Some interpretation of the trace anomaly in dense neutron star matter

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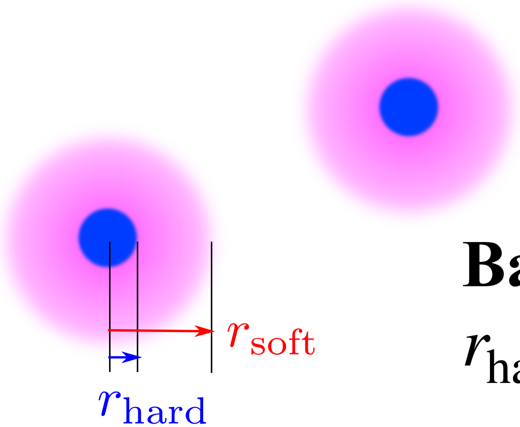
— Lunch Club Seminar in Gießen —

Dense Matter

How dense can nuclear matter be?



Interaction Cloud Size

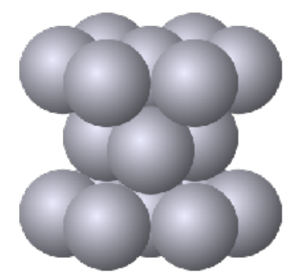
$r_{\rm soft} \sim 1/(2m_\pi) \sim 0.7\ {\rm fm}$

Baryon Number Distribution Size

$r_{\rm hard} \sim 0.5\ {\rm fm}$

Closest Packed State (hcp/fcc) Filling rate ~ 74%

$$0.74 \times \left(\frac{4\pi}{3} r_{\rm hard}^3\right)^{-1} \approx 1.4\ {\rm fm}^{-3} \approx 8.3\, n_{\rm sat}$$



Nuclear matter cannot exist at this density!

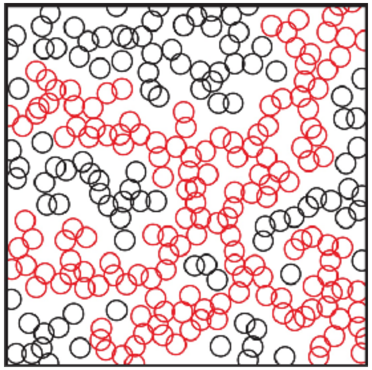
Dense Matter

How dense can nuclear matter be?

Percolation threshold

3D critical filling density ~ 34%

(Excellent Wikipedia!)



(From Wikipedia)

Interaction-mediated Percolation

$$0.34 \times \left(\frac{4\pi}{3} r_{\rm soft}^3\right)^{-1} \approx 0.24\ {\rm fm}^{-3} \approx \boxed{1.5\, n_{\rm sat}}$$

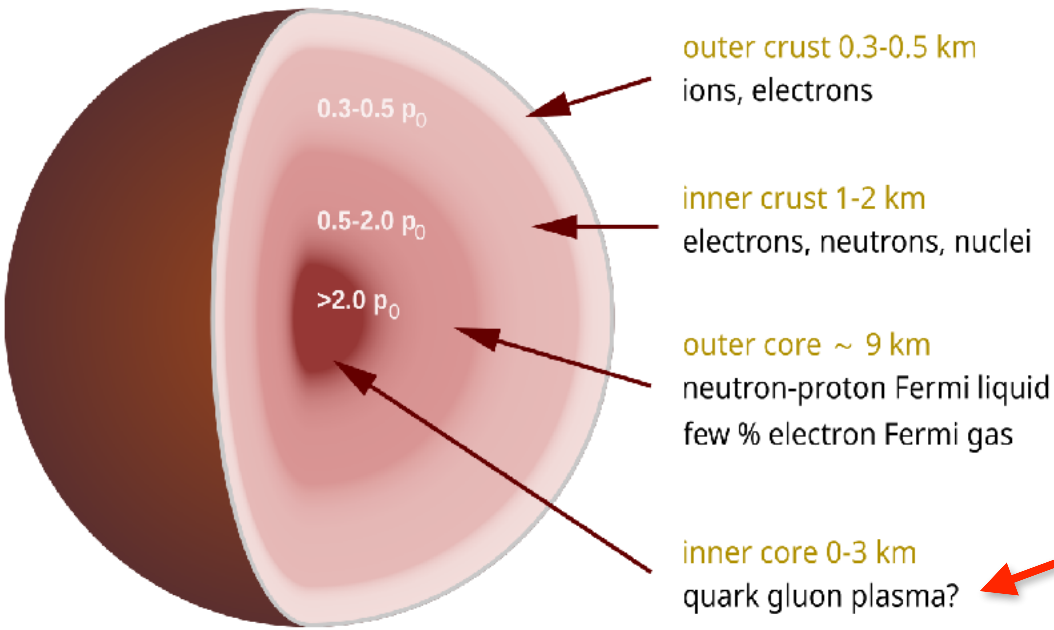
Standard nuclear-physics calculations may break down at this density due to the lack of multi-body interactions.

See: Fukushima-Kojo-Weise (2020) for more details.

Dense Matter in NS

Structures

[Wikipedia]



quark matter!

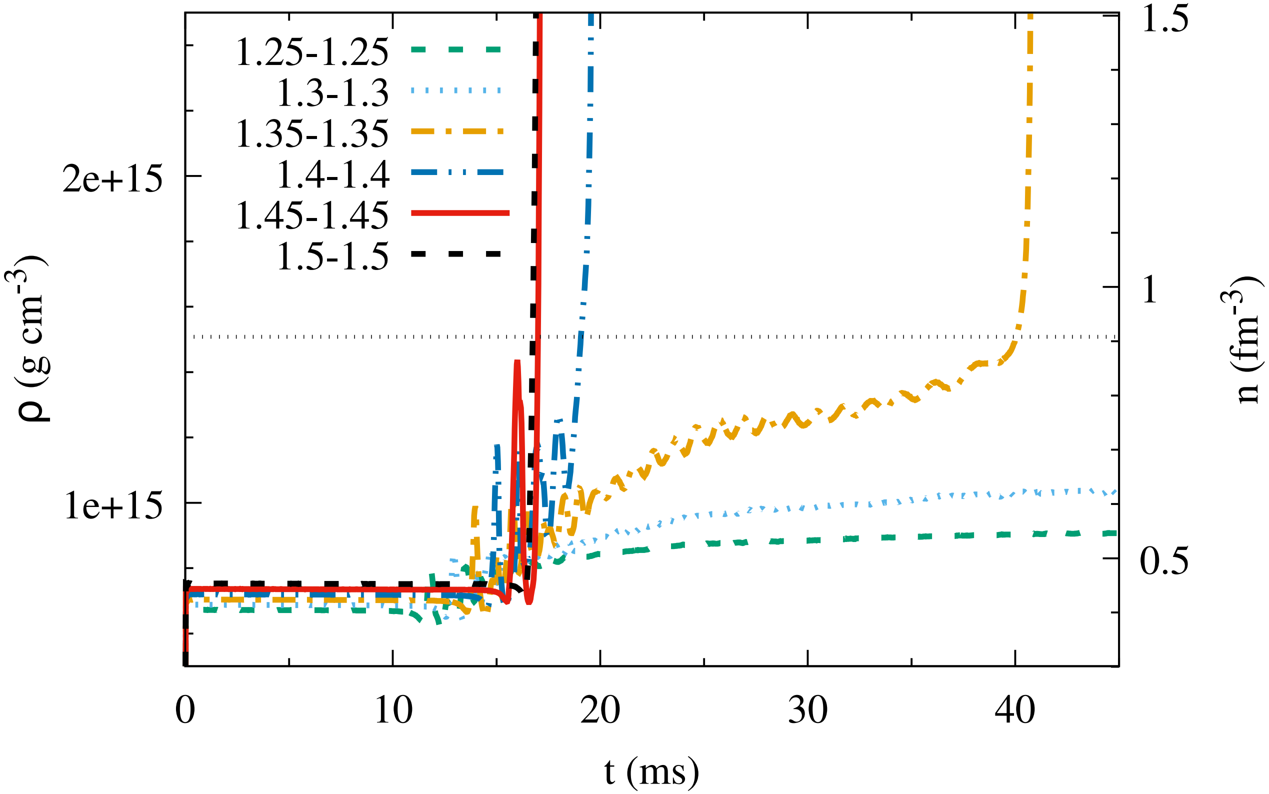
(Somebody must really correct the Wikipedia…)

