When equity fails - An appraisal of revenue sharing as the last resort*

Karsten Bocks[†] Christian Haas[‡] Thomas Heyden[†§]

December 6, 2019

Abstract

We study the trade-off venture capitalists encounter in a financing framework under moral hazard. The venture capitalist has the option to supply funds either within a revenue-sharing contract or via equity but faces a hidden effort problem. While projects with a low degree of moral hazard yield higher returns to the venture capitalist when financed by equity, revenue-sharing contracts become superior as moral hazard increases. At high moral hazard levels, revenue sharing becomes the sole financing option and hence can raise welfare. We apply our model in the context of initial coin offerings as a modern form of revenue sharing.

JEL Classification: D86, G24, G32

Keywords: Venture Capital, Revenue Sharing, Moral Hazard, ICO, Crypto Tokens

*We thank Christina Bannier, Vincent Glode, Itay Goldstein, Luke Taylor, David Yermack, Andreas Brøgger, Daniel Herold, Kim Heyden, Paul Momtaz, Ariah Klages-Mundt, the participants of the Crypto Valley Conference on Blockchain Technology 2019 in Zug, Switzerland, the UWA Blockchain, Cryptocurrency and FinTech Conference 2019 in Perth, Australia, the Annual Meeting of the German Finance Association 2019 in Essen, Germany, and the UniCredit Doctoral Seminar 2018 in Eltville, Germany, for helpful comments. Parts of this research were developed while Thomas Heyden was a visitor at the Wharton School. A previous version of this paper was circulated under the title "A theory on pre-ICO venture capital involvement".

†Chair of Banking & Finance, Justus Liebig University Giessen, Licher St. 62, 35394 Giessen, Germany.

‡Chair of International Economics, Justus Liebig University Giessen, Licher St. 66, 35394 Giessen, Germany.

§*Corresponding author:* thomas.heyden@wirtschaft.uni-giessen.de.

1 Introduction

Start-ups have a high need for growth capital. As they neither have a long history of success nor (tend to) have high free cash flows, internal financing as a financing method is not feasible and requires them to seek external financing. In addition, many entrepreneurs most often do not have enough capital on their own and thus need to search the market for an investor matching their needs (e.g., Denis, 2004; Chemmanur and Fulghieri, 2014). High information asymmetry inherent in start-up financing implies high risk and indirect costs of financing for investors. Usually, this makes it nearly impossible for start-ups to issue debt or receive a bank loan. This leaves them with raising funds via equity from external investors, traditionally. As they cannot enter the stock market easily, they must look for business angels, venture capitalists (VCs), and other investors that are willing to take high risks in exchange for potentially major gains (Hall and Lerner, 2010). A number of influential papers deal with the inherent moral hazard problem of venture capital investments stemming from information asymmetry (see, among others, Amit et al., 1998; Bergemann and Hege, 1998; Gompers, 1995; Kaplan and Strömberg, 2001; Schmidt, 2003; Ueda, 2004).

The fact that young and innovative start-ups are predominately financed with equity is widely documented in the literature. Thus, equity is often perceived to be "the last resort" (e.g., Feldman et al., 2002). This view has been challenged by crowdfunding, which has been scarcely available since the beginning of the new millennium. Crowdfunding reduces the investment amount to be collected by a single investor and further allows for different contracting forms, one of them revenue-based (Agrawal et al., 2015). Indeed, a contract forcing the start-up to give up (at least some of) its revenues is not a new invention. Actually, revenue-, or royalty-based, financing has been common practice for start-ups with a high need for financing since the late 1990s.[1] Similar to the later crowdfunding, funds raised were rather low in comparison to other financing methods. In comparison to other financiers, the field of specialized investors conducting revenue-based financing is a niche, especially in the VC market. This might well be due to the character of revenue-based financing combining

[1]For a review of the development of corporate finance instruments, see Bernthal (2019). According to him, revenue sharing dates back to the 1930's oil industry.

a fluctuating interest component similar to equity financing with the debt-like feature of paying back a royalty irrespective of the residual profit. More specifically, revenue-based financing costs the entrepreneur high shares of their revenues and, being debt-like claims, they are prioritized over stakes on the residual profit.[2] This might, on the one hand, shy away potential future equity investors for the time of the royalty payments but, on the other hand, does not dilute ownership or control rights.[3] Knowing that debt-like features can have a disciplining effect on the entrepreneur, revenue sharing can even help enabling financing in a high moral hazard environment. Advantageous in a hidden effort framework, giving up a revenue share and bearing the cost on their own can incentivize even those entrepreneurs with high cost to exert effort. While traditional revenue-sharing contracts are similar to loans in terms of the payback up to a royalty cap, we suggest a more general approach to this hybrid financing method. Traditional VC literature points not only to screening abilities but also to the monitoring function of VC investment (e.g., Lerner, 1995; Hellmann and Puri, 2002; Chemmanur et al., 2011). Underlining the importance of the VCs' engagement beyond supplying capital, Bernstein et al. (2016) find that an active involvement in the companies increases their innovativeness. This speaks for raising funds with a VC instead of a lender and points to the potential of revenue sharing within VC contracting. While in recent times, revenue-sharing components have been observable in crowdfunding, tokenization takes revenue-based financing onto a digital level. As Initial Coin Offerings (ICOs) show similarities to the revenue-sharing framework that we suggest, it moves these hybrid financing contracts to the secondary market and offers an easy exit, which makes it attractive for VCs.

In this paper, we propose a theoretical model and demonstrate that a VC investor who

[2]Lighter Capital, a lender active since 2010, is supposed to be the market leader in revenue-based financing for tech start-ups as of the date of this writing. According to their founder, their funding volume is, on average, some hundreds of thousands of dollars with claiming an annual interest rate of 15-30%. At the same time, the fintech can shorten entrepreneurs' fund raising time by relying on data-driven screening. See https://www.businesswire.com/news/home/20190516005167/en/ and https://www.inc.com/jessica-stillman/overlooked-financing-option-for-your-business.html.

[3]However, there is anecdotal evidence that traditional VC equity and revenue-based financing are undergone in combination. As covered, for instance, by The Wall Street Journal in 2010, see https://www.wsj.com/articles/SB10001424052748704679204575646940403312602.

considers investing in a start-up will, under certain conditions, either propose a revenue-sharing contract to provide fresh capital, or invest by buying an equity share of the start-up. More specifically, we employ a simple hidden effort model in which moral hazard is correlated with the entrepreneur's cost of effort to reduce marginal production cost. Our findings imply that there is a range of projects which cannot be financed by equity due to high moral hazard but could still be financed via a revenue-sharing contract. While the incentivization of the entrepreneur is easier through the revenue-sharing contract, this financing method does not strictly dominate equity financing for all levels of moral hazard. Thus, we further plead for a convertible feature in revenue-sharing contracts to leave the VC the option to opt into common equity once the interests are further aligned or asymmetries are reduced. In this context, we provide a brief demonstration of our model in a discrete time multi-periods setting and find that it is indeed optimal for the VC to convert into common equity eventually.

There is a broad strand of VC literature that strives to understand the characteristics and implications of the variation in contractual designs. In general, our paper contributes to this literature by proposing a specific contracting option to enable easier incentivization of the entrepreneur. While some earlier theoretical articles provide an explanation for the use of convertible securites (e.g., Casamatta, 2003; Chemmanur and Chen, 2014; Hellmann, 1998, 2006; Kaplan and Strömberg, 2003), others shed light on why entrepreneurs might prefer equity over debt (e.g., De Bettignies and Brander, 2007; Ueda, 2004). As opposed to most of the literature above, our aim is to suggest a particular kind of (convertible) debt-like security design, rather than providing an intuition as for why VCs—and entrepreneurs all the same—adopt certain contracts. Of course, debt-like convertible securities are far from being a novel proposition. However, the structure of these contracts typically involves principal payments depending on some predetermined factor or exogenous measure. In our model, revenue-sharing contracts attach the payment amount to the level of revenues. Hence, the principal payment becomes endogenous.

To the best of our knowledge, the concept of revenue-sharing contracts has not yet been introduced in the traditional VC literature. Albeit Wang and Zhou (2004) mention that early-stage and high-risk firms often propose revenue-sharing contracts to VCs, they do not incorporate this feature in their model of staged financing. Another practical form of revenue sharing is within crowdfunding finance as documented by Agrawal et al. (2015). Further, we strengthen its right to exist as a noteworthy hybrid financing method and serious investment option for VCs. Our findings suggest that revenue sharing can increase economic welfare by enabling the financing of projects that might not have been realized otherwise. In the broadest sense, we also contribute to the infant ICO literature. Our paper is somewhat related to Garratt and van Oordt (2019), who argue that ICOs represent a way to better align the incentives of the entrepreneur and outside investors. Despite the fraudulent practices of some so-called entrepreneurs over the last couple of years, we follow Fisch and Momtaz (2019) and make an argument for this financing form if under proper regulation.[4]

This paper proceeds as follows: Section 2 states the general setup of the model. It illustrates the equity investment case and the revenue-sharing case. Section 3 displays the results and compares the payouts from the different cases. Before Section 5 concludes, Section 4 connects revenue sharing to ICOs as its most recent application and discusses the effect of our results on this digitized financing form.

2 The model

2.1 General setup

The model setup borrows elements of Malinova and Park (2018).[5] A risk-neutral entrepreneur requires fresh capital in order to form a start-up company, which in turn will be the monopolistic supplier of a new product. The amount of money the entrepreneur needs is a fixed set-up cost C_I. After the entrepreneur obtains C_I, the start-up encounters linear demand of the form $q(p) = x - p$. Each unit produced incurs cost c_j with $j = \{L, H\}$, where the entrepreneur has the option to lower marginal costs by exerting effort for cost

[4]See Chod et al. (2019) for an argument for the governance mechanism of tokenization in platform business models and addressing potential underinvestment if investor protection is missing.

[5]In particular, we adapt the basic model framework of Malinova and Park (2018) by (i) abstracting from uncertainty (as it is not a central driver of the main mechanisms proposed in this paper), (ii) introducing market power of the investor, and (iii) incorporating entrepreneur-specific effort costs related to (marginal) cost reductions.

C_e, such that $c_H > c_L$.[6] The entrepreneur will then maximize the profit of the start-up by optimally choosing q.

In the following, we will refer to the entrepreneur and the VC in the female and male form, respectively. As the entrepreneur is capital restricted, she solely depends on external financing. The capital is provided by a risk-neutral VC who has two options. First, he can buy a share of the company's equity, which entitles him to a share of the start-up's profit. Second, he can invest C_I for an enforceable revenue-sharing contract, which entitles him to a share of the start-up's realized revenue.[7]

The sequence of decisions is such that, at first, the VC chooses between buying an equity share or a revenue share and then offers a contract to the entrepreneur. Second, if she accepts, the entrepreneur decides whether to exert effort or not. The demand is realized and the entrepreneur sets q in order to maximize her profit. Further assumptions are that the rate of return required by the investor and the entrepreneur is zero. Also, the VC is a well-informed investor as he engages in screening activity before his decision.[8] This allows him to observe the entrepreneur's type, i.e. reduce uncertainty about her cost of effort to a minimum.[9] As Kaplan and Strömberg (2001) point out, it is common practice for VCs to do so quite successfully. Amit et al. (1998) and Gompers and Lerner (2001) support this argument as an experienced VC should have a comparative advantage in screening over

[6]Effort costs (C_e) could be interpreted as (opportunity) costs of extending absolute working time or increasing working intensity in order to extend effective working time in processes that lead to a reduction in marginal costs of production. Examples of such processes are rationalization of production or conducting R&D to reduce per-unit inputs.

[7]Although there appears to be somewhat of a consensus amongst the theoretical literature that VC investors should optimally employ convertible preferred equity (see, among others, Basha and Walz, 2001; Cornelli and Yosha, 2003; Da Rin et al., 2013; Kaplan and Strömberg, 2003, Schmidt, 2003), we abstract from explicitly incorporating this feature into our model. We do so for two reasons: First, the main reason for using convertible preferred equity is investor downside protection. As payoffs are certain in the model, downside protection would pose an unnecessary complication. Second, while convertible preferred stock might be optimal theoretically, Cumming (2005a, 2005b) provides evidence that the majority of VC funds in the US and in Canada actually employ common equity. This argument is based on data that covers more than 12,000 VC and private equity financing in the period from 1991 to 2003.

[8]Gompers et al. (2019) provide a very detailed description of how this screening typically works.

