Profitable Entry into an Unprofitable Market

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Abstract

This paper shows how market entry into an unprofitable market can be profitable for a firm. A firm’s expansion into a new market can have a beneficial feedback effect for that firm in its “old market”. By entering into a new market, the firm increases its produced quantity and has higher incentives to invest in process R&D. This is a credible signal to the competitors that the firm will be more aggressive in its R&D investments. This weakens the competitors since they scare off and invest less in process R&D. This feedback effect of expanding in foreign markets increases the profits of the expanding firm in its “old market” and if this profit gain exceeds the losses through market entry, then the market entry is profitable for the firm. I also consider how the results change under Bertrand vs Cournot regime and how results change if price discrimination is possible or not. Beside that I show how higher R&D costs or lower demand in a market can lead to lower profits of one firm, but higher profits of the other firm.

Keywords: research and development, price discrimination, product differentiation, process innovation, interbrand competition, strategic commitment, separated markets

JEL Classification Codes: L13, D43, O30

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1. Introduction

One of the most important instruments that firms use to grow and increase their sales and/or profits, is expanding into new markets. Expansion into a new market can have several benefits for a firm: it can benefit from economies of scale, diversify its product portfolio, increase its revenues, reduce its risks and so forth. However, any market entrance usually needs some irreversible initial investments for advertising, building up a distribution channel etc which might be higher than the benefits and deter the firms from penetrating into new markets. Usually firms decisions whether to enter into a new market or not is based on whether the profits through market entrance covers the initial costs or not. Many market niches are served just because the costs for a firm to use an existing product and modify it for the new market is cheap enough to be less than the additional revenues through the new market. For example if we consider a car producer, the costs of developing a platform is a substantial part of the fixed costs of developing a new car model. Thus, once a car producers has spent the costs of developing a platform for one specific model, they use this platform for different other models which target other consumers.\textsuperscript{1} How ever some niches are not served at all because firms can not cover the irreversible initial costs for entering into those markets. This paper shows that even though a market niche or geographic market is too small or too competitive for a firm to cover the initial investments of market entry, it can still be profitable for the firm to enter into that market.

This paper focuses on another aspect of market entrance: As a firm - which I will call from now on firm 1 - conquers a new market, it has some irreversible initial investments on the one hand and higher total sales volume on the other hand. The higher sales volume leads to higher incentives of firm 1 for investments in marginal cost reducing process R&D (hereafter simply called R&D investments). Thus making irreversible initial investments for advertising, distribution channel and so forth to penetrate into a new market, can be used as a creditable signal for higher R&D investments to the competitor in the old market – hereafter called firm 2. If firm 1 and firm 2 have the same production technology and thus the same pre-R&D-marginal-costs, firm 2 faces a weaker position in the “race” for investing in process R&D. In this case the optimal strategy for firm 2 is to invest less in process R&D, set higher prices and focus on its loyal customers in a differentiated market. Hence, by expanding into a new market, firm 1 increases its profits in the old market through weakening its competitors and can gain or ensure a more dominant position in the old

\textsuperscript{1} For example BMW uses the same platform for its SUV model X3 and the limousine 3 series which aim different consumer groups.
market. This strategic motive in the market entry of the firms in new markets has gone unnoticed in this form in the previous literature to my knowledge.

The paper also considers the influence of different competition regimes on the results and show that no matter under Bertrand or Cournot regime, even though a market entrance is not profitable per se, it can be better for a firm to enter into that market. This is of course always the case when the profit gains in the old market are higher than the losses in the new market. Even if price discrimination is not possible, profits of firm 1 can also increase through market entrance into a per se unprofitable new market. However, this phenomena is caused here for a different reason: since firm 1 has to set the same price in the new market (where it is a monopolist) and the old market (where it competes with firm 2), firm 1 has incentives to increase its prices in the old competitive market in order to not harm its profits in the new monopoly market. Since firm 2 which is firm 1’s competitor in market $M$ anticipates this, it also sets higher prices in the old market. Thus market entry into the new market softens the competitors pricing behavior in the old market and leads this way to higher profits in the old market. Another points that will be considered in this paper is how results change if the firms are asymmetric, specially when firm 1 has a cost disadvantage.

The R&D literature is based on the pioneering works of Schumpeter (1942) and Arrow (1962). Some of the central questions are how changes in different variables influence the R&D incentives of the firms and R&D investments depend on the regime of competition such as Cournot or Bertrand competition. Several papers compare differentiated Bertrand vs. Cournot competition and find out that prices are lower (and hence output and welfare are higher) under Bertrand competition than under Cournot competition with differentiated goods.2 The different assumptions in our model lead to contrary results under certain circumstances. A number of papers such as Qiu (1997), Breton et al. (2004), and Hinloopen and Vandekerckhove (2007) show that output and welfare effects of R&D are higher under Bertrand competition if interbrand competition is not very tough. There is also a number of papers considering the relation between degree of competition and intensity of R&D investments empirically. Bertschek (1995) shows that tougher competition (represented by higher imports) leads to more investments in innovation.

Other empirical papers such as Link (1980) and Acs and Audretsch (1987) test the two essential Schumpeterian theorems – that innovation is stimulated through the existence of large firms and imperfect competition. Since the market entry of firm 1 causes an asymmetry in the old market and therefore firm 1 become the dominant firm in that market, our model is also related in the

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2 See for example Singh and Vives (1984) and Vives (1985)
broader sense to the literature which treat also dominant firm and Schumpeterian theories. Acs and Audretsch (1987) for example find that large firms tend to have the relative innovative advantage in industries which are capital-intensive, concentrated and produce a differentiated good. As we see in our model the specific structure with two separated markets connected through a firm which is active in both markets leads to an opposite result. The innovative advantage of firm 1 increases with the degree of business stealing effect which in turn is higher the less differentiated the goods are.

Brander and Spencer (1983) show in their model the strategic commitment effect of R&D investments in a symmetric two-stage Nash duopoly model. In this paper the expansion of firm 1 into a new market has also commitment component for the R&D investments of firm 1. Our model considers different asymmetries and also considers the case where different asymmetries appear at the same time. Buehler and Schmutzler (2008) show in their model of a bilateral duopoly that vertical integration increases own investment and intimidates competitors from investing in process R&D. Buehler and Schmutzler show an intimidation effect on R&D investments of the competitor which is similar to the effect shown in this model. However the intimidation effect in this model is due to market entry into a new market, while in Buehler and Schmutzler the intimidation effect is due to vertical integration.

