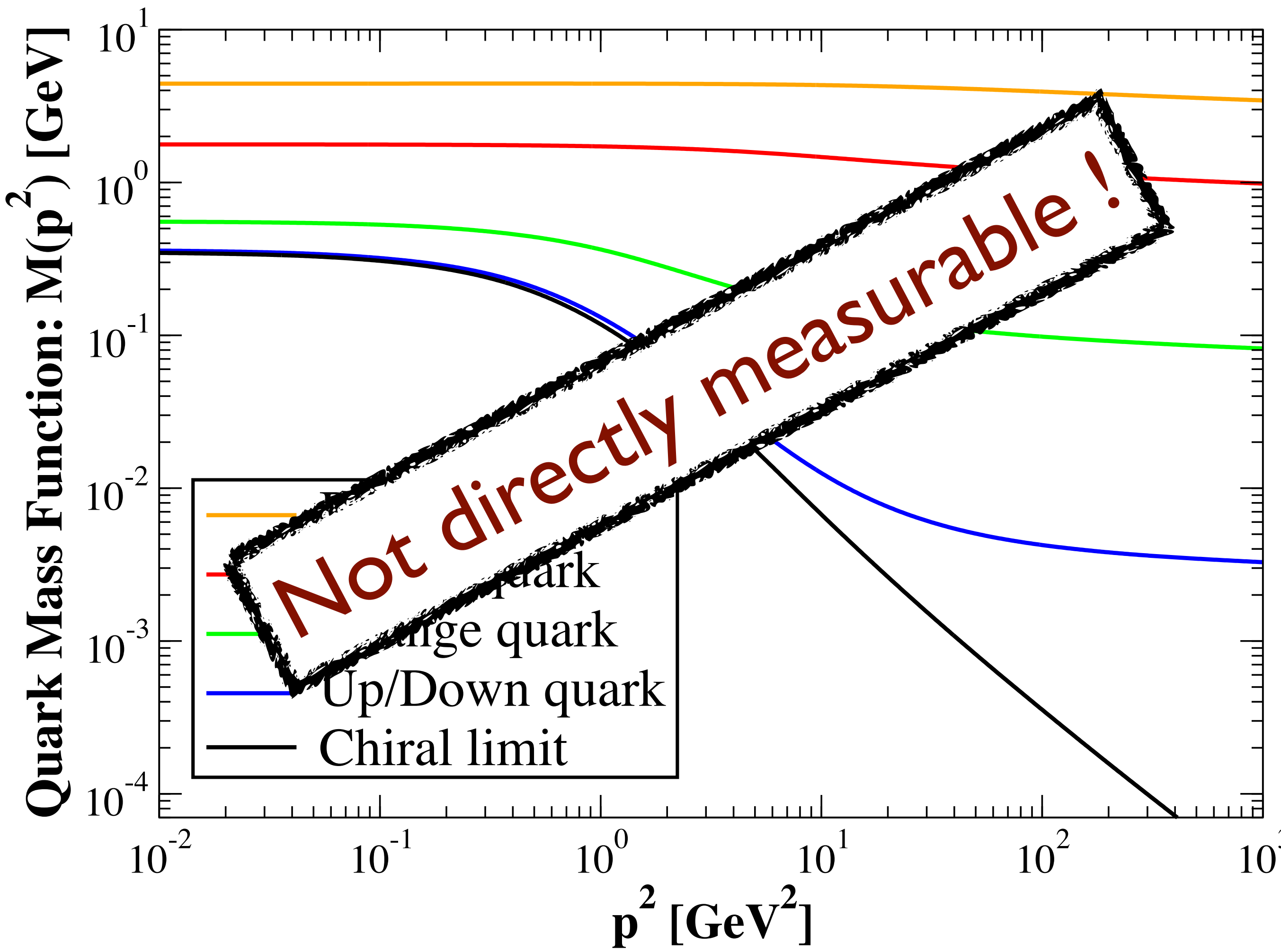
$$= Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$



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- Dynamical mass: $M_{strong} \approx 350$ MeV
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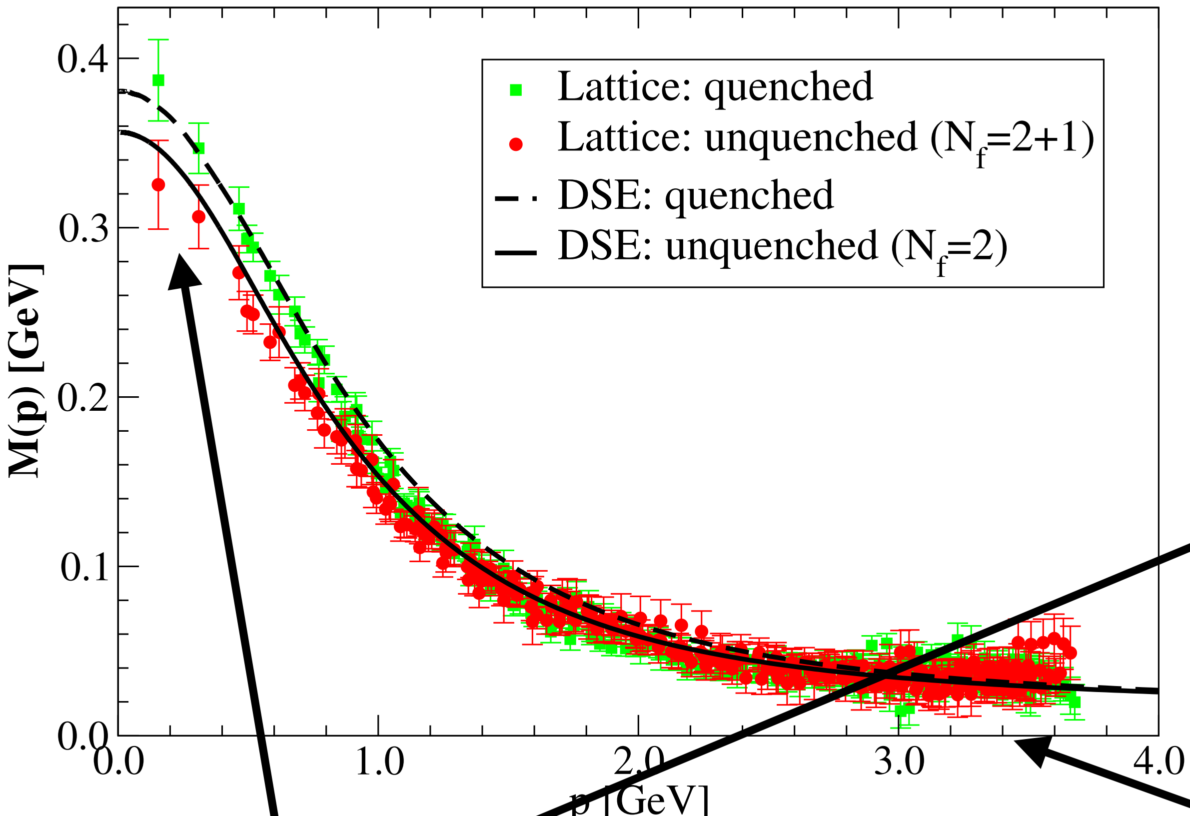
$$-\langle \bar{\Psi}\Psi \rangle = Z_2 Z_m N_c \int_p Tr S(p)$$

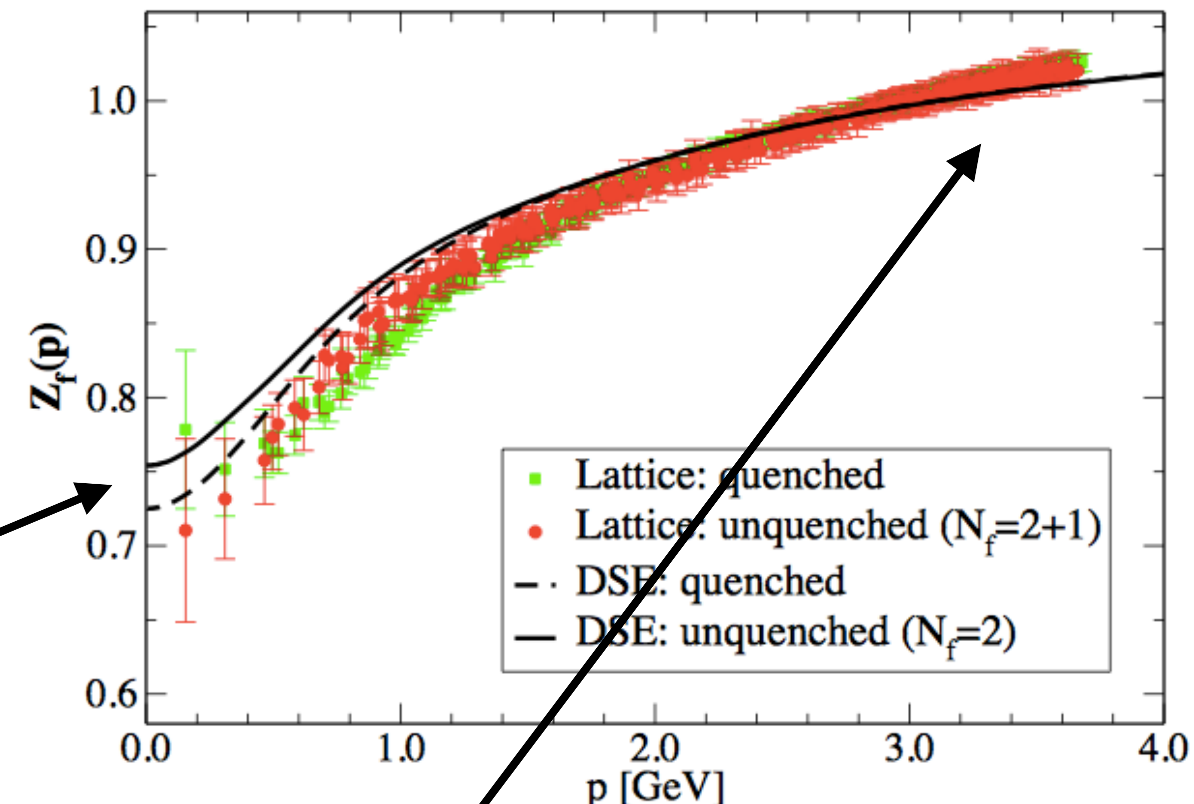
Quark dressing - comparison with lattice

Beyond rainbow-ladder (details see tomorrow):

$$S(p) = Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





'constituent quark':
large mass; very composite

'current quark':
- small mass; non-composite

cf talk of Guo

Chiral symmetry I

Noether Theorem:

Consider field Ψ with $\mathcal{L}(\Psi, \partial\Psi)$ and unitary transformation with generators λ^a:

$$\Psi \to \exp(-i\,\Theta_a \lambda^a)\Psi \approx \Psi + i\Theta_a \lambda^a \Psi$$

then we find a conserved current with

$$\partial_\mu J^a_\mu(\mathbf{x}) = 0 \quad \text{with} \quad J^a_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} \lambda^a \Psi$$

For conserved currents, the related charge

$$Q^a = \int d^3x J^a_0(\mathbf{x})$$

is conserved if

$$\frac{\partial Q^a}{\partial t} = \int d^3x \frac{\partial J^a_0}{\partial t} = -\int d^3x \nabla \mathbf{J}^a = 0$$

If $\mathbf{J}^a$ does not vanish at infinity we say that the corresponding symmetry is broken and one can show that there are associated massless bosons, the **Goldstone bosons**. proof —> see later

Chiral symmetry II - QCD with N_f=3

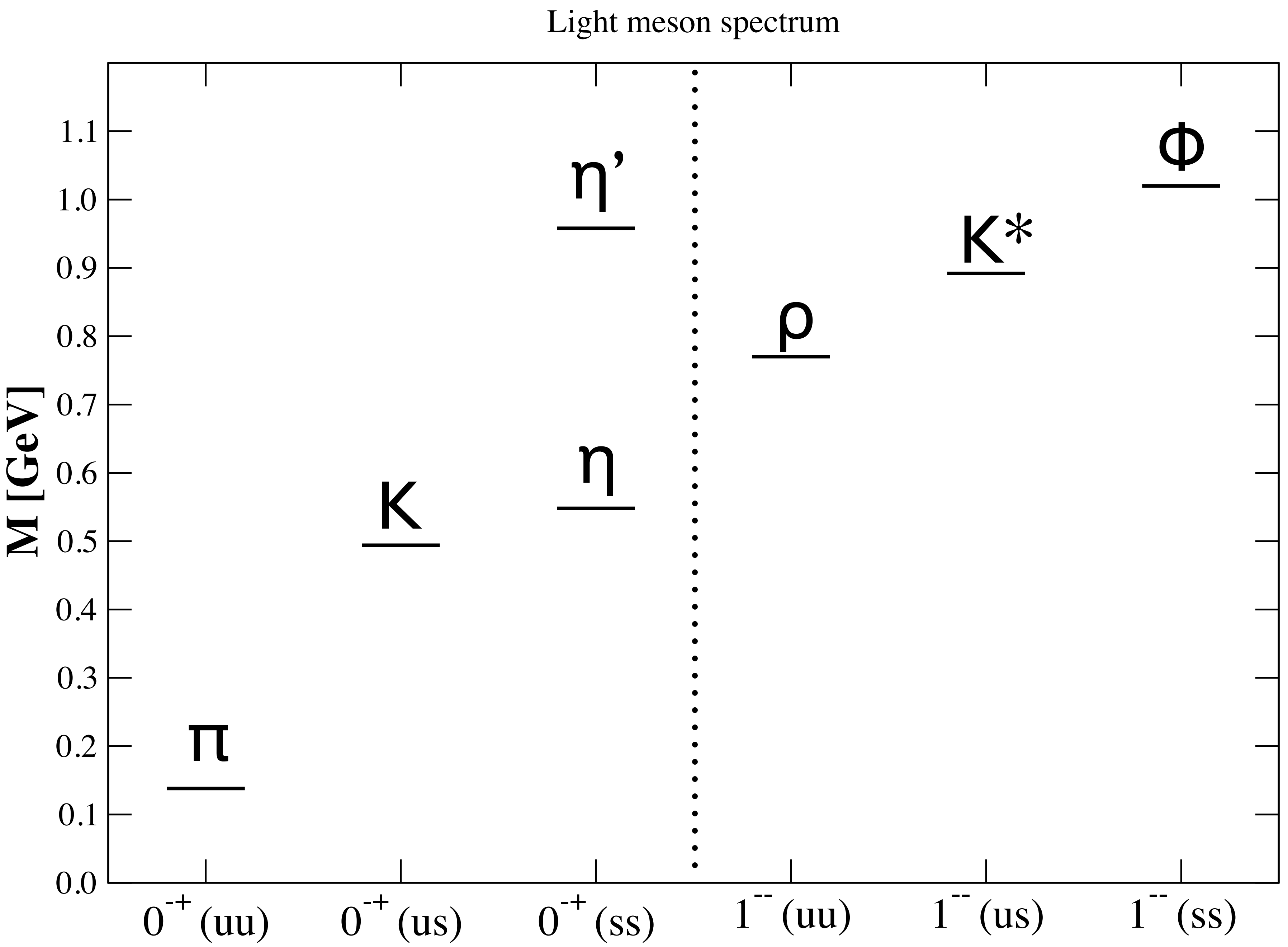
With $\Psi^T = (u, d, s)$ the QCD flavour symmetry is given by

$$U_V(3) \times U_A(3) = U_V(1) \times SU_V(3) \times U_A(1) \times SU_A(3)$$

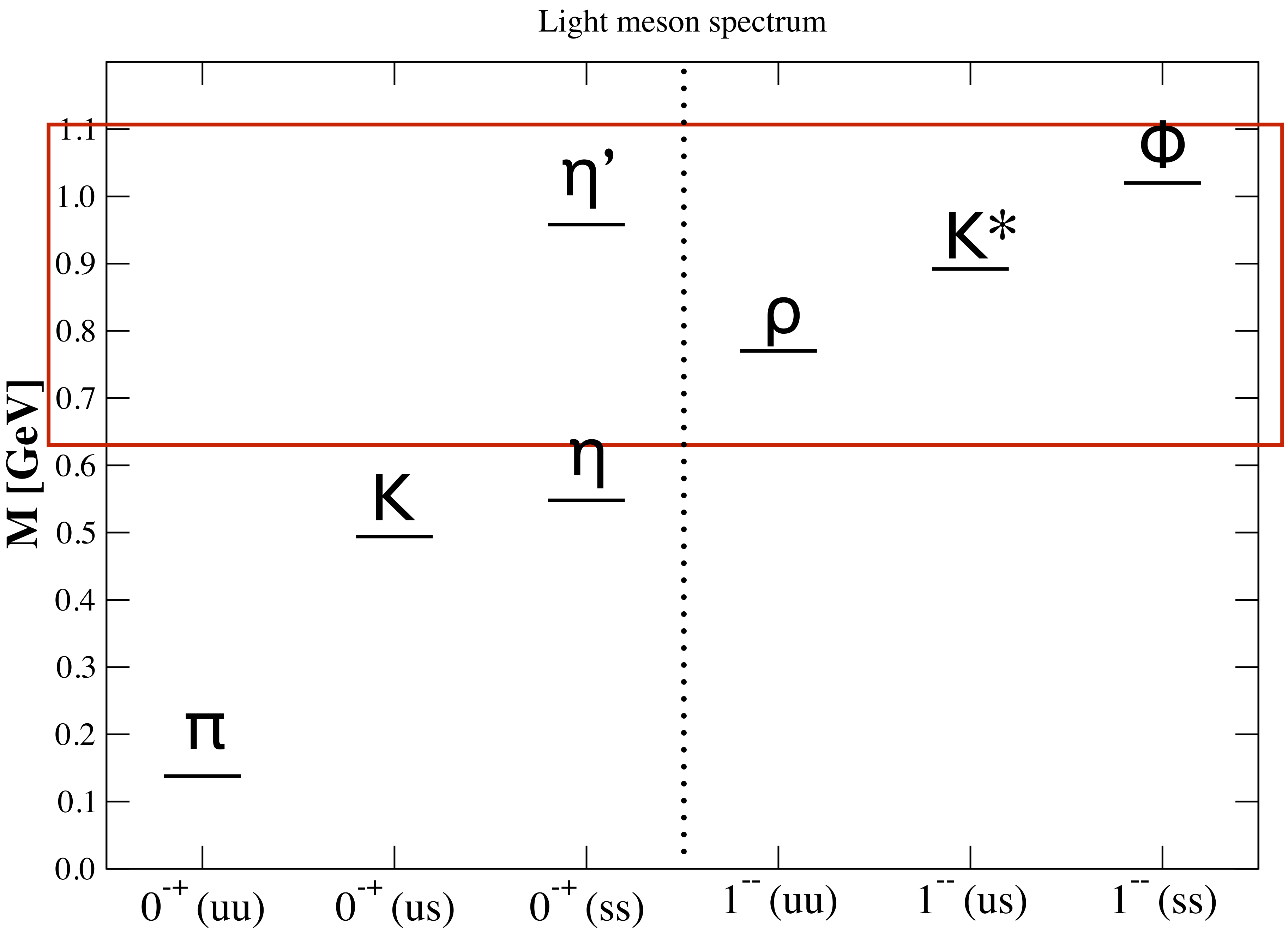
transform	$\mathcal{L}$ inv. iff	current $J_\mu^{(a)}$	charge Q
$U_V(1)$ $e^{i\Theta}$	for all M	$J_\mu = \bar{\Psi}\gamma_\mu\Psi$ $\partial_\mu J_\mu = 0$	baryon number
$SU_V(3)$ $e^{i\Theta^a\lambda^a}$	$m_u = m_d = m_s$	$J_\mu^a = \bar{\Psi}\gamma_\mu\lambda^a\Psi$ $\partial_\mu J_\mu^a = i\bar{\Psi}[\lambda^a, M]\Psi$	isospin hypercharge
$U_A(1)$ $e^{i\Theta\gamma_5}$	$M = 0$	$J_\mu^5 = \bar{\Psi}\gamma_\mu\gamma_5\Psi$ $\partial_\mu J_\mu^5 = 2i\bar{\Psi}m\gamma_5\Psi - g^2/(16\pi^2)\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}^c F_{\gamma\delta}^c$	broken, no GB (QCD anomaly)
$SU_A(3)$ $e^{i\Theta^a\lambda^a\gamma_5}$	$M = 0$	$J_\mu^{5,a} = \bar{\Psi}\gamma_\mu\gamma_5\lambda^a\Psi$ $\partial_\mu J_\mu^{5,a} = i\bar{\Psi}\{\lambda^a, M\}\Psi - \delta_{a3}e^2/(32\pi^2)\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}^{QED}F_{\gamma\delta}^{QED}$	broken, GB (QED-anomaly)

$M = diag(m_u, m_d, m_s)$

Experimental light meson spectrum

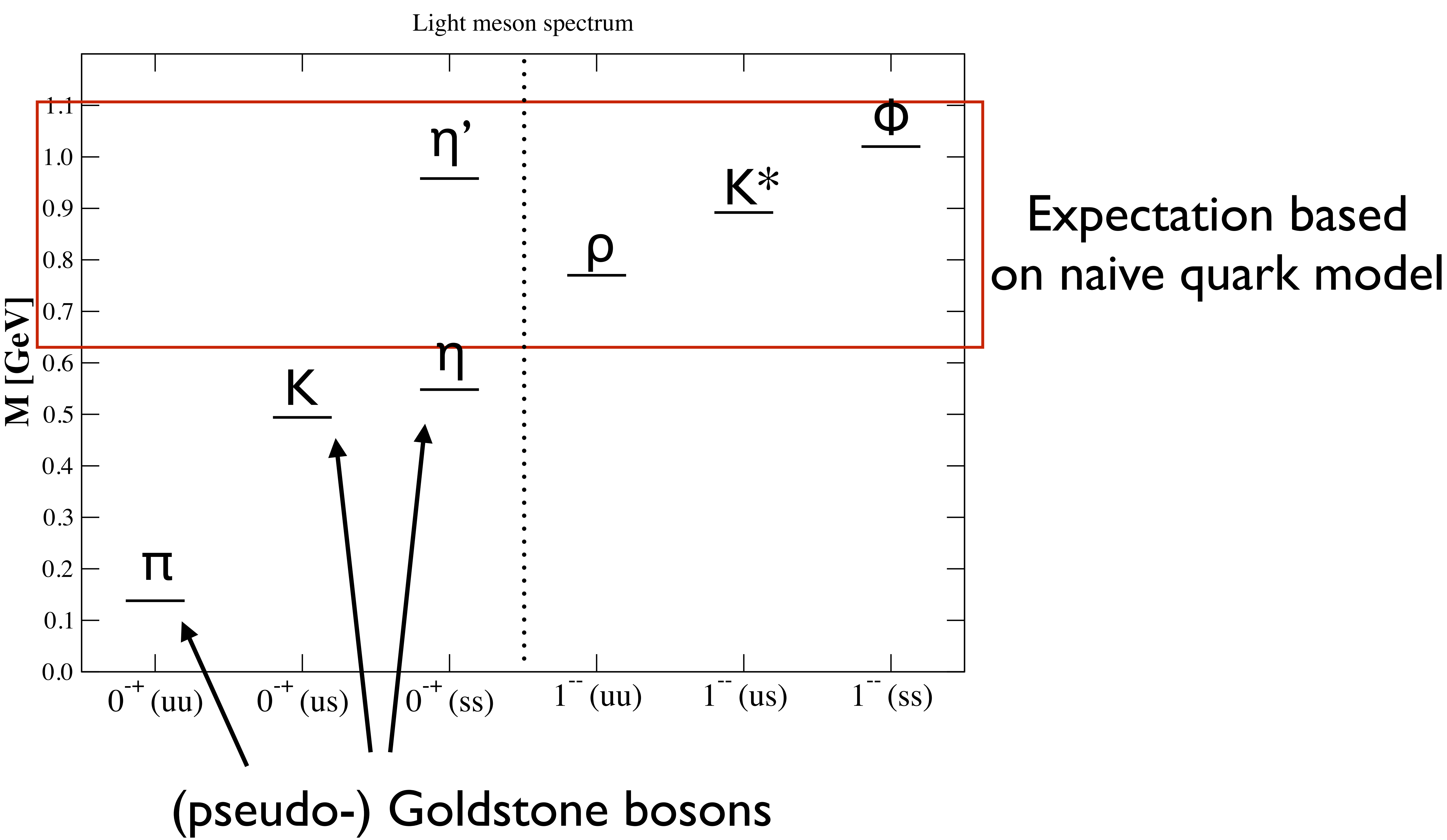


Experimental light meson spectrum

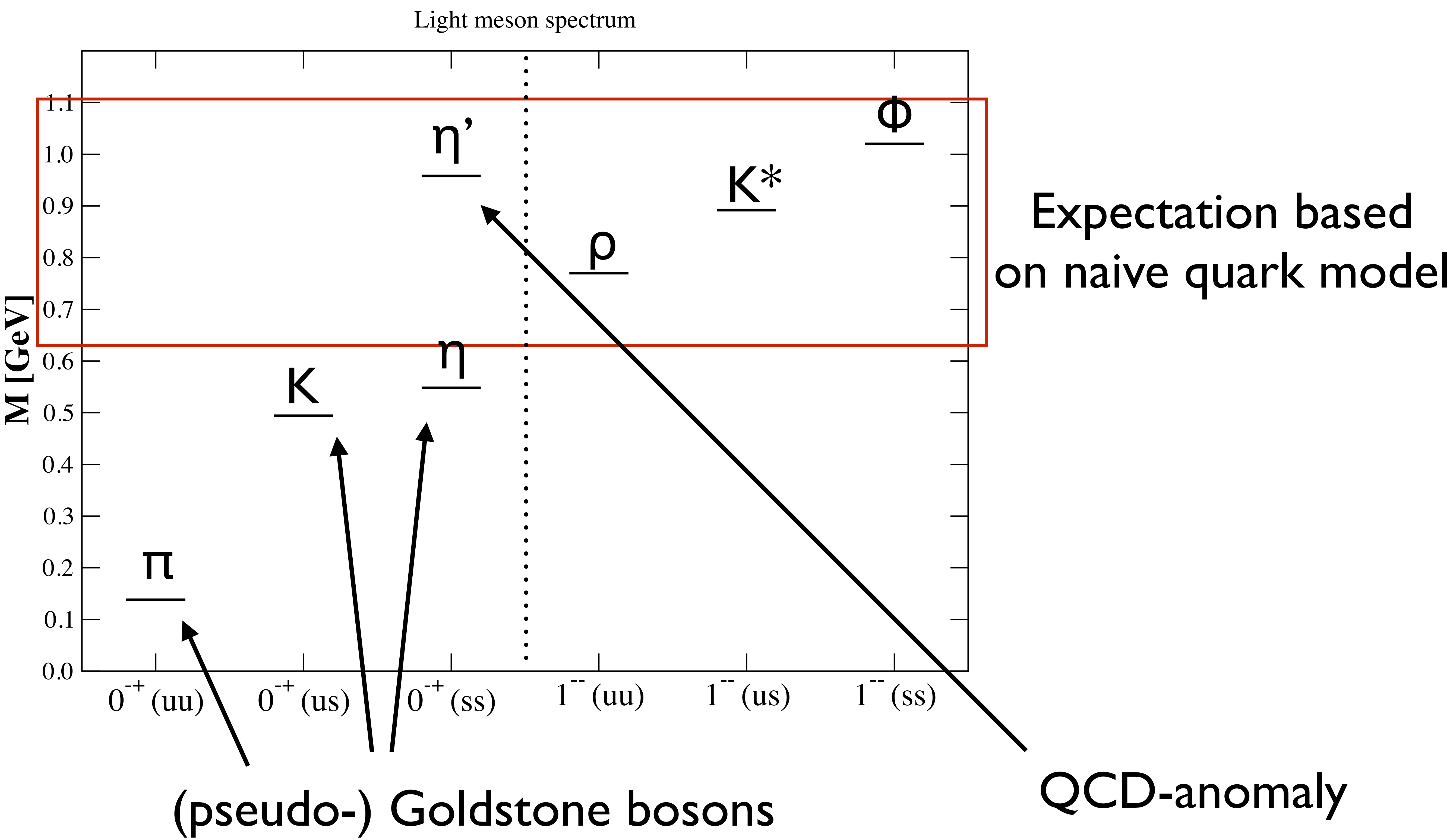


Expectation based on naive quark model

Experimental light meson spectrum



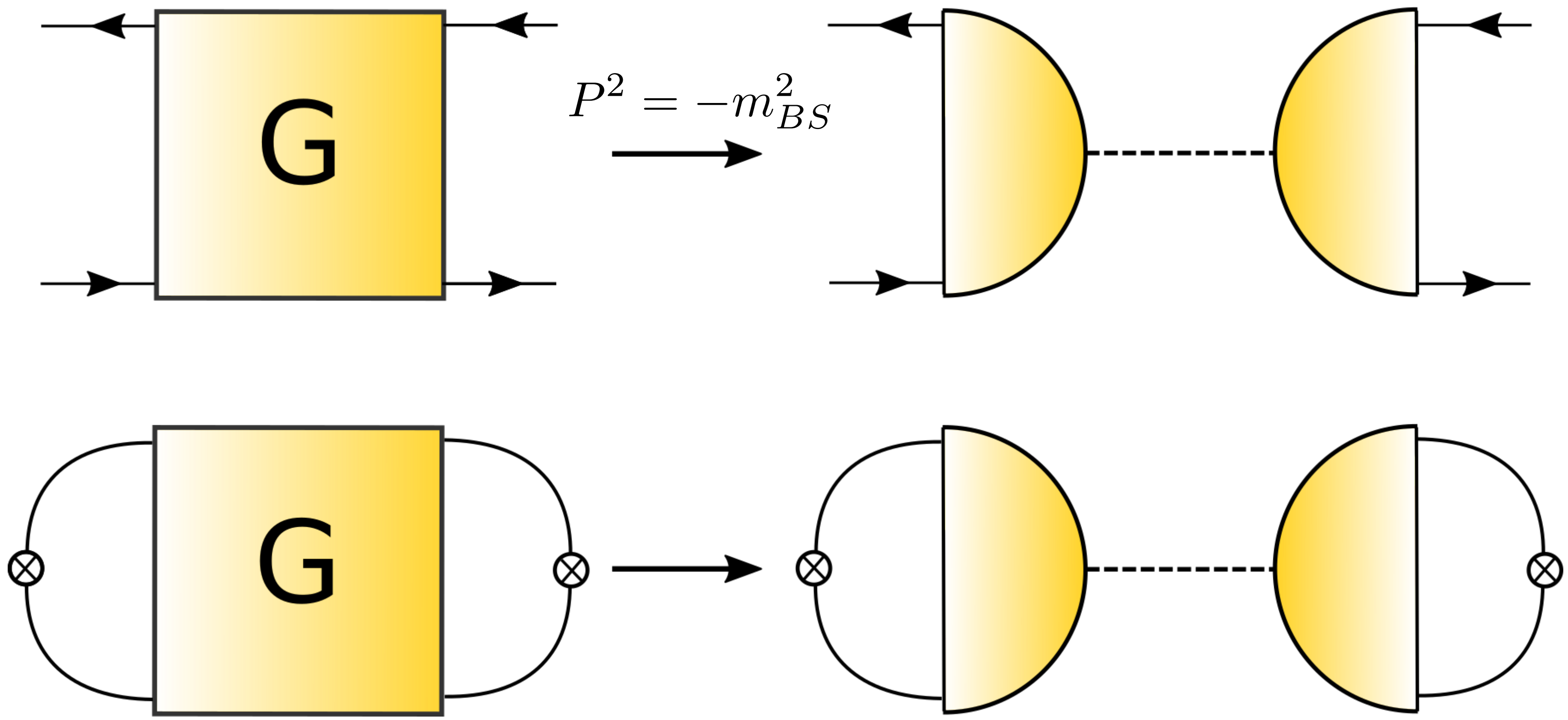
Experimental light meson spectrum



Extracting spectra from QCD-correlators

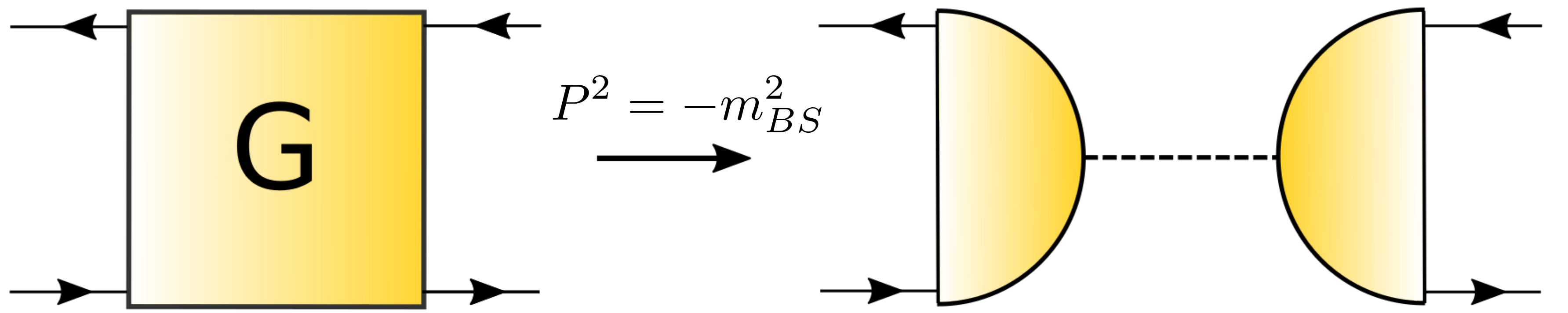
functional:

Lattice:

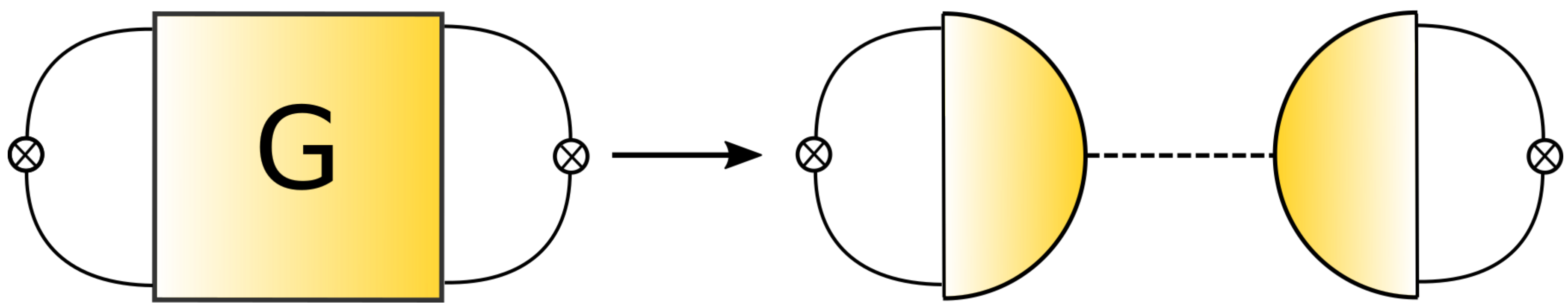


Extracting spectra from QCD-correlators

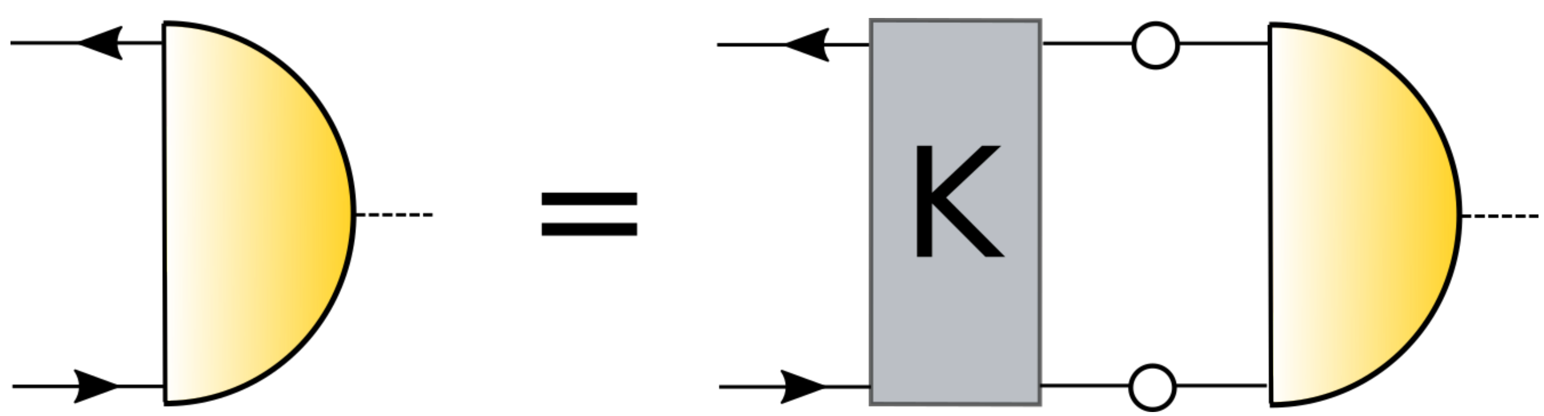
functional:



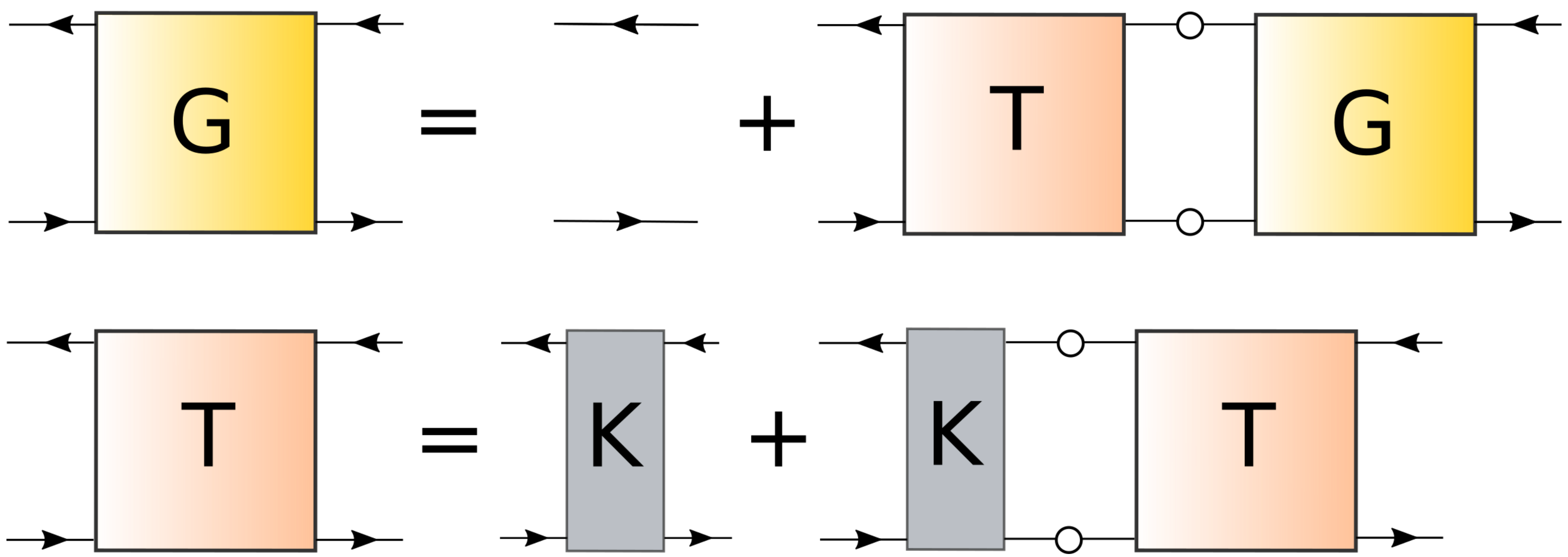
Lattice:



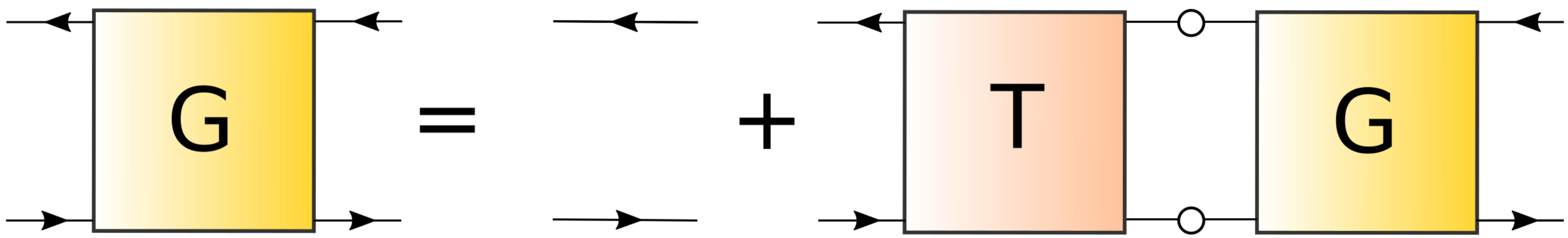
exact BSE:

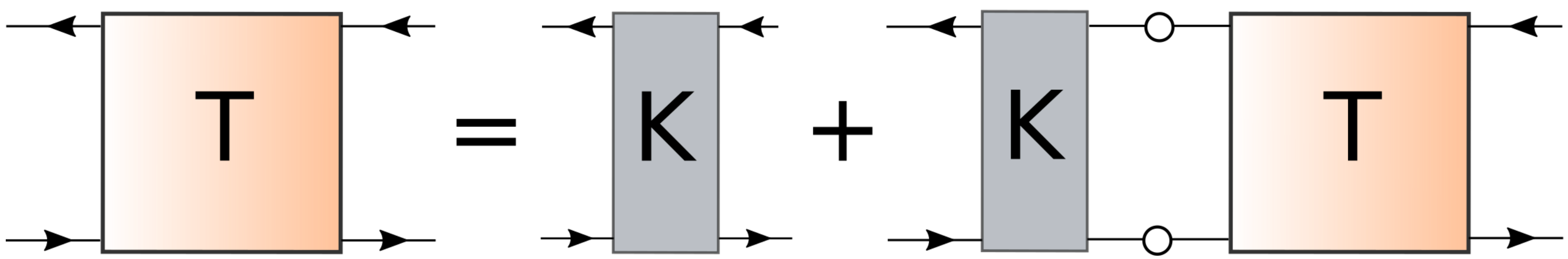


Bound states and Bethe-Salpeter equations

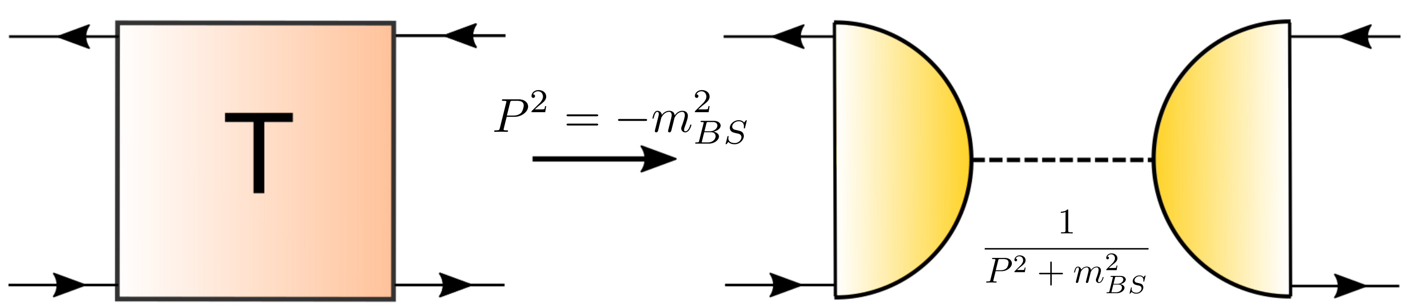


Bound states and Bethe-Salpeter equations

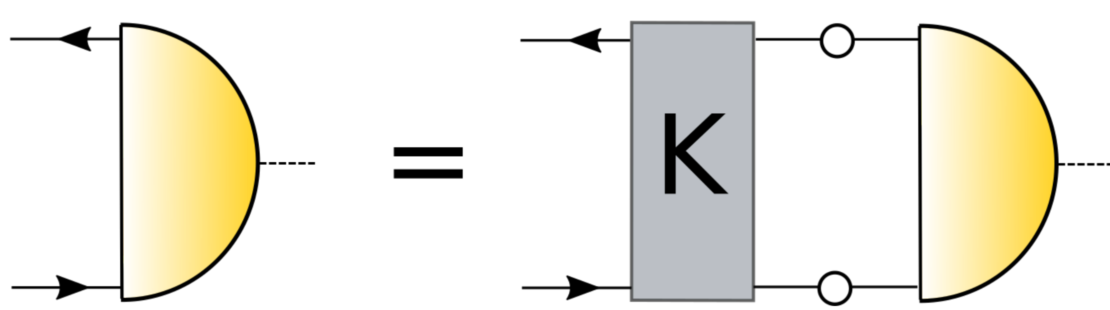




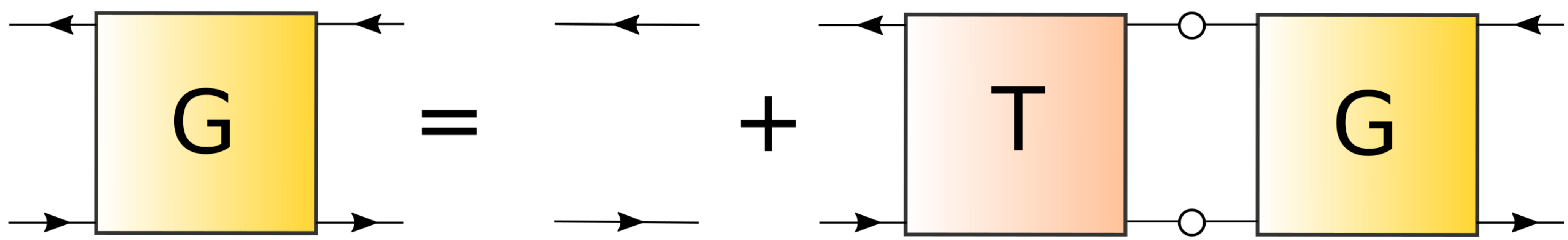
Bound states appear as poles in T:

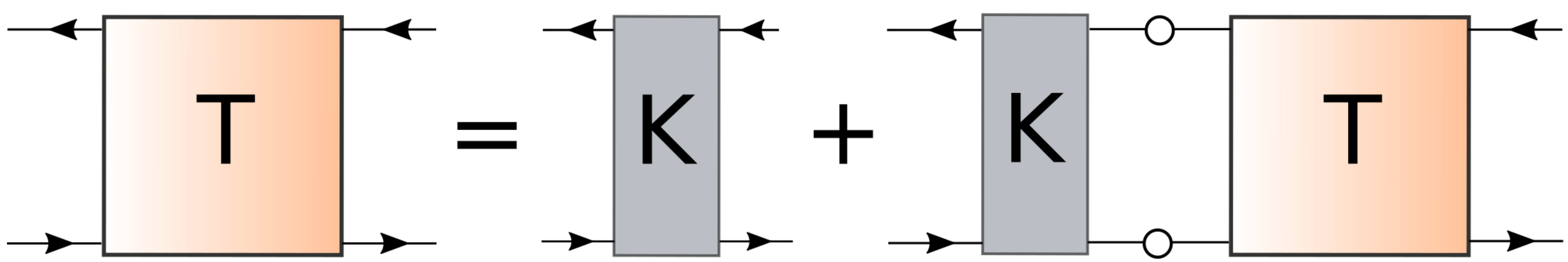


BSE:

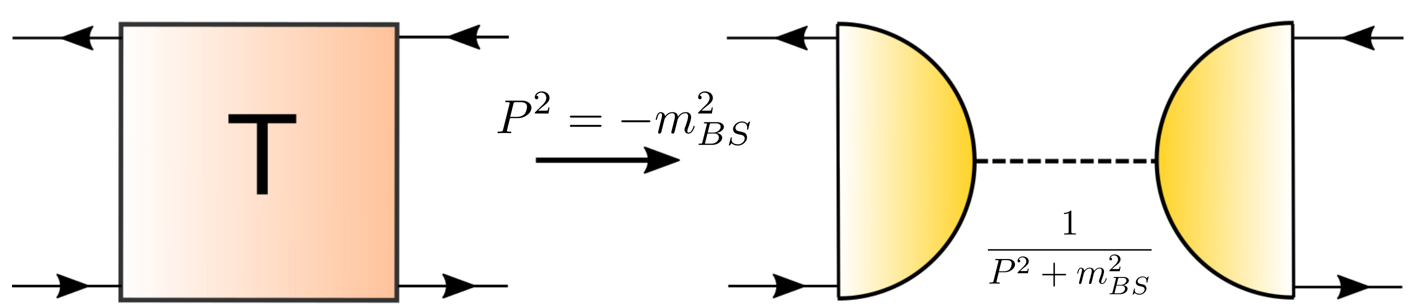


Bound states and Bethe-Salpeter equations



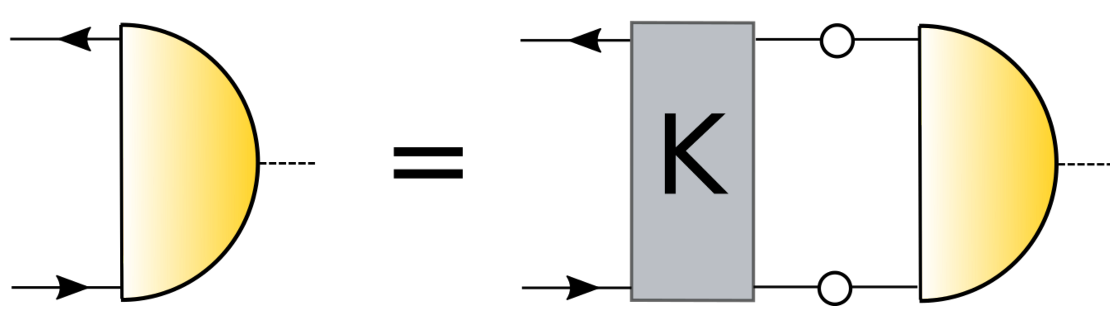


Bound states appear as poles in T:

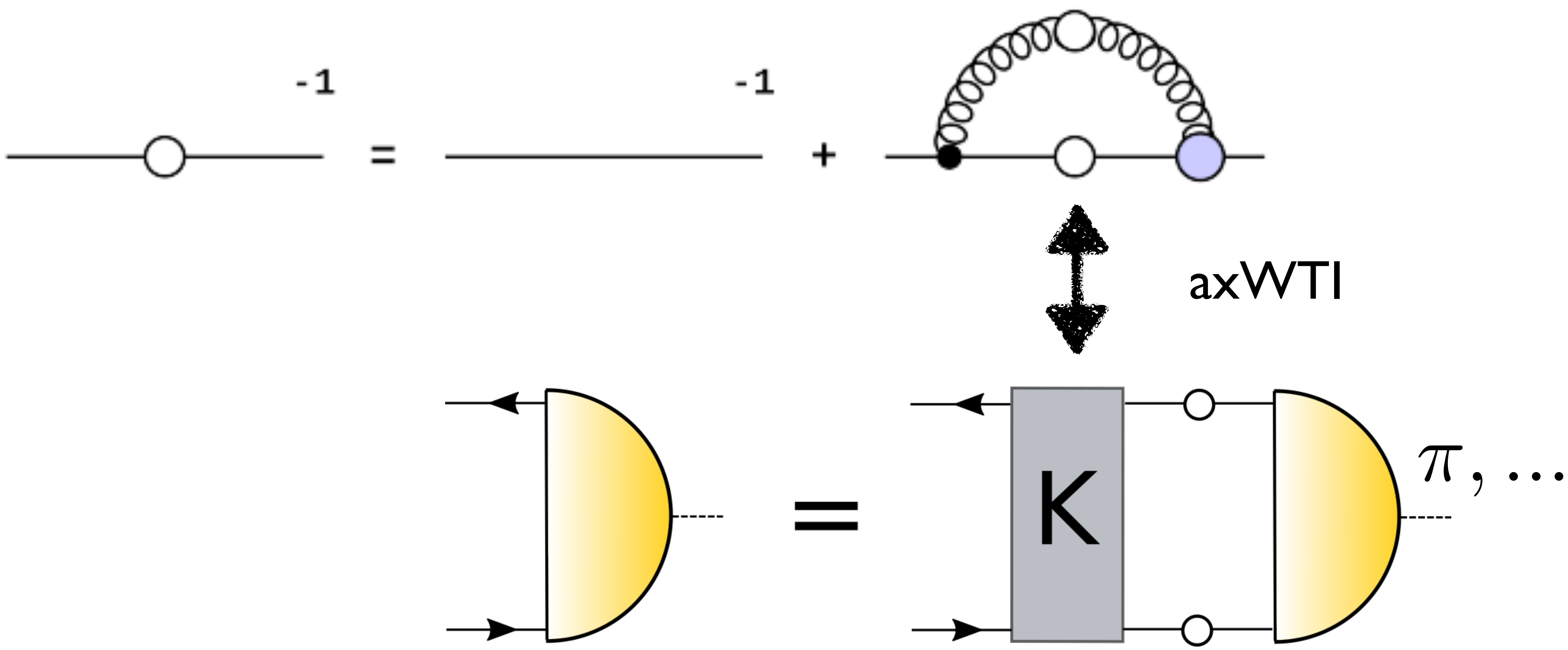


BS-wave functions = residue of bound state pole

BSE:



DSEs and Bethe-Salpeter equation



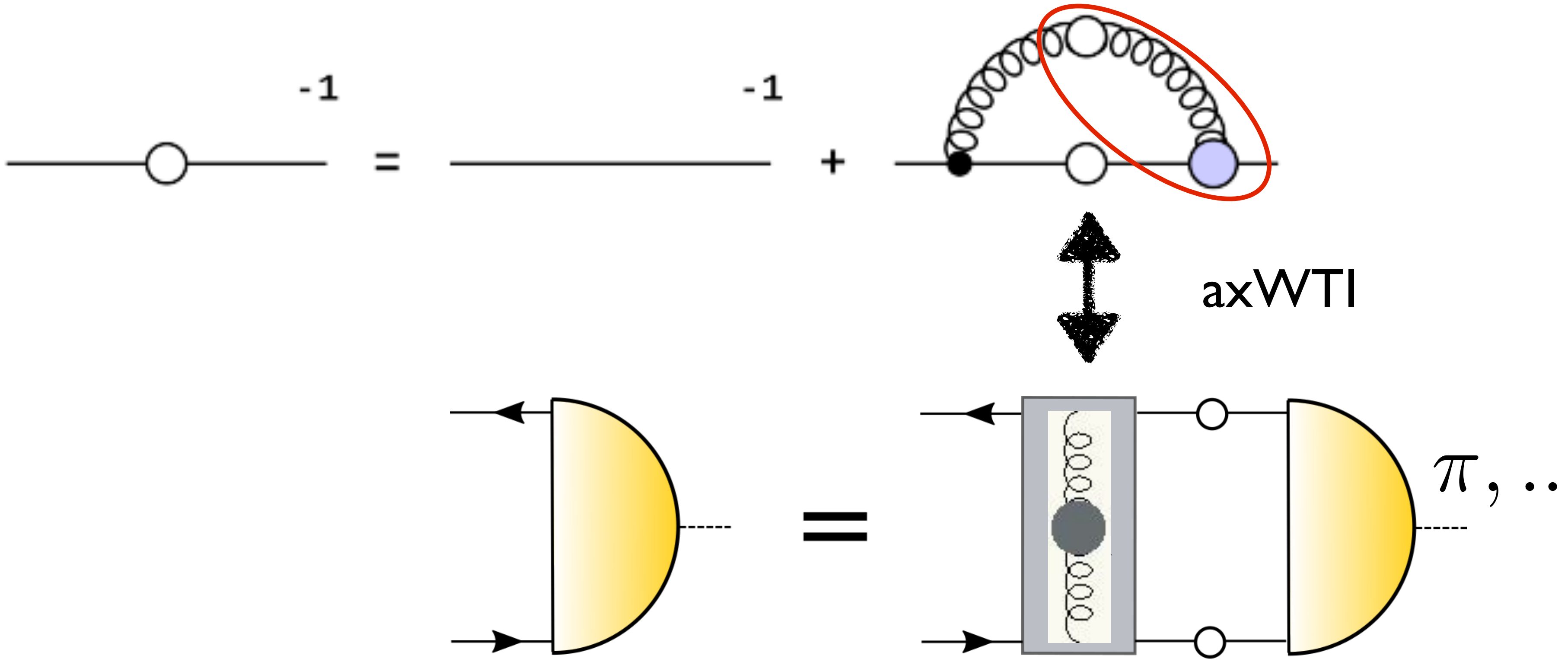
Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i \int (K \gamma_5 S_- + K S_+ \gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu} \Gamma_\nu \gamma_5 + \int \gamma_5 \gamma_\mu S_- D_{\mu\nu} \Gamma_\nu$$

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

DSEs and Bethe-Salpeter equation



RL: QED-structure of binding force

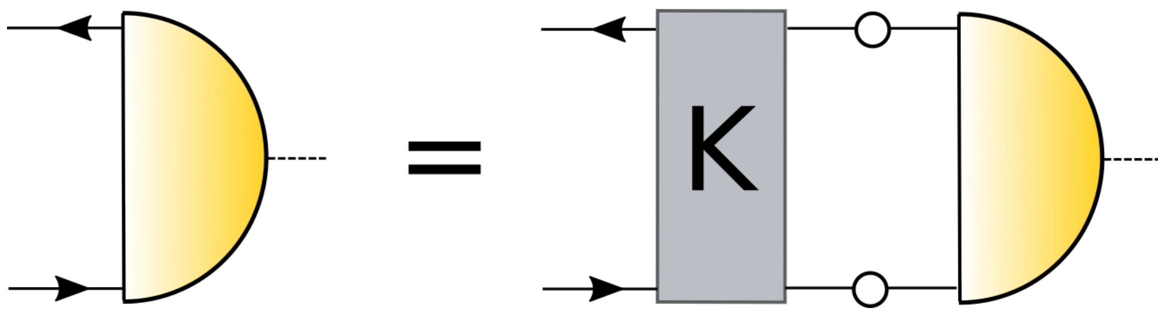
Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i\int (K\gamma_5 S_- + K S_+\gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu}\Gamma_\nu\gamma_5 + \int \gamma_5\gamma_\mu S_- D_{\mu\nu}\Gamma_\nu$$

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

Pion-BSE



$P^2 = -m_{BS}^2$

$$\mathbf{1} \times EV = operator \times EV$$

- Structure: eigenvalue equation
- Eigenvector is 'Bethe-Salpeter wave function':

$$[\Gamma_\pi(P,p)]^e_{\alpha,\beta,A,B,a,b} = \{\gamma_5 \left[F_1(P,p) + F_2(P,p)\, i\not{P} + F_3(P,p)\, pP\, i\not{p} + F_4(P,p)\, [\not{p}, \not{P}]\right]\}_{\alpha,\beta} \times \frac{\delta_{AB}}{\sqrt{3}} \times r^e_{ab}$$

(pseudo-) scalar: 4 Dirac tensor structures
(axial-)vector: 8

Bethe-Salpeter wave function

$$[\Gamma_\pi(P,p)]^e_{\alpha,\beta,A,B,a,b} = \{\gamma_5 \,[F_1(P,p) + F_2(P,p)\, i\not{P} + F_3(P,p)\, pP\, i\not{p} + F_4(P,p)\,[\not{p}\,, \not{P}]]\}_{\alpha,\beta} \times \frac{\delta_{AB}}{\sqrt{3}} \times r^e_{ab}$$

- why four tensor structures ?

  quark legs ⟶ Dirac-structure γ_μ
  pseudoscalar ⟶ no Lorenz-index, overall γ_5
  two independent momenta P_μ, p_μ

- comparison with quark model:

  same flavor and color part of wave function
  relativistic: spin and spatial wave function combined !!

Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

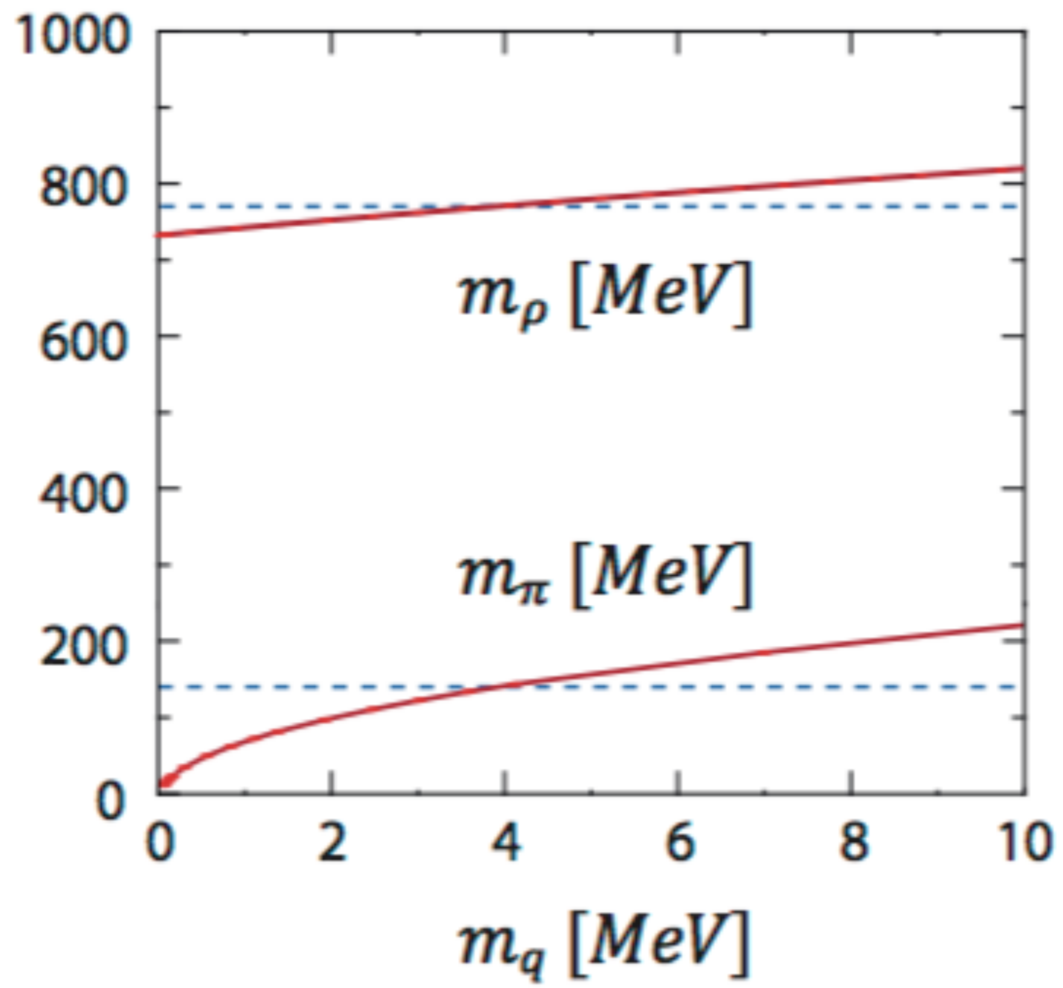
$$\begin{aligned}\Gamma_\pi(P,p) = \gamma_5 \, [&F_1(P,p) \quad \text{s-wave} \\ &+ F_2(P,p) i \not{P} \\ &+ F_3(P,p) pP i \not{p} \quad \text{p-wave} \\ &+ F_4(P,p)[\not{p}, \not{P}]]\end{aligned}$$

~~$P : (-1)^{L+1}$~~

- mesons: 'exotic' quantum numbers possible:

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

Pions as Goldstone bosons



- Gell-Mann-Oakes-Renner: $f_\pi^2 m_\pi^2 = -2\, m \langle \bar{\Psi}\Psi \rangle$
- Pion BS-amplitude: $f_\pi \Gamma_\pi(P^2 = 0, p) = B(p^2)\gamma_5$

Pion decay constant does not vanish in chiral limit !

Excited states: no GB, decay constant must vanish in chiral limit!

Hoell, Krassnigg, Roberts, PRC 70 (2004)

Proof of Goldstone theorem

We start by parametrising the matrix elements between the vacuum and bound states λ of the axial and pseudoscalar current ($P^2 = -m_\lambda^2$ fixed):

$$\langle 0|\, j_5^\mu(x)\,|\lambda\rangle = -iP^\mu f_\lambda\, e^{-ix\cdot P}, \qquad \langle 0|\, j_5(x)\,|\lambda\rangle = -ir_\lambda\, e^{-ix\cdot P}. \tag{1}$$

The first quantity encodes the transition from a pseudoscalar meson to an axialvector current and thereby defines its electroweak decay constant f_λ. The pseudoscalar analogue r_λ is not associated with a measurable quantity.
Using now the PCAC-relation (see above)

$$-i\partial_\mu\, j_{5,a}^\mu = Z_4\, i\bar\psi\,\{\mathsf{m},\mathsf{t}_a\}\,\gamma_5\,\psi \xrightarrow{\mathsf{m}=m_q} 2m_q\, j_{5,a}\,, \tag{2}$$

where $Z_4 = Z_2\, Z_m$ and $j_{5,a}(z) = Z_4\,\bar\psi(z)\,i\gamma_5\,\mathsf{t}_a\,\psi(z)$ is the pseudoscalar density, we arrive at

$$\boxed{f_\lambda\, m_\lambda^2 = 2m_q\, r_\lambda\,,} \tag{3}$$

which is valid for all flavour non-singlet pseudoscalar mesons (in the singlet case there would be an additional term from the axial anomaly).
We proceed with the axial vector Ward takahashi identity (axWTI)

$$Q^\mu\Gamma_5^\mu(k,Q) + 2m\Gamma_5(k,q) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-) \tag{4}$$

with momenta $k_\pm = k \pm Q/2$, the incoming total momentum Q and the average quark momentum k. A derivation of the vector identity can be found e.g. in Peskin and Schroeder, *Introduction to Quantum Field Theory*, chapter 7.4., from which the axWTI follows by analogy.

Proof of Goldstone theorem

The pseudoscalar and axialvector vertices each contain pole contributions from bound states (similar to the rho-meson contribution to the vector vertex):

$$\Gamma_5^\mu = Q^\mu \sum_\lambda \frac{2if_\lambda}{Q^2+m_\lambda^2}\,\mathbf{\Gamma}_\lambda + \widetilde{\Gamma}_5^\mu\,, \qquad \Gamma_5 = \sum_\lambda \frac{2ir_\lambda}{Q^2+m_\lambda^2}\,\mathbf{\Gamma}_\lambda + \widetilde{\Gamma}_5\,. \tag{5}$$

see later !

Here the quantities with tilde are regular objects and Γ_λ are the Bethe-Salpeter amplitudes of the respective bound states.

Plugging now Eq.(5) into Eq.(4) and using (3) we arrive at

$$Q^\mu\,\Gamma_5^\mu + 2m_q\,\Gamma_5 = \sum_\lambda 2if_\lambda\,\mathbf{\Gamma}_\lambda + Q^\mu\,\widetilde{\Gamma}_5^\mu + 2m_q\,\widetilde{\Gamma}_5 = S^{-1}(k_+)\,i\gamma_5 + i\gamma_5\,S^{-1}(k_-)\,. \tag{6}$$

Observe that all hadronic poles contained in the vertices have disappeared, which is consistent because the right-hand side of the axial WTI does not exhibit any such poles. In the chiral limit $m_q \to 0$ and for $Q^\mu \to 0$ this becomes

$$\sum_\lambda f_\lambda\,\mathbf{\Gamma}_\lambda(k,0) = B(k^2)\gamma_5\,. \tag{7}$$

The sum goes over all pseudoscalar 0^{-+} mesons with identical flavour quantum numbers, i.e., ground states and radial excitations.

In the chiral limit, $B(k^2)$ is only nonzero if chiral symmetry is spontaneously broken. Then there is at least one mode with $f_\lambda \neq 0$. From (3) we must have $m_\lambda \to 0$ in that case, i.e. a massless Goldstone boson.

Proof of GMOR

For excited states with $m_\lambda \neq 0$ the decay constants have to vanish in the chiral limit because of Eq. (3). Therefore the sum in Eq. (7) breaks down and we arrive at

$$f_\pi \, \mathbf{\Gamma}_\pi(k,0) = B(k^2)\gamma_5 \, . \tag{8}$$

Now we multiply on both sides with $S(k)\gamma_5 S(k)$, take the trace and integrate over momentum k. We then find

$$f_\pi \int_k tr\{S(k)\gamma_5 S(k)\Gamma_\pi(k,0)\} = \int_k B \; tr\left\{\frac{(i\not{k}\,A+B)\gamma_5(i\not{k}\,A+B)\gamma_5}{p^2A^2+B^2}\right\} \tag{9}$$

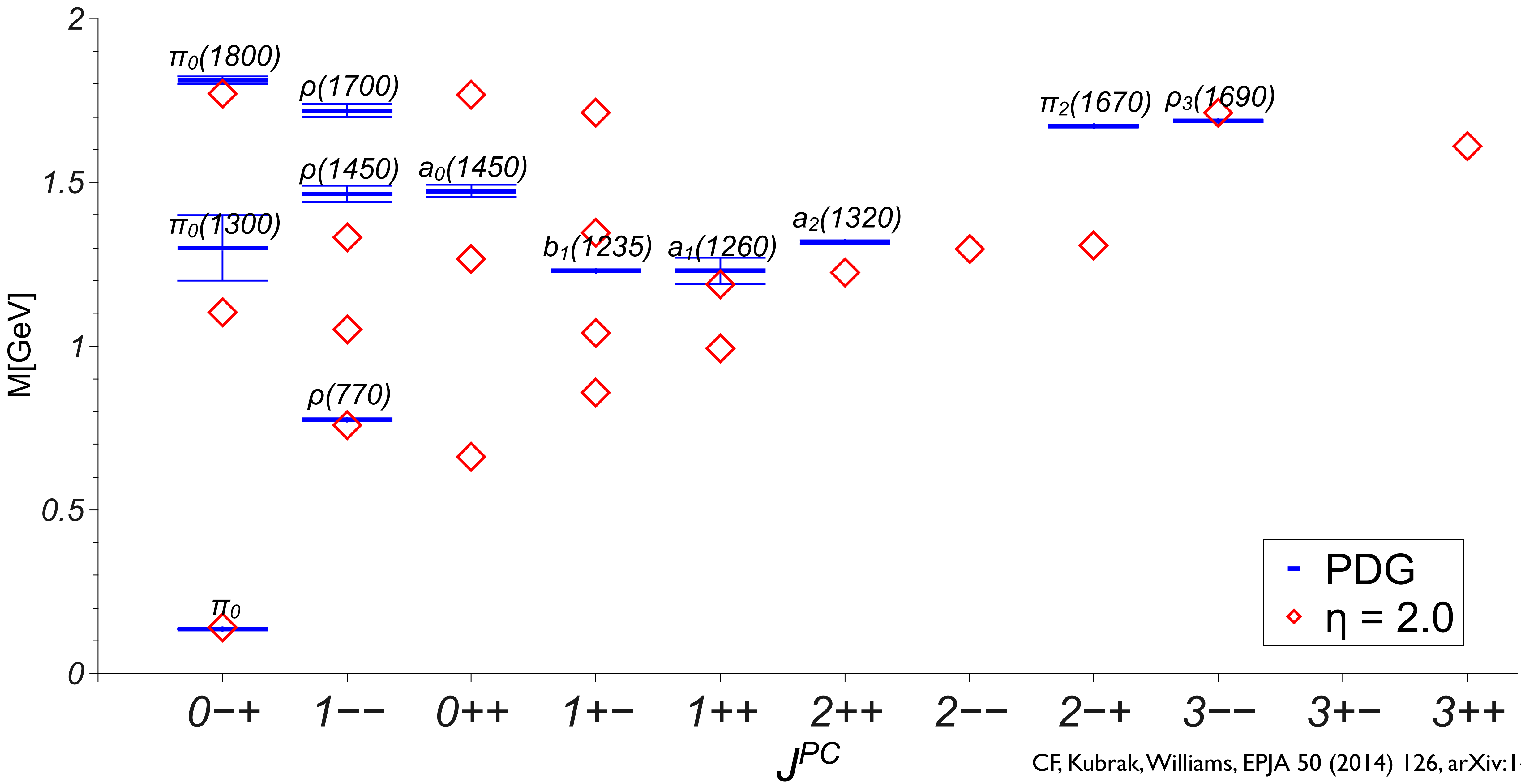
$$f_\pi r_\pi = -\langle\bar{\Psi}\Psi\rangle \tag{10}$$

and substituting this back into Eq. (3) we arrive at the Gell-Mann-Oakes-Renner relation

$$\boxed{f_\pi^2 m_\pi^2 = -2m_q\langle\bar{\Psi}\Psi\rangle} \tag{11}$$

see Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602], sections 3.4 and 4.2

Light meson spectrum

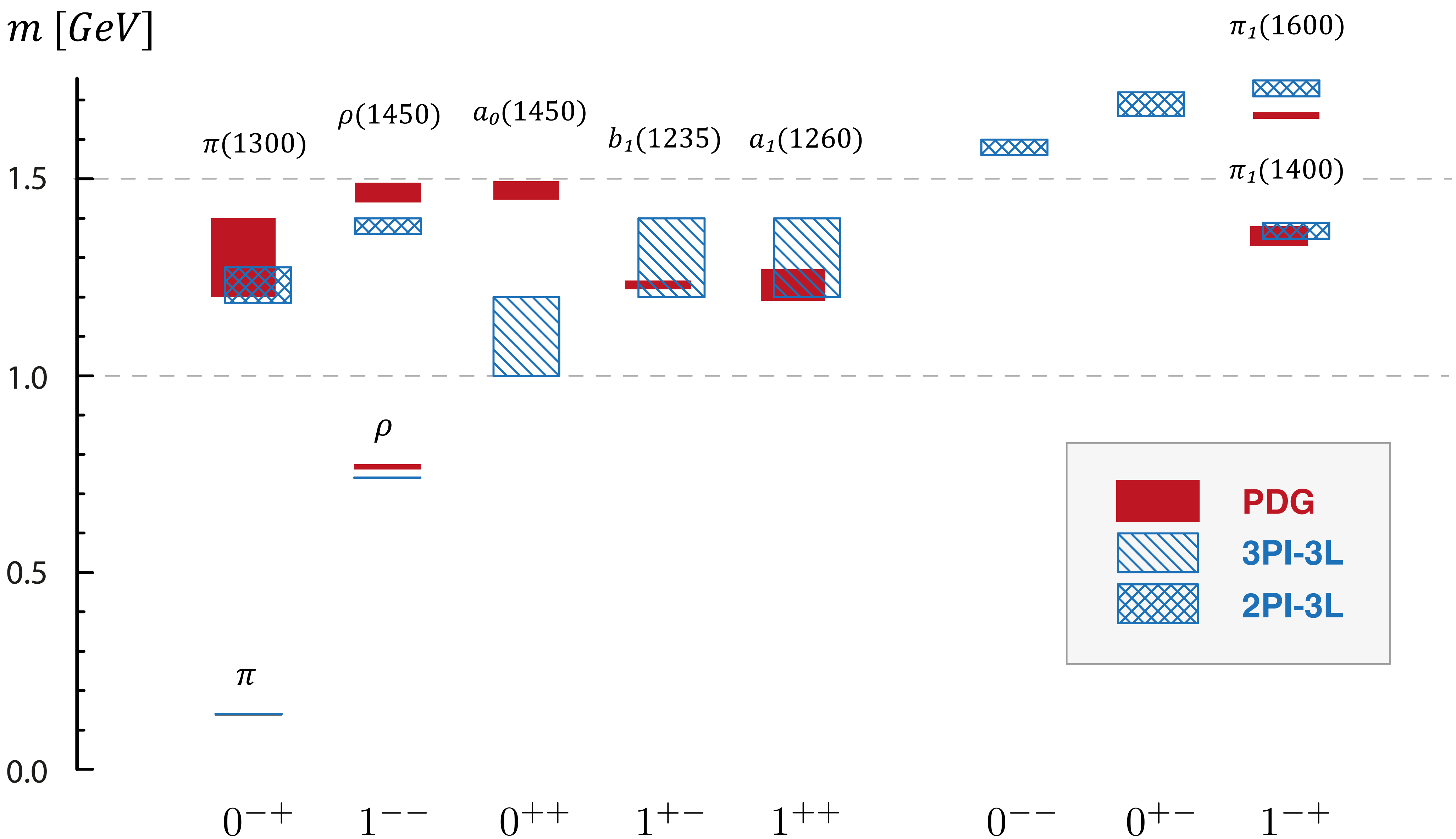


CF, Kubrak, Williams, EPJA 50 (2014) 126, arXiv:1406.4370

Williams, CF, Heupel, PRD93 (2016) 034026

- good channels (ground state): 0^{-+}, 1^{--}
- acceptable channels (ground state) : 2^{++}, 3^{--},...
- clear deficiencies in other channels and excited states

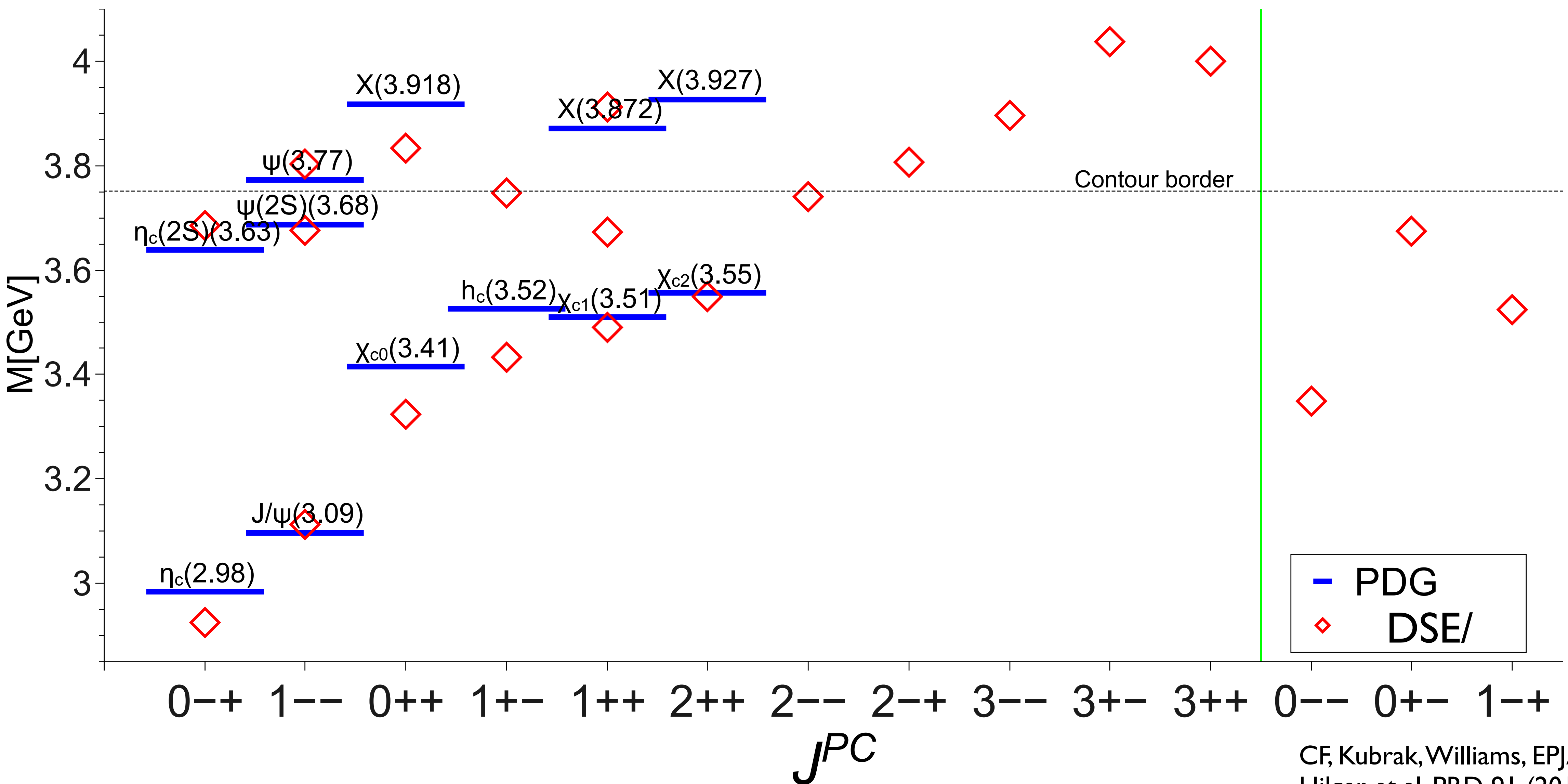
Light meson spectrum



CF, Kubrak, Williams, EPJA 50 (2014) 126, arXiv:1406.4370
Williams, CF, Heupel, PRD93 (2016) 034026

- good channels (ground state): 0^{-+}, 1^{--}
- acceptable channels (ground state) : 2^{++}, 3^{--},...
- clear deficiencies in other channels and excited states
- drastic improvement beyond rainbow-ladder !

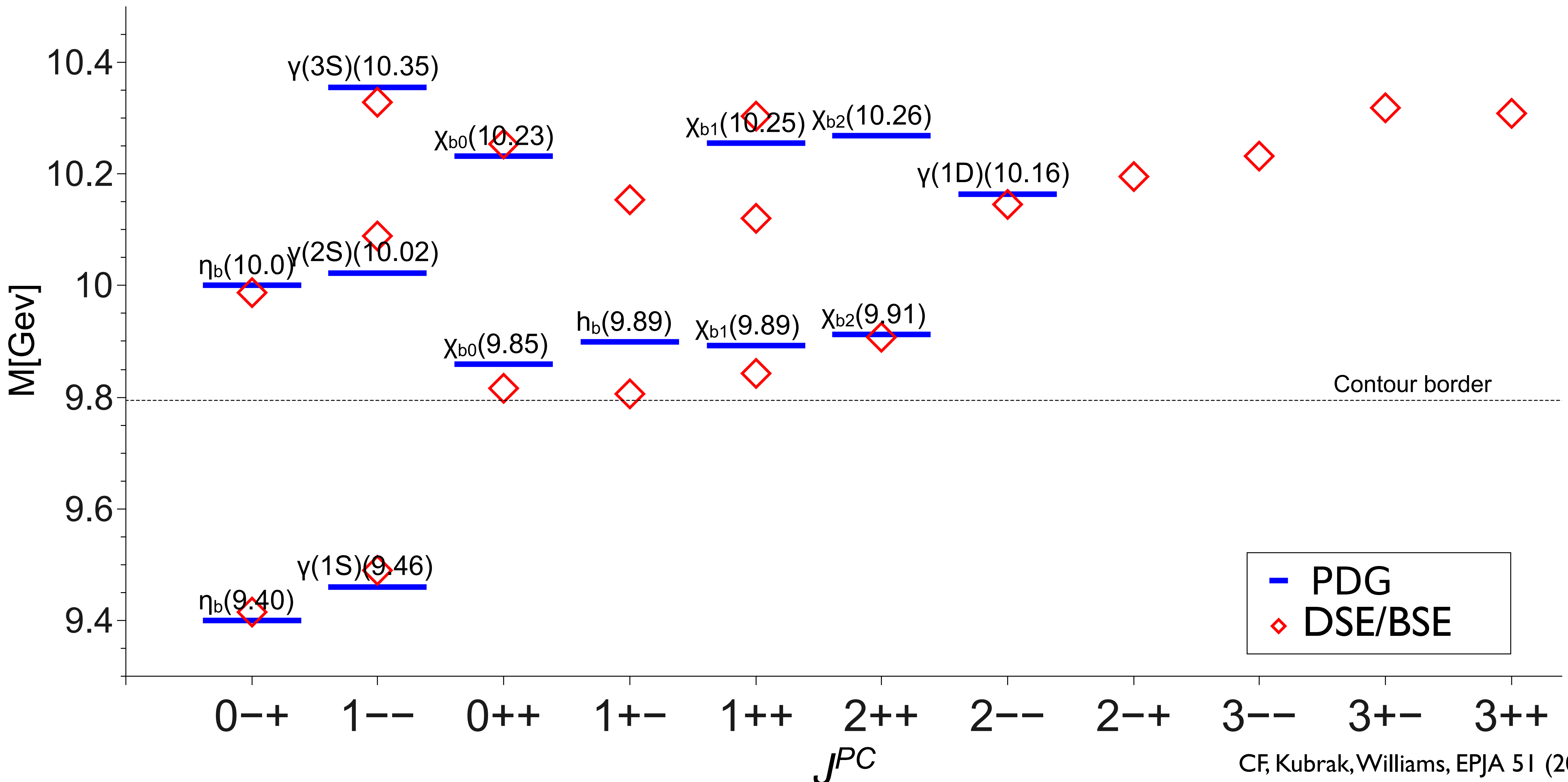
Rainbow-ladder: heavy meson spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}$,...: prediction for tensor state
- acceptable channels : $0^{-+}, 1^{++}$,...
- deficiencies in other channels: 'imbalance' of spin-structure

Rainbow-ladder: heavy meson spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: 1^{--}, 2^{++}, 3^{--},….: prediction for tensor state
- acceptable channels : 0^{-+}, 1^{++},...
- deficiencies in other channels: 'imbalance' of spin-structure

Hadron physics with functional methods

Christian S. Fischer

Justus Liebig Universität Gießen

Lecture 3

Overview

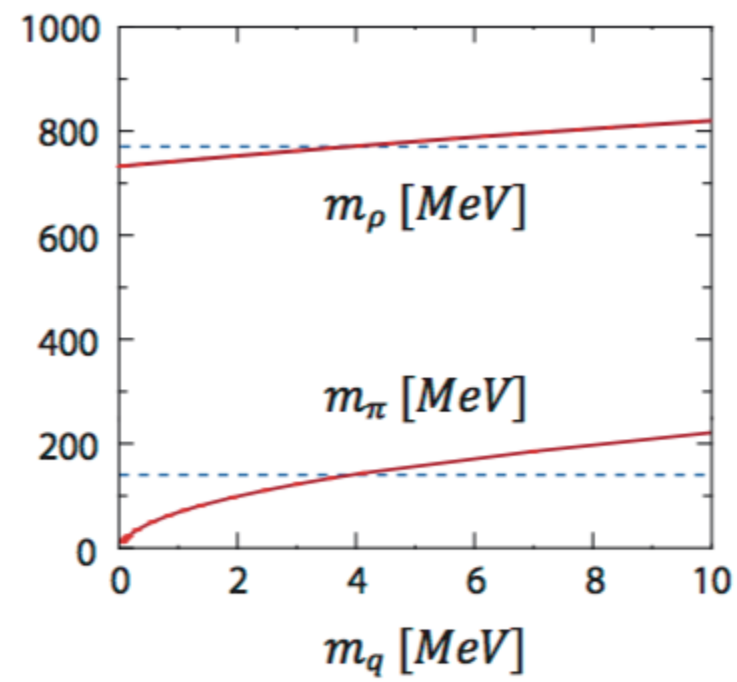
1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Confinement and dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
3. Bound states and applications (rainbow-ladder approximation)
   - Mesons
   - Baryons
   - Tetraquarks
4. Form factors and decays (rainbow-ladder approximation)
   - Meson and baryon form factors
   - The anomalous magnetic moment of the muon
5. Beyond rainbow ladder
   - Confinement and glueballs
   - Hadron results beyond rainbow-ladder

