

# Probing the QCD phase diagram in heavy-ion collisions

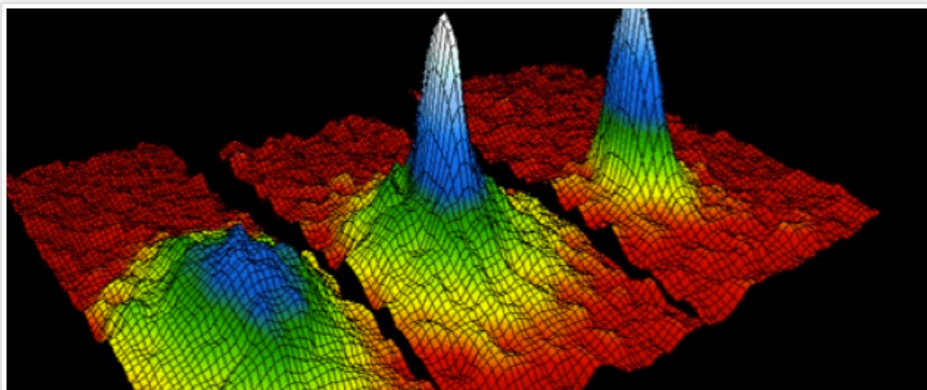
**Bernd-Jochen Schaefer**



University of Giessen, Germany

**Forschungsschwerpunkte**

Modelle und Simulation



**FWF**

Der Wissenschaftsfonds.

KARL-FRANZENS-UNIVERSITÄT GRAZ  
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# Heavy-Ion Collision Experiments

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Aim: create hot and dense QCD matter - elucidate its properties

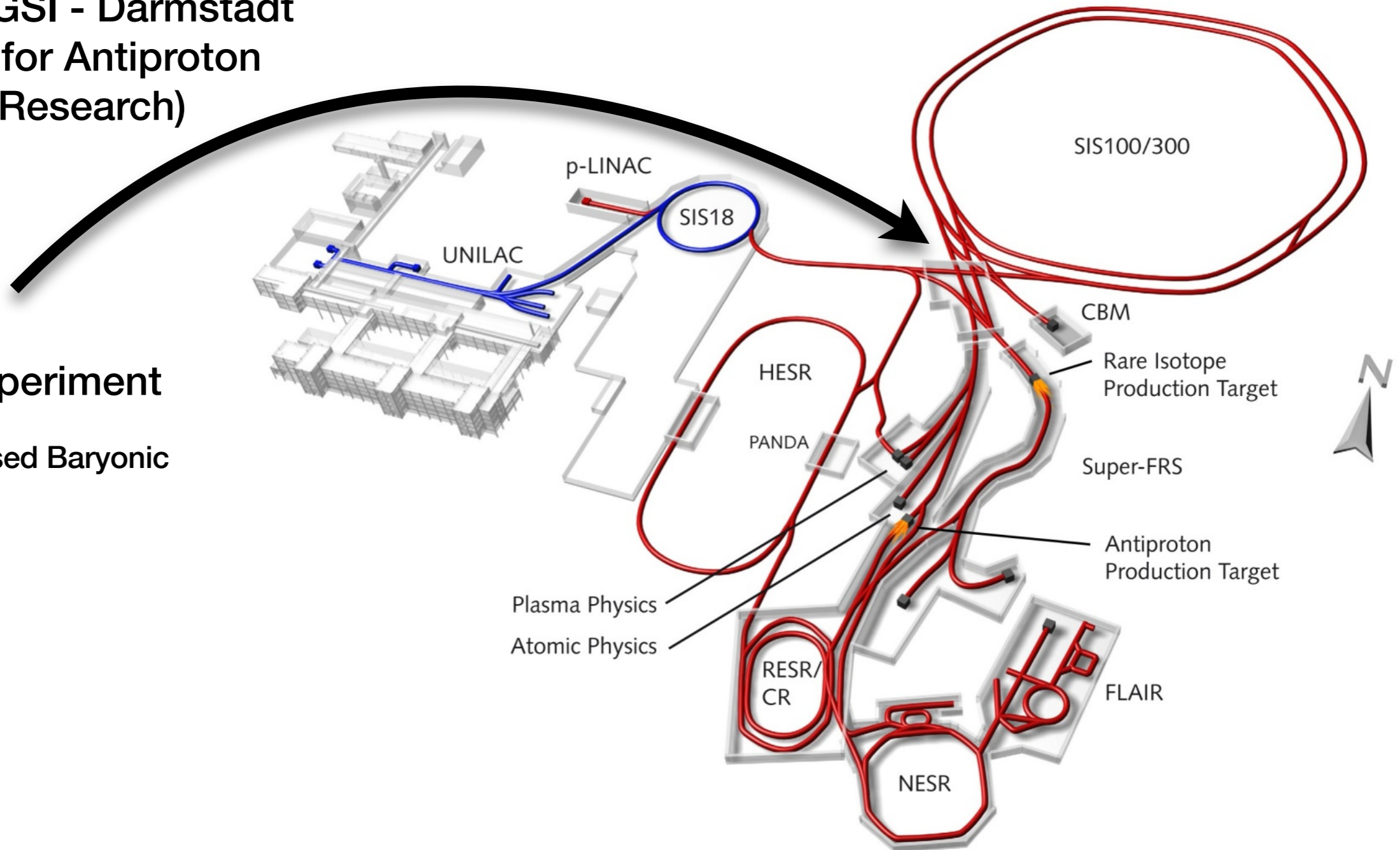
QCD under extreme conditions is a very active field (July 2013)

- Goals of HIC Experiments: learn QCD matter Equation of State
- Understanding fundamental phenomena:
  - color confinement
  - nature of chiral and deconfinement transition
  - early Universe history
  - nuclear matter
  - properties of stars
  - ...

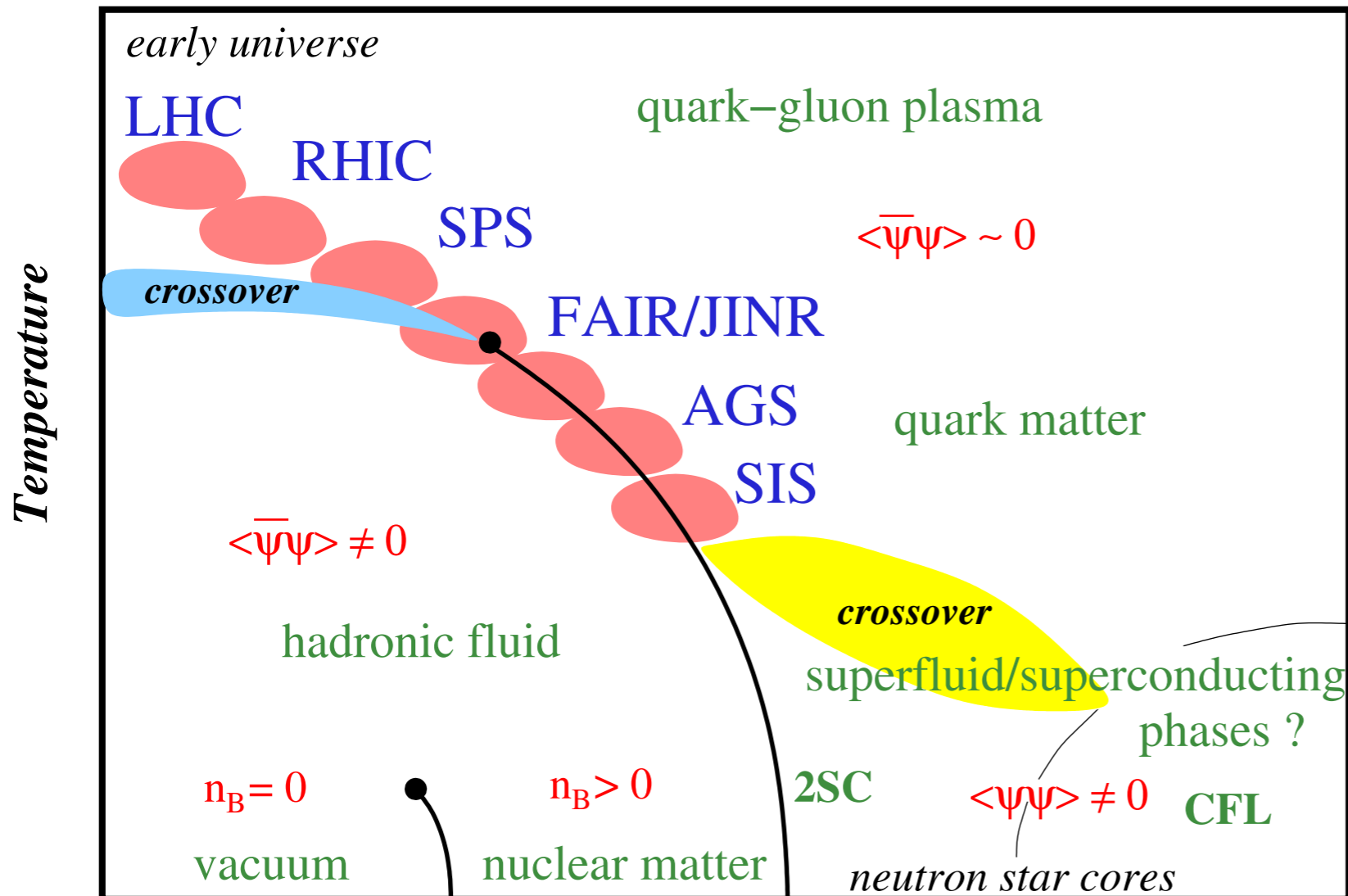
# Heavy-Ion Collision Experiments

FAIR @ GSI - Darmstadt  
(Facility for Antiproton  
and Ion Research)

CBM experiment  
( Compressed Baryonic  
Matter )

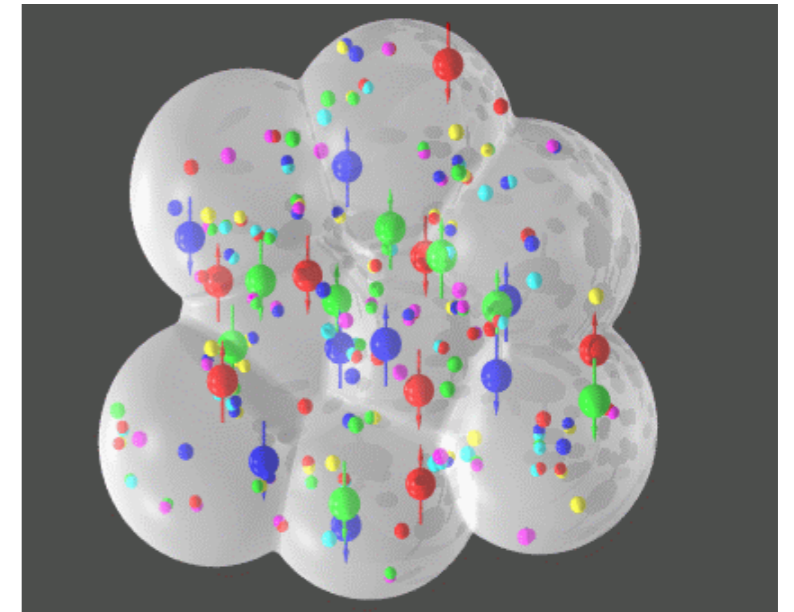
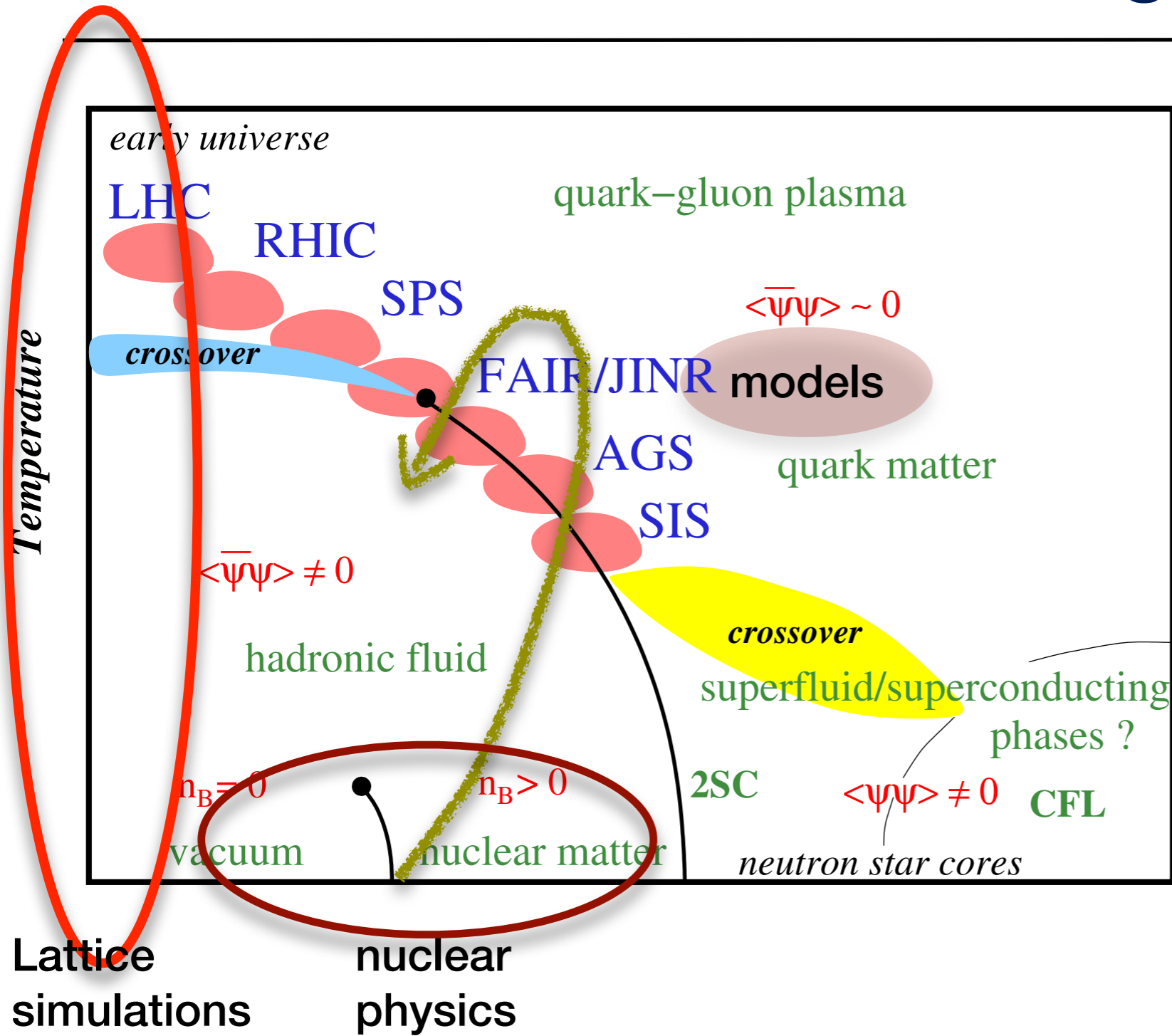


# QCD Phase diagram

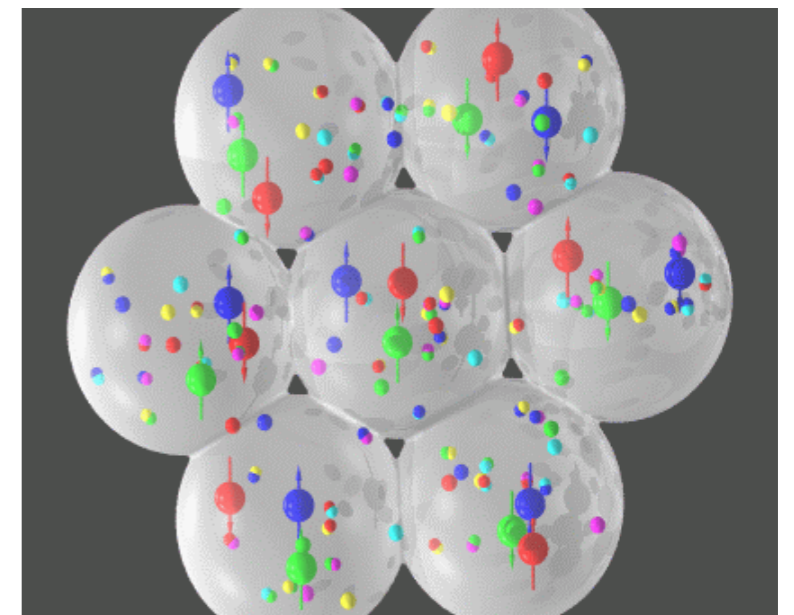


- Existence / location of critical point (CEP)
- details of phase transitions
- properties of quarks and gluons in plasma (QGP)

# QCD Phase diagram

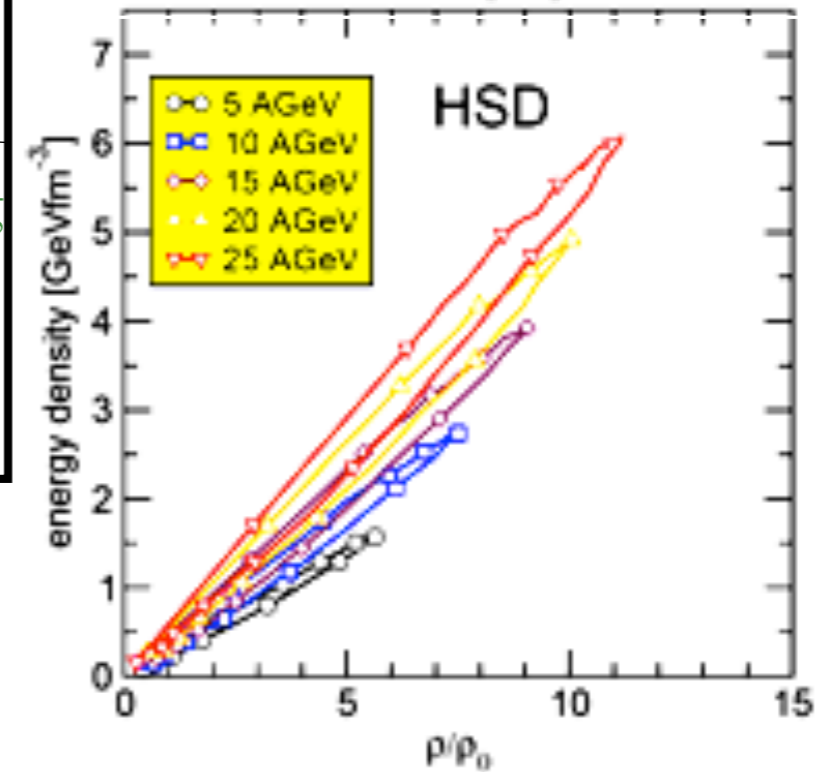
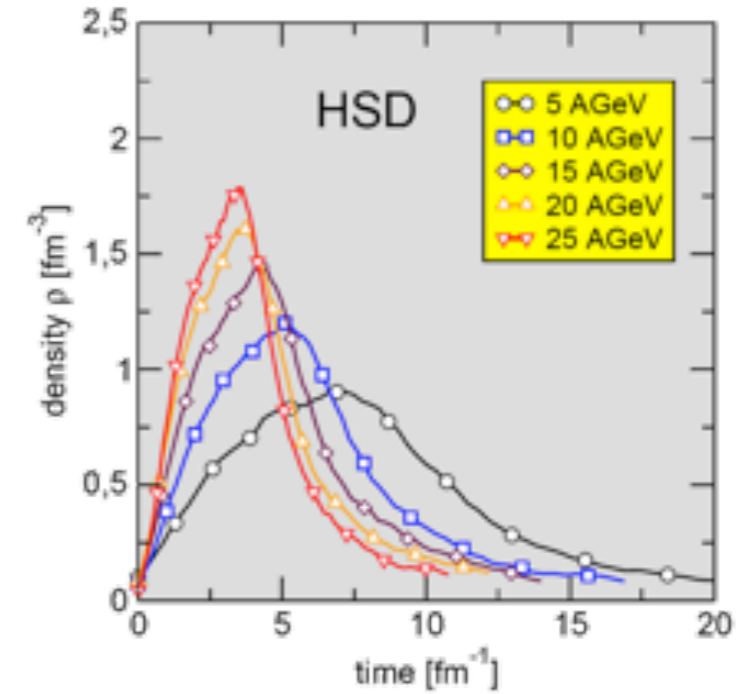
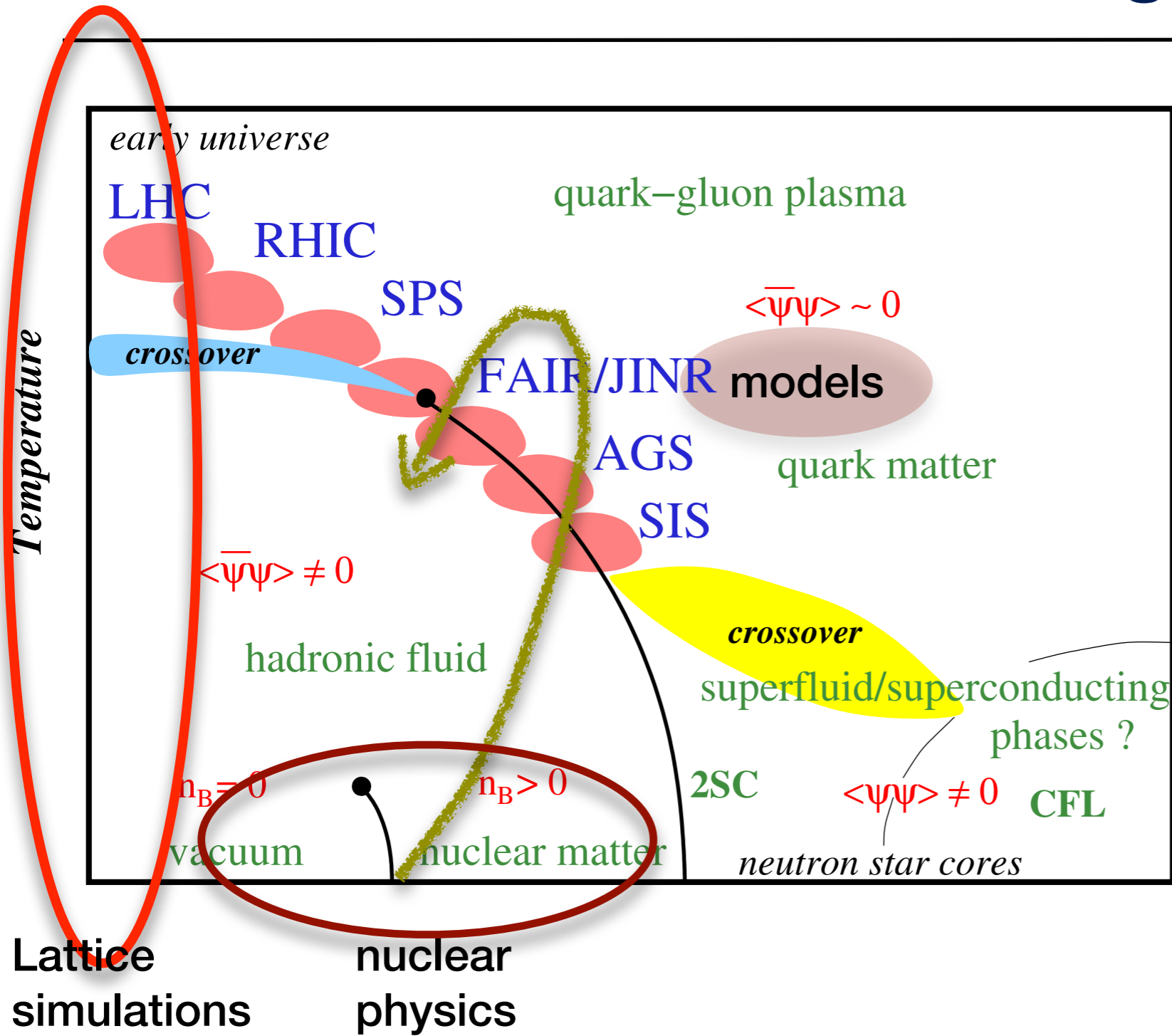