The density at the deepest core can reach $> 5\rho_{\text{sat}}$ and the binary neutron star merger can reach $> 8\rho_{\text{sat}}$

Dense Matter in NS

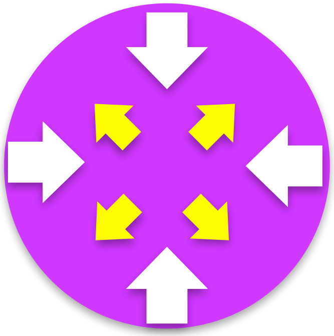
Highest accessible density

Fujimoto-Fukushima-Hotokezaka-Kyutoku (2024)



Neutron Star

Force Balance



Gravitational force is supported by the pressure from inside.

Hydrostatic condition for $r \sim r + dr$

$$\frac{dp(r)}{dr} = -G\frac{M(r)}{r^2}\varepsilon(r)$$

***M*(*r*) represents the integrated mass in *r*-sphere.**

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

(In Newtonian gravity)

Neutron Star

$$\frac{dp(r)}{dr} = -G\frac{M(r)\,\varepsilon(r)}{r^2}$$

Tolman-Oppenheimer-Volkoff Eq.

General Relativistic extension →

$$\frac{dp(r)}{dr} = -G\frac{M\varepsilon}{r^2}\left(1+\frac{p}{\varepsilon}\right)\left(1+\frac{4\pi r^3 p}{M}\right)\left(1-\frac{2GM}{r}\right)^{-1}$$