Bits and pieces to remember from Lecture 2

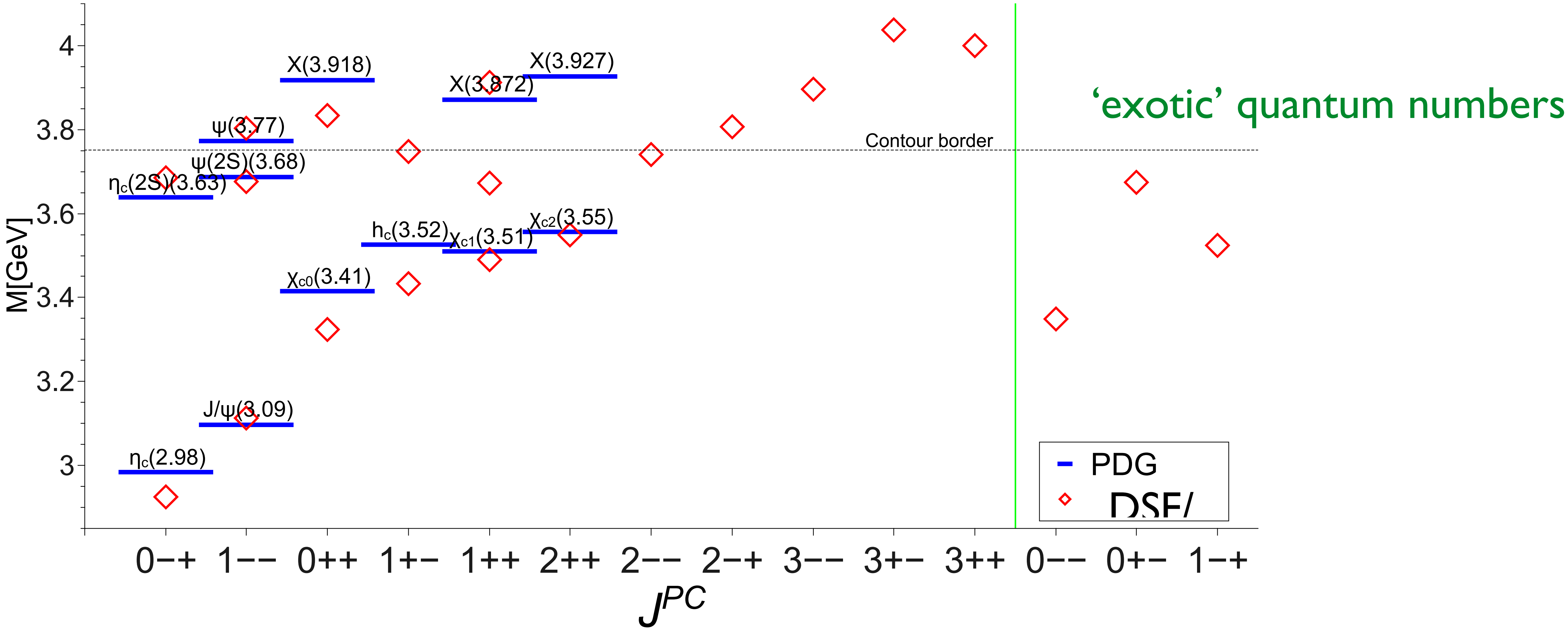
chiral symmetry and Goldstone bosons:

$$\sum_\lambda f_\lambda \Gamma_\lambda(k,0) = B(k^2)\gamma_5$$

$$f_\lambda m_\lambda^2 = 2 m_q r_\lambda$$



meson spectrum from DSEs/BSEs:



Baryon spectroscopy from QCD

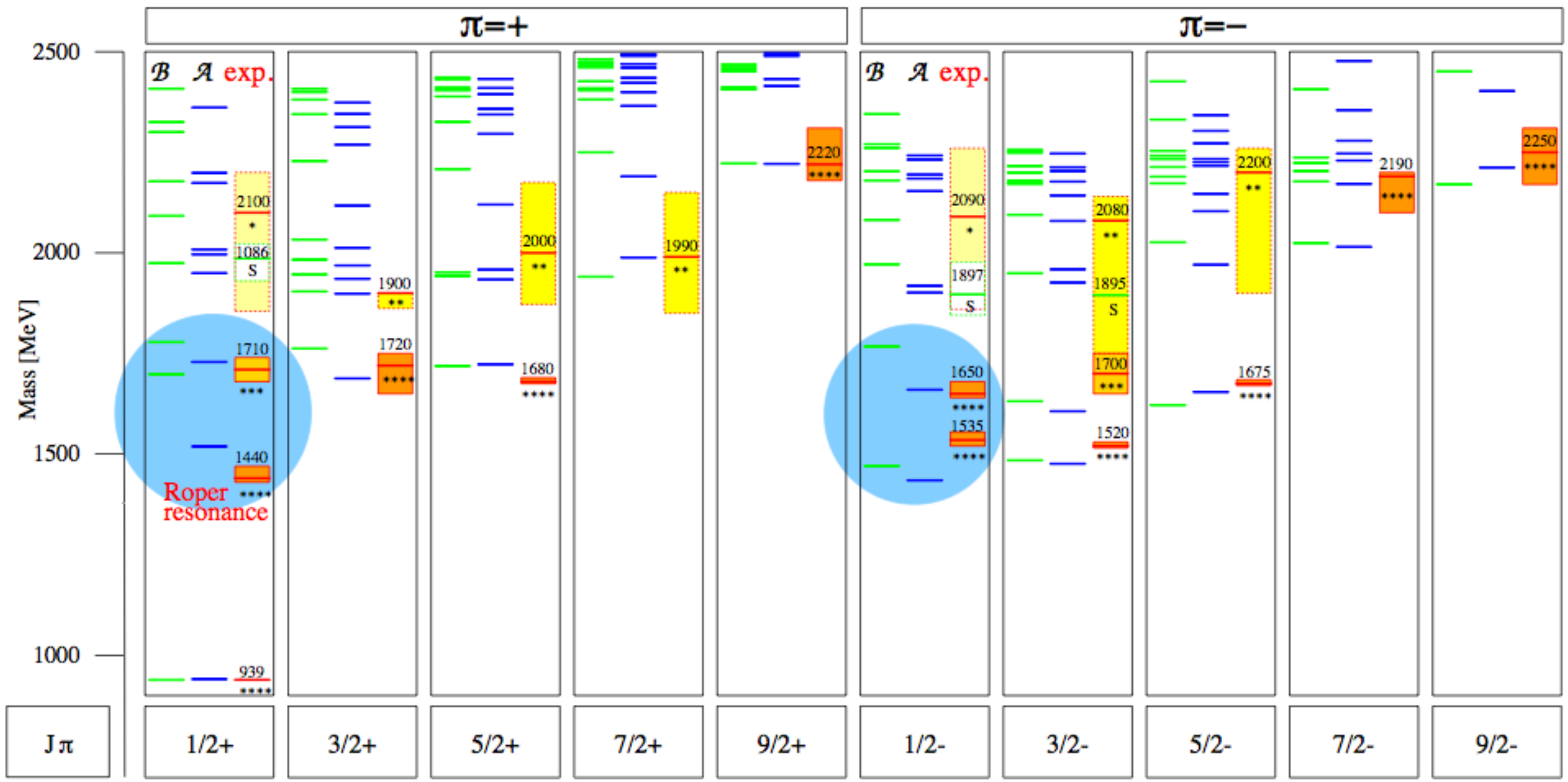
- Underlying QCD forces
  - two-body vs. three-body ⟶ Δ vs Y - configuration
  - confinement ⟶ Regge trajectories ?!
  - spin structure ⟶ (Hyper)-Fine structure
  - meson cloud effects ⟶ GB-exchange vs QCD
  - heavy/heavy-light systems ⟶ Flavor dependence
- 'Missing resonances' ⟶ 3-quark vs. quark-diquark
- Coupled-channel effects

Strategies to deal with this situation:

Nonperturbative QCD: Lattice, Functional methods	Effective theories with hadronic dof

Klemt, Richard, Rev.Mod.Phys. 82 (2010) 1095

Light baryon spectrum - quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances' - three-body vs. quark-diquark
- level ordering: $N\frac{1}{2}^{\pm}$ vs. $\Lambda\frac{1}{2}^{\pm}$

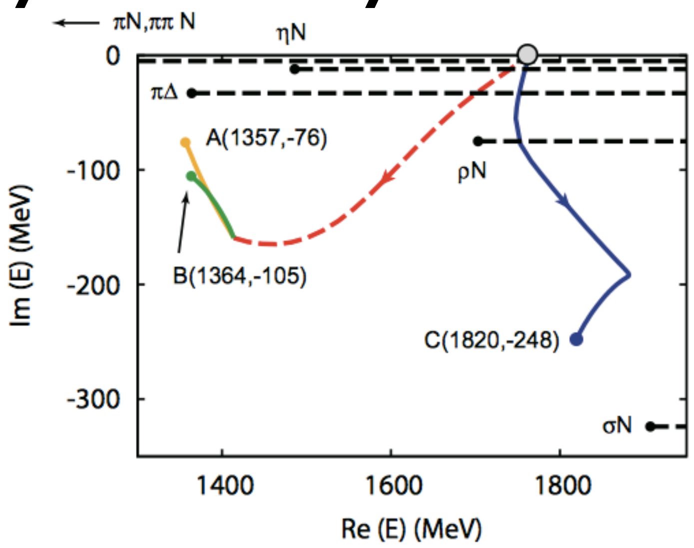
Explaining the Roper

- Quark model: p(2S), but generically too large mass
  e.g. Loring, Metsch, Petry, EPJA 10 (2001) 395 and many others…

- Hybrid ? Evidence from lattice to the contrary
  Dudek, Edwards, PRD 85 (2012) 054016

- Dynamically generated by coupled channels (no 'bare' state)
  Krehl, Hanhart, Krewald and Speth, PRC C 62 (2000) 025207
  Doring, Hanhart, Huang, Krewald and Meissner, NPA 829 (2009) 170

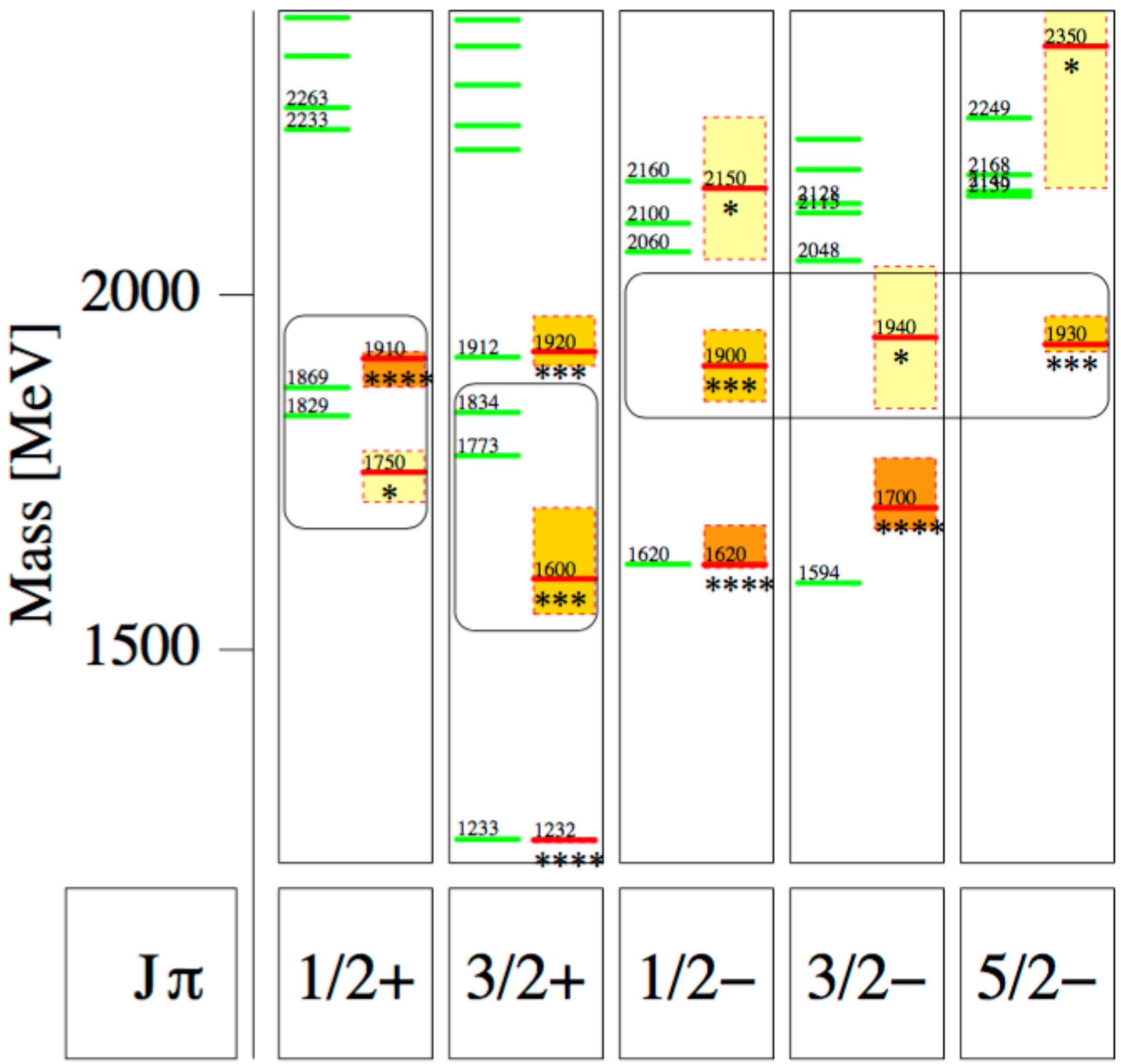
- Dynamically modified by coupled channels



Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama and Sato, PRL 104 (2010) 042302

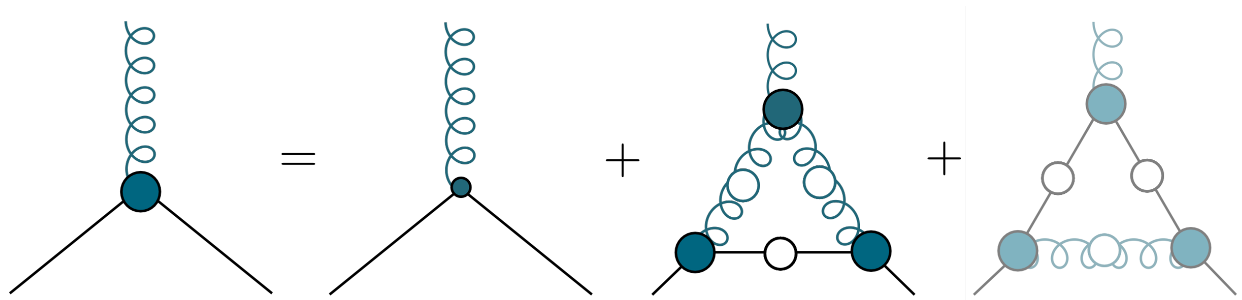
- 'bare' state via DSE/Faddeev (NJL, QCD inspired model)
  Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
  Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17

Strange baryon spectrum quark model



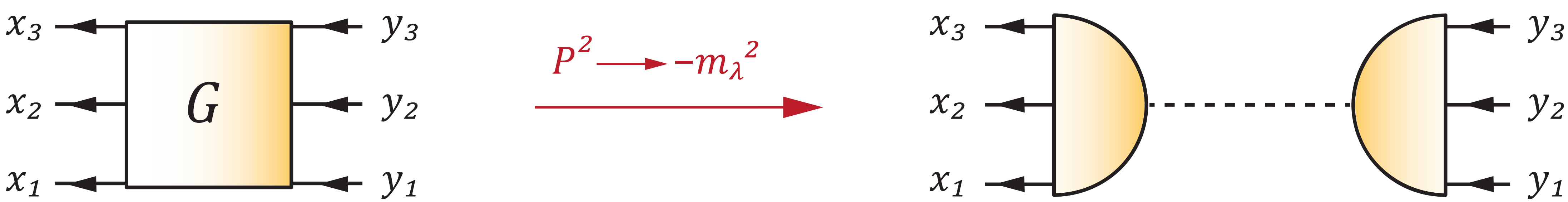
- light, strange and heavy spectrum probe QCD physics at different scales
- need flavor dependent QCD forces to explain spectrum
- models: parametrization via exchange of Goldstone-bosons

Ronniger, Metsch, EPJA 47 (2011) 162
see also Glozmann, Riska, Plessars et al.

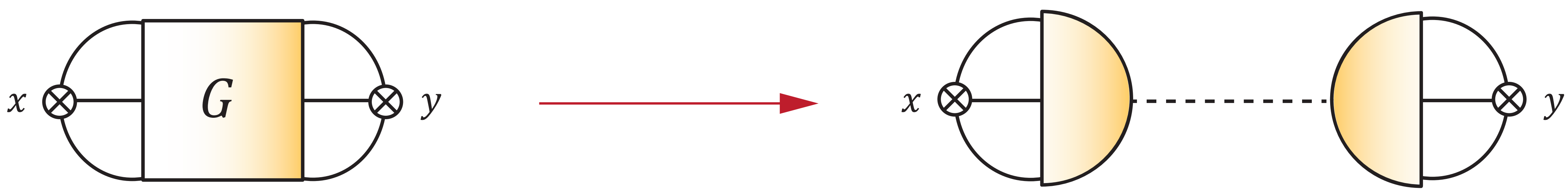


- can be determined from QCD

Extracting spectra from correlators

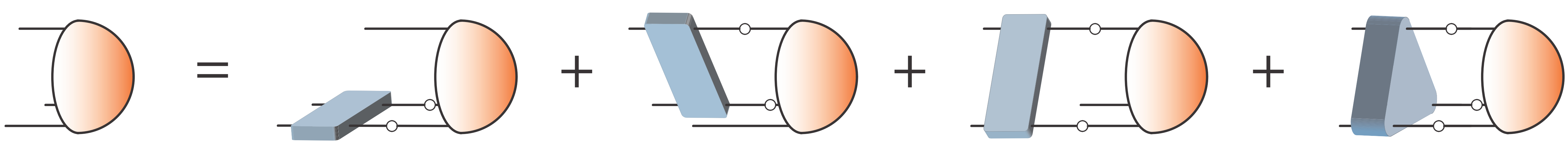


DSE/BSE



Lattice

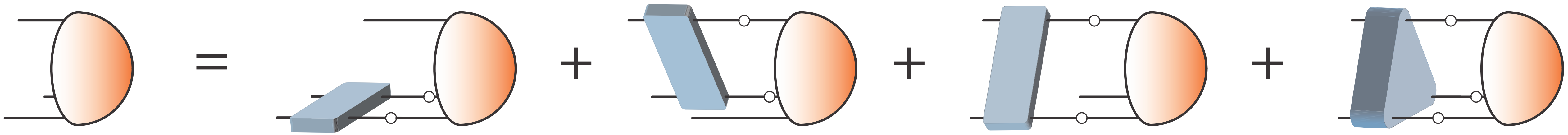
BSE for baryons (derived from equation of motion for G)



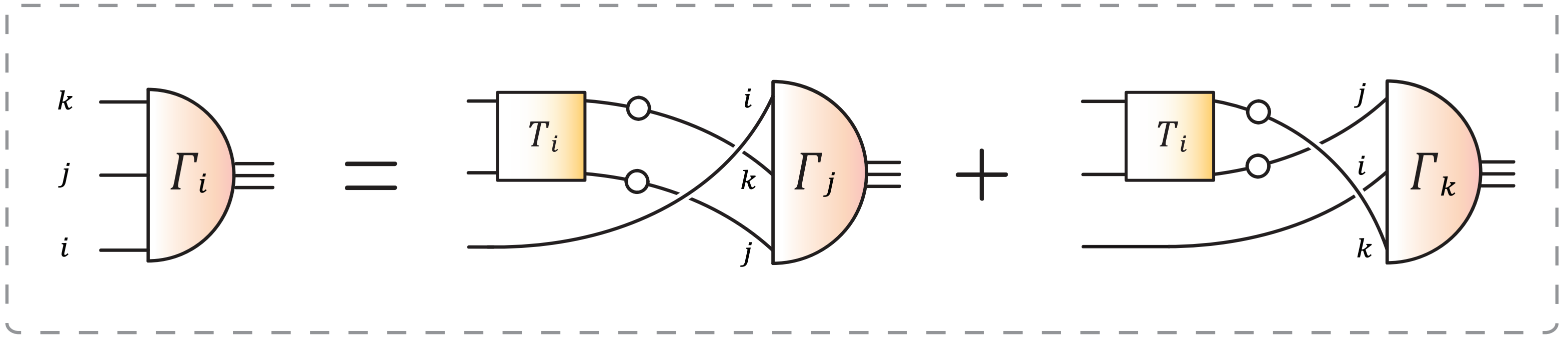
- exact equation for baryon 'wave function'

Diquark-Quark approximation

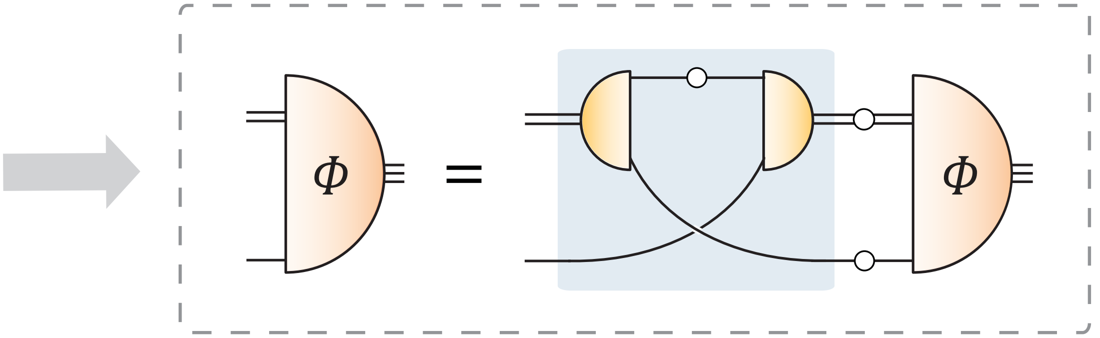
BSE for baryons (derived from equation of motion for G)



Faddeev equation (no three-body forces)



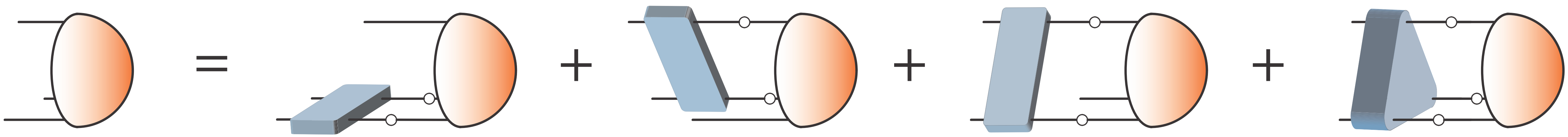
Diquark-quark



- Input in both cases: quark propagator and interaction

Diquark-Quark approximation

BSE for baryons (derived from equation of motion for G)



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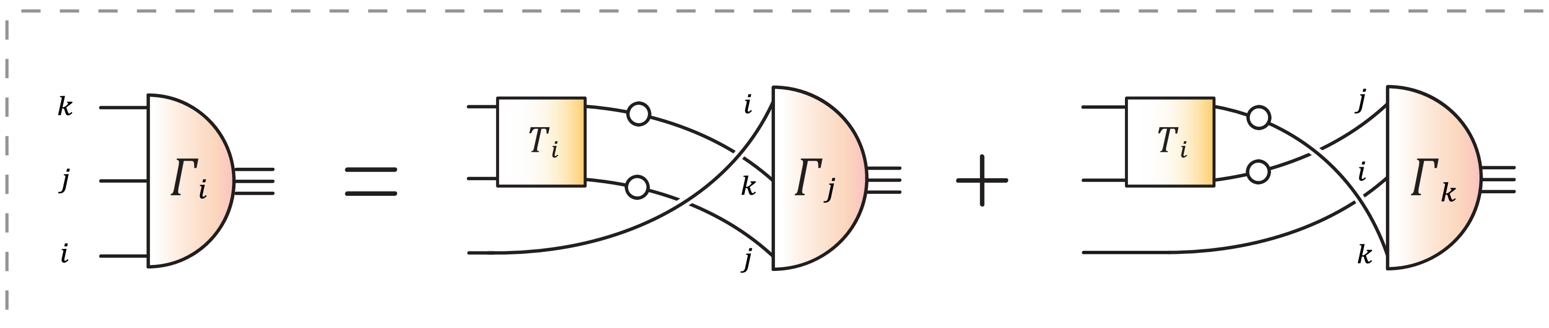
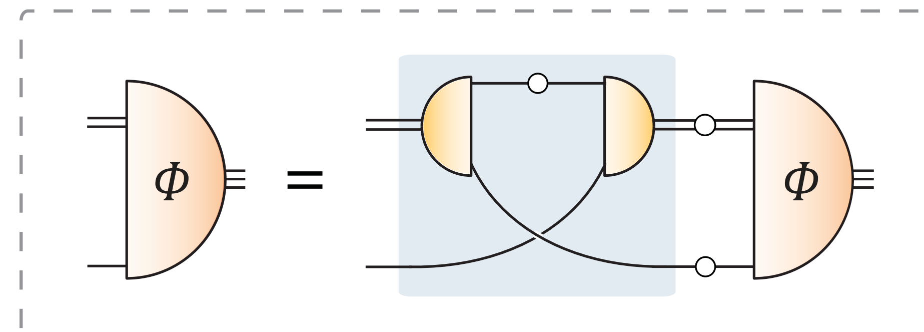
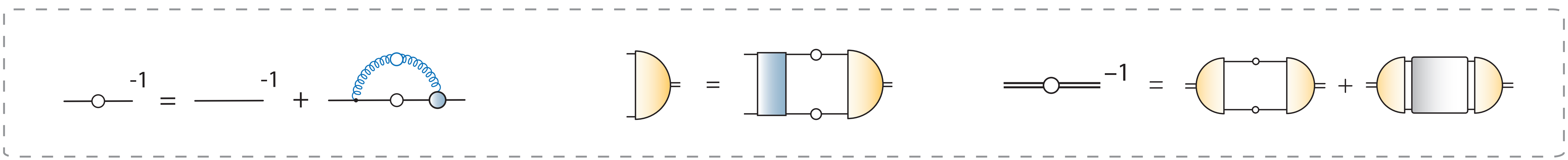


Diquark-quark





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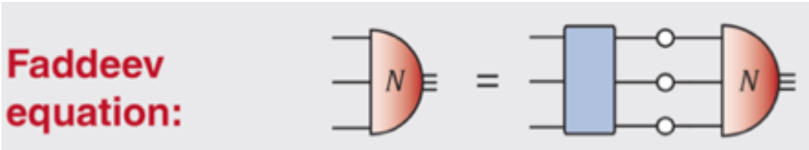
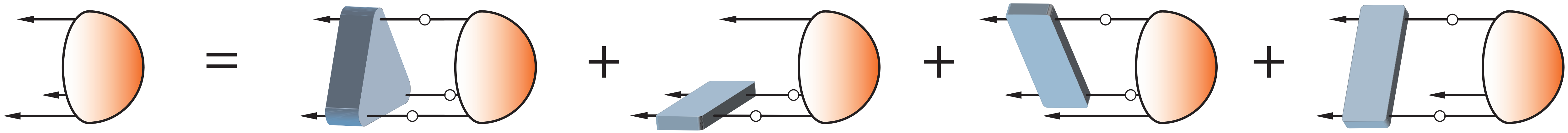


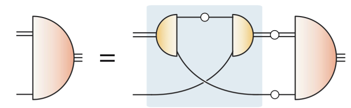


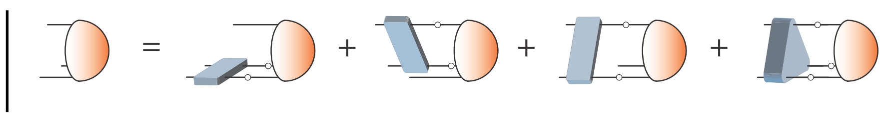
- relativistic bound state:
  - 64 tensor structures for nucleon: s, p, d - wave
  - 128 tensor structures for Delta: s, p, d, f - wave

$$D_i\,\gamma_5\,C \otimes D_j\,\Lambda_+(P)\,, \qquad D_i = \{\mathbb{1},\ \not p,\ \not q,\ \not P,\ [\not p,\not P],\ [\not q,\not P],\ [\not p,\not q],\ [\not p,\not q,\not P]\}\,,$$
$$\gamma_5\,D_i\,\gamma_5\,C \otimes \gamma_5\,D_j\,\Lambda_+(P)\,, \qquad \Lambda_\pm(P) = \tfrac{1}{2}(\mathbb{1} \pm \hat{\not P})\,,$$

DSE/BSE/Faddeev landscape (2015)

level of complexity →





			I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P=\pm$	N, Δ masses	✓	✓	✓	✓	✓
		N, Δ em. FFs	✓	✓	✓	✓	
		$N \to \Delta\gamma$	✓	✓	✓		
	$P=+$	N^*, Δ^* masses	✓	✓			
		$\gamma N \to N^*/\Delta^*$	✓	✓			
	$P=-$	N^*, Δ^* masses		✓			
		$\gamma N \to N^*/\Delta^*$					
strange		ground states		✓			
		excited states					
		em. FF					
		TFFs					
c/b		ground states					
		excited states					
			Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF

DSE/BSE/Faddeev landscape

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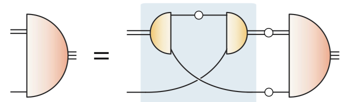

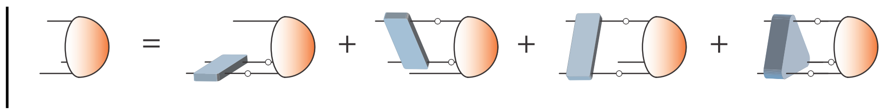

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		$\gamma N \to N^*/\Delta^*$	✓	✓			
	$P = -$	N^*, Δ^* masses	✓	✓	✓	✓	
		$\gamma N \to N^*/\Delta^*$					
strange		ground states	✓	✓	✓	✓	
		excited states	✓	✓	✓	✓	
		em. FF			✓	✓	
		TFFs				✓	
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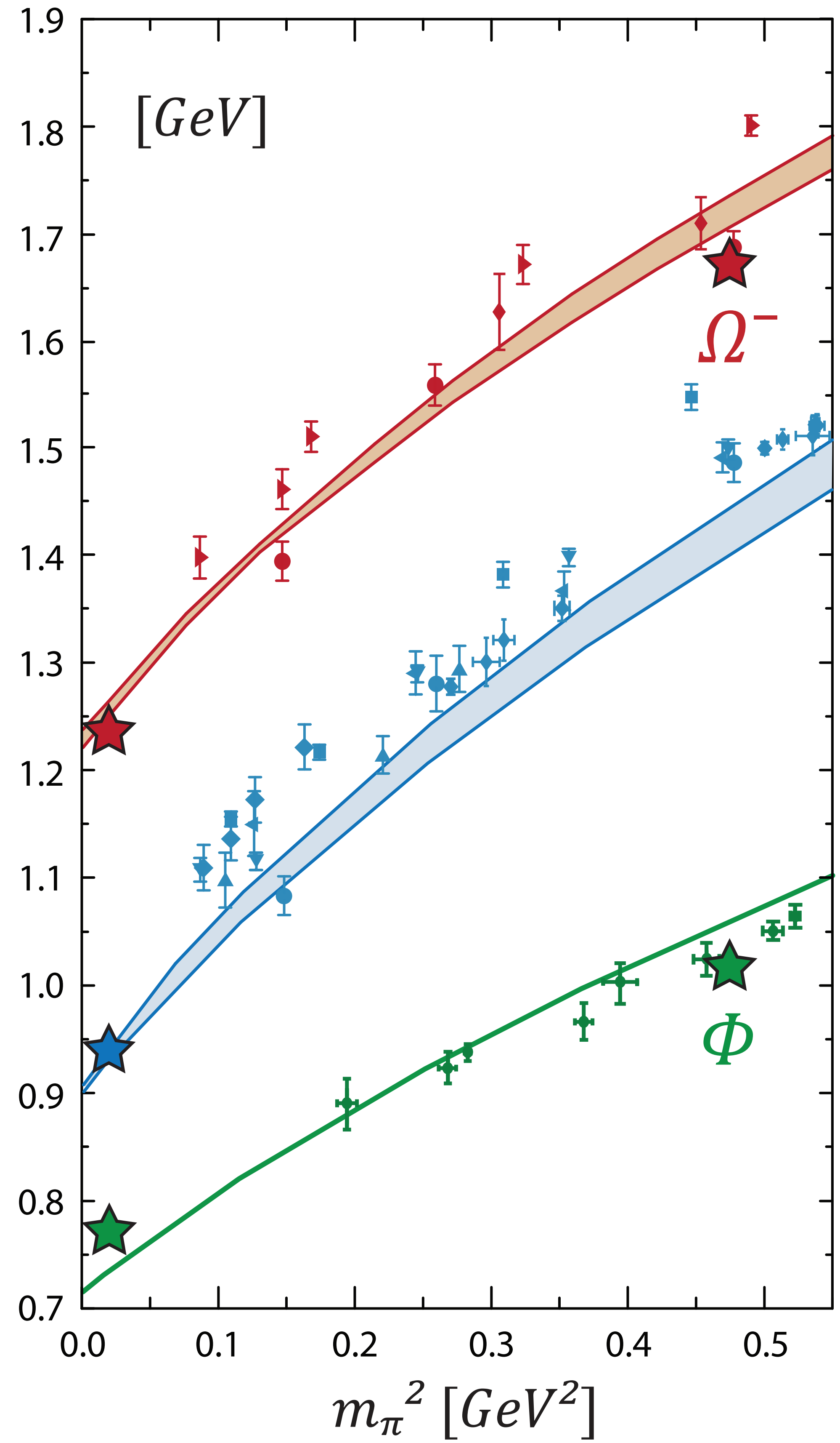
		excited states		✓		✓	
			Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF

Baryon masses

- first covariant three-body calculations !
- grosso modo: consistent description of mesons and baryons
- wave functions contain sizable p-wave contributions



Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

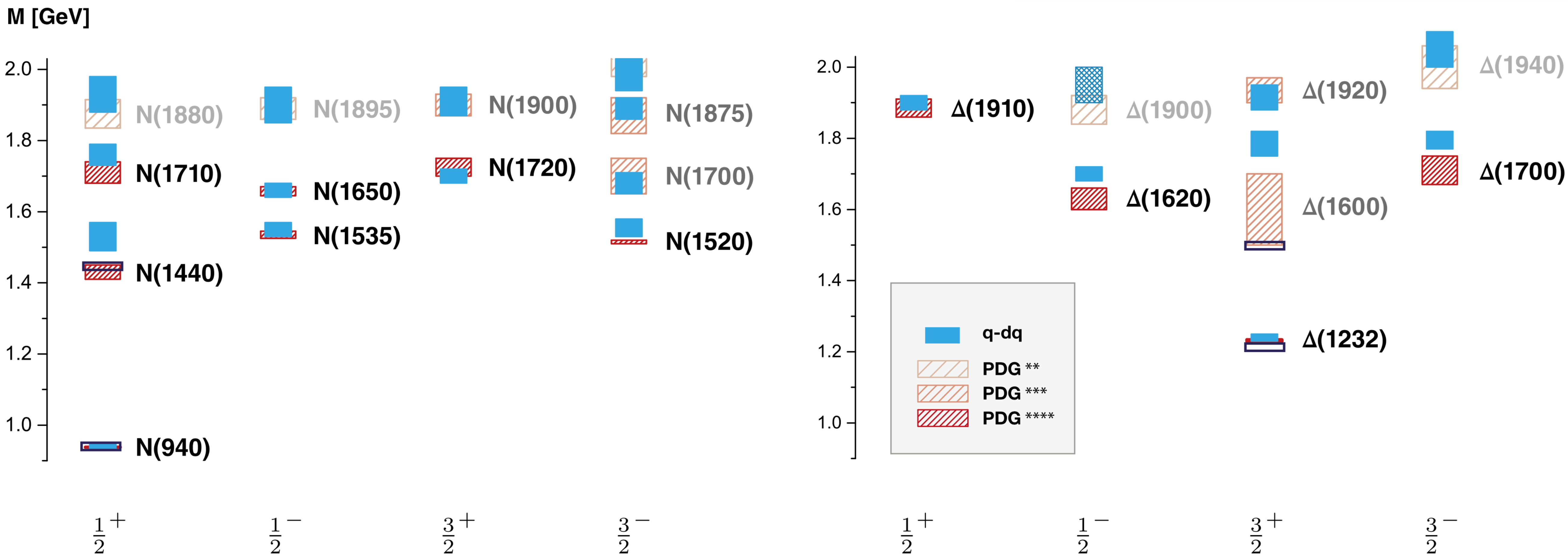
Nucleon:
Eichmann et al., PRL 104 (2010), PRD 84 (2011)

ρ–meson:
Maris & Tandy, PRC 60 (1999)

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
Eichmann, PRD 84 (2011)
Sanchis-Alepuz, Eichmann, Villalba-Chavez, Alkofer, PRD (2012)

Light baryon spectrum:

- 3 parameters + $m_{u,d,s}$ (all fixed in meson sector)



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)

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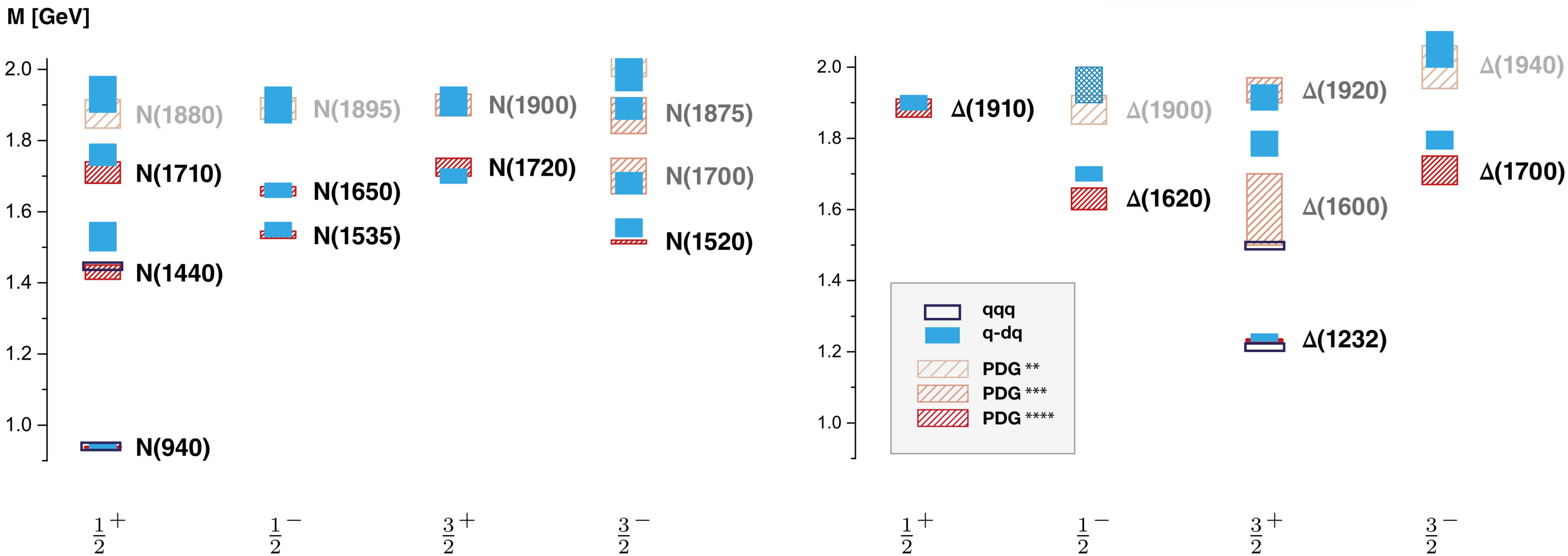

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Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- three-body agrees with diquark-quark where applicable

Relativistic proton

$J^P = \left(\frac{1}{2}\right)^+$

non-relativistic

three quarks with spin 1/2: $S = 1/2$ or $S = 3/2$

parity $P = (-1)^L$: $L = 0$ or $L = 2$

relativistic

64 components in wave function:
- 8 s-wave (L=0)
- 36 p-wave (L=1)
- 20 d-wave (L=2)

$P = (-1)^L$

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

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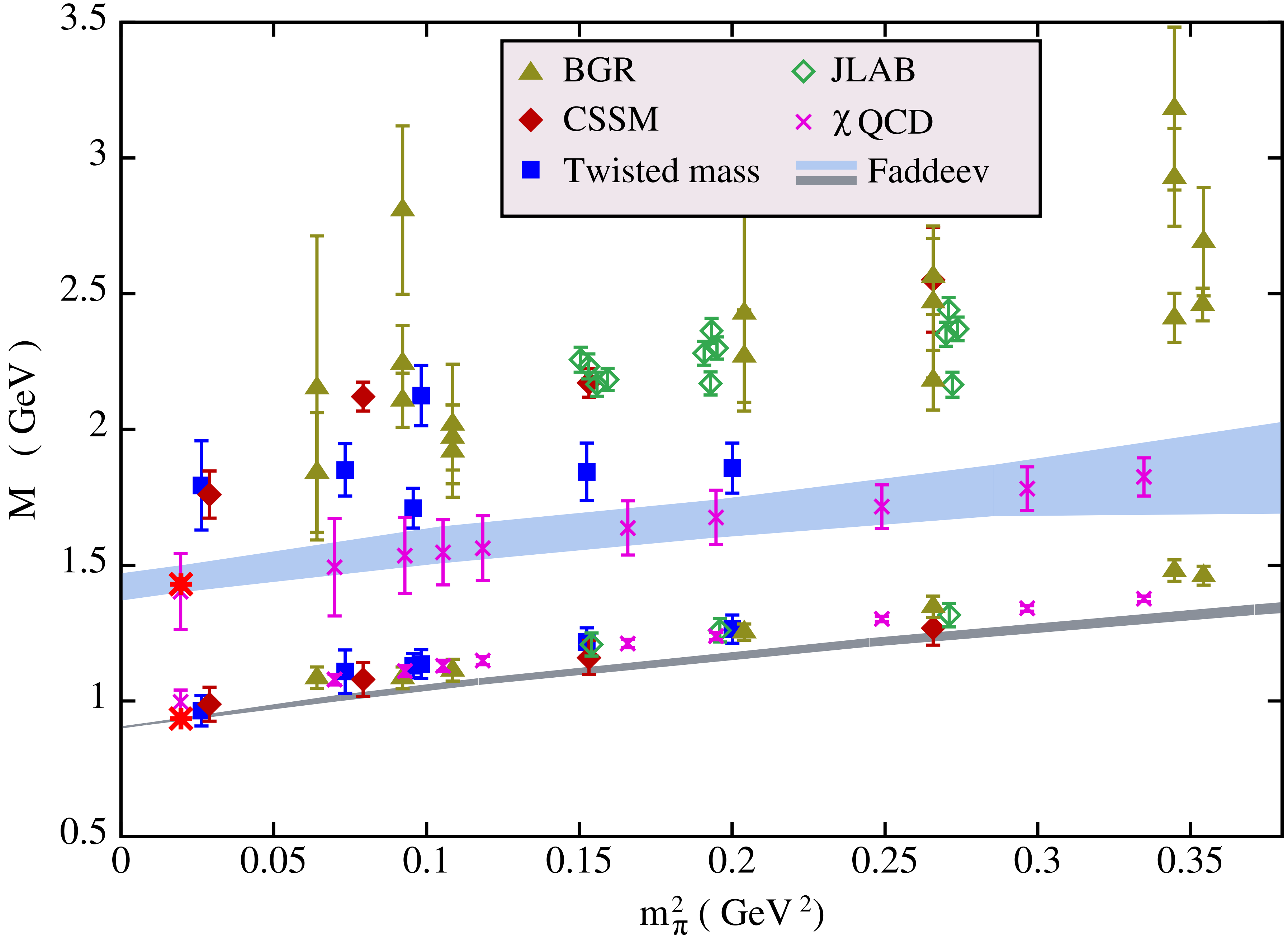
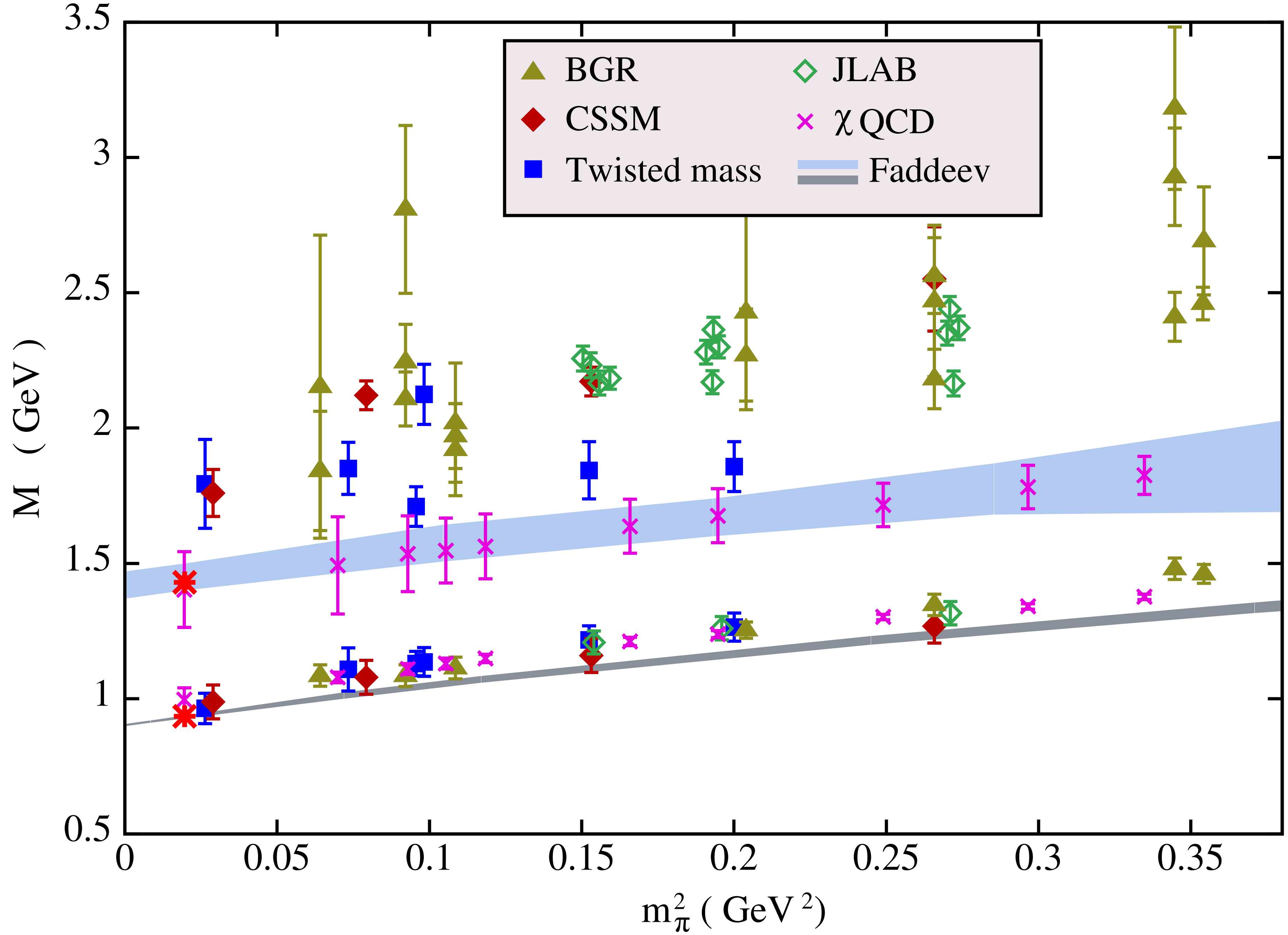
Mass evolution



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016)

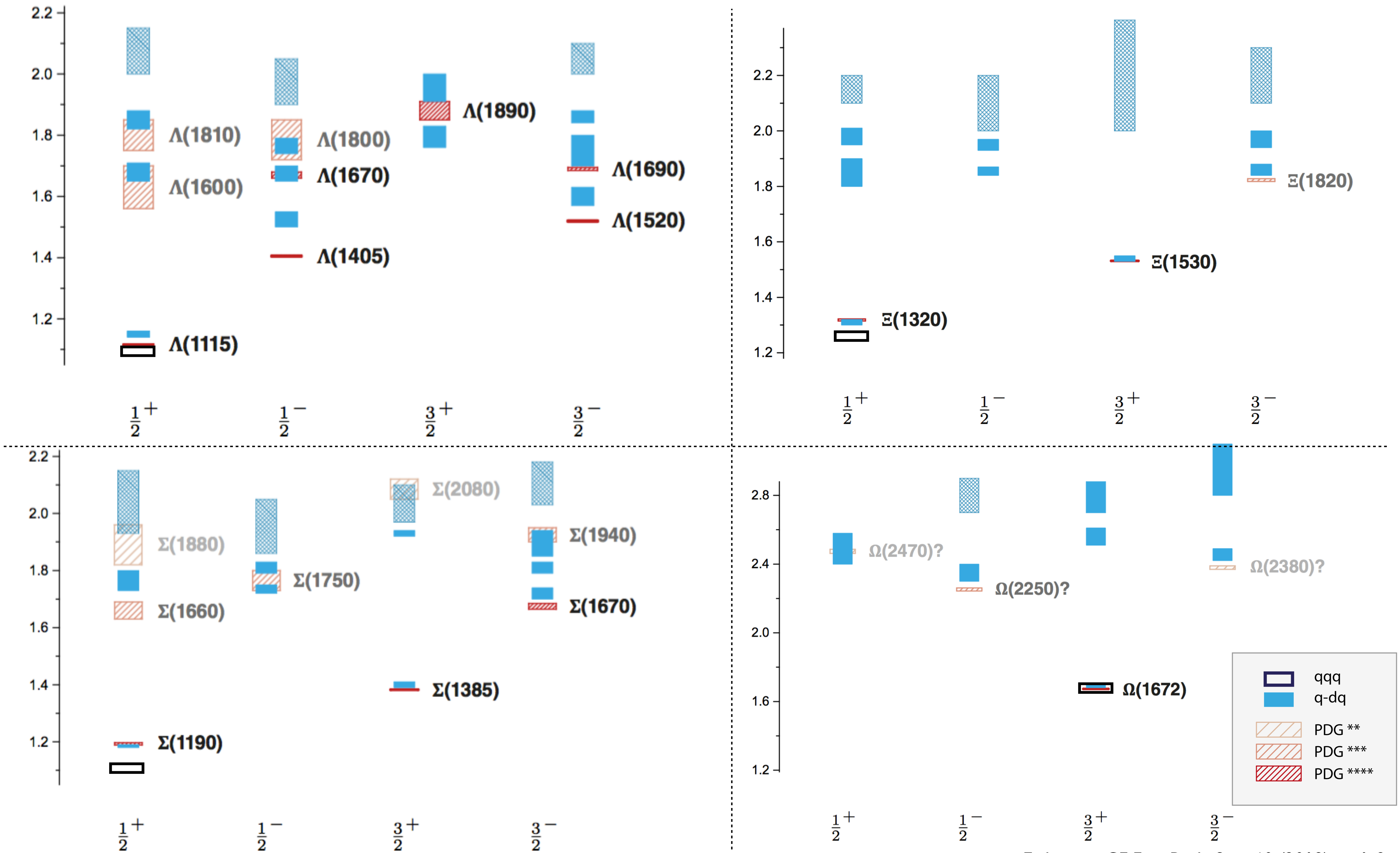
Mass evolution



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016)

- Mass evolution as expected for three-body state…

Strange baryon spectrum: DSE-RL (preliminary !)



Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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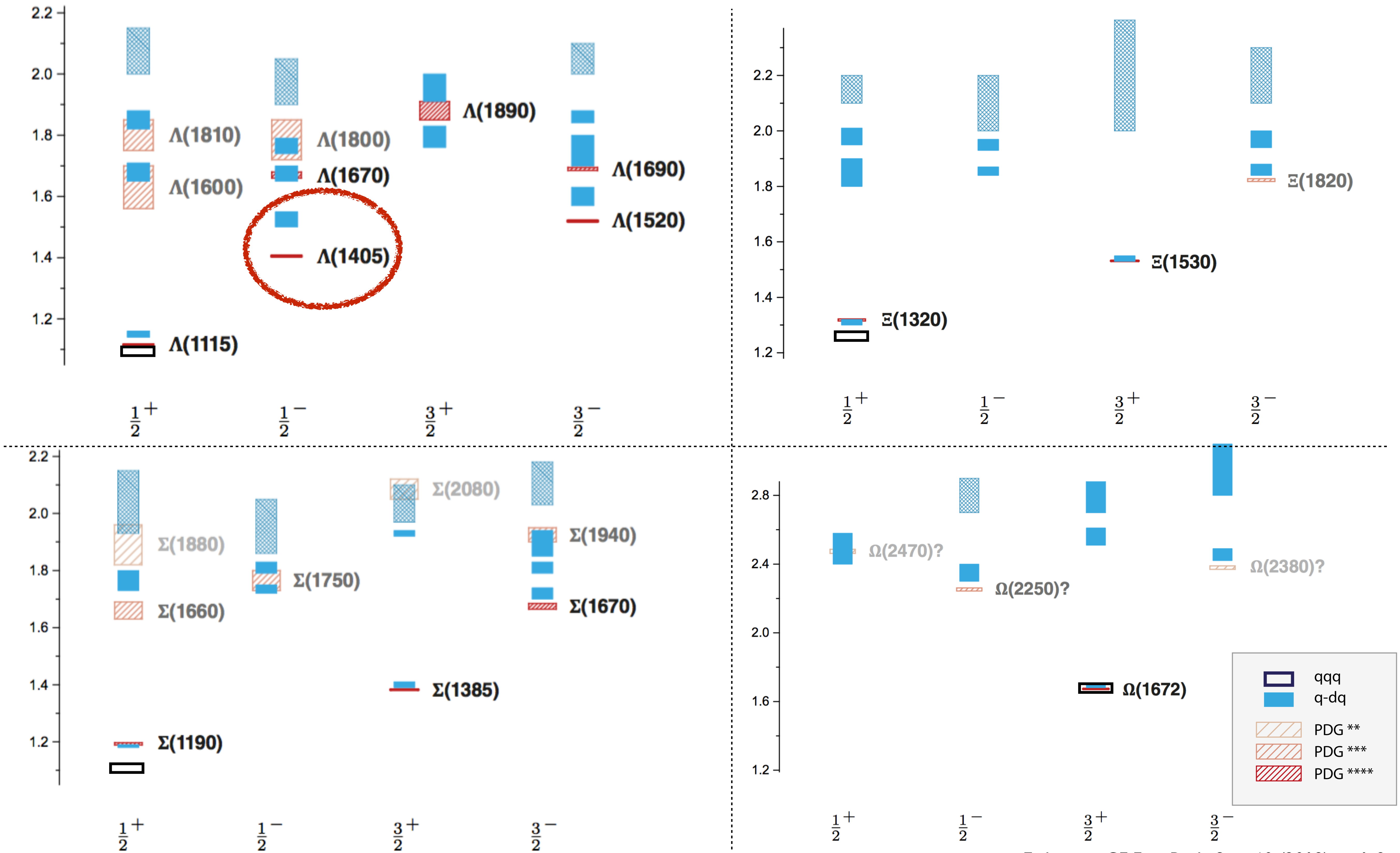

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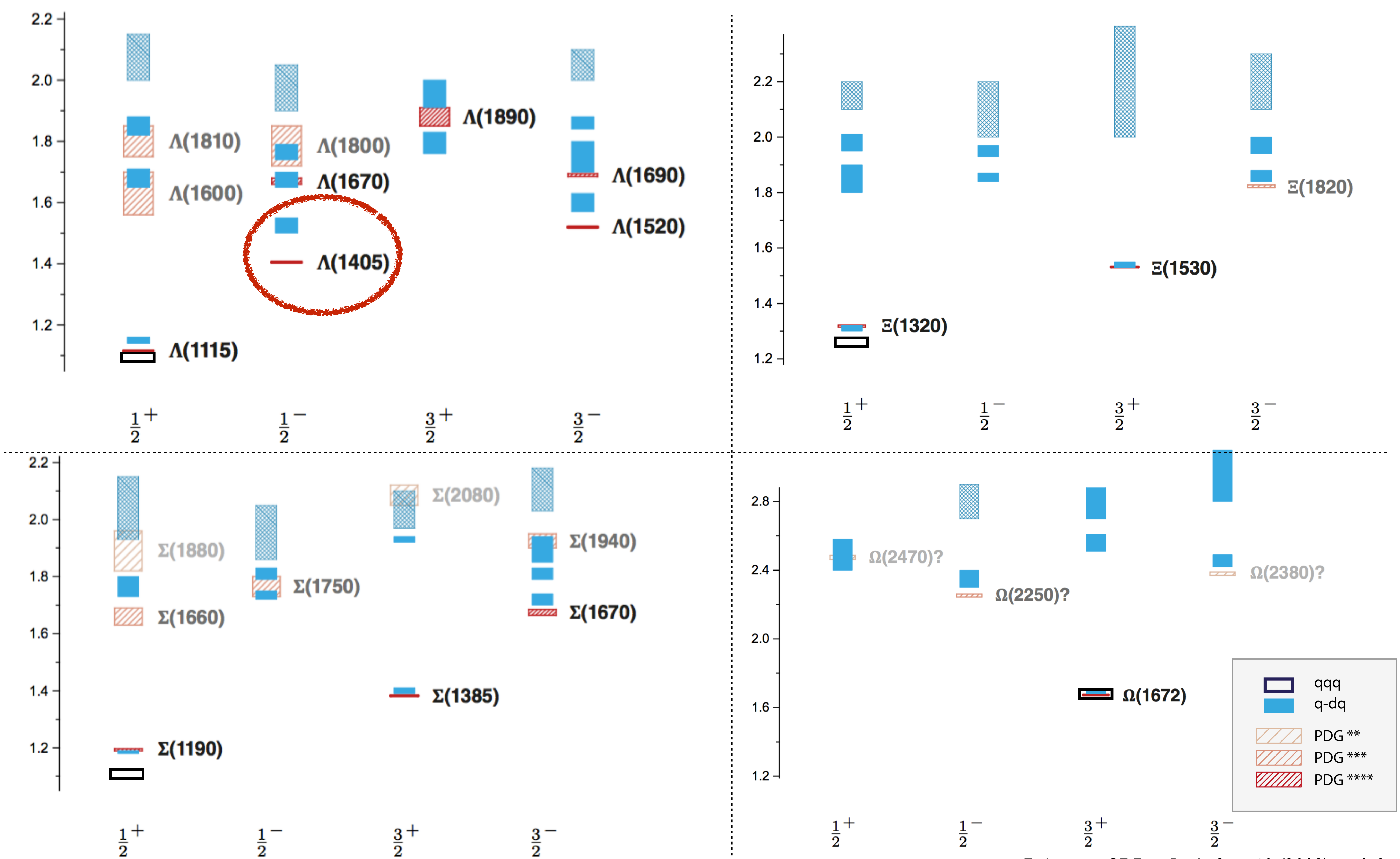

New states: Bonn-Gatchina (talk of M. Matveev at N*2019)

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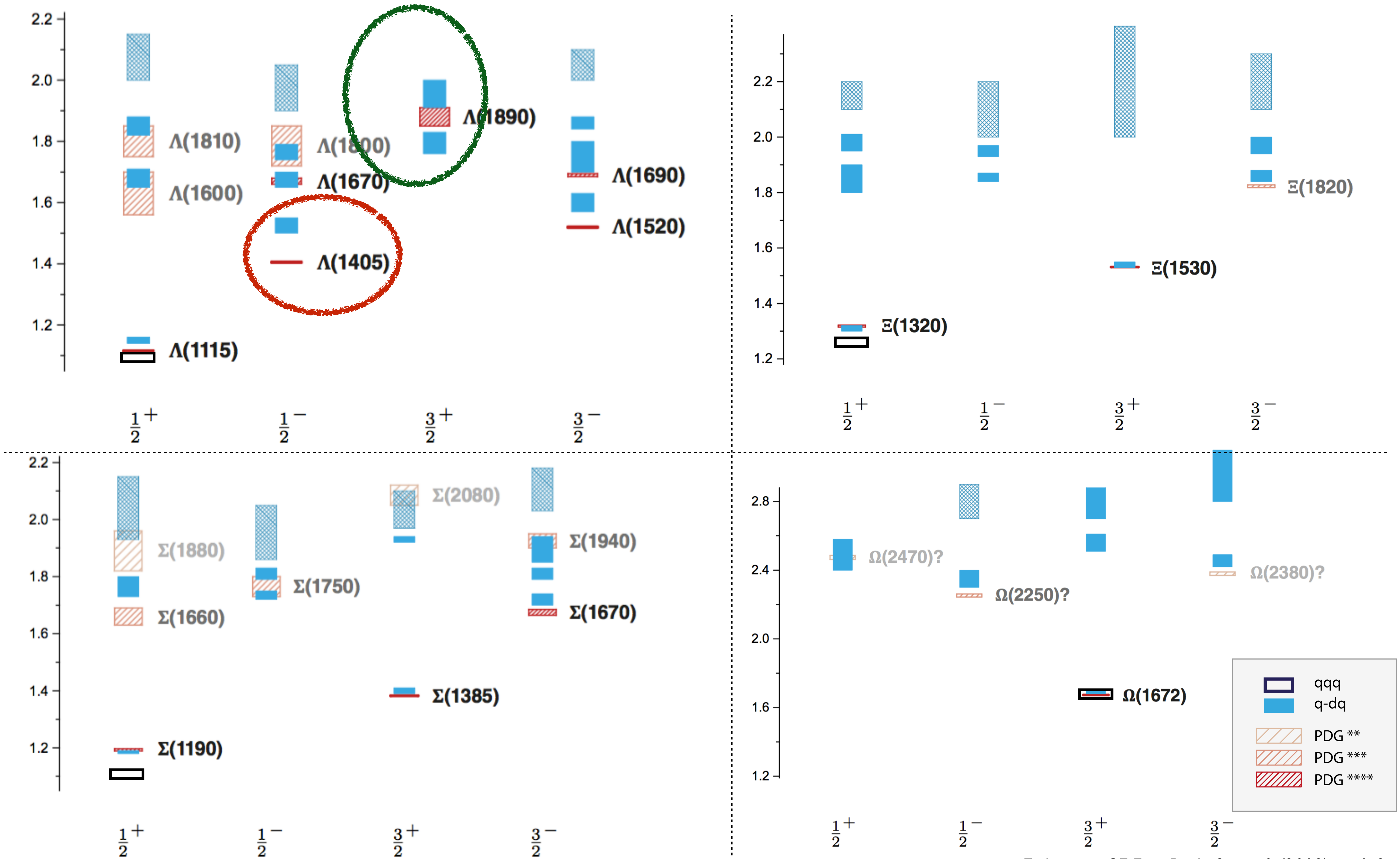

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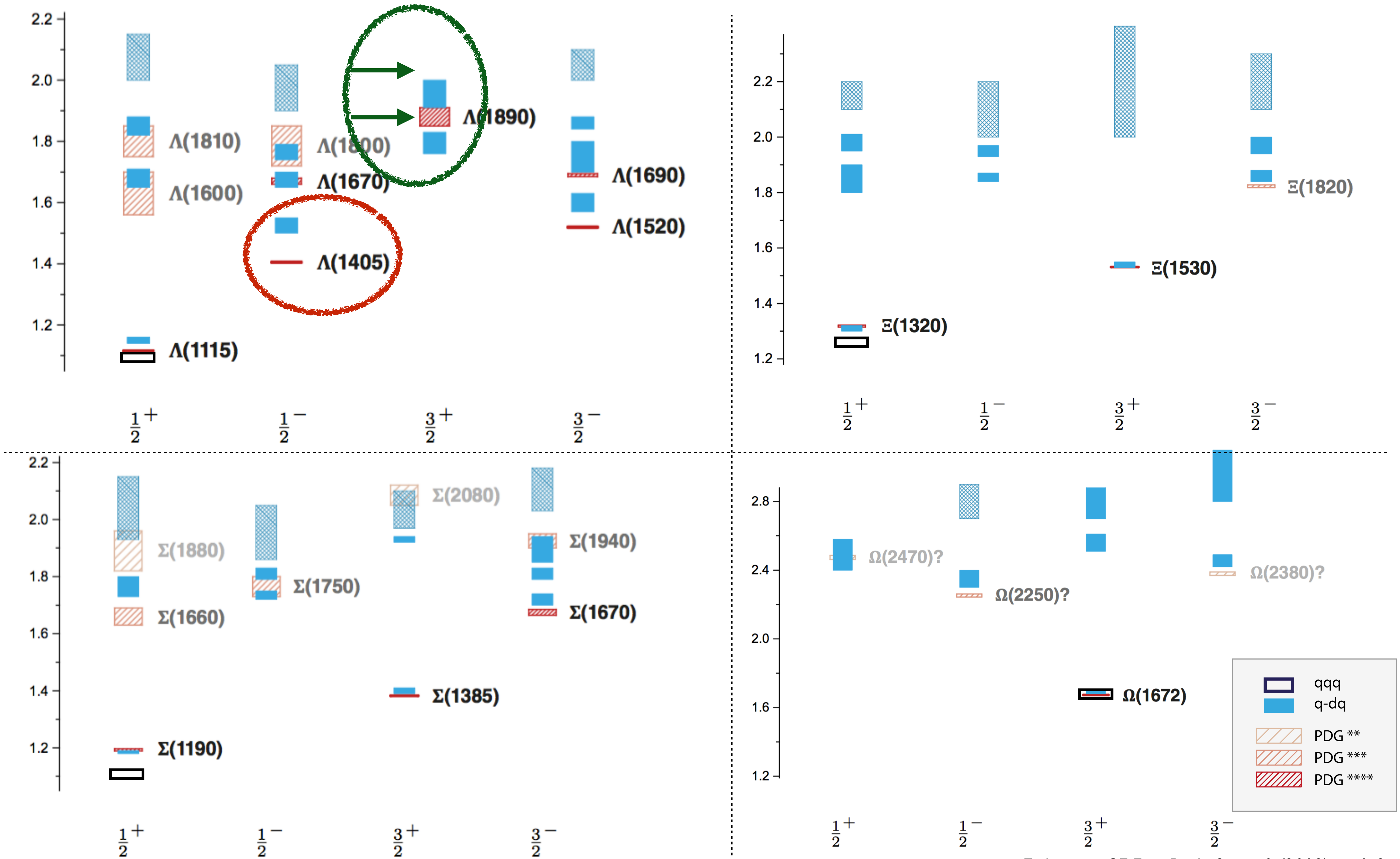

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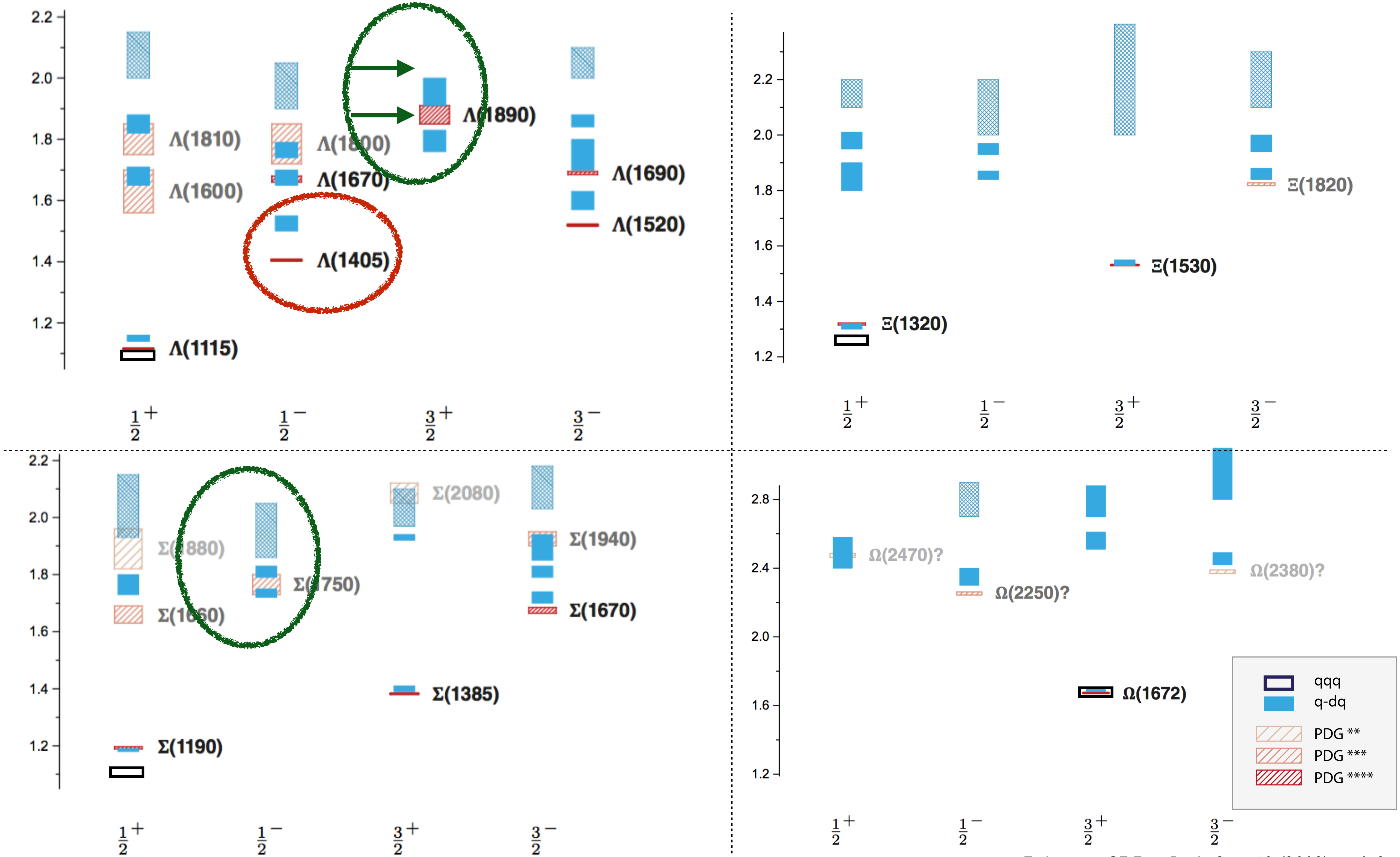

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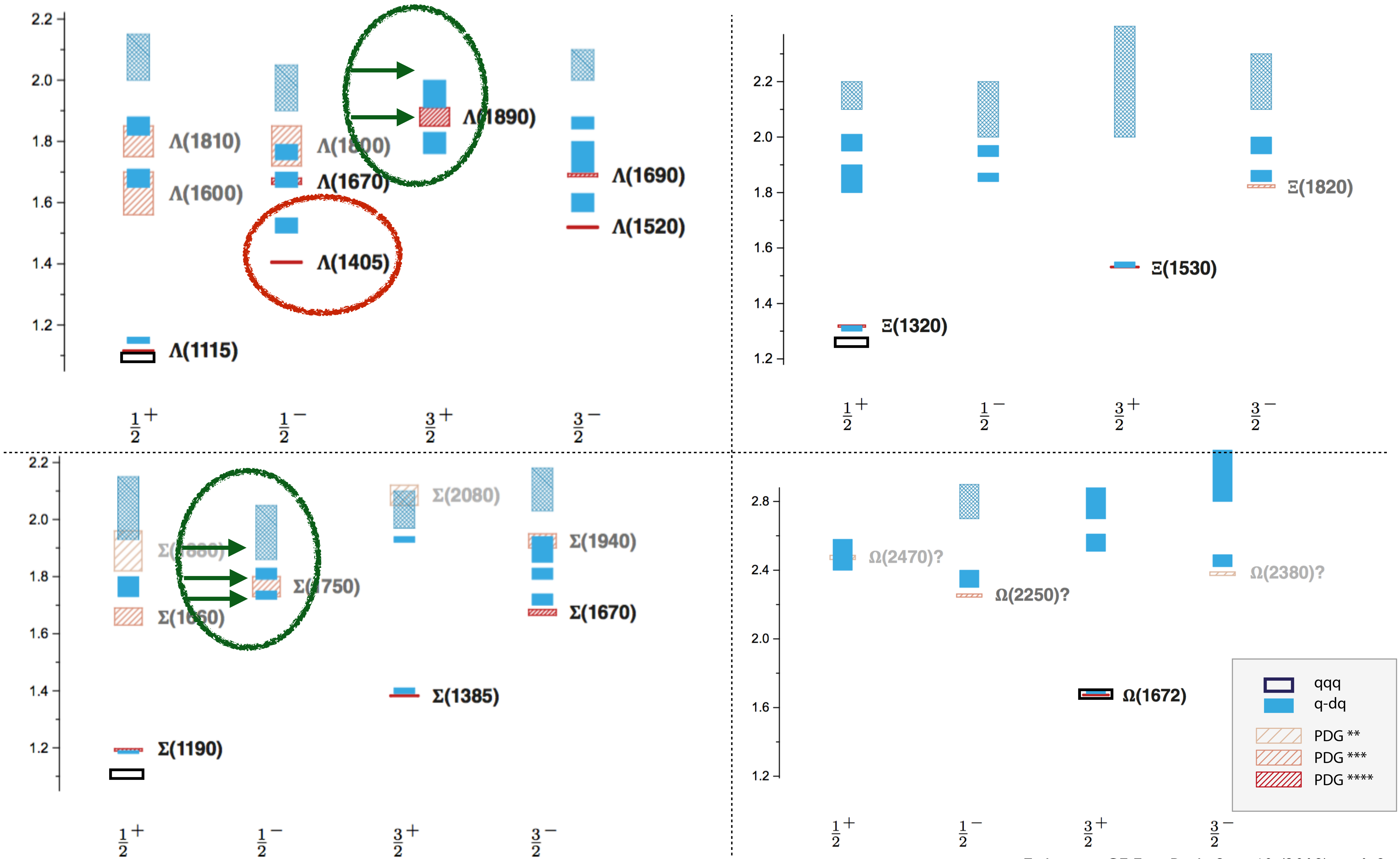

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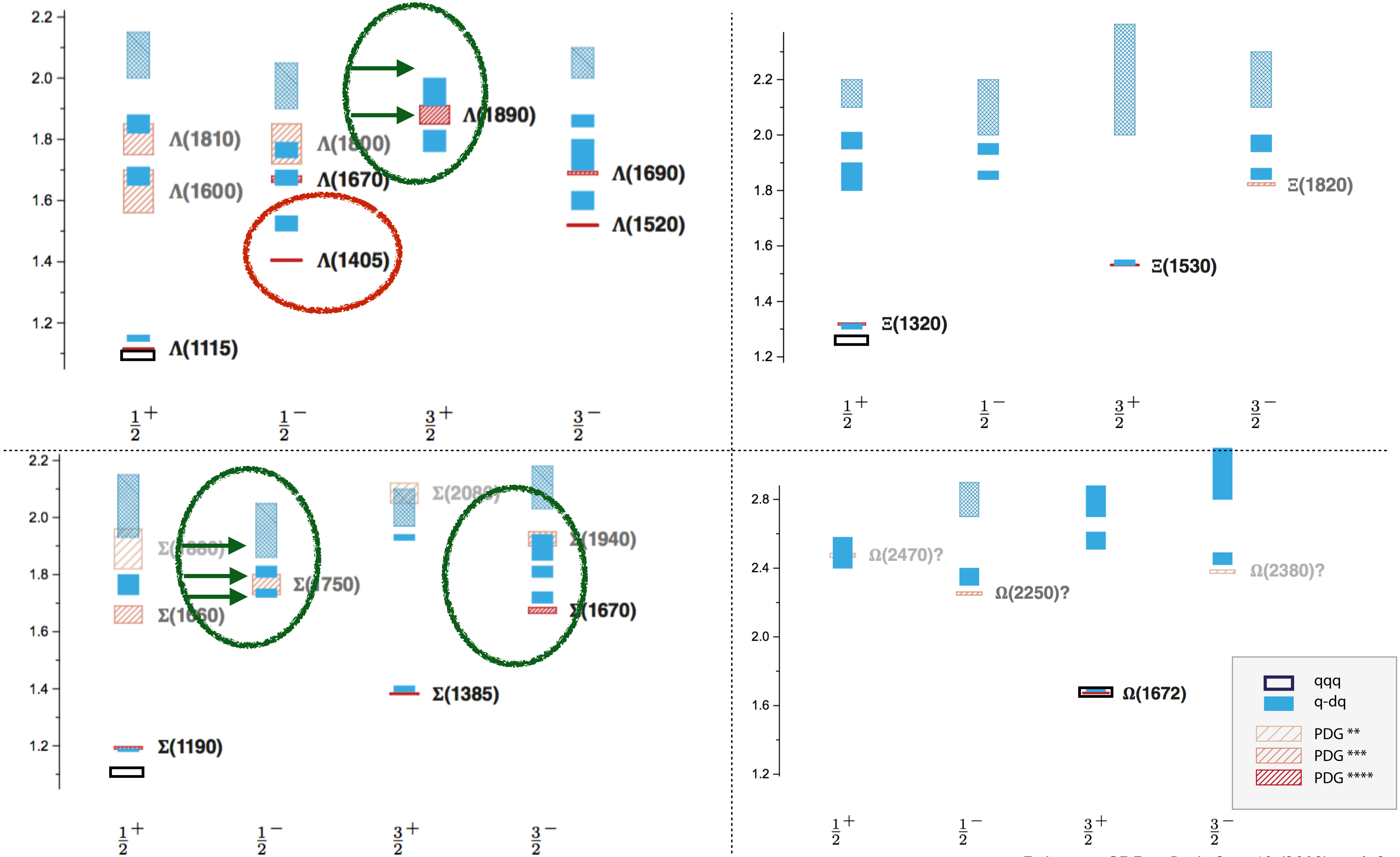

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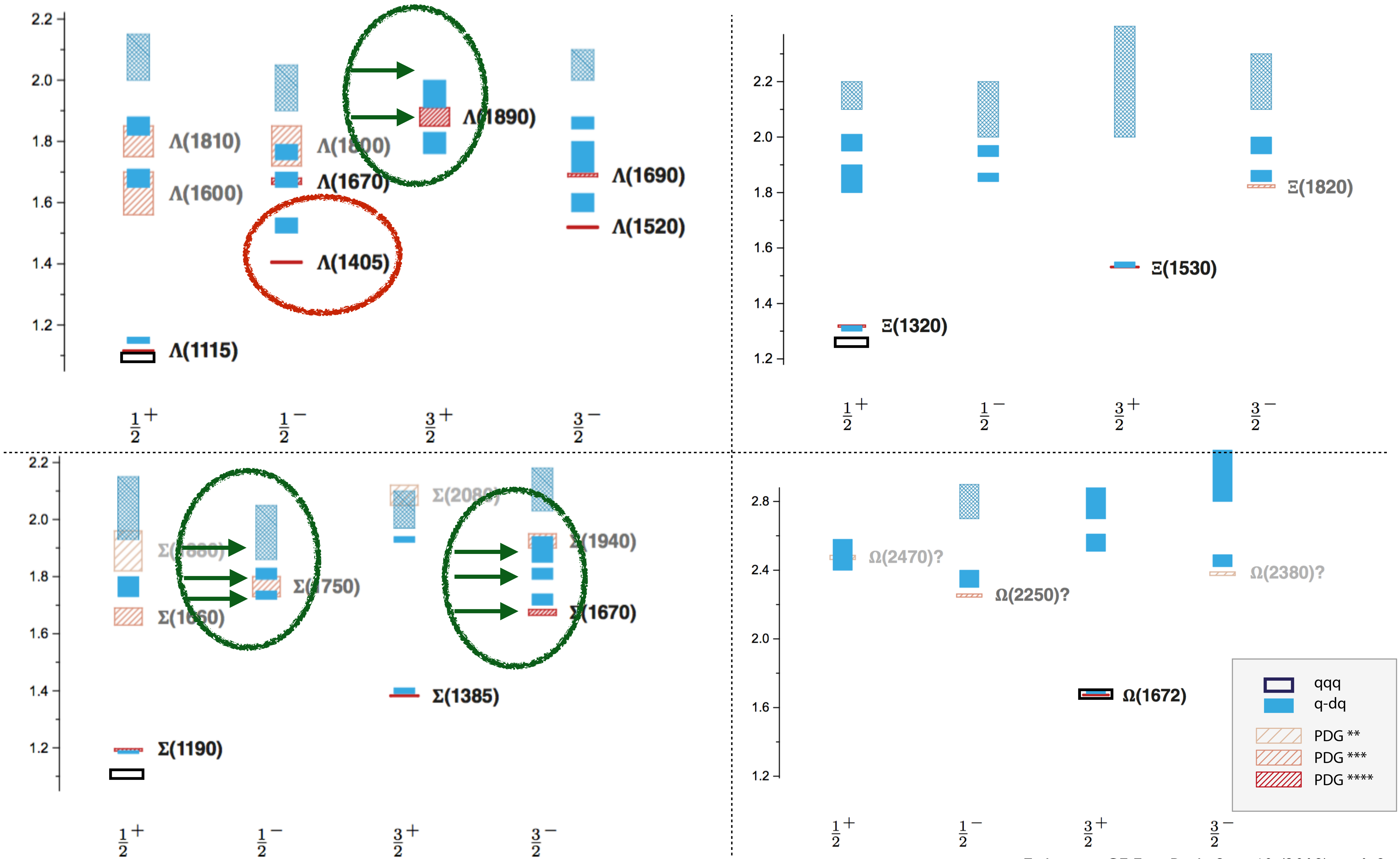

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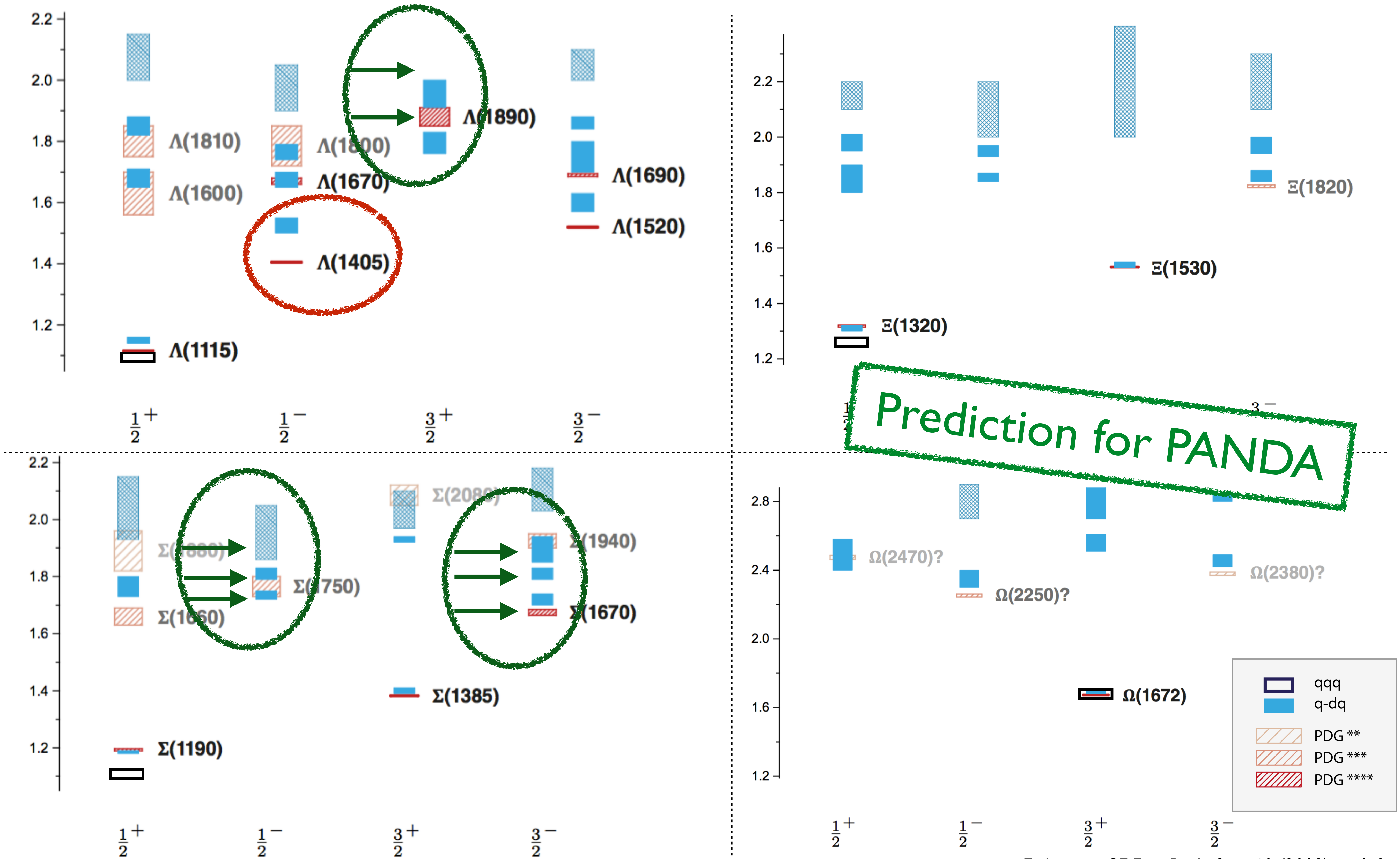

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Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better

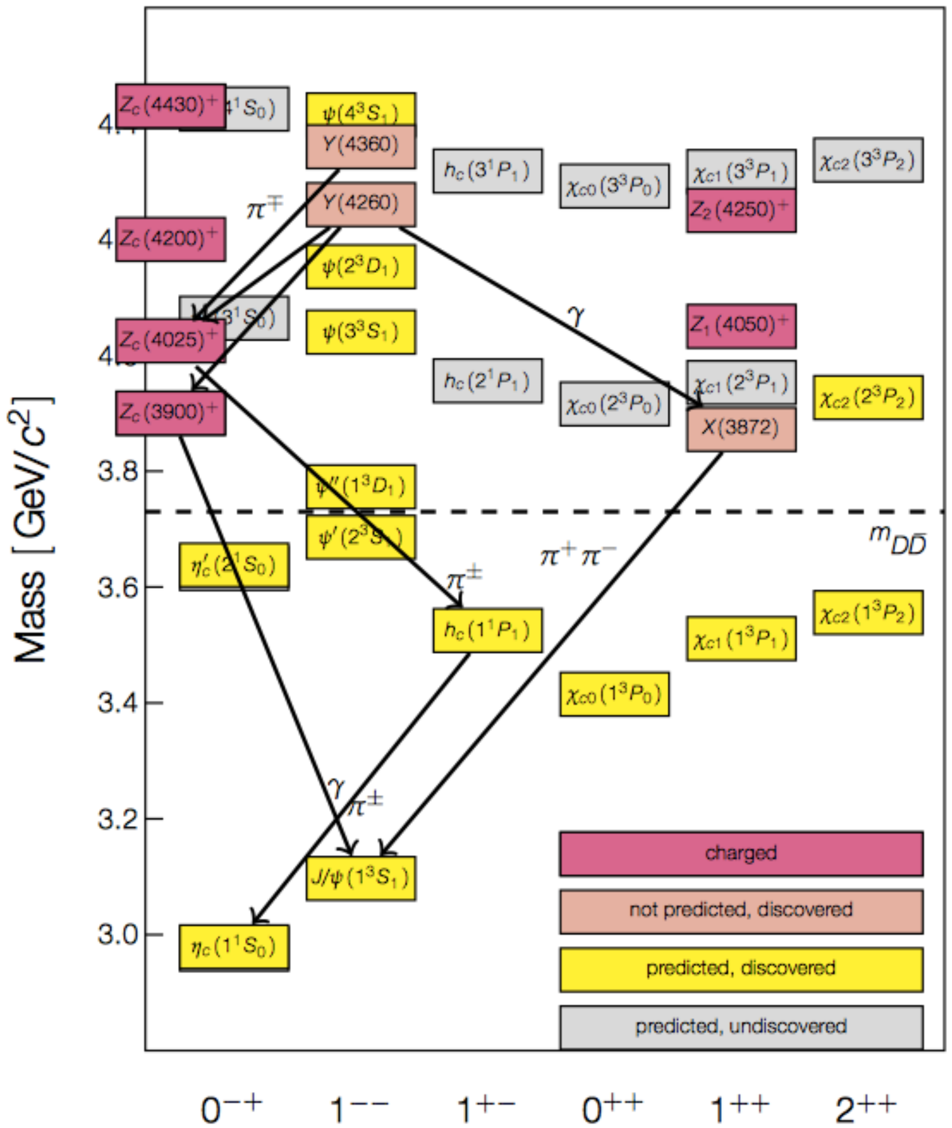
2. Properties of QCD and correlation functions
   - Confinement and dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…

3. Bound states and applications (rainbow-ladder approximation)
   - Mesons
   - Baryons
   - Tetraquarks

4. Form factors and decays (rainbow-ladder approximation)
   - Meson and baryon form factors
   - The anomalous magnetic moment of the muon

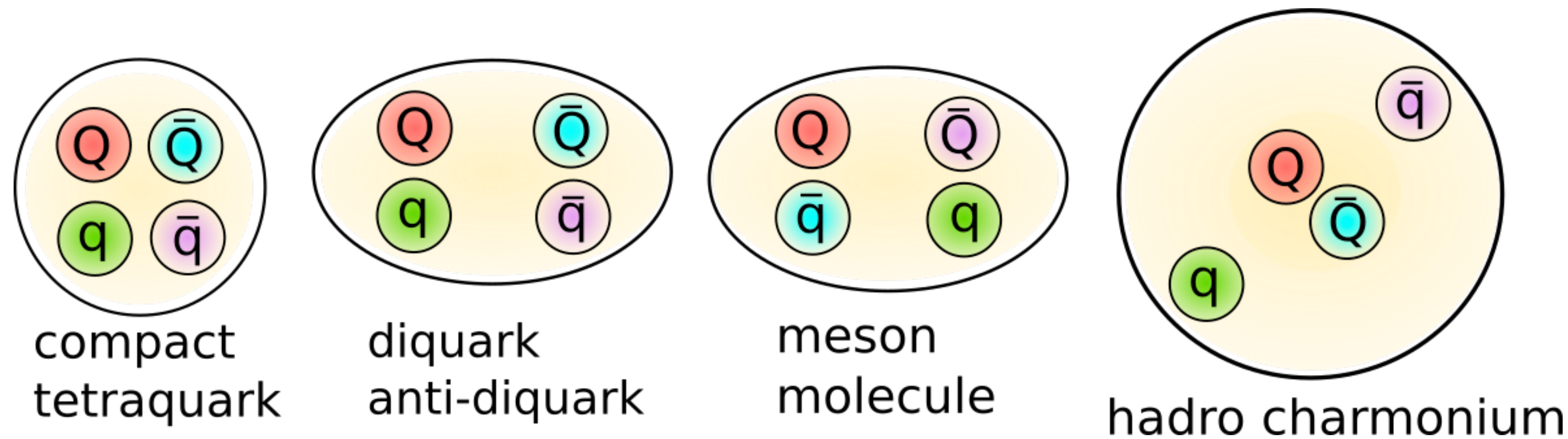
5. Beyond rainbow ladder
   - Confinement and glueballs
   - Hadron results beyond rainbow-ladder

Tetraquark candidates in charmonium region



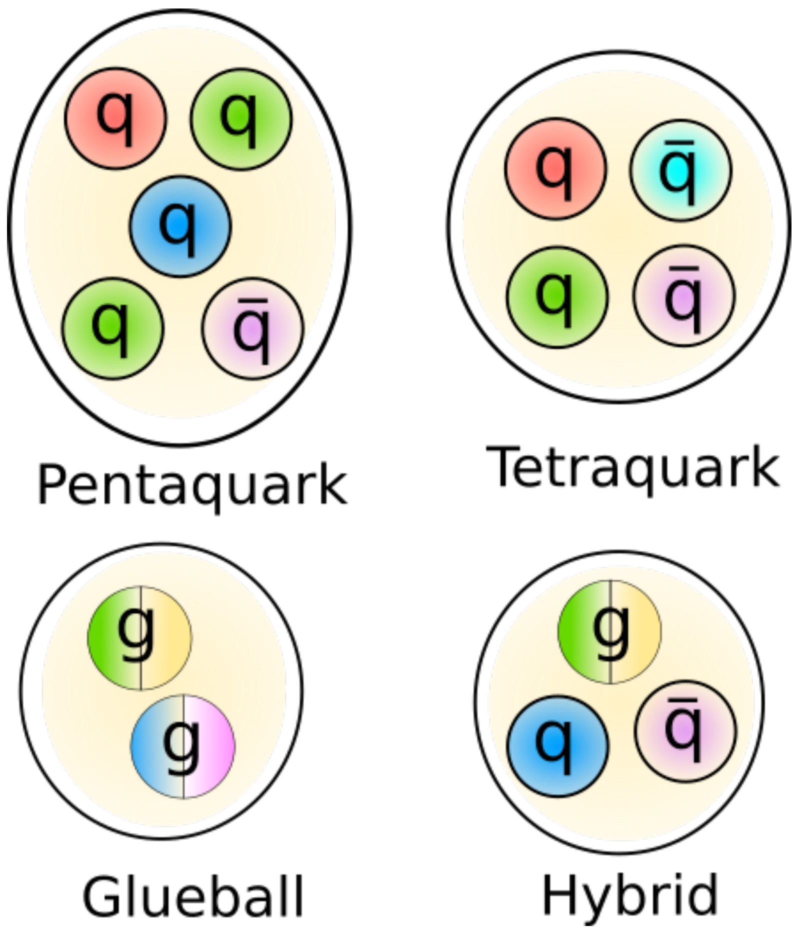
Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??