plasma phase

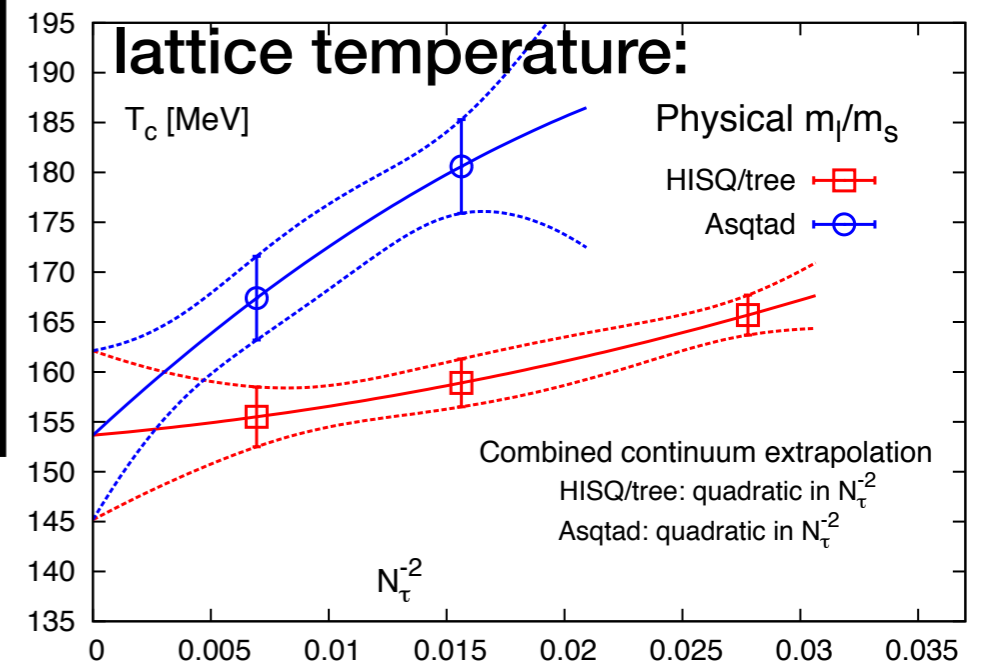
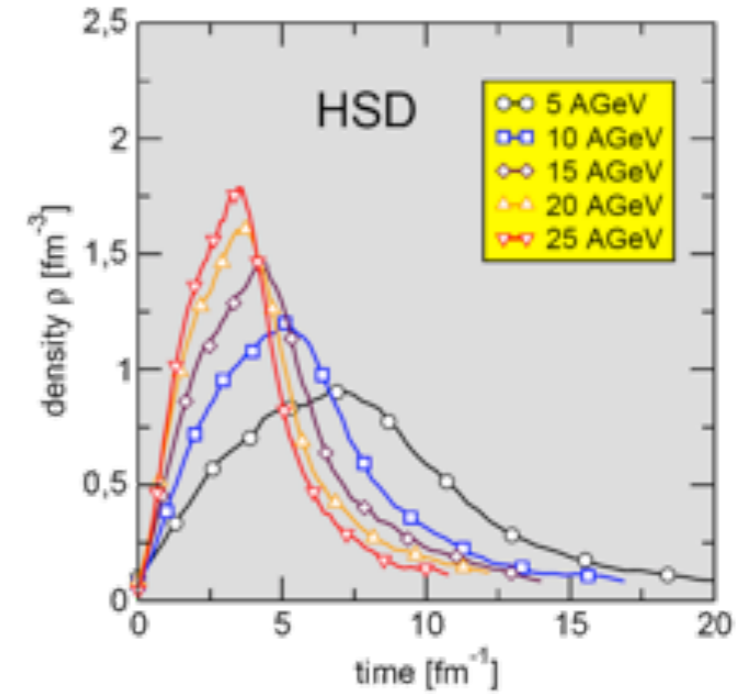
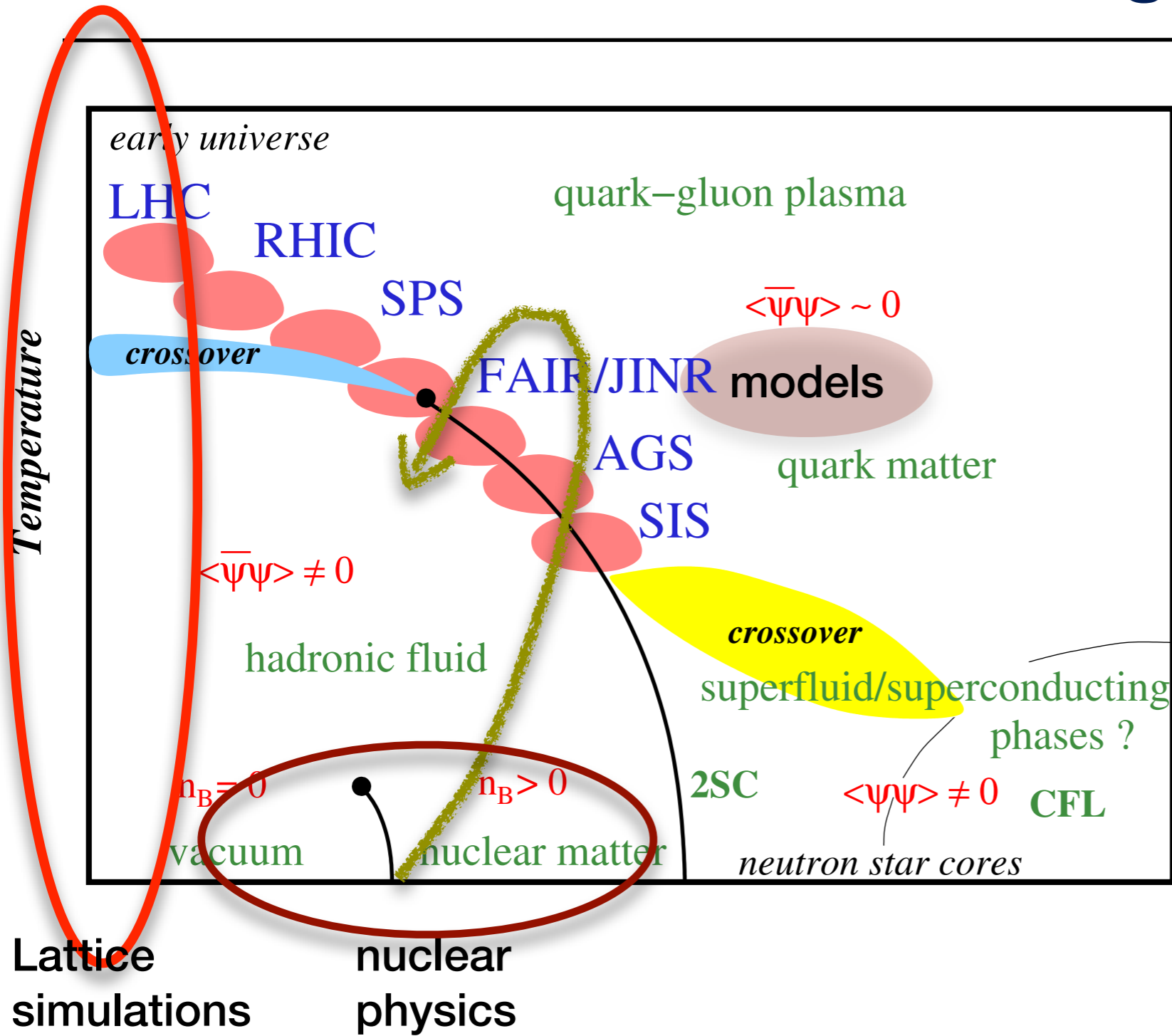


nuclear matter

# QCD Phase diagram



# QCD Phase diagram



courtesy of F. Karsch

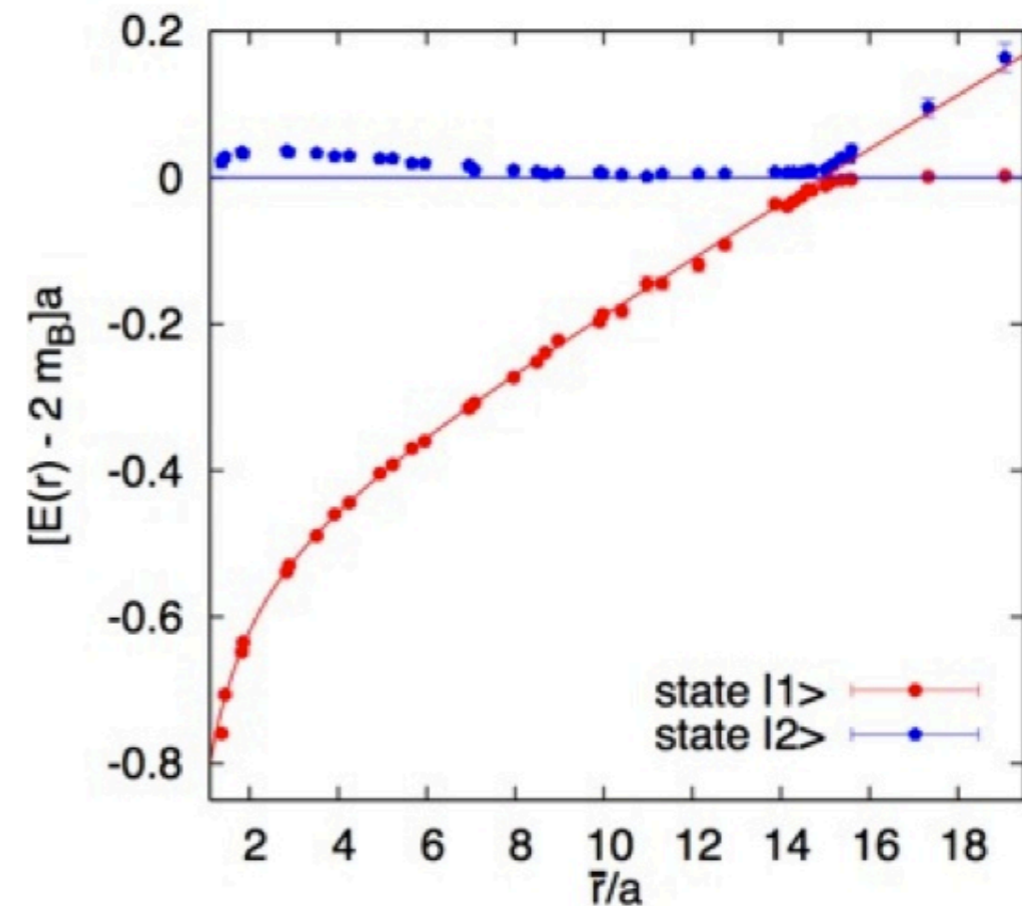
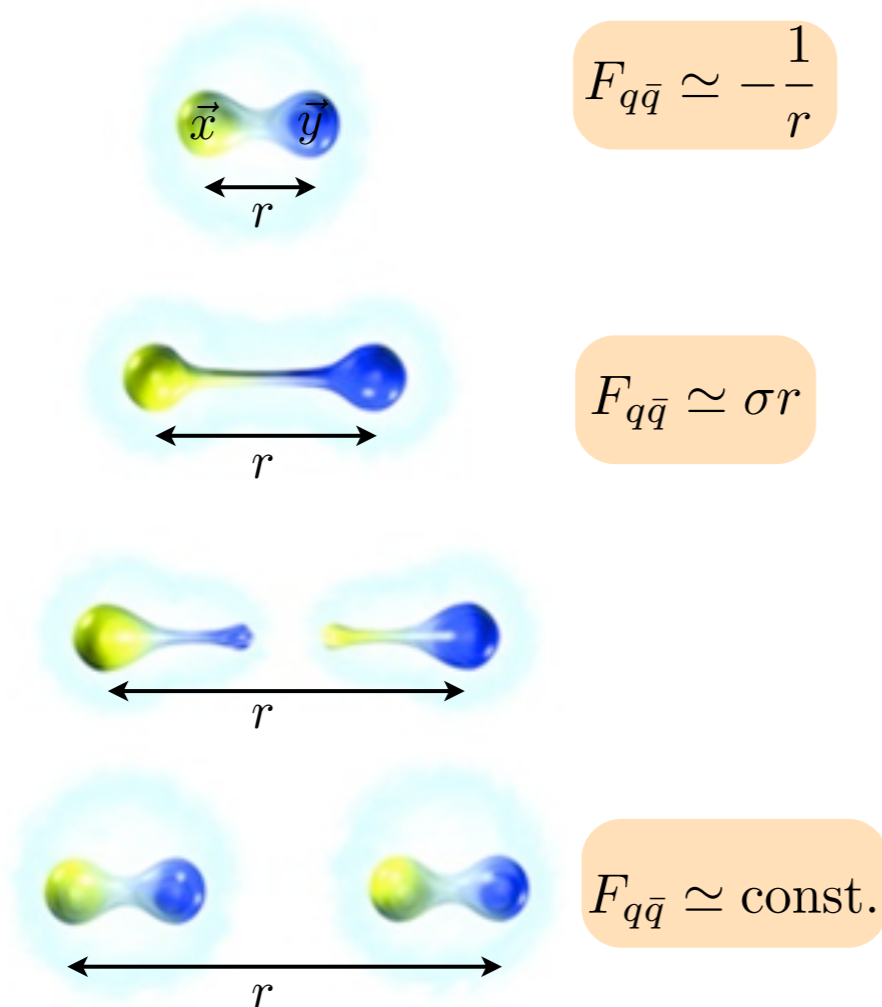


# Confinement

Two important properties of low-energy QCD: **1. Confinement**

color confinement: not quarks are detected but baryons, mesons, etc...

Yang-Mills theory with infinitely heavy test quarks: string tension



Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513

# QCD order parameters

## 1. quark confinement: Polyakov loop $\Phi$ (and $\bar{\Phi}$ )

deconfinement transition in Euclidean spacetime

$$\text{traced Polyakov loop: } l(\vec{X}) = \frac{1}{N_c} \text{Tr}_c \mathcal{P} \exp \left\{ -ig \int_0^{\beta=1/T} dx_4 A_4(x_4, \vec{X}) \right\}$$

under gauge transformation:  $l \rightarrow z_k l$

**Center symmetry**

center of  $SU(N_c)$ :  $Z_{N_c}$  elements of the center commute with all group elements

$$\longrightarrow z_k = \exp(2\pi i k / N_c) \mathbf{1} \quad k = 0, \dots, N_c - 1$$

# QCD order parameters

## 1. quark confinement: Polyakov loop $\Phi$ (and $\bar{\Phi}$ )

deconfinement transition in Euclidean spacetime

- quark fields break center symmetry explicitly
- ➔ center symmetry exact only in pure gluonic theory (YM)
- expectation value of traced Polyakov loop:

$$\Phi = \langle l(\vec{x}) \rangle = \exp(-\beta F_q) \quad ; \quad \bar{\Phi} = \langle l^\dagger(\vec{x}) \rangle = \exp(-\beta F_{\bar{q}})$$

Free energy  $F_q$  of a static quark (similar for antiquark) in hot gluonic medium

# QCD order parameters

## 1. quark confinement: Polyakov loop $\Phi$ (and $\bar{\Phi}$ )

deconfinement transition in Euclidean spacetime

- in **confining** phase: free energy of single quark **diverges** ( $F_q \rightarrow \infty$ )  
 $\Phi \rightarrow 0$
- potential between a quark and antiquark increases linearly at long distances  
( $F_{\bar{q}q}(r \rightarrow \infty) \rightarrow \sigma r$ )  
correlations **vanish**:  $\langle l^\dagger(r \rightarrow \infty)l(0) \rangle \rightarrow 0$
- Expected behavior of **Polyakov loop in pure Yang-Mills**

### Confined (disordered) phase

- free energy  $F_q \rightarrow \infty$
- Polyakov loop  $\Phi = 0$
- correlations:

$$\langle l^\dagger(r \rightarrow \infty)l(0) \rangle \rightarrow 0$$

### Deconfined (ordered) phase

- free energy  $F_q < \infty$
- Polyakov loop  $\Phi \neq 0$
- correlations:

$$\langle l^\dagger(r \rightarrow \infty)l(0) \rangle \rightarrow |\langle l \rangle|^2 \neq 0$$

# QCD order parameters

## 2. chiral symmetry breaking: chiral condensate $\langle \bar{q}q \rangle$

- chiral symmetry in vacuum **spontaneously** broken (this is the source of hadron masses)
- classical QCD symmetry:  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_A$  in **chiral limit**
- spontaneous chiral symmetry breaking

$$SU(N_f)_{L+R \equiv V} \times U(1)_B$$

→  $N_f^2 - 1$  massless Nambu-Goldstone bosons ( $N_f > 1$ )

- chiral condensate

$$\langle \bar{q}q \rangle = \langle \bar{q}_R q_L + \bar{q}_L q_R \rangle \quad q_{R/L} = \frac{1}{2} (1 \pm \gamma_5) q \quad \text{right/left projected fields}$$

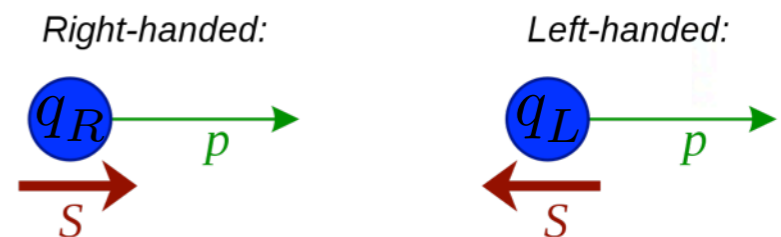
- Expected behavior of **chiral condensate**

Broken (ordered) phase

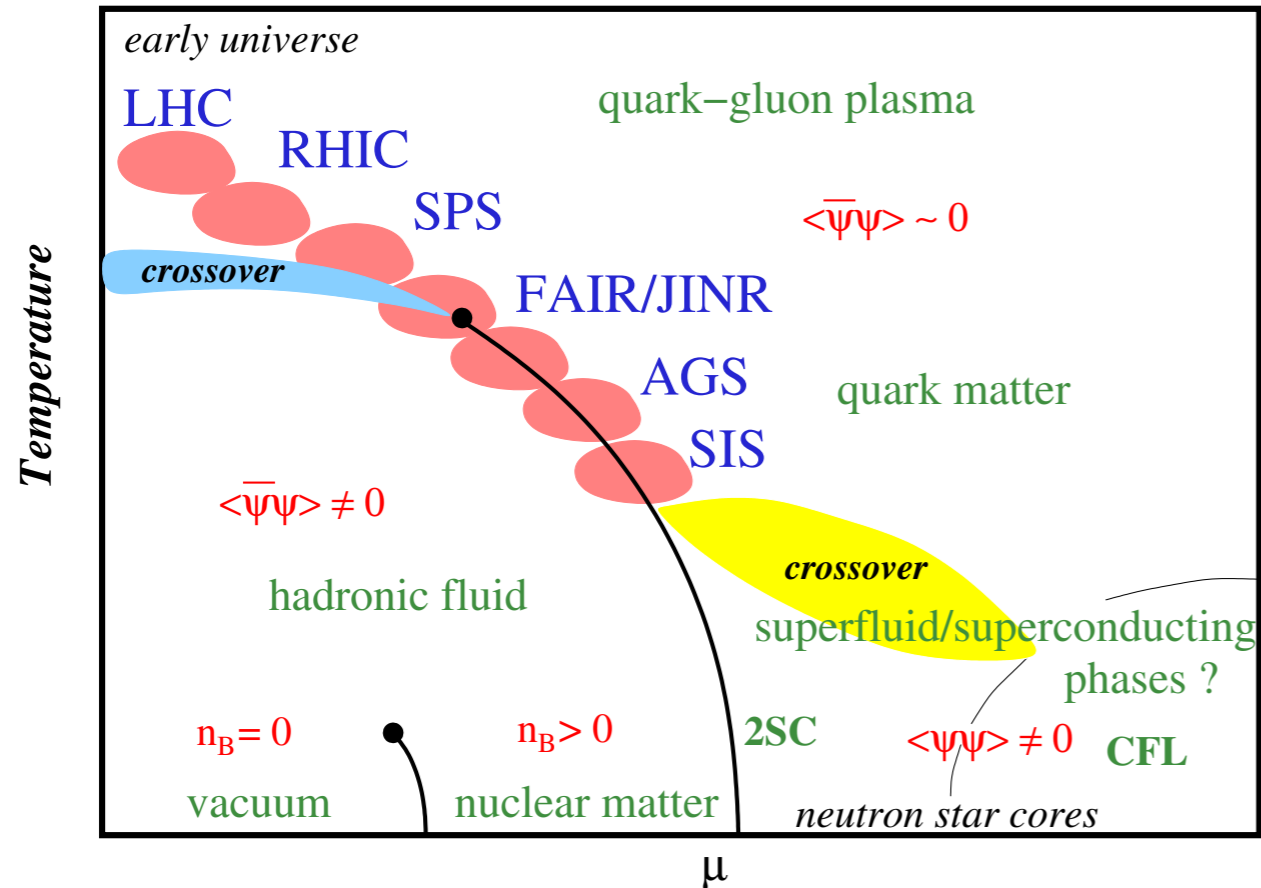
- condensate  $\langle \bar{q}q \rangle \neq 0$

