Exploring the QCD phase structure with the Functional Renormalization Group

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FWF Der Wissenschaftsfonds.

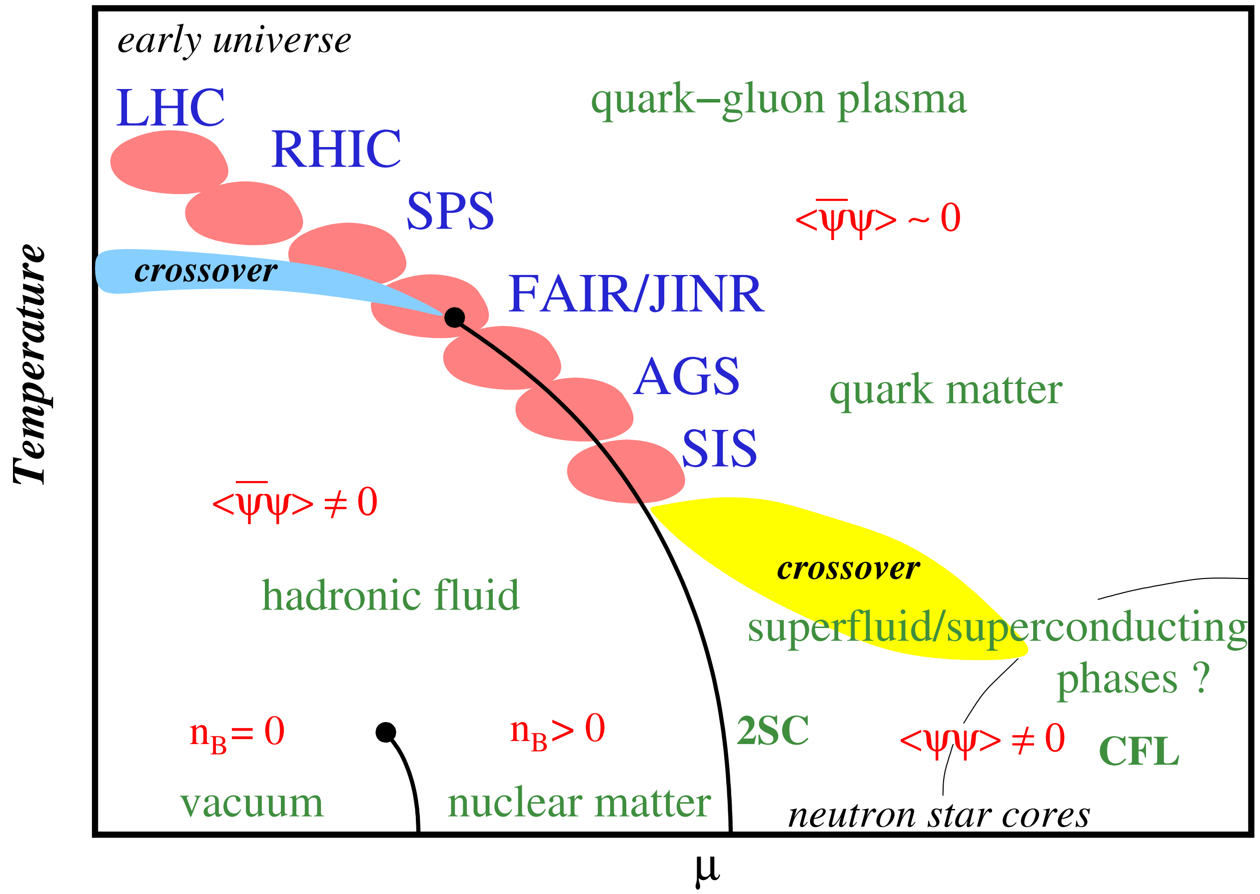
Strong Interactions in Quantum Field Theory

Bildungshaus Schloss Retzhof, Leitring, Austria. June 27th-28th 2013

UNI GRAZ

Austria-Croatia-Hungary-Triangle Workshop 2013

Conjectured QC$_3$D phase diagram



at densities/temperatures of interest
only model calculations available

➜ can one improve the model calculations?

➜ remove model parameter dependency

Theoretical questions:

related to chiral & deconfinement transition

- **CEP:** existence/location/number
- **Quarkyonic phase:** coincidence of both transitions at $\mu = 0$ & $\mu > 0$?
- relation between chiral & deconfinement?
  chiral CEP/deconfinement CEP?
- **finite volume effects?**
  ➜ lattice comparison
- **role of fluctuations?** so far mostly mean-field results **effects of fluctuations are important**

  e.g. size of critical region around CEP
- **What are good experimental signatures?**

  ➜ higher moments more sensitive to criticality deviation from HRG model for

<u>non-perturbative continuum functional methods</u>

➜ complementary to lattice

⇒ no sign problem μ>0 ⇒ chiral symmetry/fermions/small masses/chiral limit

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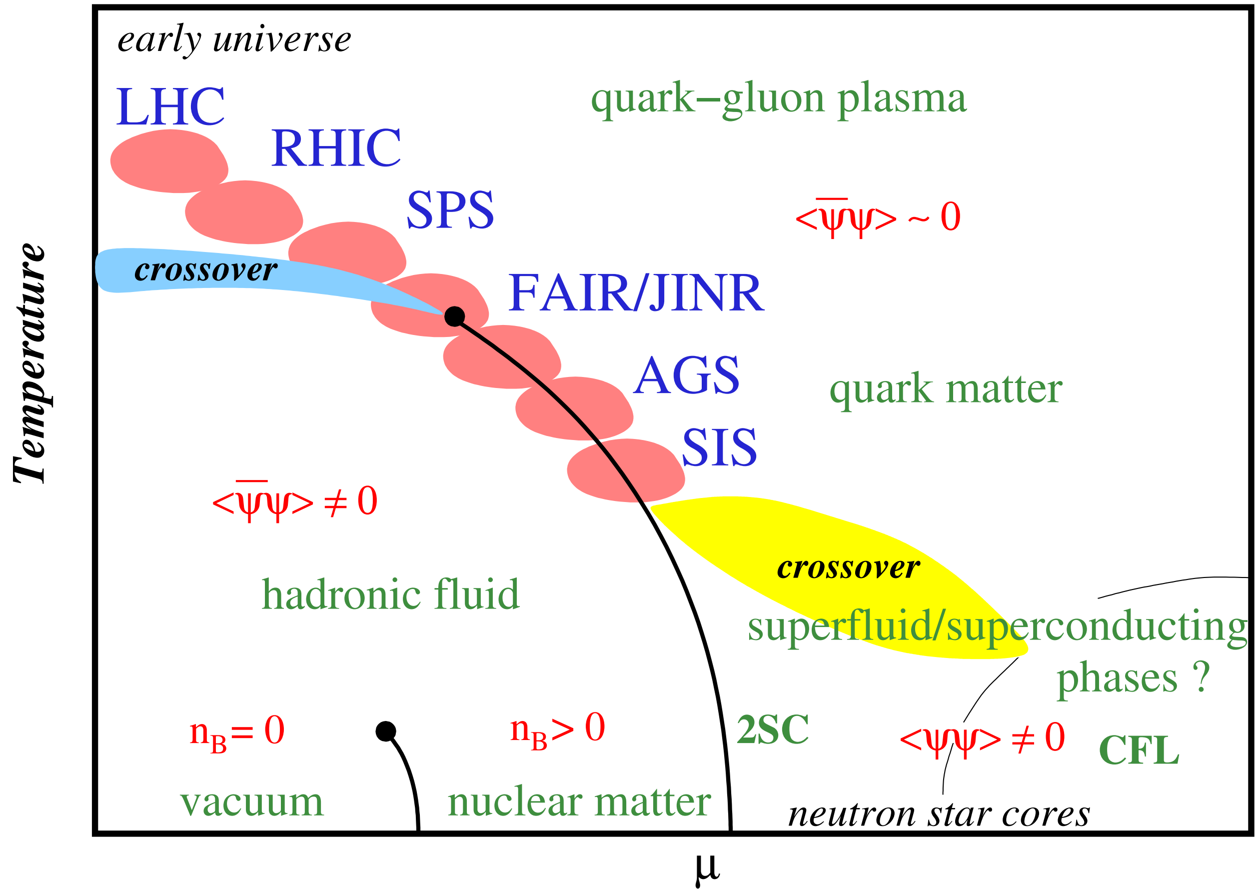

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Method of choice: **Functional Renormalization Group (FRG)**

e.g. (Polyakov)-quark-meson model truncation

- good description for chiral sector
- implementation of gauge dynamics (deconfinement sector)

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Agenda

- QCD-like model studies

  chiral and deconfinement aspects

  two and three flavor

- Mean-field approximation vs. Renormalization Group methods

- role of baryons - two color QCD

Polyakov-Quark-Meson (PQM) Model

Chiral effective model:

- quarks: ψ
- mesons: σ, $\vec{\pi}$
- gauge fields: A_μ^a in $D_\mu = \partial_\mu + iA_\mu \to$ Polyakov-loop (PQM) model

PQM Lagrangian: [BJS, J. Pawlowski, J. Wambach 2007]

$$\begin{aligned}\mathcal{L}_{\mathrm{PQM}} &= \bar{\psi}\left(\not{D} + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \mu\gamma^0\right)\psi \\ &+\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V(\vec{\phi})\end{aligned}$$

Mean-Field Approximations (MFA)

Integration of quarks, neglect bosonic fluctuations

Grand potential

$$\Omega(T,\mu;\sigma) = \Omega_{\rm vac} + \Omega_{\rm T} + V_{\rm MF}(\sigma) \qquad (+\mathcal{U}_{\rm Poly}(\Phi)\ )$$

vacuum term: sharp three-momentum cutoff

$$\Omega_{\rm vac}(\Lambda) = -4\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+m_q^2}$$

for each cutoff: adjust model parameters like f_π, m_σ, m_π

standard MFA: no-sea term $\Lambda = 0$

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Chiral transition

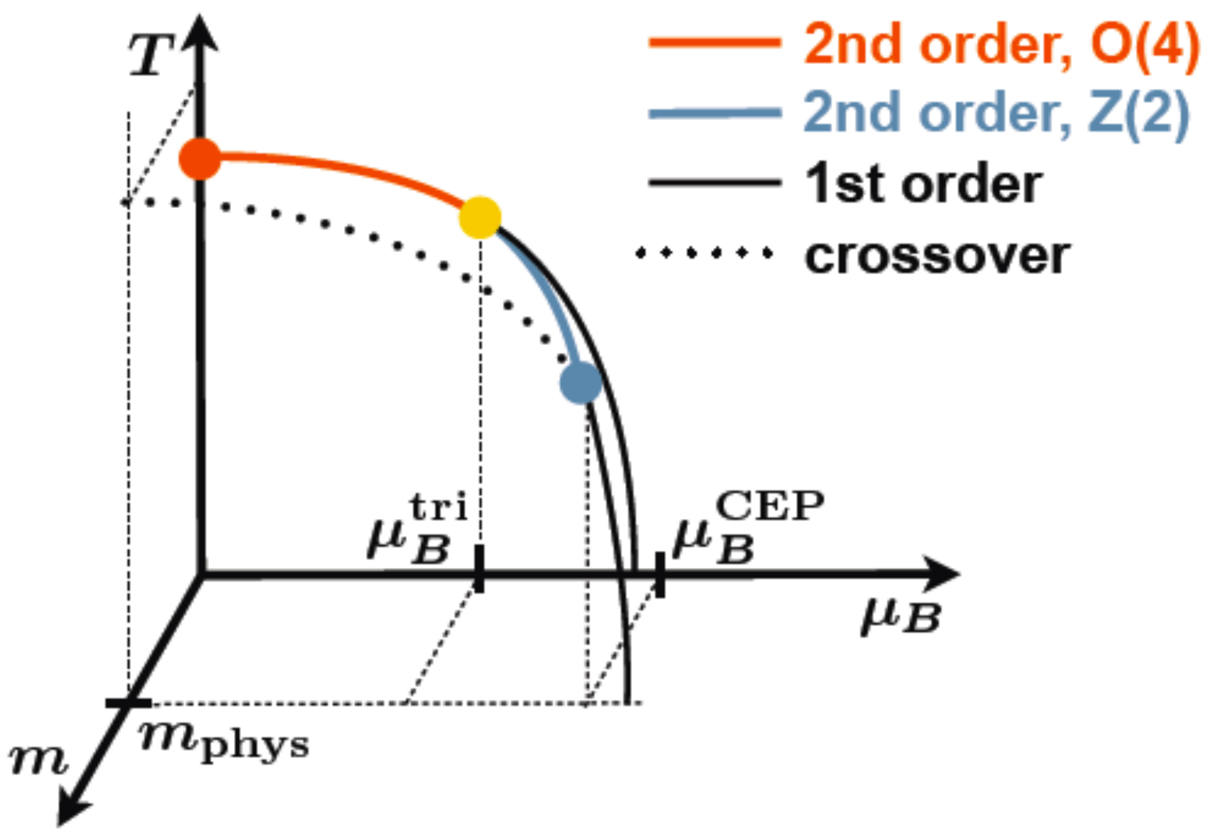
Fluctuations of oder parameter $\rightarrow \infty$ at 2nd order transition

critical fluctuations ➜ phase boundary?

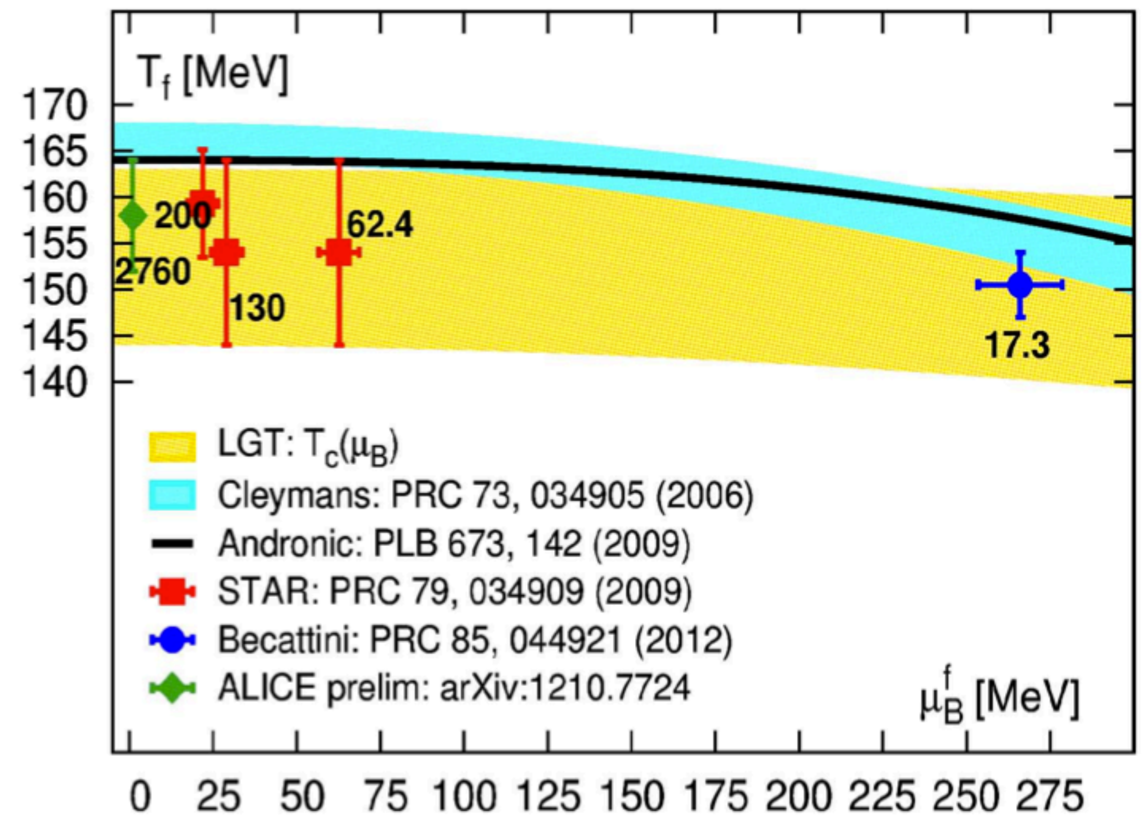
How can we probe a transition?