One condition still missing…

A relation between *p* and *ε* → **Equation of State (EOS)**

Initial

$r = 0$

$\varepsilon(r=0) = \varepsilon_c$ **free parameter**

$p(r=0) = p_c = p(\varepsilon_c)$

Final

$r = R$

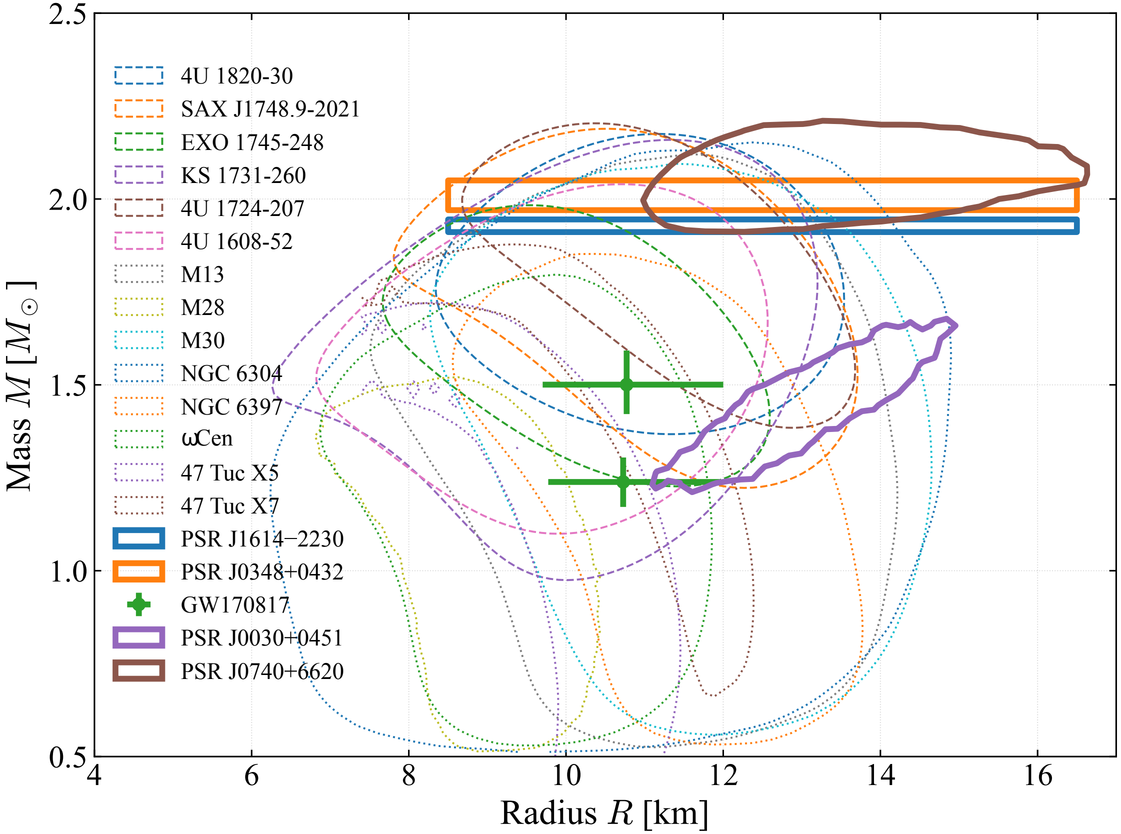
$p(r=R) = 0$

$M = \int dr 4\pi r^2 \varepsilon(r)$

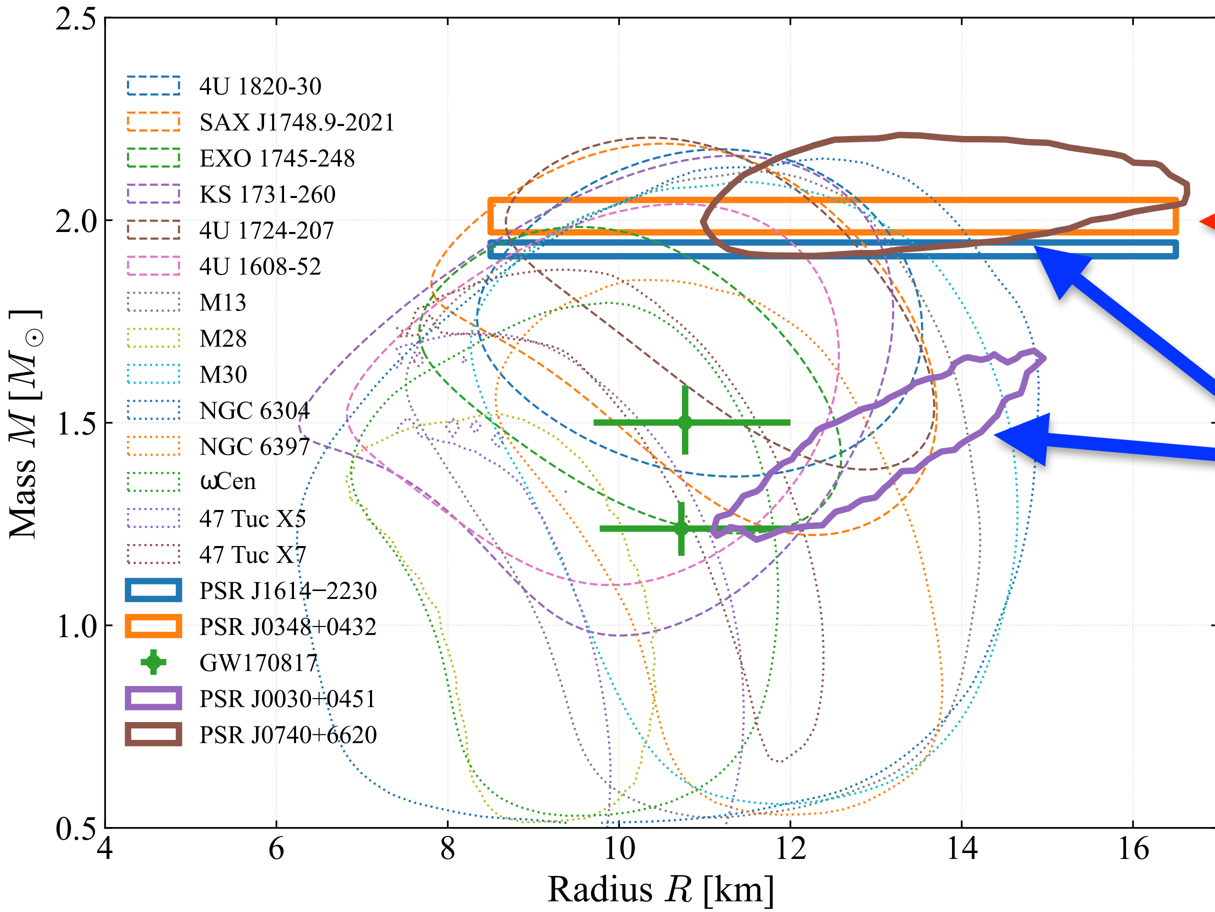
Neutron Star

Compilation of the observed data (68% Credible)

Fujimoto-Fukushima-Kamata-Murase (2024)



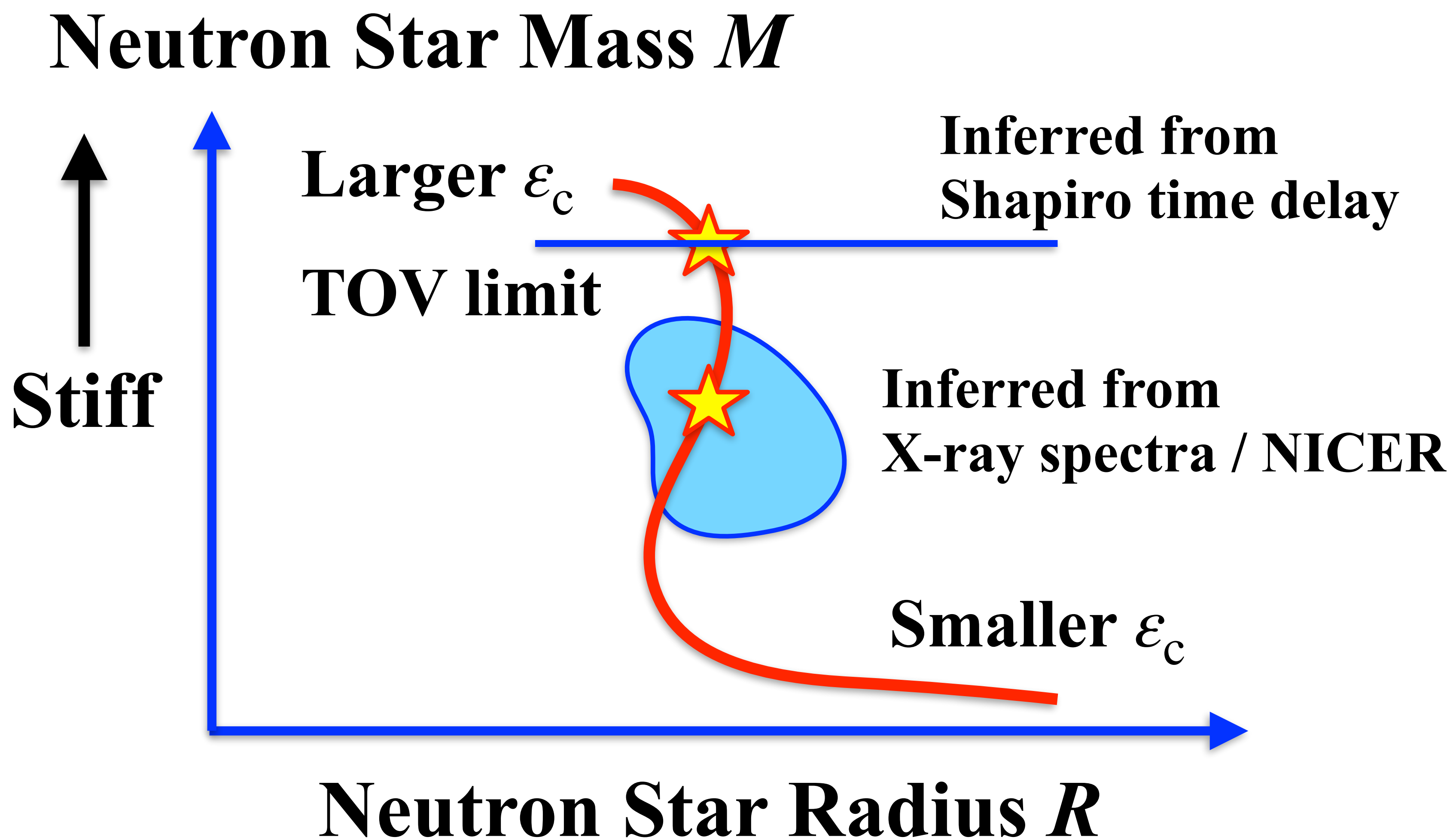
Neutron Star



EOS must be "stiff" enough to reach the Shapiro delay observation.