Symmetric (disordered) phase

- condensate  $\langle \bar{q}q \rangle = 0$



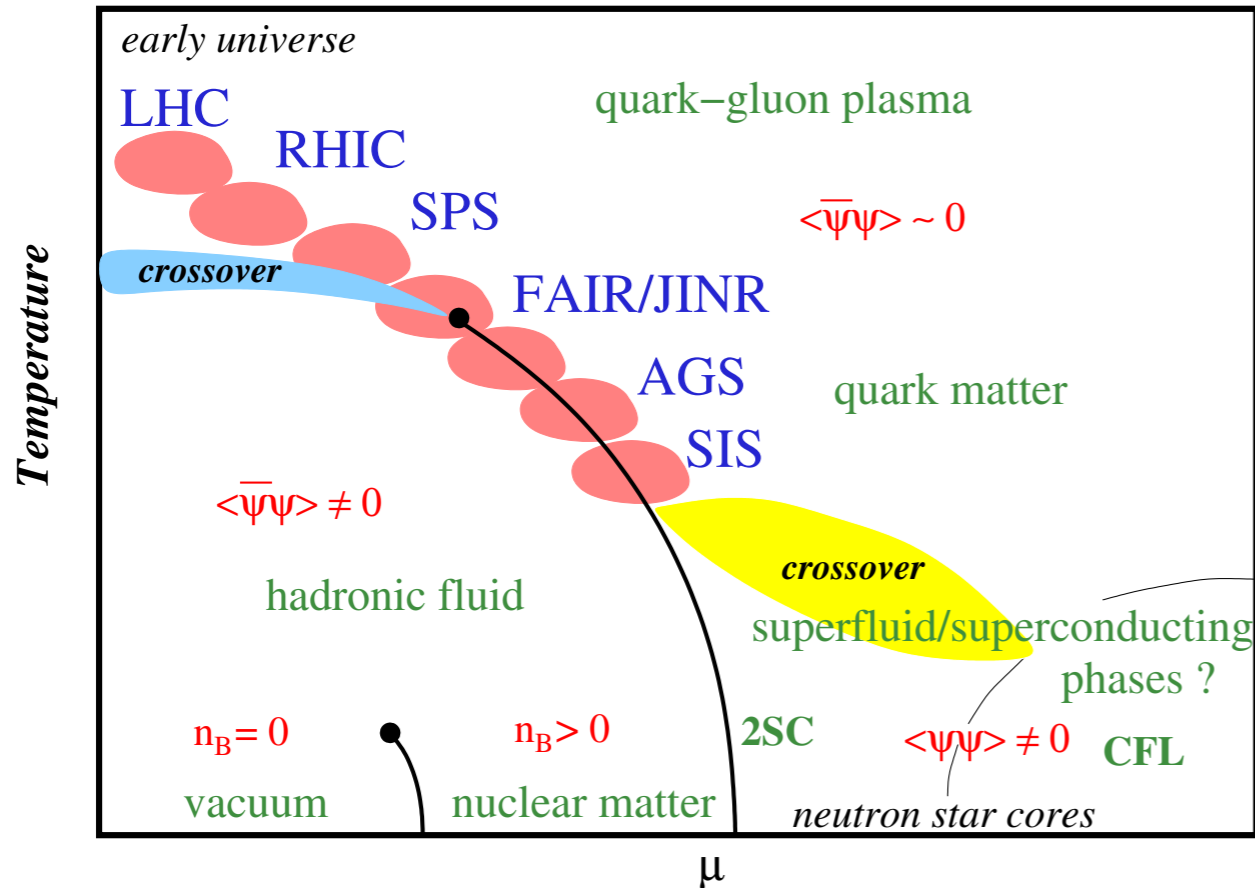
# Conjectured QC<sub>3</sub>D phase diagram



at densities/temperatures of interest  
only model calculations available

- can one improve the model calculations?
- remove model parameter dependency

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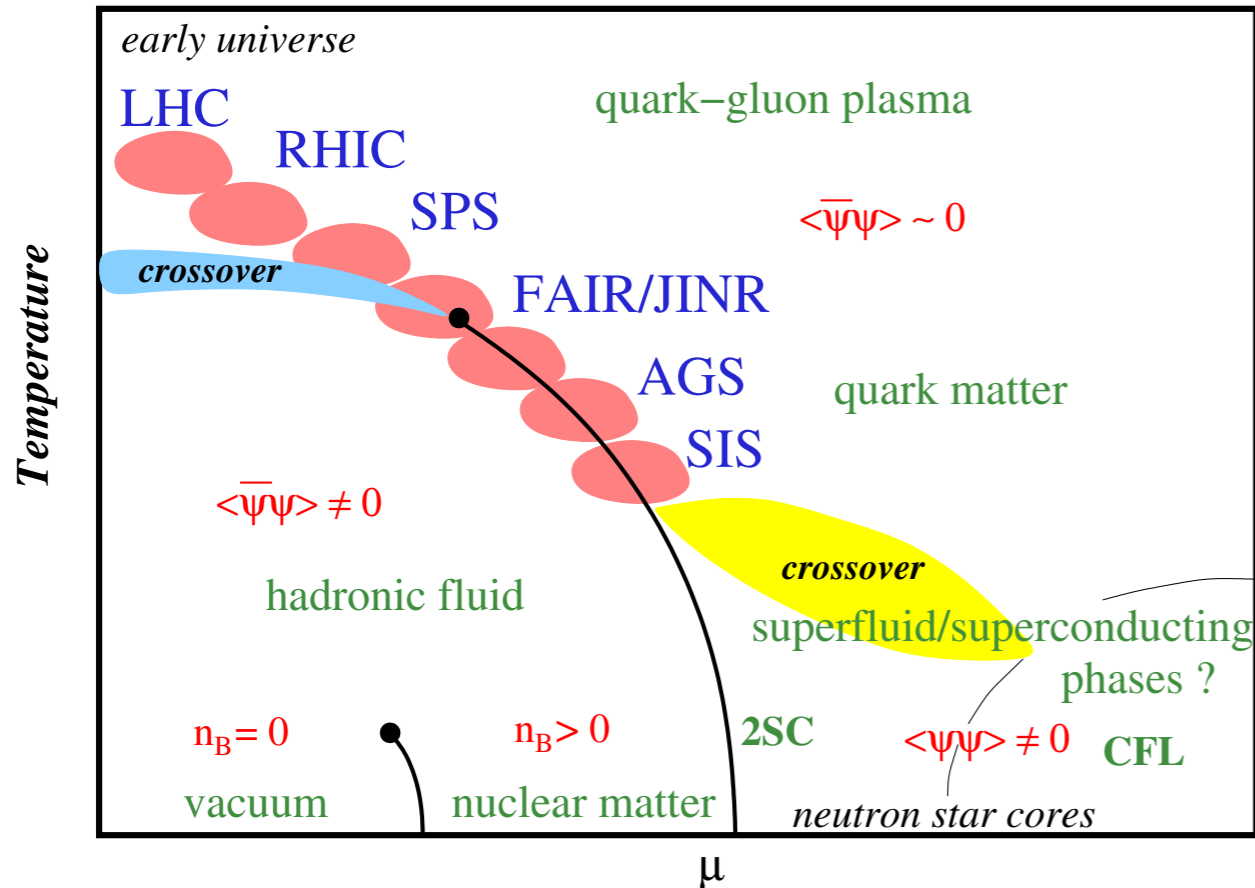
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## Theoretical questions:

related to chiral & deconfinement transition

- **CEP**: existence/location/number
- **Quarkyonic phase**: coincidence of both transitions at  $\mu = 0$  &  $\mu > 0$  ?
- relation between chiral & deconfinement?  
**chiral CEP/deconfinement CEP?**
- **finite volume effects?**  
→ lattice comparison
- **role of fluctuations?** so far mostly mean-field results **effects of fluctuations are important**  
e.g. size of critical region around CEP
- **What are good experimental signatures?**  
→ higher moments more sensitive to criticality  
deviation from HRG model for

# Conjectured QC<sub>3</sub>D phase diagram



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non-perturbative continuum functional methods

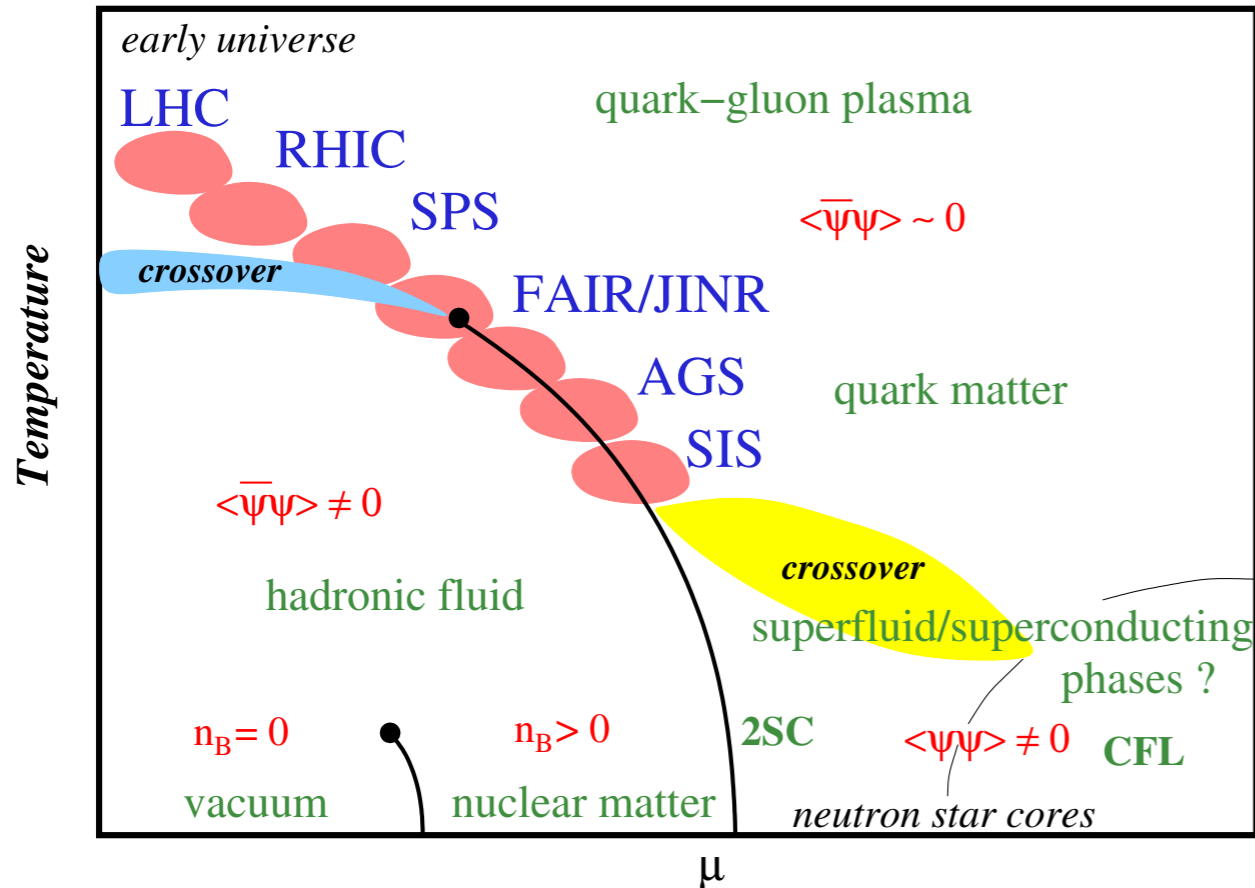
→ complementary to lattice

⇒ no sign problem  $\mu > 0$

⇒ chiral symmetry/fermions/small masses/chiral limit



# Conjectured QC<sub>3</sub>D phase diagram



## Theoretical questions:

related to chiral & deconfinement transition

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Method of choice: **Functional Renormalization Group (FRG)**

e.g. (Polyakov)-quark-meson model truncation

- good description for chiral sector
- implementation of gauge dynamics (deconfinement sector)

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# Agenda

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- **QCD-like model studies**
  - chiral and deconfinement aspects
  - two and three flavor
- **Mean-field approximation vs. Renormalization Group methods**
- **role of baryons - two color QCD**

# Polyakov-Quark-Meson (PQM) Model

Chiral effective model:

- quarks:  $\psi$
- mesons:  $\sigma, \vec{\pi}$
- gauge fields:  $A_\mu^a$  in  $D_\mu = \partial_\mu + iA_\mu \rightarrow$  Polyakov-loop (PQM) model

PQM Lagrangian:

[BJS, J. Pawłowski, J. Wambach 2007]

$$\begin{aligned} \mathcal{L}_{\text{PQM}} = & \bar{\psi} \left( \not{D} + g(\sigma + i\gamma^5 \vec{\pi} \vec{\tau}) - \mu\gamma^0 \right) \psi \\ & + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + V(\vec{\phi}) \end{aligned}$$

# Mean-Field Approximations (MFA)

Integration of quarks, neglect bosonic fluctuations

Grand potential

$$\Omega(T, \mu; \sigma) = \Omega_{\text{vac}} + \Omega_T + V_{\text{MF}}(\sigma) \quad (+\mathcal{U}_{\text{Poly}}(\Phi))$$

vacuum term: sharp three-momentum cutoff

$$\Omega_{\text{vac}}(\Lambda) = -4 \int^{\Lambda} \frac{d^3 p}{(2\pi)^3} \sqrt{\vec{p}^2 + m_q^2}$$

for each cutoff: adjust model parameters like  $f_{\pi}, m_{\sigma}, m_{\pi}$

standard MFA: no-sea term  $\Lambda = 0$

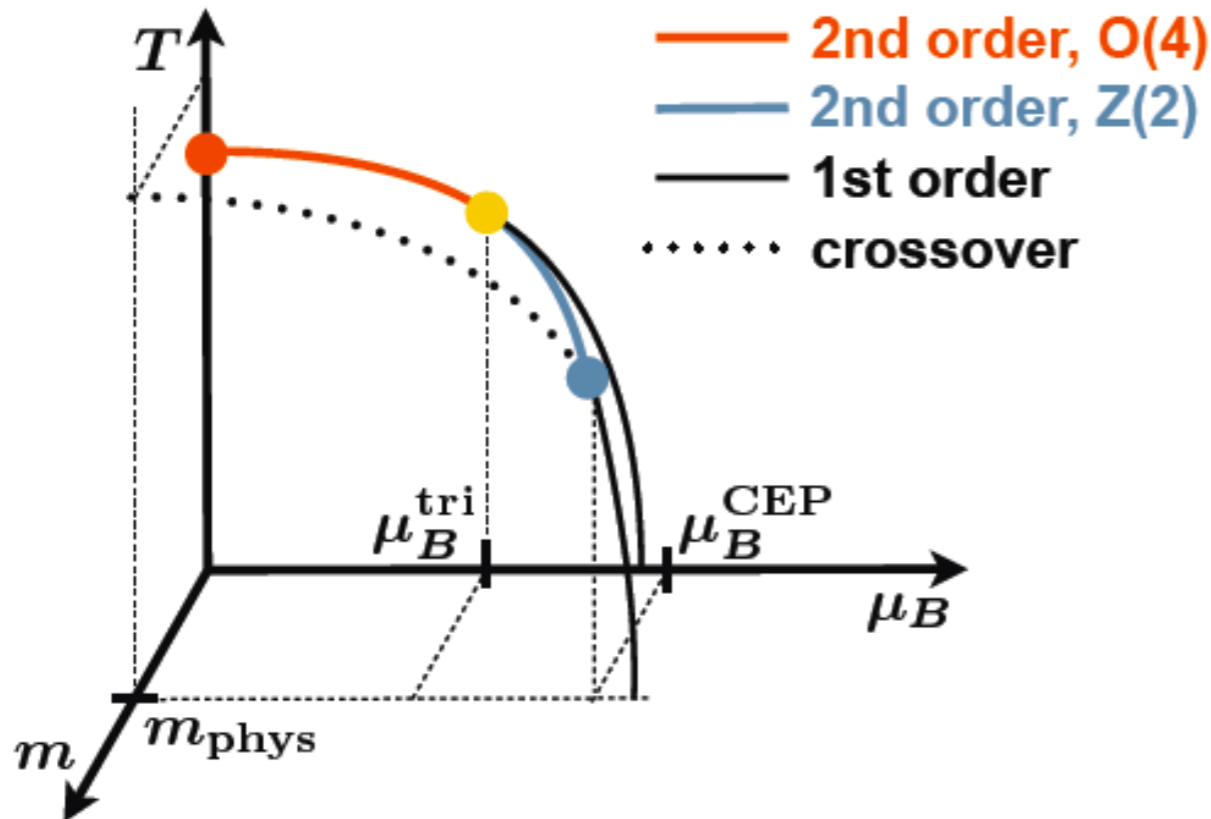
# Chiral transition

Fluctuations of order parameter  $\rightarrow \infty$  at 2<sup>nd</sup> order transition  
 critical fluctuations  $\rightarrow$  phase boundary?

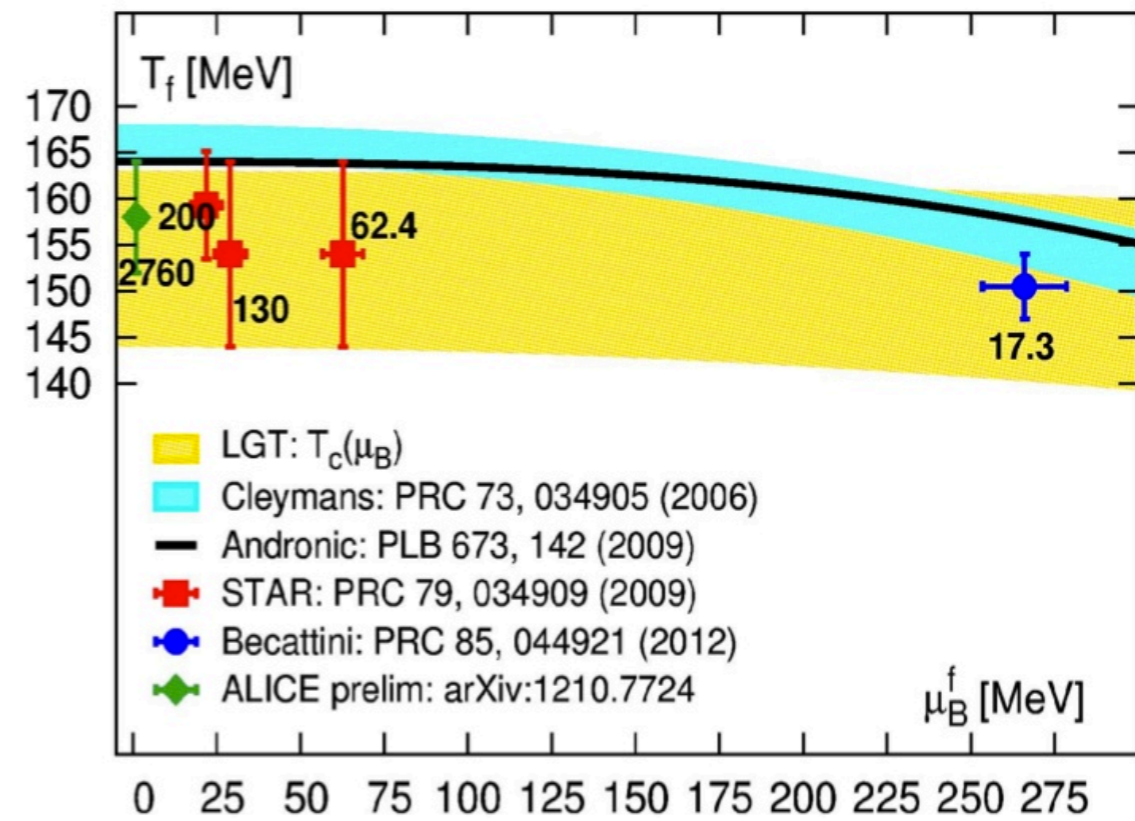
How can we probe a transition?

- singular behaviour in  $\frac{\partial^n p(X)}{\partial X^n}$  with  $X = T, \mu, \dots$
- higher order cumulants  $c_n \equiv \frac{\partial^n p(T, \mu)}{\partial (\mu/T)^n}$

... more sensitive to criticality



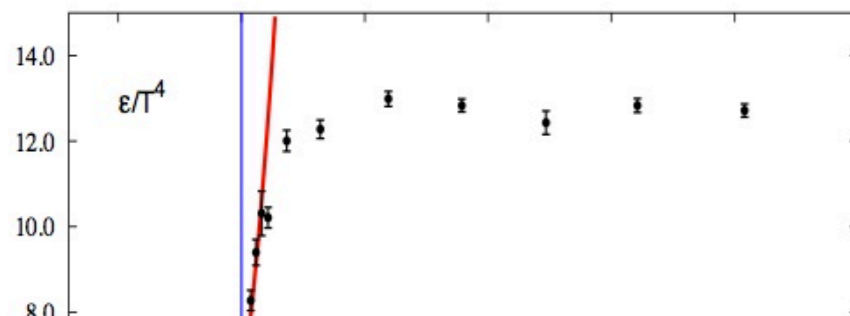
freeze-out close to chiral crossover line



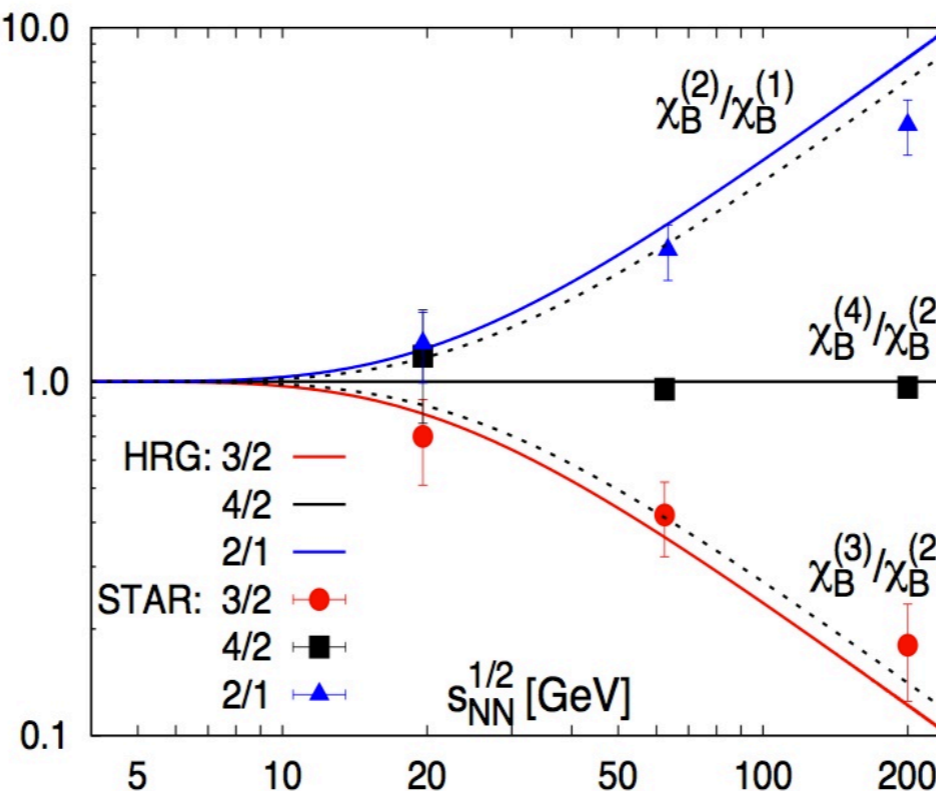
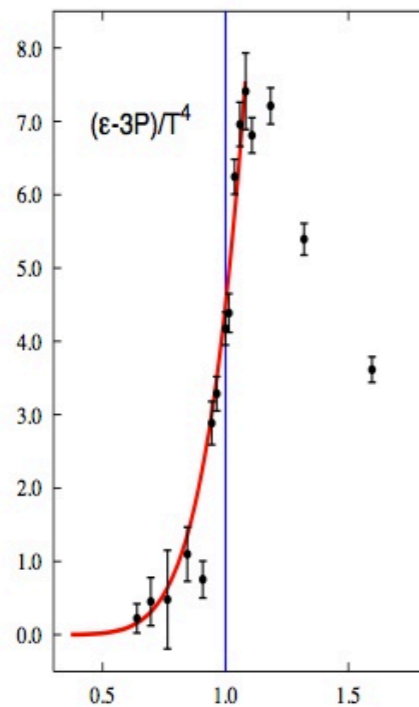
[HotQCD, QM 2012]

# Hadron Resonance Gas (HRG) Model

HRG model: good lattice data description

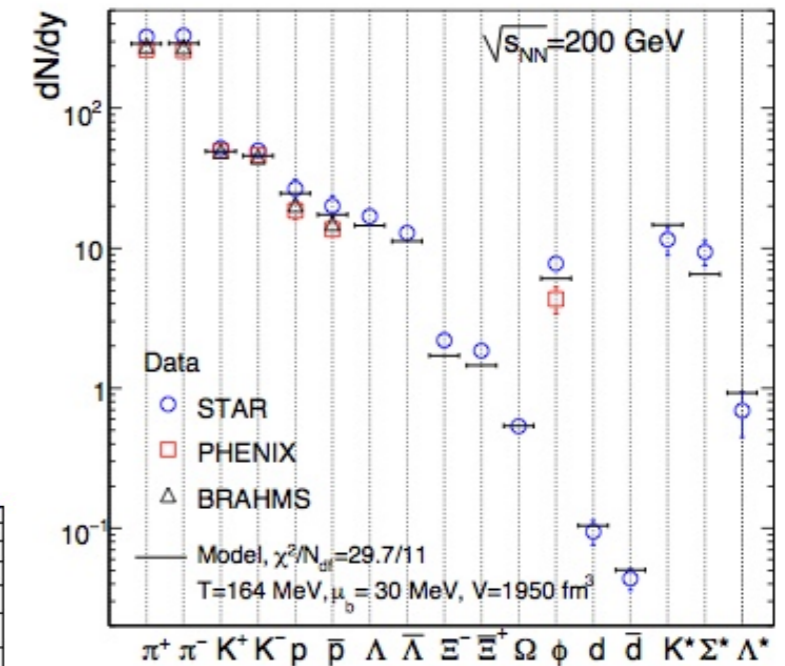


HRG model: no critical fluctuations



HRG model versus experiment

[Andronic et al. 2011]