[9]We implicitly set this screening cost to zero as they do not affect the qualitative results.

other investors. Moreover, we assume that the project is only economically viable if the entrepreneur produces at low marginal costs c_L, i.e. with c_H the project's net payout is negative:

$$\left(\frac{x - c_H}{2}\right)^2 < C_I < \left(\frac{x - c_L}{2}\right)^2 - C_e. \tag{1}$$

2.2 Equity investment

If the VC opts for buying an equity share in the traditional sense, he chooses his optimal α_E by solving

$$\max_{\alpha_E \in [0,1]} \quad VC^E_\pi(\alpha_E; \pi^m_L, C_I) = \alpha_E \pi^m_L - C_I \tag{2a}$$

$$\text{s.t.} \quad (1 - \alpha_E)\pi^m_L - C_e \geq (1 - \alpha_E)\pi^m_H, \tag{2b}$$

$$(1 - \alpha_E)\pi^m_L - C_e \geq 0, \tag{2c}$$

$$\pi^m_j = \left(\frac{x - c_j}{2}\right)^2. \tag{2d}$$

(2a) denotes the VC's objective function, i.e. his net payout. (2b) and (2c) are the entrepreneur's incentive compatibility constraint (ICC) and participation constraint (PC), respectively. (2d) reflects the anticipation of the entrepreneur's profit maximizing behavior.[10] In a frictionless world with no moral hazard, the entrepreneur realizes c_L without the need to exert effort. With $C_e = 0$, both ICC and PC become obsolete (both are fulfilled for any $\alpha_E \in [0, 1]$). As a result, the VC demands a maximum share of profits with $\alpha_E = 1$ as his objective function (2a) is monotonically increasing in α_E.

If, however, the entrepreneur does have the option of reducing marginal costs only by exerting effort, the VC faces a hidden effort problem. Note that he will only provide C_I if the entrepreneur realizes c_L. Otherwise the projects net present value (NPV) will be negative for the investor.[11] This imposes a moral hazard problem on the part of the entrepreneur. The VC thus has to maximize (2a) subject to the constraints, where the PC (2c) is redundant as

[10] The entrepreneur chooses q_j in order to maximize her profit for a given α_E set by the VC: $\max_{q_j \geq 0} \{(1 - \alpha_E)\,(p(q_j)q_j - c_j q_j)\}$. The resulting (standard monopoly) profit is shown in (2d).

[11] We use the term net payout when referring to cash-flows (to the VC) in general and the term net present value when referring to optimal cash-flows.

the ICC's right hand side is always non-negative.[12] Essentially, this means solving the ICC for α_E, which yields

$$\alpha_E \leq \frac{\Delta\pi - C_e}{\Delta\pi} =: \alpha_E(c_H, c_L, x, C_e), \tag{3}$$

where $\Delta\pi = \pi_L^m - \pi_H^m$. Choosing α_E such that the entrepreneur is made indifferent, i.e. (3) holds with strict equality, and substituting it in (2a) gives the VC's optimal value (function) as

$$\begin{aligned} NPV^E = NPV^E(c_H, c_L, x, C_e, C_I) &:= VC_\pi^E(\alpha_E(c_H, c_L, x, C_e), \pi_L^m, C_I) \\ &= \frac{\Delta\pi - C_e}{\Delta\pi}\pi_L^m - C_I = \pi_L^m - \frac{\pi_L^m}{\Delta\pi}C_e - C_I. \end{aligned} \tag{4}$$

Proposition 1. *Caused by the need to incentivize the entrepreneur, the VC cannot demand an equally high α_E as if there was no moral hazard. He has to leave a minimum of C_e to the entrepreneur and will thus receive a lower NPV.*

Proof. Follows immediately from the analysis above, particularly (4). □

Following this modeling, (3) implies that the VC chooses his share only by means of C_e and $\Delta\pi$. Furthermore, it shows intuitively that if the cost of effort is very low, the incentive constraint is easily satisfied. Thus, the VC charges a large share of the monopoly profit for providing C_I. As C_e increases, the moral hazard problem intensifies and the VC has to render part of the profit to the entrepreneur as compensation for C_e. Another driver is the reduction in marginal costs that the entrepreneur can realize by exerting effort. As c_L becomes smaller, $\Delta\pi$ increases and allows the VC to increase his share, as well.

[12] This has to be the case as $\pi_H^m > 0$ (follows with (2d) and $x > c_H$), and $(1 - \alpha_E) \geq 0$.

2.3 Revenue sharing

If the VC decides to invest C_I for a revenue-sharing contract, he chooses his optimal α_{RS} by solving

$$\max_{\alpha_{RS}\in[0,1]} \quad VC_{\pi}^{RS}(\alpha_{RS}; q_L, x, C_I) = \alpha_{RS}(x - q_L)q_L - C_I \tag{5a}$$

$$\text{s.t.} \quad (1-\alpha_{RS})(x-q_L)q_L - c_L q_L - C_e \geq (1-\alpha_{RS})(x-q_H)q_H - c_H q_H, \tag{5b}$$

$$(1-\alpha_{RS})(x-q_L)q_L - c_L q_L - C_e \geq 0, \tag{5c}$$

$$q_L \geq 0, \tag{5d}$$

$$q_H \geq 0. \tag{5e}$$

The minuend of (5a) is the VC's revenue-sharing component. As with the equity case, (5b) and (5c) are the entrepreneur's ICC and PC, respectively.

Based on α_{RS}, the entrepreneur decides whether to exert effort or not. Furthermore, she maximizes her share of the start-up's profit (total profit less share of the revenue paid to the VC) after the VC invests C_I by choosing q_j optimally

$$q_j(\alpha_{RS}, c_j, x) = \arg\max_{q_j \geq 0} \left\{(1-\alpha_{RS})(x-q_j)q_j - c_j q_j\right\}, \tag{6}$$

which yields an optimal quantity of

$$q_j(\alpha_{RS}, c_j, x) = \frac{x - \frac{c_j}{1-\alpha_{RS}}}{2}. \tag{7}$$

When choosing his optimal α_{RS} the VC anticipates the entrepreneur's behavior, i.e. (7). Now, (5a) denotes the VC's objective function under revenue-sharing financing, i.e. his NPV. As can be seen from (7), the optimal output quantity is negatively affected by α_{RS} and thus strictly below the monopoly quantity $q_j^m = (x - c_j)/2$ if $\alpha_{RS} > 0$.[13] Moreover, this effect leads to the issue that q_L and q_H might assume negative values theoretically as α_{RS} approaches one, in order to satisfy the ICC and PC. Naturally, this does not make sense economically. For this reason we impose the technical restrictions (5d) and (5e) that limit q_L and q_H to non-negative values.

[13] This effect can also be thought of as the underinvestment problem documented by Chod and Lyandres (2018).

Essentially, two different channels affect the outcome of the model. (i) The moral hazard component, and (ii) the output distortion introduced by revenue sharing. Before considering both effects at once, we shall discuss the model at first with each channel deactivated in order to provide an idea of their respective impact.

Revenue-sharing financing without moral hazard (WMH)

First of all, consider a case with no moral hazard. The VC's net payout is denoted by (5a). Again, the entrepreneur does not need to exert effort in order to reduce marginal costs, and hence can be more easily incentivized. Taking into account the downward distortion of the quantity produced, the VC chooses the optimal share by maximizing his objective function (5a) with respect to α_{RS}. This yields an unconstrained solution to the maximization problem, $\alpha_{RS}^{UC} = \alpha_{RS}^{UC}(x, c_L)$, which is implicitly defined by

$$\frac{1 + \alpha_{RS}^{UC}(x, c_L)}{(1 - \alpha_{RS}^{UC}(x, c_L))^3} - \left(\frac{x}{c_L}\right)^2 \equiv 0. \tag{8}$$

With $\alpha_{RS}^{UC} \in (0, 1)$ and $NPV^{WMH} \equiv VC_{\pi}^{RS}(\alpha_{RS}^{UC}; q_L(\alpha_{RS}^{UC}, c_L, x), x, C_I)$ the following proposition can be derived:

Proposition 2. *The output distortion under revenue-sharing financing leads to $NPV^{WMH} < NPV^{E}$ if there is no moral hazard.*

Proof. See subsection A.1 in the Appendix. □

The intuition behind this result is that the investor participates asymmetrically in the revenue but not the cost of production. Hence, output distortion is limited to the revenue-sharing case but not the equity case.

Revenue-sharing financing without output distortion (WOD)

In contrast to the case above, now suppose that moral hazard is present but quantity distortion does not occur (by assumption) under revenue-sharing financing. Therefore, the output is the monopoly quantity $q_j^m = (x - c_j)/2$ and the VC's net profit, in case the entrepreneur exerts effort, is

$$VC_{\pi}^{WOD}(\alpha_{RS}; q_L^m, x, C_I) = \alpha_{RS}(x - q_L^m)q_L^m - C_I, \tag{9}$$

where the minuend is still the VC's revenue-sharing component. As in the equity case, the VC will only invest in the start-up if the entrepreneur exerts effort and realizes c_L. Given that both quantities q_L and q_H are positive by assumption (no quantity distortion and $x > c_j$), constraints (5d) and (5e) are always satisfied and nonbinding, (5c) is redundant if (5b) holds. He thus has to set α_{RS} optimally with respect to the ICC (5b). Solving for α_{RS} yields

$$\alpha_{RS} \leq \frac{\Delta\pi - C_e}{\Delta R}, \tag{10}$$

where $\Delta R = (x - q_L^m)q_L^m - (x - q_H^m)q_H^m$ denotes the difference in revenues, i.e. the revenues realized with low and high marginal costs. Choosing α_{RS} such that the entrepreneur is made indifferent between exerting effort or not and substituting it in (5a) gives the VC's profit

$$NPV^{WOD} = \frac{\Delta\pi - C_e}{\Delta R}(x - q_L^m)q_L^m - C_I =: NPV^{WOD}(c_H, c_L, x, C_e, C_I). \tag{11}$$

Proposition 3. *Due to an easier incentivization, $NPV^{WOD} > NPV^E$ and revenue-sharing financing will dominate equity financing if the output is not distorted under revenue-sharing financing.*

Proof. See subsection A.2 in the Appendix. □

In the case of revenue sharing, the entrepreneur can fully acquire the benefits from exceeding effort and hence is easier incentivized than under equity financing.

Revenue-sharing financing: combined mechanisms

Now consider the case with both mechanism activated. Again, the VC will only invest in the start-up if the entrepreneur exerts effort and realizes c_L. He maximizes (5a) by choosing α_{RS} while taking into account the effect of α_{RS} on output, i.e. (7). Moreover, he considers the entrepreneur's participation and incentive compatibility constraint, i.e. (5c) and (5b), respectively. Depending on the type of entrepreneur the VC is facing, he is bound by either one or both of the constraints or can maximize his profits independently of the constraints imposed by the entrepreneur. In general, we can distinguish between two different cases that are possible. Depending on c_H and its proximity to c_L in particular, the VC chooses α_{RS} optimally, with $\alpha_{RS}^* = \arg\max_{\alpha_{RS} \in [0,1]} VC_\pi^{RS}(\alpha_{RS}; q_L, x, C_I)$, either

Lemma 1. *(i) if $\alpha_{RS}^{UC} \leq 1 - c_H/x$ according to*

$$\arg\max_{\alpha_{RS}\in[0,1]} VC_{\pi}^{RS}(\alpha_{RS}) = \begin{cases} \alpha_{RS}^{UC}(c_L, x) = \alpha_{RS}^{UC} & if \quad 0 \leq C_e \leq C_e^{UP} \\ \alpha_{RS}^{PC}(C_e, c_L, x) = \alpha_{RS}^{PC} & if \quad C_e^{UP} < C_e \leq C_e^{PI} \\ \alpha_{RS}^{IC}(C_e, c_H, c_L, x) = \alpha_{RS}^{IC} & if \quad C_e > C_e^{PI} \end{cases}, \tag{12}$$

or (ii) if $\alpha_{RS}^{UC} \geq 1 - c_H/x$ according to

$$\arg\max_{\alpha_{RS}\in[0,1]} VC_{\pi}^{RS}(\alpha_{RS}) = \begin{cases} \alpha_{RS}^{UC}(c_L, x) = \alpha_{RS}^{UC} & if \quad 0 \leq C_e \leq C_e^{UI} \\ \alpha_{RS}^{IC}(C_e, c_H, c_L, x) = \alpha_{RS}^{IC} & if \quad C_e > C_e^{UI} \end{cases}. \tag{13}$$

Proof. See subsection A.3 in the Appendix. □

In order to establish the intuition behind the underlying mechanism consider Figure 1, where graphs (i) and (ii) correspond to (12) and (13), respectively. As the basic mechanics are similar for both graphs, we shall begin by discussing (i). The intersection of the blue solid line with the (horizontal) blue dotted line corresponds to the unconstrained solution of the VC's objective function, i.e. (8). The red graph corresponds to the right hand side of the ICC, or $f^H(\alpha_{RS}; c_H, x)$. The solid black graph represents the PC (similarly the left hand side of the ICC), or $f^L(\alpha_{RS}, C_e; c_L, x)$, at $C_e = 0$.[14] As C_e increases, f^L shifts downwards which corresponds to a left shift of the graph in the positive quadrant as indicated by the dashed black graphs.

