The second eigenfunction of the 1-Laplace operator and related geometrical problems

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Abstract: There are many properties about the eigensolutions of the eigenvalue problem of the p-Laplace operator

$$-\Delta_p u = \lambda u|u|^{p-2}$$

under homogeneous Dirichlet boundary conditions known. We focus on the limit problem $p = 1$, which can be treated by the associated variational problem. The first eigenfunction of the 1-Laplace operator is quite well understood and can be characterized as multiple of the characteristic function χ_C of the Cheeger set C of Ω. The study of the second eigenfunction is more sophisticated and requires tools from nonsmooth critical point theory (in particular the 'weak slope'). We discuss several ways how to define the second eigenfunction of the 1-Laplace operator and show that it does not satisfy many of the properties of the second eigenfunction of the p-Laplace operator in general. Moreover, we show that in some cases the second eigenfunction takes the form $u_2 = c_1\chi_{C_1} - c_2\chi_{C_2}$, where (C_1, C_2) is a so called Cheeger-2-cluster of Ω, while in other cases functions of that type can't be eigenfunctions at all.