[Karsch, Redlich 2010]

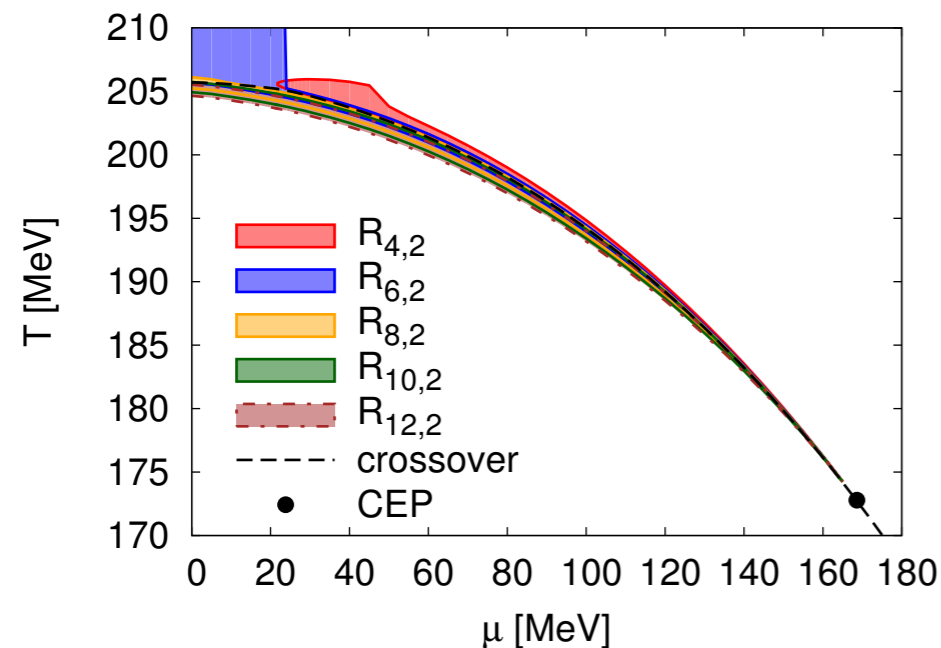
# Role of vacuum term in (P)QM models

Fluctuations of higher moments exhibit **strong variation from HRG model**

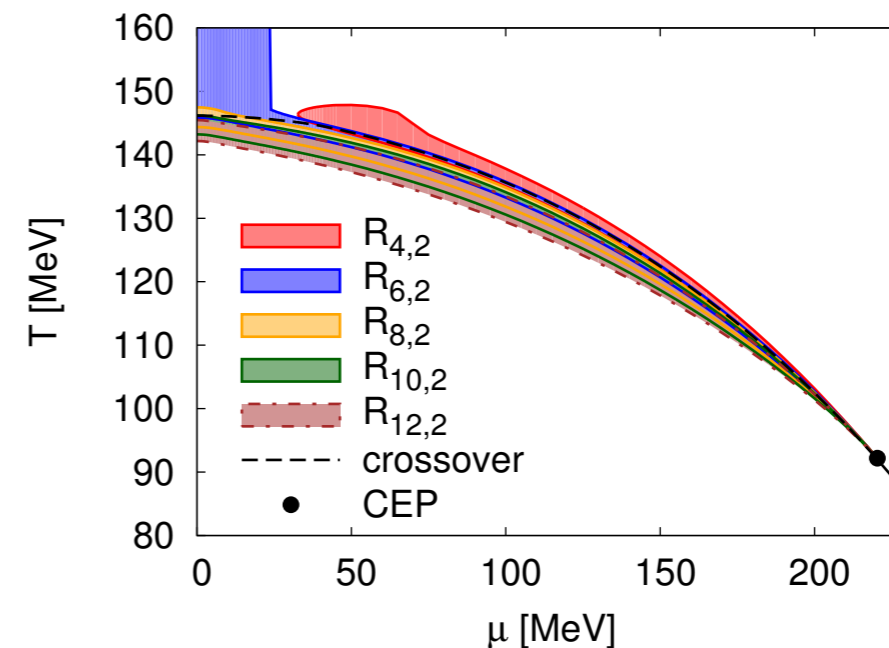
[Karsch, Redlich, Friman, Koch et al. 2011]

- → turn negative
- higher moments:  $R_{n,m}^q = c_n/c_m$
- regions where  $R_{n,2} < 0$  along crossover in the phase diagram

unquenched PQM MFA



QM MFA w/o vacuum



role of vacuum term in (P)QM models see [BJS, M. Wagner 2012]

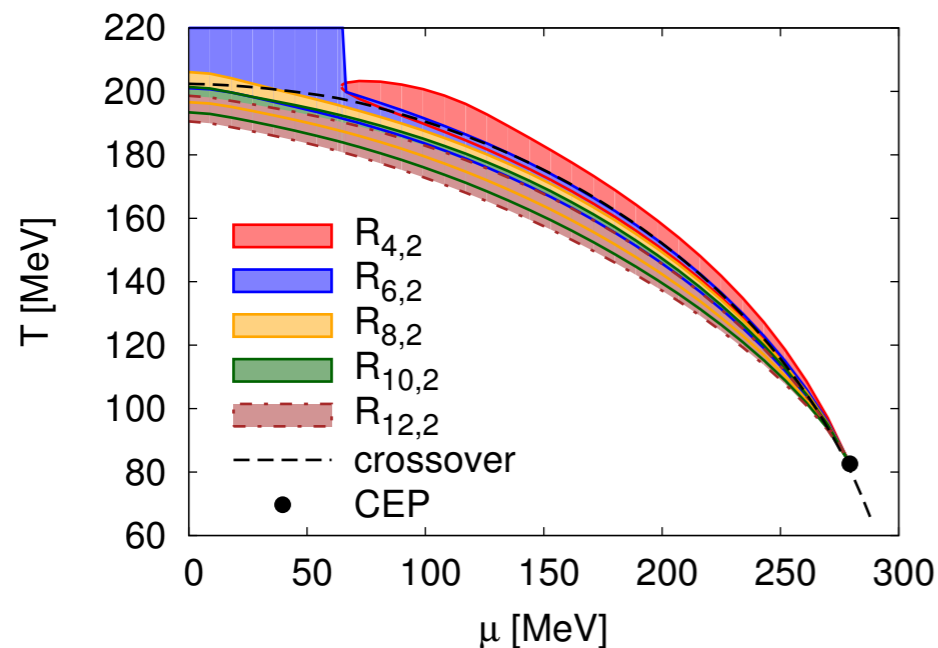
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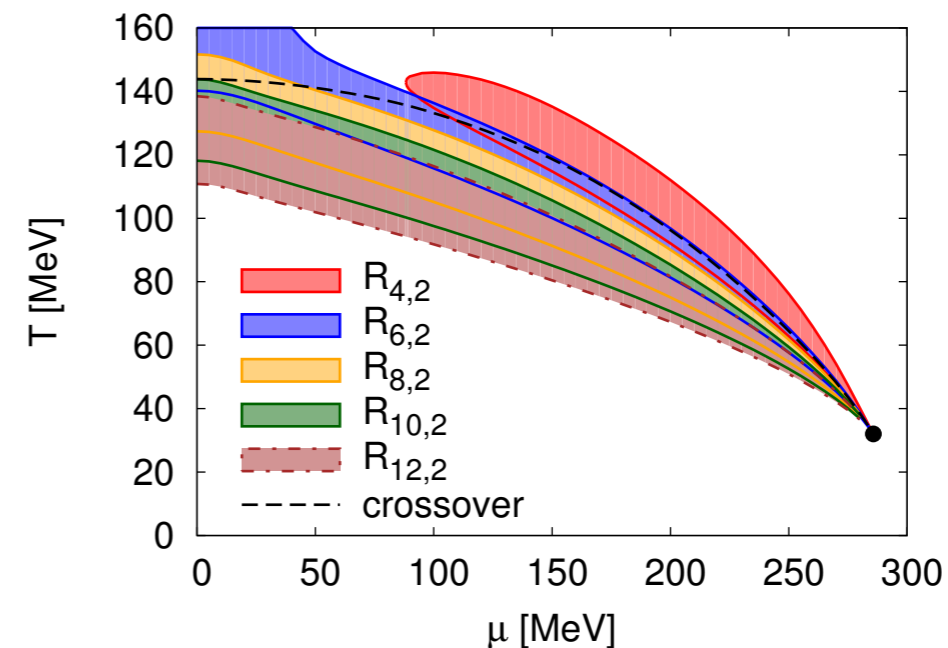
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# Mean-Field PQM

# $N_f=2+1$

Two improvements:

1. sea contribution

2. matter back-coupling

no-sea: red dashed line

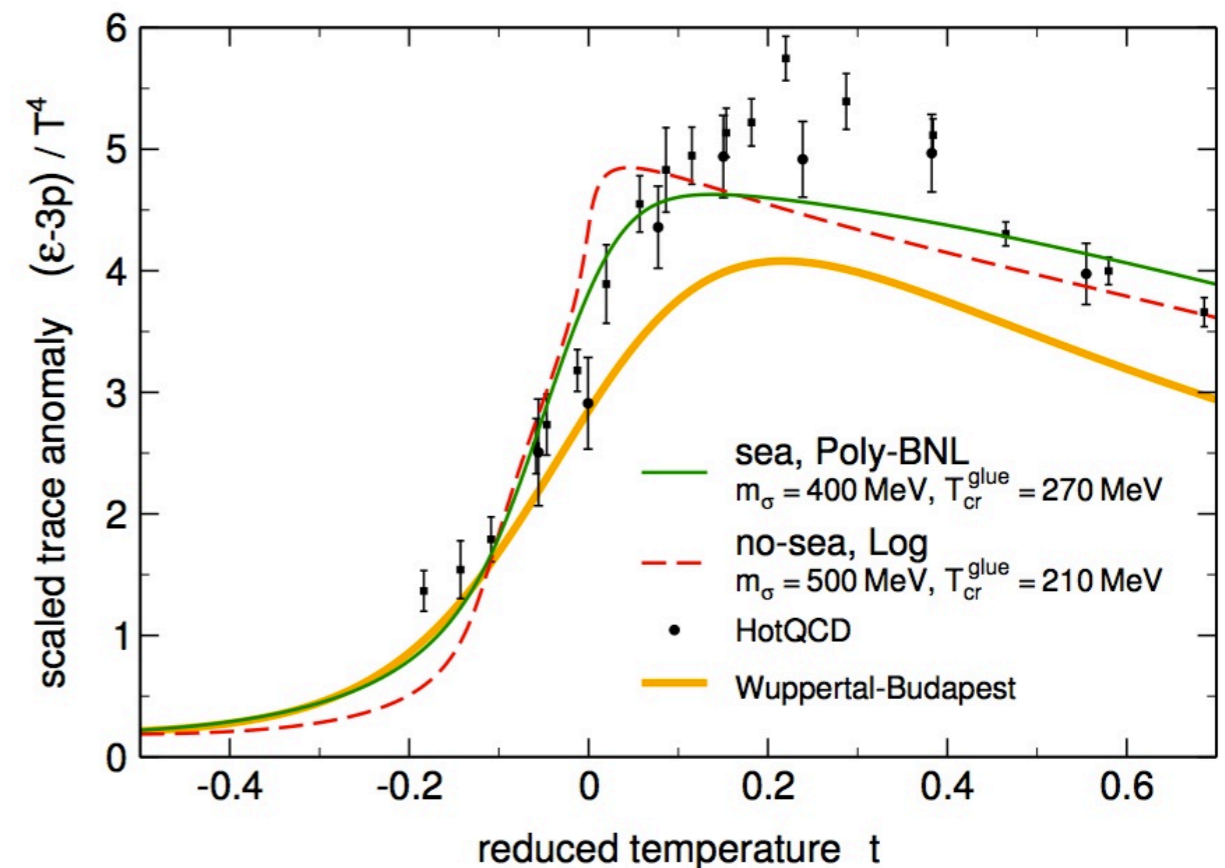
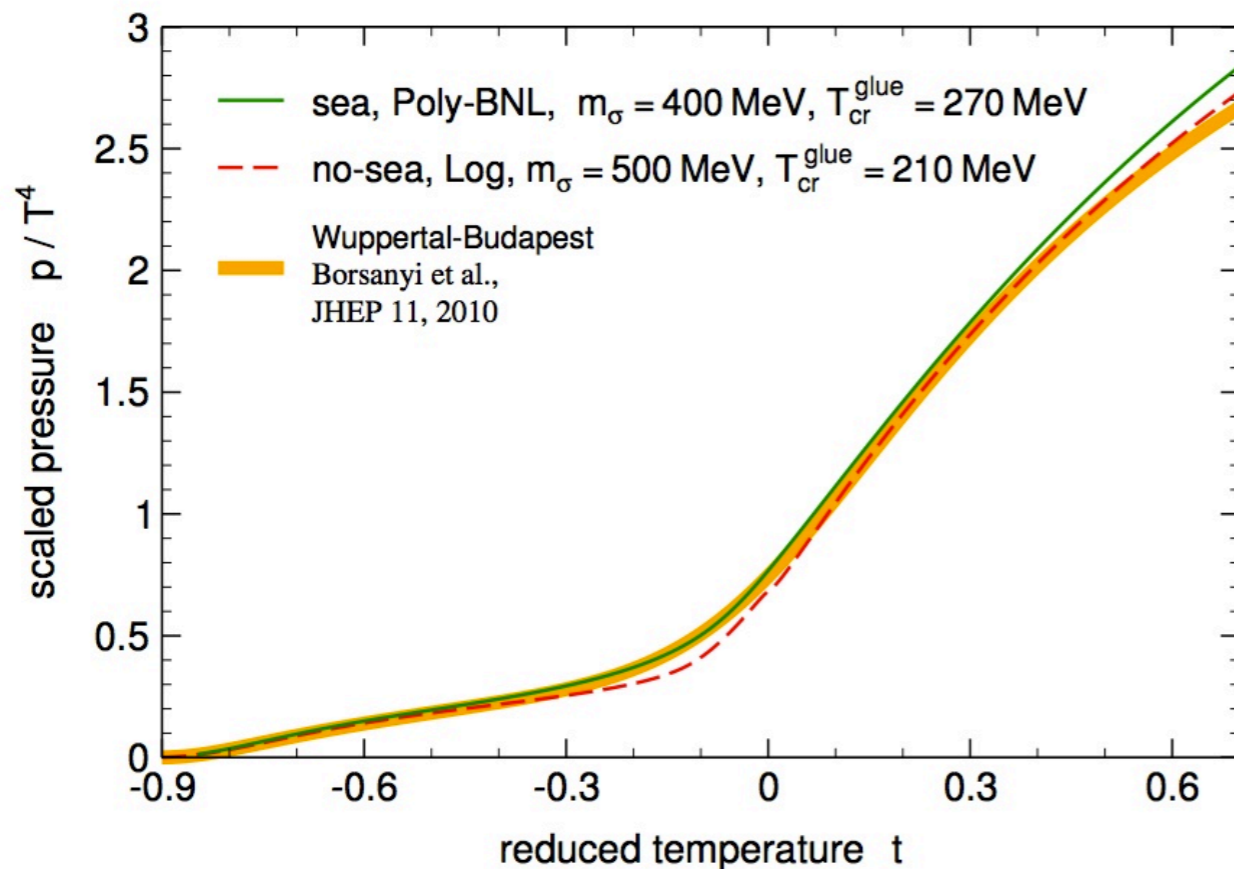
sea: green line (vacuum term included)

an effective unquenching

$$\mathcal{U}_{glue}(t_{glue}) = \mathcal{U}_{YM}(t_{YM})$$

$$\text{with } t_{YM}(t_{glue}) = 0.57 t_{glue}$$

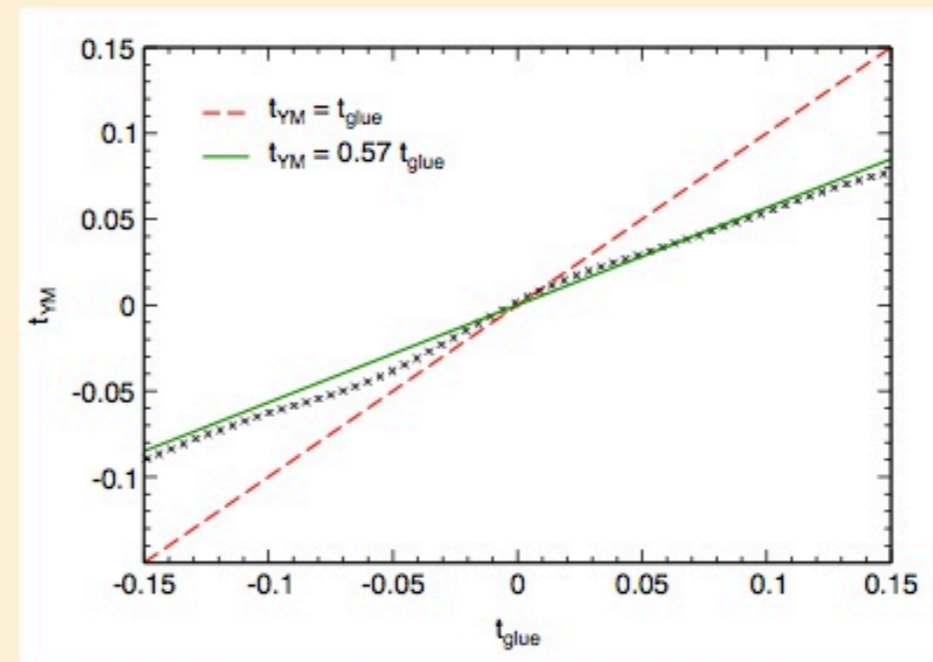
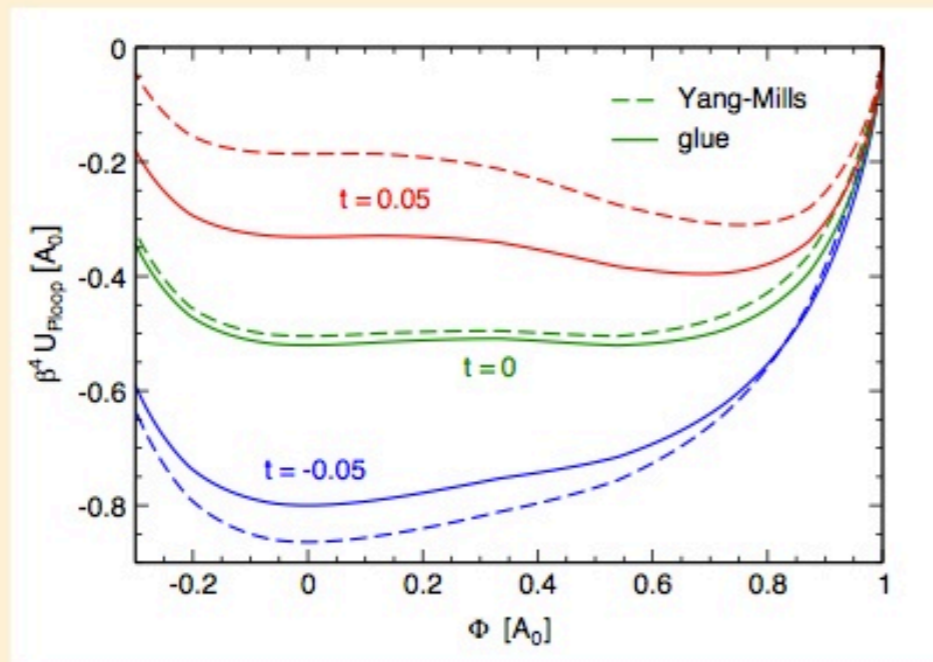
[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]



# Matching the scales with FRG

## Map between Yang-Mills and glue potential: $t_{\text{YM}}(t_{\text{glue}})$

L. M. Haas, RS, J. Braun, J. M. Pawłowski, J. Schaffner-Bielich, PRD 87, 076004, 2013



$$\rightarrow t_{\text{YM}}(t_{\text{glue}})$$

$$t_{\text{YM}} = 0.57 t_{\text{glue}} \quad \partial t_{\text{YM}} / \partial t_{\text{glue}} < 1$$

$$\Rightarrow \mathcal{U}_{\text{glue}}(t, \Phi) = \mathcal{U}_{\text{YM}}(t_{\text{YM}}(t), \Phi)$$