Although there is no moral hazard in the case of $C_e = 0$, the VC will not set $\alpha_{RS} = 1$ as the quantity would be distorted drastically. Rather, he sets the optimal $\alpha_{RS} = \alpha_{RS}^{UC}$, which can be considered as an upper bound. Moreover, while in this case neither constraint is binding, they are both satisfied. The VC will set α_{RS}^{UC} as long as $C_e \leq C_e^{UP}$. Once C_e exceeds this threshold, the unconstrained α_{RS}^{UC} is not optimal anymore. If the entrepreneur were offered a contract based on α_{RS}^{UC}, she would reject it. If she accepted, she could either exert effort and achieve a reduction of marginal costs, or she could save C_e and produce at higher marginal costs. In both cases she would yield a negative payout. Therefore, if the root of f^L is below α_{RS}^{UC} and above or equal to that of f^H (i.e. the range marked in

[14] Please refer to subsection A.3 in the Appendix for a more detailed description of f^H and f^L.

green), the VC will set $\alpha_{RS} = \alpha_{RS}^{PC}$. In this case, both conditions hold and the PC is a binding constraint. Furthermore, there is no moral hazard problem in this range of C_e as the ICC is nonbinding. Therefore, the entrepreneur will exert effort and does not require incentivization. If C_e exceeds C_e^{PI} and the VC were to offer a contract based on α_{RS}^{PC}, the entrepreneur would accept and produce at c_H. Hence, this is the case where a moral hazard problem exists. The VC solves this by setting $\alpha_{RS} = \alpha_{RS}^{IC}$, which is determined by the intersection of f^L with f^H. In this case, the ICC is the binding condition and the PC is satisfied but nonbinding.

Essentially, the difference between graphs (i) and (ii) is that, in the second graph, c_H is lower. More specifically, it assumes a value such that the root of $f^H \geq \alpha_{RS}^{UC}$. This has the following effect: While α_{RS}^{UC} similarly serves as an upperbound for the VC's choice of α_{RS}, the threshold level of C_e is lower, i.e. $C_e^{UI} < C_e^{UP}$. Now, as the reduction in marginal costs is considerably smaller than in the previous case the ICC becomes the binding constraint as soon as $C_e > C_e^{UI}$.

Considering the specification of α_{RS}, we can now derive the VC's optimal value (function) $NPV^{RS} \equiv VC_{\pi}^{RS}(\alpha_{RS}^*, q_L(\alpha_{RS}^*, x), C_I)$. Again, depending on c_H and its proximity to c_L in particular the optimal value (function) can be formulated either (i) according to

$$NPV^{RS} = \begin{cases} \alpha_{RS}^{UC} \cdot R_L(\alpha_{RS}^{UC}, c_L, x) - C_1 = NPV_{UC}^{RS}(c_H, c_L, x, C_I) & if \quad 0 \leq C_e \leq C_e^{UP}; \\ \alpha_{RS}^{PC} \cdot R_L(\alpha_{RS}^{PC}, c_L, x) - C_1 = NPV_{PC}^{RS}(c_H, c_L, x, C_e, C_I) & if \quad C_e^{UP} < C_e \leq C_e^{PI}; \\ \alpha_{RS}^{IC} \cdot R_L(\alpha_{RS}^{IC}, c_L, x) - C_1 = NPV_{IC}^{RS}(c_H, c_L, x, C_e, C_I) & if \quad C_e > C_e^{PI}, \end{cases} \tag{14}$$

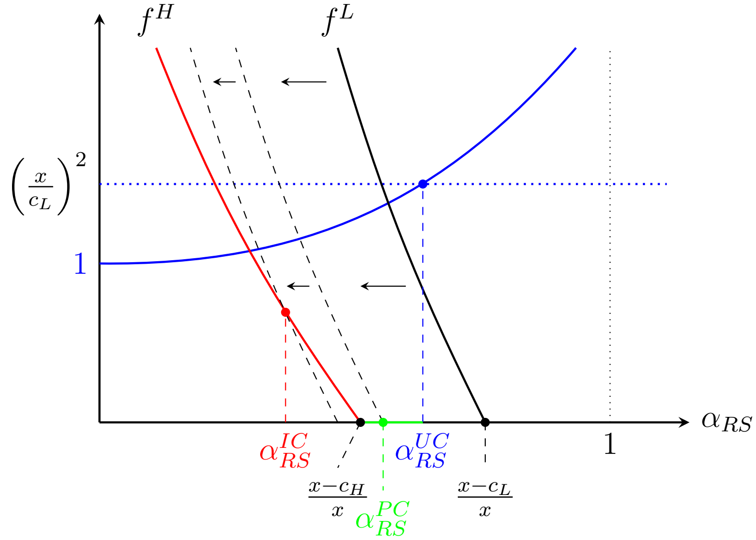
or (ii) according to

$$NPV^{RS} = \begin{cases} \alpha_{RS}^{UC} \cdot R_L(\alpha_{RS}^{UC}, c_L, x) - C_1 = NPV_{UC}^{RS}(c_H, c_L, x, C_I) & if \quad 0 \leq C_e \leq C_e^{UI}; \\ \alpha_{RS}^{IC} \cdot R_L(\alpha_{RS}^{IC}, c_L, x) - C_1 = NPV_{IC}^{RS}(c_H, c_L, x, C_e, C_I) & if \quad C_e > C_e^{UI}, \end{cases} \tag{15}$$

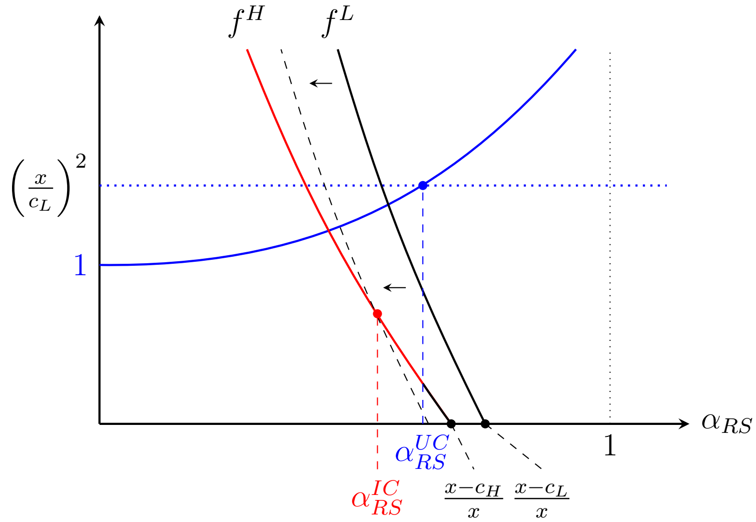
with

$$R_L(\alpha_{RS}, c_L, x) = \frac{x^2 - \left(\frac{c_L}{1-\alpha_{RS}}\right)^2}{4}. \tag{16}$$

Figure 1: Depiction of the formation of α^*_{RS}.



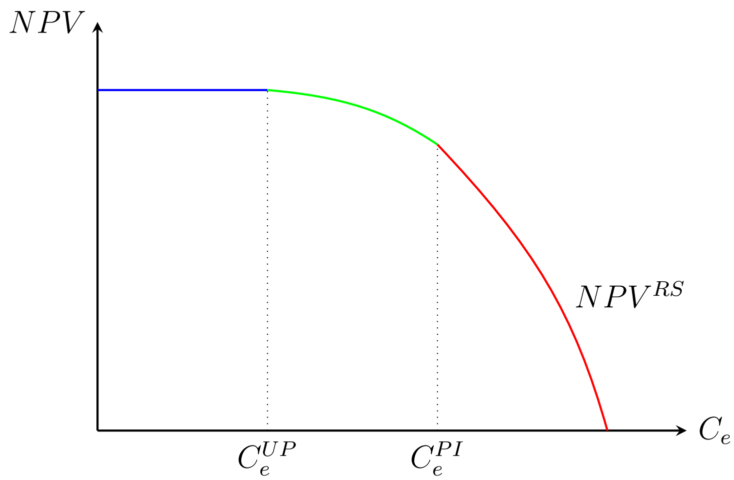
(i) Corresponds to (12).



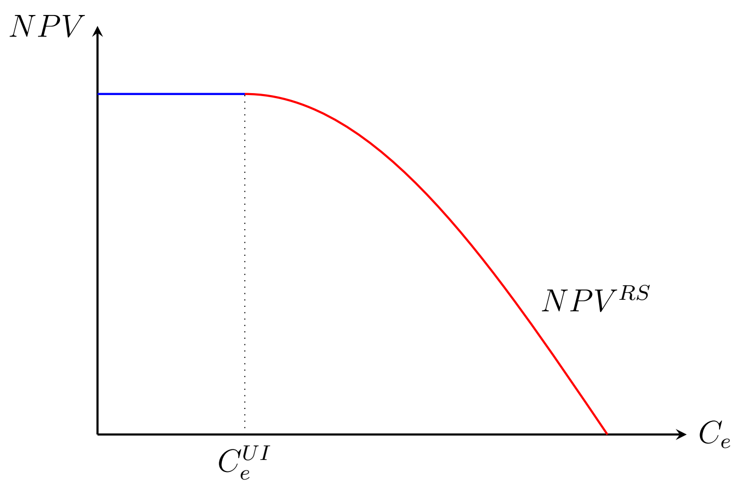
(ii) Corresponds to (13).

Consider graphs (i) and (ii) of Figure 2 that show (14) and (15), respectively. The VC's net payout starts at a certain value for very low levels of C_e and is horizontal as long as it is below the threshold value C_e^{UP} (or C_e^{UI}). This is the range in which the VC will set

Figure 2: Venture capitalist profits under revenue sharing.



(i) Corresponds to (14).



(ii) Corresponds to (15).

α_{RS}^{UC}, implying that the entrepreneur will have a positive payout that decreases in C_e and is greater than the difference in revenues R_L and R_H minus her cost of effort. As soon as C_e exceeds the respective thresholds, the VC's net payout decreases in C_e as he has to render a larger share of revenue to the entrepreneur in order to satisfy the optimization constraints.

3 Results

In the next step, we graphically compare the VC's profits from an equity investment to those from revenue-sharing financing. The results stated in Propositions 2 and 3 suggest that there is no clear relationship to be expected. As revenue sharing is superior in dealing with high levels of moral hazard and hence increases the VC's profit, the output distortion shifts profits in the opposite direction depending on the level of C_e.[15]

Figure 3: Venture capitalist profits under equity and revenue sharing.