Since the market entrance causes asymmetric R&D investments of the firms and thus different post R&D marginal costs and different prices of the firms, this paper is also related to dominant firm literature. Most of the dominant firm papers are usually based on Forchheimer's model of price leadership where there is one dominant firm and one or several fringe firm(s). The papers regarding dominant firm models are considering various aspects of competition. Ono (1982) considers a model where the dominant firm first sets the market price, the fringe firms set their outputs and the dominant firm serves the residual demand. Deneckere and Kovenock (1992) present a model where all firms use the price as strategic variable in a Bertrand-Edgeworth duopoly game. Tasnádi (2000) extends the model of Bertrand-Edgeworth price setting game to a game with infinitely many firms. In a later model (2010) he also considers a quantity setting game based on Forchheimer. Wied-Nebbeling (2007) considers degree of heterogeneity in a Bertrand setting. She considers one dominant firm with constant marginal costs and two fringe firms with increasing marginal costs. She shows in her model that fringe firms are not necessarily better off in a heterogeneous market. Other works – such as Gaskins (1970) – consider the dynamic limit price.

3 For an empirical paper on price leadership see Rassenti and Wilson (2004)
4 In an other paper Tasnádi (2004) shows that in a simple price-setting game with a large firm and many fringe firms the large firm does not accept the role of the price leader.
setting problem of the dominant firm and the short-term versus long-term price setting behavior of the dominant firm.\textsuperscript{5}

In this paper the difference between dominant and fringe firm is not necessarily caused by different pre R&D cost functions but by different amounts of investments in process R&D through the market entrance of a firm into a new (and eventually unprofitable) market. This market entry makes the firm become a dominant firm. The model considers how factors such as degree of product homogeneity, costs of R&D and the size of the markets influence the profits and R&D investments of the dominant and fringe firm. Beside the attribution to the literature mentioned above, the model reinforces some elements in the literature such as:\textsuperscript{6}

- investments are strategic substitutes;
- the greater ceteris paribus the opponent's cost reduction is, the lower are the incentives of a given firm to invest
- the lower a firm's own state of post R&D marginal costs, the greater are the incentives to invest.

In the next section we will introduce into the model and consider the results under Bertrand competition with price discrimination. Chapter 3 shows the results under Bertrand competition when price discrimination is possible and chapter 4 compares these outcomes with the case where price discrimination is not possible. In chapter 5 we will consider the results under Cournot regime and chapter 6 concludes.

2. The model

I assume two separated markets, $M$ and $N$, which differ in size. Both markets are represented by the same representative consumer, where the difference in market size arises from the different number of consumers in each market. There are $m$ consumers in market $M$, and $n$ consumers in market $N$, where $m > n$. To be more precise I assume that market $N$ is small enough to be unprofitable per se for a firm to enter that market, thus the fix costs of entry can not be covered.\textsuperscript{7}

\textsuperscript{5} See for example Cherry (2000). This subject is also mentioned in some Industrial Organization books such as Martin (1994), Shepherd (1997) and Tirole (1988).

\textsuperscript{6} For example Athey and Schmutzler (2001) show these points in a dynamic oligopoly model with firms which are market leaders through low costs or high quality.

\textsuperscript{7} However, I assume that market $N$ is big enough to be profitable for firm 1 if one adds the profits that firm 1 makes in market $M$ through the intimidation effect of serving market $N$. 

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There exist two firms, 1 and 2, where firm 1 produces good 1 and firm 2 produces good 2. The products 1 and 2 are substitutes and the degree of substitutability among them is represented by $\gamma$, where $\gamma$ can vary among the wide range of independent goods ($\gamma = 0$) and perfect substitutes ($\gamma = 1$). Both firms are active in market $M$. I assume that one of the firms – hereafter per assumption firm 1 – enters into market $N$ first, and the other firm – per assumption firm 2 – would stay out of market $N$ because market $N$ is too small to be profitable for firm 2, even if we consider the intimidation effect mentioned above.

The timing of the game is the following. In period $t_0$ the nature decides the degree of substitutability among the goods of firms 1 and 2.\footnote{I assume that $\gamma$ is an exogenous parameter for the following reason: Even if firms influence the degree of product differentiation of their products by investing in marketing campaigns, product design etc, they still can not control how substitutable the product at the end will be for consumers comparing to other products in the market.} In period $t_1$ firm 1 decides whether it wants to enter into market $N$ or not. In period $t_2$ both firm 1 and firm 2 decide whether they want to remain in market $M$ or exit the market, knowing their own and competitor's original cost functions and degree of product substitutability $\gamma$. In Period $t_3$ firms decide how much they want to invest in R&D. In period $t_4$ they set their final prices/quantities depending on the chosen R&D levels under Bertrand/Cournot regime and sell their products.

If firm 1 decides to enter into market $N$, its market entrance goes along with some fix investment $F$. The structure of the model in that case is illustrated below:
Bertrand competition whereby price discrimination is possible.

3. Bertrand Competition with price discrimination

In this chapter, I allow firm 1 to set different prices in markets $M$ and $N$. I assume that firms compete à la Bertrand and the representative consumer in both markets have a linear quadratic utility function which has the form $U = a(q_1 + q_2) - (q_1^2 + q_2^2)/2 - \gamma q_1 q_2$. As we see in the utility function, the consumers prefer ceteris paribus high product differentiation in this model. The utility function yields the following linear inverse demand function of each individual in market $M$:

$$p_{Mi} = a - q_{Mi} - \gamma q_{Mj}$$  \hspace{1cm} \forall \, i, j \in \{1, 2\} \text{ where } i \neq j \quad (1)$$

