

Problem Set 2 (due date: 28.10.2013)

1. The company Lauren Ralph produces a number of products including men's shirts, q_1 , and cologne, q_2 . Let the cost functions associated with these products be

$$c(q_1, 0) = 2 + \sqrt{q_1}$$

$$c(0, q_2) = 2 + q_2^2$$

$$c(q_1, q_2) = 3 + \sqrt{q_1} + q_2^2$$

- a) Show that production of shirts enjoys substantial product-specific scale economies but that production of cologne does not. What does this suggest about the likely structure of the cologne market?
- b) Discuss whether the scope economies between shirt and cologne production, along with the significant scale economies in shirts, imply that the cologne market will likely be quite concentrated.
2. The company Lemac leases a fast food restaurant of the Burger Inc. chain. Rent is 10 000 €. In order to sell Burger, Lemac has to hire employees (L). Each employee costs 100 €. The output of Burger y (in 1000) is based on the production function $y = L^{0.5}$.
- a) Determine the cost function of Lemac.
- b) Calculate the average costs and their minimum.
- c) Calculate the marginal costs.

3. A firm produces with the given Cobb Douglas production function $q = L^{\frac{2}{3}} K^{\frac{1}{3}}$ where K is the capital input and L is the labor input.
- a) Determine the firm's cost minimization problem and setup the Lagrangian function.
- b) Derive the cost function for the firm.
- c) Assume that the price of capital r is 0.01 and the price of labor is 100. What are total costs to produce 100 000 units of good q ?
- d) *Show that the derivative of the cost function with respect to the factor prices yields the conditional demand functions for the input factors (Shephard's Lemma).

4. Let r be the price of capital and w be the price of labor, and consider the following cost function:

$$C(r, w, q) = \left(\frac{10r}{3}\right)^{\frac{1}{3}} \left(\frac{5w}{3}\right)^{\frac{2}{3}} q^2 + F$$

- a) Derive the expression for the average costs.
- b) Determine the output level at which any scale economies are exhausted.
5. An urban rapid-transit line runs crowded trains (200 passengers per car) at rush hour, but nearly empty trains (10 passengers per car) at off-peak hours. A management consultant argues that the cost of running a car for one trip on this line is about 50 € regardless of the number of passengers. Hence, the consultant concludes that the cost per passenger is about 25 cents at rush hour, but

rises to 5 € in off-peak hours. From this she reasons that the firm had better discourage the off-peak business. Is the consultant a good economist? Why or why not?

6. If the production technology is of the Leontief-type, we have $q = \min\left\{\frac{K}{a}, \frac{L}{b}\right\}$ where K is the capital input, L is the labor input; and a and b are positive constants. If r is the price of capital and w is the price of labor, derive the cost function for a firm with this production technology. To what extent does this cost function exhibit either scale economies or diseconomies?

Hint: Solve the problem intuitively!