- singular behaviour in $\frac{\partial^n p(X)}{\partial X^n}$ with $X = T, \mu, \ldots$
- higher order cumulants $c_n \equiv \frac{\partial^n p(T,\mu)}{\partial(\mu/T)^n}$

. . . more sensitive to criticality



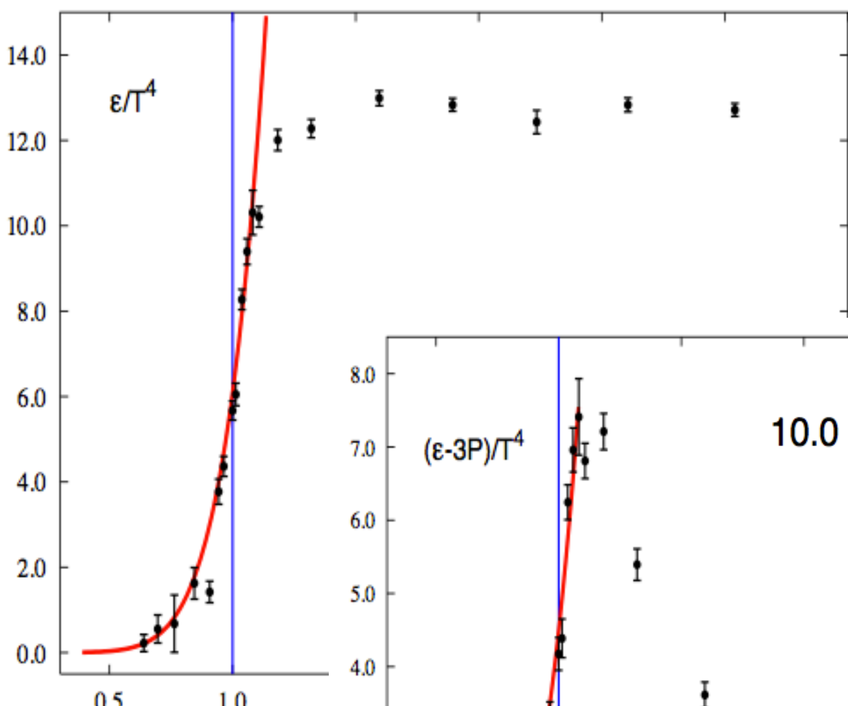
freeze-out close to chiral crossover line

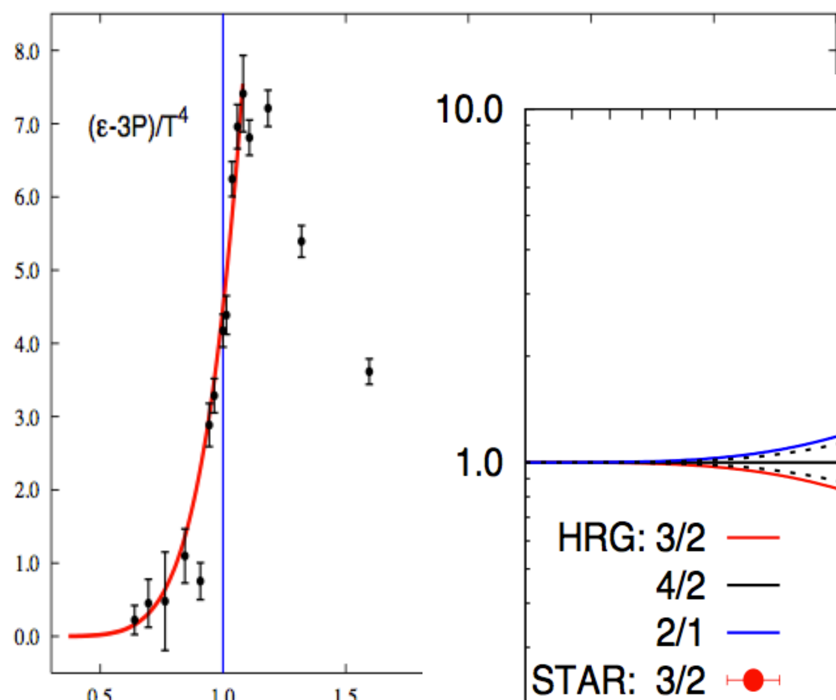


[HotQCD, QM 2012]

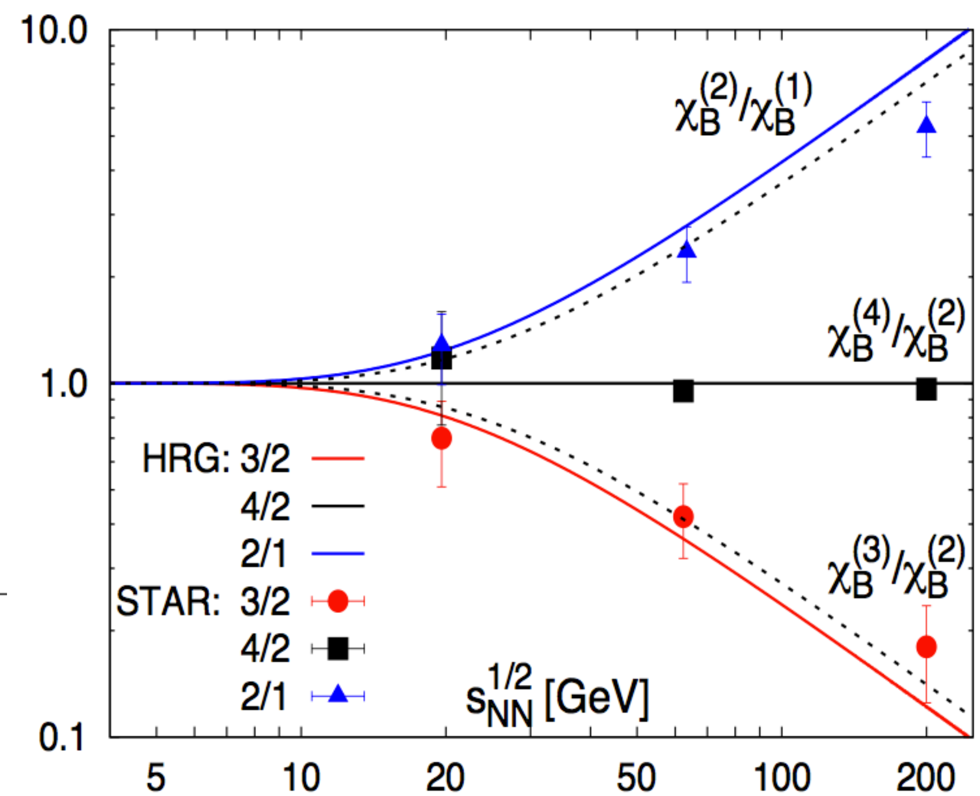
Hadron Resonance Gas (HRG) Model

HRG model: good lattice data description





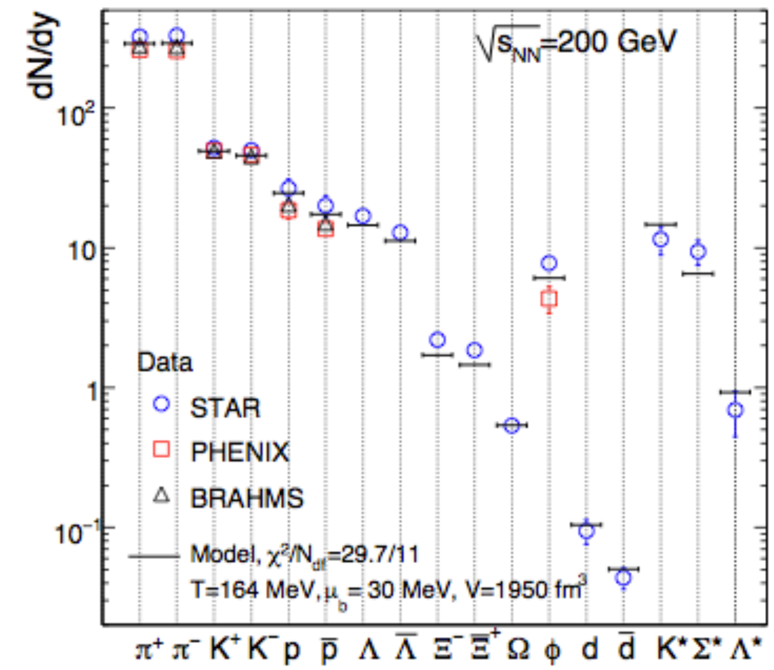
HRG model: no critical fluctuations



[Karsch, Redlich 2010]

HRG model versus experiment

[Andronic et al. 2011]



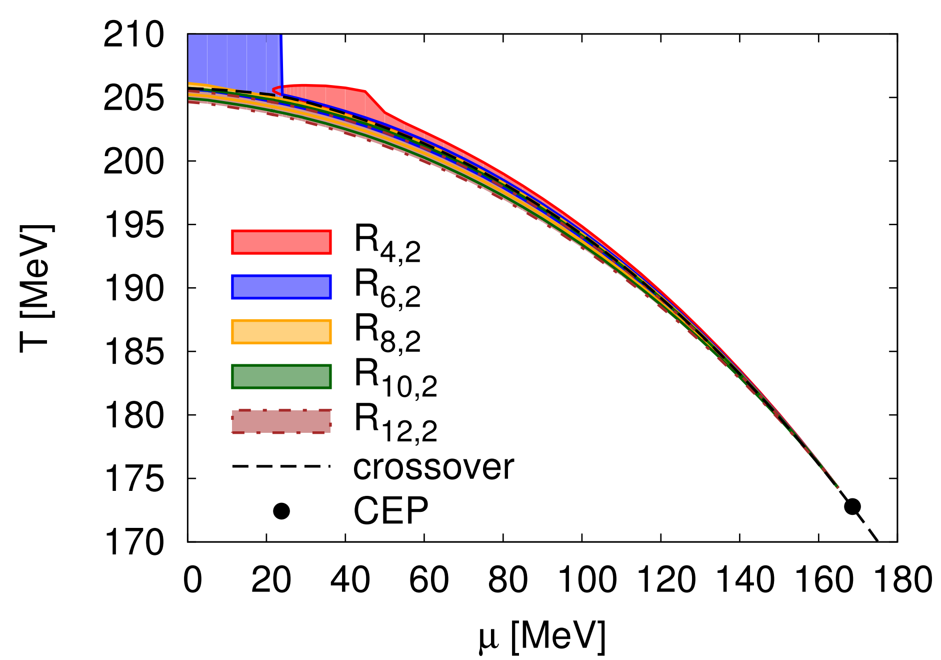
Role of vacuum term in (P)QM models

Fluctuations of higher moments exhibit **strong variation from HRG model**

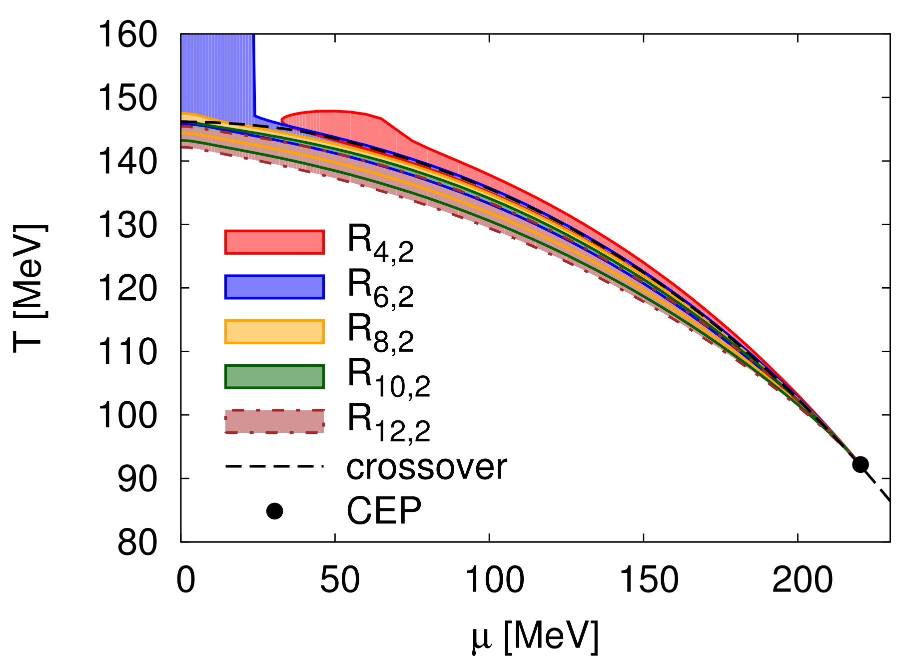
[Karsch, Redlich, Friman, Koch et al. 2011]

- $\to$ turn negative
- higher moments: $R^q_{n,m} = c_n/c_m$
- regions where $R_{n,2} < 0$ along crossover in the phase diagram

unquenched PQM MFA



QM MFA w/o vacuum



role of vacuum term in (P)QM models see [BJS, M. Wagner 2012]

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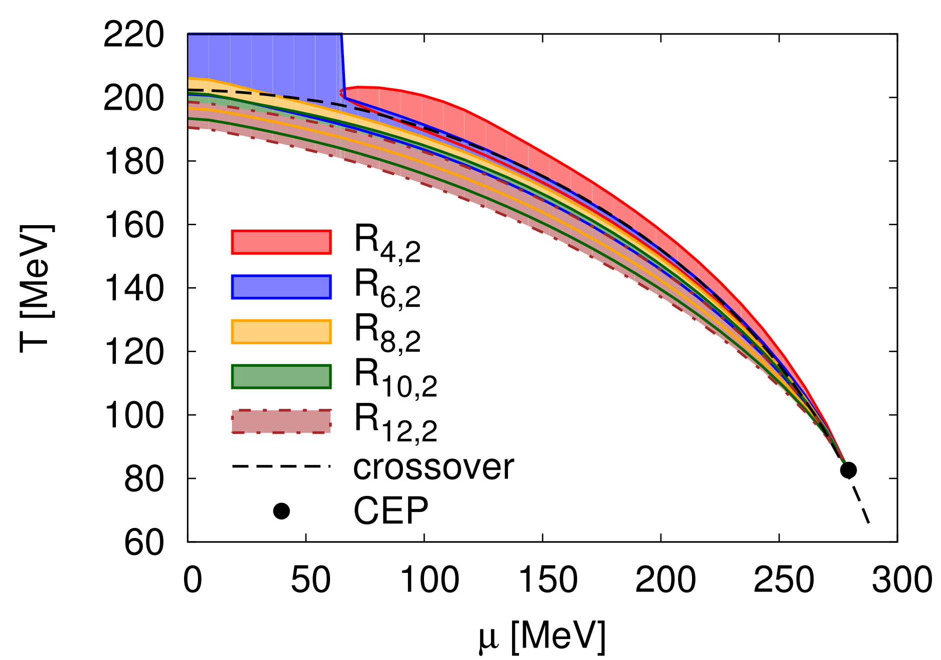