Lensing and timing (NICER) constraining the gravity strength and the radius!

EOS Reconstruction



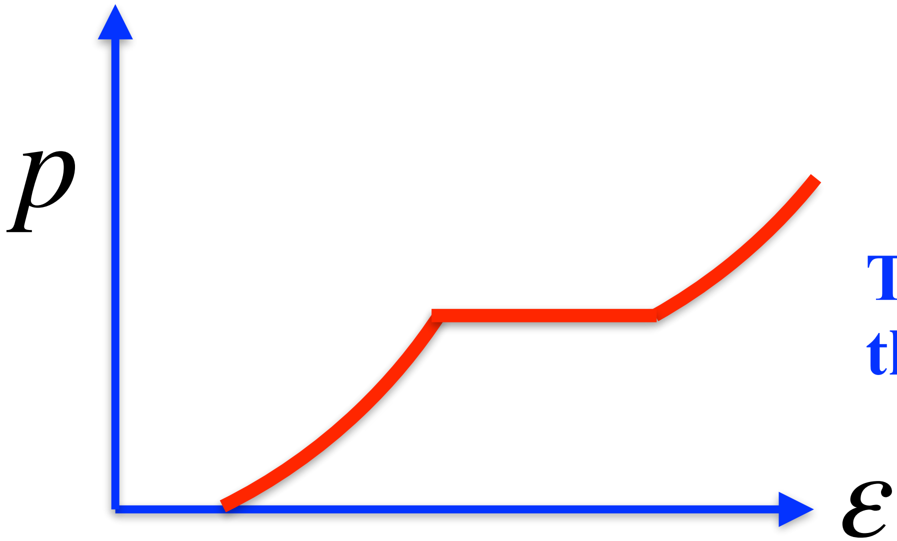
EOS Reconstruction

Mathematically proven:

$$p = p(\varepsilon) \longleftrightarrow M = M(R)$$

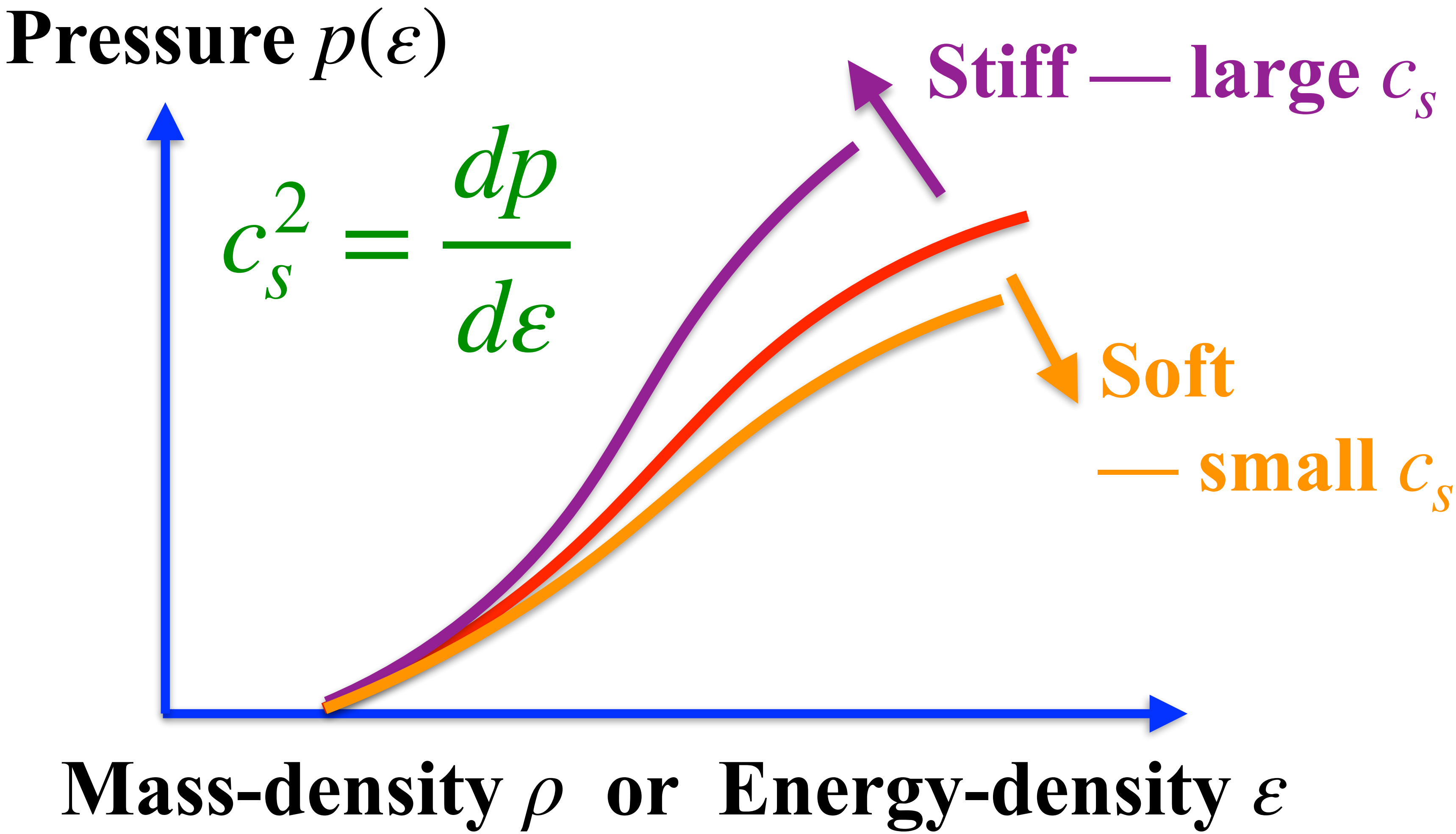
One-to-one Correspondence through TOV eq.