# Mean-Field PQM

# $N_f=2+1$

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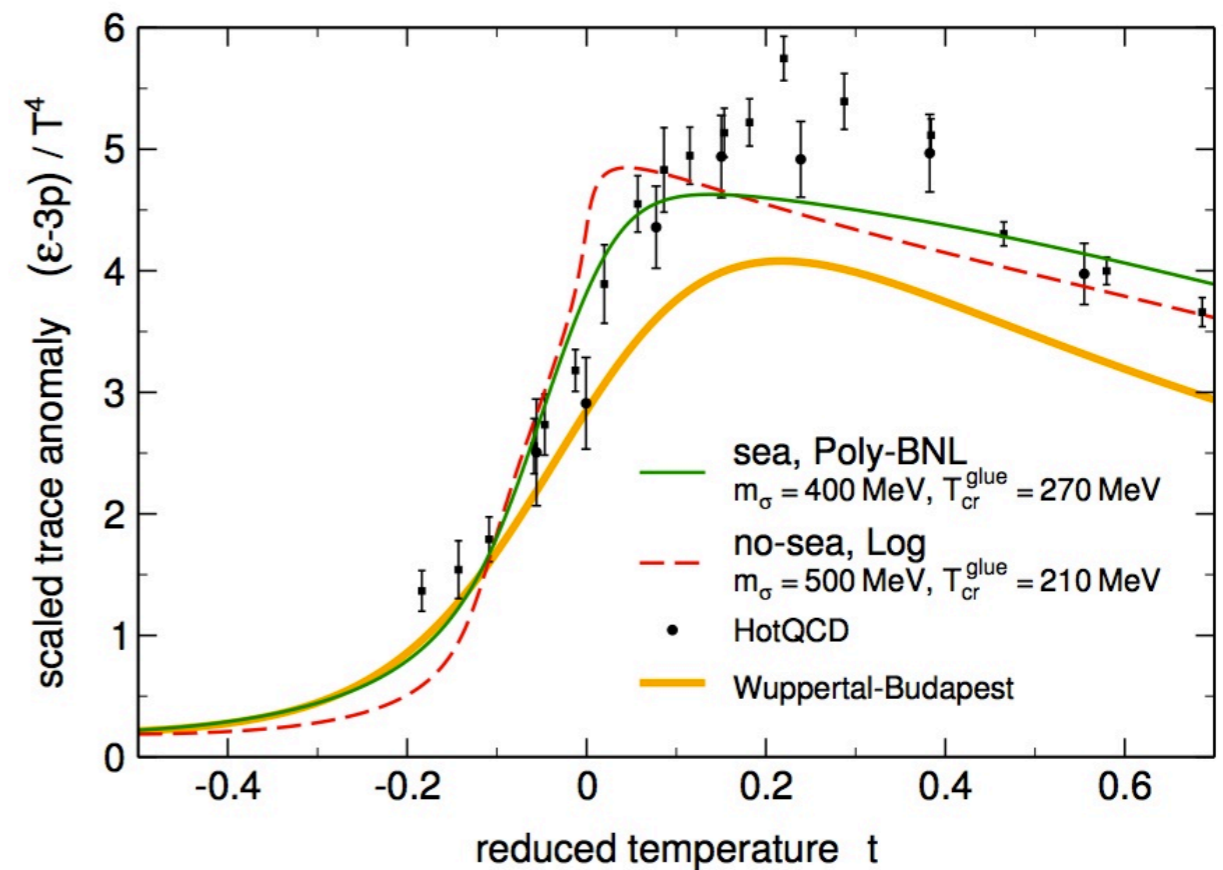
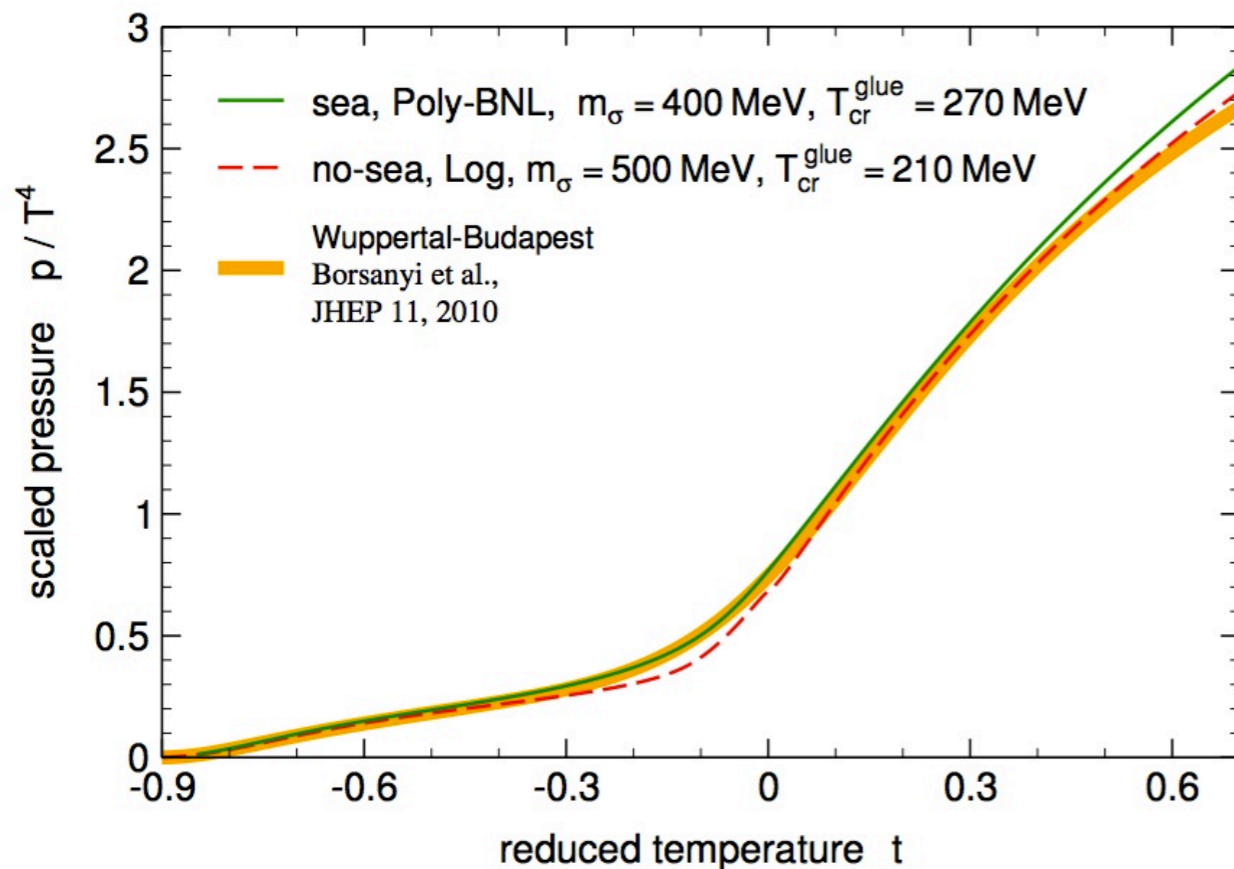
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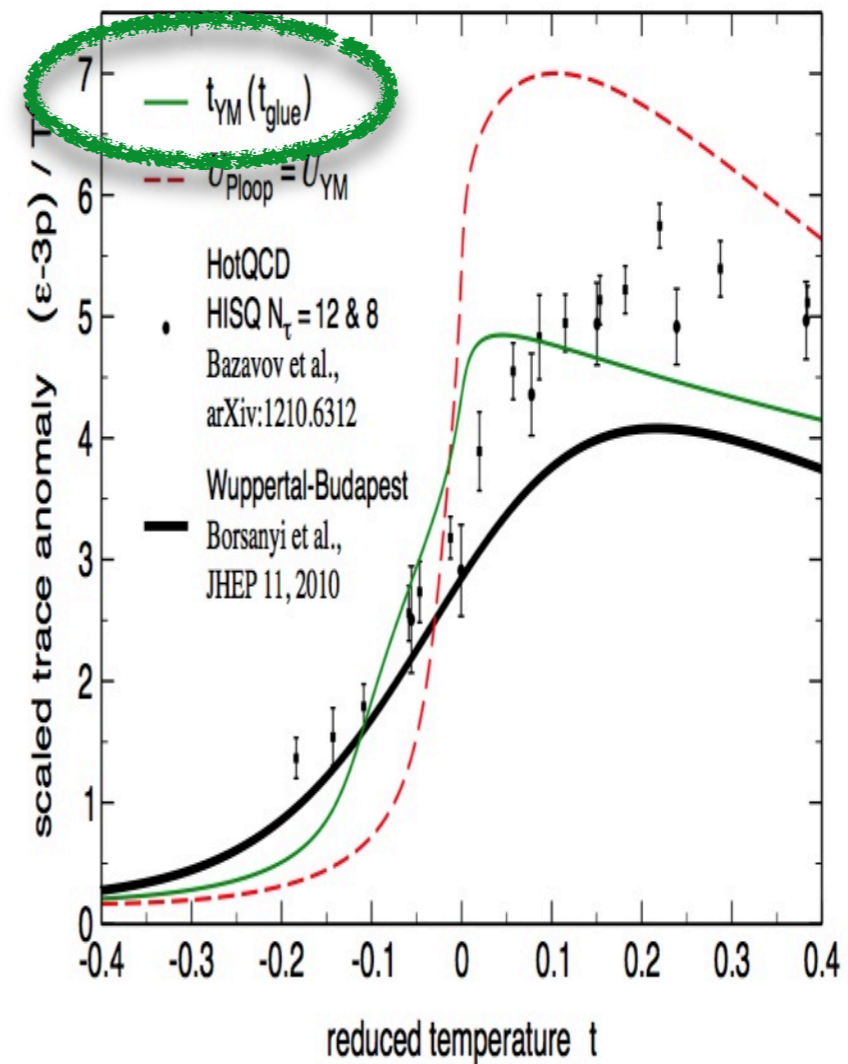
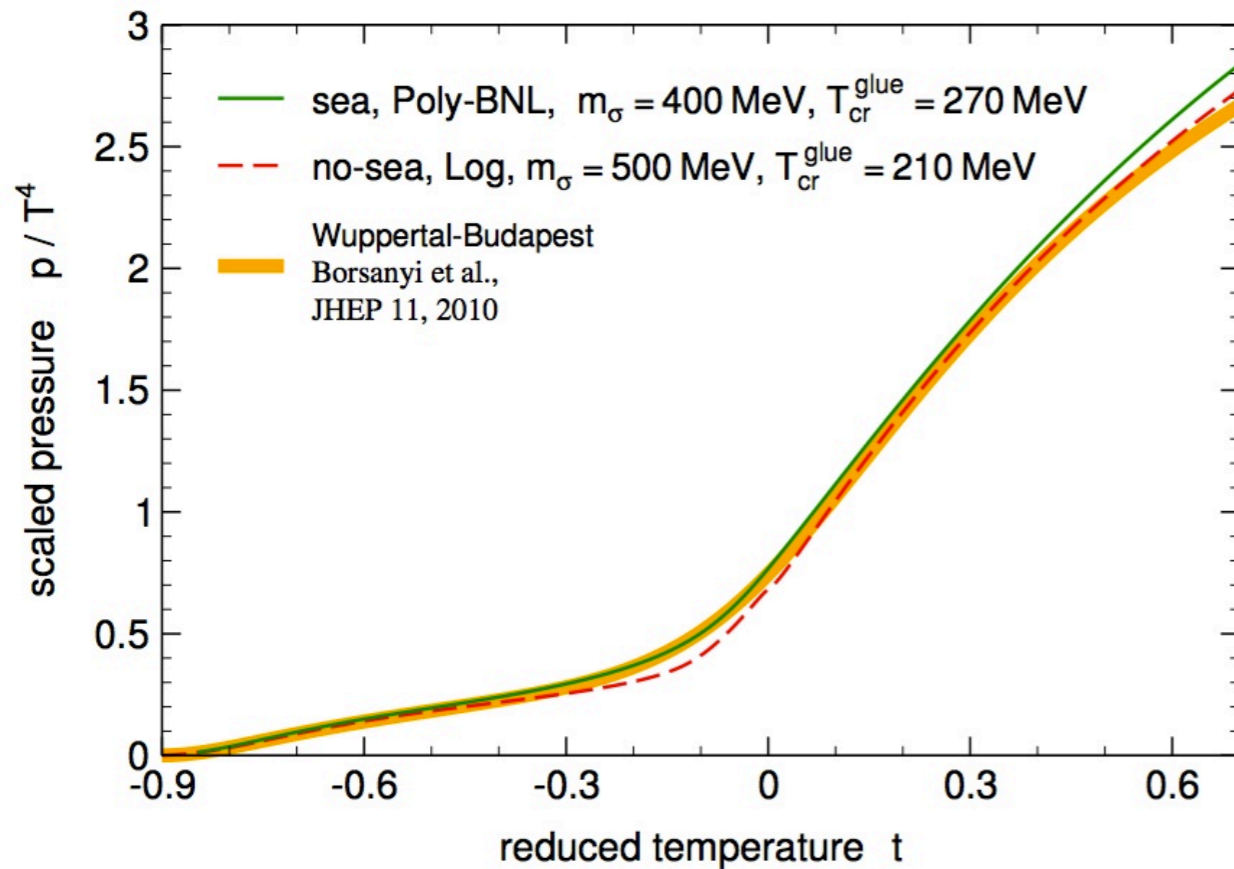
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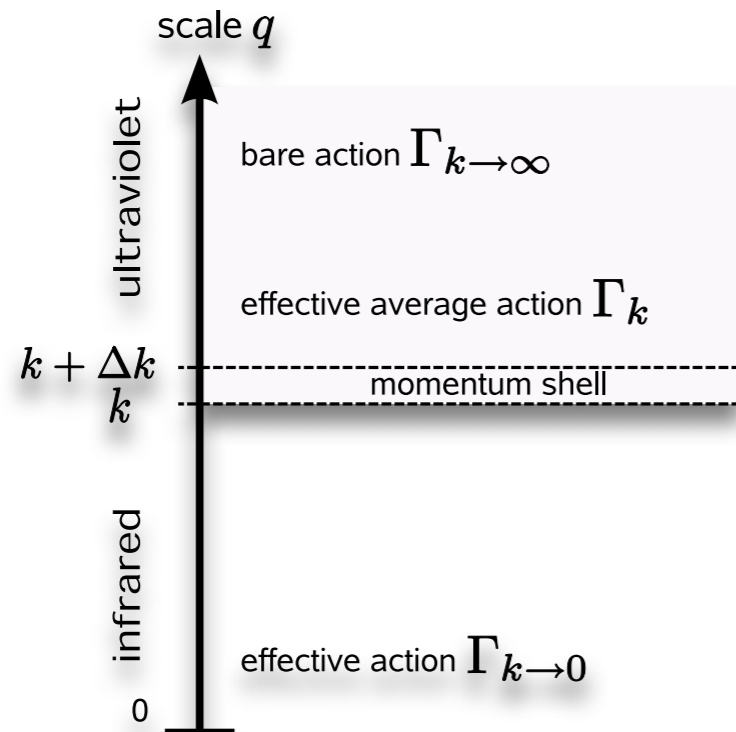
# Functional RG (FRG) Approach

■  $\Gamma_k[\phi]$  scale dependent effective action

$$t = \ln(k/\Lambda)$$

$R_k$  regulators

$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi}$$



## FRG (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k \partial_k \Gamma_k[\phi] \sim \frac{1}{2} \text{Tr} \left( \frac{R_k}{\Gamma_k^{(2)} + R_k} \right)$$

[Wetterich 1993]

■ Ansatz for  $\Gamma_k$ : Leading order derivative expansion

arbitrary potential

$$\Gamma_k = \int d^4 x \bar{q} [i \gamma_\mu \partial^\mu - g(\sigma + i \vec{\tau} \vec{\pi} \gamma_5)] q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 - c \sigma$$

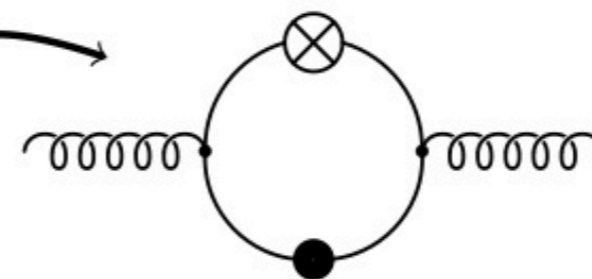
# FRG and QCD

**full dynamical QCD FRG flow:** fluctuations of  
**gluon, ghost, quark and meson** (via hadronization) fluctuations

Braun, Fister, Haas, Marhauser, Pawłowski; 2009

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[ \text{Gluon loop} - \text{Ghost loop} \right] - \left[ \text{Quark loop} + \frac{1}{2} \text{Meson loop} \right]$$

in presence of dynamical quarks:  
 gluon propagator modified:



⇒ pure Yang Mills flow + matter back-coupling

# FRG: quark-meson truncation

First step: flow for **quark-meson** model truncation: neglect **YM contributions**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[ \text{diagram 1} - \text{diagram 2} \right] - \left[ \text{diagram 3} + \frac{1}{2} \text{diagram 4} \right]$$

# Flow for quark-meson truncation

Two quark flavor

$$\Gamma_k = \int d^4x \left\{ \bar{\psi}(\not{\partial} + \mu\gamma_0 + ih(\sigma + i\gamma_5\vec{\tau}\vec{\pi}))\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \Omega_k(\sigma, \vec{\pi}) \right\}$$

$$\Omega_k(\sigma, \vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - c\sigma$$

flow for grand potential

[BJS, J.Wambach]

$$\partial_t \Omega_k(T, \mu; \phi) = \frac{k^4}{12\pi^2} \left[ \frac{3}{E_\pi} \coth\left(\frac{E_\pi}{2T}\right) + \frac{1}{E_\sigma} \coth\left(\frac{E_\sigma}{2T}\right) - \frac{2N_c N_f}{E_q} \left\{ \tanh\left(\frac{E_q - \mu}{2T}\right) + \tanh\left(\frac{E_q + \mu}{2T}\right) \right\} \right]$$

$$E_\pi^2 = 1 + 2\Omega'_k/k^2, \quad E_\sigma^2 = 1 + 2\Omega'_k/k^2 + 4\phi^2\Omega''_k/k^2, \quad E_q^2 = 1 + g^2\phi^2/k^2$$

$$\phi \sim \langle \bar{q}q \rangle, \quad \Omega'_k = \partial\Omega_k/\partial\phi \quad \text{etc}$$

- quark fluctuations: chiral symmetry breaking
- meson fluctuations: chiral symmetry restoration



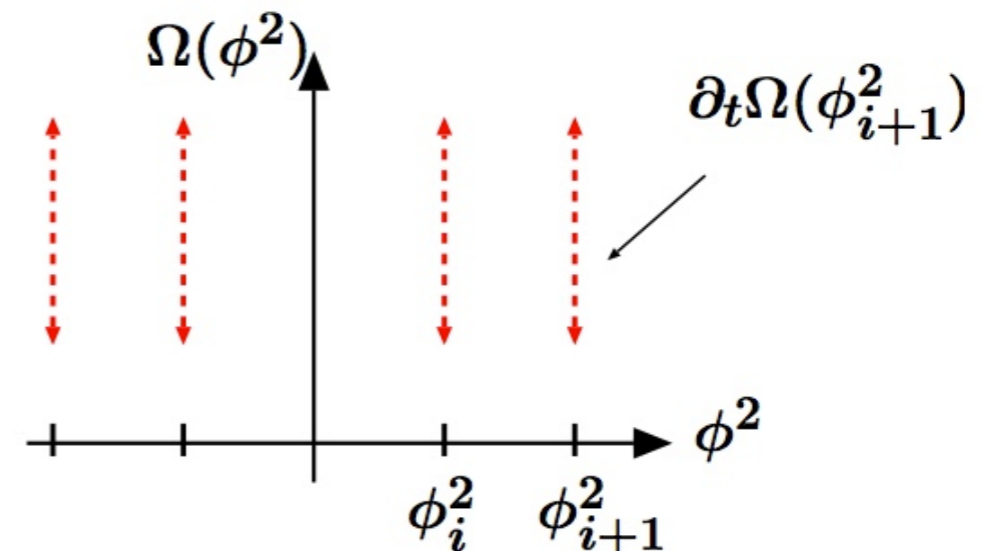
# Solving Flow Equations

two strategies

1. Taylor expansion around  
some point

$$V_k(\phi) = \sum_{n=1}^{N_{\max}} \frac{a_{k,2n}}{n!} (\phi^2 - \phi_0^2)^n$$

2. Expansion on a grid (1-dim.)



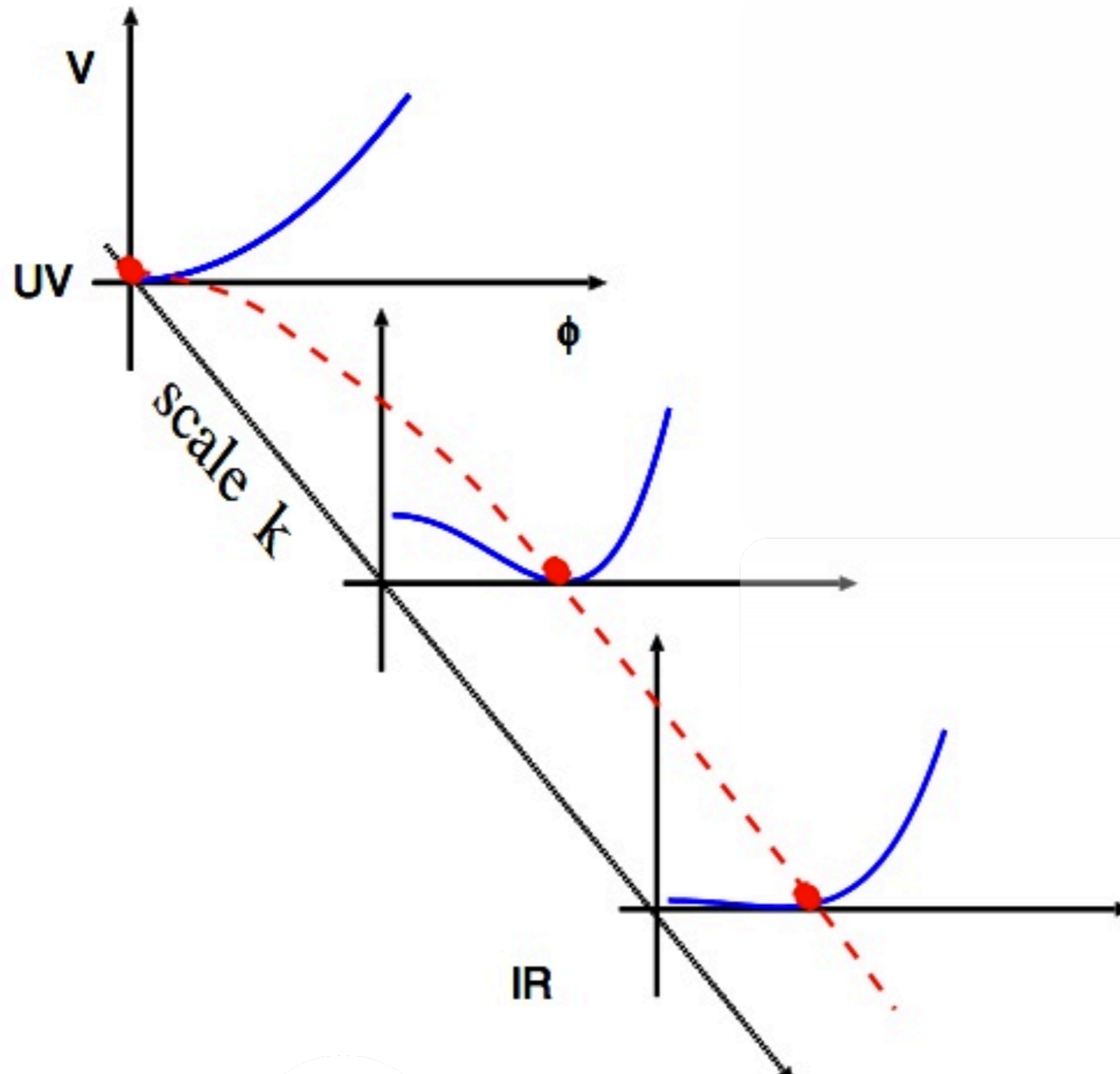
- initial condition at high UV cutoff (symmetric potential)

$$V_\Lambda(\phi) = \frac{\lambda_\Lambda}{4} (\phi^2)^2$$

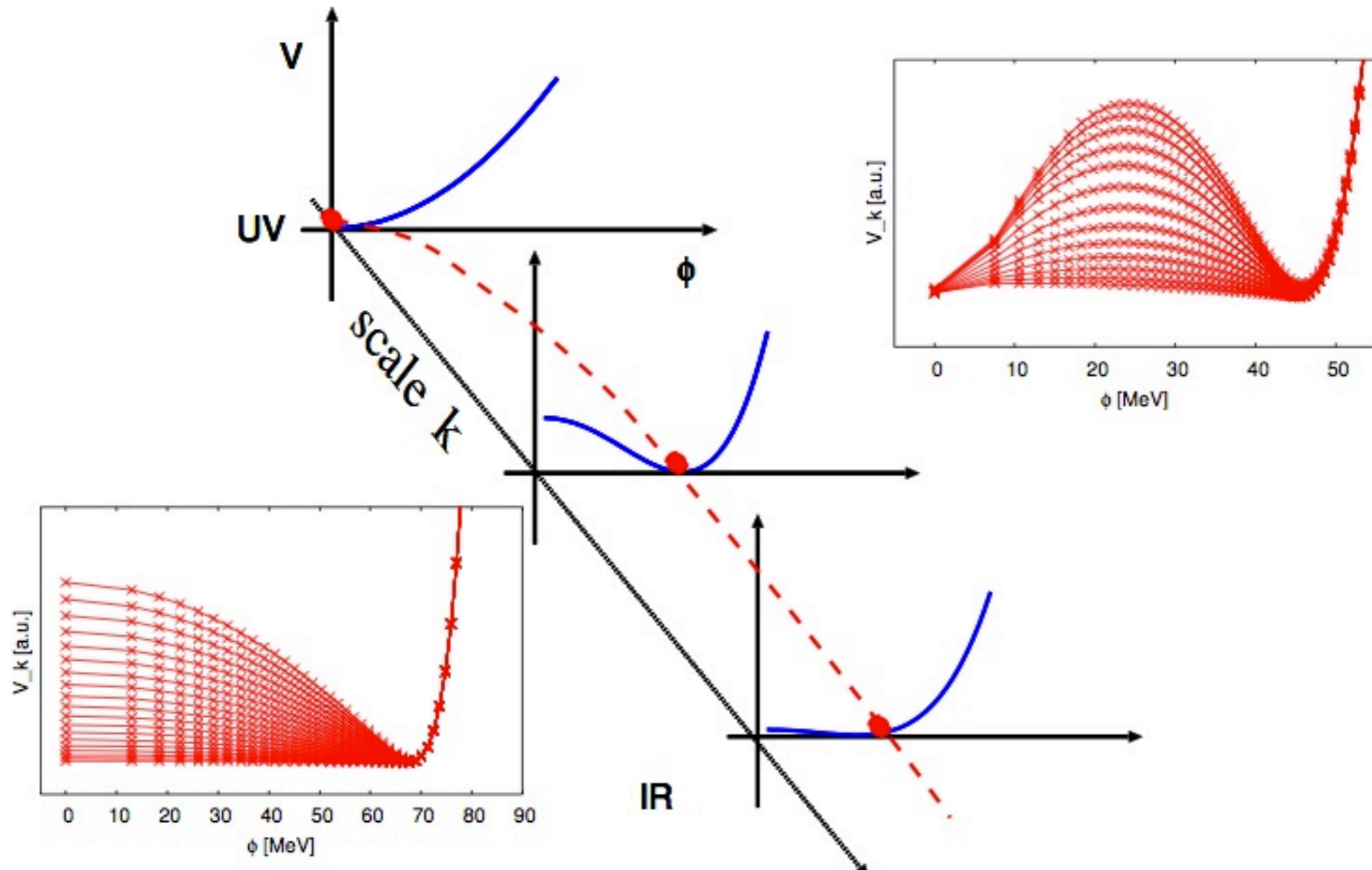
- Fix UV parametrization such to reproduce IR physics