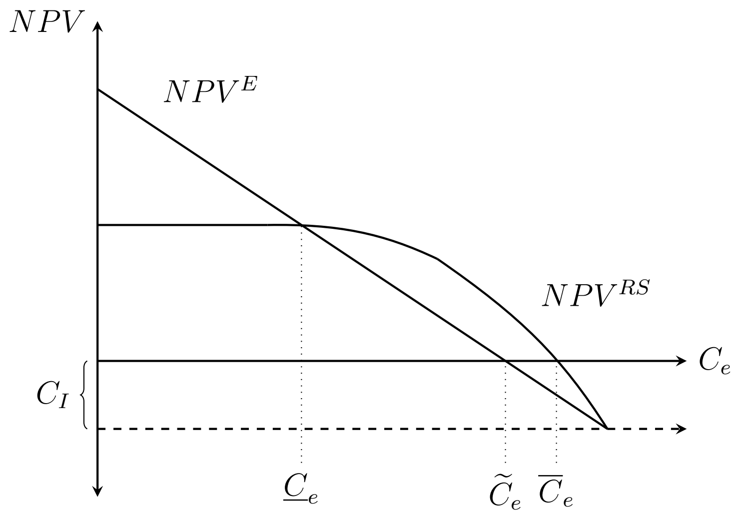


Figure 3 shows both equity and revenue-sharing value functions of the VC. While the NPV under equity financing is linearly decreasing in C_e, the output distortion causes the profit from revenue sharing to adopt a concave non-increasing, ultimately downward-sloping course. This suggests that there can be a range of C_e in which only revenue sharing is a viable financing option. If the cost of effort is below $\underline{C}_e$, equity is the preferred financing option albeit revenue sharing also leads to a positive outcome. For $C_e \in (\underline{C}_e, \widetilde{C}_e)$, equity financing would lead to a positive profit but revenue-sharing financing beats equity in this interval. In the range $C_e \in (\widetilde{C}_e, \overline{C}_e)$ equity is not a viable option. The measure of this last interval is

[15]We present the VC's value function of revenue sharing without differentiating between (14) and (15). Although they are technically different functions, their relation to the VC's value function of an equity investment are qualitatively identical.

essentially driven by the value of C_I, i.e. the amount of money the VC has to invest in the start-up. If it were zero, the set of entrepreneurs that could be financed with either equity or revenue sharing would be the same. Hence, revenue sharing can potentially increase the set of start-ups that could be realized.

The economic intuition behind this result is the following. When the cost of effort is very low, the moral hazard problem is small. This leads the VC to choose a high share α_{RS} as the entrepreneur does not require a lot to be compensated for her cost of effort. However, the high α_{RS} has a disproportionately strong negative effect on the output quantity. Now, the VC indeed retains a large share of the revenue but the decrease in output outweighs this positive revenue effect and causes the profit to be lower than under equity financing. As the moral hazard problem increases, the VC has to render more of his share to the entrepreneur in order to satisfy her participation and incentive compatibility constraint. This is easier to achieve under revenue sharing as, in addition to the fact that with decreasing α_{RS} the output distortion becomes less severe, the entrepreneur benefits from a reduction of marginal costs to a greater extent than under equity financing.

Proposition 4. *The opposite effects of output distortion and easier incentivization lead to areas in which equity dominates revenue-sharing financing and areas in which revenue-sharing finacing dominates equity.*

Proof. See subsection A.4 in the Appendix. □

While revenue sharing can, in general, increase the set of feasible start-ups, it is rather unclear whether this will have a positive effect on the aggregate market and thus on welfare. Considering that for any given $C_e < \widetilde{C}_e$ the output under revenue sharing will be below that of equity financing, cumulative net payout will also be lower. However, revenue sharing can indeed create welfare if $C_e \geq \widetilde{C}_e$.

Proposition 5. *Consumer surplus, cumulative net payout, and hence welfare are greater under equity financing compared to revenue-sharing financing if* $0 \leq C_e < \widetilde{C}_e$. *For* $\widetilde{C}_e \leq C_e < \overline{C}_e$*, the opposite applies.*

Proof. See subsection A.5 in the Appendix. □

Finally, while revenue sharing can be the preferred financing option from a VC's perspective, we should note that this is also contingent on the (relative) level of C_I. Although it may enable the realization of a larger set of start-ups, equity might still be the only financing option if the initial investment is sufficiently high.

4 A note on ICOs

The financing of Blockchain start-ups is the most recent financing application that incorporates revenue-sharing components. Ever since the Blockchain world emerged, the option has developed for firms to raise funds by issuing crypto tokens in an ICO. In exchange for any liquid funds, such as fiat or cryptocurrencies, start-ups can conduct an ICO by giving out self-created crypto tokens (see, e.g., Howell et al., 2019; Li and Mann, 2018). These virtual tokens can take different forms and, theoretically, entail different rights depending on the coding of the smart contract. While the token could potentially entail rights similar to equity titles (security tokens), the most commonly used form is the utility token that gives the buyer the right on the future product (Chod et al., 2019; Fisch, 2019; Momtaz et al., 2019; for an introduction to the ICO market, see Momtaz, 2019b). Usually, tokens are intended to be listed on crypto exchanges after the ICO. Tradability on a secondary market allows token holders to exit their investment and sell their tokens if they do not plan to trade it in for the product itself.[16]

From the start-up's perspective, a utility token sale is the sale of a voucher that entitles the buyer to consume the company's product. Hence, it is comparable to a pre-sale of the future product. According to Malinova and Park (2018), there are several viable options regarding the implementation of the ICO. On the one hand, the entrepreneur can sell a fixed amount of units of the future product to investors. The proceeds from this sale are then used to cover set-up costs (and the unit costs of the units presold). With the start of production, the entrepreneur issues another amount of tokens, with the price of these additional tokens equal to the price of the product. Another possible option is to fix the total number of

[16] As Catalini and Gans (2018) show, tradability can also create competition for the token itself, which can add to its value.

tokens prior to production and pre-sell a fraction of these tokens to investors.[17] After the pre-sale and with start of production, the start-up sells the remaining fraction of tokens to the consumers of the product. Investors who bought their tokens during the pre-sale will also sell their tokens to consumers and consequently receive a total compensation equal to a share of total revenues, where the share is identical to their fraction of the absolute number of tokens.

Therefore, ICOs can be designed as a form of revenue sharing and can, when issuing utility tokens with a fixed total number, serve as implementation and utilization of the mechanism described in the above model—i.e. improved alignment of interests in the context of start-ups with high moral hazard related to cost reductions. In addition, ICOs extend the set of potential exit strategies of VCs, when choosing revenue sharing as the initial contract. Firstly, ICOs offer the possibility of converting a contract between a VC and an entrepreneur into liquid funds by converting a revenue-sharing contract into (a fraction of the total number of) tokens—technically in a pre-sale. These tokens can then be sold in an ICO, while retaining the incentive structure. This is of high relevance when the moral hazard problem is particularly high and equity financing is a non-viable option. Secondly, ICOs can receive their legitimacy in the context of a high liquidity preference of the VC, i.e. if an early exit of the VC is intended.[18] Although a conversion of the revenue-sharing contract into equity shares, followed by an IPO or trade sale, is in principle a viable option for a wide range of entrepreneurs, it is suboptimal in early stages when moral hazard has not yet been sufficiently reduced. In early stages, ICOs that retain the character of revenue sharing can be a superior alternative to conversion of revenue-sharing contracts into equity shares for the VC.[19]

[17] The investors receive a number of tokens which equal a fixed share of the future revenues. As this makes the denomination of tokens irrelevant, the share of revenues translates into an arbitrary but set number of tokens.

[18] In a traditional equity investment, VC investors gain (at least partial) control over the portfolio company. The liquidity-preference argument is one potential reason why VCs would be willing to forego control rights, as would be the case with ICO revenue sharing.

[19] See Subsection A.6 in the Appendix for a brief formal analysis of this aspect.

5 Conclusion

In this paper we examine the possibility of revenue-sharing contracts as a means of financing for start-ups. We develop a simple theoretical model and show that, from an investor's perspective, revenue sharing can be superior to traditional equity financing if moral hazard is high. Moreover, revenue sharing can also raise welfare by enabling the financing of start-ups that would not have received an equity investment. Although revenue-sharing contracts are part of a niche market in start-up finance, they have been around for approximately 25 years. Surprisingly, there are only a few papers in the VC literature that mention revenue sharing as a viable option. One potential reason for this could be that, traditionally, revenue-sharing contracts are a form of debt financing (Bernthal, 2019) and could thus lead to a debt-overhang problem. Although common VC financing instruments tend to have debt-like features (e.g., Kaplan and Strömberg, 2001, 2003), a conventional VC fund might be reluctant to accept or propose such an agreement at a first glance as it could prevent other equity investors to provide further capital. However, it is quite imaginable that a VC were inclined to offer a revenue-sharing contract with unlimited maturity term and an option to convert into common stock. This would satisfy the usual characteristics of VC financing instruments, as documentd by Hellmann (2006). In fact, we show that it is indeed optimal for the VC to convert at some point in the future. In a multi-periods setting, this helps the VC to solve the incentive problem if the entrepreneur's cost of effort were decreasing over time.

As Kaivanto and Stoneman (2007) show, though having a different focus, claims contingent on sales can be beneficial especially to young firms in the sense of financing growth and innovation in high-risk sectors. Given that revenue sharing is used by Blockchain start-ups, we argue that ICOs might indeed be a powerful financing tool. Of course, in order for ICOs to become a serious and legit alternative, they will require stronger regulation (see also Cumming et al., 2019). As Momtaz (2019a) argues, the moral hazard problem inherent in ICOs in general is manifested in signals that are not costly to imitate. If regulators were to standardize the ICO process, moral hazard could be reduced substantially. If this were achieved, it might well be worth considering a subsidy for revenue-sharing contracts. If entrepreneurs could convey their type to authorities, this might even increase the set of feasible start-ups further. Obviously, this is by no means limited to blockchain start-ups. However, there are natural limitations in terms which industries might be suitable candidates for revenue sharing. As our model suggests, the mechanism will not work for start-ups that require a very high initial investment. Hence, we argue that revenue sharing can be a superior alternative for industries with low to moderate investment intensities.

Apart from the potential considerations for policy makers, our finding also bears an empirical implication. Depending on whether the mechanism is known among investors and entrepreneurs, it should be possible to investigate if revenue-sharing contracts are indeed negotiated between VC investors and high-risk entrepreneurs. In this case, a potential avenue could be to approximate the severeness of moral hazard using, for instance, the entrepreneur's extent of education or—in case the person is a serial entrepreneur—the number of times they have started a business before.

As with any theoretical model, there are limitations to ours. We should note that our mechanism primarily addresses costs of effort that are induced through a channel in which marginal costs are reduced. A potential extension of our model could incorporate an effort channel that enables the entrepreneur to increase the demand through a marketing channel. Furthermore and with regards to ICOs, we deactivate the price mechanism behind utility tokens and assume that tokens are just a means of trade with a fixed exchange rate between token and product. As Holden and Malani (2019) demonstrate, however, the value of utility tokens could be limited. This implies that, in the case of ICOs, there might be a second effect reducing revenues under revenue sharing in comparison to equity. Research that draws upon our theory might want to consider this.

References

Agrawal, A., C. Catalini, and A. Goldfarb (2015): "Crowdfunding: Geography, social networks, and the timing of investment decisions," *Journal of Economics & Management Strategy*, 24(2), 253–274.

Amit, R., J. Brander, and C. Zott (1998): "Why do venture capital firms exist? Theory and Canadian evidence," *Journal of Business Venturing*, 13(6), 441–466.

Bascha, A., and U. Walz (2001): "Convertible securities and optimal exit decisions in venture capital finance," *Journal of Corporate Finance*, 7(3), 285–306.

Bergemann, D., and U. Hege (1998): "Venture capital financing, moral hazard, and learning," *Journal of Banking & Finance*, 22(6-8), 703–735.

Bernstein, S., X. Giroud, and R. R. Townsend (2016): "The Impact of Venture Capital Monitoring," *Journal of Finance*, 71(4), 1591–1622.

Bernthal, J. B. (2019): "The Evolution of Entrepreneurial Finance: A New Typology," *Brigham Young University Law Review*, 773(4).

Casamatta, C. (2003): "Financing and advising: optimal financial contracts with venture capitalists," *Journal of Finance*, 58(5), 2059–2085.

Catalini, C., and J. Gans (2018): *Initial Coin Offerings and the Value of Crypto Tokens.* National Bureau of Economic Research, Working Paper.

Chemmanur, T. J., and Z. Chen (2014): "Venture capitalists versus angels: The dynamics of private firm financing contracts," *Review of Corporate Finance Studies*, 3(1-2), 39–86.

Chemmanur, T. J., and P. Fulghieri (2014): "Entrepreneurial finance and innovation: An introduction and agenda for future research," *Review of Financial Studies*, 27(1), 1–19.

Chemmanur, T. J., K. Krishnan, and D. K. Nandy (2011): "How Does Venture Capital Financing Improve Efficiency in Private Firms? A Look Beneath the Surface," *Review of Financial Studies*, 24(12), 4037–4090.

Chod, J., and E. Lyandres (2018): *A Theory of ICOs: Diversification, Agency, and Information Asymmetry*. Working Paper.

Chod, J., N. Trichakis, and A. Yang (2019): *Platform Tokenization: Financing, Governance, and Moral Hazard*. Working Paper.

Cornelli, F., and O. Yosha (2003): "Stage financing and the role of convertible securities," *Review of Economic Studies*, 70(1), 1–32.