Hereby is $a$ the maximum willingness to pay of the representative consumer. If firm 1 enters into market $N$, the inverse demand function of each consumer in market $N$ is:

$$p_N = a - q_{Na} \quad (2)$$

From (1) and (2) we can derive the quantities sold by firm 1 and firm 2 in market $M$ ($q_{1m}$ and $q_{2m}$) and the quantity sold by firm 1 in market $N$ ($q_{1n}$):

$$q_{im} = \frac{a (1-\gamma) - p_{im} + p_{jm} \gamma}{1-\gamma^2} \quad , \quad \forall \, i, j \in \{1, 2\} \text{ where } i \neq j \quad (3)$$

$$q_{1n} = a - p_{1n}$$

The profit functions of the firms have the following form:

$$\pi_1 = m(p_{1m} - c_1 + x_i)q_{1m} + n(p_{1n} - c_1 + x_i)q_{1n} - \nu x_i^2/2 - 2F \quad (4)$$

$$\pi_2 = m(p_{2m} - c_2 + x_i)q_{2} - \nu x_i^2/2 - F$$

Hereby is $F$ the fix costs of market entry such as advertising and building up a distribution channel and $\nu/2$ represents the costs of R&D. The amount of cost reduction due to R&D investments of firm $i$ is represented by $x_i$. Thus the total costs of R&D investments of firm $i$ is $\nu x_i^2/2$ \hspace{1cm} $\forall \, i \in \{1, 2\}$, whereby $\nu$ is the parameter that represents how costly R&D investments are. The effectiveness of R&D investments is negatively correlated with the parameter $\nu$. The amount of marginal cost reduction is ordinally connected with the R&D investments of the firms. For this reason I use the investments in marginal cost reduction and R&D investments as synonyms in this paper, when I
compare when case are R&D investments higher or lower. The model is solved recursively, hence we start to solve the second stage by inserting (3) into (4):

$$\pi_1 = m \left( p_{1m} - c_1 + x_1 \right) \left( a(1-y) - p_{1m} + p_2 y \right) + n \left( p_{1n} - c_1 + x_1 \right) \left( a - p_{1n} \right) - v x_1^2 - 2 F$$

$$\pi_2 = m \left( p_{2m} - c_2 + x_2 \right) \left( a(1-y) - p_2 + p_1 y \right) - v x_2^2 - F$$

(5)

The marginal costs of firms 1 and 2 in the original state before the R&D investments are represented by $c_1$ and $c_2$ respectively. Building the first order conditions from the profit functions in (5) with respect to the prices and solving them leads:

$$p_{1m} = \frac{1}{2} \left( a + c_1 - x_1 - \frac{m(a - p_2)y}{m + n(1-y^2)} \right)$$

$$p_{2m} = \frac{1}{2} \left( a(1-y) + c_2 - x_2 + p_1 y \right)$$

(6)

$$p_{1n} = \frac{a + c - x_1}{2}$$

Inserting the prices in (6) into each other lead to the prices depending on external variables and on $x_1$ and $x_2$:

$$p_{im} = \frac{a(2-y-y^2) + 2(c_i - x_i) + y(c_j - x_j)}{4 - y^2}, \forall i, j \in \{1, 2\} \quad \text{where} \quad i \neq j$$

(7)

After solving the last stage of the game, the optimal R&D investments of firms 1 and 2 ($x_1$ and $x_2$) can be determined by inserting (7) into (5). Maximizing these profit functions w.r.t. $x_1$ and $x_2$, and inserting them into each other lead to the optimal R&D investments. By inserting the optimal R&D investments into (7) we get the prices depending only on external variables. The R&D investments, prices, marginal costs and market shares depending on the degree of competition $\gamma$ are simulated in the graph below. The terms of the optimal amount of marginal cost reduction can be found in appendix A.
Graph 1: Left graph: MC reductions of firm 1 (blue) and firm 2 (orange) which is ordinally related to their R&D investments. Middle graph: post R&D marginal costs of firm 1 (dot-dashed) and firm 2 (dashed) and prices $p_{1M}$ (blue), $p_{1N}$ (purple) and $p_2$ (orange). Right graph: market shares of firm 1 (blue) and firm 2 (orange) in market M. All graphs depend on degree of competition $\gamma$. The values of the graph are: $a = 60$, $m = 0.6$, $n = 0.1$, $\nu = 2$, $c_1 = 30$, $c_2 = 30$.

In this simulation I assume $c_1 = c_2 = 30$, thus the firms have symmetric marginal costs before investing in R&D. As we can see in the graphs, there is a break in the development of R&D investments, prices and marginal costs, when degree of competition in market $M$ is higher than a certain threshold. The reason for the jump in prices, R&D investments and post R&D marginal cost of firm 1 at the threshold is that if $\gamma$ is over this threshold in period $t_2$ firm 2 decides to exit market $M$ and there for firm 1 does not need to set limit pricing in period $t_4$. Since firm 1 serves 2 markets, it has higher sales and thus R&D investments of firm 1 are always higher than firm 2. The higher $\gamma$ nature chooses in period $t_0$, the stronger are the following two countervailing effects on R&D incentives of the firms:

- Demand is lower ceteris paribus due to the assumed utility function of consumers and thus both firms have less incentives to invest in R&D.
- Business stealing effect is more significant and (specially the more productive firm) has stronger incentives to invest in R&D.

9 Note that when I mention higher or lower $\gamma$, I usually consider external changes of $\gamma$. One can assume a situation where the life cycle of a firm's product ends and it wants to introduce the new generation of the product into the market, knowing how the product of the competitor is placed. By giving a slightly different image to the new generation of that product (for example through a different image), a firm can influence the optimal level of R&D investments of itself and its competitor and – as I will show later – the prices and profits.
In the area where products are relatively differentiated, a higher degree of competition leads
to lower R&D investments and thus higher marginal costs and higher prices of both firms in market
\( M \) and higher prices of firm 1 in market \( N \), where it is a monopolist. The R&D investments decrease
because the first effect dominates in areas of low \( \gamma \) and the demand of consumers is lower ceteris
paribus, the more homogeneous the goods are. Even though a higher \( \gamma \) yields in this case sinking
R&D investments of the firms – and thus higher post-R&D-marginal costs, both firms set lower
prices in market \( M \) due to tougher competition. This lead to shrinking margins of both firms in
market \( M \). Since firm 1 invests more in R&D, it has lower post marginal costs than firm 2 in this
case. Thus if products are more homogeneous than a certain threshold, firm 2 does not operate in
market \( M \) and thus does not invest in R&D any more. This is the break point of the graph (here \( \gamma \approx
0.9143 \)). In the area where \( \gamma \) is above this break point, firm 1 is a monopolist in both markets and it
sets monopoly price in both markets and chooses the monopoly level of R&D investments/MC
reductions.

The “jump” in the break point of the graphs might appear not logical at the first glance.
However, one must take into account that in the area where \( \gamma \) is lower than the break in the graph,
firm 1 faces competition in market \( M \) and thus there is a business-stealing effect. This yields
significantly higher incentives for firm 1 to invest more in R&D and set lower prices comparing
with the area where firm 1 is monopolist in both markets. In this model firm 1 does not need limit
pricing when firm 2 is not active in market \( M \) due to the timing of the model. As mentioned in
chapter 2, firms set their amount of R&D investments and their prices and marginal costs after
knowing if the other firm is active in market \( M \) or not.