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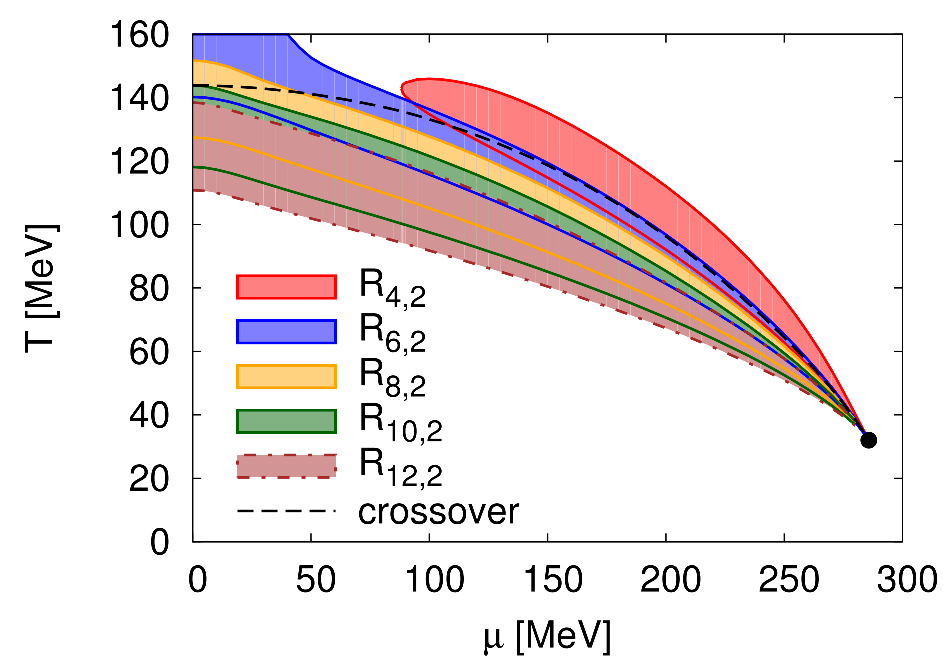


role of vacuum term in (P)QM models see [BJS, M. Wagner 2012]

Mean-Field PQM N_f=2+1

Two improvements:

1. sea contribution

no-sea: red dashed line

sea: green line (vacuum term included)

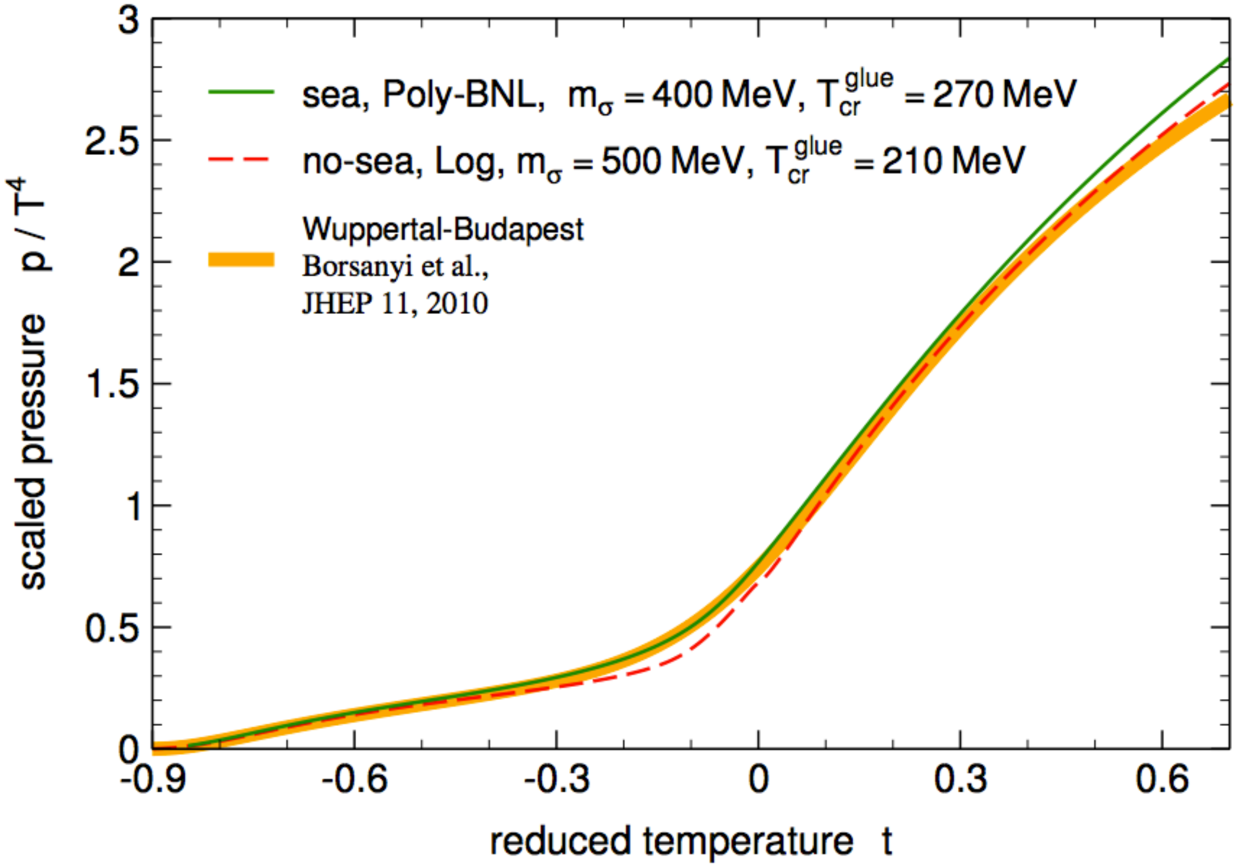
2. matter back-coupling

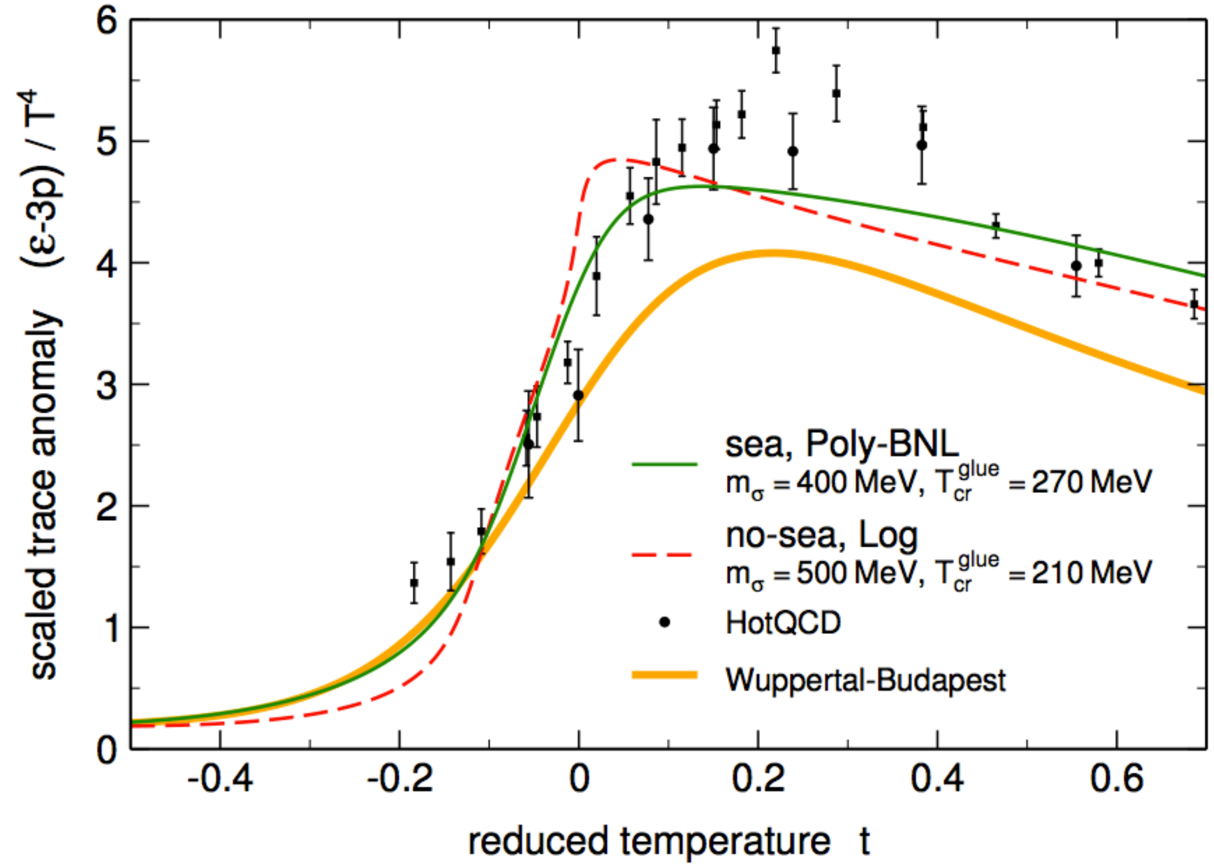
an effective unquenching

$$\mathcal{U}_{glue}(t_{glue}) = \mathcal{U}_{\text{YM}}(t_{\text{YM}})$$

with $t_{\text{YM}}(t_{glue}) = 0.57\, t_{\text{glue}}$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]

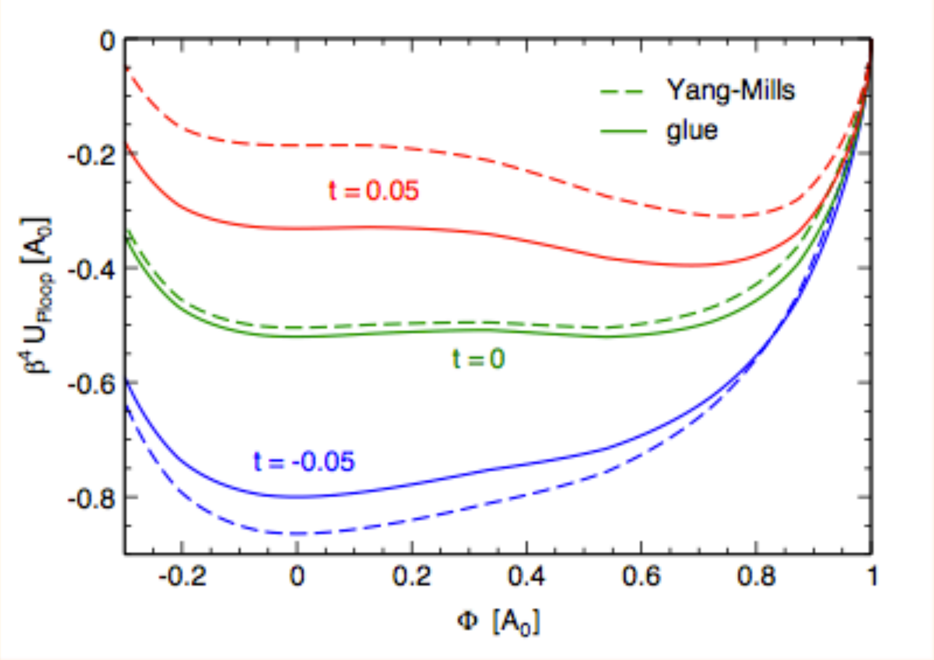


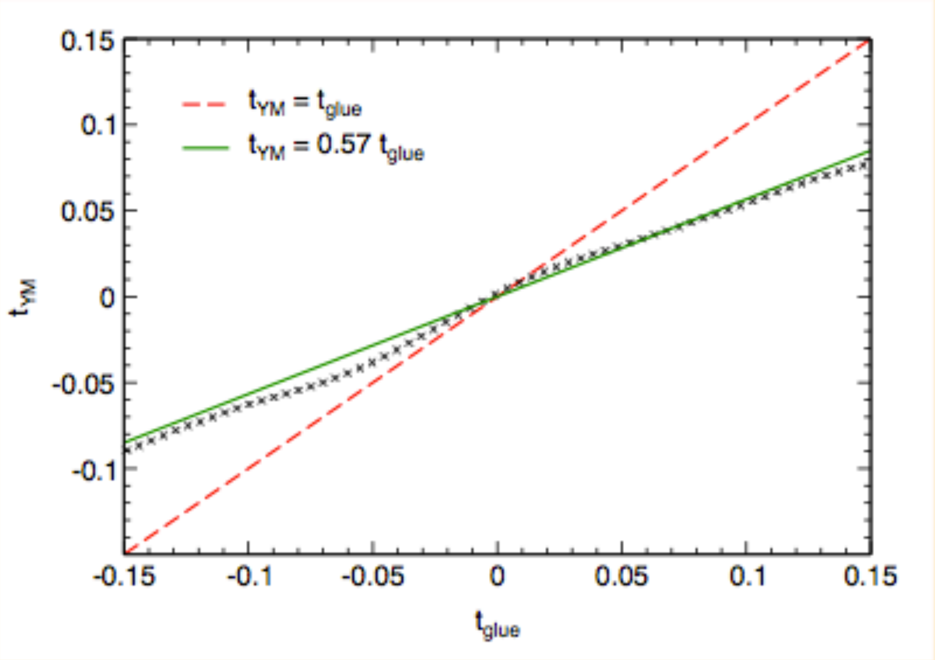


Matching the scales with FRG

Map between Yang-Mills and glue potential: $t_{\rm YM}\left(t_{\rm glue}\right)$

L. M. Haas, RS, J. Braun, J. M. Pawlowski, J. Schaffner-Bielich, PRD 87, 076004, 2013





$\rightarrow\ t_{\rm YM}\left(t_{\rm glue}\right)$ $\qquad$ $t_{\rm YM} = 0.57\, t_{\rm glue} \qquad \partial t_{\rm YM}/\partial t_{\rm glue} < 1$

$$\Rightarrow \quad \mathcal{U}_{\rm glue}\left(t, \Phi\right) = \mathcal{U}_{\rm YM}\left(t_{\rm YM}(t)\,, \Phi\right)$$