Cumming, D. J. (2005a): "Agency costs, institutions, learning, and taxation in venture capital contracting," *Journal of Business Venturing*, 20(5), 573–622.

——— (2005b): "Capital structure in venture finance," *Journal of Corporate Finance*, 11(3), 550–585.

Cumming, D. J., S. Johan, and A. Pant (2019): "Regulation of the Crypto-Economy: Managing Risks, Challenges, and Regulatory Uncertainty," *Journal of Risk and Financial Management*, 12(3), 126.

Da Rin, M., T. Hellmann, and M. Puri (2013): "A survey of venture capital research," in *Handbook of the Economics of Finance*, vol. 2, pp. 573–648. Elsevier.

De Bettignies, J.-E., and J. A. Brander (2007): "Financing entrepreneurship: Bank finance versus venture capital," *Journal of Business Venturing*, 22(6), 808–832.

Denis, D. J. (2004): "Entrepreneurial finance: an overview of the issues and evidence," *Journal of Corporate Finance*, 10(2), 301–326.

Feldman, M., I. Feller, J. Bercovitz, and R. Burton (2002): "Equity and the technology transfer strategies of American research universities," *Management Science*, 48(1), 105–121.

Fisch, C. (2019): "Initial coin offerings (ICOs) to finance new ventures," *Journal of Business Venturing*, 34(1), 1–22.

Fisch, C., and P. P. Momtaz (2019): *Venture Capital and the Performance of Blockchain Technology-Based Firms: Evidence from Initial Coin Offerings (ICOs)*. Working Paper.

Garratt, R., and M. R. van Oordt (2019): *Entrepreneurial incentives and the role of initial coin offerings*. Bank of Canada, Working Paper.

Gompers, P. A. (1995): "Optimal investment, monitoring, and the staging of venture capital," *Journal of Finance*, 50(5), 1461–1489.

Gompers, P. A., W. Gornall, S. N. Kaplan, and I. A. Strebulaev (2019): "How do venture capitalists make decisions?," *Journal of Financial Economics, Forthcoming.*

Gompers, P. A., and J. Lerner (2001): "The venture capital revolution," *Journal of Economic Perspectives*, 15(2), 145–168.

Hall, J., and J. Lerner (2010): "Venture capitalists' decision criteria in new venture evaluation," *Journal of Business Venturing*, 8(1), 25–42.

Hellmann, T. (1998): "The allocation of control rights in venture capital contracts," *The RAND Journal of Economics*, pp. 57–76.

——— (2006): "IPOs, acquisitions, and the use of convertible securities in venture capital," *Journal of Financial Economics*, 81(3), 649–679.

Hellmann, T., and M. Puri (2002): "IPOs, acquisitions, and the use of convertible securities in venture capital," *Journal of Finance*, 57(1), 169–197.

Holden, R., and A. Malani (2019): *The ICO Paradox: Transactions Costs, Token Velocity, and Token Value*. National Bureau of Economic Research, Working Paper.

Howell, S. T., M. Niessner, and D. Yermack (2019): "Initial Coin Offerings: Financing Growth with Cryptocurrency Token Sales," *Review of Financial Studies, Forthcoming.*

Kaivanto, K., and P. Stoneman (2007): "Public provision of sales contingent claims backed finance to SMEs: A policy alternative," *Research Policy*, 36(5), 637–651.

Kaplan, S. N., and P. Strömberg (2001): "Venture capitals as principals: contracting, screening, and monitoring," *American Economic Review*, 91(2), 426–430.

Kaplan, S. N., and P. Strömberg (2003): "Financial contracting theory meets the real world: An empirical analysis of venture capital contracts," *Review of Economic Studies*, 70(2), 281–315.

Lerner, J. (1995): "Venture Capitalists and the Oversight of Private Firms," *Journal of Finance*, 50(1), 301–318.

Li, J., and W. Mann (2018): *Initial Coin Offerings and Platform Building*. Working Paper.

Malinova, K., and A. Park (2018): *Tokenomics: when tokens beat equity*. Working Paper.

Momtaz, P. P. (2019a): *Entrepreneurial Finance and Moral Hazard: Evidence from Token Offerings*. Working Paper.

——— (2019b): "Token Sales and Initial Coin Offerings: Introduction," *Journal of Alternative Investments*, 21(4), 7–12.

Momtaz, P. P., K. Rennertseder, and H. Schröder (2019): *Token Offerings: A Revolution in Corporate Finance?* Working Paper.

Schmidt, K. M. (2003): "Convertible securities and venture capital finance," *Journal of Finance*, 58(3), 1139–1166.

Ueda, M. (2004): "Banks versus venture capital: Project evaluation, screening, and expropriation," *Journal of Finance*, 59(2), 601–621.

Wang, S., and H. Zhou (2004): "Staged financing in venture capital: moral hazard and risks," *Journal of Corporate Finance*, 10(1), 131–155.

A Appendix

A.1 Proof of Proposition 2

Proof. In order to show that the VC will indeed prefer equity over revenue-sharing financing in the absence of moral hazard, it is sufficient to show that the VC's profit under equity financing, see (4), is larger than the (cumulative) net payout under revenue-sharing financing, see (44):

$$
\begin{aligned}
& NPV^E(C_e, C_I, c_H, c_L)\big|_{C_e=0} > CP_{L,RS}(\alpha_{RS}, C_e, C_I, c_L, x)\big|_{C_e=0} \\
\Leftrightarrow \quad & \left(\frac{x-c_L}{2}\right)^2 - C_I > \frac{x^2 - 2xc_L + \left(\frac{c_L}{1-\alpha_{RS}}\right)^2 (1-2\alpha_{RS})}{4} - C_I \\
\Leftrightarrow \quad & c_L^2 > \left(\frac{c_L}{1-\alpha_{RS}}\right)^2 (1-2\alpha_{RS}) \\
\Leftrightarrow \quad & (1-\alpha_{RS})^2 > 1 - 2\alpha_{RS} \\
\Leftrightarrow \quad & \alpha_{RS} > 0.
\end{aligned}
$$

Therefore for all $\alpha_{RS} > 0$, the VC indeed prefers equity financing over revenue sharing. Obviously, with $\alpha_{RS} = 0 \Rightarrow VC_\pi^{RS} = 0$ and hence smaller than $VC^E > 0 \Leftrightarrow x > c_L$. □

A.2 Proof of Proposition 3

Proof. In order to show that the VC will indeed prefer revenue sharing over equity in the absence of output distortion the following inequality must hold:

$$NPV^{WMH} = \frac{\Delta\pi - C_e}{\Delta R}(x - q_L^m)q_L^m - C_I > \frac{\Delta\pi - C_e}{\Delta\pi}\pi_L^m - C_I = NPV^E \tag{17}$$

Re-arranging the condition, we obtain

$$
\begin{aligned}
& \frac{\Delta\pi - C_e}{\Delta R} R_j^m > \frac{\Delta\pi - C_e}{\Delta\pi}\pi_L^m \\
\Leftrightarrow \quad & \frac{\Delta\pi - C_e}{\Delta R} R_j^m - \left(1 - \frac{C_e}{\Delta\pi}\right)\pi_L^m > 0 \\
\Leftrightarrow \quad & \frac{R_L^m}{\Delta R} > \frac{\pi_L^m}{\Delta\pi},
\end{aligned} \tag{18}
$$

where $\Delta\pi = \pi_L^m - \pi_H^m$, $\Delta R = R_L^m - R_H^m$, and $R_j^m = (x - q_j^m)q_j^m$, i.e. the monopoly revenue with $j = \{L, H\}$.

Factoring out (18) shows that (17) indeed holds, as it breaks down to

$$c_H > c_L.$$

□

A.3 Proof of Lemma 1

The derivation of profit maximizing revenue sharing from VC's perspective is structured as follows:

a) α_{RS} is derived i) for the unconstrained problem, ii) such that the participation constraint holds with equality, iii) such that the incentive compatibility constraint holds with equality. Furthermore, it is shown that for low levels of C_e, the incentive compatibility constraint is identical to the participation constraint (and hence technically redundant). Results are summarized in Lemma 2.

b) Conditions are derived under which either solution i), ii), or iii) of a) applies. Results are stated in Lemma 1'.

For later purposes, let

$$\begin{aligned}
f^L(\alpha_{RS}, C_e; c_L, x) &:= (1 - \alpha_{RS}) \frac{x^2 - \left(\frac{c_L}{1-\alpha_{RS}}\right)^2}{4} - c_L \frac{x - \frac{c_L}{1-\alpha_{RS}}}{2} - C_e, \\
f^H(\alpha_{RS}; c_H, x) &:= (1 - \alpha_{RS}) \frac{x^2 - \left(\frac{c_H}{1-\alpha_{RS}}\right)^2}{4} - c_H \frac{x - \frac{c_H}{1-\alpha_{RS}}}{2}, \\
\text{and} \quad f^{IC}(\alpha_{RS}, C_e; c_H, c_L, x) &:= f^L(\alpha_{RS}, C_e; c_L, x) - f^H(\alpha_{RS}; c_H, x),
\end{aligned}$$

so that the constraints can be rewritten as

$$\begin{aligned}
(PC) &\quad f^L(\alpha_{RS}, C_e; c_L, x) \geq 0, \\
(ICC) &\quad f^L(\alpha_{RS}, C_e; c_H, c_L, x) \geq f^H(\alpha_{RS}; c_H, x) \quad \Leftrightarrow \quad f^{IC}(\alpha_{RS}, C_e; c_H, c_L, x) \geq 0.
\end{aligned}$$

Unconstrained maximization: Note that the objective function $VC_\pi^{RS}(\alpha_{RS}; q_L, x, C_I)$ is strictly concave in α_{RS},

$$\text{as shown by:} \quad \frac{\partial^2 VC_\pi^{RS}(\alpha_{RS}; q_L, x, C_I)}{\partial \alpha_{RS}^2} = -\frac{2c_L^2}{(1-\alpha_{RS})^3} - \frac{2c_L^2}{(1-\alpha_{RS})^3} - \alpha_{RS}\frac{6c_L^2}{(1-\alpha_{RS})^4}$$
$$< 0 \;\forall c_L > 0, \alpha_{RS} \in [0,1), \tag{19}$$

and there exists a maximum at $\alpha_{RS}^{UC} = \alpha_{RS}^{UC}(c_L, x)$, for given c_L and x, implicitly defined by

$$\frac{1+\alpha_{RS}^{UC}(c_L,x)}{(1-\alpha_{RS}^{UC}(c_L,x))^3} - \left(\frac{x}{c_L}\right)^2 \equiv 0, \tag{20}$$

follows with (19) and with:

$$\frac{\partial VC_\pi^{RS}(\alpha_{RS}; q_L, x, C_I)}{\partial(\alpha_{RS})} = x^2 - \frac{c_L^2}{(1-\alpha_{RS})^2} - \alpha_{RS}\frac{2c_L^2}{(1-\alpha_{RS})^3} = 0$$
$$\Rightarrow \underbrace{\frac{1+\alpha_{RS}}{(1-\alpha_{RS})^3}}_{=:f^{UCL}(\alpha_{RS})} = \underbrace{\left(\frac{x}{c_L}\right)^2}_{=:f^{UCR}(c_L,x)}, \tag{21}$$

with $0 < \alpha_{RS}^{UC}$,

$$\text{follows with:} \quad f^{UCR}(c_L,x) = \left(\frac{x}{c_L}\right)^2 \in (1,\infty) \text{ if } 0 < c_L < x, \tag{22}$$
$$f^{UCL}(\alpha_{RS})\big|_{\alpha_{RS}=0} = 1, \tag{23}$$
$$\frac{df^{UCL}(\alpha_{RS})}{d\alpha_{RS}} = \frac{1}{(1-\alpha_{RS})^3} + \frac{3\,(1+\alpha_{RS})}{(1-\alpha_{RS})^4} > 0, \tag{24}$$

and $\alpha_{RS}^{UC} < 1 - c_L/x < 1$,

$$\text{follows with (22), (24), and:} \quad f^{UCL}(\alpha_{RS})\big|_{\alpha_{RS}=1-c_L/x} = \frac{x^2\,(2x-c_L)}{(c_L)^3} > \left(\frac{x}{c_L}\right)^2 \text{ if } c_L < x, \tag{25}$$

$\partial\alpha_{RS}^{UC}(c_L,x)/\partial x > 0$ (follows with the definition (20), $\partial f^{UCR}(\cdot)/\partial x > 0$, and (24)), and $\partial\alpha_{RS}^{UC}(c_L,x)/\partial c_L < 0$ (follows with the definition (20), $\partial f^{UCR}(\cdot)/\partial c_L < 0$, and (24)). The solution to the unconstrained maximization problem is given by $\alpha_{RS} = \alpha_{RS}^{UC}$.