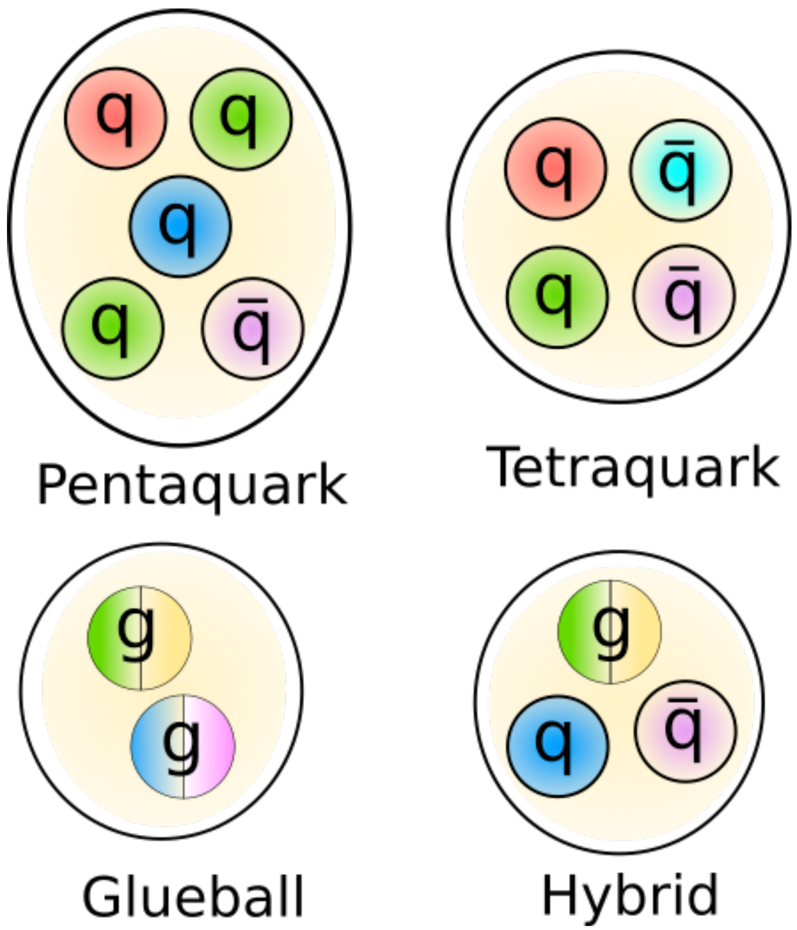


Related to details of underlying QCD forces between quarks

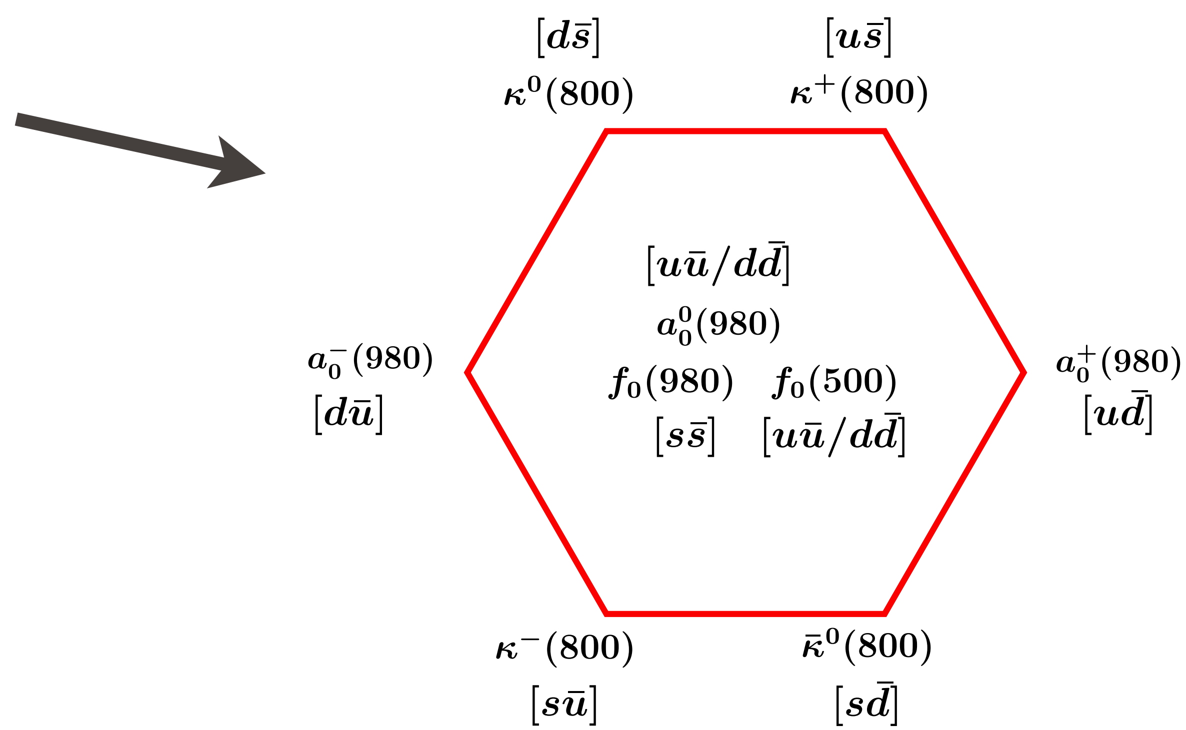
Tetraquarks in the light meson sector



K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)



Light meson sector: scalars!



$f_0(980)$ [j] $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 40$ to 100 MeV

$f_0(980)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	476
$K\overline{K}$	seen	36
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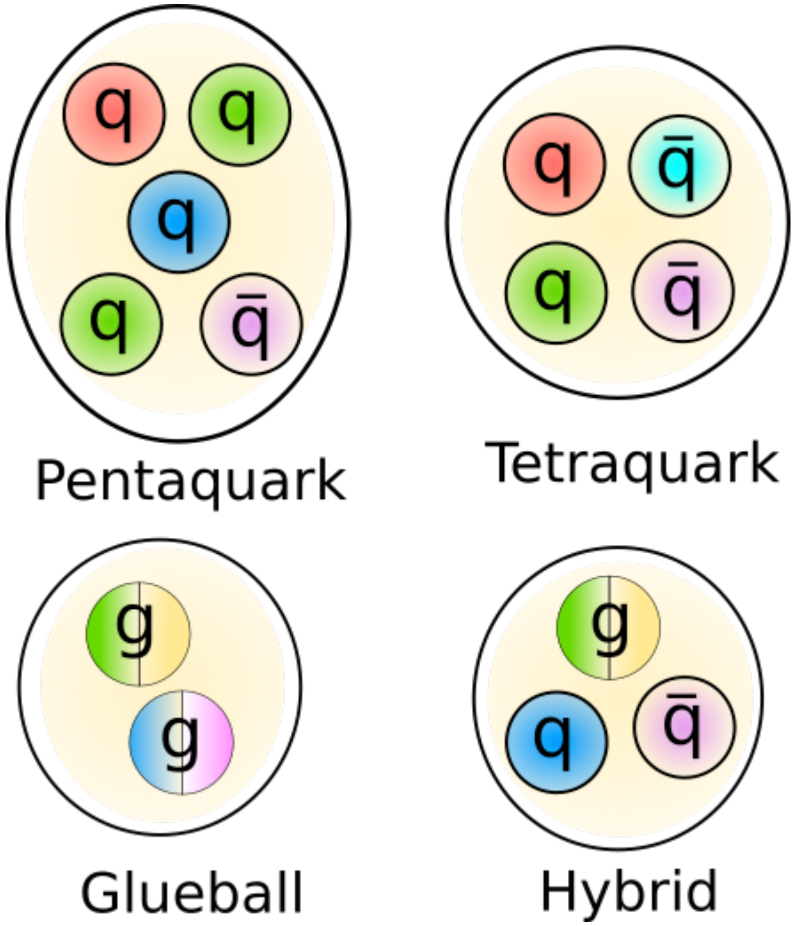

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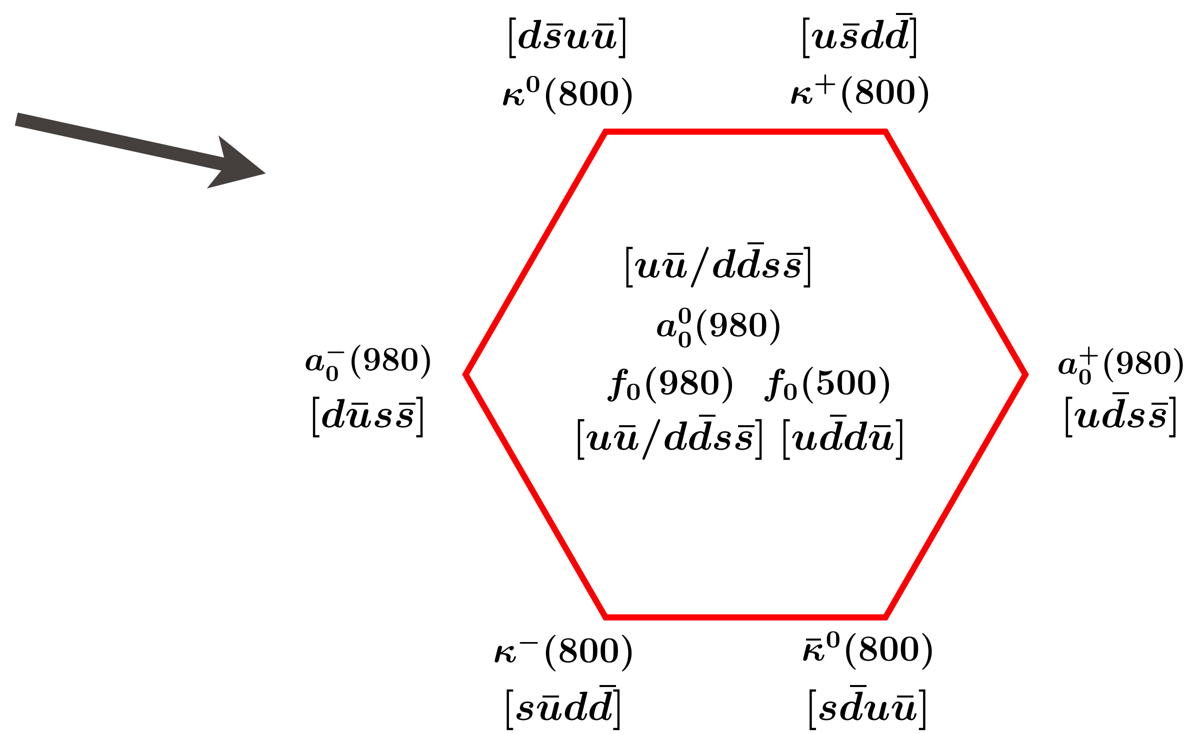

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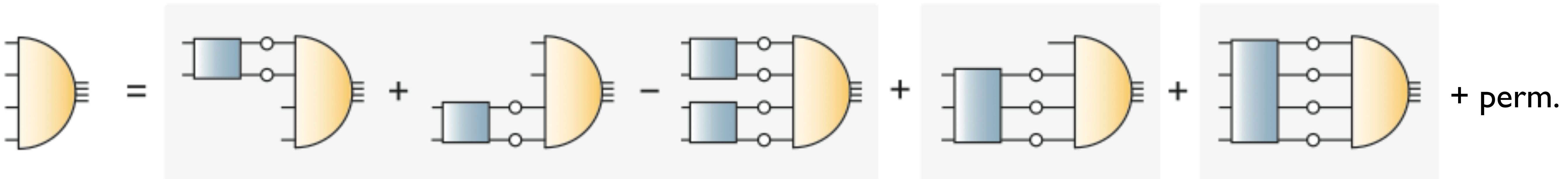
Tetraquarks from the four-body interaction

Exact equation:



Two-body interactions

Three- and four-body interactions

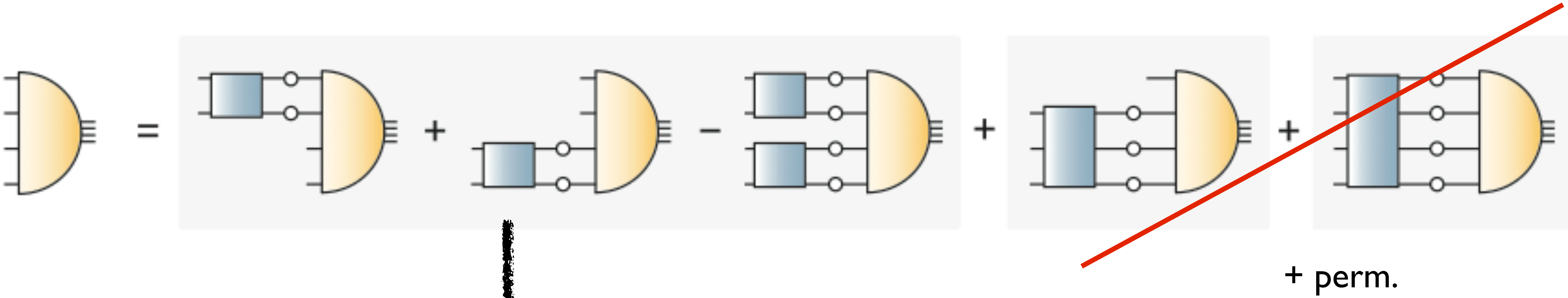
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

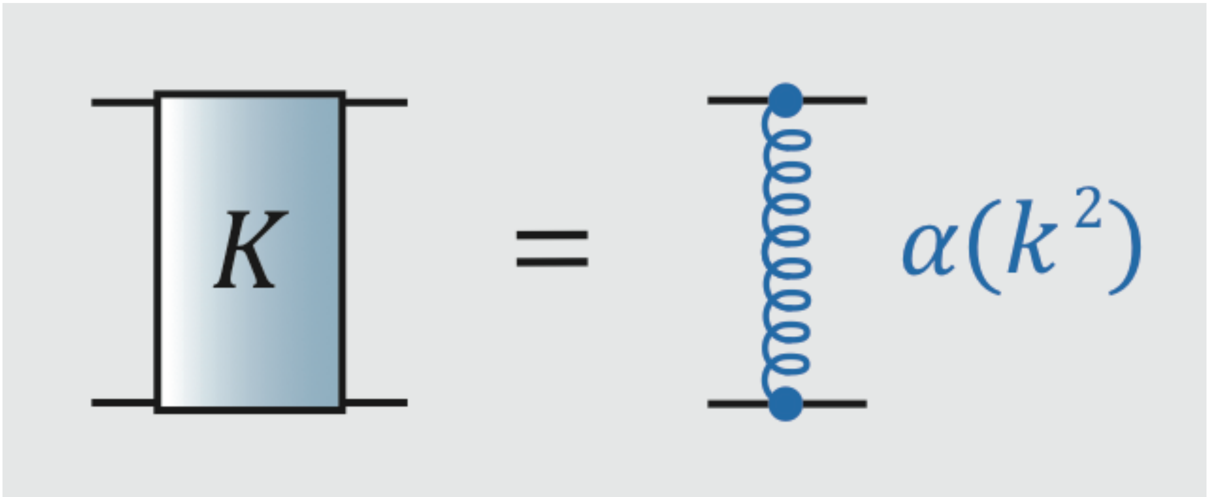
Heupel, Eichman, CF, PLB 718 (2012) 545-549

Eichman, CF, Heupel, PLB 753 (2016) 282-287

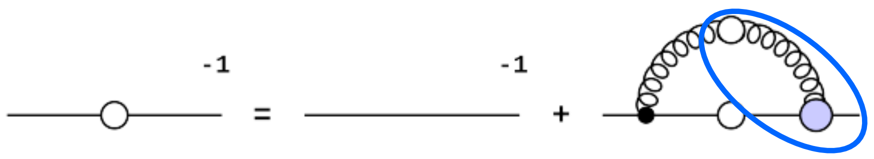
- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

Solving the four-body equation





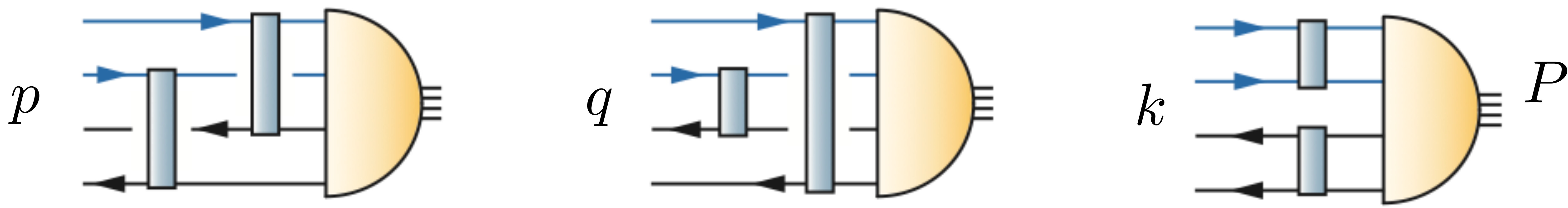
- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\ldots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

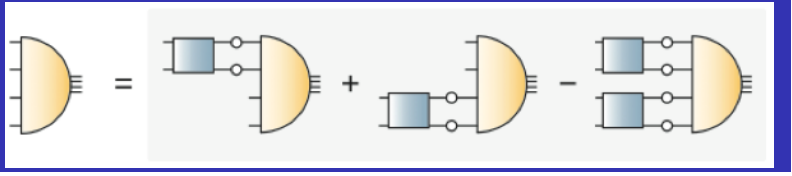
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

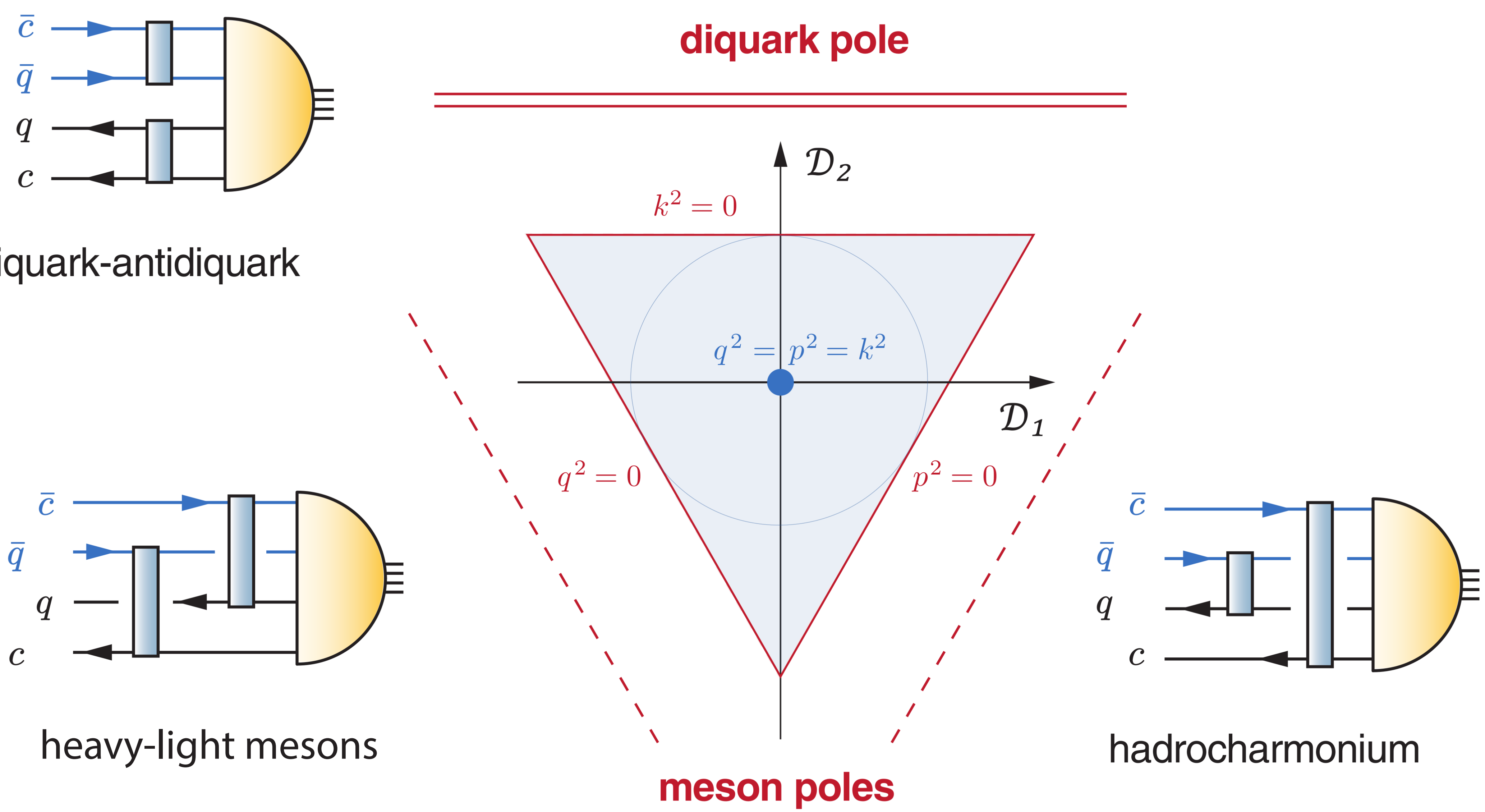
- good approximation: keep s-waves only; 16 tensor structures

Four-body equation: permutations

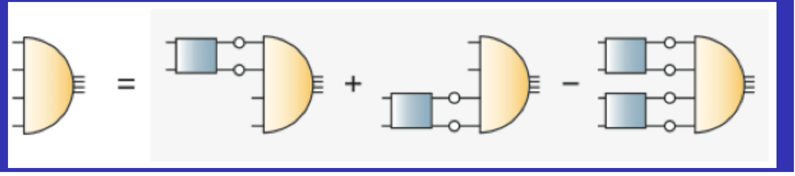


- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ p, q, k : relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

  $\mathcal{D}_2 \sim q^2 - p^2$



Four-body equation: permutations



- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ $\qquad p, q, k$: relative momenta
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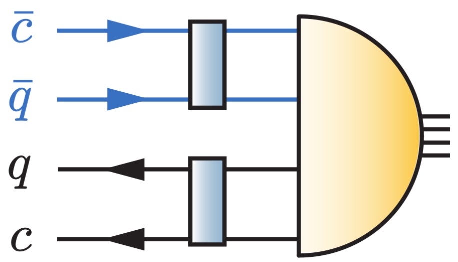
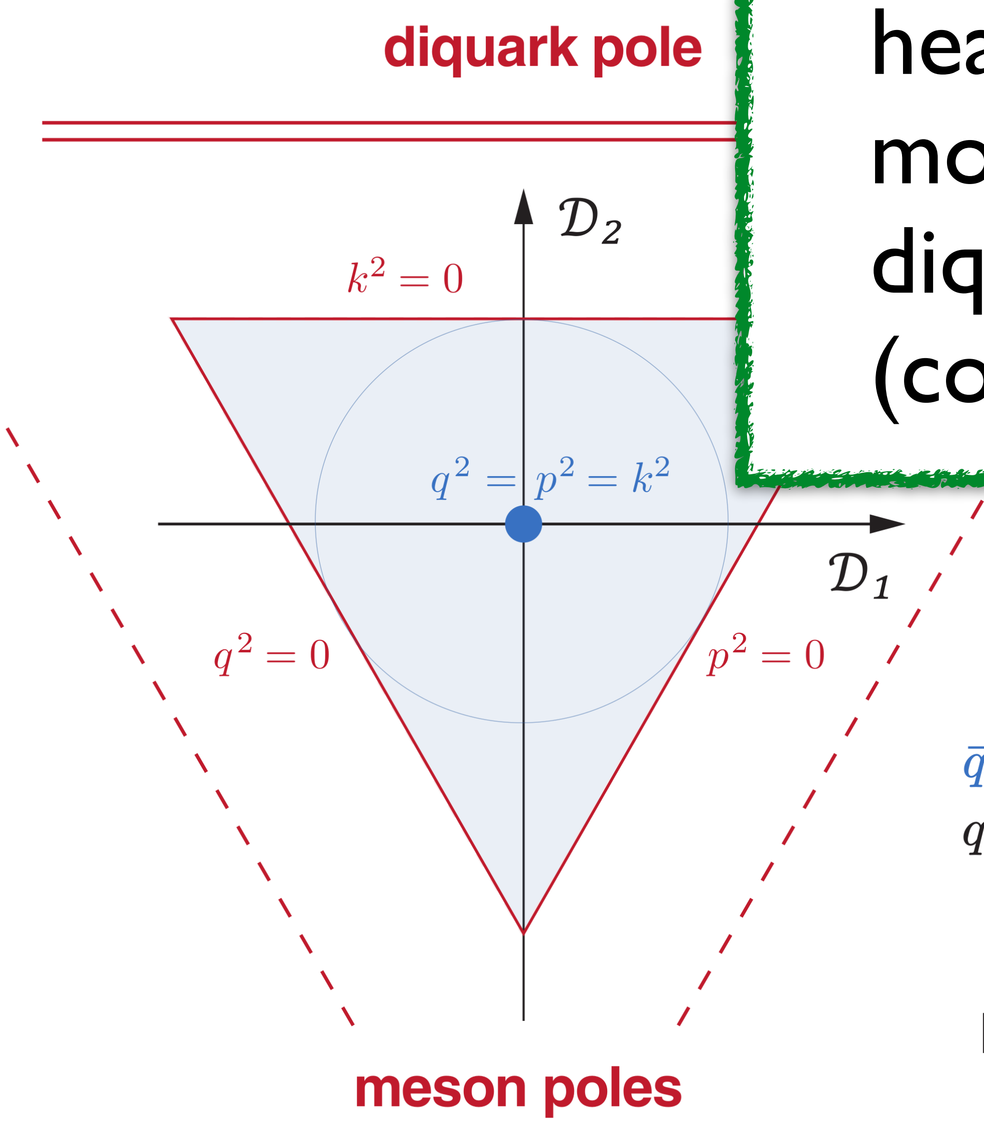
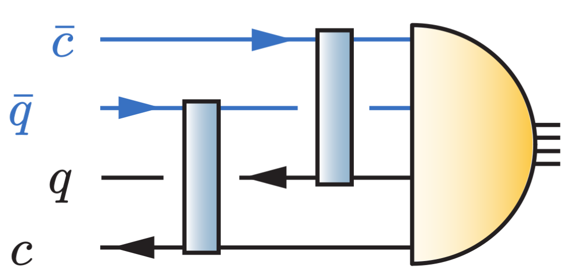


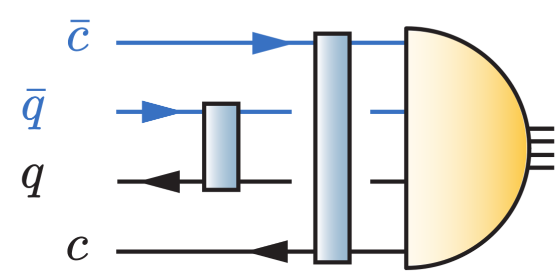
diquark-antidiquark



- model independent: heavy-light meson poles more important than diquark poles (color factor !)



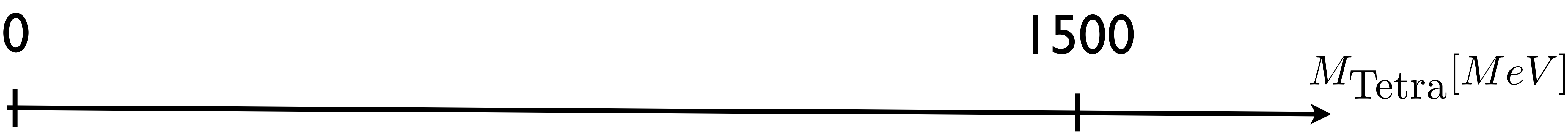
heavy-light mesons



hadrocharmonium

Bound state masses

- Different levels of approximations:



Bound state masses

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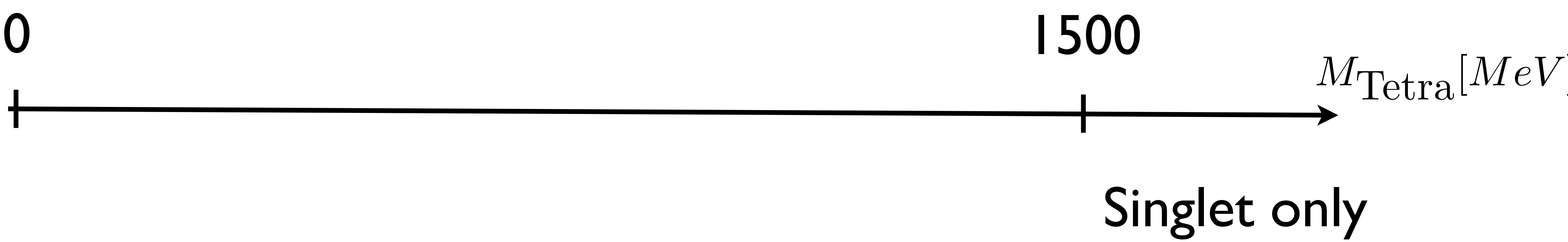
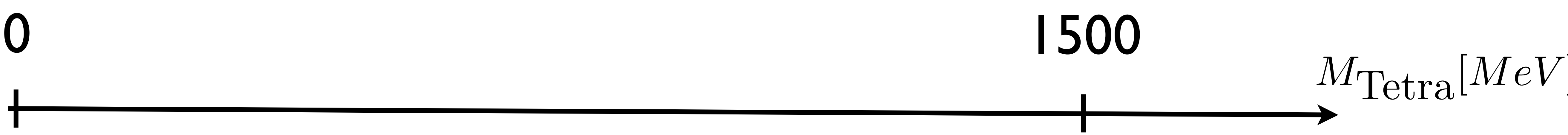


Bound state masses

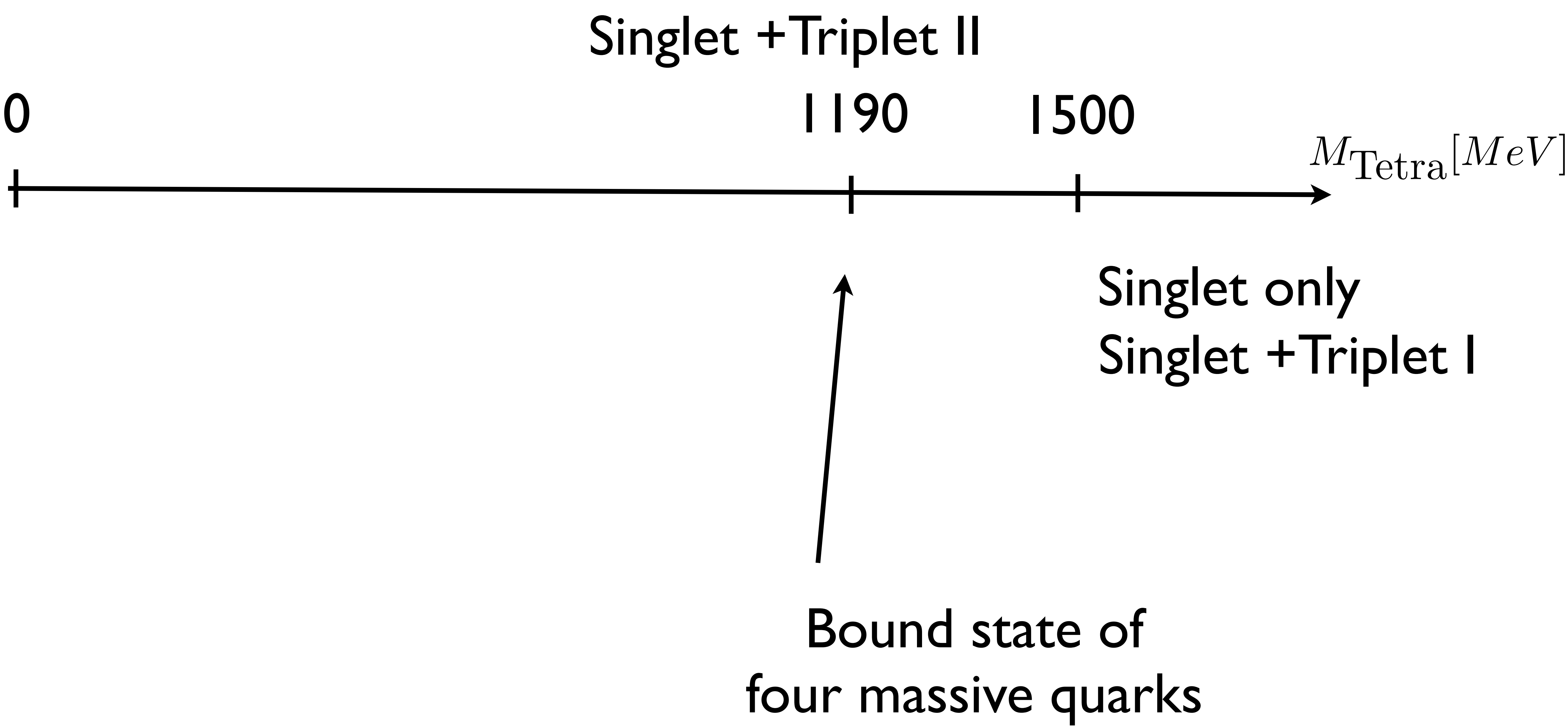
- Different levels of approximations:



Singlet only
Singlet +Triplet I

Bound state masses

- Different levels of approximations:



Bound state masses

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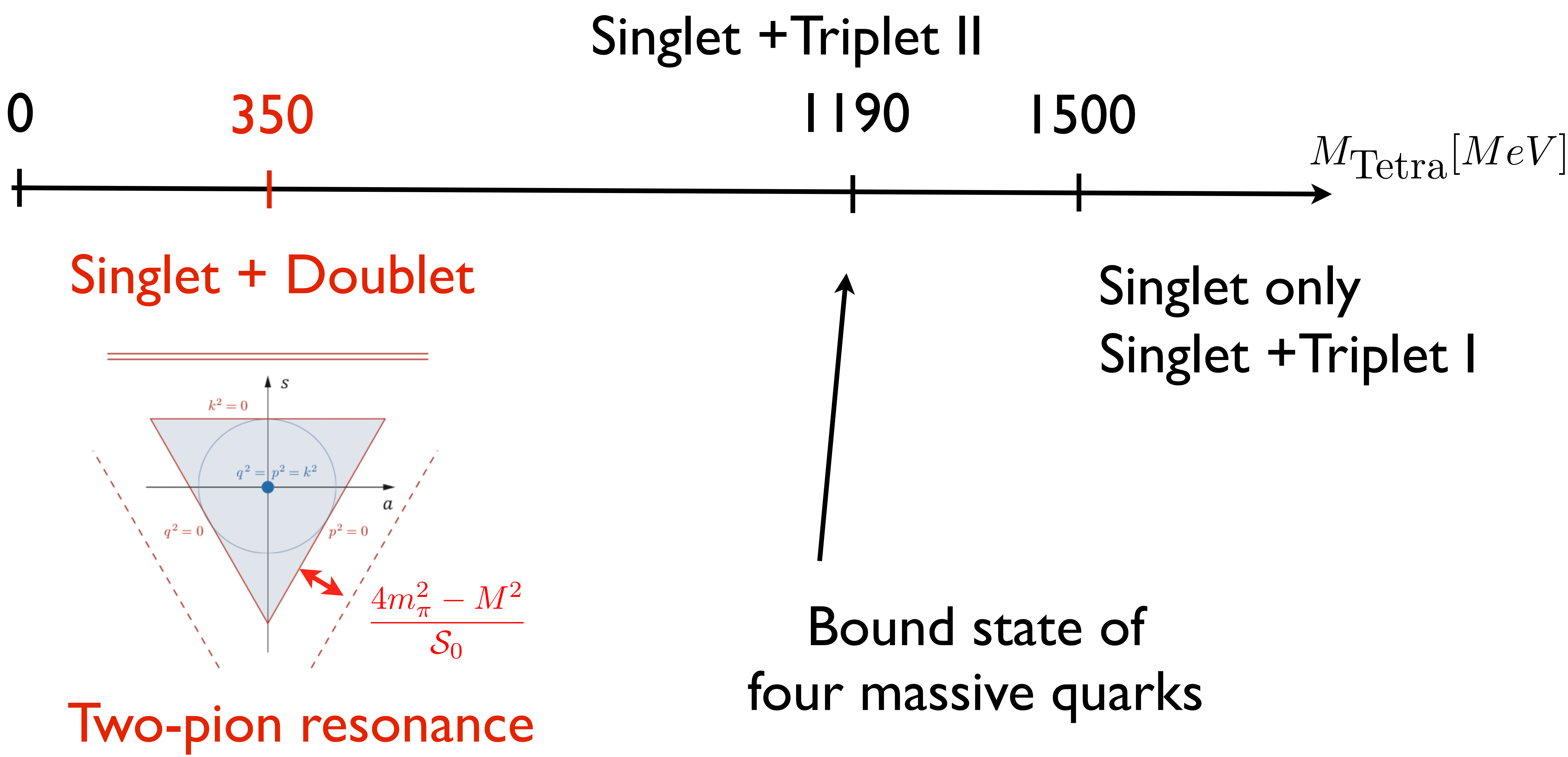
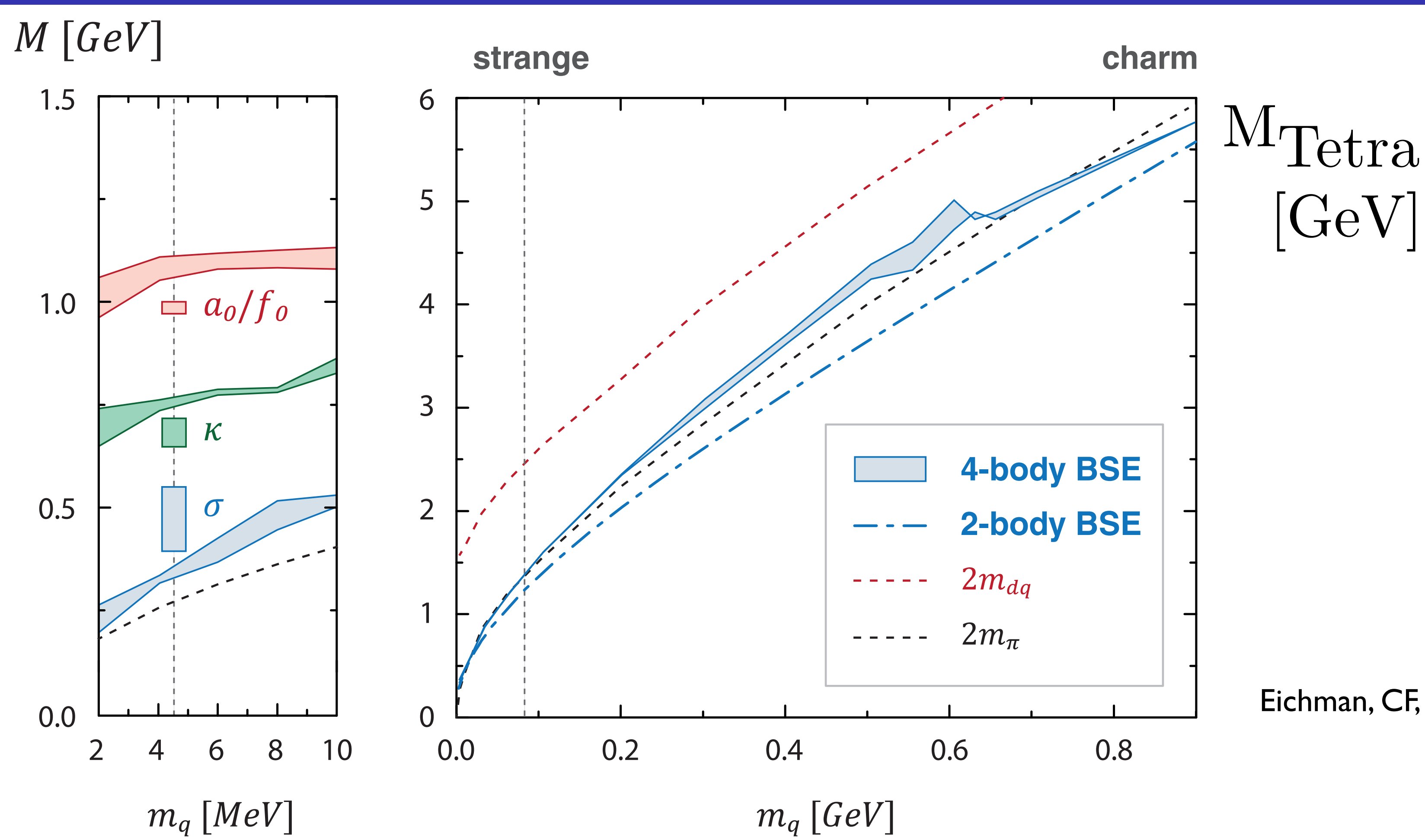