Mean-Field PQM N_f=2+1

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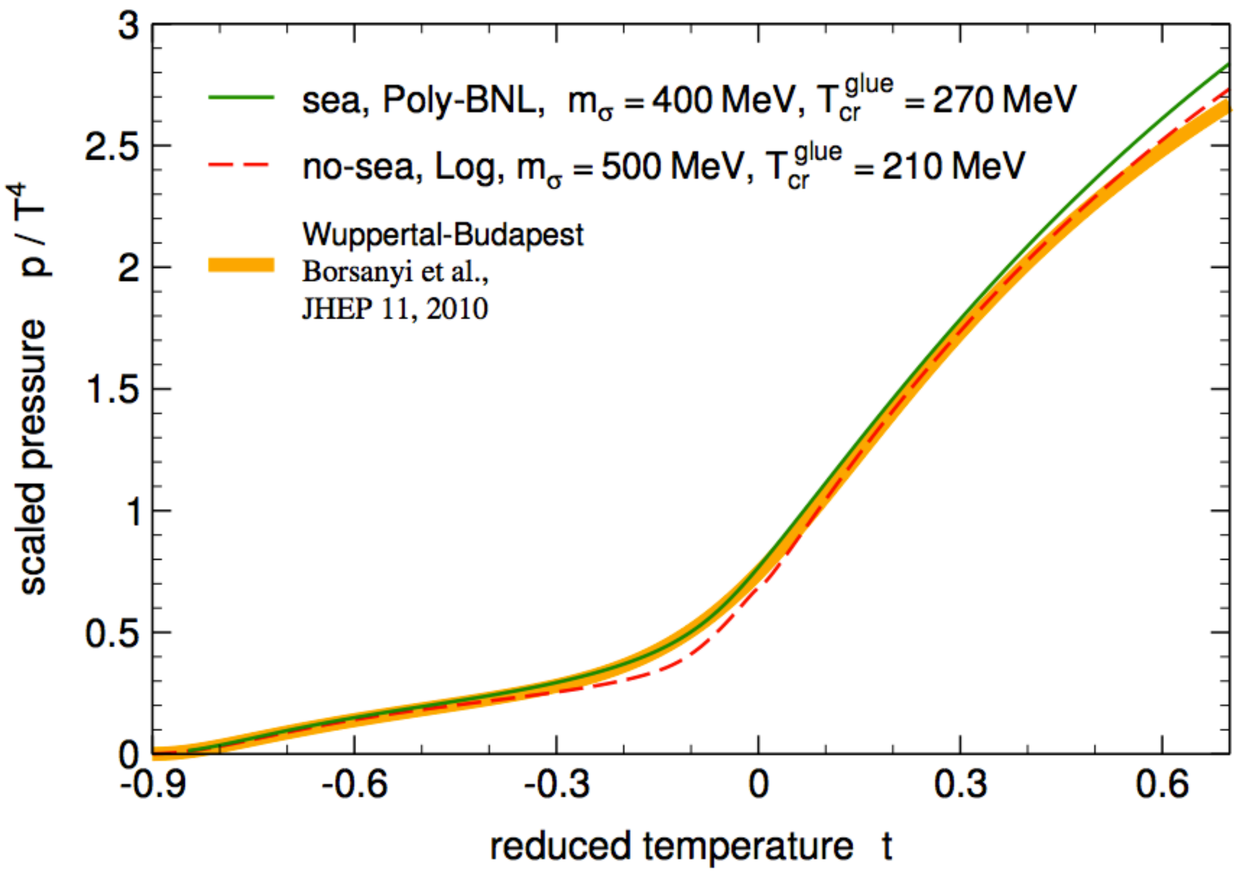
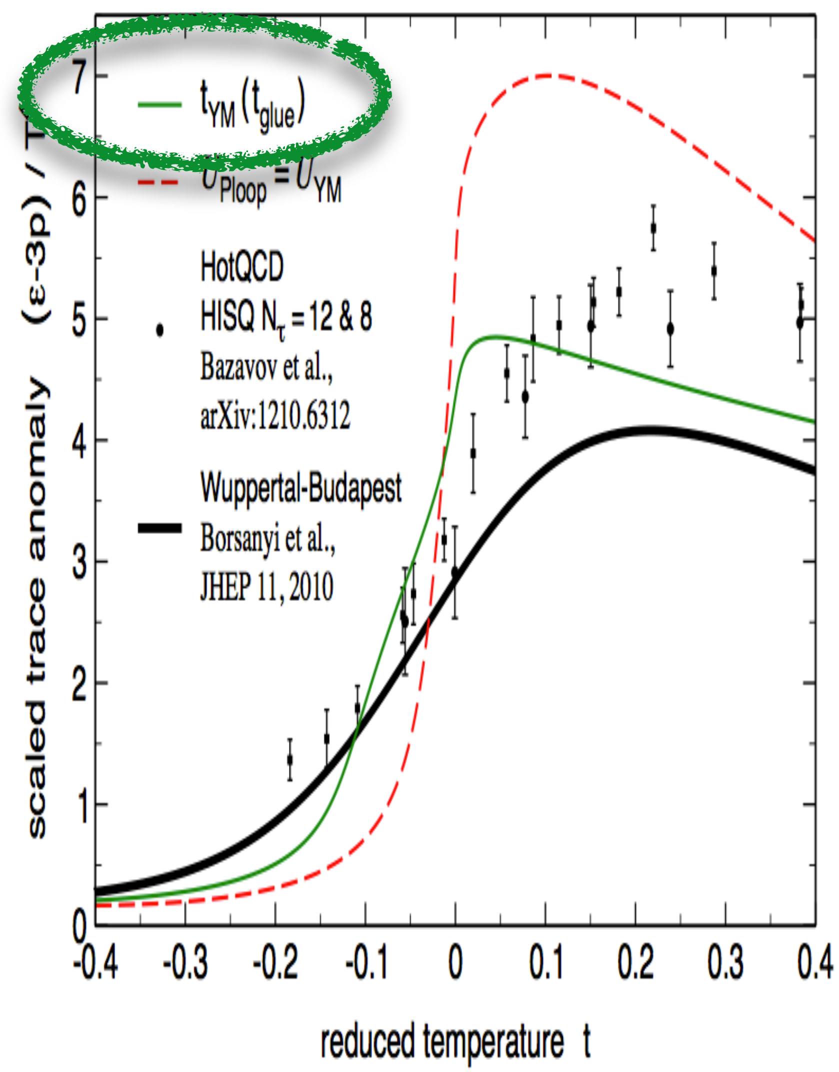
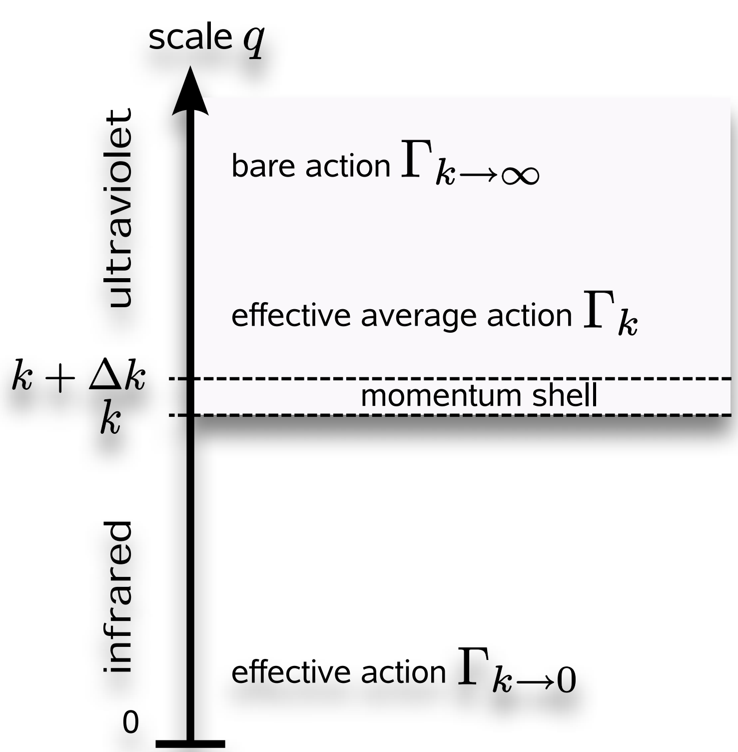




Functional RG (FRG) Approach

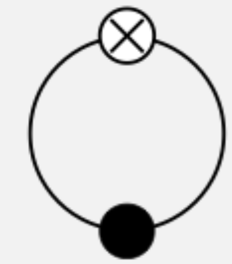
■ $\Gamma_k[\phi]$ scale dependent effective action $\qquad t = \ln(k/\Lambda) \qquad R_k$ regulators $\qquad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$



FRG (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



[Wetterich 1993]

■ Ansatz for Γ_k: **Leading order derivative expansion** $\qquad$ **arbitrary potential V**

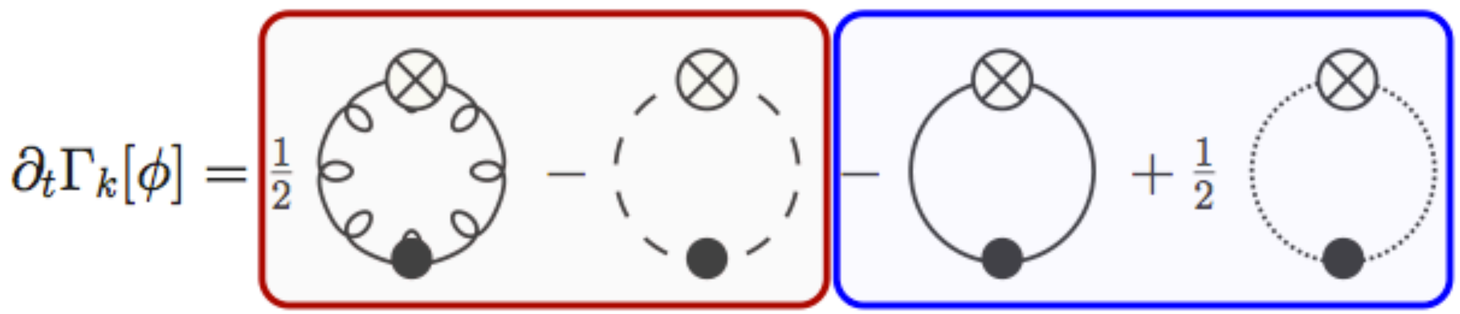
$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

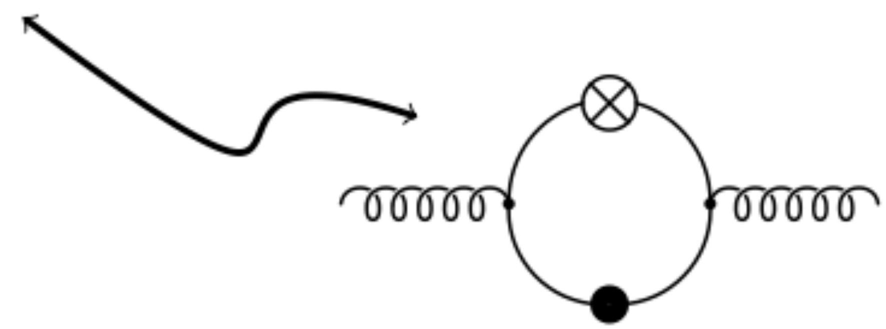
FRG and QCD

full dynamical QCD FRG flow: fluctuations of
gluon, ghost, quark and meson (via hadronization) fluctuations

Braun, Haas, Marhauser, Pawlowski; 2009

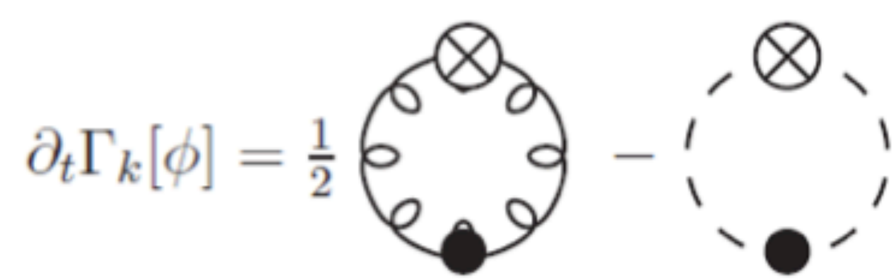


in presence of dynamical quarks:
gluon propagator modified:



$\Rightarrow$ pure Yang Mills flow + matter back-coupling

pure Yang Mills flow
replaced by eff. Polyakov-loop potential $\mathcal{U}_{\text{Pol}}$:
(fit to lattice YM thermodynamics)



FRG and QCD

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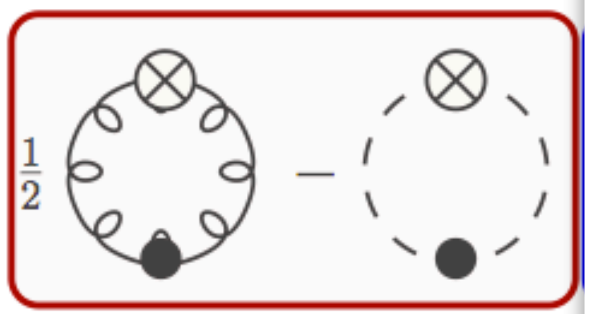
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$ 

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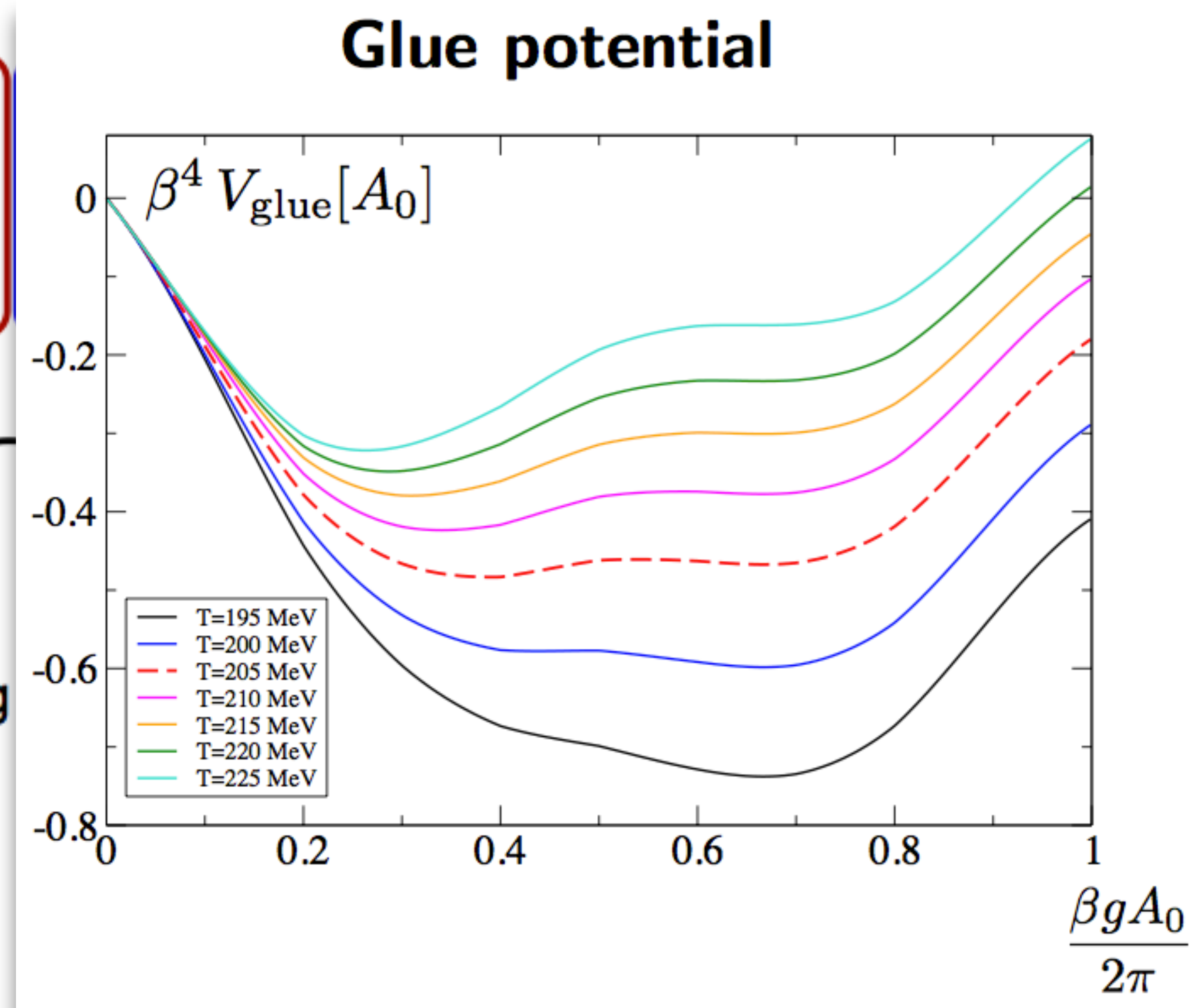