However, note that any solution to the maximization problem (i.e. any α_{RS} set by the VC) has to fulfill both the participation and the incentive compatibility constraint, i.e. $f^L(\alpha_{RS}, C_e; c_L, x) \geq 0$ and $f^{IC}(\alpha_{RS}; c_H, x) \geq 0$. With

$$\frac{\partial f^H(\alpha_{RS}; c_H, x)}{\partial \alpha_{RS}} < 0 \quad \forall\, \alpha_{RS} \in [0, \alpha_{RS}^{PC}),$$

and

$$\frac{\partial f^{IC}(\alpha_{RS}, C_e; c_H, c_L, x)}{\partial \alpha_{RS}} < 0 \quad \forall\, \alpha_{RS} \in [0, \frac{1 - c_H}{x})$$

it follows that if a constraint holds (with equality) for some $\alpha_{RS}^x \in [0, 1]$, the constraint is still fulfilled for some $\alpha_{RS} < \alpha_{RS}^x$.[20] As the objective function (5a) is strictly concave in α_{RS} and has a maximum at α_{RS}^{UC}, choosing some $\alpha_{RS} > \alpha_{RS}^{UC}$ is not profit maximizing. Hence, if one or both constraints hold with equality for some $\alpha_{RS}^z > \alpha_{RS}^{UC}$, choosing $\alpha_{RS} = \alpha_{RS}^{UC}$ is compatible with the constraints and profit is maximized. In contrast, if one or both of the constraints are fulfilled for some $\alpha_{RS}^z < \alpha_{RS}^{UC}$ choosing $\alpha_t = \alpha_{RS}^{UC}$ would maximize the objective function but violates one or both constraints. Hence in that case, choosing the largest α_{RS} that is compatible with both constraints maximizes profit under the given constraints (profit maximization by choosing the largest α_{RS} follows from $\partial VC_\pi^{RS}(\alpha_{RS}; q_L, x, C_I)/\partial \alpha_{RS} > 0\ \forall\ \alpha_{RS} < \alpha_{RS}^{UC}$).

In a next step, the minimum levels of α_{RS} that are compatible with either constraint are derived.

Derivation of the minimum level of α_{RS} imposed by the participation constraint: Solving $f^L(\alpha_{RS}, C_e; c_L, x) = 0$ for α_{RS} under the technical constraint $x - c_L/(1 - \alpha_{RS}) \geq 0 \Leftrightarrow q_L \geq 0$, yields

$$\alpha_{RS} = 1 - \left[2C_e + \frac{c_L}{x} + 2\left[C_e\left(\frac{c_L}{x} + C_e\right)\right]^{\frac{1}{2}} \right] =: \alpha_{RS}^{PC}(C_e, c_L, x) = \alpha_{RS}^{PC}, \tag{26}$$

where $\alpha_{RS}^{PC}(C_e, c_L, x)$ has the domain $C_e \in [0, \infty)$, $c_L, x \in \mathbb{R}^+$, and the range $\alpha_{RS}^{PC} \in (-\infty, 1-$

[20] The domains for the partial derivatives above are chosen according to economic intuition. $f^H(\alpha_{RS}; c_H, x)$ and $f^L(\alpha_{RS}, C_e; c_L, x)$ are economically meaningful on these domains only. α_{RS}^{PC} is derived below (see (26)).

$\frac{c_L}{x}]$.[21] Furthermore,

$$\frac{\partial \alpha_{RS}^{PC}(C_e, c_L, x)}{\partial C_e} = -2\left[1 + \frac{1}{2}\left[C_e\left(\frac{c_L}{x} + C_e\right)\right]^{\frac{1}{2}}\right] < 0, \tag{27}$$

$$\frac{\partial^2 \alpha_{RS}^{PC}(C_e, c_L, x)}{\partial C_e^2} = -\frac{1}{2}\left[C_e\left(\frac{c_L}{x} + C_e\right)\right]^{\frac{1}{2}} < 0, \tag{28}$$

$\partial \alpha_{RS}^{PC}(C_e, c_L, x)/\partial c_L < 0$, and $\partial \alpha_{RS}^{PC}(C_e, c_L, x)/\partial x > 0$. The participation constraint imposes $\alpha_{RS} \leq \alpha_{RS}^{PC}$.

Derivation of the minimum level of α_{RS} imposed by the incentive compatibility constraint: Firstly, note that the incentive compatibility constraint is binding for some α_{RS} below a threshold level $\alpha_{RS}^{IC'}$ only. With $\alpha_{RS} > \alpha_{RS}^{IC'}$, $f^H(\alpha_{RS}; c_H, x) < 0$. From an economic perspective $f^H(\alpha_{RS}; c_H, x) < 0$ is meaningless (as outputs are bound to be non-negative, $q_H, q_L \geq 0$). In particular, with $\alpha_{RS} > \alpha_{RS}^{IC'}$, the incentive compatibility constraint is technically identical to the participation constraint and redundant in the maximization problem. Solving $f^H(\alpha_{RS}; c_H, x) = 0$ for α_{RS} under the technical constraint $x - c_H/(1 - \alpha_{RS}) \geq 0 \Leftrightarrow q_H \geq 0$, yields

$$\alpha_{RS} = 1 - \frac{c_H}{x} =: \alpha_t^{IC'}(c_H, x) = \alpha_t^{IC'},$$

where $\alpha_t^{IC'}(c_H, x)$ has the domain $c_H, x \in \mathbb{R}^+$, and the range $\alpha_t^{IC'} \in (0, 1)$. Furthermore, $\partial \alpha_t^{IC'}(c_H, x)/\partial c_H < 0$, and $\partial \alpha_t^{IC'}(c_H, x)/\partial x > 0$. To derive (for later purposes) the level of C_e such that the incentive compatibility constraint is binding at a maximum α_{RS}, it is sufficient to solve $f^L(\alpha_{RS}, C_e; c_L, x) = \alpha_{RS}^{IC'}$ for C_e:

$$C_e = \frac{x}{4c_H}\left(c_H^2 - c_L^2\right) =: C_e^{PI}(c_H, c_L, x) = C_e^{PI}.$$

The incentive compatibility constraint is defined as $f^L(\alpha_{RS}, C_e; c_L, x) \geq f^H(\alpha_{RS}; c_H, x) \Leftrightarrow f^{IC}(\alpha_{RS}, C_e; c_H, c_L, x) \geq 0$. Solving $f^{IC}(\alpha_{RS}, C_e; c_H, c_L, x) = 0$ for α_{RS} yields

$$\alpha_{RS} = 1 - \frac{\frac{1}{4}(c_H^2 - c_L^2)}{\frac{1}{2}x(c_H - c_L) - C_e} =: \alpha_{RS}^{IC}(C_e, c_H, c_L, x) = \alpha_{RS}^{IC}, \tag{29}$$

[21] In contrast to the technical constraint $q_L \geq 0$, α_{RS}^{PC} is not bound to be non-negative as it does not lead to misleading results.

where $\alpha_{RS}^{IC}(C_e, c_H, c_L, x)$ has the domain $C_e \in \left[C_e^{PI}, \frac{1}{2}x(c_H - c_L)\right)$, $c_H, c_L, x \in \mathbb{R}^+$, and the range $\alpha_{RS}^{IC} \in (0, 1 - c_H/x]$. Furthermore,

$$\frac{\partial \alpha_{RS}^{IC}(C_e, c_L, x)}{\partial C_e} = -\frac{\frac{1}{4}(c_H^2 - c_L^2)}{\left(\frac{1}{2}x(c_H - c_L) - C_e\right)^2} < 0, \tag{30}$$

$$\frac{\partial^2 \alpha_{RS}^{IC}(C_e, c_L, x)}{\partial C_e^2} = -\frac{\frac{1}{2}(c_H^2 - c_L^2)}{\left(\frac{1}{2}x(c_H - c_L) - C_e\right)^3} < 0, \tag{31}$$

$\partial \alpha_{RS}^{IC}(C_e, c_L, x)/\partial c_L < 0$, and $\partial \alpha_{RS}^{IC}(C_e, c_L, x)/\partial x > 0$. Note that for all $C_e \in [0, C_e^{PI})$ $f^H(\alpha_{RS}; c_H, x) < 0$ and hence the incentive compatibility constraint is identical to the participation constraint. The incentive compatibility constraint imposes $\alpha_{RS} \leq \alpha_{RS}^{IC}$. Furthermore, note that $\alpha_{RS}^{PC} \geq \alpha_{RS}^{IC}$ if $\alpha_{RS}^{IC}(C_e, c_H, c_L, x)$ is defined:

$$\alpha_{RS}^{PC} \geq \alpha_{RS}^{IC} \quad \Leftrightarrow \quad C_e \geq C_e^{PI}. \tag{32}$$

The following Lemma summarizes the analysis above:

Lemma 2. *Unconstrained maximization yields $\alpha_{RS}^{UC} \in (0,1)$. The participation constraint imposes $\alpha_{RS} \leq \alpha_{RS}^{PC}$. The incentive compatibility constraint i) imposes $\alpha_{RS} \leq \alpha_{RS}^{IC}$, and ii) is defined for $C_e \geq C_e^{PI} > 0$ only.*

Proof. Follows from the analysis in this subsection. □

With the preceding analysis, the following Lemma (which contentwise corresponds to Lemma 1):

Lemma 1'. *If $\alpha_{RS}^{UC} > 1 - c_H/x$ and*

a) for sufficiently low levels of C_e, particularly $C_e \in [0, C_e^{UP}]$, α_{RS}^{UC} applies.

b) for intermediate levels of C_e, particularly $C_e \in [C_e^{UP}, C_e^{PI}]$, α_{RS}^{PC} applies.

c) for positive and sufficiently high levels of C_e, particularly $C_e \in [C_e^{PI}, \infty)$, α_{RS}^{IC} applies.

Furthermore, if $\alpha_{RS}^{UC} < 1 - c_H/x$ and

d) for sufficiently low levels of C_e, particularly $C_e \in [0, C_e^{UI}]$, α_{RS}^{UC} applies.

e) for positive and sufficiently high levels of C_e, particularly $C_e \in [C_e^{UI}, \infty)$, α_{RS}^{IC} applies.

Proof. a) With $C_e = 0$, $\alpha_{RS}^{PC} = 1 - c_L/x$ (see (26)), and $\alpha_{RS}^{IC}(C_e, c_H, c_L, x)$ is not defined (see domain for $\alpha_{RS}^{IC}(C_e, c_H, c_L, x)$ in (29)). α_{RS}^{UC} is defined by (20). Let

$$z(\alpha_{RS}, c_L, x) := f^{UCL}(\alpha_{RS}) - f^{UCR}(c_L, x). \tag{33}$$

Substituting $\alpha_{RS} = 1 - c_L/x$ into $z(\alpha_{RS}, c_L, x)$ yields

$$\frac{2 - c_L/x}{(c_L/x)^3} - \left(\frac{x}{c_l}\right)^2 > 0 \Leftrightarrow x > c_L,$$

where the second inequality is true by assumption ($c_L < c_H < x$). Together with $\partial z(\alpha_{RS}, c_L, x)/\partial \alpha_{RS} > 0$ it follows that $\alpha_{RS}^{UC}(c_L, x)\big|_{C_e=0}$ is always smaller than $\alpha_{RS}^{PC}(C_e, c_L, x)\big|_{C_e=0} = 1 - c_L/x$. However, with $\partial \alpha_{RS}^{PC}(C_e, c_L, x)/\partial C_E < 0 \; \forall \; C_e > 0$, $\alpha_{RS}^{PC} \in [-\infty, 1 - c_L/x]$ (for both see properties of (26)), and $\alpha_{RS}^{UC} \in (0, 1)$ (see properties of (20)), there exists a unique $C_e = C_e^{UP}$ such that $\alpha_{RS}^{PC} = \alpha_{RS}^{UC}$. Note that with $\alpha_{RS}^{UC} > 1 - c_H/x$, $f^H(\alpha_{RS}; c_H, x) < 0$ and therefore the incentive compatibility constraint is identical to the participation constraint at α_{RS}^{UC}. Hence, for all $C_e \in [0, C_e^{UP}]$ both constraints are fulfilled and profit is maximized by choosing $\alpha_{RS} = \alpha_{RS}^{UC}$.