Mass evolution of tetraquark



Eichman, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results: $m_\sigma \sim 350\,\text{MeV}$

$$m_\kappa \sim 750\,\text{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\text{GeV}$$

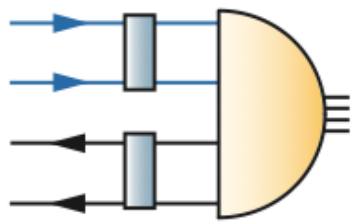
$$m_{a_0,f_0} \sim 1080\,\text{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\text{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichman, CF, PLB 718 (2012) 545-549

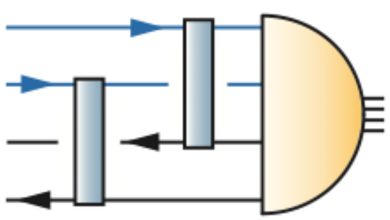
Outlook: heavy-light systems

Dynamical situation in **S4**-doublet:



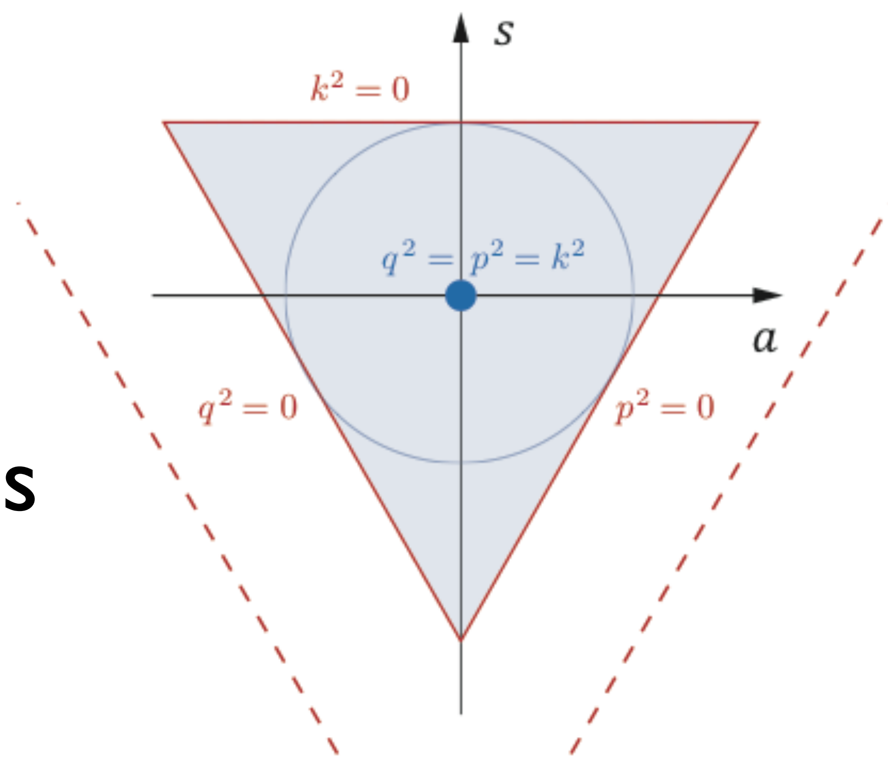
heavy-light diquarks

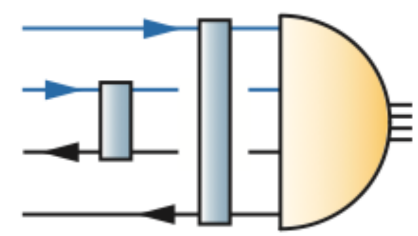
$cq - \bar{c}\bar{q}$



meson-molecules

$c\bar{q} - \bar{c}q$



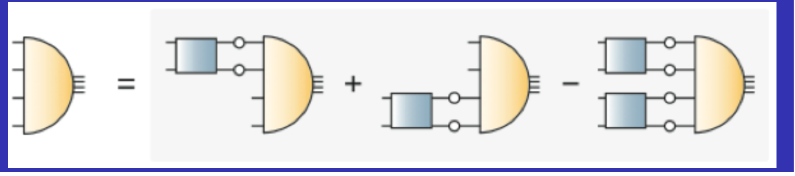


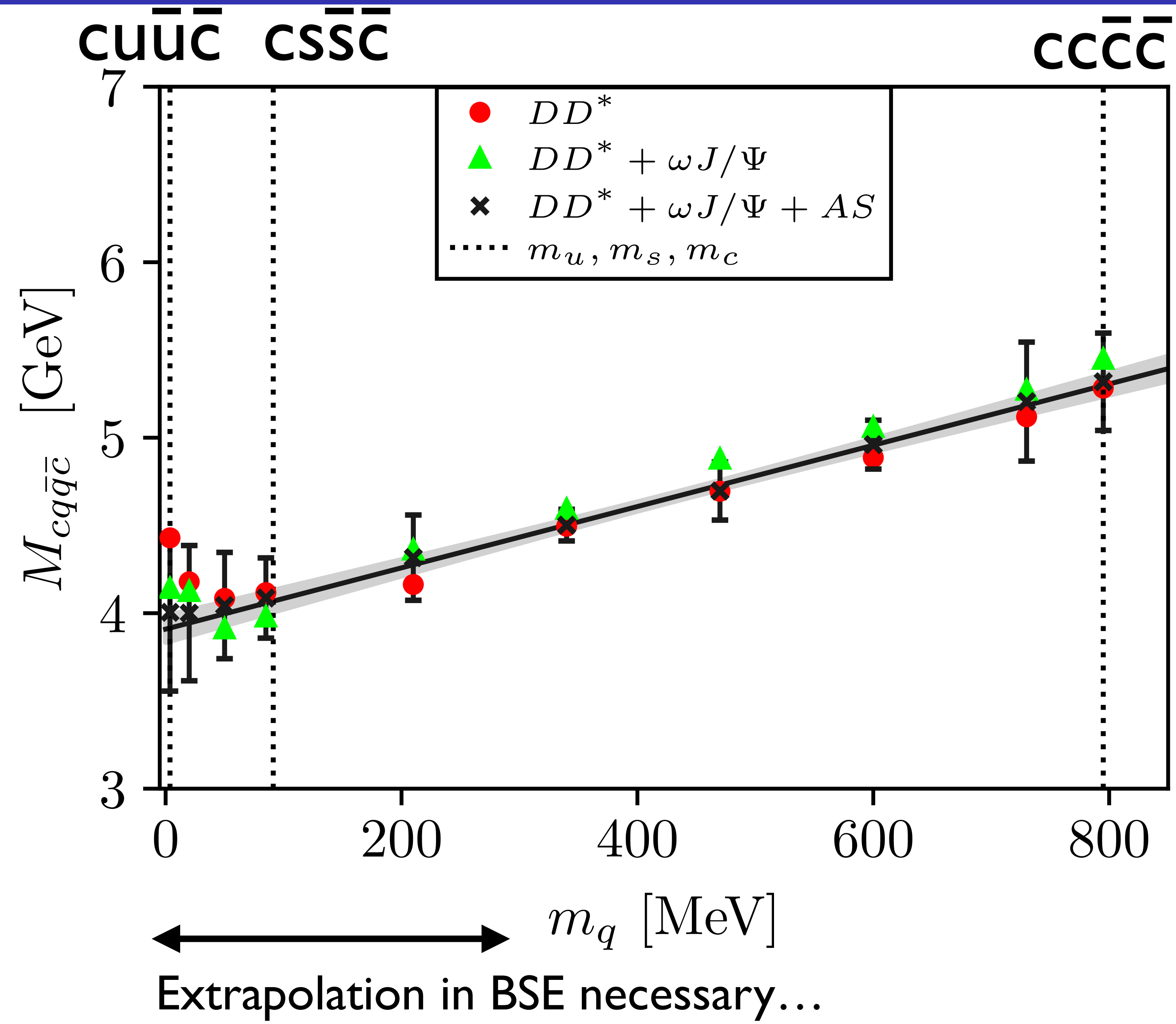
hadro-charmonium

$c\bar{c} - q\bar{q}$

Dynamical decision of most important clustering!

$J^{PC} = 1^{++}$





m_c fixed

m_q varied

- DD* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74)\,\mathrm{MeV} \longrightarrow X(3872)$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Confinement and dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
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   - Mesons
   - Baryons
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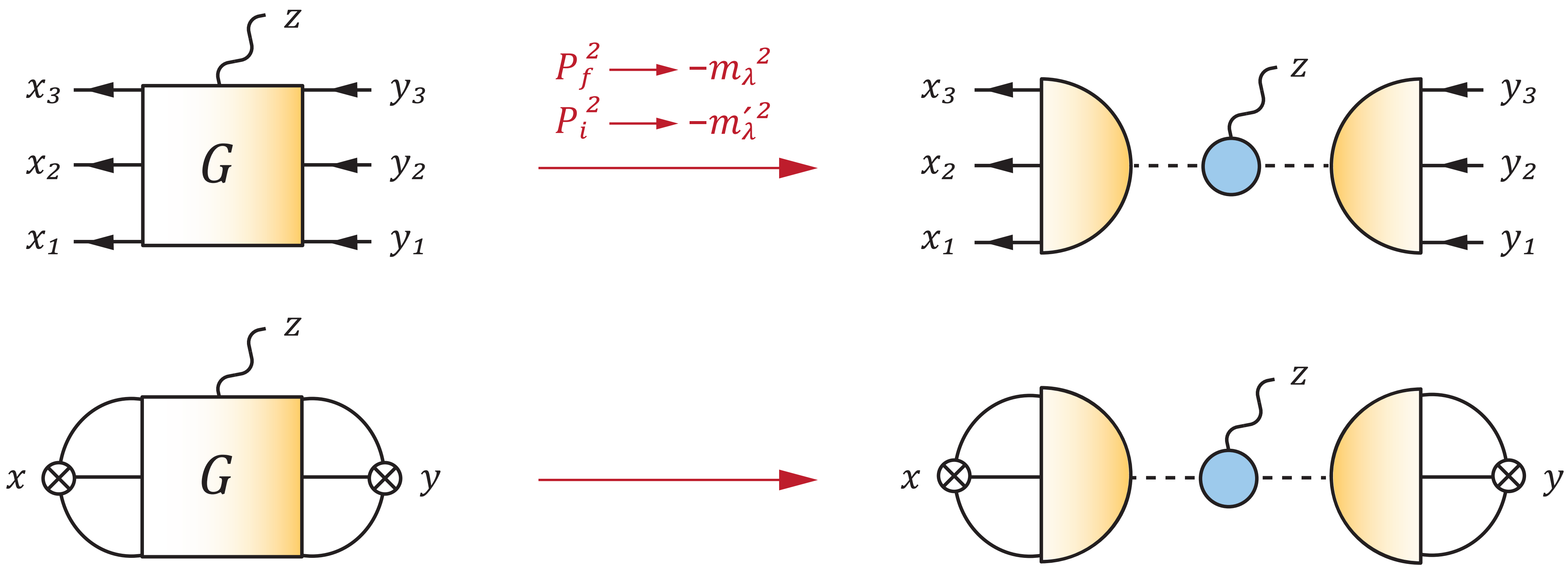
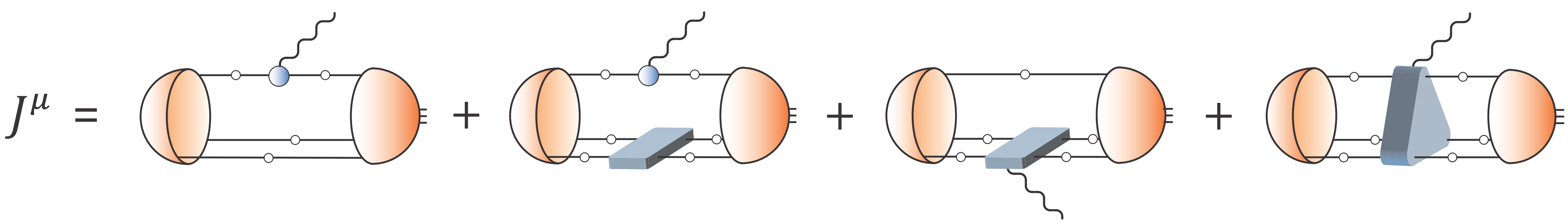
   - Hadron results beyond rainbow-ladder



DSE/BSE

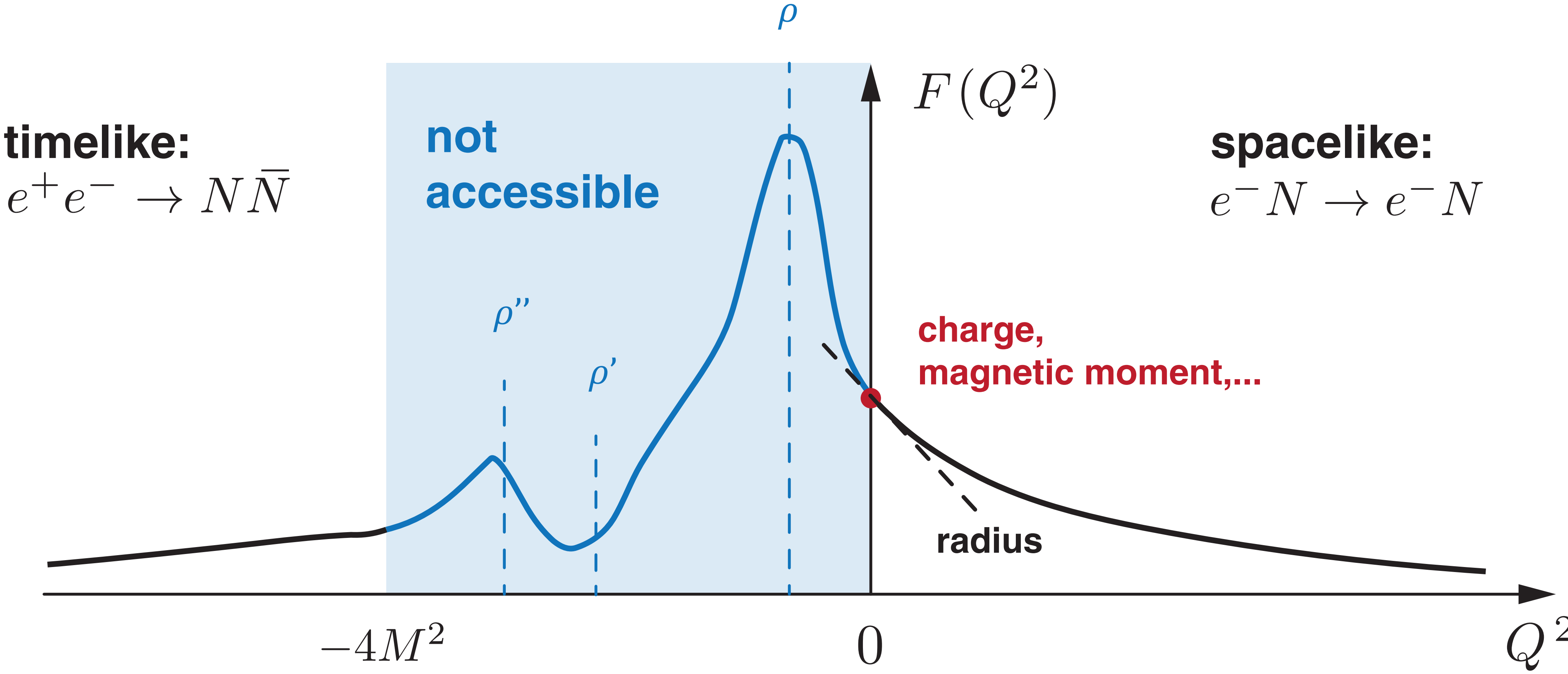
Lattice

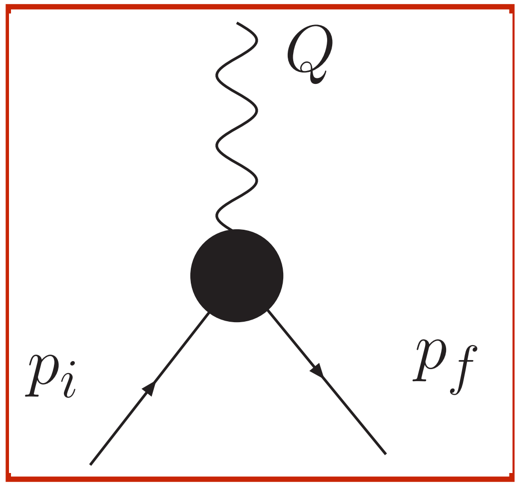
Form factor from BSEs (derived from equation of motion for G and 'gauging')



- exact equation for baryon form factors

Physics from form factors I





$$Q = (0,0,0,2\sqrt{Q^2+M^2})$$
$$p_i = (0,0,Q,-\sqrt{Q^2+M^2})$$
$$p_f = (0,0,Q,\sqrt{Q^2+M^2})$$

$$Q = (0,0,Q,0)$$
$$p_i = (0,0,-Q/2,\sqrt{Q^2+M^2})$$
$$p_f = (0,0,Q/2,\sqrt{Q^2+M^2})$$

Physics from form factors II

- Example: pion electromagnetic form factor

$$\mathcal{J}^\mu(p_i,p_f) = (p_i+p_f)^\mu F(Q^2)$$

charge radius

with $$F(Q^2) = F(0) - \frac{r^2}{6}Q^2 + \cdots$$

electric charge

- Example: nucleon electromagnetic form factor

$$\mathcal{J}^\mu(p_i,p_f) = i\bar{u}(p_f)\left(F_1(Q^2)\gamma^\mu + \frac{iF_2(Q^2)}{4M}[\gamma^\mu, \not{Q}]\right)u(p_i)$$

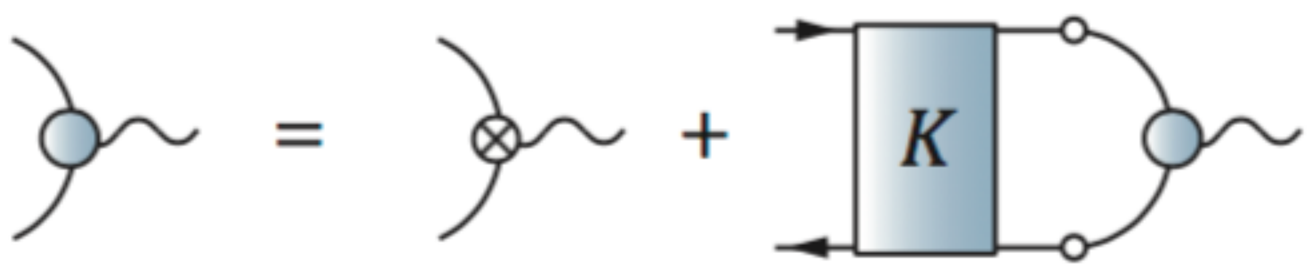
with $$F_1(Q^2) = F_1(0) - \frac{{r_1}^2}{6}Q^2 + \cdots$$ electric charge

$$F_2(Q^2) = F_2(0)\left[1 - \frac{{r_2}^2}{6}Q^2 + \cdots\right]$$ charge radii

anomalous magnetic moment

Currents coupling to quarks

Exact equation for any vertex in QCD coupling a colorless current to a quark-antiquark pair:



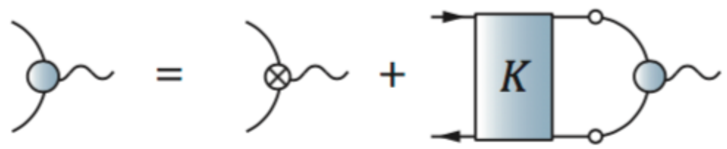
Q

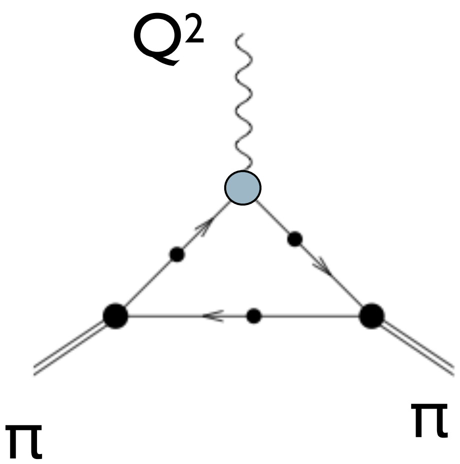
- 'inhomogenuous' Bethe-Salpeter equation
- contains meson poles for on-shell total momenta $Q^2 = -m_{BS}^2$
- physics content determined by quantum numbers

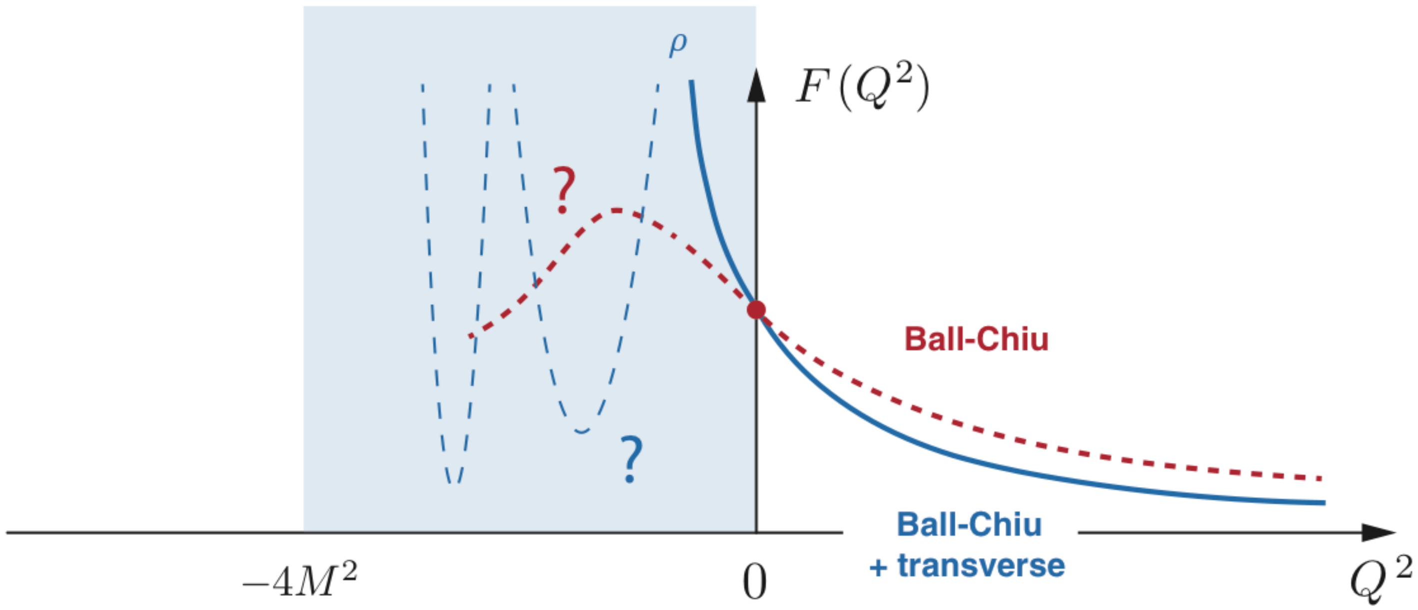
e.g. vector-quark-antiquark vertex contains vector meson poles

Quark-photon vertex and pion form factors

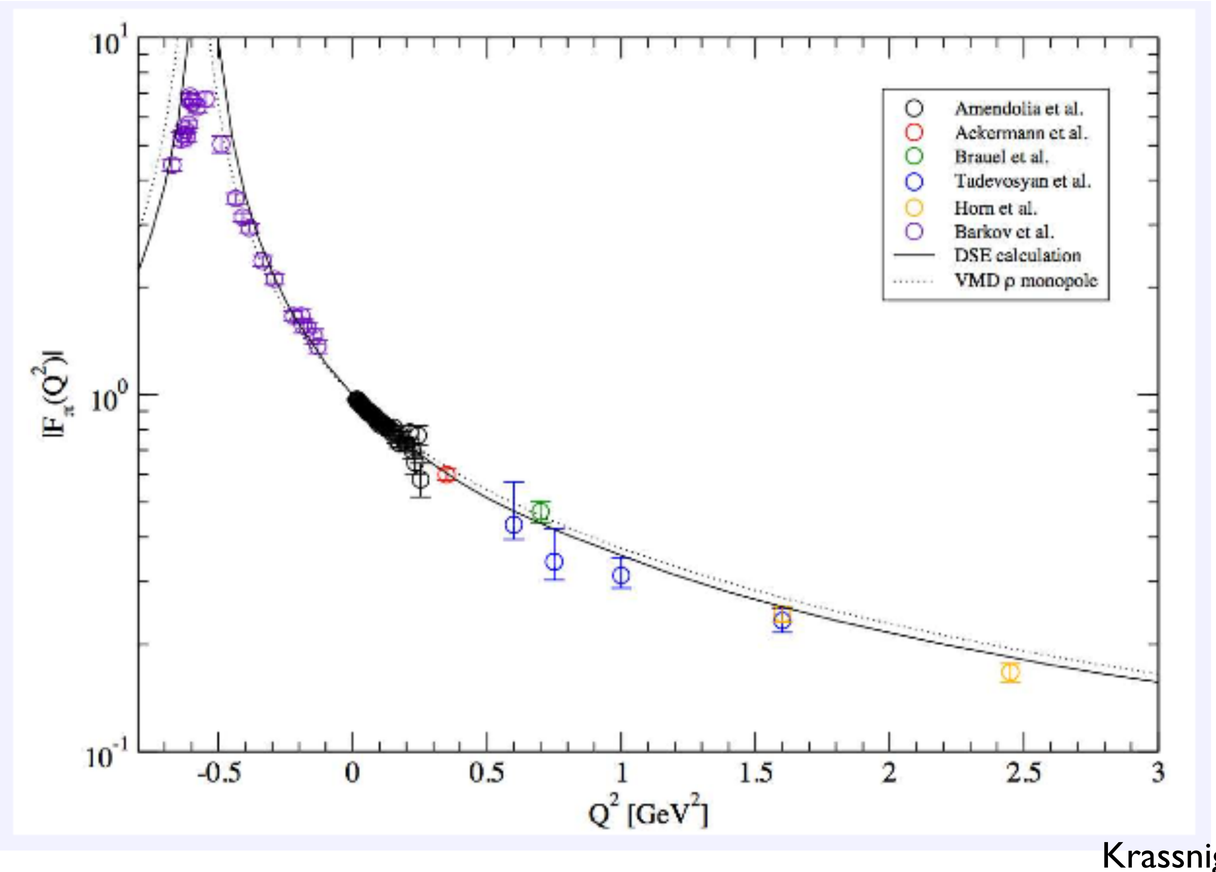
Pion form factor:







Eichmann, Acta Phys. Polon. Supp. 7 (2014) 3



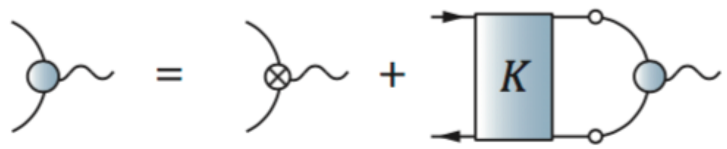
Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

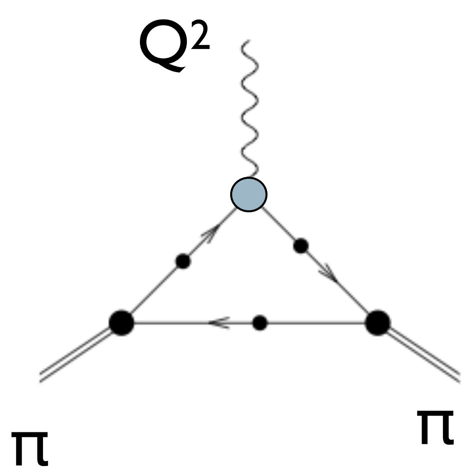
cf. proof of Goldstone theorem

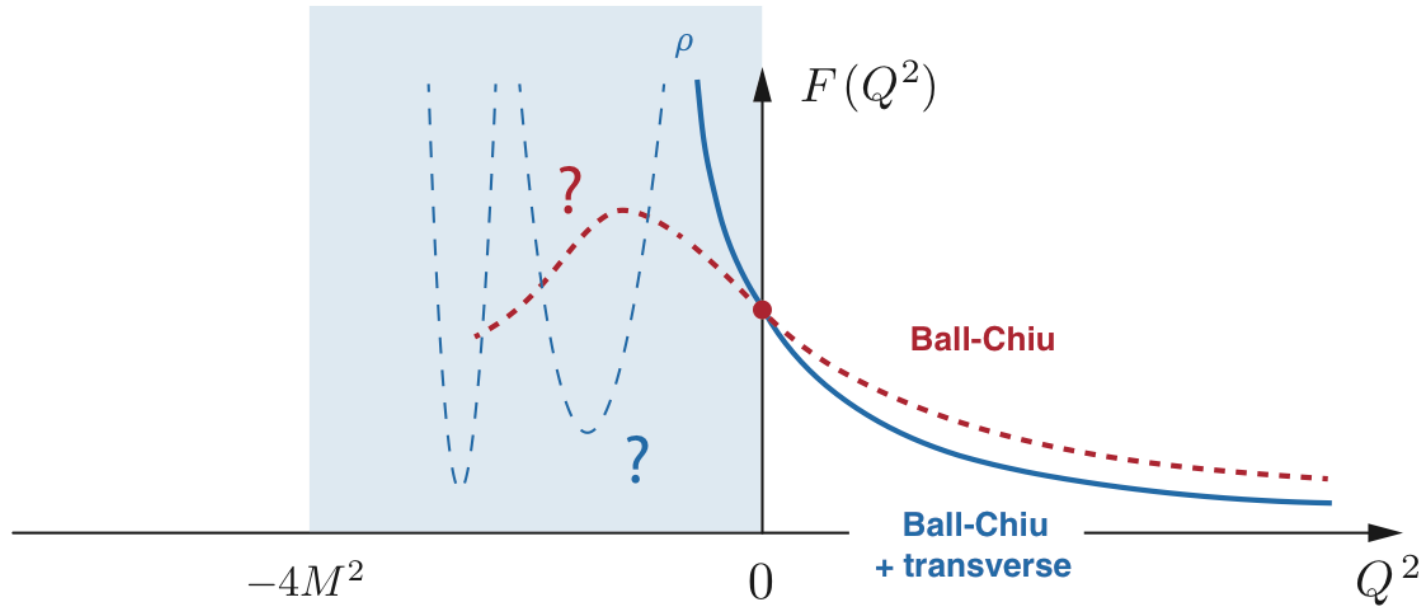
Vector meson poles dynamically generated!

Quark-photon vertex and pion form factors

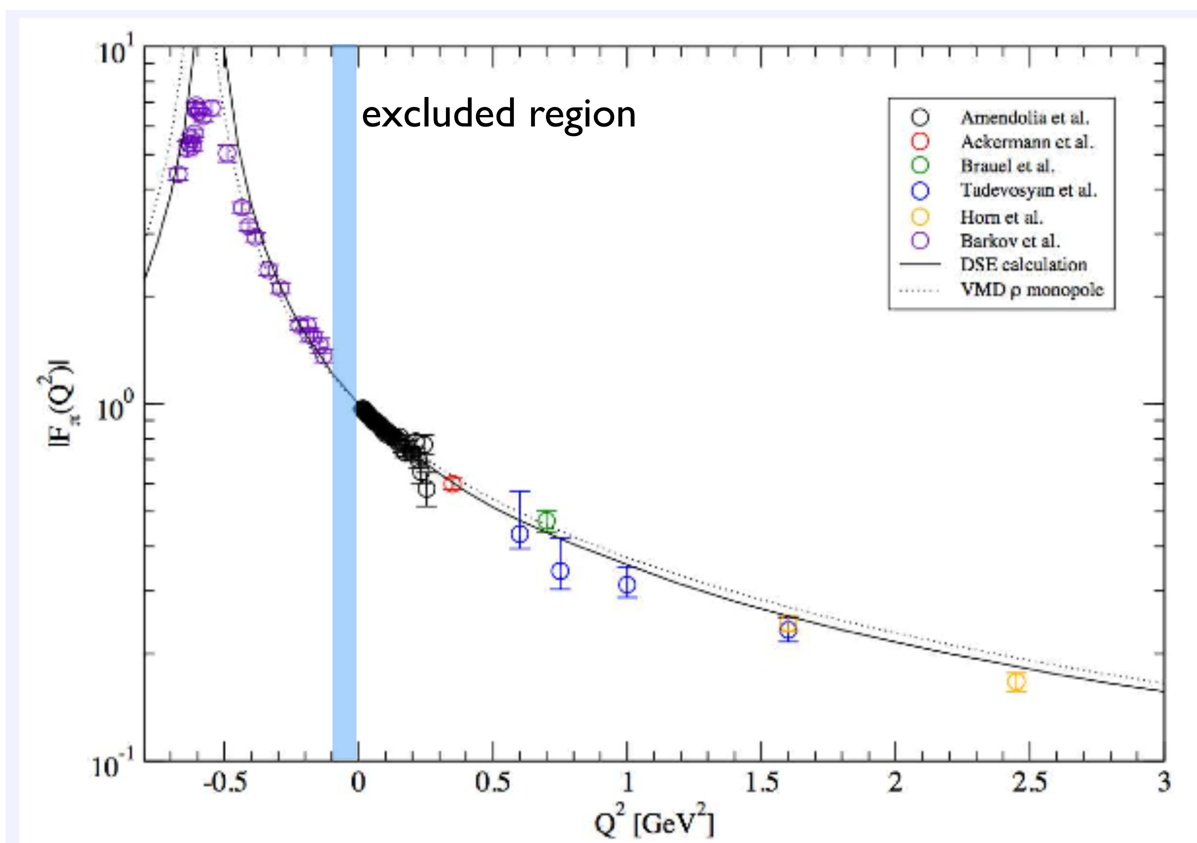
Pion form factor:







Eichmann, Acta Phys. Polon. Supp. 7 (2014) 3



Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

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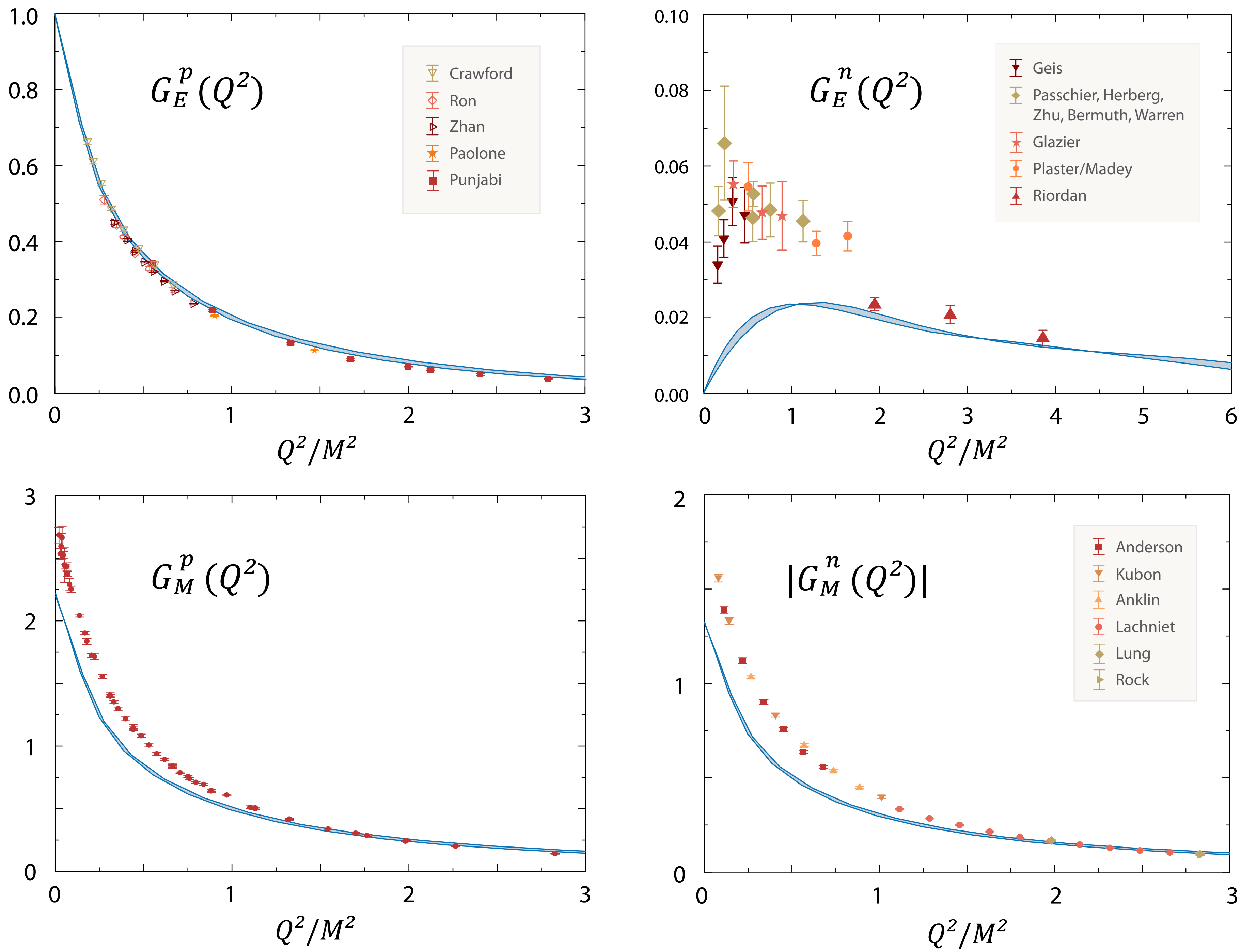
Vector meson poles dynamically generated!

Nucleon form factors and magnetic moments



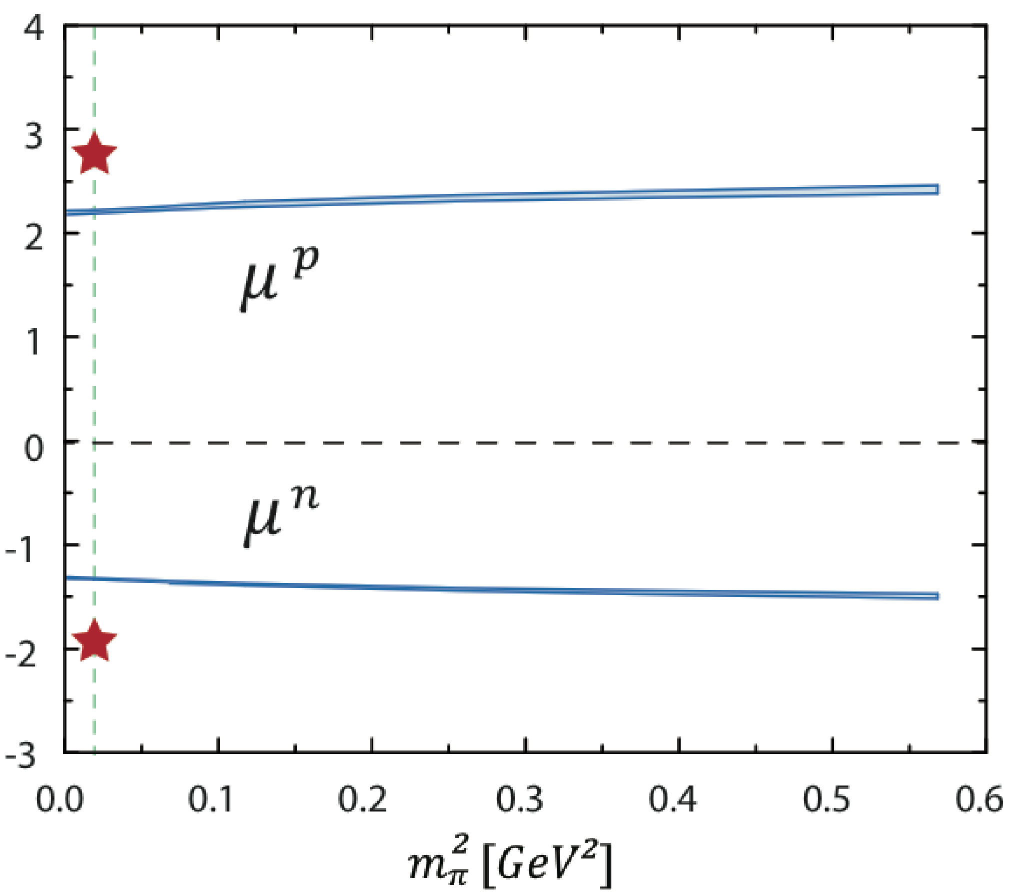
- missing pion cloud effects
- similar for axial form factors

Eichmann, PRD 84 (2011)

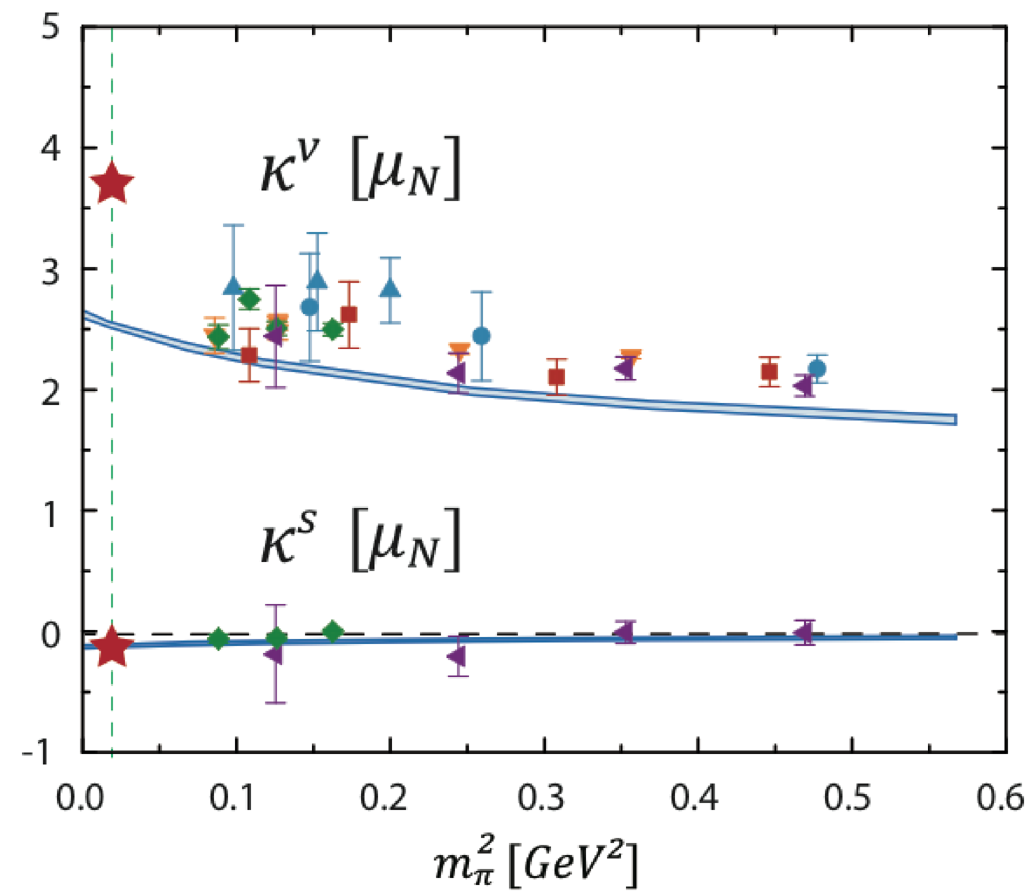
Eichmann and CF, EPJ A48 (2012) 9

Magnetic moments

Magnetic moments (p, n):