[Haas, Stiele, Braun, Pawlowski, Schaffner-Bielich, (2013)]

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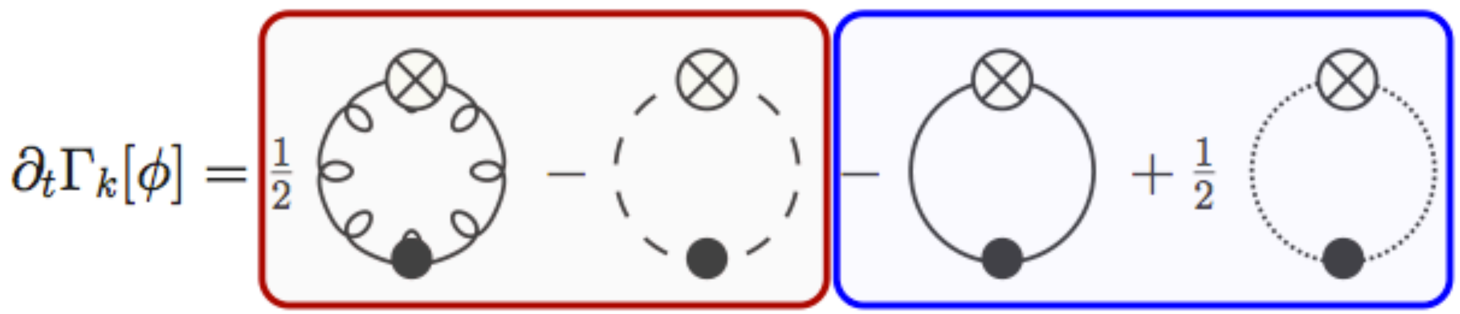

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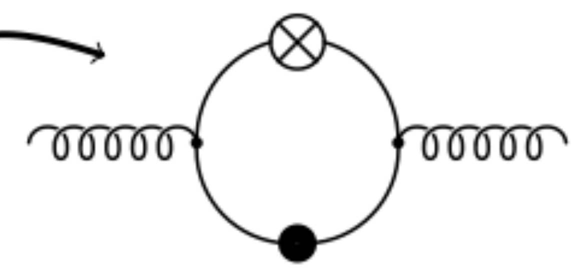

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$\rightarrow \mathcal{U}_{\mathrm{Pol}}(\Phi)$

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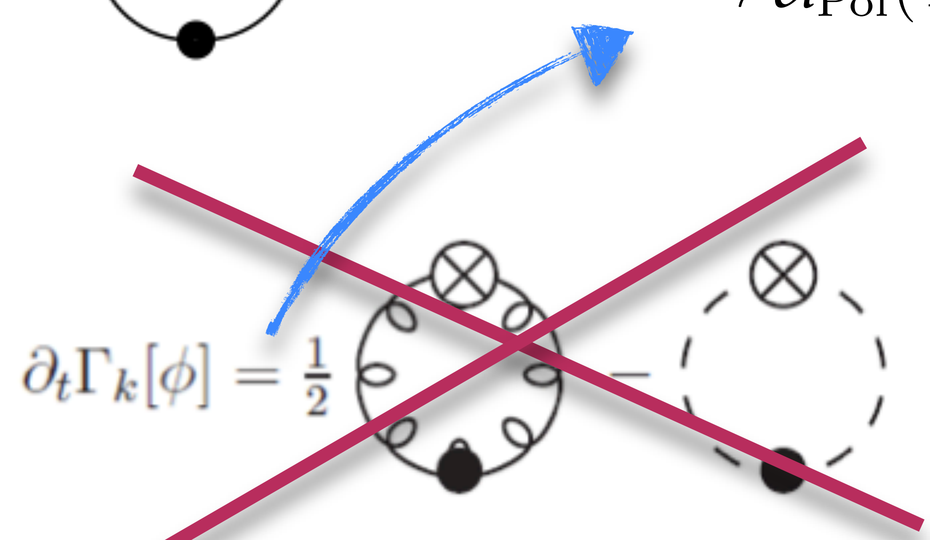

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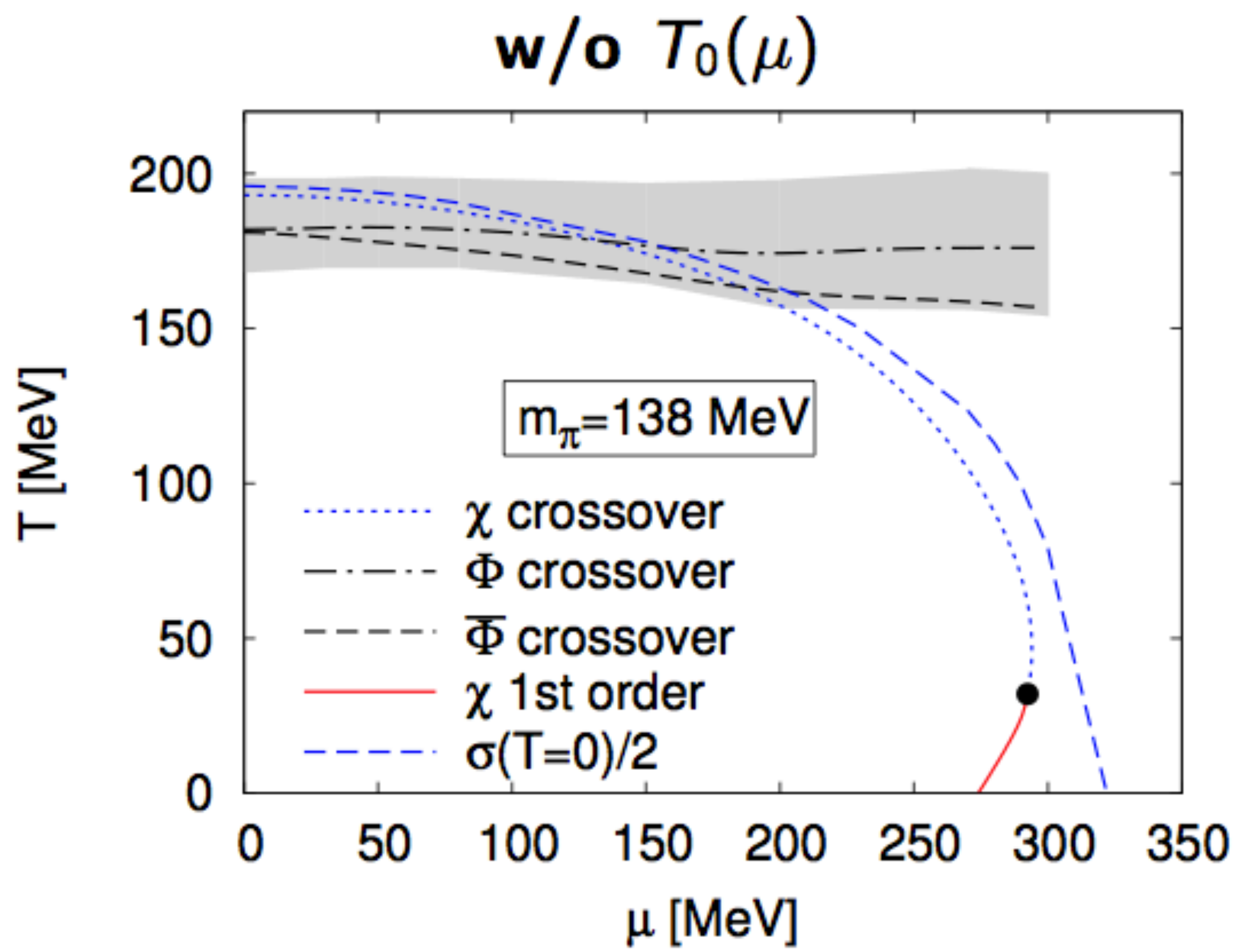
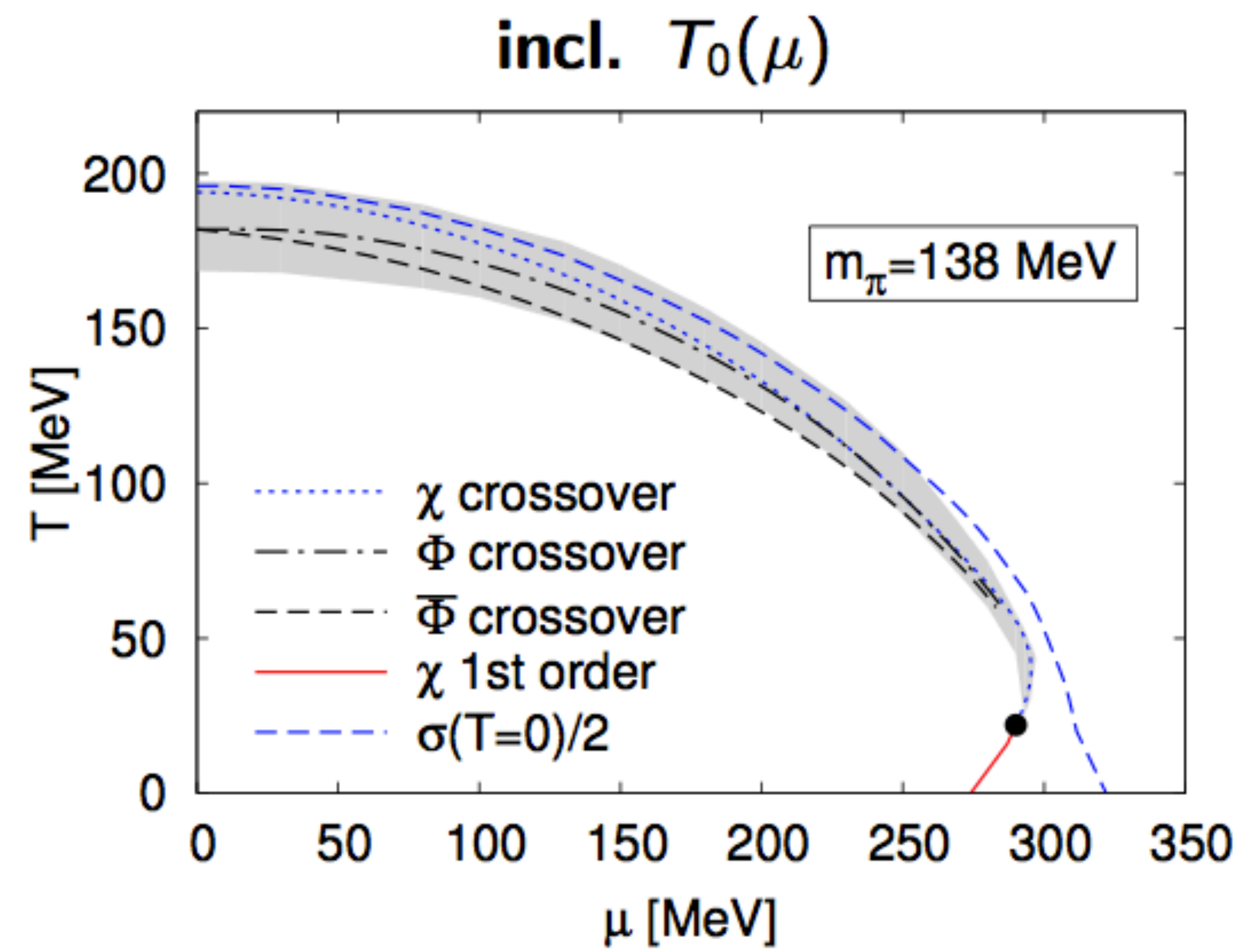
FRG Quark-Meson $N_f = 2$





[Herbst, Pawlowski, BJS 2010,2013]