b) Note that $\alpha_{RS}^{IC}(C_e, c_H, c_L, x)$ is defined for $C_e \geq C_e^{PI}$ only. With $C_e = C_e^{PI}$, $\alpha_{RS}^{IC} = 1 - c_H/x$, and with $C_e > C_e^{PI}$, $\alpha_{RS}^{IC} < 1 - c_H/x$ (see (29)). As follows from a), if C_e is sufficiently small, α_{RS}^{UC} applies. However, with $\partial \alpha_{RS}^{PC}(C_e, c_L, x)/\partial C_E < 0 \; \forall \; C_e > 0$, $\alpha_{RS}^{PC} \in [-\infty, 1 - c_L/x]$ (for both see (26)), and $\alpha_{RS}^{UC} \in (0, 1)$ (see (20)), there exists a unique $C_e = C_e^{UP}$ such that $\alpha_{RS}^{PC} = \alpha_{RS}^{UC}$. Furthermore, for all $C_e \in (C_e^{UP}, C_e^{PI}]$ α_{RS}^{PC} applies as the incentive compatibility constraint is fulfilled (it is identical to the participation constraint, see properties of (29)) and α_{RS}^{UC} is larger than α_{RS}^{PC} and hence the participation constraint is not fulfilled with $\alpha_{RS} = \alpha_{RS}^{UC}$.

c) With $C_e \geq C_e^{PI}$, $\alpha_{RS}^{IC}(C_e, c_H, c_L, x)$ is defined (see (29)) and is smaller than α_{RS}^{PC} (see (32)) and α_{RS}^{UC} (follows with $\alpha_{RS}^{UC} > 1 - c_H/x$). Therefore, with $\alpha_{RS} = \alpha_{RS}^{IC}$, the participation constraint is fulfilled (holds with (in)equality if $C_e = (>) C_e^{PI}$) and the incentive compatibility constraint is just fulfilled. With $\alpha_{RS} = \alpha_{RS}^{UC}$ the incentive compatibility constraint would not be fulfilled, i.e. $f^H(\alpha_{RS}; c_H, x) < f^L(\alpha_{RS}, C_e; c_L, x)$. With $\alpha_{RS} = \alpha_{RS}^{PC}$ the incentive compatibility constraint would not be fulfilled if $C_e > C_e^{PI}$, and just be fulfilled if $C_e = C_e^{PI}$.

d) and e) Note that for $C_e = C_e^{PI}$, $\alpha_{RS}^{PC} = \alpha_{RS}^{IC} = 1 - c_H/x$ (see (26) and (29)) and for $C_e > C_e^{PI}$, $\alpha_{RS}^{PC} > \alpha_{RS}^{IC}$ (see (32)). As a consequence, with $\alpha_{RS}^{UC} < 1 - c_H/x$ if any of the constraints, the incentive compatibility is binding, and hence α_{RS}^{IC} applies. Furthermore, with $\partial\alpha_{RS}^{IC}(C_e, c_H, c_L, x)/\partial C_E < 0 \ \forall\ C_e \in [C_e^{PI}, \frac{1}{2}x(c_H - c_L))$ and with $\alpha_{RS}^{IC} \in [0, 1 - c_H/x]$, there exists a unique $C_e = C_e^{IU}$ such that $\alpha_{RS}^{IC} = \alpha_{RS}^{UC}$. Therefore, with $C_e \leq C_e^{IU}$, $\alpha_{RS} = \alpha_{RS}^{UC} \leq \alpha_{RS}^{IC}$; with $C_e \geq C_e^{IU}$, $\alpha_{RS} = \alpha_{RS}^{IC} \leq \alpha_{RS}^{UC}$.

□

A.4 Proof of Proposition 4

The analysis in this subsection uses results derived in subsection A.3 in this Appendix and summarized in Lemmata 1 and 2. Furthermore, we define NPV^{RS} as (piecewise-defined) function of the exogenous variables:

$$NPV^{RS}(c_H, c_L, x, C_e, C_I) = \begin{cases} NPV_l^{RS}(c_H, c_L, x, C_e, C_I) & if \quad \alpha_{RS}^{UC} \leq 1 - c_H/x; \\ NPV_h^{RS}(c_H, c_L, x, C_e, C_I) & if \quad \alpha_{RS}^{UC} \geq 1 - c_H/x, \end{cases} \tag{34}$$

where $NPV_l^{RS}(c_H, c_L, x, C_e, C_I)$ and $NPV_h^{RS}(c_H, c_L, x, C_e, C_I)$ are defined as:

$$NPV_l^{RS}(c_H, c_L, x, C_e, C_I) = \begin{cases} NPV_{UC}^{RS}(c_H, c_L, x, C_I) & if \quad 0 \leq C_e \leq C_e^{UP}; \\ NPV_{PC}^{RS}(c_H, c_L, x, C_e, C_I) & if \quad C_e^{UP} < C_e \leq C_e^{PI}; \\ NPV_{IC}^{RS}(c_H, c_L, x, C_e, C_I) & if \quad C_e > C_e^{PI}, \end{cases} \tag{35}$$

and

$$NPV_h^{RS}(c_H, c_L, c, C_e, C_I) = \begin{cases} NPV_{UC}^{RS}(c_H, c_L, x, C_I) & if \quad 0 \leq C_e \leq C_e^{UI}; \\ NPV_{IC}^{RS}(c_H, c_L, x, C_e, C_I) & if \quad C_e > C_e^{UI}, \end{cases} \tag{36}$$

To show that the set $\mathbb{C}_e^{IS} = \left\{C_e \mid NPV^{RS} > NPV^E\right\}$ is non-empty for $C_I > 0$, the following steps are conducted:

a) It is shown that $NPV^E(\cdot)$ is linearly decreasing in C_e and $NPV^{RS}(\cdot)$ is non-increasing and non-decreasing in C_e for $C_e \leq C_e^{PI}$ and hence when $NPV_{UC}^{RS}(\cdot)$ applies, and strictly decreasing and strictly concave for $C_e > C_e^{PI}$ in the case of $\alpha^{UC} > 1 - c_H/x$ and hence when $NPV^{RS}_{PC}(\cdot)$ and $NPV^{RS}_{IC}(\cdot)$ apply, and for $C_e > C^{IU}_e$ in the case of $\alpha^{UC} < 1 - c_H/x$ and hence when $NPV^{RS}_{IC}(\cdot)$ applies. From that it follows that $NPV^E(\cdot)$ and $NPV^{RS}(\cdot)$ either do not intersect, are tangential for a unique C_e, intersect once for some C_e, or intersect twive in the space $\{C_e, NPV; c_L, c_H, x, C_I\}$.

b) It is shown that if $C_I = 0$ they intersect twice in the positive quadrant in the space $\{C_e, NPV; c_L, c_H, x, C_I\}$ by showing that

   i) $NPV^E(\cdot) > NPV^{RS}(\cdot)$ if $C_e = 0$,

   ii) if $C_I = 0$, there exists a unique C_e such that $NPV^E(\cdot) = NPV^{RS}(\cdot) = 0$, and

   iii) the slope of $NPV^{RS}(\cdot)$ is smaller than the slope of $NPV^E(\cdot)$ at this C_e.

   From that it follows that they intersect twice in the positive quadrant: The graph of $NPV^{RS}(\cdot)$ cuts the graph of $NPV^E(\cdot)$ from below at the first intersection and from above at the second intersection. Hence the set $\mathbb{C}^{IS}_e$ is non-empty.

c) Additionally, with

$$\frac{\partial NPV^E(\cdot)}{\partial C_I} < 0, \quad \frac{\partial NPV^{RS}(\cdot)}{\partial C_I} < 0, \quad \text{and} \quad \frac{\partial^2 NPV^E(\cdot)}{\partial C_I^2} = \frac{\partial^2 NPV^{RS}(\cdot)}{\partial C_I^2} = 0$$

   it follows that for some $C_I \in (0, NPV^{RS}_{UC})$ there exists a subset $\mathbb{C}'^{IS}_e \subseteq \mathbb{C}^{IS}_e$ for that $\mathbb{C}'^{IS}_e = \{C_e \mid NPV^{RS} \geq 0 \ \wedge \ NPV^E < 0\}$.

Proof. a) $NPV^E(\cdot)$ is linearly downward sloping in the space $\{C_e, NPV; c_L, c_H, x, C_I\}$:

$$\frac{dNPV^E}{dC_e} = \frac{\partial NPV^E(\cdot)}{\partial C_e} = -\frac{(x - c_L)^2}{(x - c_L)^2 - (x - c_H)^2} < 0 \quad \text{if} \quad c_L < c_H < x. \tag{37}$$

For sufficiently low levels of C_e and hence if constraints are non-binding, $NPV^{RS}(\cdot) = NPV^{RS}_{UC}(\cdot)$, NPV^{RS} is independent of C_e,

$$\frac{dNPV^{RS}}{dC_e} = \frac{\partial NPV^{RS}_{UC}(c_H, c_L, x, C_I)}{\partial C_e} = 0$$

NPV^{RS} is decreasing in C_e for sufficiently high levels and hence with binding constraints (see Lemma 1). It is strictly concave for sufficiently high levels of C_e, particularly $C_e \in (C^{PI}_e, \infty)$ if $\alpha^{UC} > 1 - c_H/x$, and $C_e \in (C^{UI}_e, \infty)$ if $\alpha^{UC} < 1 - c_H/x$, in the space $\{C_e, NPV; c_L, c_H, x, C_I\}$. If $NPV_{PC}^{RS}(\cdot)$ applies:

$$\frac{dNPV^{RS}}{dC_e} = \frac{\partial NPV_{PC}^{RS}(\cdot)}{\partial C_e} = \underbrace{\frac{\partial \alpha_{PC}^{RS}(C_e, c_L, x)}{\partial C_e}}_{<0,\text{ see (27)}} \cdot$$

$$\underbrace{\left[R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x) + \alpha_{PC}^{RS}(C_e, c_L, x) \cdot \frac{\partial R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x)}{\partial \alpha_{PC}^{RS}(C_e, c_L, x)} \right]}_{>0 \Leftrightarrow x > c_L/(1-\alpha) \Leftrightarrow q_L(\alpha_{RS}, c_L, x) > 0} < 0,$$

and

$$\frac{d}{dC_e}\frac{dNPV^{RS}}{dC_e} = \frac{\partial^2 NPV_{PC}^{RS}(\cdot)}{\partial C_e^2} = \underbrace{\frac{\partial^2 \alpha_{PC}^{RS}(C_e, c_L, x)}{\partial C_e^2}}_{<0,\text{ see (28)}} \cdot$$

$$\underbrace{\left[R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x) + \alpha_{PC}^{RS}(C_e, c_L, x) \cdot \frac{\partial R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x)}{\partial \alpha_{PC}^{RS}(C_e, c_L, x)} \right]}_{>0 \Leftrightarrow x > c_L/(1-\alpha) \Leftrightarrow q_L(\alpha_{RS}, c_L, x) > 0}$$

$$+ \underbrace{\frac{\partial^2 \alpha_{PC}^{RS}(C_e, c_L, x)}{\partial C_e^2}}_{<0,\text{ see (28)}} \left[\underbrace{\frac{\partial R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x)}{\partial \alpha_{PC}^{RS}(C_e, c_L, x)} \cdot \frac{\partial \alpha_{PC}^{RS}(C_e, c_L, x)}{\partial C_e}}_{>0,\text{ see (27) and (38)}} \right.$$

$$+ \underbrace{\frac{\partial \alpha_{PC}^{RS}(C_e, c_L, x)}{\partial C_e} \cdot \frac{\partial R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x)}{\partial \alpha_{PC}^{RS}(C_e, c_L, x)}}_{>0,\text{ see (27) and (38)}}$$

$$\left. + \underbrace{\frac{\partial^2 R_L(\alpha_{PC}^{RS}(C_e, c_L, x), c_L, x)}{\partial \alpha_{PC}^{RS}(C_e, c_L, x)^2} \cdot \frac{\partial \alpha_{PC}^{RS}(C_e, c_L, x)}{\partial C_e} \cdot \alpha_{PC}^{RS}(C_e, c_L, x)}_{>0,\text{ see (27) and (39)}} \right] < 0,$$

noting that

$$\frac{\partial R_L(\alpha_{RS}, c_L, x)}{\partial \alpha_{RS}} = -\frac{c_L^2}{2\,(1-\alpha_{RS})^3} < 0, \tag{38}$$

$$\frac{\partial^2 R_L(\alpha_{RS}, c_L, x)}{\partial \alpha_{RS}^2} = -\frac{3c_L^2}{2\,(1-\alpha_{RS})^4} < 0. \tag{39}$$