Lindblom (1992)



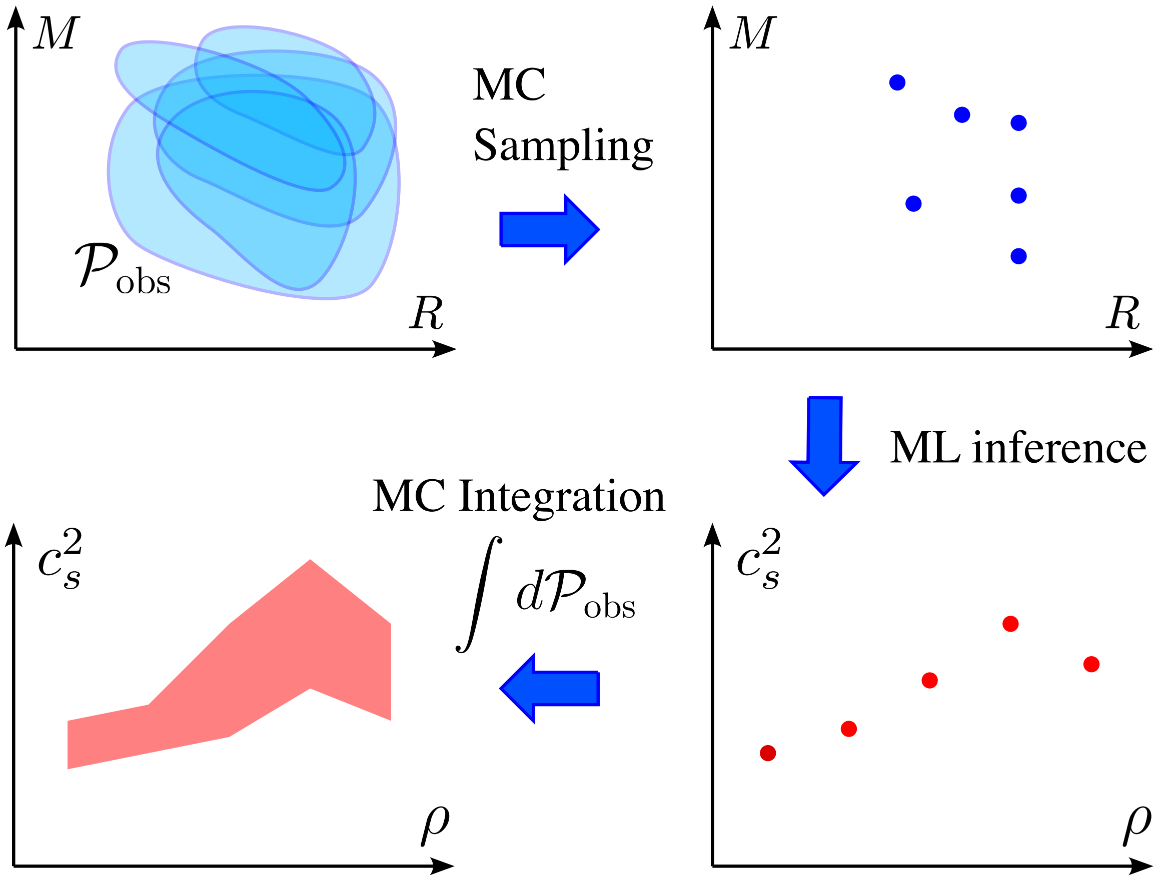
This is the case even with the 1st-order phase transition.

EOS Reconstruction



EOS Inference Program

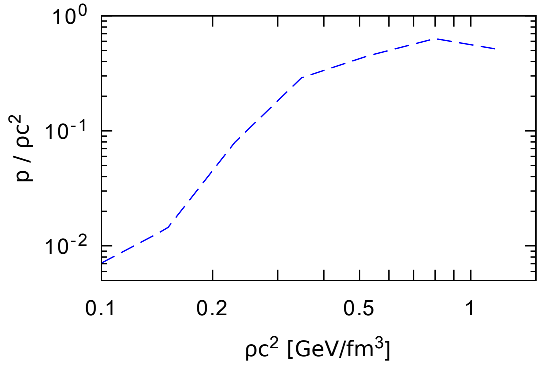
Fujimoto-Fukushima-Kamata-Murase (2018-2024)



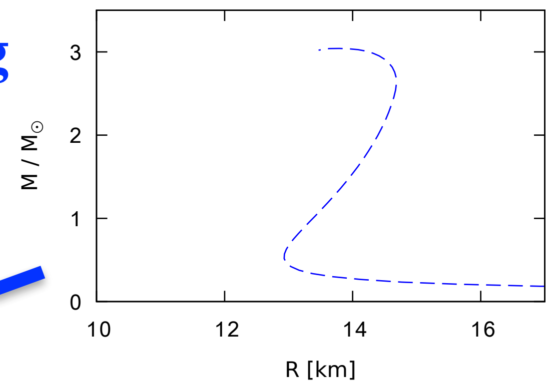
EOS Inference Program

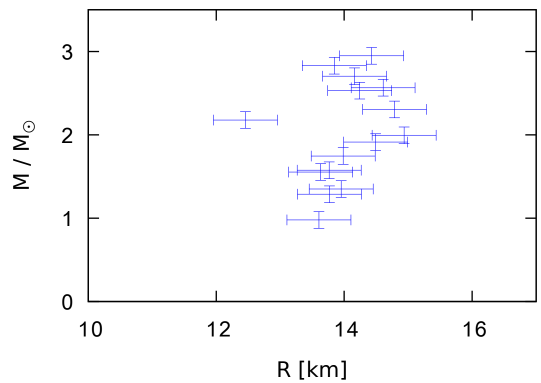
Fujimoto-Fukushima-Kamata-Murase (2018-2024)