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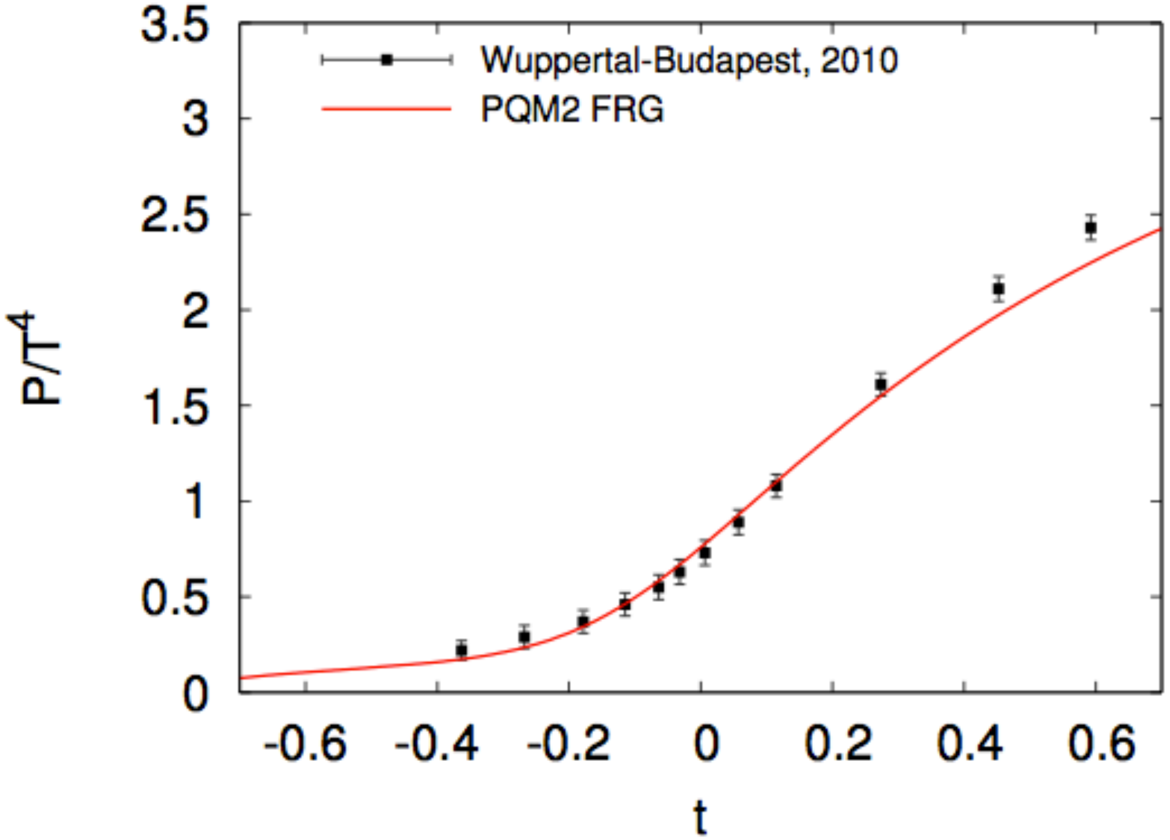
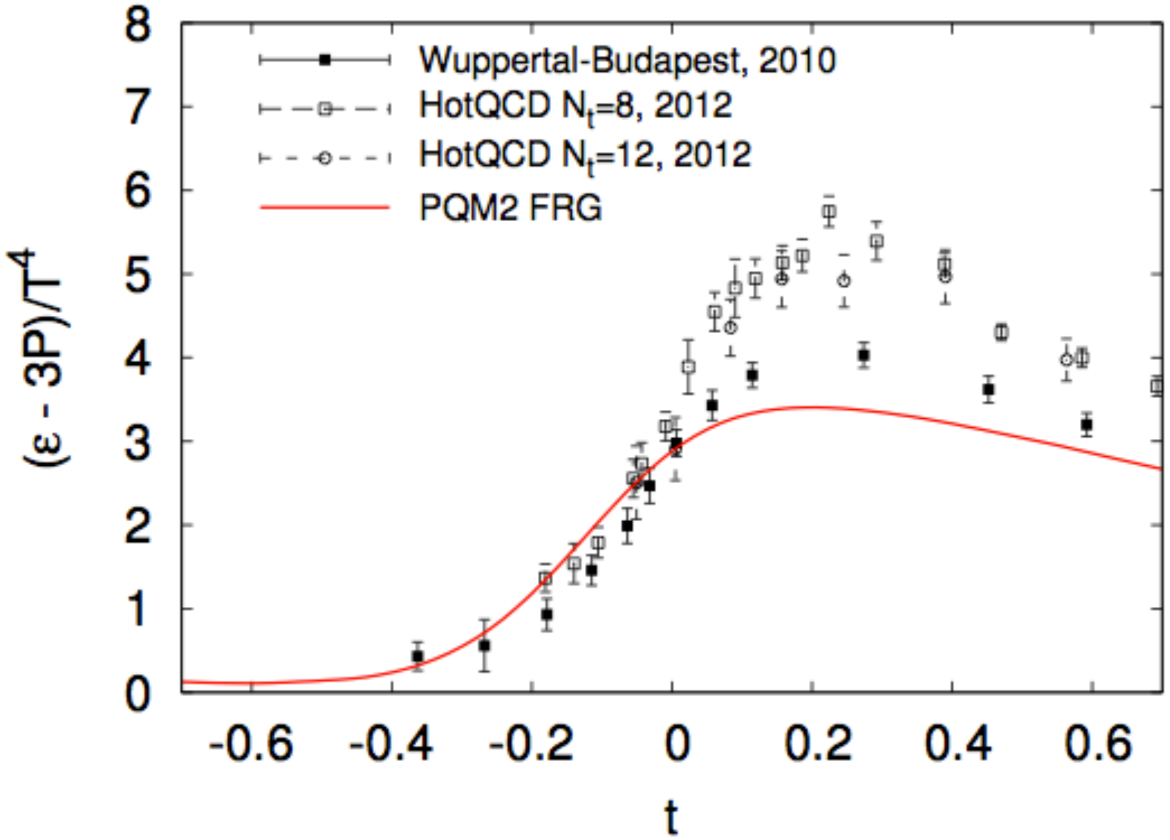
FRG Quark-Meson

Pressure and interaction measure in comparison with lattice data (polynomial Polyakov-loop potential)

$N_f = 2$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]



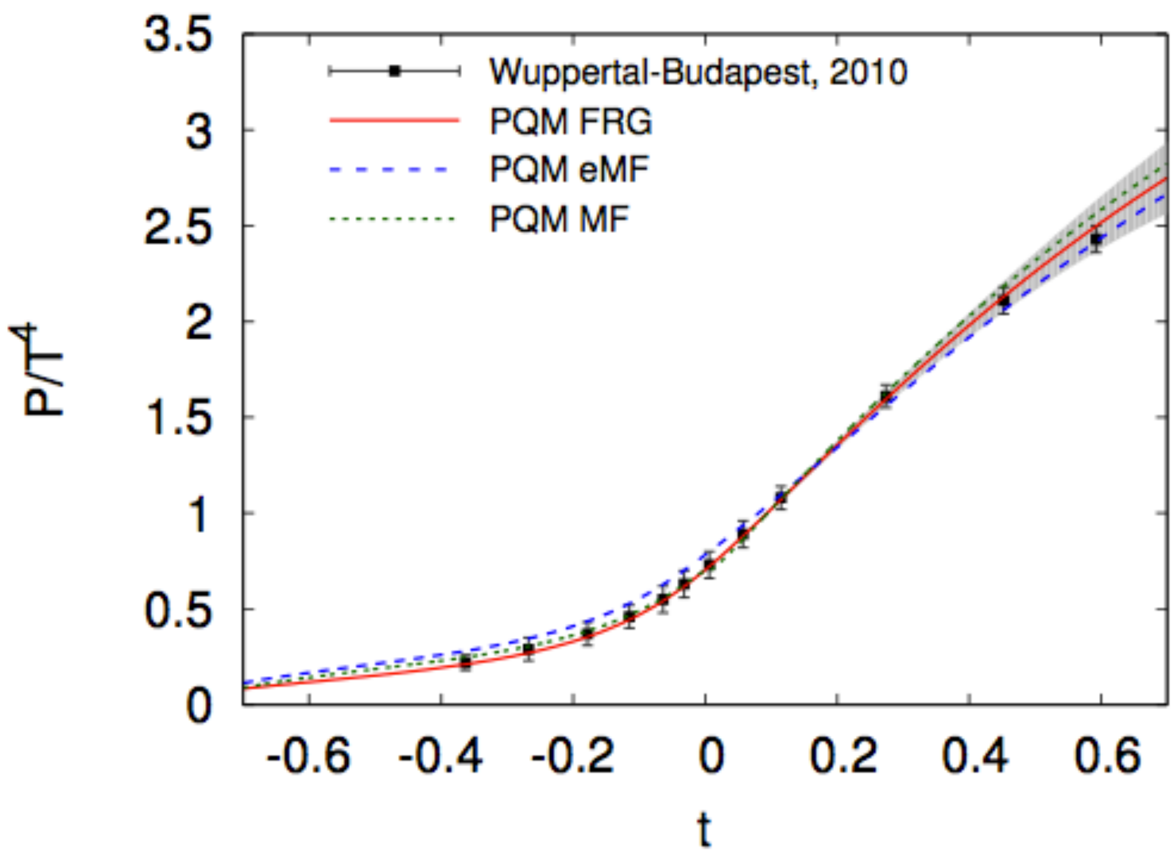


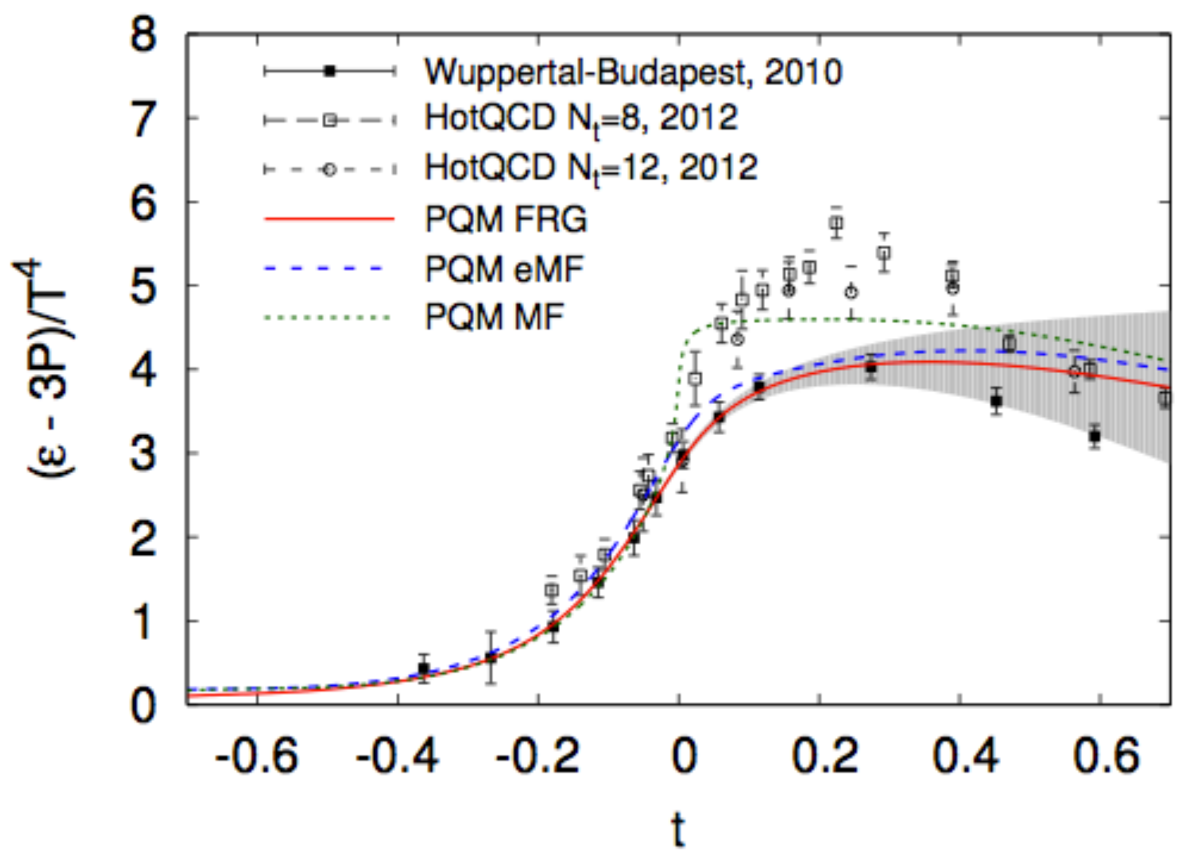
FRG Quark-Meson

Pressure and interaction measure in comparison with lattice data (**polynomial** Polyakov-loop potential)

$N_f = 2+1$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]





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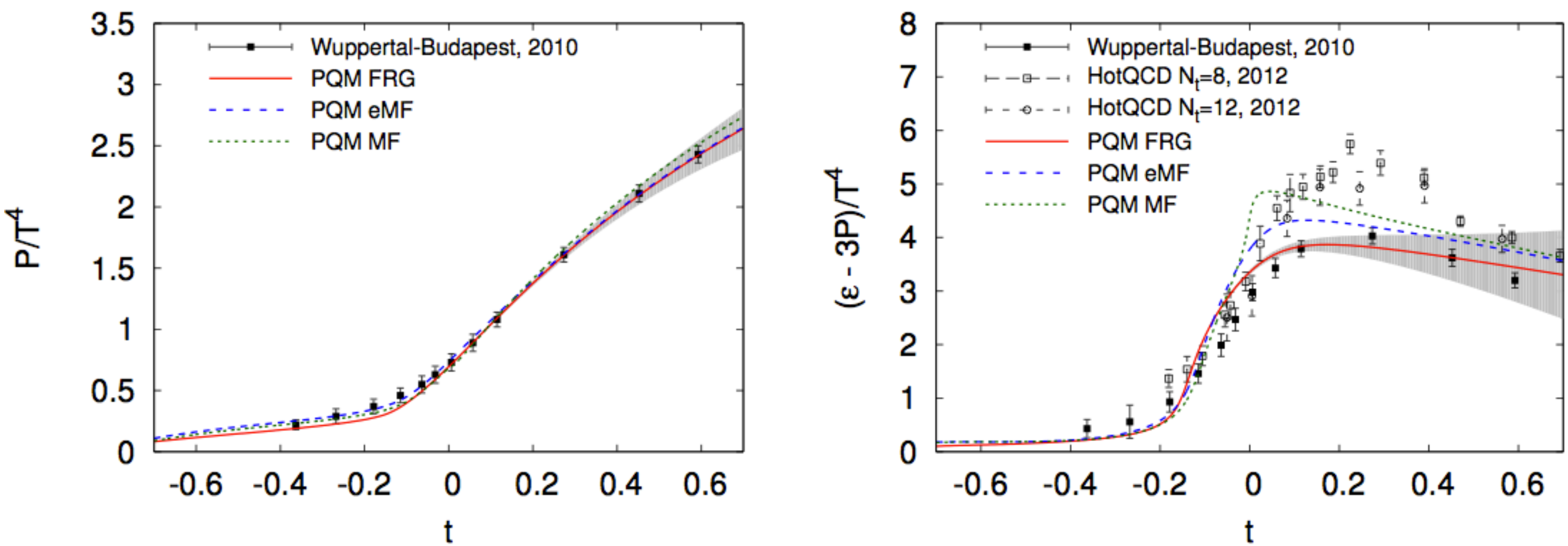
FRG Quark-Meson

Pressure and interaction measure in comparison with lattice data (**logarithmic** Polyakov-loop potential)

$N_f = 2+1$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]



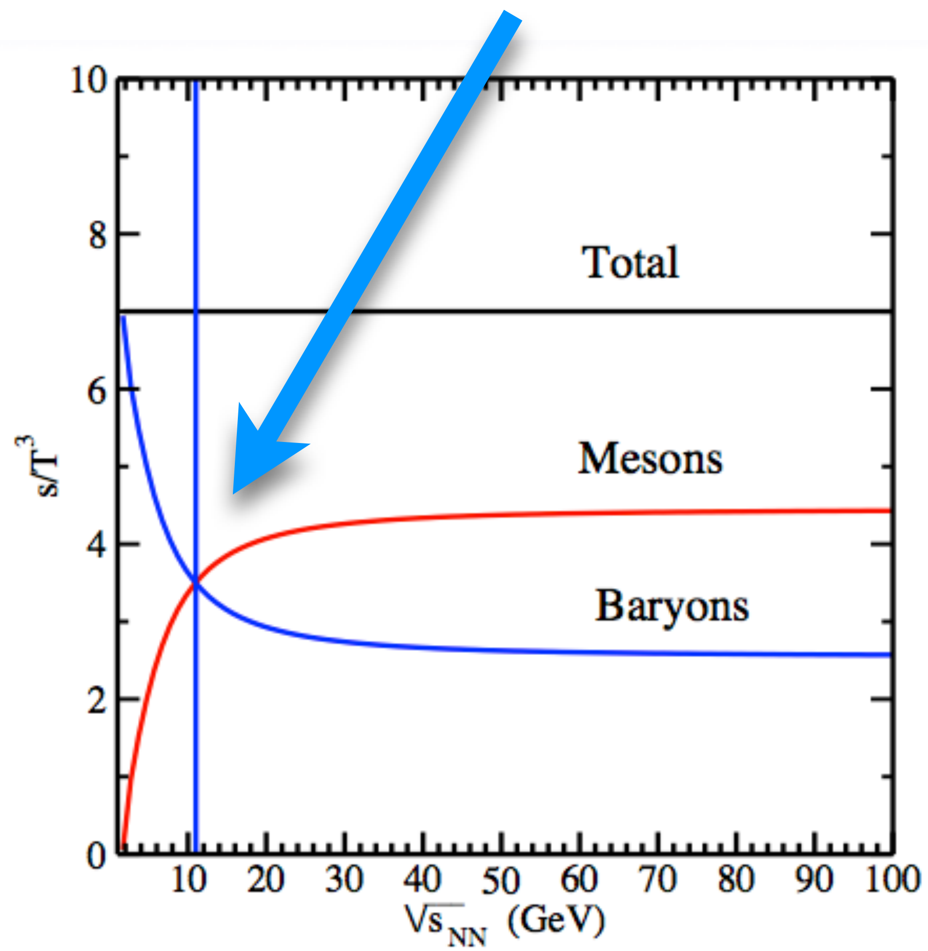
Role of Baryons?