If $NPV_{IC}^{RS}(\cdot)$ applies:

$$\frac{dNPV^{RS}}{dC_e} = \frac{\partial NPV_{IC}^{RS}(\cdot)}{\partial C_e} = -\left[\frac{x^2}{4} - \frac{4c_L^2}{\left(c_H^2 - c_L^2\right)^2}\left(\frac{x}{2}\left(c_H - c_L\right) - C_e\right)^2\right] \cdot \frac{\frac{1}{4}\left(c_H^2 - c_L^2\right)}{\left(\frac{1}{2}x\left(c_H - c_L\right) - C_e\right)^2} + \frac{8c_L^2}{\left(c_H^2 - c_L^2\right)^2}\left(\frac{x}{2}\left(c_H - c_L\right) - C_e\right)\left[1 - \frac{\frac{1}{4}\left(c_H^2 - c_L^2\right)}{\frac{x}{2}\left(c_H - c_L\right) - C_e}\right] < 0, \tag{40}$$

and

$$\frac{d}{dC_e}\frac{dNPV^{RS}}{dC_e} = \frac{\partial^2 NPV_{IC}^{RS}(\cdot)}{\partial C_e^2} = \underbrace{\frac{\partial^2 \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}{\partial C_e^2}}_{<0,\text{ see (31)}} \cdot \underbrace{\left[R_L(\alpha_{IC}^{RS}(C_e, c_H, c_L, x), c_L, x) + \alpha_{IC}^{RS}(C_e, c_H, c_L, x) \cdot \frac{\partial R_L(\alpha_{IC}^{RS}(C_e, c_H, c_L, x), c_L, x)}{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}\right]}_{>0 \Leftrightarrow x > c_L/(1-\alpha) \Leftrightarrow q_L(\alpha_{RS}, c_L, x) > 0}$$
$$+ \underbrace{\frac{\partial^2 \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}{\partial C_e^2}}_{<0,\text{ see (31)}}\left[\underbrace{\frac{\partial R_L(\alpha_{IC}^{RS}(C_e, c_H, c_L, x), c_L, x)}{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)} \cdot \frac{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}{\partial C_e}}_{>0,\text{ see (30) and (38)}}\right.$$
$$+ \underbrace{\frac{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}{\partial C_e} \cdot \frac{\partial R_L(\alpha_{IC}^{RS}(C_e, c_H, c_L, x), c_L, x)}{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}}_{>0,\text{ see (30) and (38)}}$$
$$\left. + \underbrace{\frac{\partial^2 R_L(\alpha_{IC}^{RS}(C_e, c_H, c_L, x), c_L, x)}{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)^2} \cdot \frac{\partial \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}{\partial C_e} \cdot \alpha_{IC}^{RS}(C_e, c_H, c_L, x)}_{>0,\text{ see (30) and (39)}}\right] < 0.$$

b) i) Follows immediately with the proof of Proposition 2, see subsection A.4 in the Appendix.

ii) Note that with $C_e = \Delta\pi$, $NPV^E(\cdot) = 0$ if $C_I = 0$:

$$NPV^E(\cdot)\big|_{C_I=0} = \frac{\Delta\pi - C_e}{\Delta\pi}\pi_L^m = 0 \Leftrightarrow C_e = \Delta\pi.$$

Furthermore, with $C_e = \Delta\pi = \frac{1}{2}x(c_H - c_L) - \frac{1}{4}(c_H^2 - c_L^2)$, $NPV^{RS} = NPV_{IC}^{RS}(\cdot) = 0$ if $C_I = 0$:

$$NPV_{IC}^{RS}(\cdot)\big|_{C_e=\Delta\pi, C_I=0} = \left(1 - \frac{\frac{1}{4}(c_H^2 - c_L^2)}{\frac{1}{4}(c_H^2 - c_L^2)}\right)\frac{x^2 - c_L\left(\frac{\frac{1}{4}(c_H^2 - c_L^2)}{\frac{1}{4}(c_H^2 - c_L^2)}\right)^2}{4} = 0.$$

iii) Combining (37) and (40) yields:

$$\begin{aligned}
\left.\frac{dNPV^E}{dC_e}\right|_{C_e=\Delta\pi} &= -\frac{(x-c_L)^2}{(x-c_L)^2-(x-c_H)^2} > -\frac{x^2-c_L^2}{c_H^2-c_L^2} = \left.\frac{dNPV^{RS}}{dC_e}\right|_{C_e=\Delta\pi} \\
\Leftrightarrow &\quad (x^2-c_L^2)(c_H-c_L) > (c_H^2-c_L^2)(x-c_L) \\
\Leftrightarrow &\quad x > c_H.
\end{aligned}$$

Therefore the slope of $NPV^{RS}(\cdot)$ is indeed smaller than the slope of $NPV^E(\cdot)$ if $C_e = \Delta\pi = \frac{1}{2}x(c_H - c_L) - \frac{1}{4}(c_H^2 - c_L^2)$.

□

A.5 Proof of Proposition 5

Proof. Note that for $0 \leq C_e < \widetilde{C}_e$, both, equity financing and revenue-sharing financing are possible (see discussion of Figure 2, and subsection A.4 in the Appendix). Consumer surplus in the low cost scenario, with quantity q_L^*, is given by

$$CS_L = \int_0^{q_L^*} p(q) - p(q_L^*)\ dq = \frac{(x - p(q_L^*))\, q_L^*}{2}. \tag{41}$$

Note that the entrepreneur's choice of the optimal quantity is not affected by α_E, as the entrepreneur's optimization problem reads:

$$\max_{q_L \geq 0} \left\{(1-\alpha_E)\left(p(q_L)q_L - c_L q_L\right)\right\} = (1-\alpha_E) \max_{q_L \geq 0} \left\{(p(q_L)q_L - c_L q_L)\right\}.$$

In contrast, as derived above, the entrepreneurs quantity choice depends on α_{RS}, see (7). She chooses to produce the monopoly quantity if $\alpha_{RS} = 0$ (see (7)), and reduces the quantity with increasing α_{RS} (follows immediately from differentiation of (7)). Together with $dCS_L/dq_L^* > 0$ (follows from differentiation of (41)) it follows that consumer surplus is greater under equity financing than under revenue-sharing financing if $0 \leq C_e < \widetilde{C}_e$, i.e. if both financing options are feasible. In contrast, with $\widetilde{C}_e \leq C_e < \overline{C}_e$ only revenue-sharing financing is possible and hence consumer surplus is greater under revenue-sharing financing.

Cumulative net payout in the low cost scenario is defined as

$$CP_L = [(1-\alpha_{RS})p(q_L)q_L - c_L q_L - C_e] + [\alpha_{RS} p(q_L)q_L - C_I]. \tag{42}$$

Under equity financing, the entrepreneur chooses the monopoly quantity and hence cumulative net payout is given by

$$CP_{L,E} = \left(\frac{x - c_L}{2}\right)^2 - C_e - C_I. \tag{43}$$

In contrast, under revenue-sharing financing, the entrepreneur chooses a smaller quantity than the monopoly quantity (as argued above). As a result cumulative net payout, given by

$$CP_{L,RS} = \frac{x^2 - 2xc_L + \left(\frac{c_L}{1-\alpha_{RS}}\right)^2 (1 - 2\alpha_{RS})}{4} - C_e - C_I =: CP_{L,RS}(\alpha_{RS}, C_e, C_I, c_L, x), \tag{44}$$

is equal to cumulative net payout under equity financing if $\alpha_{RS} = 0$, and lower if $\alpha_{RS} > 0$ (follows immediately with (43) and (44)).

In contrast, with $\widetilde{C}_e \leq C_e < \overline{C}_e$ only revenue-sharing financing is possible and hence cumulative net payout is greater under revenue-sharing financing. □

A.6 Conversion of unlimited maturity revenue-sharing contract into equity share

The purpose of this subsection is to demonstrate that the VC indeed converts (as it is profit maximizing) a revenue-sharing contract into an equity share if costs of effort decrease significantly over time.

Consider a multi-periods (discrete time) setting of the model outlined above. The VC has to invest C_I once, the entrepreneur has to invest time dependent effort costs $C_{e,t}$ each period in order to produce at c_L. Let time dependent effort costs be given as

$$C_{e,t} = f^C(z) \geq 0, \tag{45}$$

with $\Delta f^C(z)/\Delta z < 0$, and $\lim z \to \infty \Rightarrow f^C(z) = C_e^{min} < \underline{C}_e$, where z corresponds to the number of periods between 0 and t in which the entrepreneur has (exerted effort and) produced at c_L. Again, we assume that the VC has to ensure the entrepreneur produces with lower marginal costs. This implies that the VC only offers contracts that fulfill the participation and the incentive compatibility constraint in each period.[22] Therefore, the preceding analysis, particularly Section 2, can be interpreted as the one-period outcome of a contract described above. To keep the analysis as simple as possible, we will keep notation and definitions used and derived above, in particular NPV^E and NPV^{RS}, although these terms in the multi-periods framework refer to the cash flow to the VC in one period. We assume the VC can offer a revenue-sharing contract with a one-sided option of converting his revenue share into an equity share. Multi-periods net present value (MNPV) of the VC between period 0 and ∞ is given by

$$MNPV = \sum_{t=0}^{\infty} \delta^t I^{RS}(t) NPV^{RS}(c_H, c_L, x, f^C(t), C_I) + \delta^t I^E(t) NPV^E(c_H, c_L, x, f^C(t), C_I), \tag{46}$$

with $0 \leq \delta \leq 1$ as discount factor of the VC, and $I^j(t)$, $j \in \{RS, E\}$, as indicator functions where $I^{RS}(t) = 1$, $I^E(t) = 0$ if in period t cash flows to the VC come from a revenue-sharing contract and $I^{RS}(t) = 0$, $I^E(t) = 1$ if cash flows come from equity share, respectively. Then, the VC offers at $t = 0$ a revenue-sharing contract if $C_{e,t}$ is above the threshold level $\overline{C}_e$. In each period the VC considers to convert his revenue-sharing contract taking into account current and future effects on his cash flow. The VC chooses to convert his revenue-sharing contract into an equity share in period $t \geq 1$ if the following inequality holds in t but is not fulfilled in $t-1$:

$$\begin{aligned} & NPV^E(c_H, c_L, x, f^C(t), C_I) - NPV^{RS}(c_H, c_L, x, f^C(0), C_I) \\ & > \frac{\delta}{1-\delta} \left[NPV^E(c_H, c_L, x, f^C(t+1), C_I) - NPV^E(c_H, c_L, x, f^C(t), C_I) \right]. \end{aligned} \tag{47}$$

The left hand side of (47) corresponds to the effect of a conversion of the revenue-sharing contract in t on current profits of the VC. Note that for all $C_e \lesseqqgtr C_e^S$ it follows that $\left[NPV^E(c_H, c_L, x, f^C(t), C_I) - NPV^{RS}(c_H, c_L, x, f^C(0), C_I) \right] \gtreqqless 0$, where C_e^S is implicitly de-

[22] We implicitly assume that the entrepreneur discounts future profits sufficiently strongly. In an infinite period setting, the existence of a one-sided option to convert the revenue-sharing contract into an equity share (as described below) is sufficient to avoid the incentive compatibility problem if the entrepreneur's discount rate is sufficiently small.

fined by

$$NPV^{RS}(f^C(0)) \equiv NPV^E(C_e^S). \tag{48}$$

The ride hand side of (47) corresponds to the effect of a conversion of the revenue-sharing contract in t (as opposed to a conversion in $t+1$) on future profits of the VC. Note that it is always non-negative. With $\lim t \to \infty$ and if the entrepreneur exerts effort $\forall t \geq 0$ $\Rightarrow f^C(t) = C_e^{min}$. With that, and $NPV^E > NPV^{RS}$ if $C_e = C_e^{min} < \underline{C}_e$ it follows that there exists a period $t \geq 1$ such that (47) holds with strict inequality and hence the VC will convert his revenue-sharing contract into an equity share (for $\lim t \to \infty$, the left hand side of (47) is positive and the right hand side of (47) is equal to zero).