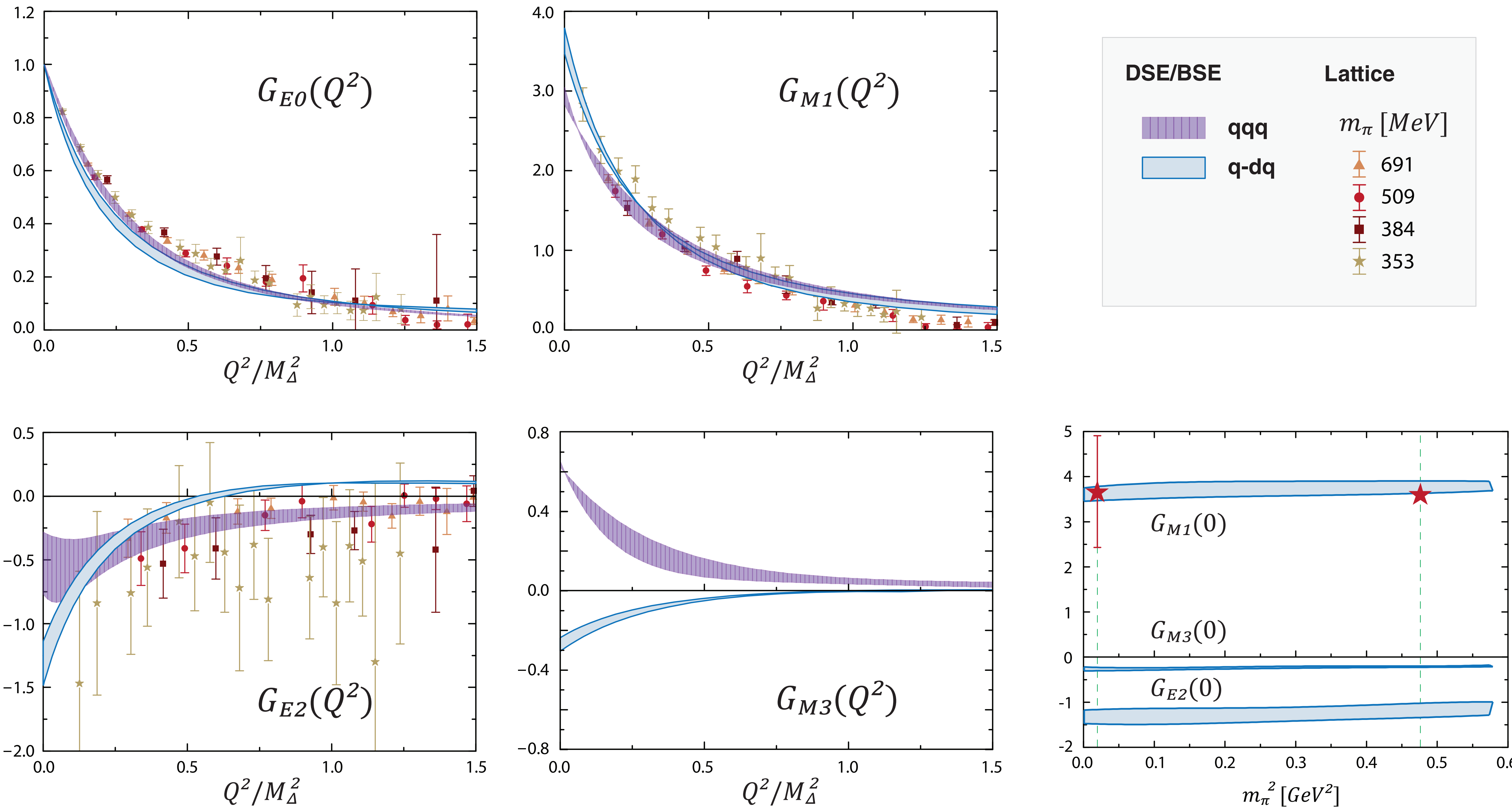
Isovector (p-n), isoscalar (p+n):



- missing pion cloud effects in isovector moment κ^v
- no pion cloud effects in isoscalar moment κ^s

Eichmann, PRD 84 (2011)

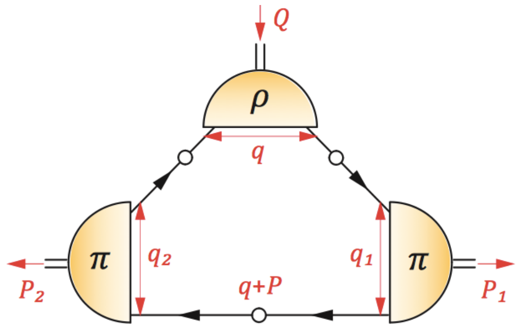
Δ-form factors

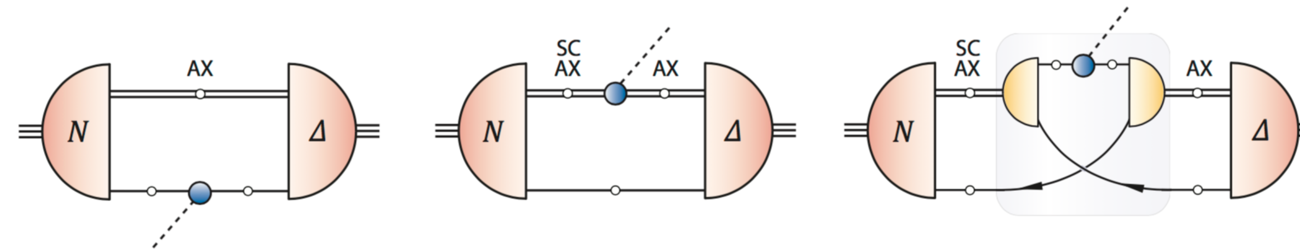


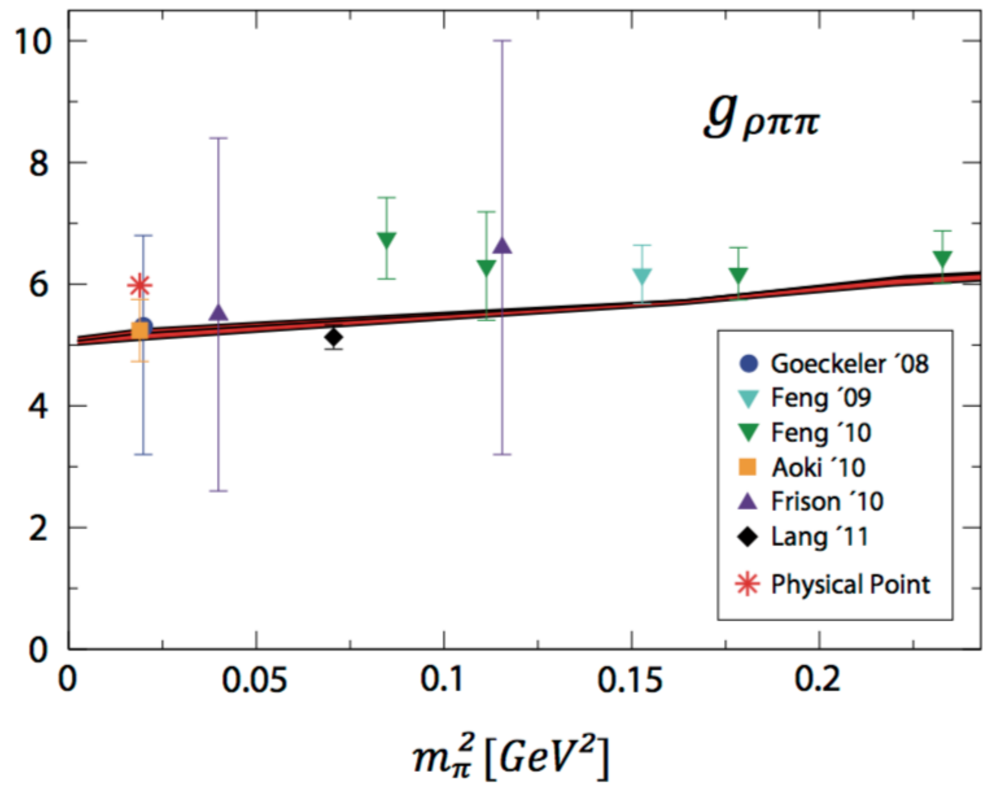
- may serve to distinguish between qqq and q-dq !

Sanchis-Alepuz, Williams, Alkofer, PRD87 (2013)
Nicmorus, Eichmann, Alkofer, PRD82 (2010)

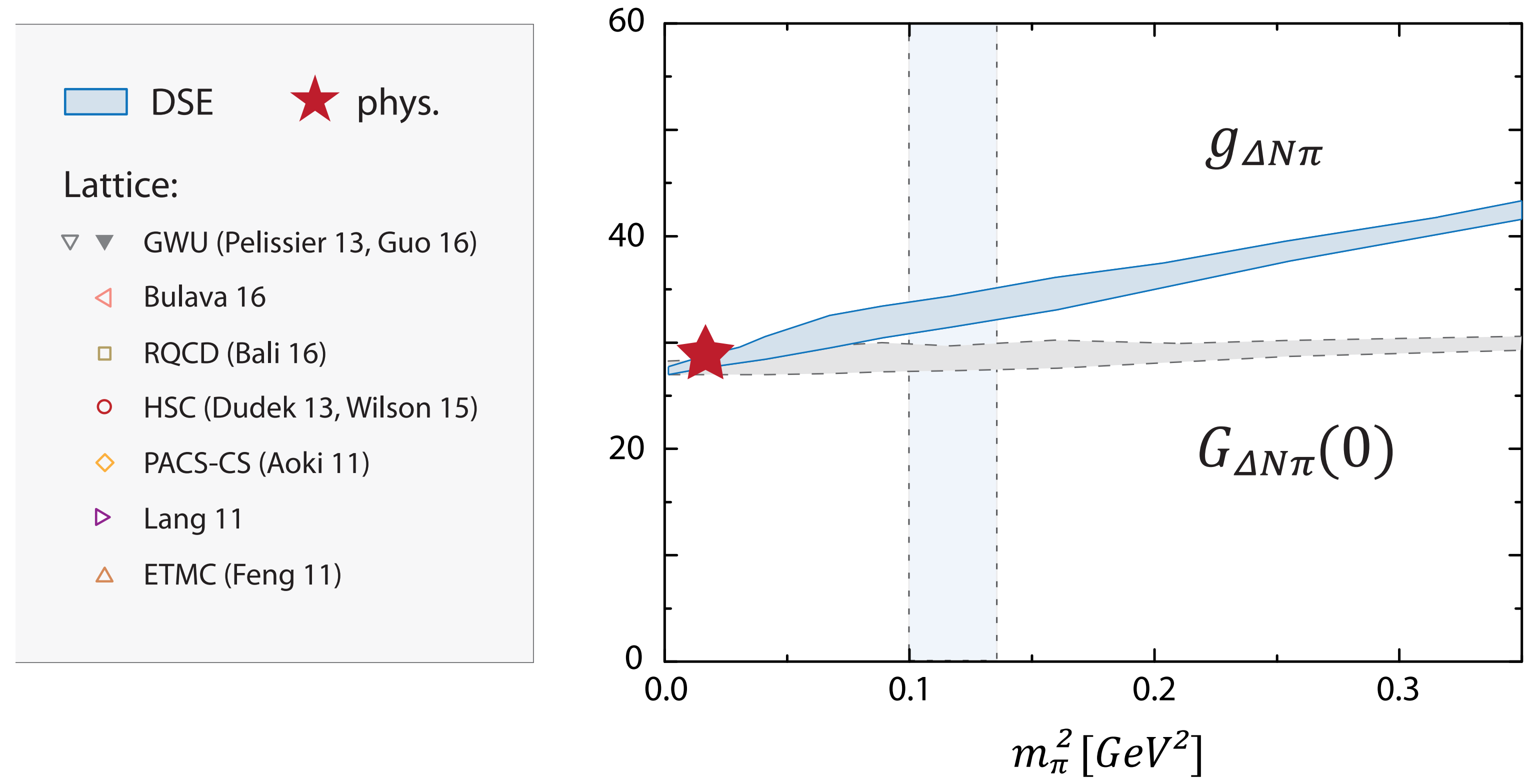
Decays: ρππ and ΔNπ







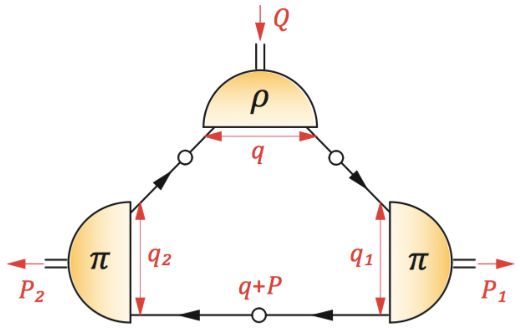
$$g_{\Delta N\pi} = G_{\Delta N\pi}(Q^2 = -m_\pi^2)$$

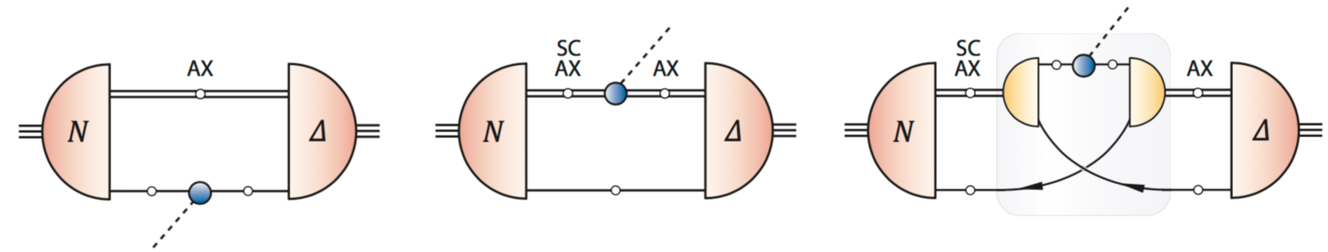


Mader, Eichmann, Blank, Krassnigg PRD84 (2011)

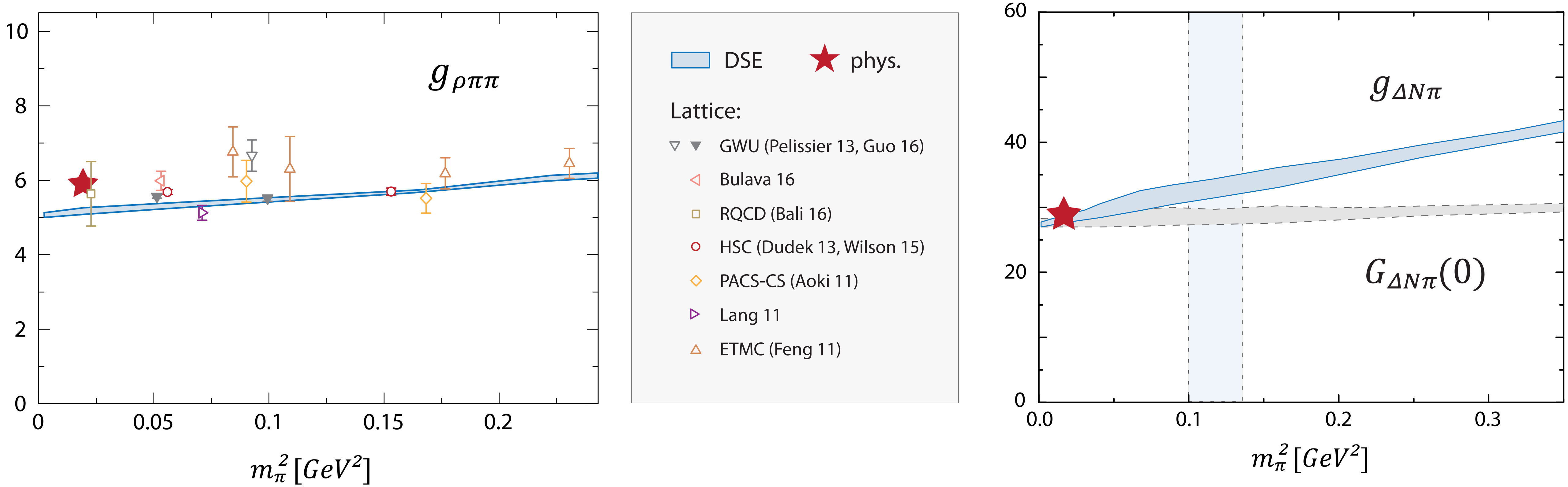
- Decay constants approx. correct in rainbow-ladder
  (although bound states have no width)
- Good agreement with lattice and experiment

Decays: ρππ and ΔNπ





$$g_{\Delta N\pi} = G_{\Delta N\pi}(Q^2 = -m_\pi^2)$$



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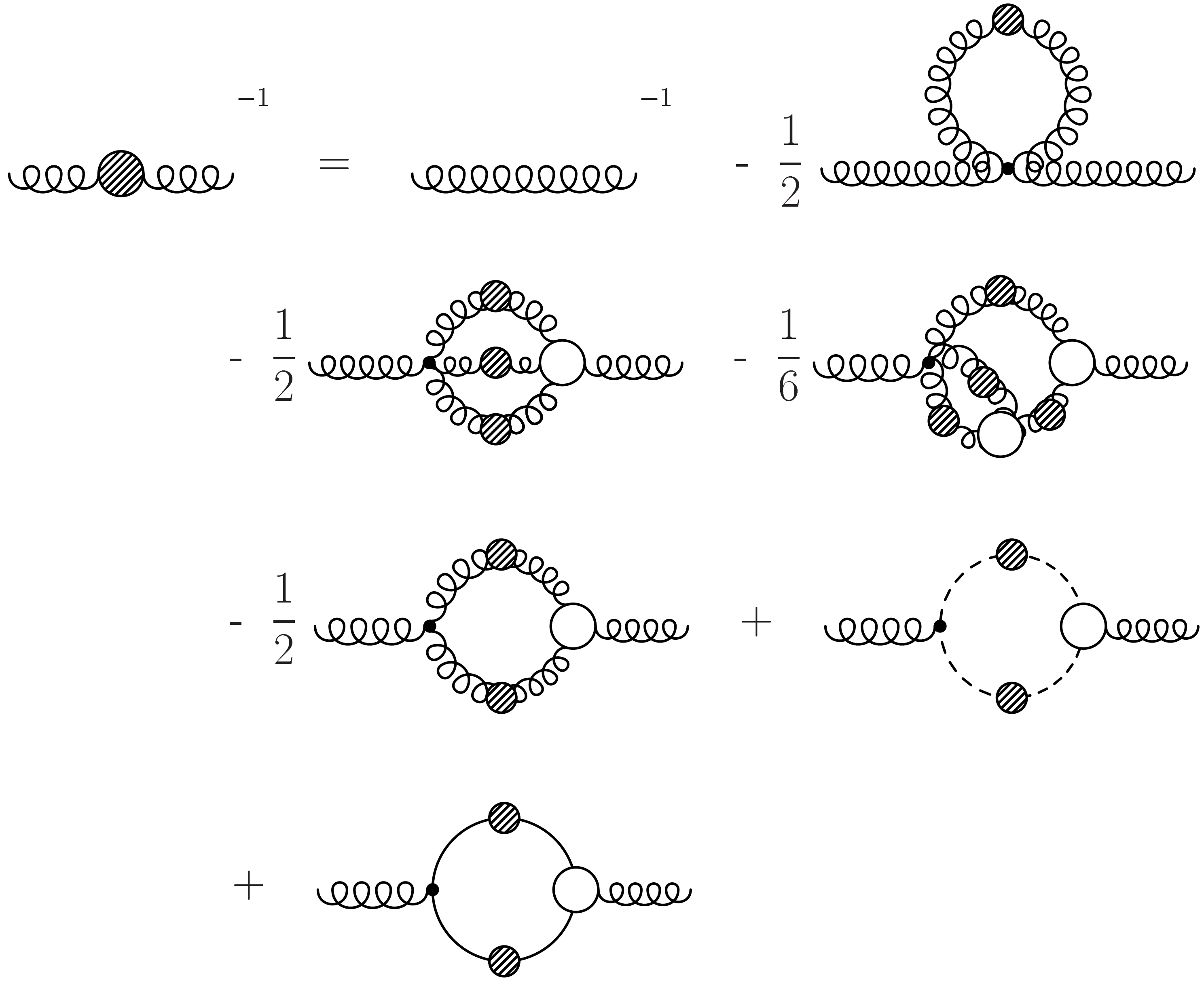
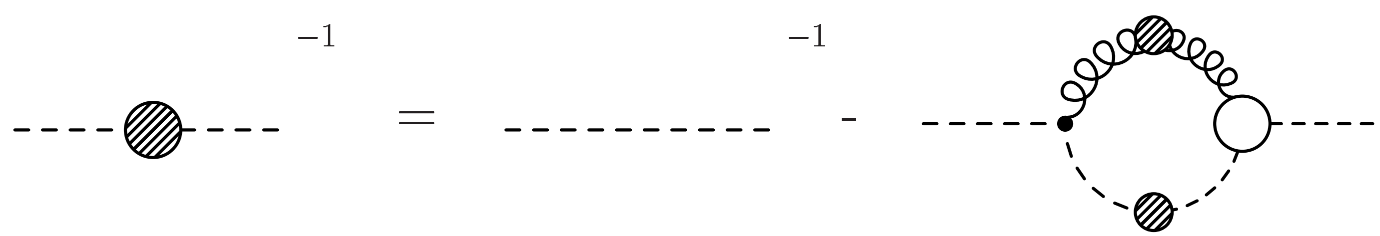
DSEs: Quark and gluon propagators

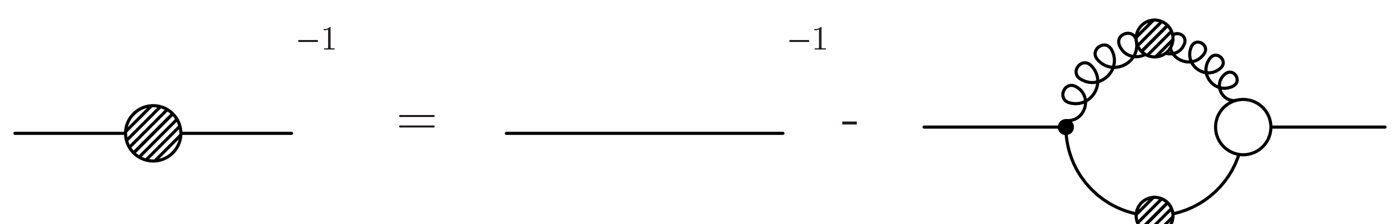
gluon:



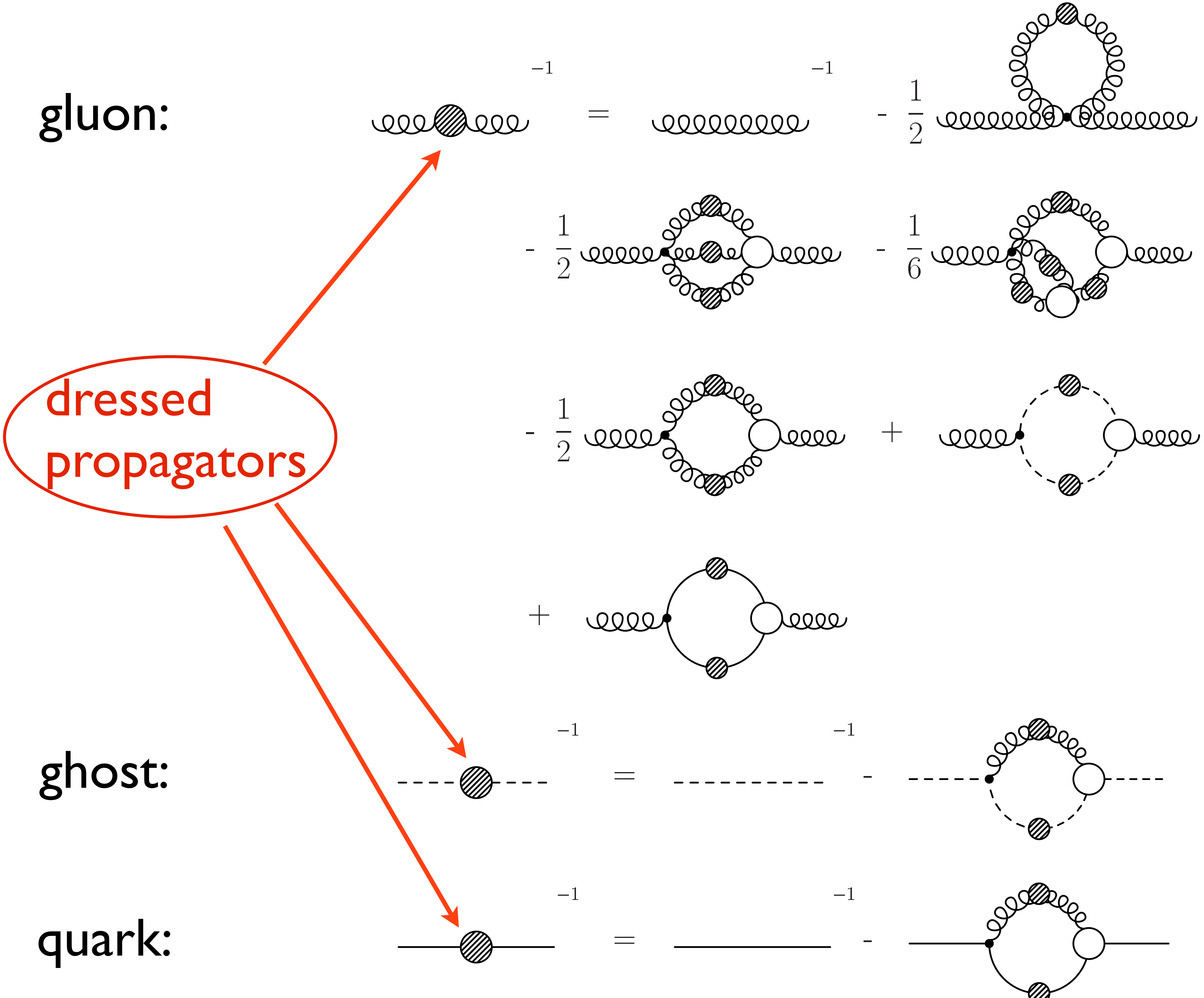
ghost:



quark:



DSEs: Quark and gluon propagators



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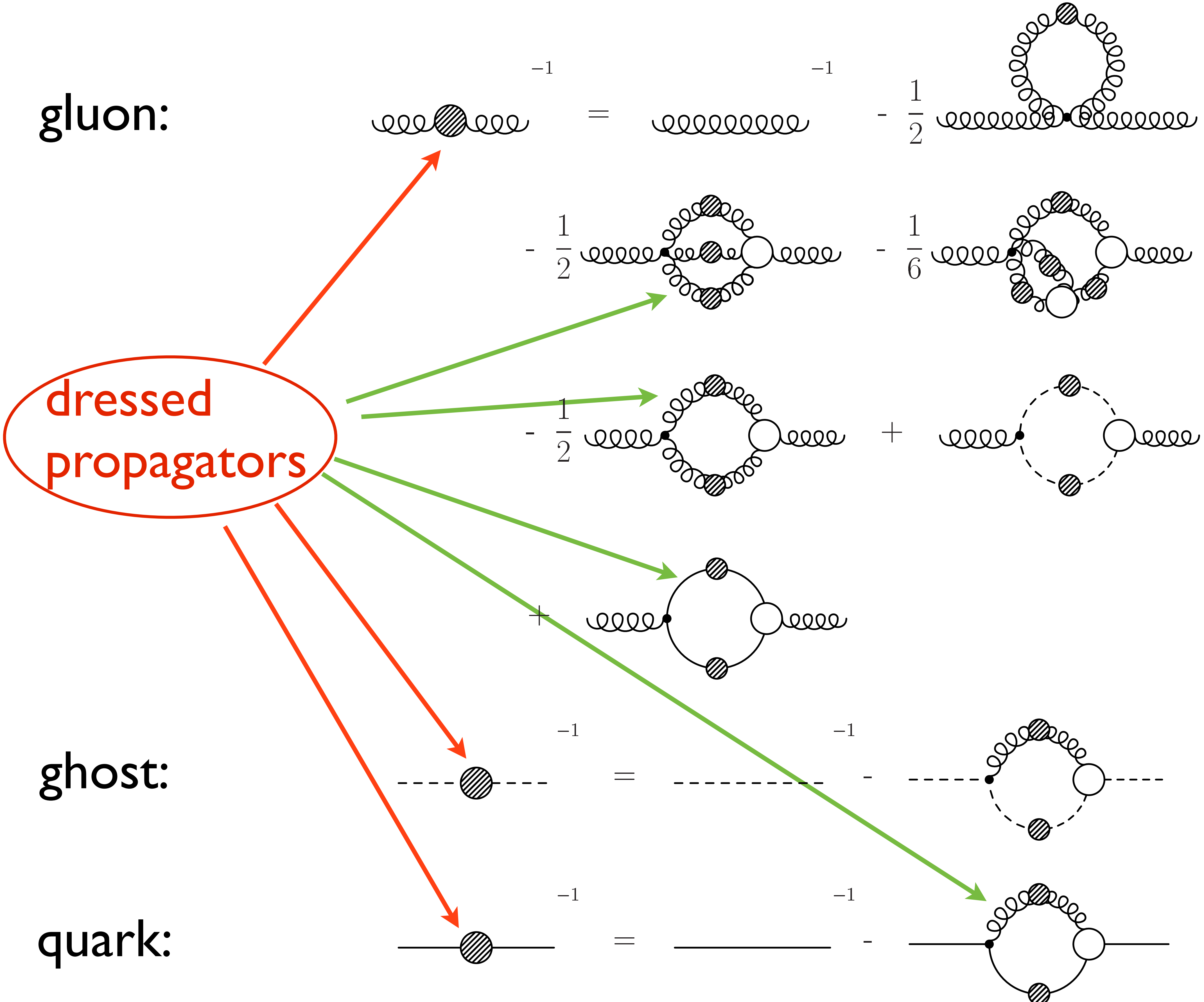
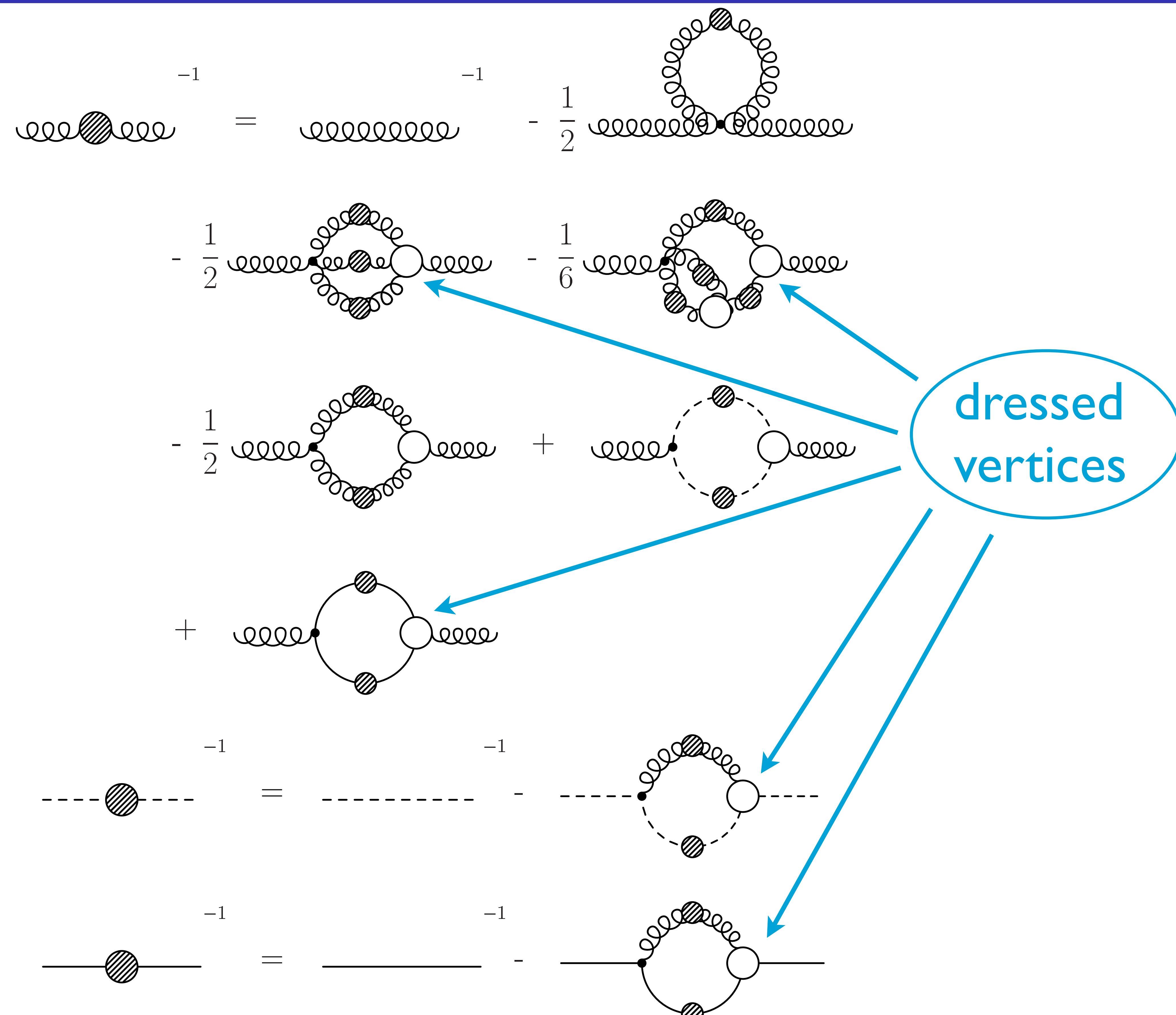


DSEs: Quark and gluon propagators

gluon:

ghost:

quark:



Truncation strategies beyond rainbow-ladder

Goal: solve DSEs for n-point functions with n <= m

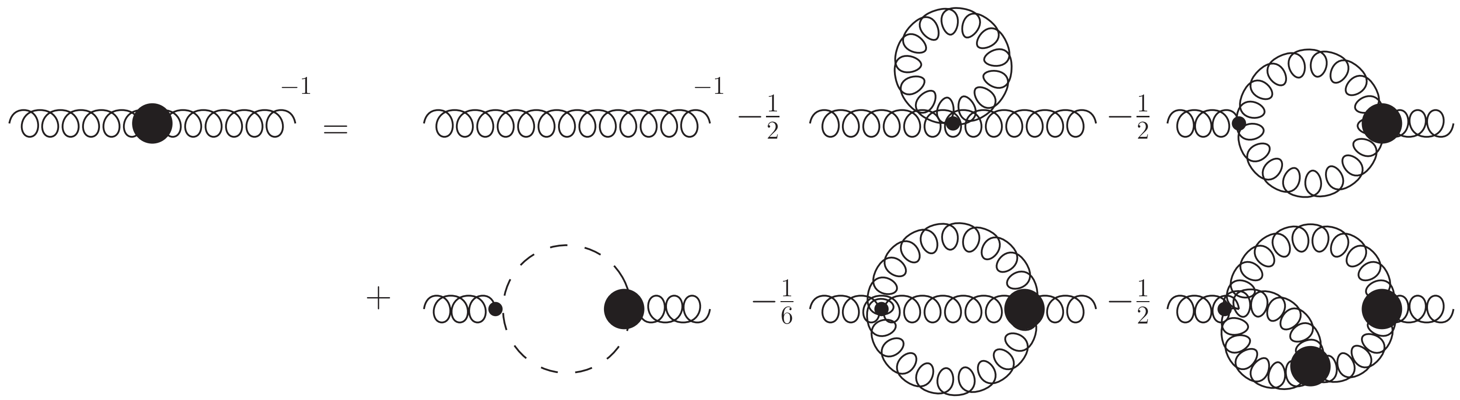
What to do with n>m ?

- set all n-point functions with n>m to zero:
  rigorous field expansion of effective action
  (*problem for m<4: loose contact with perturbation theory…*)

- use Slavnov-Taylor IDs and perturbation theory for n=m+1;
  set all n-point functions with n>m+1 to zero
  (*problem: also ST IDs need to be approximated…*)

cross check with lattice QCD !

Landau gauge gluon propagator

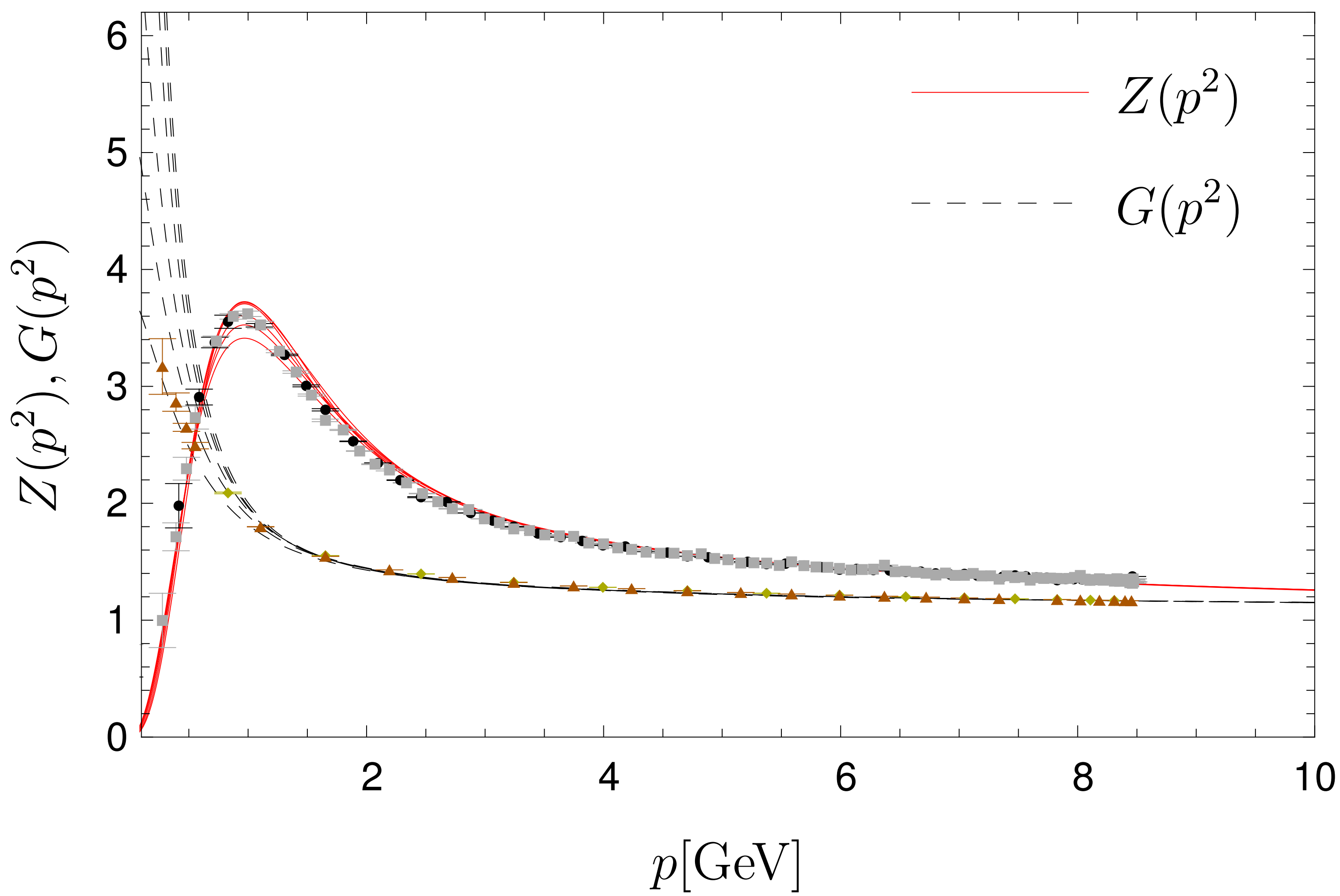


$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

- spacelike momenta:
  good agreement with lattice

- fully dressed gluon appears massive

  Cornwall PRD 26 (1982);
  Cucchieri, Mendes PoS Lat2007 297
  Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
  Boucaud et al. JHEP 0806 (2008) 099;
  CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

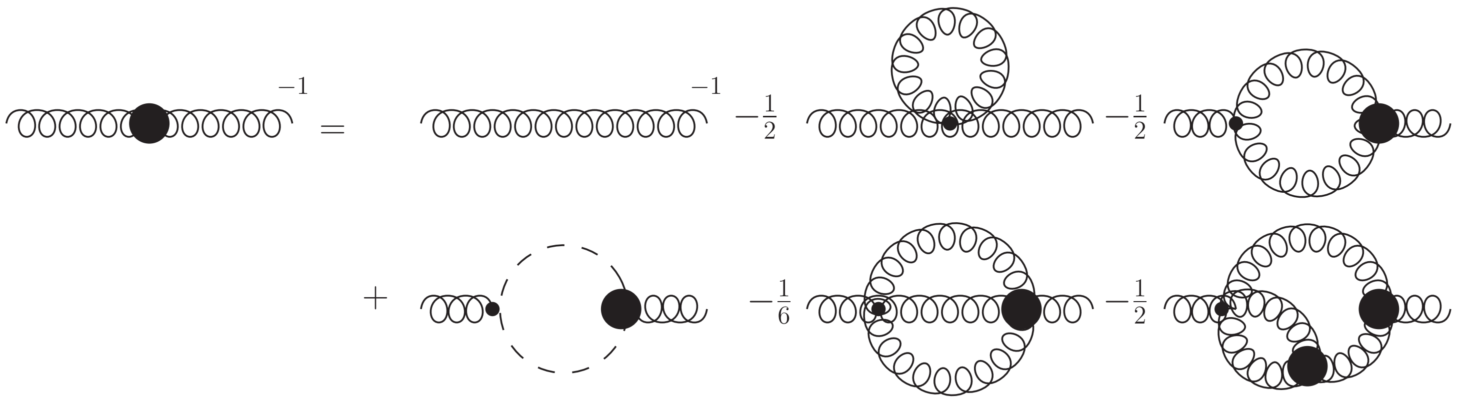


DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

- time-like momenta: work in progress

  CF, Huber, PRD 102 (2020) 094005, arXiv:2007.11505

Landau gauge gluon propagator



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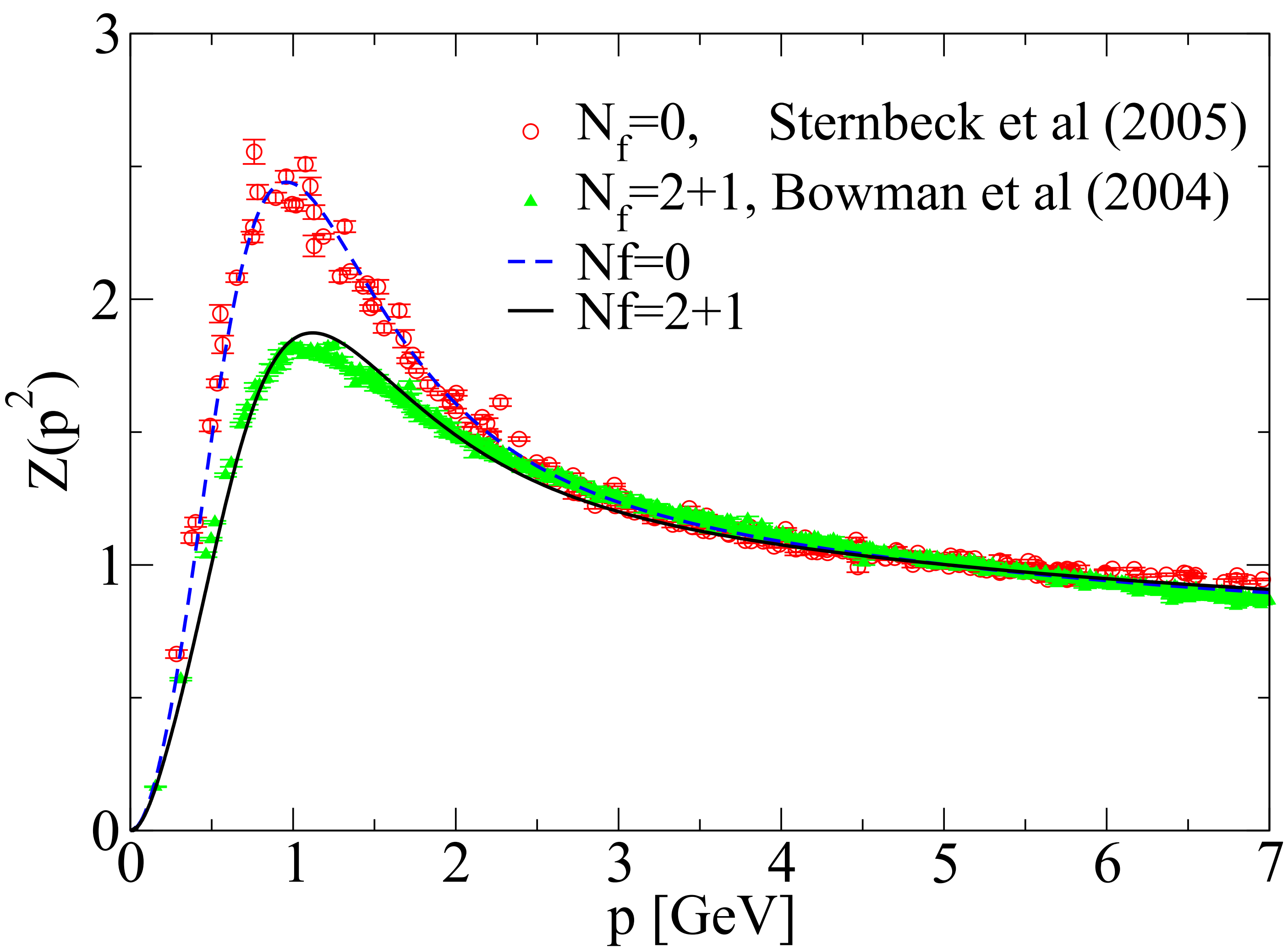