The next graph simulates R&D investments, prices and marginal costs, whereby firm 2 has a
superior production technology and thus lower pre-R&D marginal costs, so \( c_2 < c_1 \).

Graph 2: Left graph: Marginal cost reductions of firm 1 (blue) and firm 2 (orange) which is ordinally related to
their R&D investments. Right graph: marginal costs of firm 1 (dot-dashed) and firm 2 (dashed) and prices \( p_{1M} \)
\[ p_{1N} \text{ (purple) and } p_2 \text{ (orange) depending on degree of competition, whereby } c_2 < c_1. \] The values of the graph are: \( a = 60, m = 1, n = 0.2, \nu = 2.8, c_1 = 30, c_2 = 28. \)

In graph 2 each firm has a different advantage: firm 1 credibly commits itself through its market entrance into market \( N \) to invest more aggressive in process R&D, while firm 2 has lower pre R&D marginal costs, thus a more efficient production technology before firms invest in process R&D. If the goods are independent, then firm 1 invests more in R&D in our simulation and decreases its marginal costs stronger. If the nature chooses the goods in period \( t_0 \) more homogeneous, this leads ceteris paribus to lower demand in market \( M \) due to the demand function and thus both firms invest less in R&D and set lower prices. However, if the goods are more homogeneous, firm 2's superior technology gains a higher importance in the competition. If the products are homogeneous enough, a relative small size of market \( N \) and a relative big marginal cost advantage of firm 2 is given, then the “power constellation” changes and firm 1 has less incentives to invest in R&D due to technological advantage of firm 2 and firm 1 can even be driven out of market \( M \). Analogously to the first graph, firm 2 sets monopoly price after squeezing out the competitor out of market \( M \). Note that in opposite to the first graph, even though firm 1 is driven out of market \( M \), it still invests in R&D. The reason is that firm 1 is still active in market \( N \), where it is a monopolist.

The graph below shows how the firms' R&D investments depend on \( m \) - the number of consumers in market \( M \) which determines the size of market \( M \).

Graph 3: MC reductions of the firms depending on size of market \( M \) under Bertrand competition with price discrimination. The Values of this graph are: \( a = 60, c_1 = 36, c_2 = 36, n = 0.3, \nu = 3, \gamma = 0.4 \) (link graph) and \( \gamma = 0.75 \) (right graph).
An increasing size of market $M$ usually increases the incentives of both firms to invest in R&D simply because the firms' sales are higher in a bigger market. As $m$ is still rather small, a growth in size of market $M$ yields higher R&D investments of both firms. However a bigger size of market $M$ amplifies the firms incentives to gain market share through lower prices, hence it has a similar effect like increasing business stealing effect. As we can see in the right graph, if both $\gamma$ and $m$ are high firm 1 invests aggressive enough in R&D to scare off firm 2 from investing and thus forcing firm 2 to invest less in R&D. In this case, a further growth of market $M$ yields less R&D investments of firm 2 while firm 1 invests more in R&D.

**Proposition 1**

An increasing size of market $M$ can lead to lower profits of firm 2. This is the case, when both the R&D costs $\nu$ and the size of market $M$ are relatively high.

The proof is shown in Appendix B.

The reason is the same as in the graph before: an increasing size of market $M$ leads more aggressive R&D investments of firm 1. If the combination of $\gamma$ and $m$ is high enough a further increasing size of market $m$ makes firm 1 aggressive enough in its R&D investments, that the profits of firm 2 decrease even though the market size has grown.

4. Bertrand competition without price discrimination

In this chapter I change an assumption and allow arbitrage between the markets $M$ and $N$. For simplicity I assume that there are no transportation costs. Thus firm 1 can not set different prices in markets $M$ and $N$, and therefor $p_{1m} = p_{1n} = p_1$. Inserting the demands from (3) into (4) and maximizing them with respect to the prices and solving them, results into the following optimal prices:

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\[ p_1 = \frac{2(m+n)(a+c_1-x_1) - m(a-c_2+x_2)\gamma - (a(m+2n)+2n(c_1-x_1))\gamma^2}{4(m+n)-(m+4n)\gamma^2} \]

\[ p_2 = \frac{(m+n(1-\gamma^2))(2c_2-2x_2+(c_1-x_1)\gamma) + a(1-\gamma)(m(2+\gamma)+n(2+\gamma-\gamma^2))}{4(m+n)-(m+4n)\gamma^2} \] (8)

Inserting back (8) into (4) and maximizing the profits with respect to the R&D investments, and inserting them into each other yields the optimal R&D investments of the firms. The terms can be found in appendix C. The devolution of the graph of simulation in this case – which has the same parameter values as graph 1 – is very similar to that of graph 1. However in this graph, firm 2 is driven out of market \( M \) by \( \gamma = 0.9143 \) (vs \( \gamma = 0.8981 \) in the “price-discrimination-case” where firm 1 could set different prices). The reason why firm 2 is driven here out of market \( M \) at a higher \( \gamma \) is that firm 1 has to set the same price in the market where it is a monopolist as in the market where it competes with firm 2. Hence firm 1 sets here ceteris paribus tendentiously a higher price in market \( M \), but lower price in market \( N \) than in the price-discrimination-case. Thus when firm 1 can not use price discrimination, this yields higher sales of firm 1 in market \( N \) and lower sales in market \( M \). In this case the higher prices in market \( M \) yields to a “later” market exit of firm 2, thus under “tougher” circumstances.

However, it can happen that the higher quantity sold by firm 1 in market \( N \) is more than the lower quantity it sells in market \( M \), and firm 1 invests more aggressive in R&D. In that case, a market exit of firm 2 can be forced even under less likely circumstances, even though firm 1 sets here higher prices in market \( M \) than under the price-discrimination-case.

As in graph 1, investment decisions are strategic substitutes, that is, a firm's incentive to invest in marginal-cost-reducing process R&D is lower when the competitor is investing more in process R&D.

Both firms know that firm 1 has higher R&D incentives than firm 2 because of the higher amount it sales due to its market entrance into market \( N \). As the products become more similar, less consumer tastes are served and thus demand in market \( M \) decreases ceteris paribus, thus both firms invest less in R&D. However the better substitutes the products are, the more significant is the business stealing effect. When business stealing effect is already significant, a further homogeneity of the products yields firm 1 using its strategic advantage and investing more in R&D, even though the demand in market \( M \) shrinks ceteris paribus. In this case, firm 2 invests less since R&D investments are strategic substitutes, and if competition is over a certain threshold, firm 2 can be driven out of market.
The corresponding prices of the firms in equilibrium are shown in the graph below:

Graph 4: prices of firm 1 (blue) and firm 2 (orange) in Bertrand competition without price discrimination. The values of the graph are: $a = 60$, $n = 0.2$, $v = 2.8$, $c_2 = 30$, $c_1 = 30$, $m = 1$ (left) and $m = 0.4$ (right).