Proof of principle



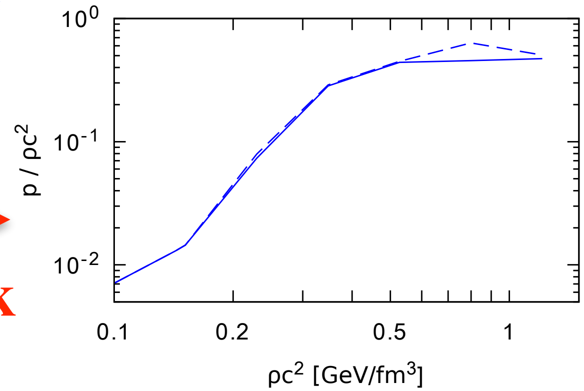
Corresponding M-R relation





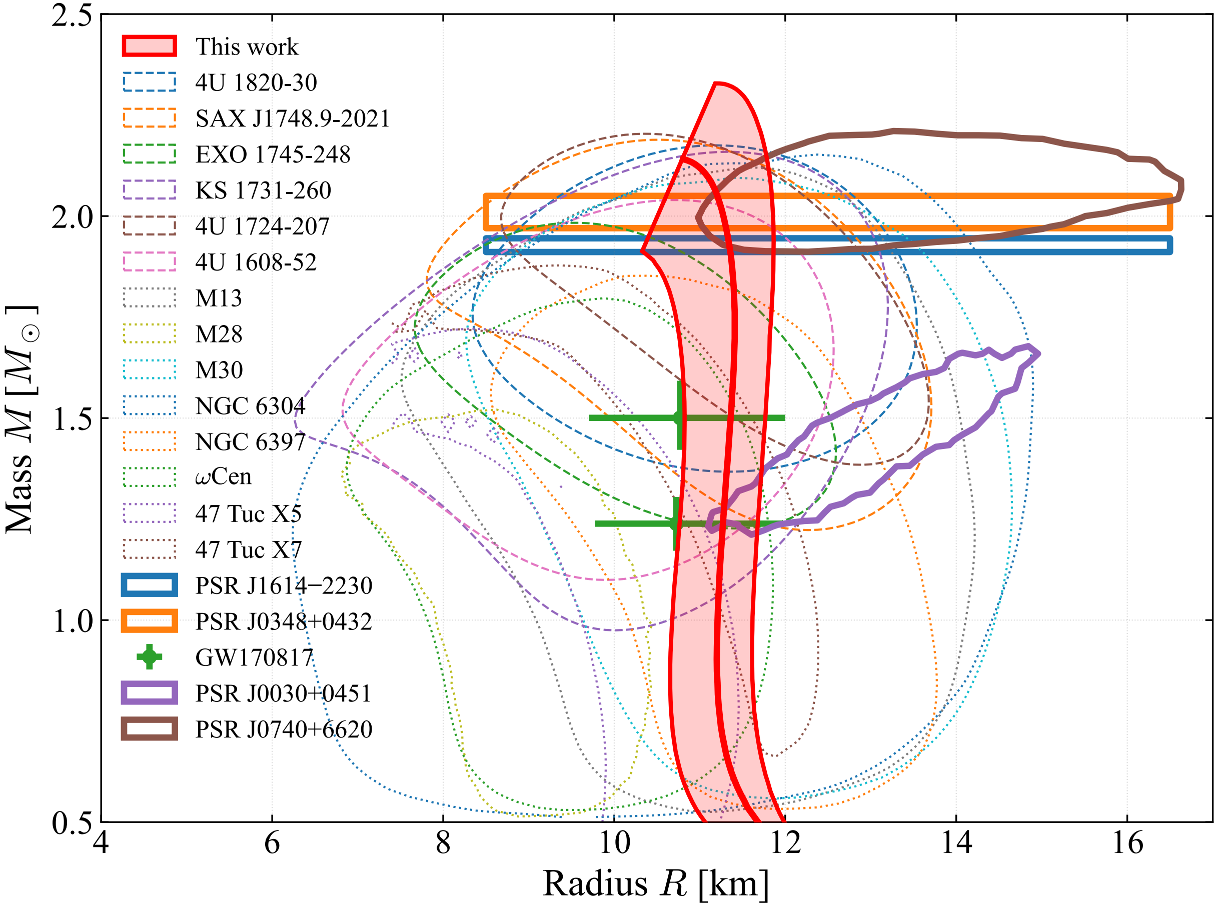
Mimic the astro data

Validity check



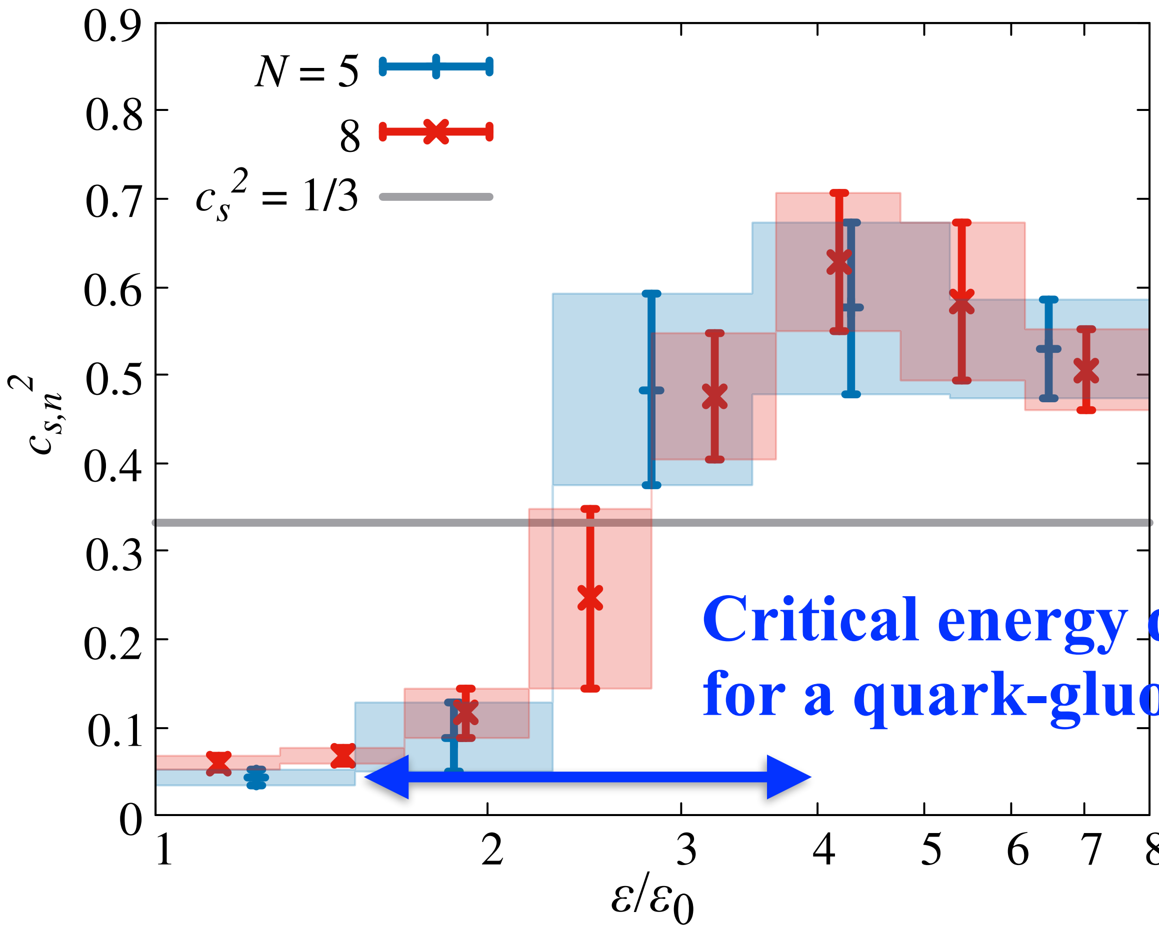
EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



EOS Inference Program

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



Not minimum, but maximum!?