Hopfer, CF and Alkofer, JHEP 1411 (2014) 035

- spacelike momenta: good agreement with lattice
- fully dressed gluon appears massive

Cornwall PRD 26 (1982);
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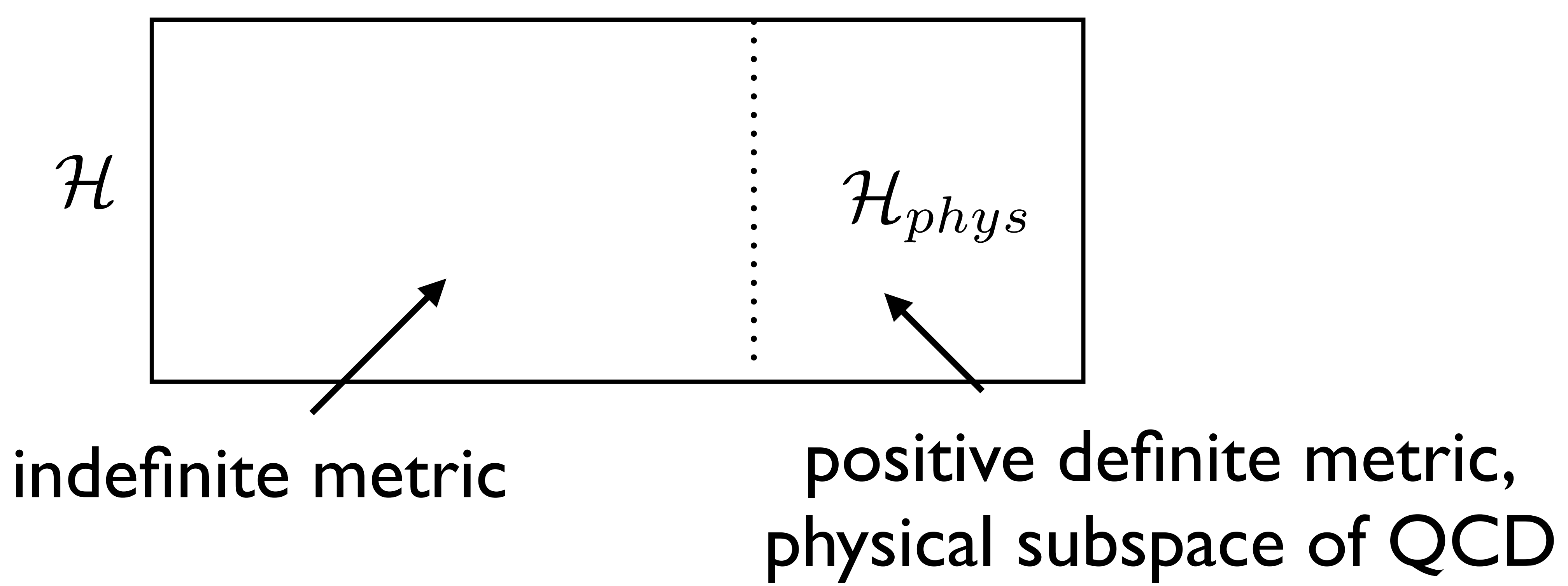
Confinement and positivity violation



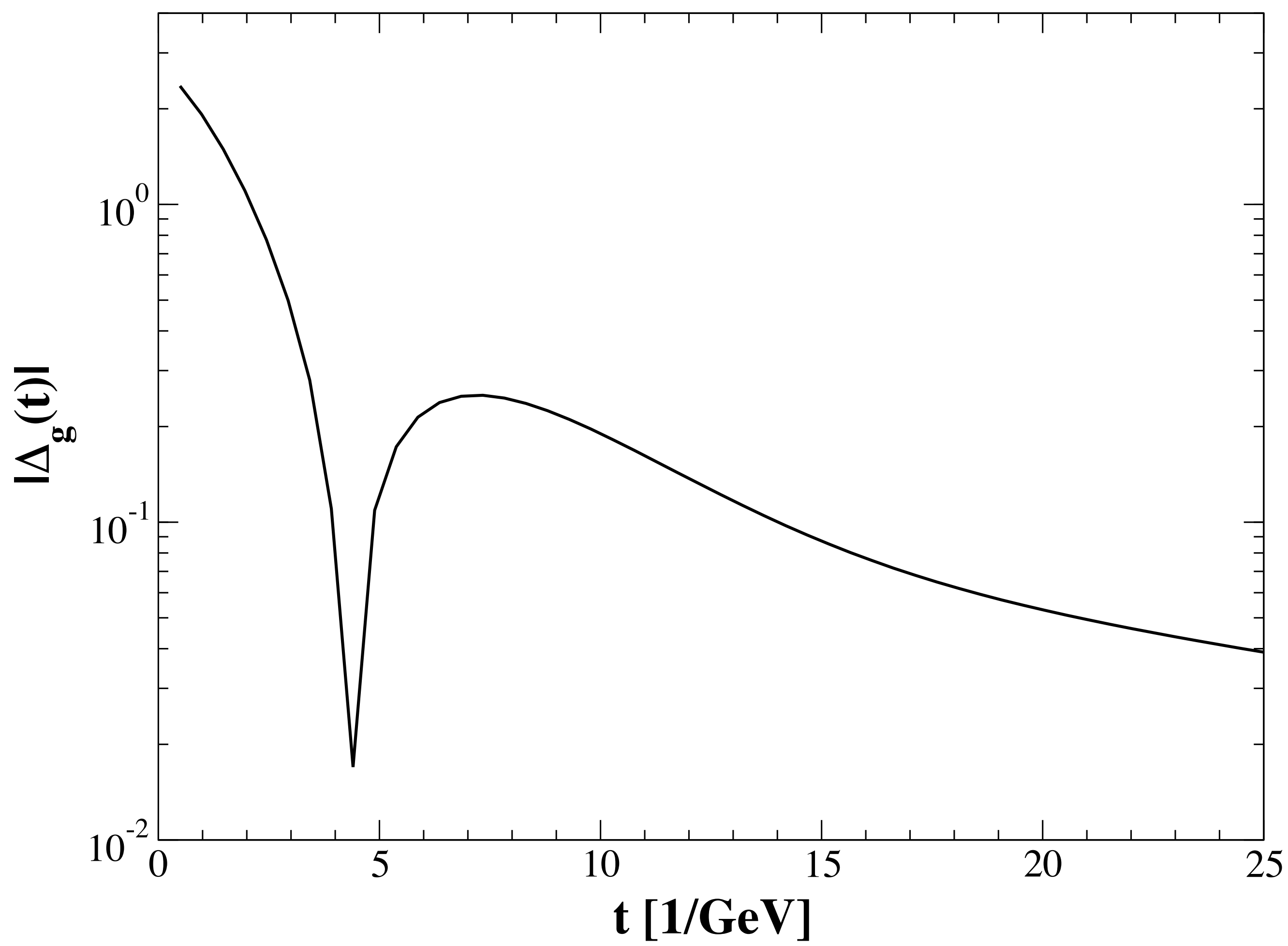
- State space of (gauge fixed) QCD: indefinite metric
- Physical asymptotic states need to live in $\mathcal{H}_{phys}$
- Conversely, if we would know that a particle lives not in $\mathcal{H}_{phys}$, it is confined.

Axiomatic QFT (Osterwalder-Schrader):

physical particle $\longrightarrow$ $D(t, \mathbf{p}) \geq 0$

Positivity violations

Schwinger function: $$\Delta_g(t) = \int \frac{dp_0}{2\pi}\, e^{itp_0} \left(\frac{Z(p^2)}{p^2}\right)\Big|_{\vec{p}=0}$$



Alkofer, Detmold, CF, Maris,
PRD 70 (2004) 014014

- Violation of positivity: color screening

Gluons cannot exist as asymptotic states

Positivity violations

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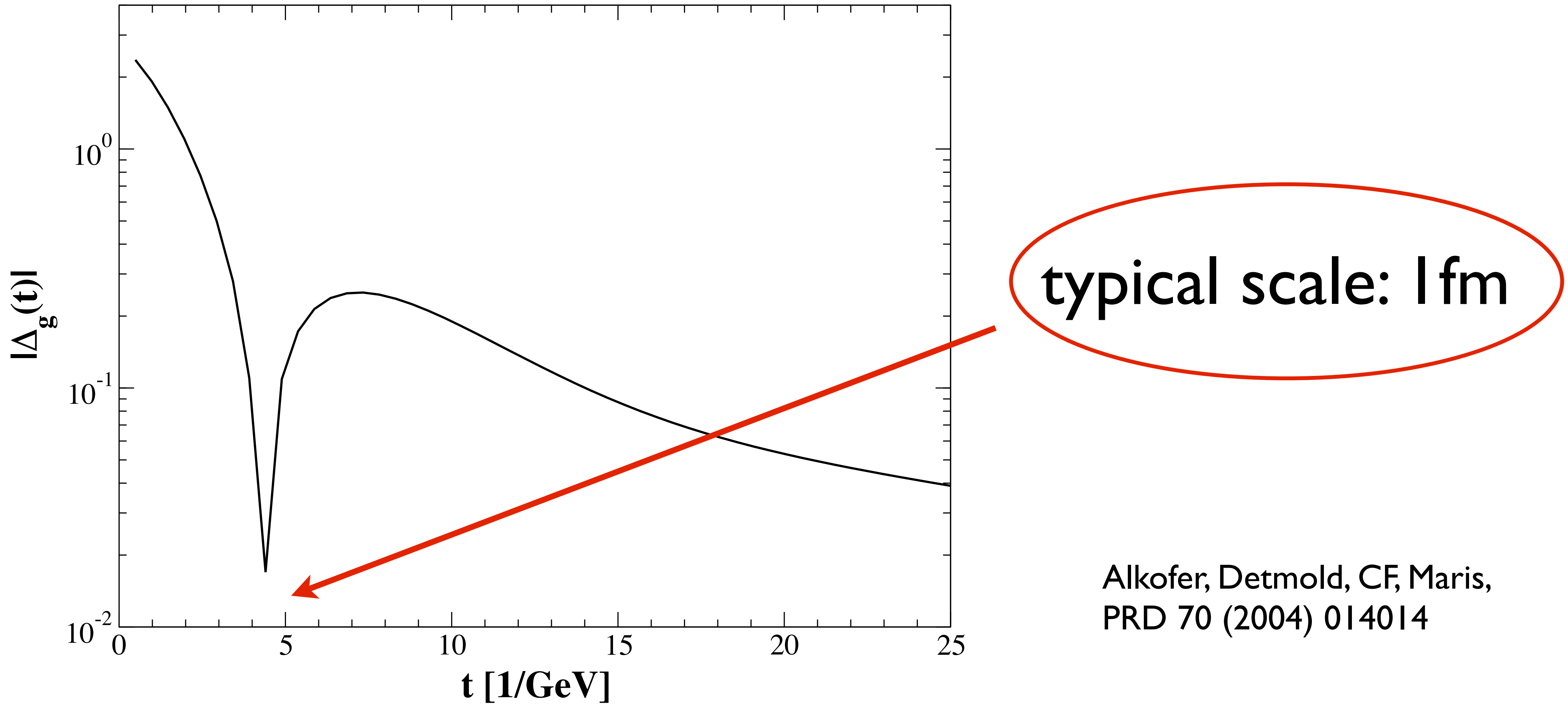

Alkofer, Detmold, CF, Maris, PRD 70 (2004) 014014

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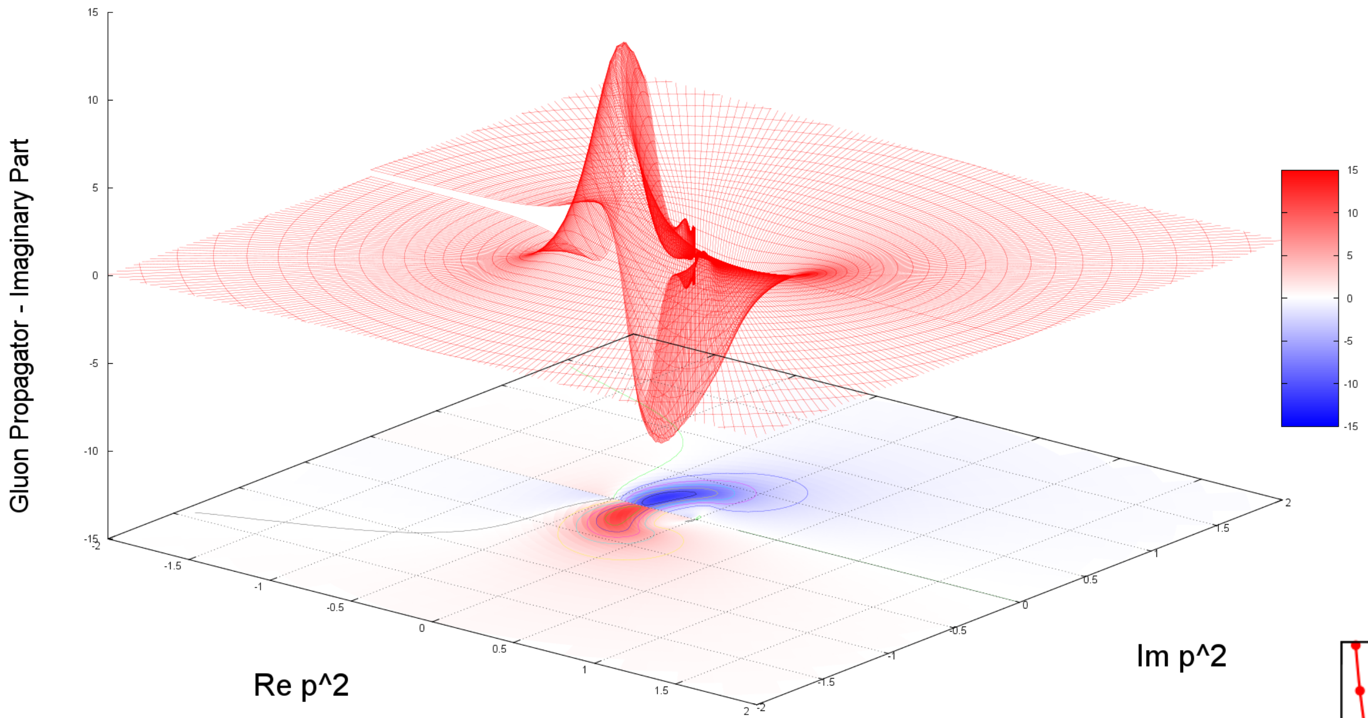
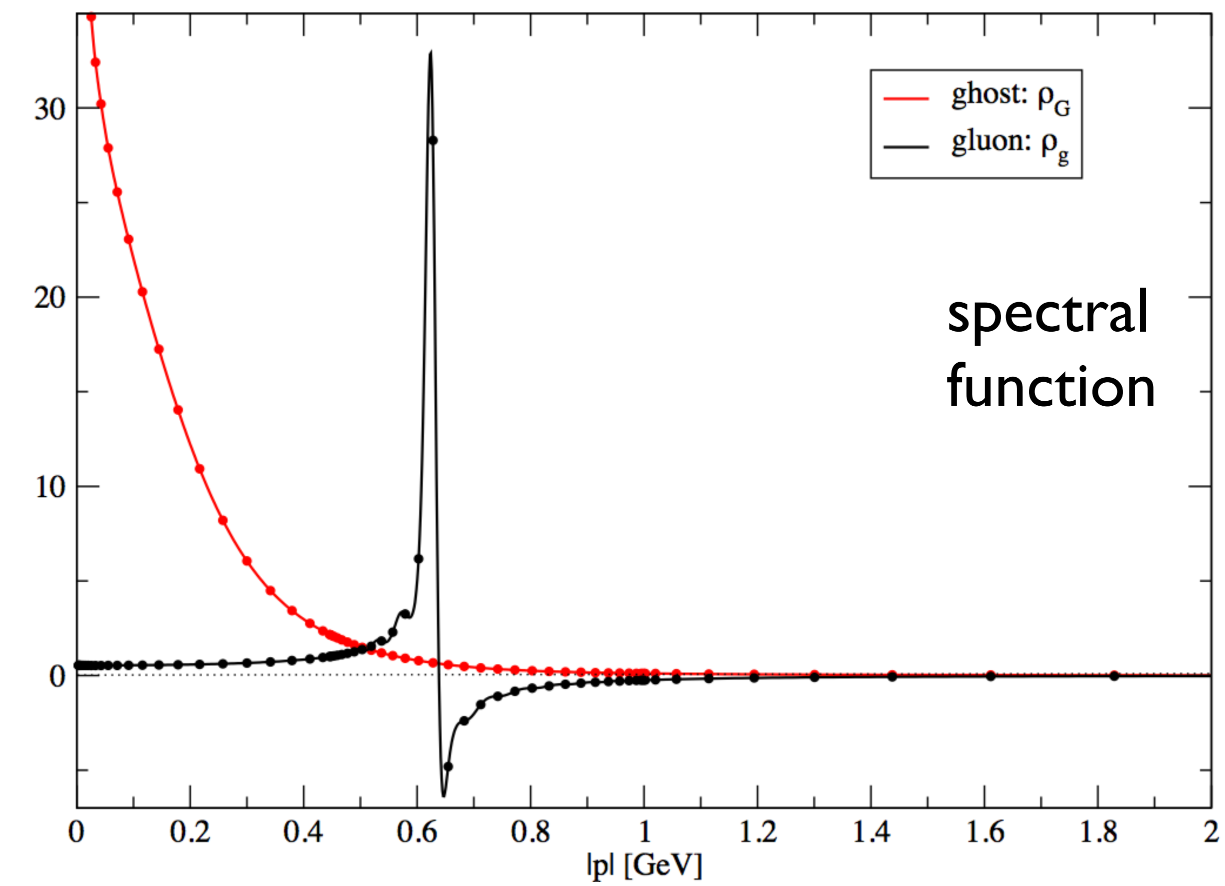




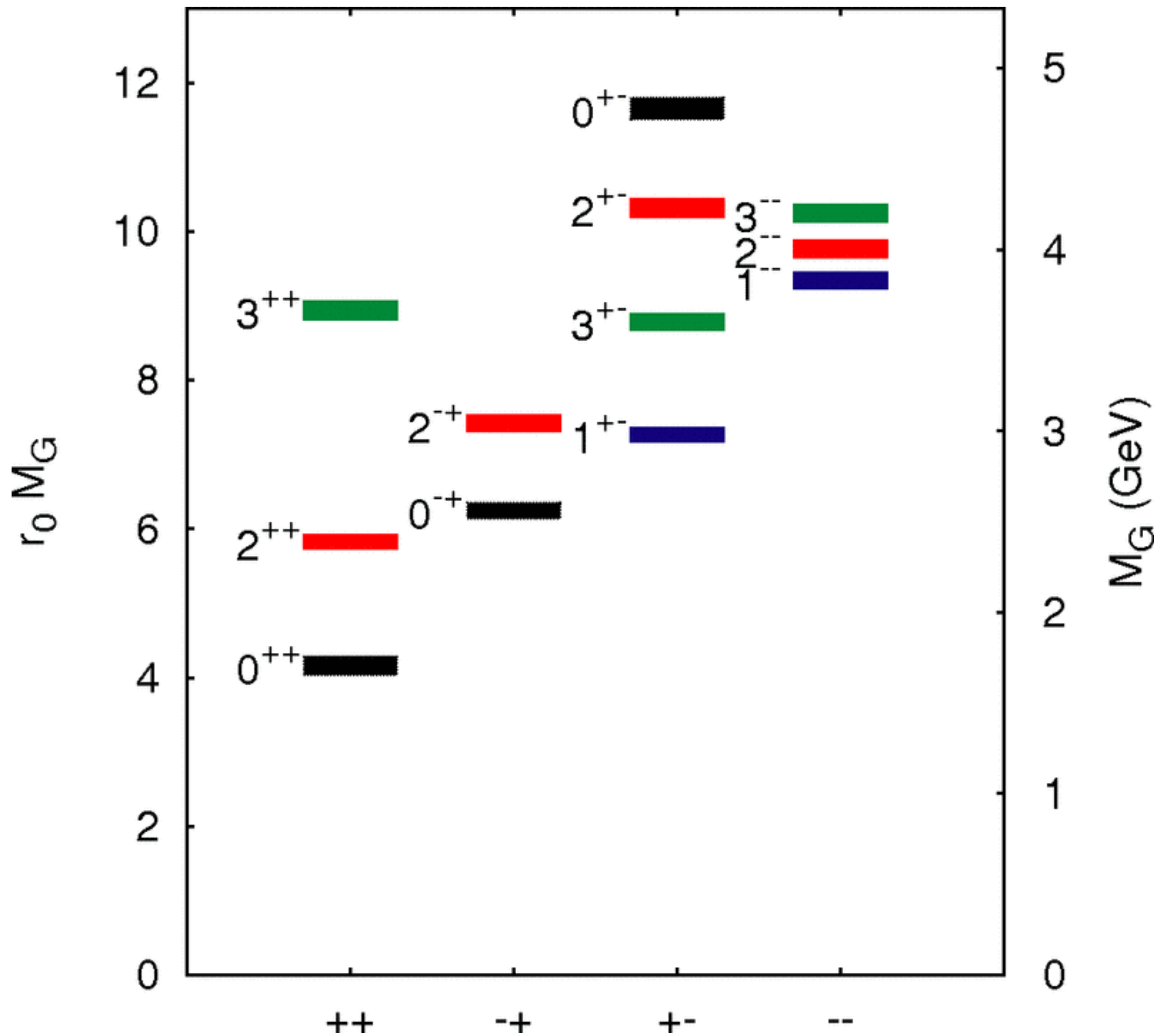
- spectral function: positivity violations
- $600\,\mathrm{MeV} < m_g < 700\,\mathrm{MeV}$

Cornwall PRD 26 (1982); Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099

Gluon cannot appear in detector!

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001
CF, Huber, PRD 102 (2020) 094005, arXiv:2007.11505

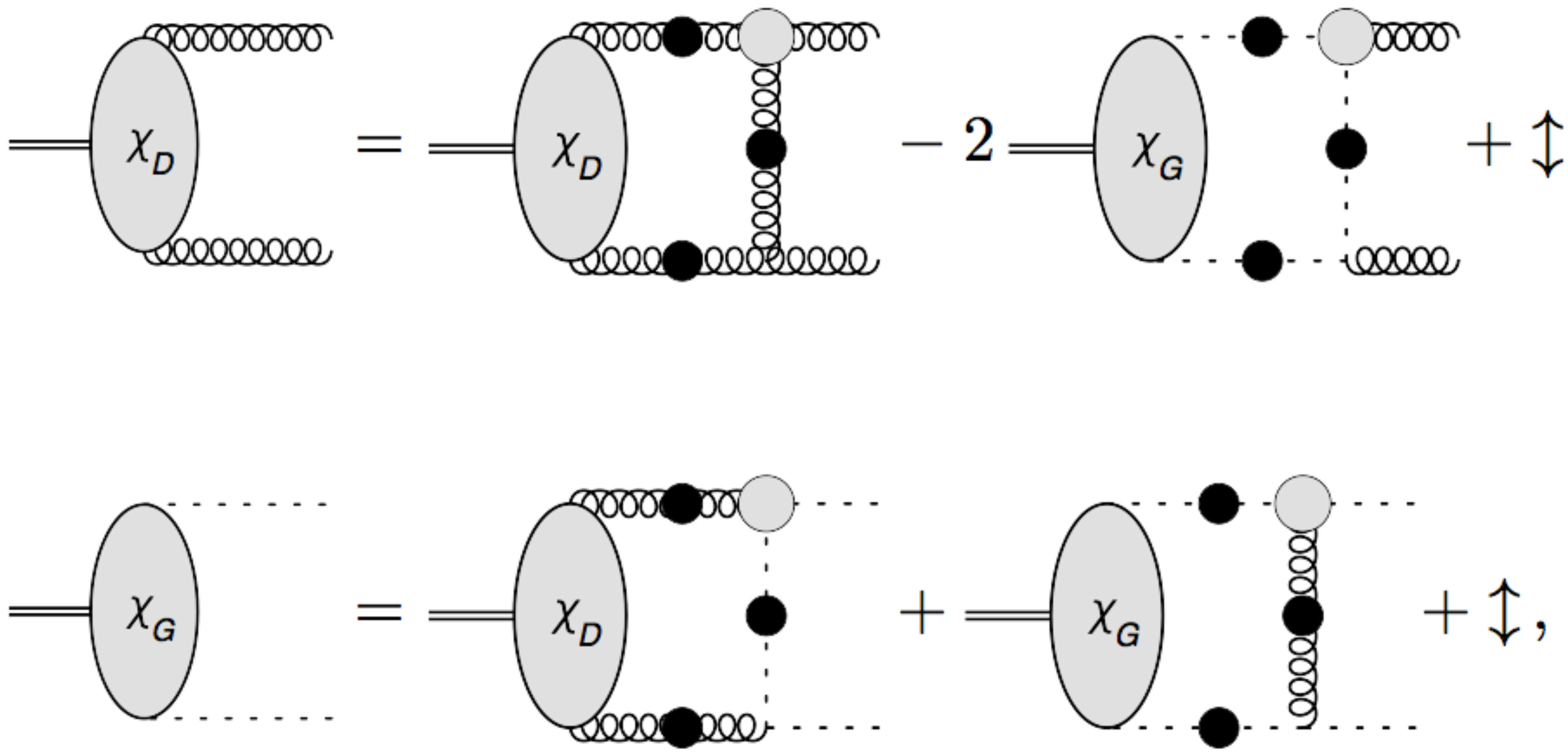
Glueballs from the lattice



Morningstar and Peardon, PRD 60 (1999)

- Lattice: reliable results for theory without dynamical quarks

Glueballs from DSE/BSEs



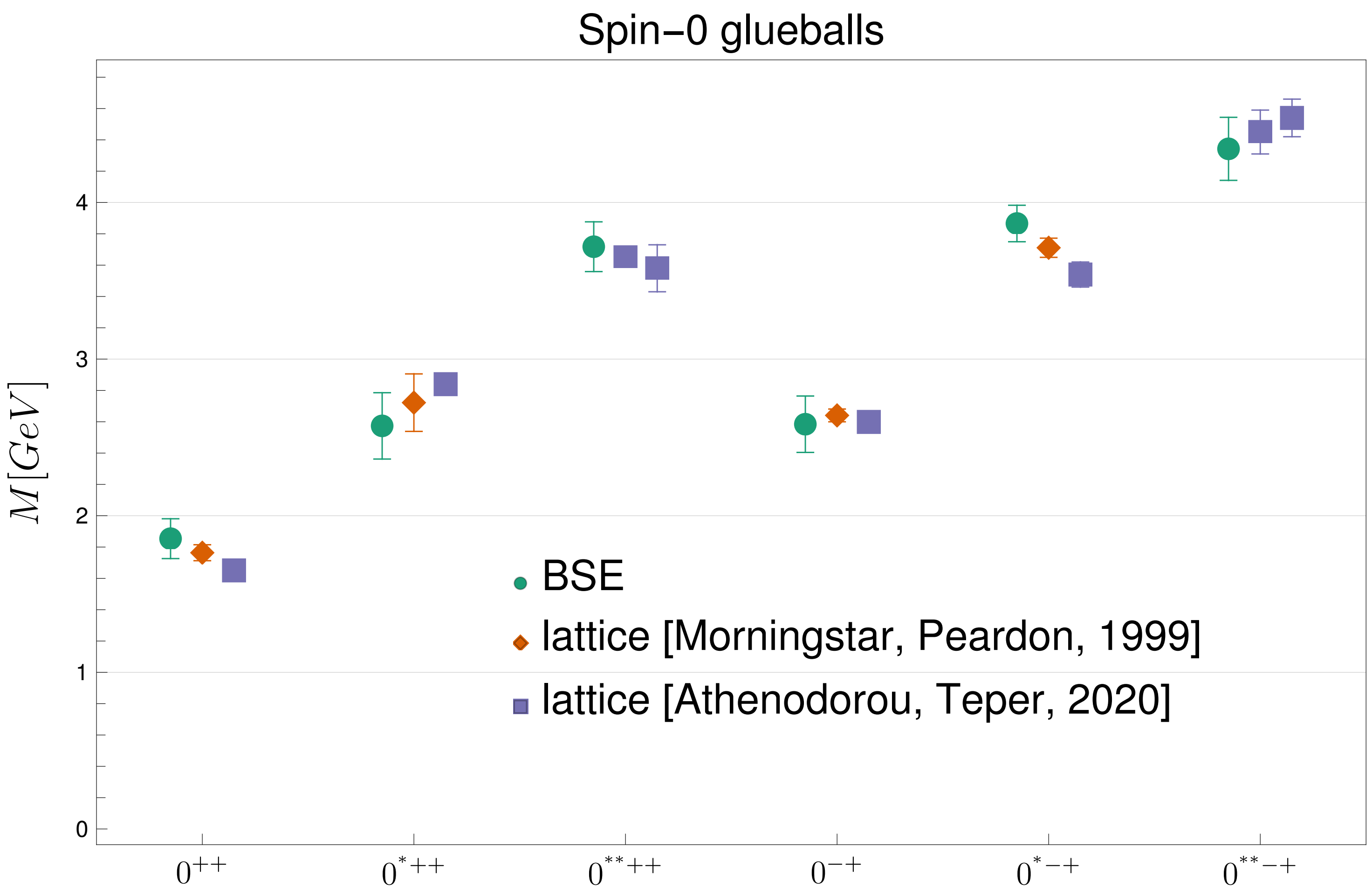
- Mixing of two-gluon amplitudes with ghost-antighost
- exploratory: simple models

Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001
Souza et al. , EPJA 56 (2020) no.1, 25

- new: high quality input from 3PI truncation

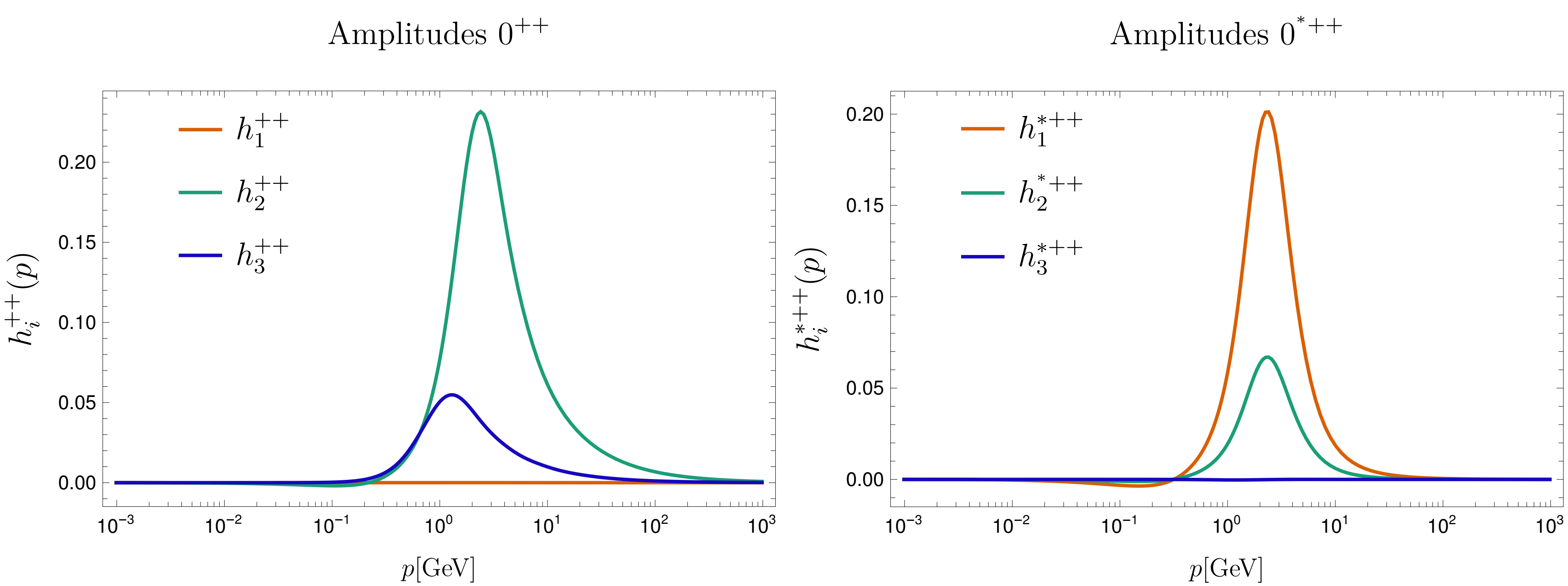
Huber, PRD 101 (2020) 114009
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020)

Glueballs: results



- spectrum: very good agreement lattice vs. DSE/BSE

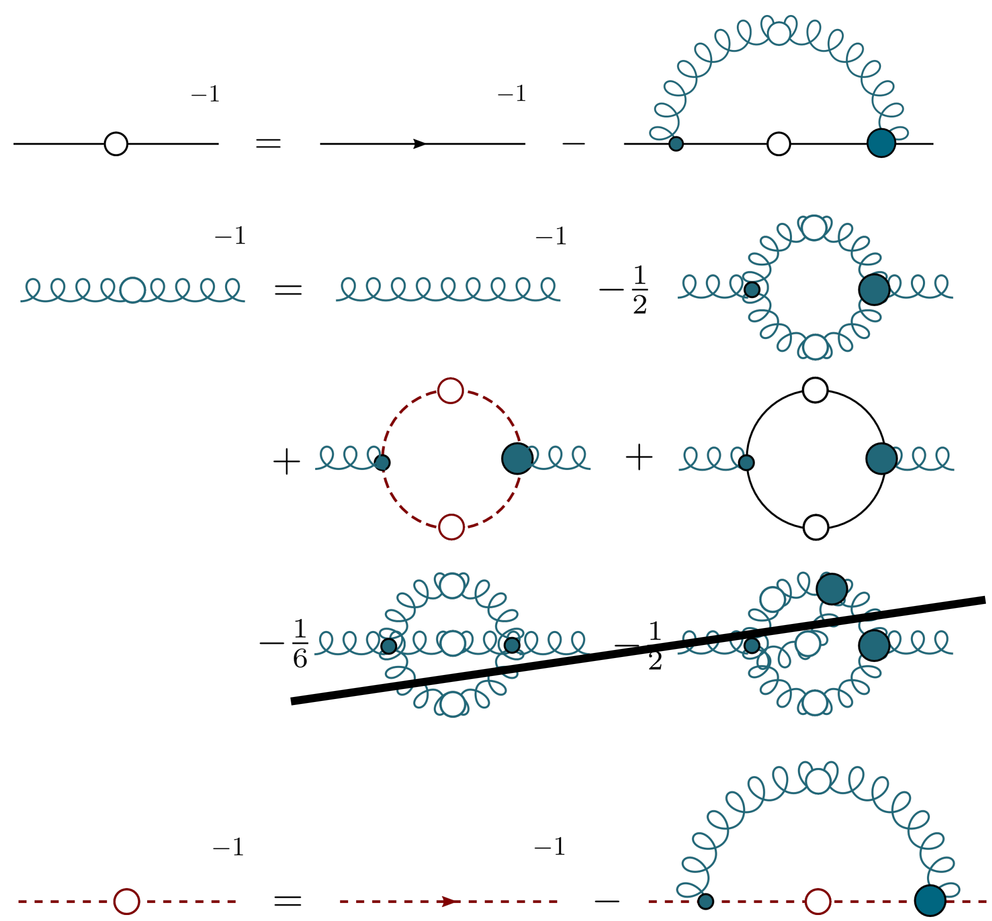
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]



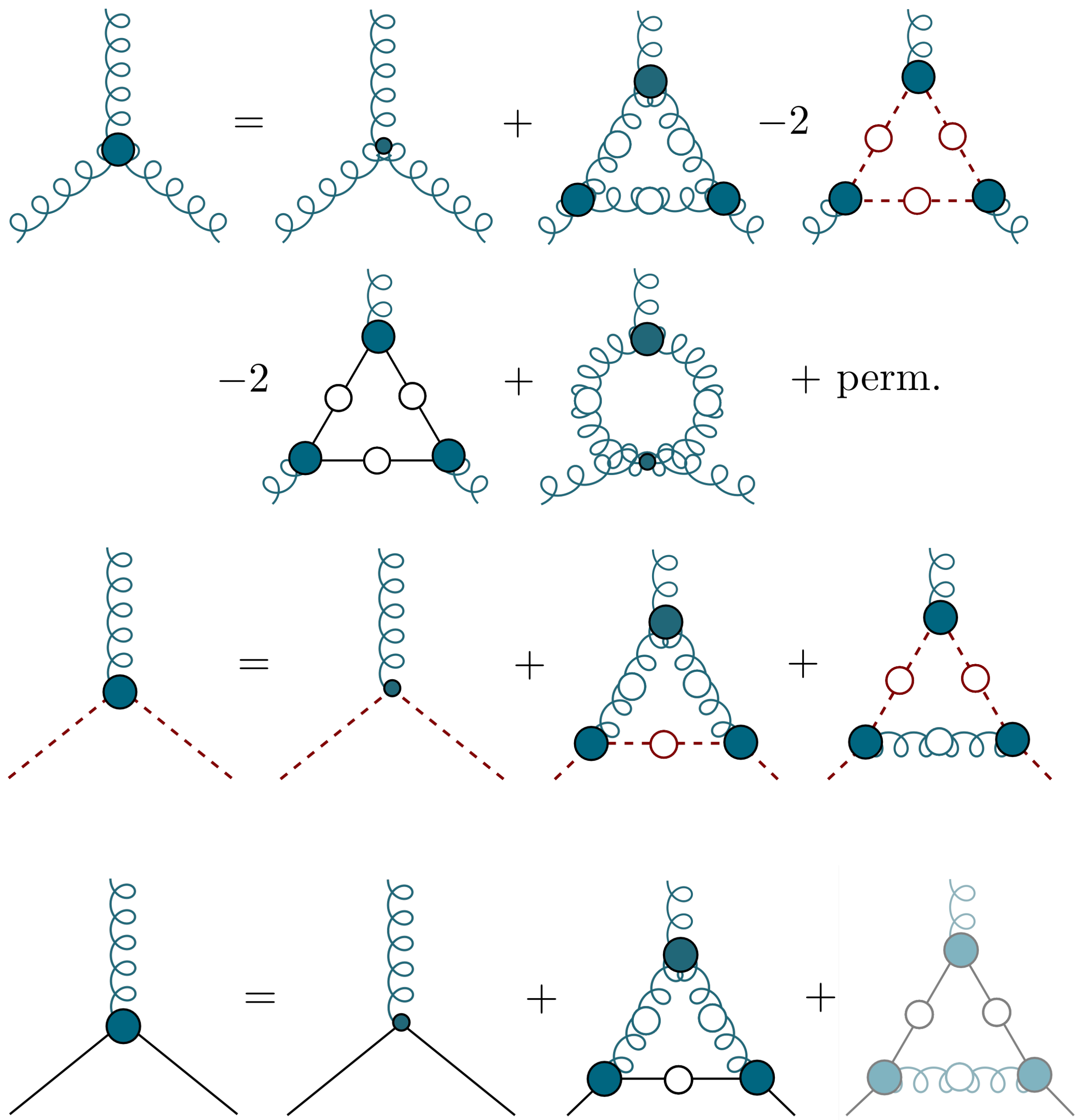
- excited states: different internal structure

3PI-truncation

propagators



vertices



Williams, CF, Heupel, PRD93 (2016) 034026

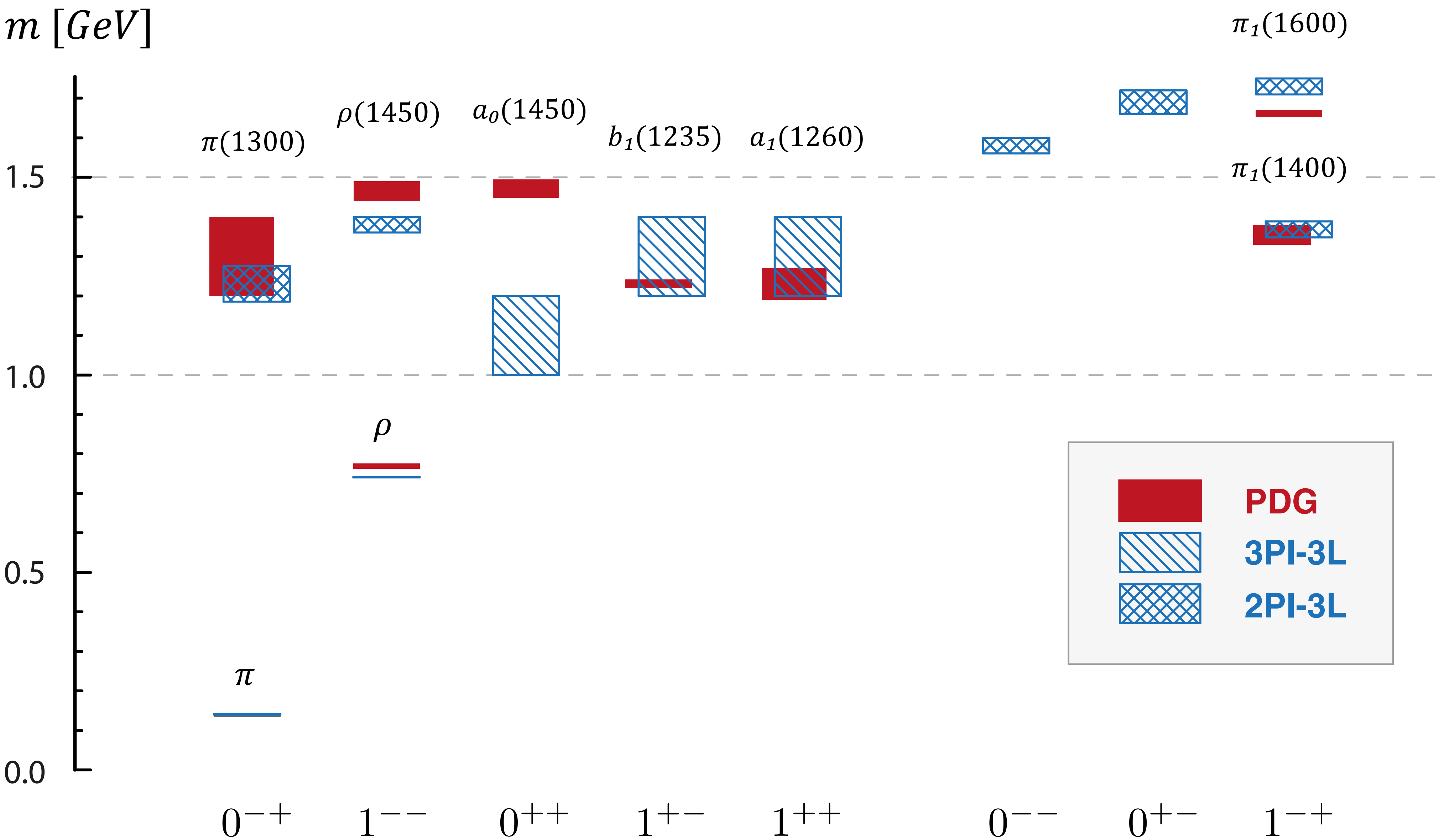
for different BRL approaches see e.g. work of Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Gao, Huber, Maas, Mitter, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

Light meson spectrum

Williams, CF, Heupel, PRD93 (2016) 034026

- excellent agreement with experiment
- to do: extension to heavy meson sector

Light meson spectrum



Williams, CF, Heupel, PRD93 (2016) 034026

- excellent agreement with experiment
- to do: extension to heavy meson sector

Summary: Hadron physics with functional methods

Main goals:

- one framework for all areas of hadron physics: mesons, baryons, 'exotic states', form factors, hadronic contributions to standard model
- access to DXSB, confinement,…

Main challenge:

- systematic control over error budget: intrinsic + cp to other methods like lattice QCD

Main results:

- NOT high precision physics
- BUT competitive contributions in many areas of hadron physics