The higher degree of homogeneity the nature chooses in stage $t_0$ for the goods 1 and 2, the more firm 1 faces two problems: the demand in market $M$ is less ceteris paribus and firm 1's optimal prices in markets $M$ and $N$ diverge more from each other due to the tougher competition in market $M$. So the unit price firm 1 chooses for both firms is more away of the optimal prices firm 1 would have set in markets $M$ and $N$ if it could prices discriminate. The lower demand in market $M$, the stronger are firm 1's incentives to keep its price high to benefit from high margins in market $N$ where it is a monopolist. Hence firm 1's prices react less “elastic” than firm 2's prices on an increasing degree of competition in market $M$. As we can see in the right graph above, if goods are relatively independent and the size of market $m$ is not big, then firm 1 sets lower prices because of its lower post R&D costs due to its higher R&D investments. If competition is in the middle ranges, firm 1 prefers to set higher prices in order to avoid setting too low prices in market $N$, where firm 1 is a monopolist. If the goods are comparatively homogeneous, the business stealing effect gets a higher weight. In this case since the prices are already relatively low due to the tough competition, it is worth for firm 1 to use its higher sales for high R&D investment and aggressive price-setting in order to gain more customers from firm 2 and eventually even forcing firm 2 to stay out of the market.

The graph below demonstrates how amount of marginal cost reduction and thus amount of R&D investments depend on size of market $M$. Other than in graph 3, I assume here that firm 2 has a technological advantage in $t=0$, thus before the firms invest in R&D.
Graphik 5: Marginal cost reduction of the firm 1 (blue) and firm 2 (orange) depending on the size of market $M$. The values of the graph are $a = 60$, $c_1 = 30$, $c_2 = 24$, $v = 2$, $n = 0.2$, $\gamma = 0.1$ (left graph), $\gamma = 0.4$ (middle graph) and $\gamma = 0.75$ (right graph).

Intuitively, one would always expect from an increasing size of market $M$ more aggressive marginal costs reduction and thus lower R&D investments of the firms. However in the constellation simulated in the graph above – with firm 1 serving two markets and firm 2 having lower pre R&D marginal costs – this is not always the case. While a bigger size of market $M$ always leads to higher R&D investments of firm 2, it only leads to higher R&D investments of firm 1 if the products of firm 1 and 2 are relatively differentiated. However, if degree of competition is high enough, an increasing size of market $M$ can lead to lower R&D investments of firm 1. The reason is that firm 2's advantage of lower pre-R&D marginal costs is more significant, the more homogeneous the products are and in this case firm 2 invests more aggressive in R&D. In this case firm 1 prefers to set a high price and focus rather on its monopoly market $N$ and is not willing to engage into a tough competition in market $M$ since it faces an aggressive competitor with a relatively homogeneous product. Hence, a bigger size of market $M$ leads to lower relative competitive advantage of firm 1. For this reason, firm 1 tends to focus more on market $N$. This yields higher “convergence” of firm 1's price to the monopoly price in market $N$ and thus lower R&D investments of firm 1. If degree of competition is high enough and market $M$ is already comparatively big, a further incremental of firm $M$ can even yield to lower R&D investments of firm 1.

10 This effect could be also observed already in graph 4, where firm 1 would even stay out of market $M$ if the degree of product homogeneity was over a certain threshold.
Proposition 2

A rise of the R&D costs $\nu$ can lead to higher profits of firm 2.

The proof is shown in the graphic below. The graph is the simulation of one example where higher R&D costs $\nu$ leads to higher profits of firm 2.

We get the profit functions depending only on external variables by inserting the optimal prices and R&D level into \((5)\). The graphic below shows the profit functions depending on R&D costs $\nu$:

Graph 6: Profits of the firms when firm 1 enters into market $N$ depending on costs of R&D investments $\nu$ under Bertrand competition without price discrimination. The Values of this graphic are: $a = 60$, $n = 0.3$, $m = 0.8$, $\gamma = 0.6$, $F = 40$, $c_1 = 30$, $c_2 = 30$ (left graph) and $c_2 = 26$ (right graph).

The dependency between firm 1’s profit function and R&D costs $\nu$ is intuitive: the more costly R&D investments are, the less are the profits of firm 1. For firm 2 however, the relation is not intuitive at the first glance. As the simulation shows, it can be the case that the profit of firm 2 increases as R&D costs $\nu$ increases\(^{11}\). Since higher R&D costs lead to lower R&D incentives of both firms, the advantage of firm 1 – which is higher R&D incentives – becomes less significant. This in turn makes firm 2 relatively more competitive and thus increases its profits. As we can also see in the graph it can even happen that if R&D is cheap enough, firm 1 can become very aggressive in its R&D investments so that firm 2 can even be driven out of the market. The right graph shows that the same pattern can be seen even if firm 2 has a cost advantage.

---

\(^{11}\) This is not the case if the goods are highly independent. In that case the profits of both firms decrease if costs of R&D increase.
5. Cournot Competition

Cournot Competition with price discrimination

In this chapter, I will consider how the results change if we consider competition under Cournot regime. Since Cournot competition and product differentiation have similar effects of weakening competition among firms, I will assume homogeneous products under Cournot competition (so $\gamma = 1$) in order to get the effects of Cornot competition more clearly. I start again with solving the model recursively by inserting (1) and (2) into (4) to get the profit functions of the firms depending on quantities $q_{m1}$, $q_{m2}$, and $q_2$ under the assumption that price discrimination is possible.

$$
\pi_1 = m q_{m1} [(a - q_{m1} - q_{m2} \gamma - c + x_1)] + n q_{m1} [(a - q_{m1} - c + x_1)] - v \frac{x_1^2}{2} - 2 F
$$

$$
\pi_2 = m q_2 [(a - q_2 - q_{m1} \gamma - c + x_2)] - v \frac{x_2^2}{2} - F
$$

Building the first order conditions from the profit functions in (9) with respect to the quantities and inserting the quantities into each other yield:

$$
q_{1m} = \frac{a - c + 2 x_1 - x_2}{3}
$$

$$
q_{1n} = \frac{(a - c + x_1)}{2}
$$

$$
q_2 = \frac{a - c + 2 x_2 - x_1}{3}
$$

Under monopoly the quantity in market $m$ is $q_{im} = \frac{(a - c) + x_i}{2}$ and as competition mode changes to a duopoly Cournot competition, the quantity sold by each firms decreases to:

$$
q_{im} = \frac{(a - c) + 2 x_j - x_i}{3}, \quad \forall i, j \in \{1, 2\}.
$$

An investment in R&D yields lower costs and thus higher sales. This effect is stronger the better substitutes the goods are.