$$\phi_{k \rightarrow 0} \equiv f_\pi \sim 93 \text{ MeV}$$

# Scale evolution towards the IR



# Scale evolution towards the IR

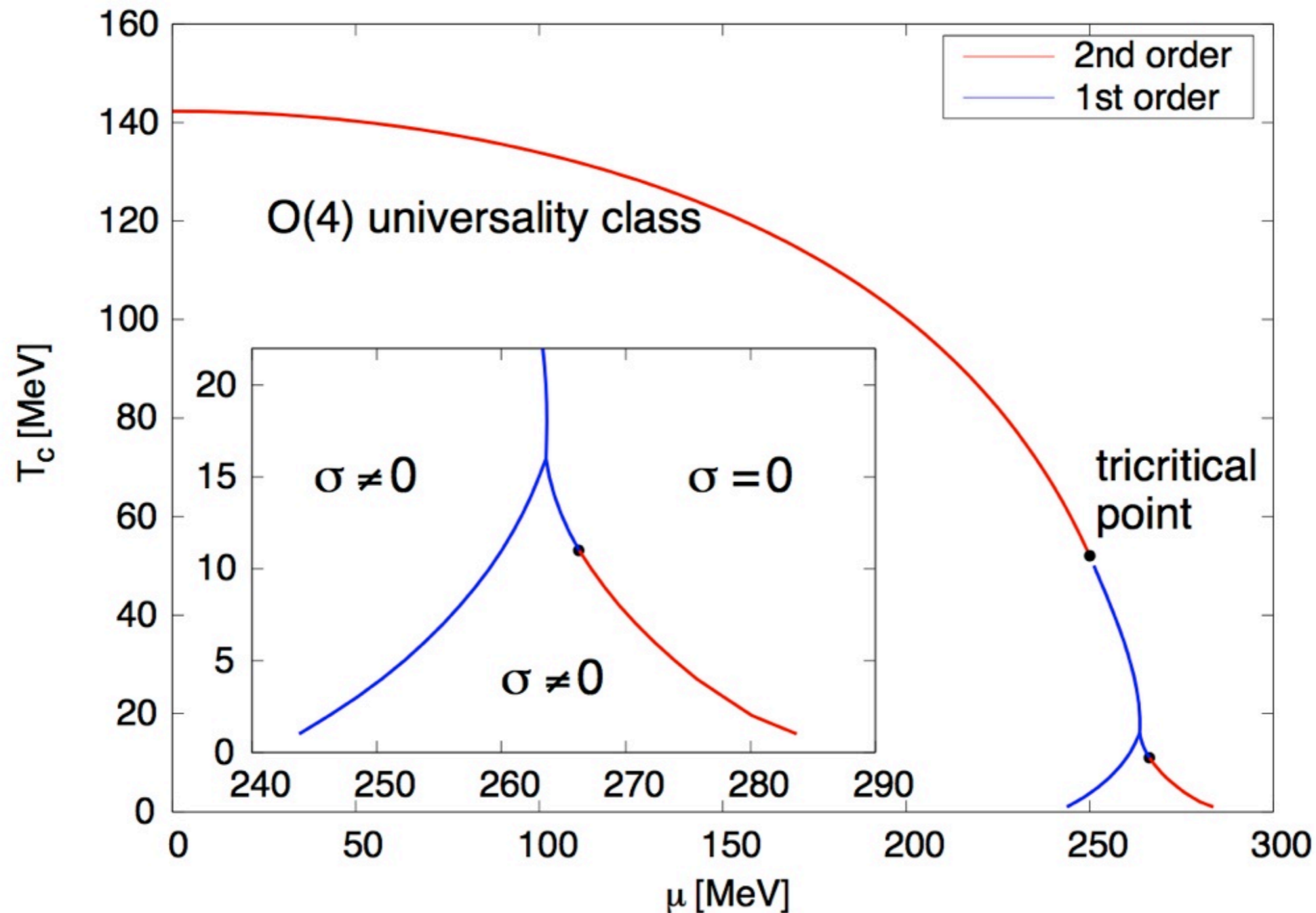


# Phase diagram $N_f=2$ QM truncation

$O(4) \sim SU(2) \times SU(2)$

chiral limit

no spinodal lines!



# FRG and QCD

**full dynamical QCD FRG flow:** fluctuations of **gluon, ghost, quark and meson** (via hadronization) fluctuations

Braun, Haas, Marhauser, Pawłowski; 2009

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left( \text{Diagram 1} - \text{Diagram 2} \right)$$

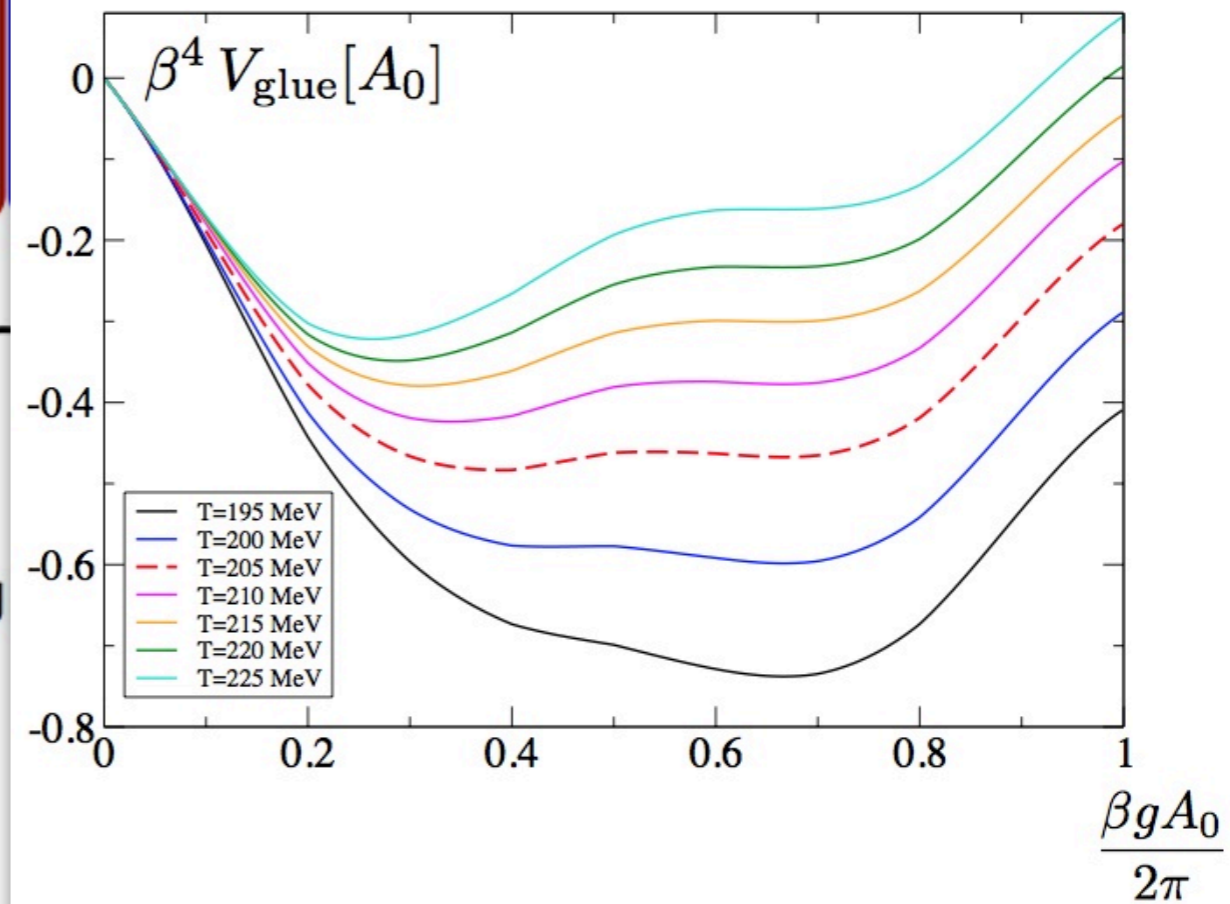
in presence of dynamical quarks:  
gluon propagator modified:

⇒ pure Yang Mills flow + matter back-coupling

## pure Yang Mills flow

replaced by eff. Polyakov-loop potential  $\mathcal{U}_{\text{Pol}}$ :  
(fit to lattice YM thermodynamics)

## Glue potential



[Haas, Stiele, Braun, Pawłowski, Schaffner-Bielich, (2013)]

# FRG and QCD

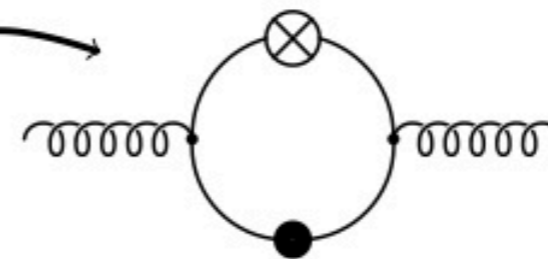
**full dynamical QCD FRG flow:** fluctuations of  
**gluon, ghost, quark and meson** (via hadronization) fluctuations

Braun, Haas, Marhauser, Pawłowski; 2009

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[ \text{Gluon loop} - \text{Ghost loop} \right] - \left[ \text{Quark loop} + \frac{1}{2} \text{Meson loop} \right]$$

in presence of dynamical quarks:  
 gluon propagator modified:

⇒ pure Yang Mills flow + matter back-coupling



→  $\mathcal{U}_{\text{Pol}}(\Phi)$

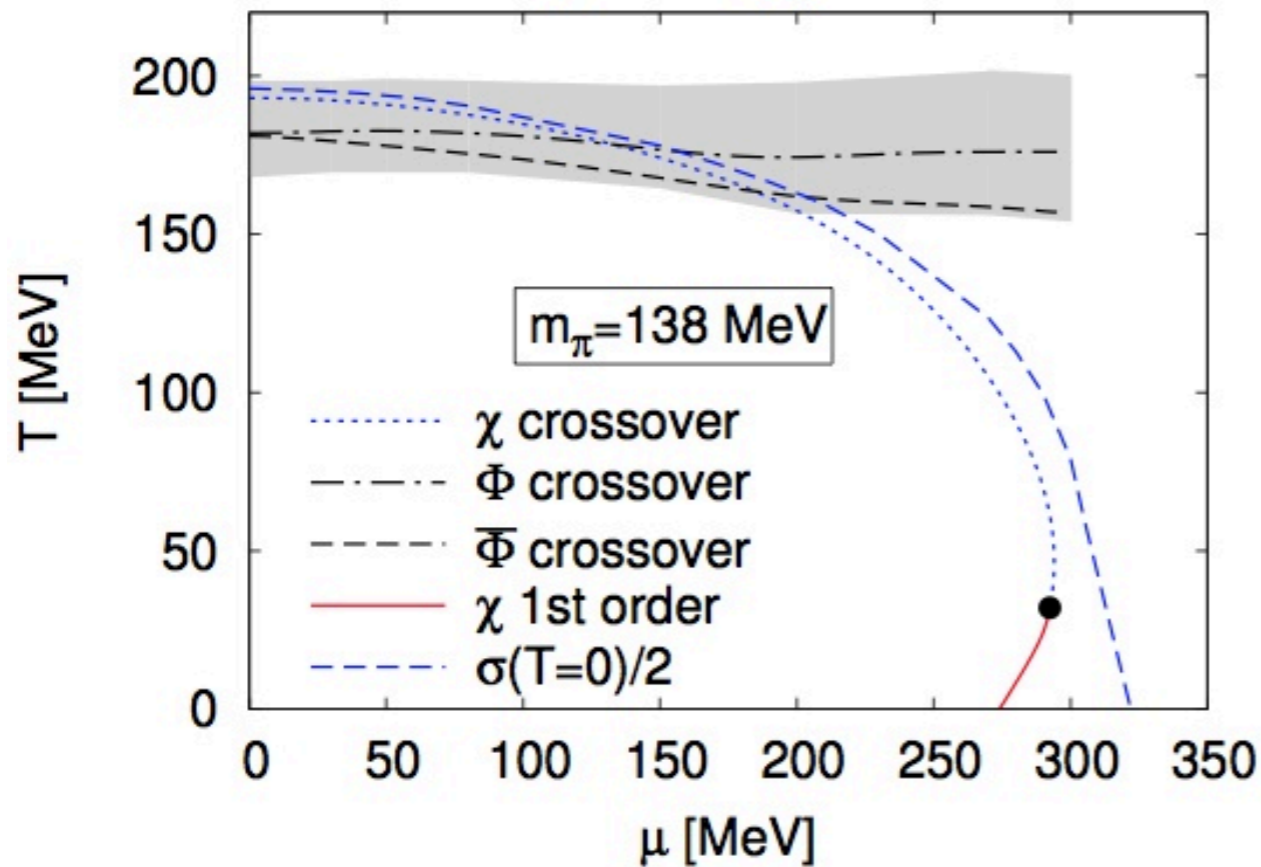
## pure Yang Mills flow

replaced by eff. Polyakov-loop potential  $\mathcal{U}_{\text{Pol}}$ :  
 (fit to lattice YM thermodynamics)

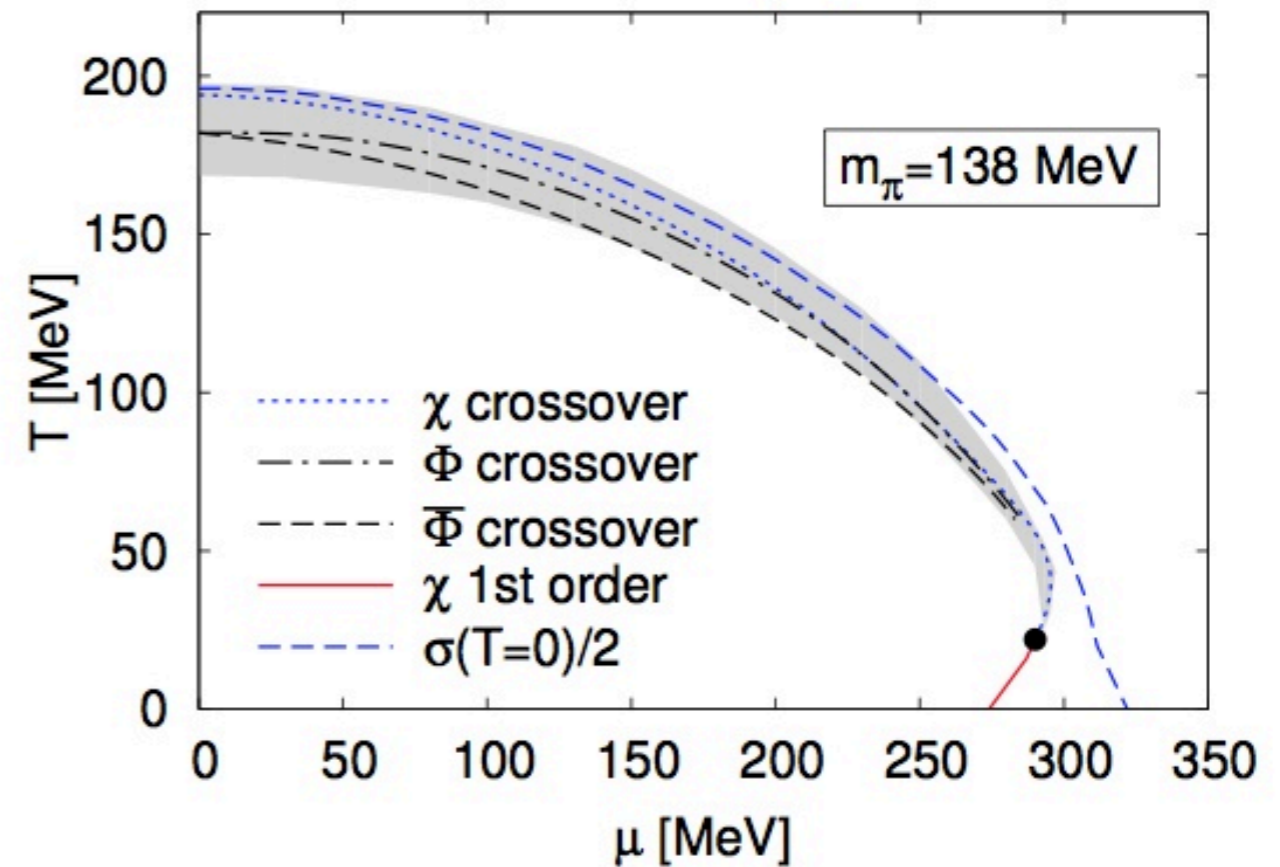
~~$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left[ \text{Gluon loop} - \text{Ghost loop} \right]$$~~

# FRG: Quark-Meson with Polyakov

w/o  $T_0(\mu)$



incl.  $T_0(\mu)$



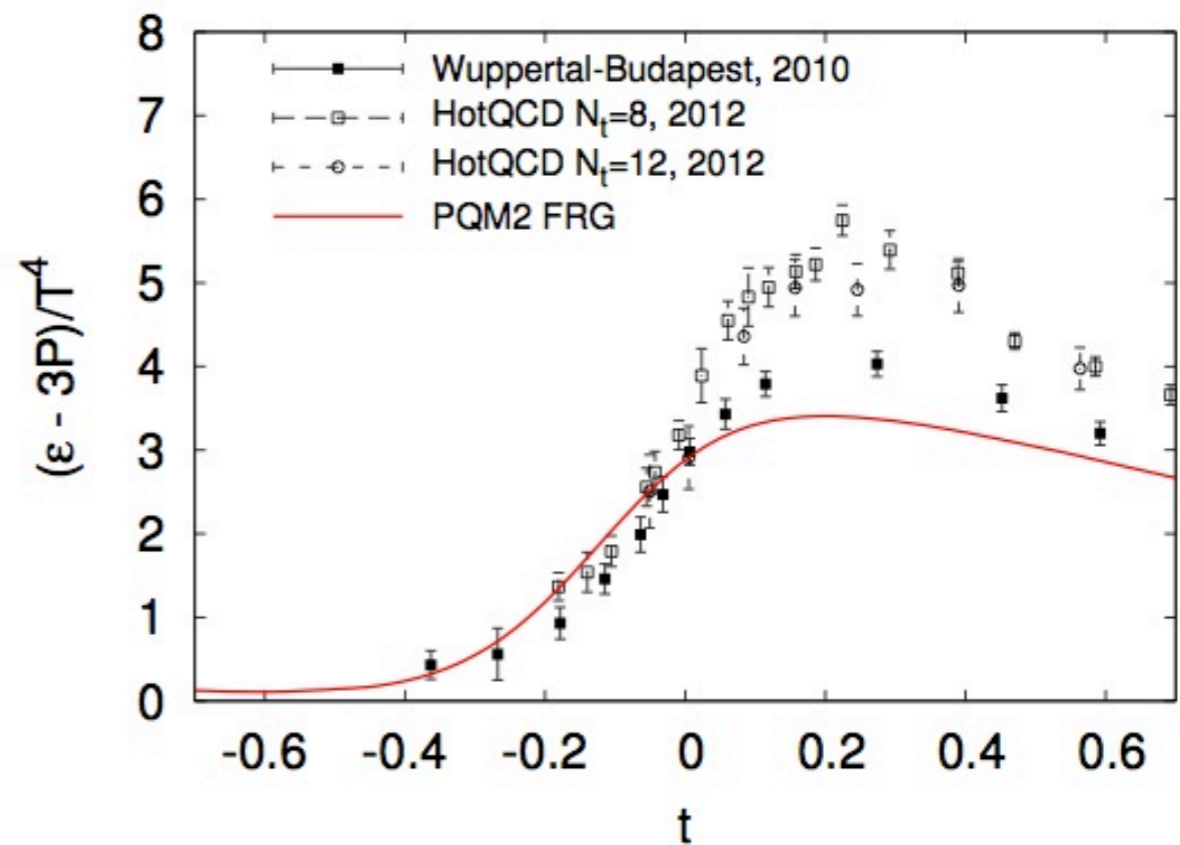
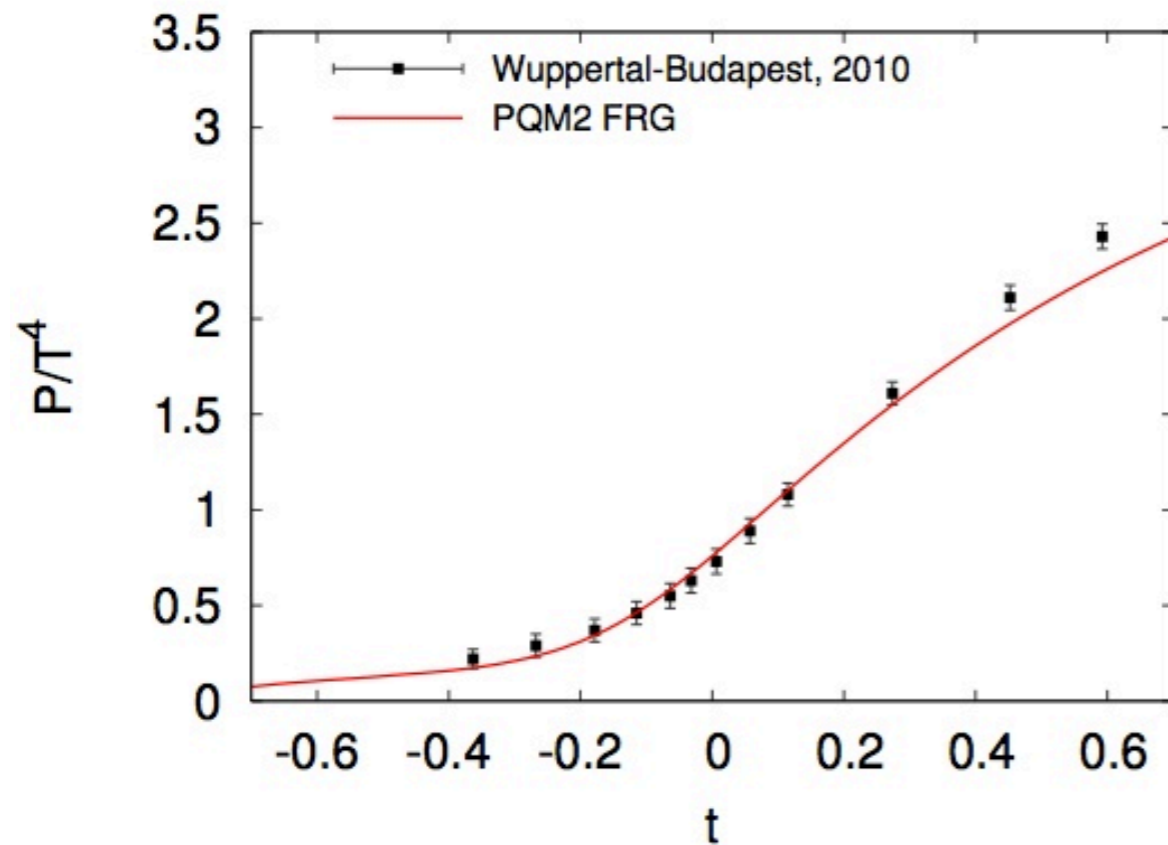
[Herbst, Pawłowski, BJS 2010,2013]