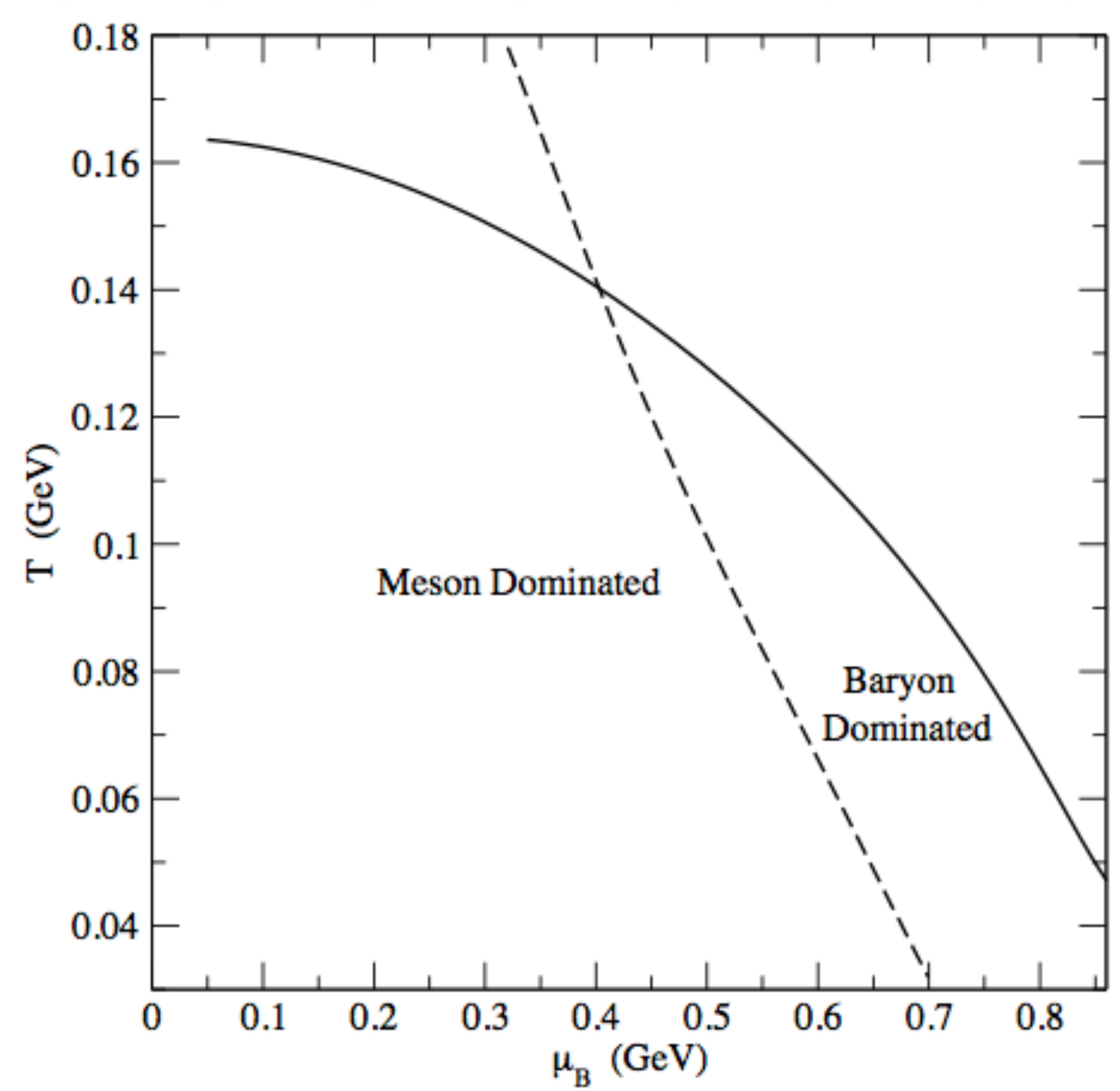
Statistical-thermal model analysis

High chemical potential: baryons more important but usually neglected in FRG

in stat. model: e.g. peak in

ratios “the horn”





$\mathrm{N_c} = 2$ QCD

QC2D becomes simpler: **no sign problem**
lattice simulations ⇔ functional methods

inclusion of baryonic degrees of freedom simpler:

Scalar diquarks play a dual role as **bosonic baryons**

relativistic analog of models for ultracold quantum gases

color-neutral bound states of two quarks (**bosonic [anti]diquarks**)

enlarged flavor symmetry: $SU(4) \cong SO(6)$ ($\mu = 0$)

replaces usual chiral $SU(2)_L \times SU(2)_R \times U(1)_B$

- Symmetry breaking: $SU(2N_f) \to Sp(N_f)$ [or $SO(6) \to SO(5)$

➜ 5 Goldstone bosons: 3 pions and 2 (anti)diquarks

Quark-Meson-Diquark (QMD) Model

Chiral effective model:

- quarks: ψ
- mesons: σ, $\vec{\pi}$
- diquarks (baryons): $\mathrm{Re}\Delta, \mathrm{Im}\Delta$
- gauge fields: A^a_μ in $D_\mu = \partial_\mu + iA_\mu \to$ Polyakov-loop extended (PQMD) model

arXiv:1306.2897

QMD Lagrangian:

[N. Strodthoff, BJS, L. von Smekal 2012]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QMD}} &= \bar{\psi}\left(\not{D} + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \mu\gamma^0\right)\psi \\
&+ \frac{g}{2}\left(\Delta^*(\psi^T C\gamma^5\tau_2 S\psi) + \Delta(\psi^\dagger C\gamma^5\tau_2 S\psi^*)\right) \\
&+ \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V(\vec{\phi}) \\
&+ \frac{1}{2}\left((\partial_\mu - 2\mu\,\delta^0_\mu)\Delta\right)(\partial_\mu + 2\mu\,\delta^0_\mu)\Delta^*
\end{aligned}
$$

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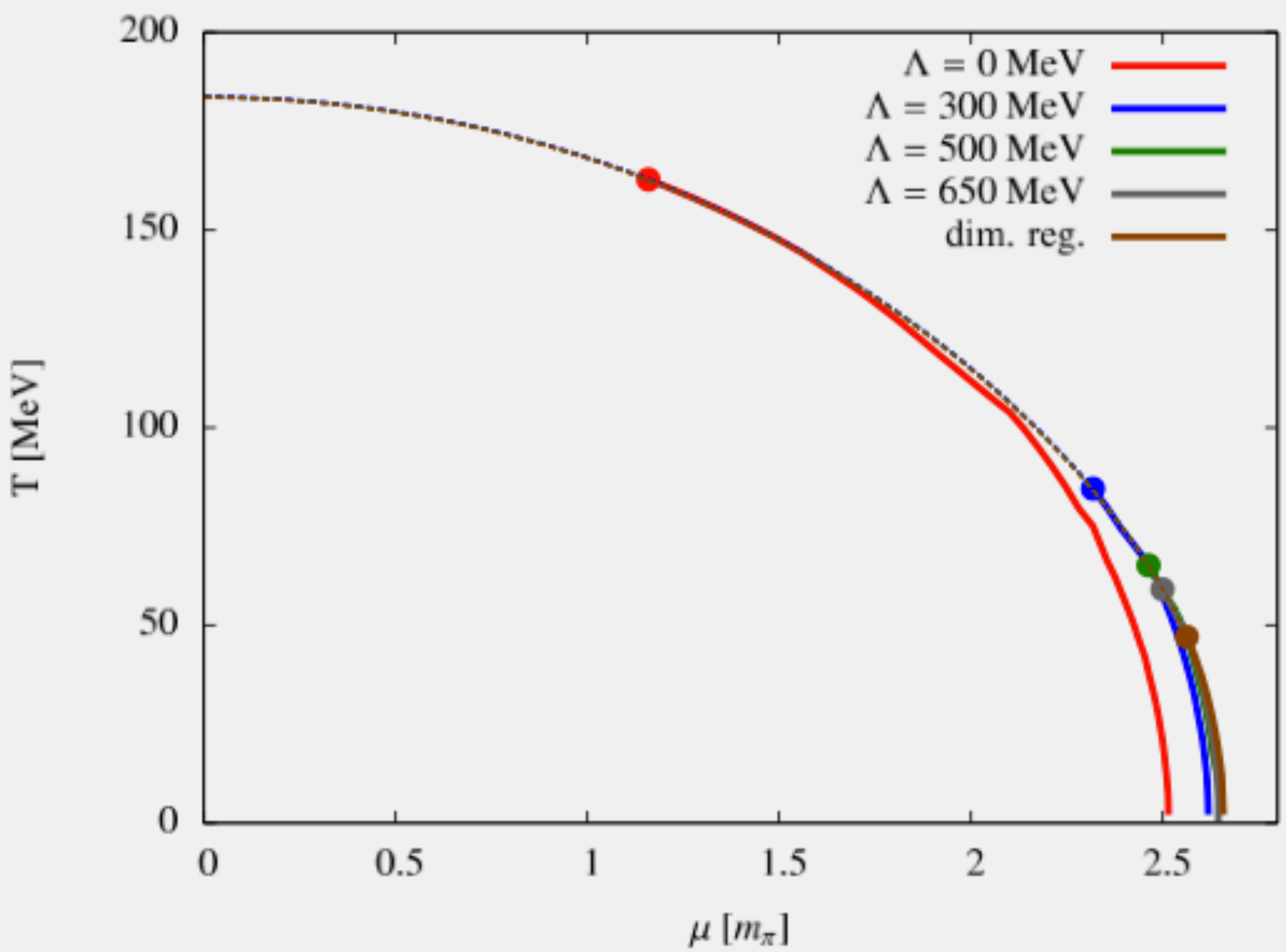
Phase diagram in MFA

Influence of Ω_{vac} on CEP for various Λ's compared to dimensional regularization



[N. Strodthoff, BJS, L. von Smekal; 2012]

- no diquark condensation ($d = 0$)
- $O(6)$-symmetric potential $V_{\text{MF}} = V_{\text{MF}}(\phi^2)$ where $\vec{\phi} = (\sigma, \vec{\pi}, \text{Re}\Delta, \text{Im}\Delta)$
  ➜ 1-dim. field variable

- 1^{st} order chiral transition
- CEP around $\mu \approx 2.5 m_\pi$

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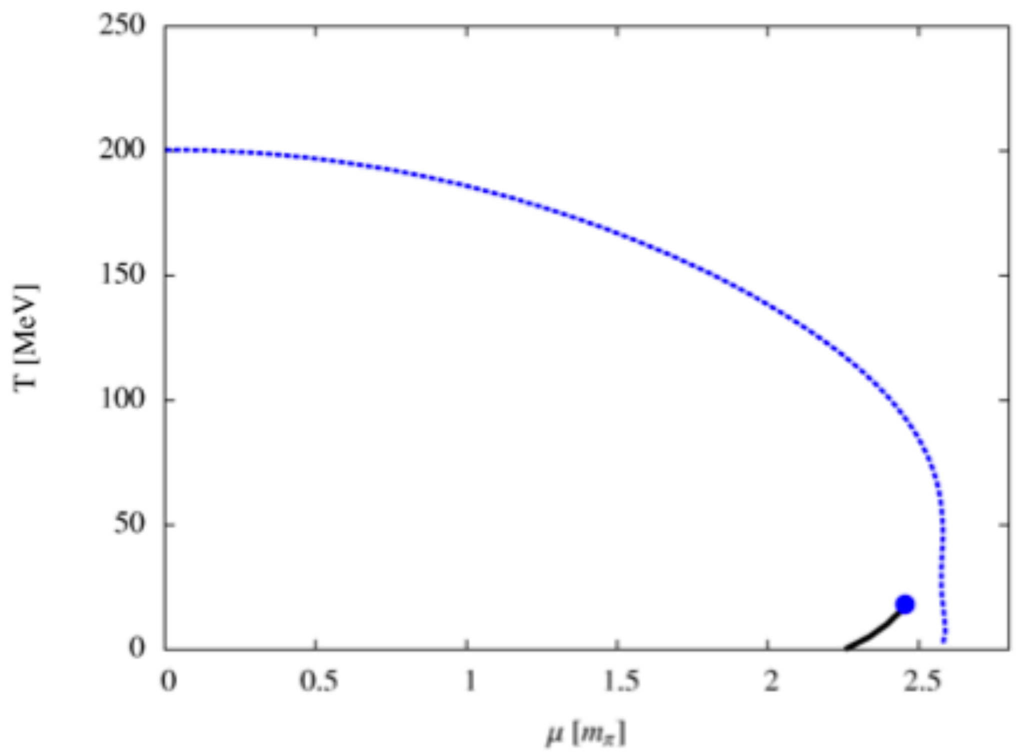
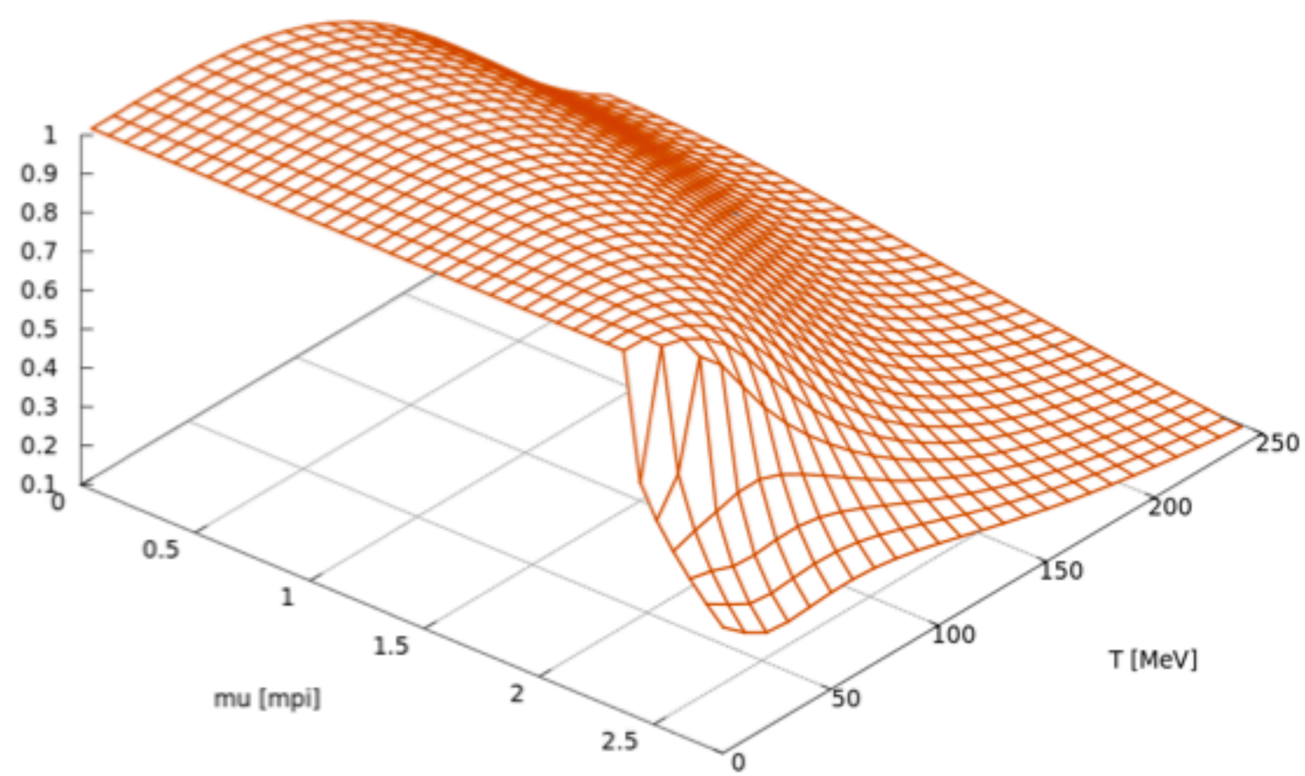
Phase diagram with FRG