The prices are in this case:
\[ p_{1m} = p_{2} = \frac{a + 2c - x_1 - x_2}{3} \]  
\[ p_{1n} = \frac{a + c - x_1}{2} \]  

The optimal R&D investment of each firm can be determined by inserting (10) into (9) and maximizing both profit functions w.r.t. R&D investments. This leads to the following reaction functions:

\[ x_1 = \frac{(a - c)(8m + 9n) - 8mx_2}{9(2\nu - n) - 16m} \]  
\[ x_2 = \frac{4m(a - c - x_1)}{9\nu - 8m} \]  

Inserting the reaction functions into each other leads to the optimal R&D levels:

\[ x_1 = \frac{(a - c)(8m(4m + 3n) - 3(8m + 9n)\nu)}{27\nu(n - 2\nu) - 32m^2 + 24m(4\nu - n)} \]  
\[ x_2 = \frac{8(a - c)m(4m + 3n - 3\nu)}{24m(4\nu - n) + 27\nu(n - 2\nu) - 32m^2} \]  

For a better visualization, I plot the R&D investments in the Graph below depending on the size of market \( M \).

Graph 8: R&D investments of the firms under Cournot competition when price discrimination is possible. The Values of this graphic are: \( a = 60 \), \( c_1 = 36 \), \( c_2 = 36 \), \( n = 0.3 \), \( \nu = 3 \).

Analogously to the case of Bertrand competition, if market \( M \) is not yet big, a bigger size of market \( M \) simply leads to higher R&D investments of both firms. If the size of market \( M \) exceeds a certain threshold, then an increasing size of market \( M \) leads only to higher R&D investments of firm 1. Similar to the case of Bertrand competition with relatively homogeneous products, firm 2 invests...
less the bigger market $M$ becomes. The reason is the same as under Bertrand competition and is discussed there.

The quantities and prices of the firms in equilibrium are

\[
q_{1n} = \frac{9(a-c)v(4m-3v)}{32m^2+24m(n-4v)+27v(2v-n)}
\]

\[
q_{1m} = \frac{3(a-c)v(3n+6v-8m)}{32m^2+24m(n-4v)+27v(2v-n)}
\]

\[
q_2 = \frac{6(a-c)v(4m+3n-3v)}{32m^2+24m(n-4v)+27v(2v-n)}
\]

And the prices are:

\[
p_{1m} = p_2 = \frac{2a(4m-3v)(4m+3n-3v)+3c\times(12v-16m-3n)}{32m^2+24m(n-4v)+27v(2v-n)}
\]

\[
p_{1n} = a + \frac{9(a-c)v(4m-3v)}{32m^2+24m(n-4v)+27v(2v-n)}
\]

As we can conclude from (14) and (15), the quantities, prices and R&D investments of firm 2 depend also on the size of market $N$ where firm 2 is not active. Therefore any change in market size of market $N$ influences price, quantity and R&D investment of firm 2.

The graph below compares the R&D investments of both firms under Cournot and Bertrand competition depending on degree of competition.

Graph 9: Comparison of the R&D investments of both firms when firm 1 enters into market $N$ under Cournot competition vs. under Bertrand competition depending on $\gamma$. The green (red) line represents $x_1$ ($x_2$) under Cournot competition, the blue (orange) line represents $x_1$ ($x_2$) under Bertrand regime. The Values of this graphic are: $a = 60$, $c_1 = 30$, $c_2 = 30$, $n = 0.1$, $m = 0.6$, $F = 30$, $v = 2$.

For a better comparability I have used the same values in the graph above as in graph 1. It
has already been shown that under Bertrand competition and for the used parameter values firm 2 is driven out of market $M$ if $\gamma$ is over the threshold $0.9143$. For the values of this simulation, firm 1 does not drive firm 2 out of market $M$ for any degree of competition under Cournot competition.

If $\gamma$ is above or around the threshold value $0.9143$ firm 1 invests more and firm 2 invest less under Bertrand than under Cournot competition. For other values of $\gamma$ both firms invest less under Bertrand competition than Cournot competition. This is contradicting with the standard results in the literature such as Qiu (1997), where both firms invest more under Bertrand competition. Thus this standard result is not valid when the constellation of our model is given.

5.1 An example for the (Un-)profitable market entrance under Cournot competition

In order to proof that a market entrance into an unprofitable market can be profitable, we need to know how much the profits of firm 1 are if it does not enter into market $N$. Then we can compare the total profits of firm 1 with the case where it does enter into market $N$. If firm 1 stays out of market $N$ and $c_1 = c_2 = c$, then the firms are symmetric. Their reaction functions are:

$$q_i = \frac{a - c - q_j + x_i}{2} \forall \{i, j\} \in \{1, 2\}$$  \hspace{1cm} (16)

Inserting the reaction functions into each other results into the quantities

$$q_i = \frac{a - c + 2x_i - x_j}{3} \forall \{i, j\} \in \{1, 2\}$$  \hspace{1cm} (17)

Inserting back the quantities into the profit functions and maximizing the profit functions with respect to the R&D investments yields the following optimal R&D investments

$$x_i = x_j = \frac{4m(a-c)}{9\sqrt{4m} - 4m} \forall \{i, j\} \in \{1, 2\}$$  \hspace{1cm} (18)

The profits of both firms are:

$$\pi_i = \pi_j = \frac{(a - c)^2 m(9\sqrt{4m} - 8m)\sqrt{9\sqrt{4m} - 8m}}{(4m - 9\sqrt{4m})^2} - F \forall \{i, j\} \in \{1, 2\}$$  \hspace{1cm} (19)

By penetrating into market $N$ and paying the irreversible fixed costs $F$ for brand advertising, building a distribution channel etc, firm 1 commits itself to higher R&D-investments due to its higher sold quantity. Since firm 2 knows this, it will react to the higher R&D incentives of firm 1 and will itself invest less in R&D. Thus firm 1 faces a less efficient competitor in market $M$ and the market penetration of firm 1 in market $N$ yields higher profits for firm 1 in market $M$. The graph
below illustrates inter alia how profitable firm 1’s market entrance into market $N$ per se is and how it changes the total profits of firm 1 depending on the costs of R&D. The cost functions, price and quantity of firm 2 are also simulated to make sure that firm 2 is active in the market in the considered area.

