Problem Set 8 (due date: 16.12.2013)

1. Two crude oil producers decide independently on quantities x_1, x_2 . Variable costs are proportional to quantity and are c_1, c_2 per ton crude oil. World demand is given by $P = B - bx$.

a) Calculate the equilibrium values $\boldsymbol{x_1^{NC}, x_2^{NC}, p^{NC}}$.

b) Calculate the market share and the profits of producer *i*.

c) How are market share and profit of producer *i* changed if he can lower his production costs?

d) Set $\boldsymbol{B = 1, b = 1}$ and $\boldsymbol{c_i + c_j = 1}$ and calculate $\boldsymbol{\Pi^1 + \Pi^2}$. How is this sum altered if $\boldsymbol{c_i}$ rises and $\boldsymbol{c_j}$ falls, so that $\boldsymbol{c_i + c_j = 1}$ remains?

e) With how much quantity would you start producing as firm 1 if there is no possibility of later reversal of this quantity? What is the market price? (Hint: This is the Stackelberg case.).

2. $\boldsymbol{N}$ $(\boldsymbol{> 2})$ identical planter decide independently how much tons of coffee they want to produce and sell. The demand for coffee is given by $\boldsymbol{x = p^{-\epsilon}, \epsilon > 1}$. Variable costs per ton of coffee are $\boldsymbol{c}$. Calculate the market equilibrium $\boldsymbol{\left(x_1^{NC}, \dots x_N^{NC}, p^{NC}\right)}$.

3. Two hotels of equal quality have capacity of beds of $\boldsymbol{\bar{x}_1}$ and $\boldsymbol{\bar{x}_2}$. Demand for hotel nights is x(p) = S – p, marginal costs are c.

a) What is the price if there is price competition and $\bar{x}_i > S, i = 1,2$?

b) Assume that customers book in the order of their reservation prices and that they book first at the cheaper hotel. If hotel 1 charges $p_1 = S - \bar{x}_1 - \bar{x}_2$, is it optimal for hotel 2 to charge a different price if $\max\{\bar{x}_1, \bar{x}_2\} \leq (S - c)/3$?

4. Consider the same situation as in problem 4 with the exception that customers book in a different order. Now the order is random and independent of their willingness to pay (proportional rationing). Again $p_1 = S - \bar{x}_1 - \bar{x}_2$. Has hotel 2 an incentive to charge a different price if

a) $\bar{x}_1 = \bar{x}_2 = (S - c)/3$?

b) $\max\{\bar{x}_1, \bar{x}_2\} \leq (S - c)/4$?

5. Each of two hotels of equal quality has a capacity of 50 beds. Off-season demand for hotel nights is x(p) = 100 – p, marginal costs are 0. Assume that rationing occurs according to the efficient rationing rule.

a) ** Discuss whether the values from the unlimited capacity Bertrand game and from the standard Cournot game, respectively, can arise as equilibrium.

b) ***What prices and quantities might constitute an equilibrium?