# FRG: Quark-Meson with Polyakov

Pressure and interaction measure in comparison with lattice data (polynomial Polyakov-loop potential)

$$N_f = 2$$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]



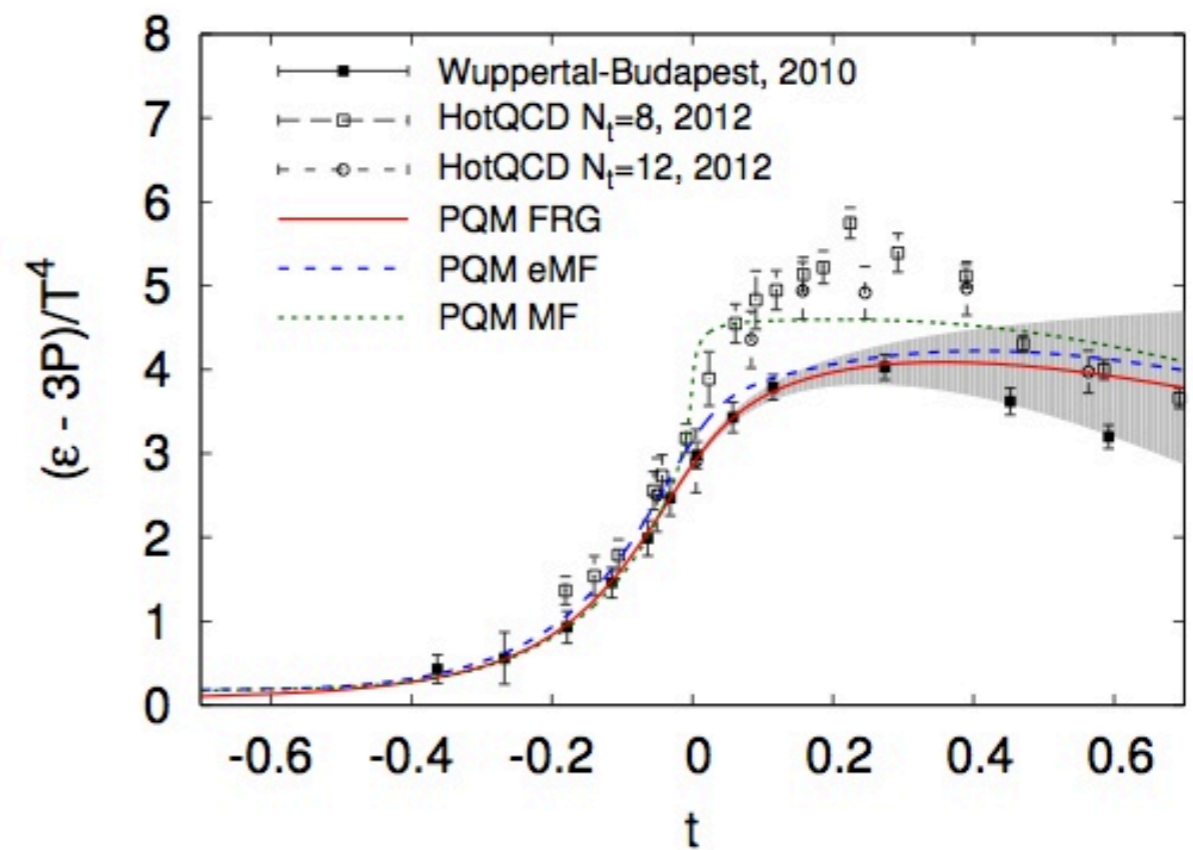
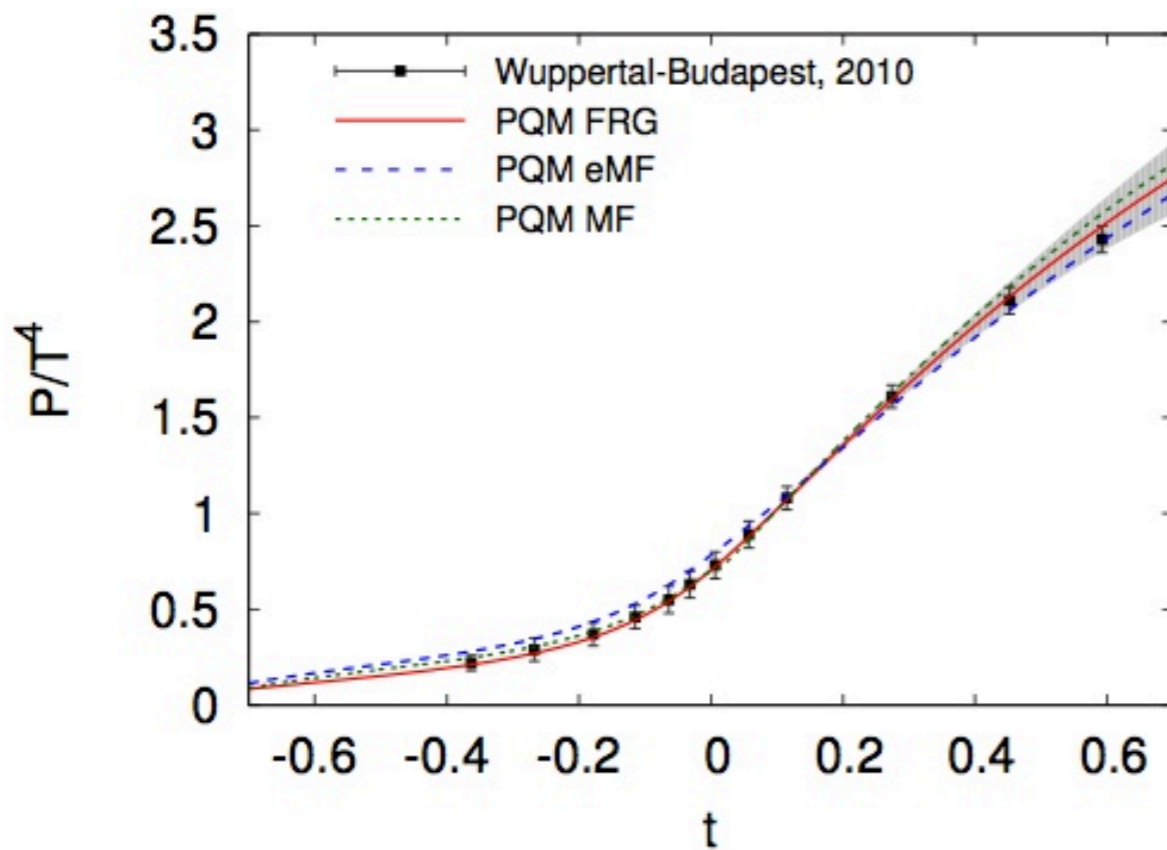


# FRG: Quark-Meson with Polyakov

Pressure and interaction measure in comparison with lattice data (**polynomial** Polyakov-loop potential)

$$N_f = 2+1$$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]

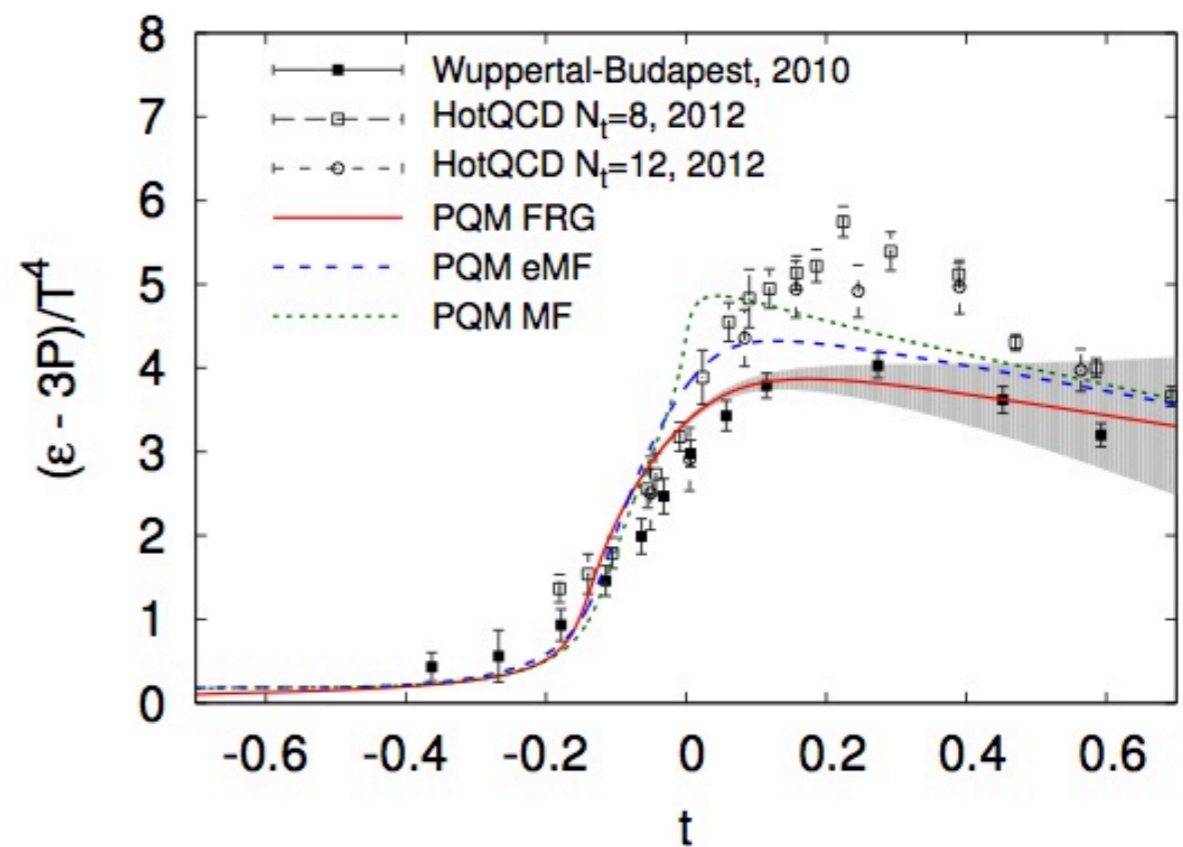
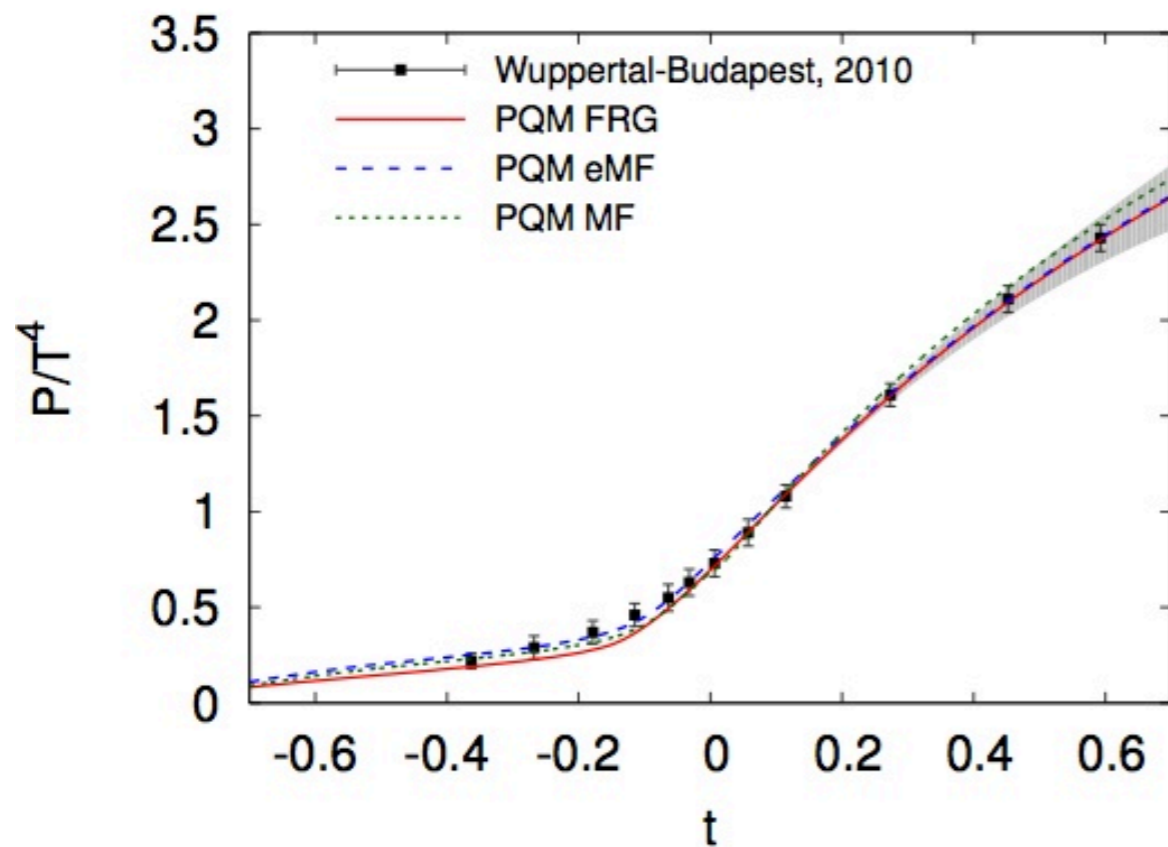


# FRG: Quark-Meson with Polyakov

Pressure and interaction measure in comparison with lattice data (**logarithmic** Polyakov-loop potential)

$$N_f = 2+1$$

[Herbst, Mitter, Stiele, Pawlowski, BJS, Schaffner-Bieleich in preparation 2013]

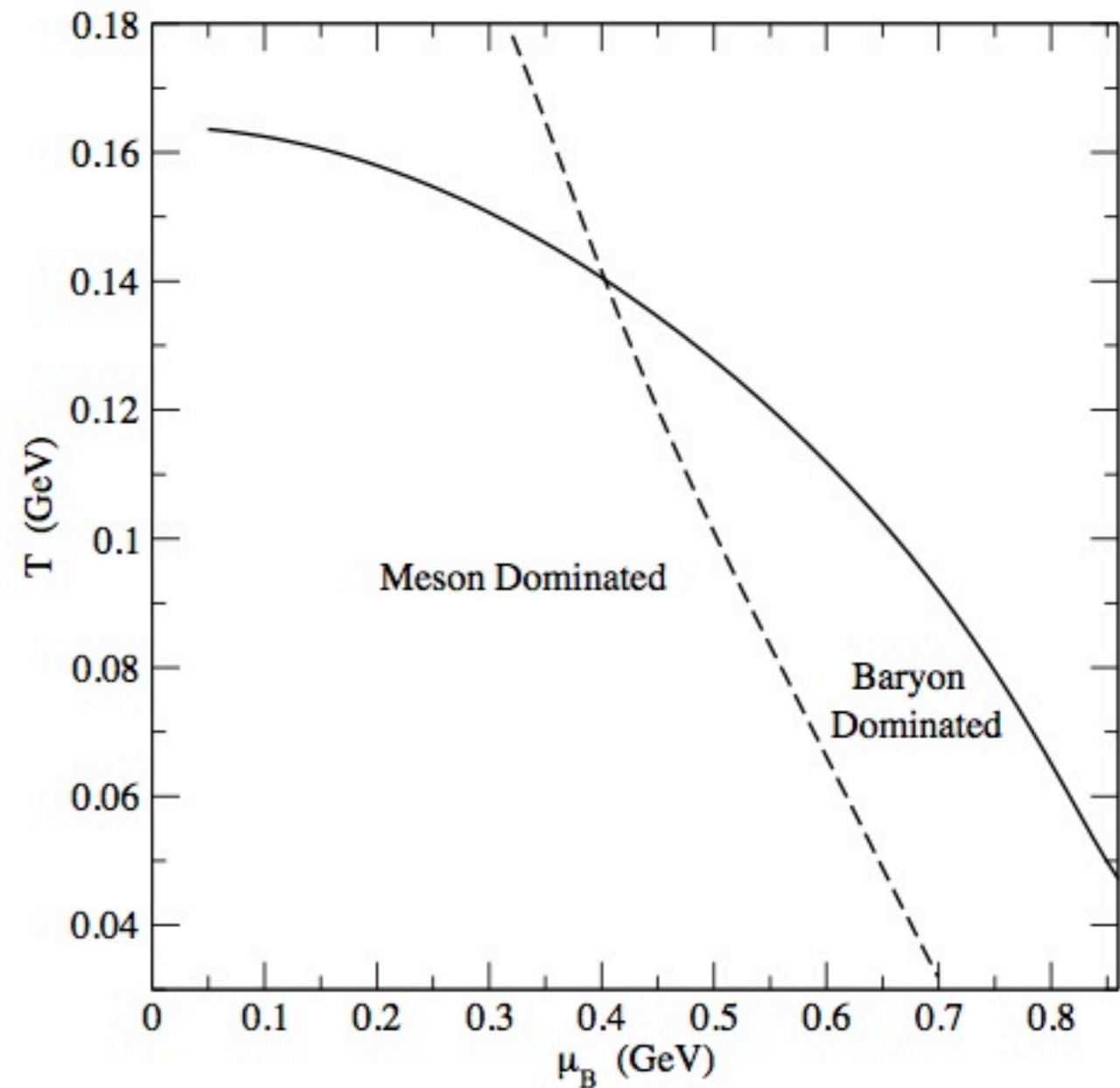
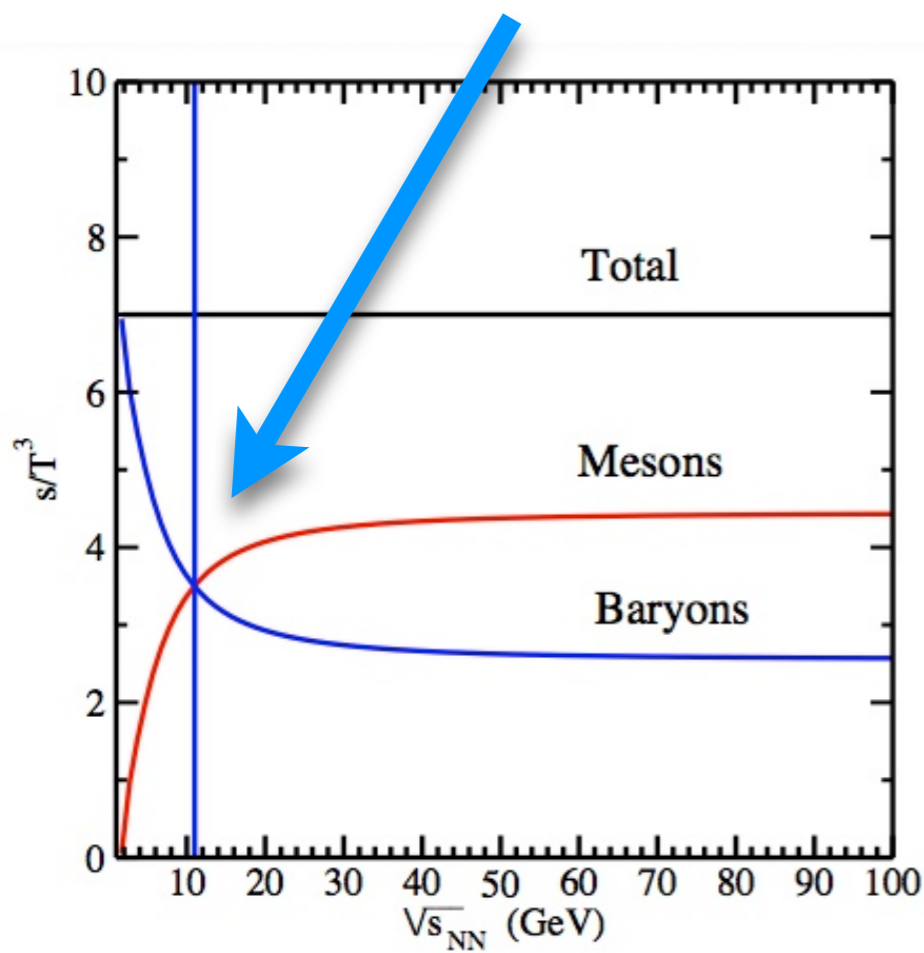


# Role of Baryons?

Statistical-thermal model analysis

High chemical potential: **baryons more important but usually neglected in FRG**

in stat. model: e.g. peak in ratios “the horn”



# $N_c = 2$ QCD

QC2D becomes simpler: **no sign problem**

lattice simulations  $\Leftrightarrow$  functional methods

inclusion of baryonic degrees of freedom simpler:

**Scalar diquarks** play a dual role as **bosonic baryons**

relativistic analog of models for ultracold quantum gases

color-neutral bound states of two quarks (**bosonic [anti]diquarks**)

enlarged flavor symmetry:  $SU(4) \cong SO(6)$  ( $\mu = 0$ )

replaces usual chiral  $SU(2)_L \times SU(2)_R \times U(1)_B$

■ Symmetry breaking:  $SU(2N_f) \rightarrow Sp(N_f)$  [or  $SO(6) \rightarrow SO(5)$ ]

→ 5 Goldstone bosons: 3 pions and 2 (anti)diquarks

# Quark-Meson-Diquark (QMD) Model

Chiral effective model:

- quarks:  $\psi$
- mesons:  $\sigma, \vec{\pi}$
- diquarks (baryons):  $\text{Re}\Delta, \text{Im}\Delta$
- gauge fields:  $A_\mu^a$  in  $D_\mu = \partial_\mu + iA_\mu \rightarrow$  Polyakov-loop extended (PQMD) [arXiv:1306.2897] model

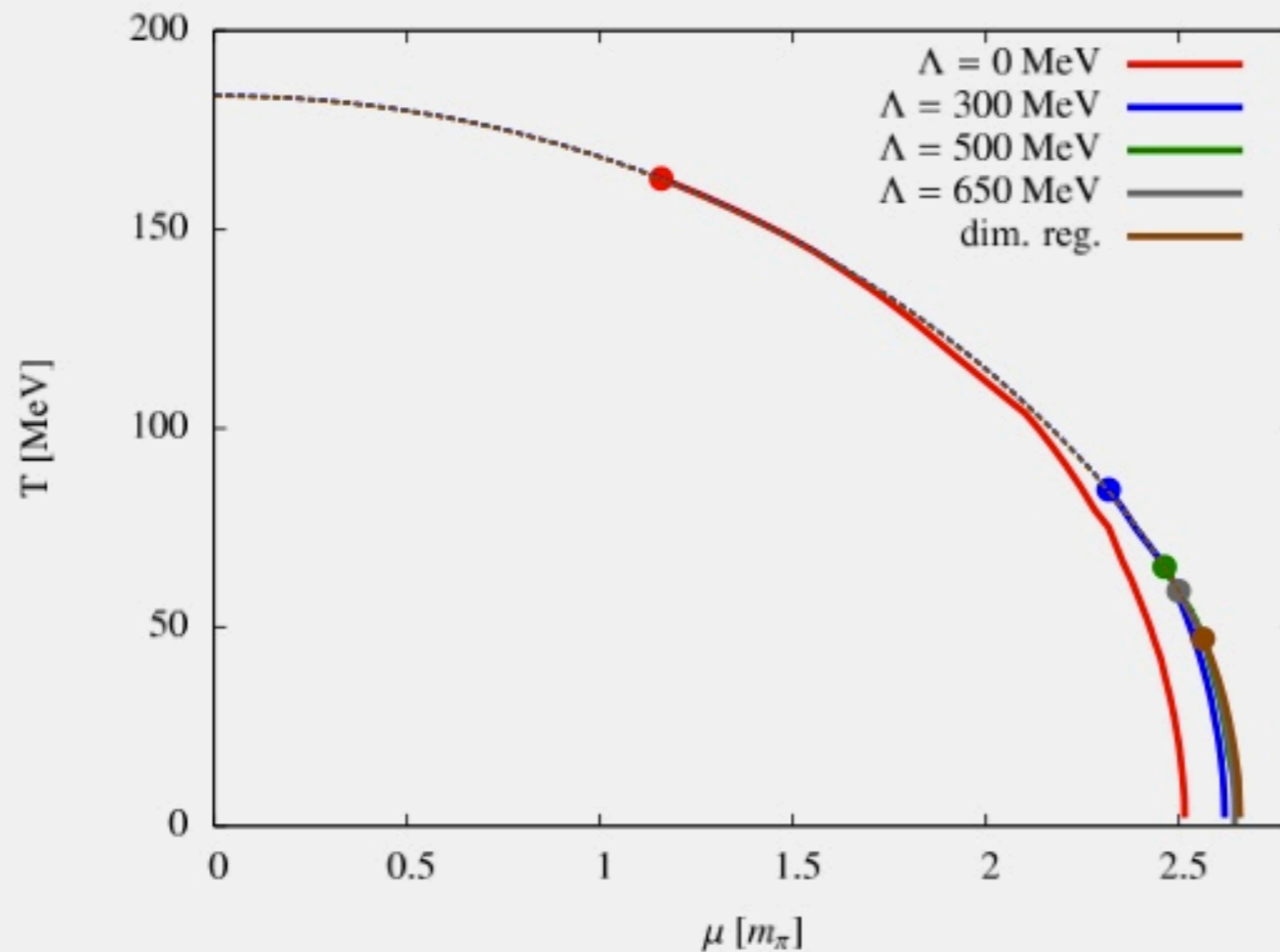
QMD Lagrangian:

[N. Strodthoff, BJS, L. von Smekal 2012]

$$\begin{aligned}\mathcal{L}_{\text{QMD}} = & \bar{\psi} \left( \not{D} + g(\sigma + i\gamma^5 \vec{\pi} \vec{\tau}) - \mu\gamma^0 \right) \psi \\ & + \frac{g}{2} \left( \Delta^* (\psi^T C \gamma^5 \tau_2 S \psi) + \Delta (\psi^\dagger C \gamma^5 \tau_2 S \psi^*) \right) \\ & + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + V(\vec{\phi}) \\ & + \frac{1}{2} \left( (\partial_\mu - 2\mu \delta_\mu^0) \Delta \right) (\partial_\mu + 2\mu \delta_\mu^0) \Delta^*\end{aligned}$$

# Phase diagram in MFA

Influence of  $\Omega_{\text{vac}}$  on CEP for various  $\Lambda$ 's compared to dimensional regularization

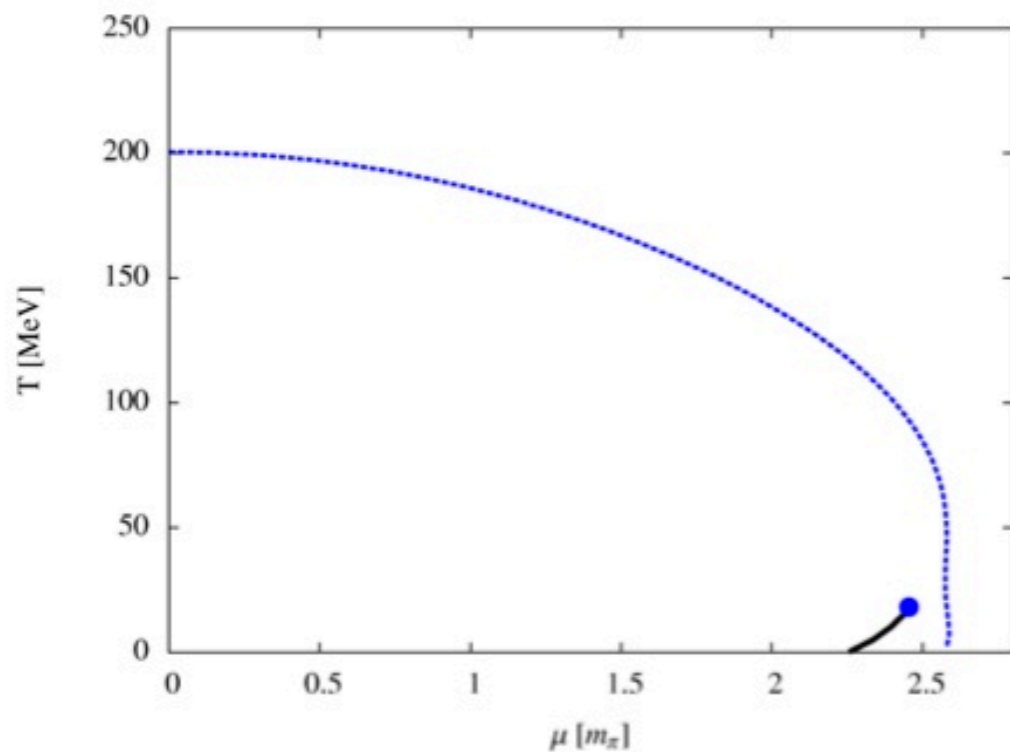


- no diquark condensation ( $d = 0$ )
- $O(6)$ -symmetric potential  $V_{\text{MF}} = V_{\text{MF}}(\phi^2)$  where  $\vec{\phi} = (\sigma, \vec{\pi}, \text{Re}\Delta, \text{Im}\Delta)$  → 1-dim. field variable
- 1<sup>st</sup> order chiral transition
- CEP around  $\mu \approx 2.5m_\pi$

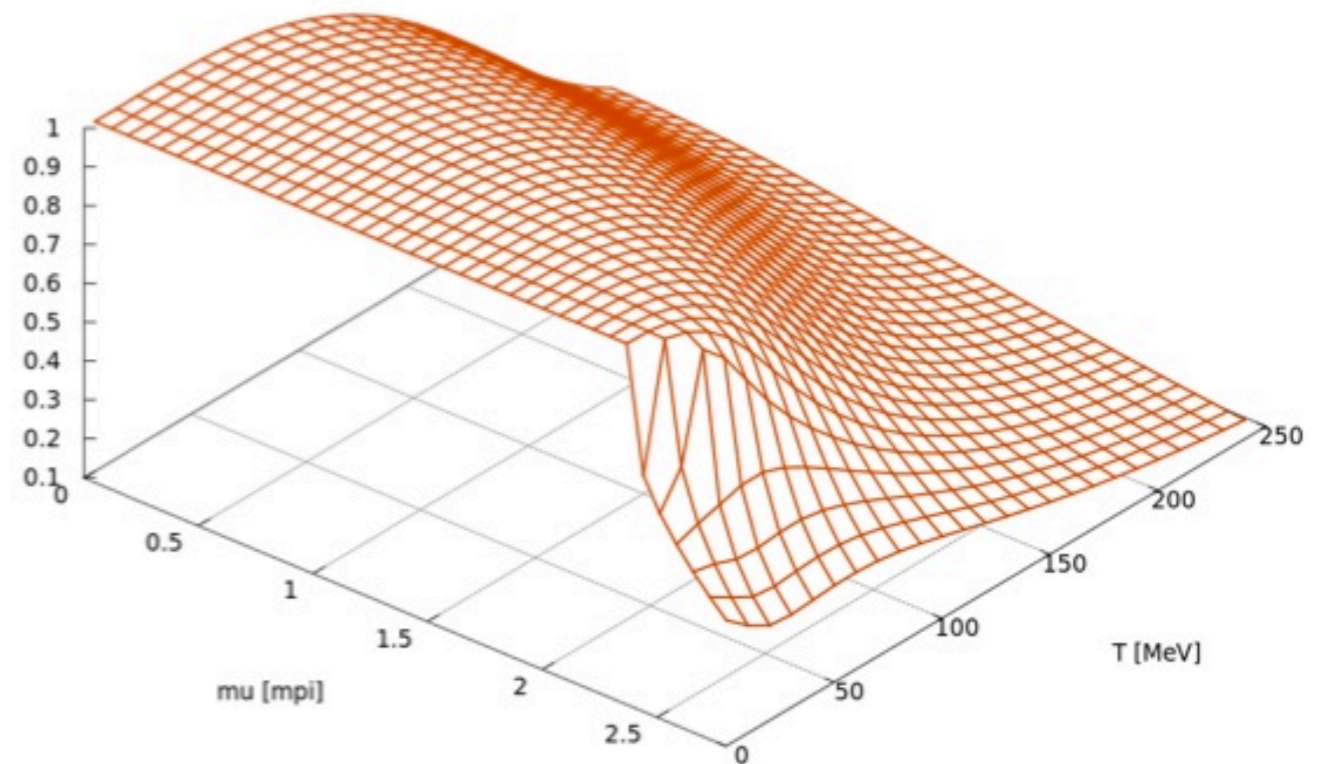
[N. Strodthoff, BJS, L. von Smekal; 2012]

# Phase diagram with FRG

- no diquark condensation:
- $O(6)$ -symmetric potential  $U_k = U_k(\phi^2)$



- chiral condensate  $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$



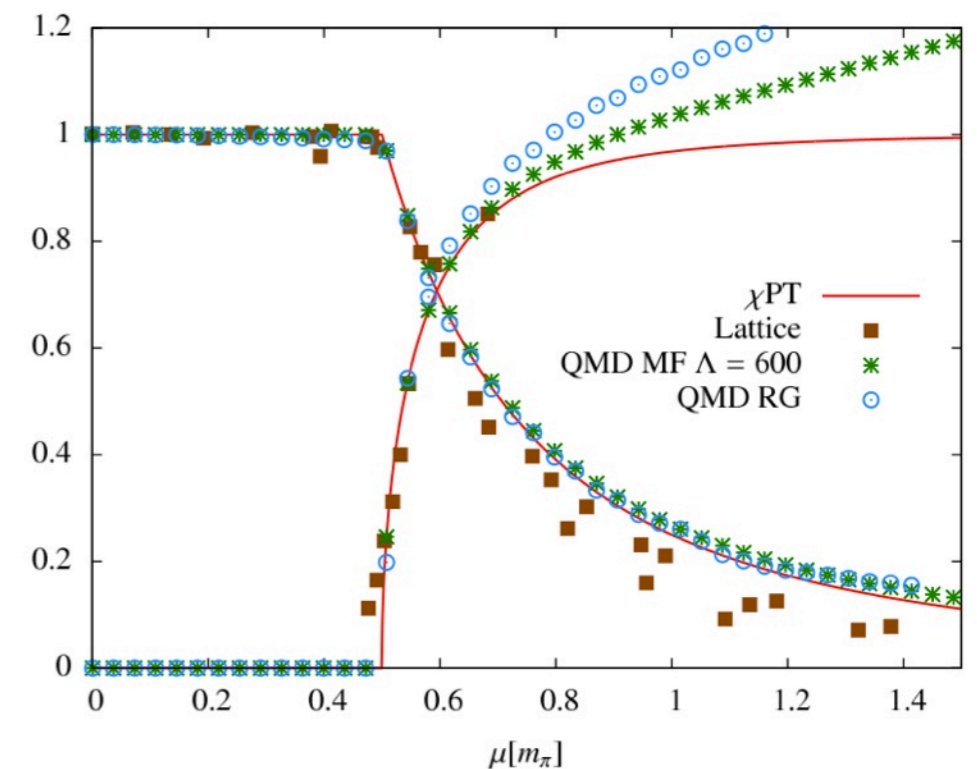
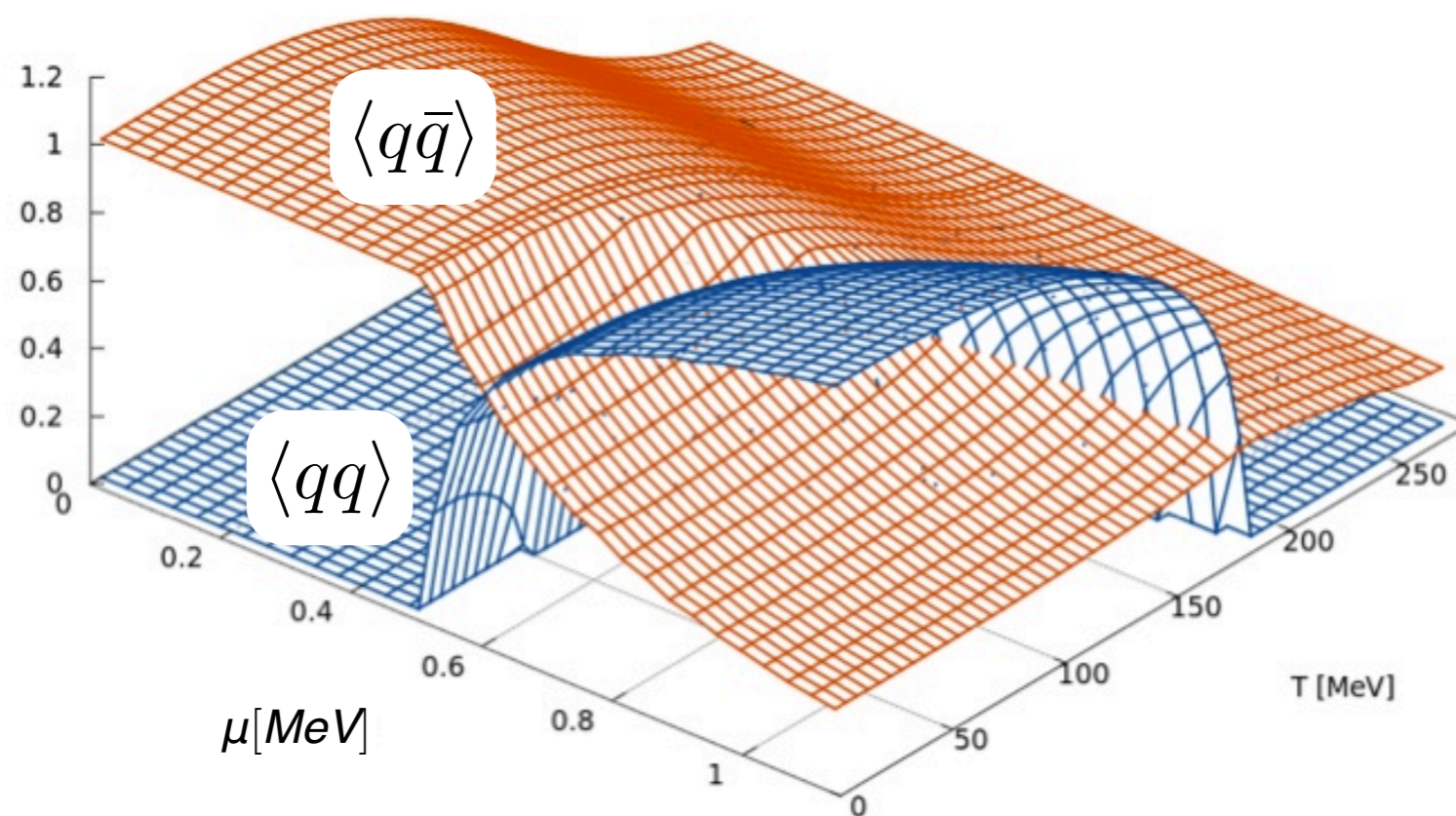
- ▷ "typical" RG phase diagram  
back-bending 1<sup>st</sup> order line with a CEP

[Strodthoff, BJS, von Smekal; 2012]

# Including diquarks

Symmetry breaking  $SO(6) \longrightarrow SO(3) \times SO(2)$

need two condensates: chiral:  $\langle q\bar{q} \rangle$  and diquark condensate:  $\langle qq \rangle$



diquark condensation at  $\mu_c = m_B/N_c$

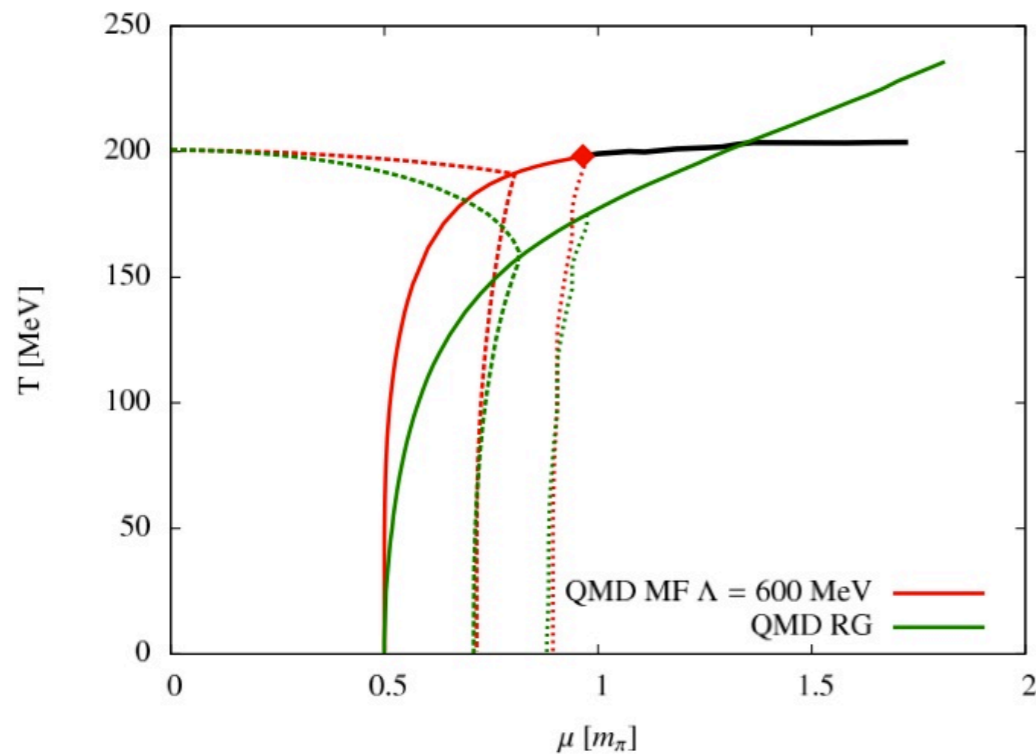
[N. Strodthoff, BJS, L. von Smekal 2012]



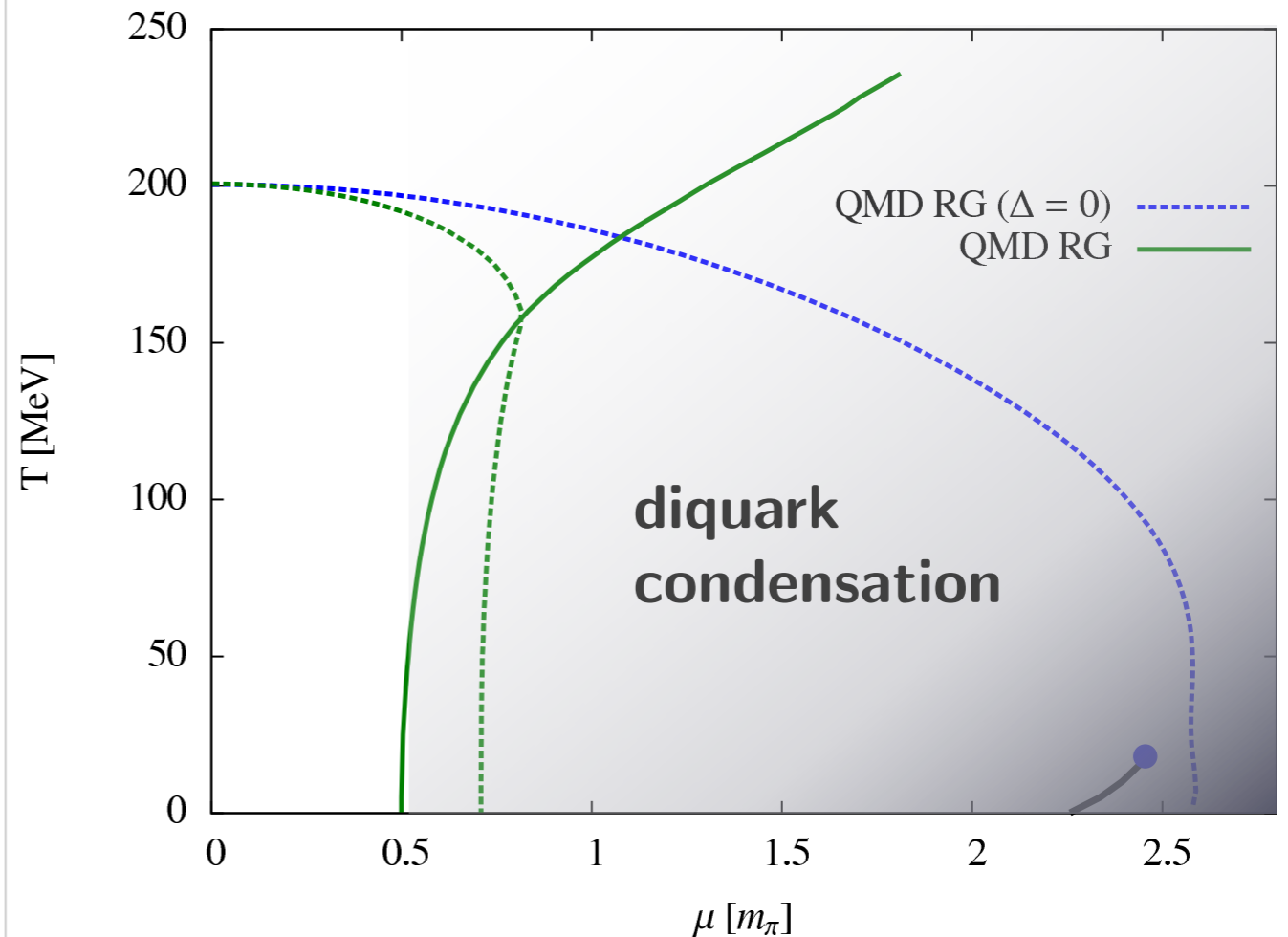
# Phase diagrams

[N. Strodthoff, BJS, L. von Smekal 2012]

FRG versus MFA: TCP is a MFA artefact



with collective baryonic fluctuations



- no low- $T$  1<sup>st</sup> order transition,  
no CEP at  $\mu \sim 2.5 m_\pi$  !

# Summary & Conclusions

- **QCD-like model studies for two and three flavors**
- **effects of quantum and thermal fluctuations on QCD phase structure**
- **QCD for two colors: towards an understanding of role of baryons on phase structure**
- **existence of critical points in phase diagram**

**functional approaches (e.g. FRG) are suitable and controllable tools to investigate the QCD phase diagram and its boundaries**

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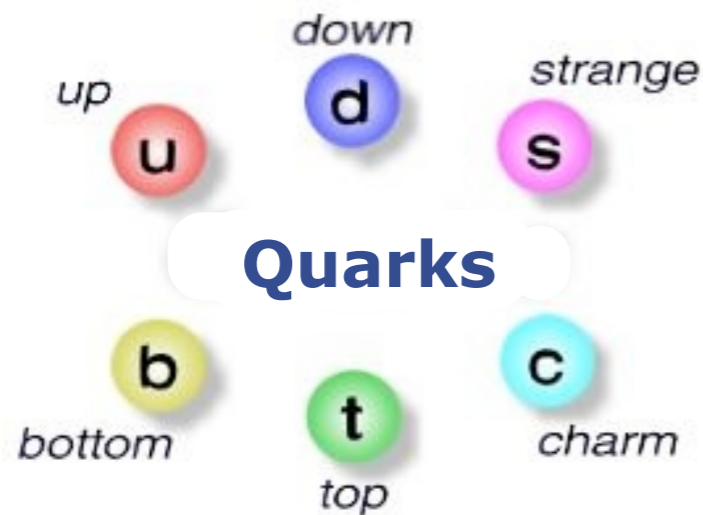
# Backup material

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# Dynamical chiral symmetry breaking

Two important properties of low-energy QCD: **2. Dynamical chiral SB**

dynamical **quark mass generation** via weak and strong force



Yoichiro Nambu,  
Nobel prize 2008

		u	d	s	c	b	t
$M_{\text{weak}}$	$[MeV/c^2]$	3	5	80	1200	4500	176000
$M_{\text{strong}}$	$[MeV/c^2]$	350	350	350	350	350	350
$M_{\text{total}}$	$[MeV/c^2]$	350	350	450	1500	4800	176000