- no diquark condensation:
- $O(6)$-symmetric potential $U_k = U_k(\phi^2)$



- chiral condensate $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$



▷ "typical" RG phase diagram

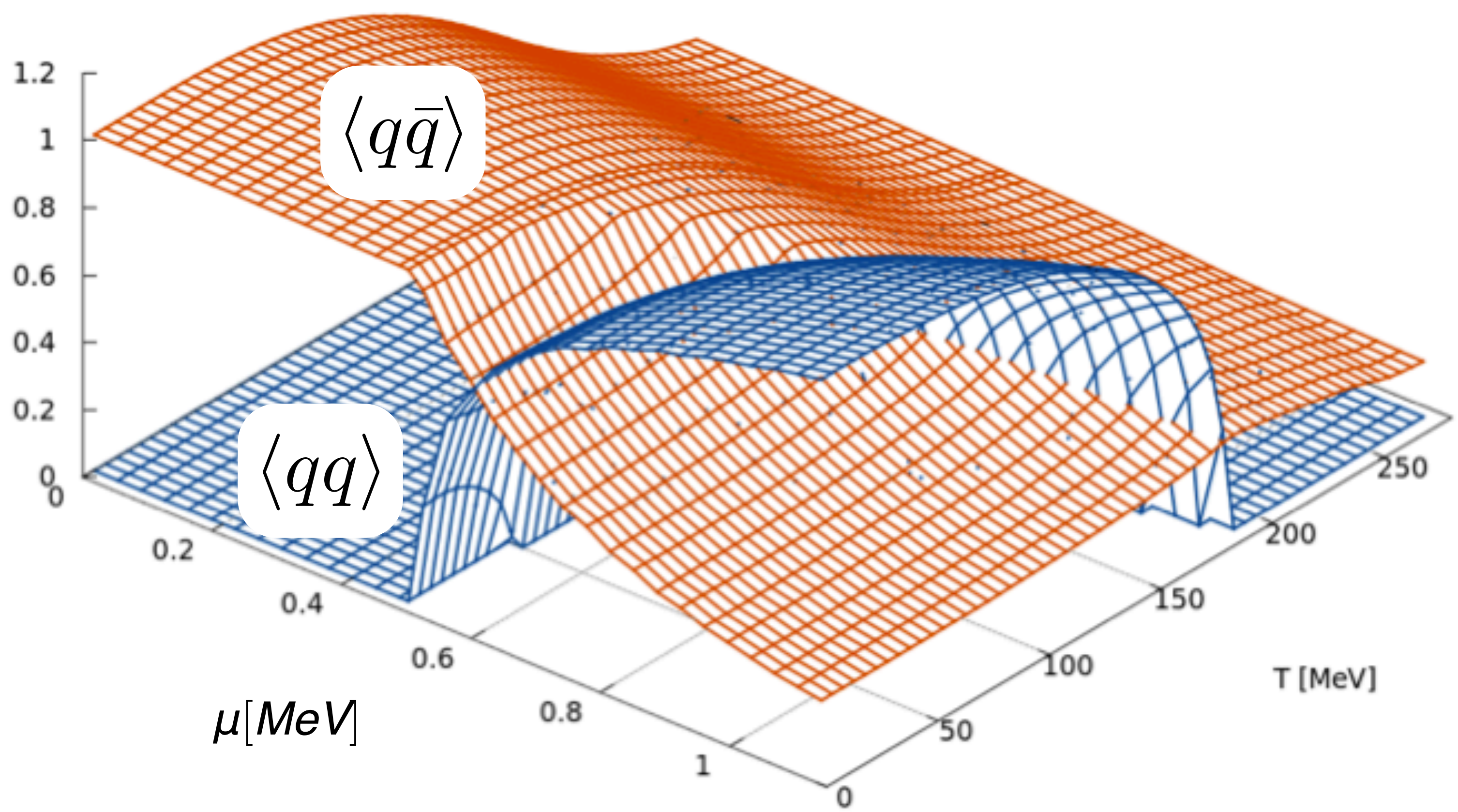
back-bending 1st order line with a CEP

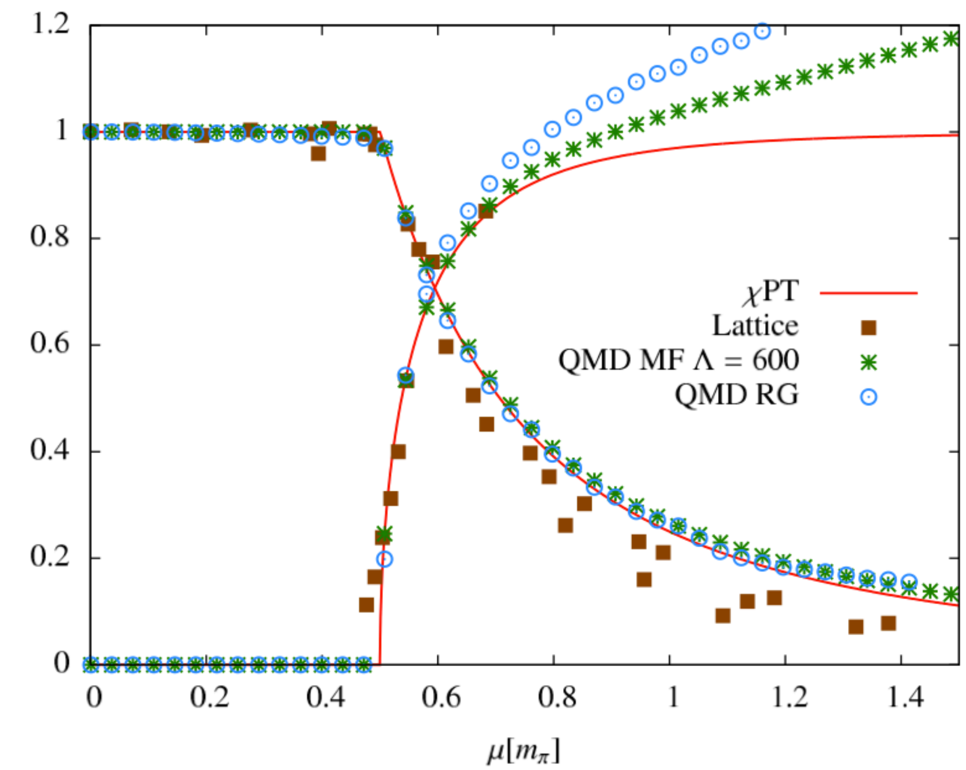
[Strodthoff, BJS, von Smekal; 2012]

Including diquarks

Symmetry breaking $\quad SO(6) \longrightarrow SO(3) \times SO(2)$

need two condensates: chiral: $\langle q\bar{q}\rangle$ and diquark condensate: $\langle qq\rangle$



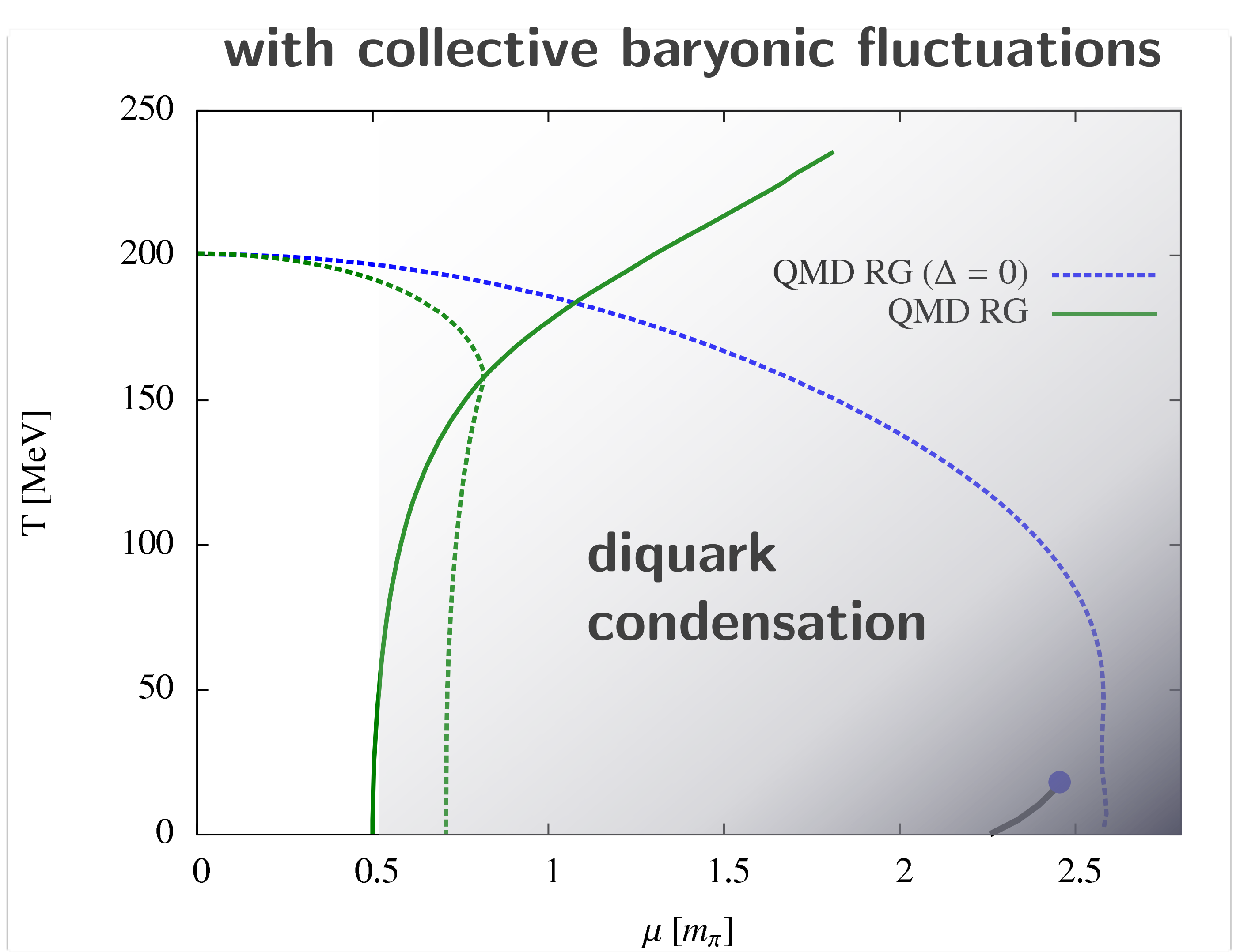


diquark condensation at $\mu_c = m_B/N_c$

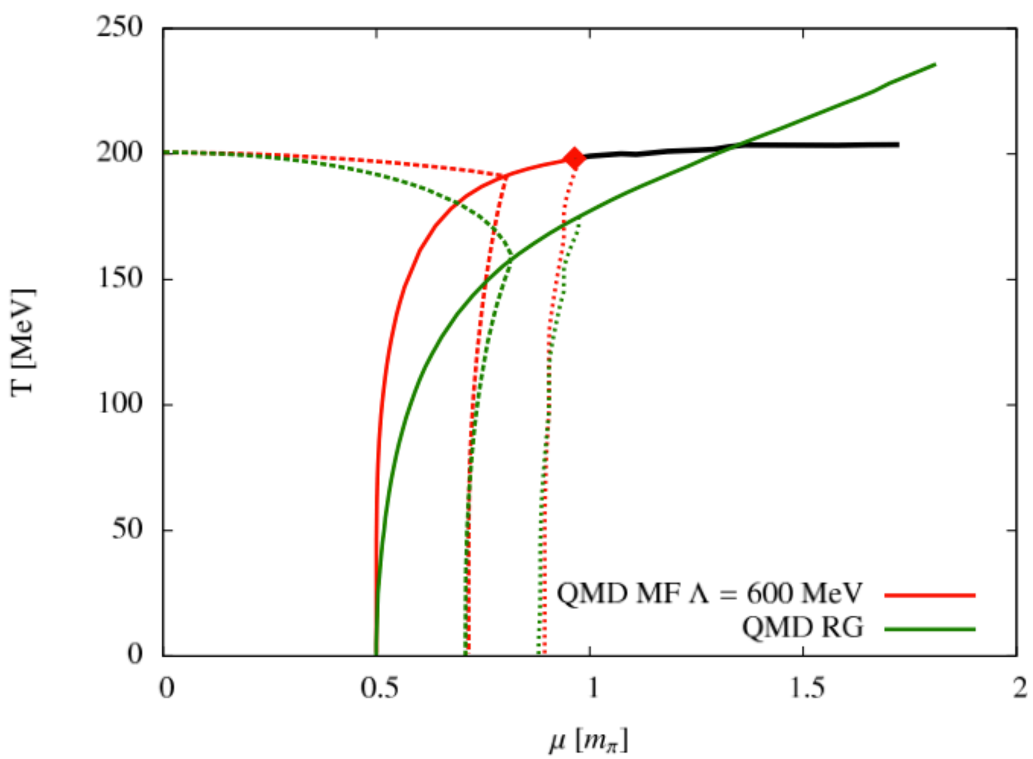
[N. Strodthoff, BJS, L. von Smekal 2012]

Phase diagrams

[N. Strodthoff, BJS, L. von Smekal 2012]



FRG versus MFA: TCP is a MFA artefact



- no low-T 1^{st} order transition, no CEP at $\mu \sim 2.5\ m_\pi$!

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Summary & Conclusions

- QCD-like model studies for two and three flavors
- effects of quantum and thermal fluctuations on QCD phase structure
- QCD for two colors: towards an understanding of role of baryons on phase structure
- existence of critical points in phase diagram

functional approaches (e.g. FRG) are suitable and controllable tools to investigate the QCD phase diagram and its boundaries

Backup material

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