Graph 10: Orange: prices of firm $a$ and $b$ in market $m$, dashed and dot dashed: cost functions of firms 1 and 2 after R&D investments, red: quantities sold by firm $b$, purple: additional profit that firm 1 makes through entering into market $N$, blue: how does the total profits of firm 1 changes due to market entrance in market $N$:

$\Delta \pi_1, c-\Delta \pi_1, c, n_1, p$

As we can see in the graph above, if $v$ is for example 1.1 firm 1 would make losses in market $N$ if it enters into that market (purple line). However the total profits of firm 1 increase (blue line). Inserting the values and calculating the numerical example shows that market entrance into market $N$ per se creates losses of approximately 2.1334, while through market entrance the total profits of firm 1 increases by about 6.7776.

6. Conclusion

This paper has shown that in some cases, market entrance into a new market can be profitable even though the ex ante fix costs of market entry for advertising, distribution channel and so forth can not be covered. When firm 1 enters into market $N$, it commits itself to higher production volume and thus higher R&D. Since the firms' R&D investments are strategic substitutes, firm 1's commitment to higher R&D investments yields lower investments of firm 2 in process R&D. Firm 1 faces therefor a weaker competitor in market $M$ through its market entry into market $N$. The innovative advantage of firm 1 increases, the better the degree of product
homogeneity and the bigger the size of market $M$ is. This can even yield to market exit of firm 2 if both firms have similar pre-R&D cost functions and the degree of product substitutability $\gamma$ exceeds a certain threshold.

However if firm 2 has a pre-R&D cost advantage, which is significant comparing to the higher R&D incentives of firm 1, a better the degree of product homogeneity and a bigger size of market $M$ can emphasize the cost advantage of firm 2 and be disadvantageous for firm 1. If the products are homogeneous enough and the size of market $N$ is relative small, it can happen that higher competition forces firm 1 to stay out of market $M$, and concentrate on market $N$ only.

The paper has shown how changes in variables influence R&D investments and profits of the firms. Among others, the paper shows that higher costs of process R&D can lead to lower profits of firm 1, but to higher profits of firm 2. Beside that the model proved that for some values of $\gamma$, both firms invest less under Bertrand competition than Cournot competition in this model. This finding contradicts the standard results in literature the.

We also showed that for high $\gamma$ and $m$, it can happen that a further incremental of Market $M$'s size results into lower profits of firm 2 even though the number of competitors remain the same and the firms simply operate in a bigger market. We also have shown that an increasing size of market $M$ has two effects under Bertrand competition without price discrimination with asymmetric firms where firm 2 has lower pre-R&D marginal costs: it increases the incentives of both firms to invest in R&D and it increases the relative weight of firm 2's pre-R&D cost advantage. As $m$ increases, both firms invest more in R&D, however firm 2's increasing R&D investments grow faster than firm 1's R&D investments. This effect is stronger, the better substitutes the goods become.

In a nutshell the paper shows that market entry of a firm into a new market and asymmetries among the firm can be connected with surprising results.
Appendix

Appendix A

Marginal cost reductions of the firms in Bertrand with price discrimination in the area of competition

\[ x_a = \frac{((a-c)(v(-2+g)(-1+g)(2+g)2(-4+g^2+a(-2+g))2(1+g)(2+g)+2(-2+g)^22(-4+a(-4+g^2))))}{(2v2(-4+g^2)3(-1+g^2)-2(-2+g^2)2(-4+a(-4+g^2))-v(-4+g^2)(-8(-2+g^2)2+a(-4+g^2)2(-1+g^2)))} \]

\[ x_b = \frac{-((a-c)(2v(-2+g)(-1+g)(2+g)-2(-2+g)(-4+a(-4+g^2))))}{(2v2(-4+g^2)3(-1+g^2)-2(-2+g^2)2(-4+a(-4+g^2))-v(-4+g^2)(-8(-2+g^2)2+a(-4+g^2)2(-1+g^2)))} \]

MC reductions of the firm 1 in Bertrand with price discrimination if firm 2 exits market \( M \):

\[ x_a = \frac{(a-c_a)(m+n)}{(2v-m-n)} \]

Appendix B

proof of proposition 1:

In order to proof that an increasing size of market \( M \) can lead to lower profits of firm 2, I present the simulation of one example where this was the case. Inserting the prices – which are simulated in graph 1 – into (5) leads to the profits of the firms depending on external variables. The graph below shows the plot of the profit functions of the firms depending on the size of market \( M \).
Graph: Profit of the firms depending on size of market \( M \). The values of this graph are: \( a = 60 \), \( c_1 = 30 \), \( c_2 = 30 \), \( n = 0.5 \), \( v = 2 \), \( F = 30 \), \( \gamma = 0.3 \) (left graph) and \( \gamma = 0.5 \) (right graph).

As we can see in the graph, for high \( \gamma \) and \( m \), it can happen that an incremental of Market \( M \)'s size results into lower profits of firm 2.

**Appendix C**

Price setting behavior of the firms under Bertrand competition when price discrimination is possible

\[
x_1 = \frac{-c v (1 + g) (-4 + g^2) (4 m (-2 + g^2) + (2 + g) (-1 + g) (2 + g) (-n g + 2 v (2 + g)))}{(8 m^2 (-2 + g^2) 2 - 2 m (n - 4 v) (-4 + g^2) (-2 + g^2) 2 - (n - 2 v) v (-4 + g^2) 3 (-1 + g^2))} + \\
\frac{(a (-8 m^2 (-2 + g^2) 2 + v (-4 + g^2) 2 (-1 + g^2) (n (-4 + g + g^2) - 2 v (-2 + g + g^2)) + 2 m (8 - 6 g^2 + g^4) (n (-2 + g^2) - 2 v (-3 + g + 2 g^2))))}{(8 m^2 (-2 + g^2) 2 - 2 m (n - 4 v) (-4 + g^2) (-2 + g^2) 2 - (n - 2 v) v (-4 + g^2) 3 (-1 + g^2))}
\]

\[
x_2 = \frac{a + c + ((a - c) (8 m^2 (-2 + g^2) 2 - n v (-4 + g^2) 3 (-1 + g^2) - 2 m (8 - 6 g^2 + g^4) (n (-2 + g^2) - 2 v (-2 + g + g^2))))}{2 (8 m^2 (-2 + g^2) 2 - 2 m (n - 4 v) (-4 + g^2) (-2 + g^2) 2 - (n - 2 v) v (-4 + g^2) 3 (-1 + g^2))}
\]

**Appendix D**

The graph below demonstrates R&D investments depending on \( \gamma \), the degree of competition in market \( M \).
Graph: Optimal R&D investments of the firms depending on degree of competition $\gamma$. The blue lines are the R&D investments of firm 1 and the orange lines are the R&D investments of firm 2. The values of the graph are: $a = 60$, $c = 30$, $n = 0.2$, $m = 1$, $v = 2.8$, $c_1 = 30$, $c_2 = 30$ (left graph) and $c_2 = 28$ (right graph). Changing the values does not change the “shape” of the graph.