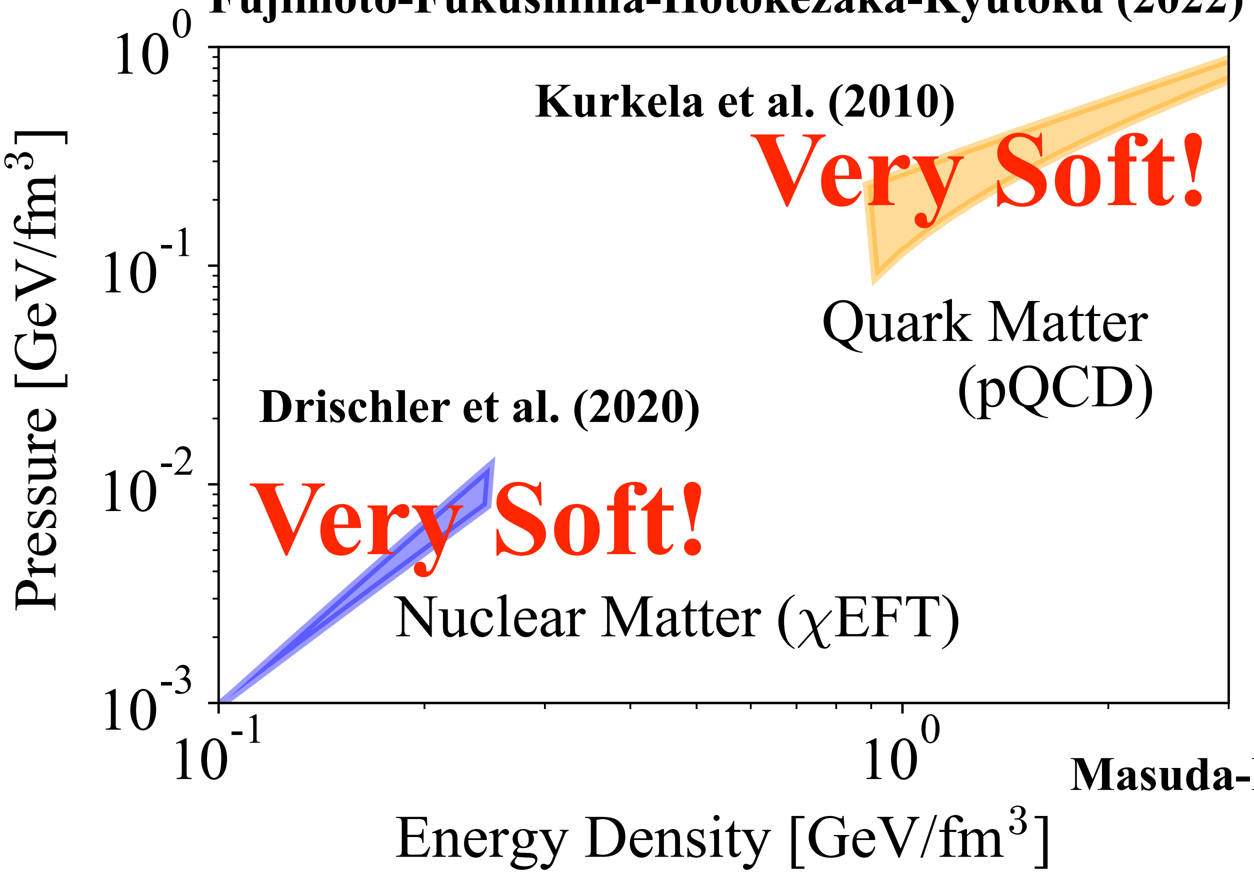
Critical energy density for a quark-gluon plasma

$$\varepsilon_0 = 150\,\text{MeV fm}^{-3}$$

Interpretation ?

Interpolation between Low and High Density Regions

Fujimoto-Fukushima-Hotokezaka-Kyutoku (2022)



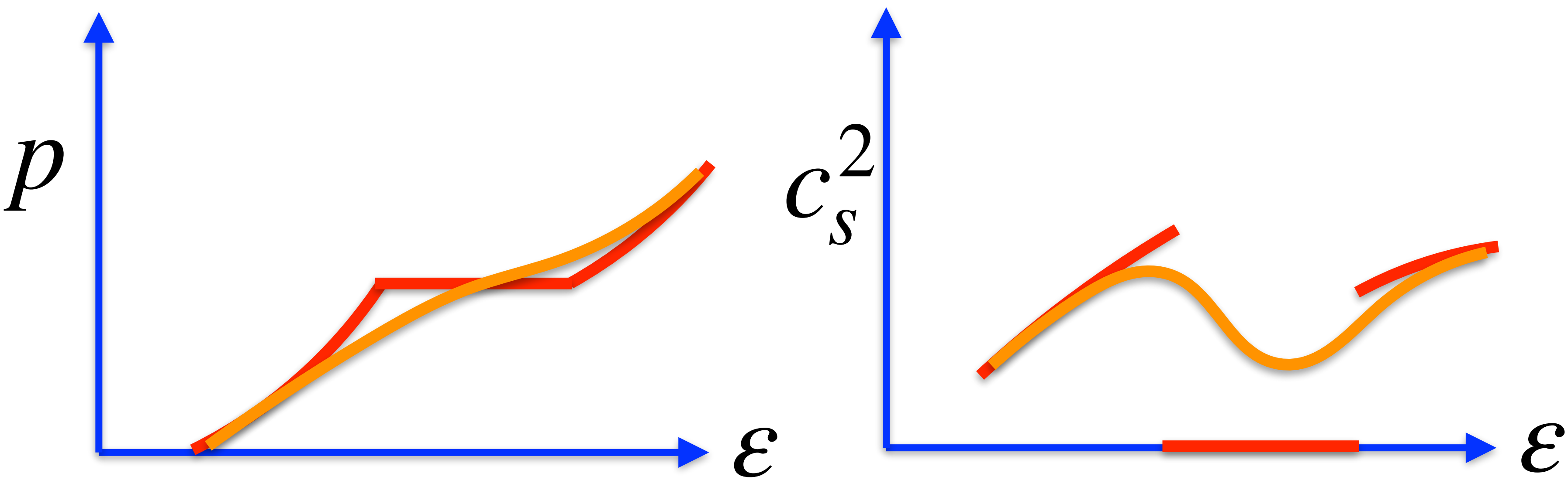
$\bar{\Lambda}/\mu_q = 2 \sim 4$

Interpolated EOS must be stiff enough to support massive NS… so, a peak in the sound velocity is *unavoidable*!

Masuda-Hatsuda-Takatsuka (2012)

Interpretation ?