Appendix E

Price setting of the firms in Bertrand without price discrimination under simultaneous R&D decision

\[
p_1 = - (a (-4 (m+n)^2 (m-2 v)^4 m (m+n) v g^2 + (m+n) (m^2+2 m n-3 m v-8 n v) g^2 + m (m^4 n) v g^3+(m+2 n) (m+4 n) v g^4) (2 m^2 (-2+g^2)-4 n (n-v) (-1+g^2)^2-m (2 n-v) (4-5 g^2+g^4))+v (-4 (m+n)+(m+4 n) g^2) (-2 c_a (m+n-n g^2) (-2 (m+n) (m-2 v)+m^2+2 m n-5 m v-8 n v) g^2+(m+4 n) v g^4)+cb m g (-2 m^2 (-2+g^2)+4 n (n-v) (-1+g^2)^2+m (2 n-v) (4-5 g^2+g^4)))/(-4 m^5 (-2+g^2)^2+32 n^3 (n-2 v) v (-1+g^2)^2+8 m n^2 (-1+g^2)^3 (2 n^2+5 n v (-4+g^2)-6 v^2 (-4+g^2))+4 m^2 n (-1+g^2)^2 (-3 v^2 (-4+g^2)^2+8 n^2 (-2+g^2)+4 n v (18-10 g^2+g^4))+4 m^4 (-2+g^2) (n (8-8 g^2+g^4)-v (8-6 g^2+g^4))+m^3 (-1+g^2) (-v^2 (-4+g^2)^2+2 n v (-2+g^2) (56-22 g^2+g^4)+4 n^2 (24-24 g^2+5 g^4))),
\]

\[
p_2 = (-a (2 m^2 (-2+g^2)+4 n v (-1+g^2)^2+m (-1+g^2) (4 n+v (-4+g^2))) (2 m^2 (-2+g^2)-4 n^2 (n+v (-2+g))-1+g^2)^3-m n (-1+g^2) (v (2+g) (8+(8+g) g)+6 n (-2+g^2))+m^2 (v (-2+g) (-1+g) (2 g)^2-2 n (6-6 g^2+g^4)))+v (m+n-n g^2) (-4 (m+n)+(m+4 n) g^2) (2 cb (m^2 (-2+g^2)-2 n (n-2 v) (-1+g^2)^2-m (n-v) (4-5 g^2+g^4))+ca g (2 m^2 (-2+g^2)+4 n v (-1+g^2)^2+m (-1+g^2) (4 n+v (-4+g^2))))/(-4 m^5 (-2+g^2)^2+32 n^3 (n-2 v) v (-1+g^2)^2+8 m n^2 (-1+g^2)^3 (2 n^2+5 n v (-4+g^2)-6 v^2 (-4+g^2))+4 m^2 n (-1+g^2)^2 (-3 v^2 (-4+g^2)^2+8 n^2 (-2+g^2)+4 n v (18-10 g^2+g^4))+4 m^4 (-2+g^2) (n (8-8 g^2+g^4)-v (8-6 g^2+g^4))+m^3 (-1+g^2) (-v^2 (-4+g^2)^2+2 n v (-2+g^2) (56-22 g^2+g^4)+4 n^2 (24-24 g^2+5 g^4))),
\]
Appendix F

Price setting of the firms in Bertrand with price discrimination simultaneous

\[
p_a = \frac{-c v (1 + g)(-4 + g^2)(4 m(2 + g) + n(1 + g)(2 + g)(-n + 2 v(2 + g)))}{(8 m^2(2 - 2 + g^2) - 2 m(n - 4 v)(4 + g^2)(2 - (n - 2) v)(-4 + g^2) 3(-1 + g^2))} + \]

\[
a(8 m^2(2 - 2 + g^2)2 + v(-4 + g^2)(2(-1 + g^2)(n(-4 + g + g^2)) - 2 v(-2 + g + g^2)) + 2 m(8 - 6 g^2 + 4)(n^2 - 2 + g^2) - 2(3 + g + 2 g^2))}{(8 m^2(2 - 2 + g^2)2 - 2 m(n - 4 v)(-4 + g^2)(2 - 2 + g^2)2 - (n - 2) v)(-4 + g^2) 3(-1 + g^2))}
\]

\[
p_m = \frac{a + c + ((a - c)(8 m^2(2 - 2 + g^2)2 - n(-4 + g^2)3(-1 + g^2)) - 2 m(8 - 6 g^2 + 4)(n(-2 + g^2) - 2 v(-2 + g^2)))}{2(8 m^2(2 - 2 + g^2)2 - 2 m(n - 4 v)(-4 + g^2)(2 - 2 + g^2)2 - (n - 2) v)(-4 + g^2) 3(-1 + g^2))}
\]

\[
p_a = \frac{-2 c v(-2 + g)(1 + g)(2 + g)(2 m(-2 + g) + n(-2 + g)(1 + g)(2 + g)(-n + v(2 + g)))}{(8 m^2(2 - 2 + g^2)2 - 2 m(n - 4 v)(-4 + g^2)(2 - 2 + g^2)2 - (n - 2) v)(-4 + g^2) 3(-1 + g^2))} + \]

\[
-a(2 m(-2 + g) + v(4 - 5 g^2 + 4 d))(4 m(-2 + g) - (4 + g^2)(n(-2 + g) - 2 v(-2 + g + 2 g^2)))}{(8 m^2(2 - 2 + g^2)2 - 2 m(n - 4 v)(-4 + g^2)(2 - 2 + g^2)2 - (n - 2) v)(-4 + g^2) 3(-1 + g^2))}
\]
References