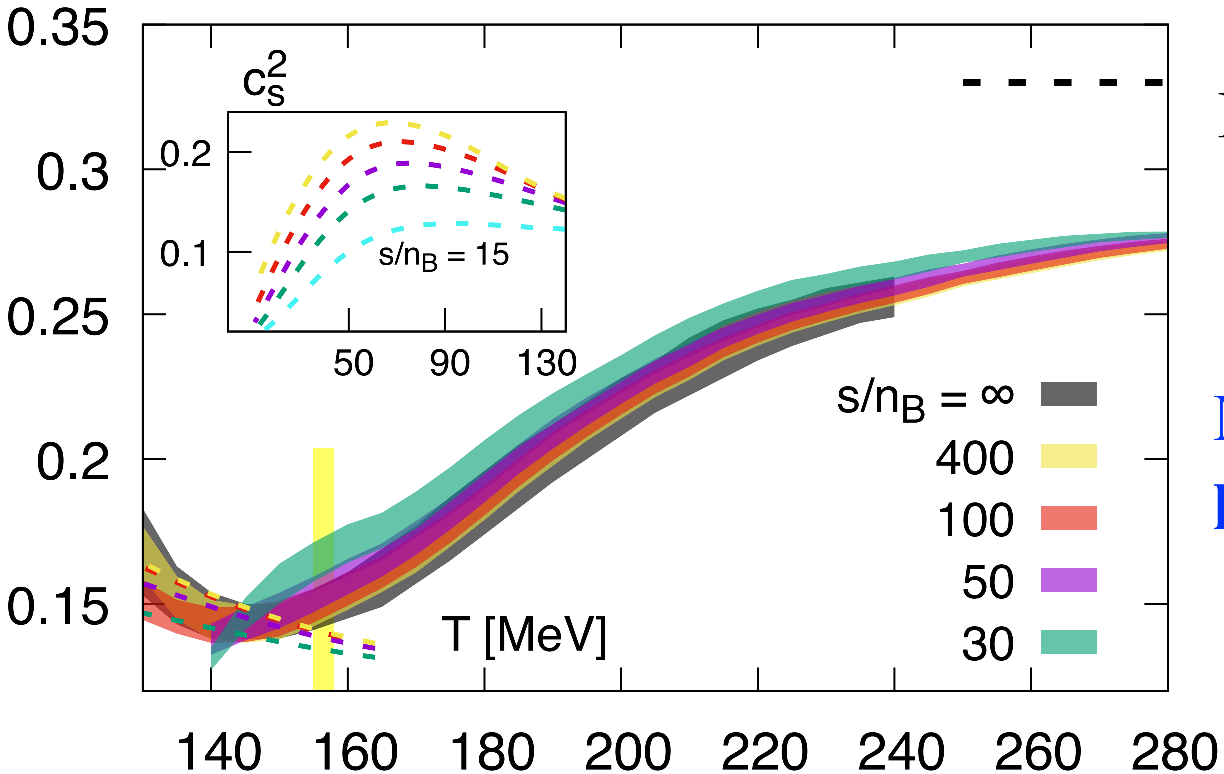
[1st-order-like EoS]



Phase transition is manifested by a minimum in the speed of sound.

Interpretation ?

[High-Temperature QCD — QGP Crossover]



HotQCD Collab. (2212.09043)

Minimum around phase transition

Interpretation ?

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Measure of conformality: $$\Delta = \frac{1}{3} - \frac{p}{\varepsilon}$$

Gavai-Gupta-Mukherjee (2004)

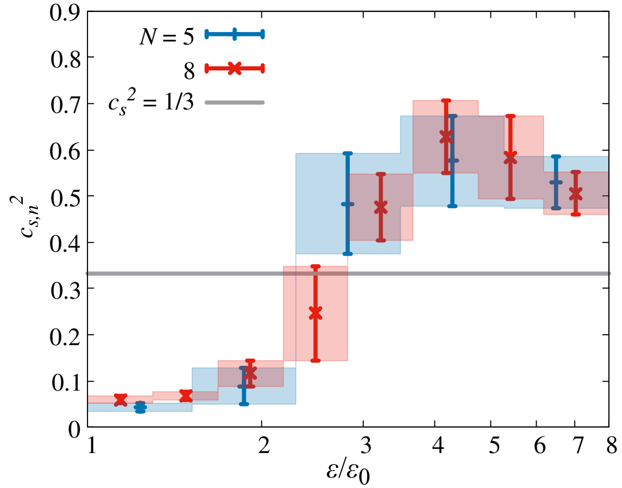
$$c_s^2 = \frac{dp}{d\varepsilon} = c_{s,\,\mathrm{deriv}}^2 + c_{s,\,\mathrm{non-deriv}}^2$$

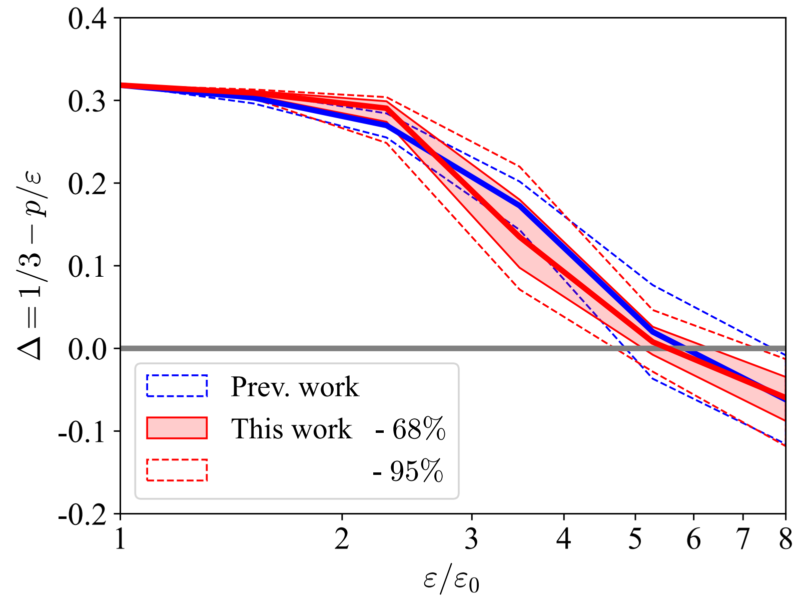
$$c_{s,\,\mathrm{deriv}}^2 = -\,\varepsilon \frac{d\Delta}{d\varepsilon} \qquad c_{s,\,\mathrm{non-deriv}}^2 = \frac{1}{3} - \Delta$$

Derivative **Non-Derivative**

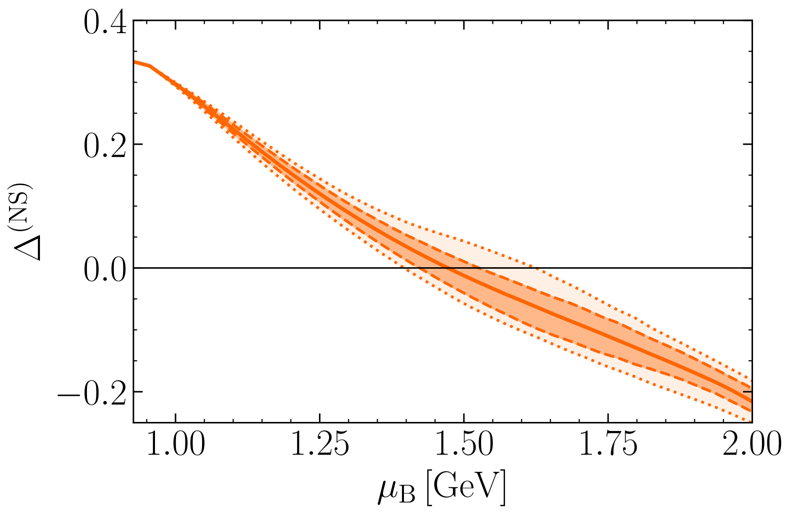
Dominant at high density making a peak!

Interpretation ?





Brandes-Fukushima-Iida-Yu (2024)

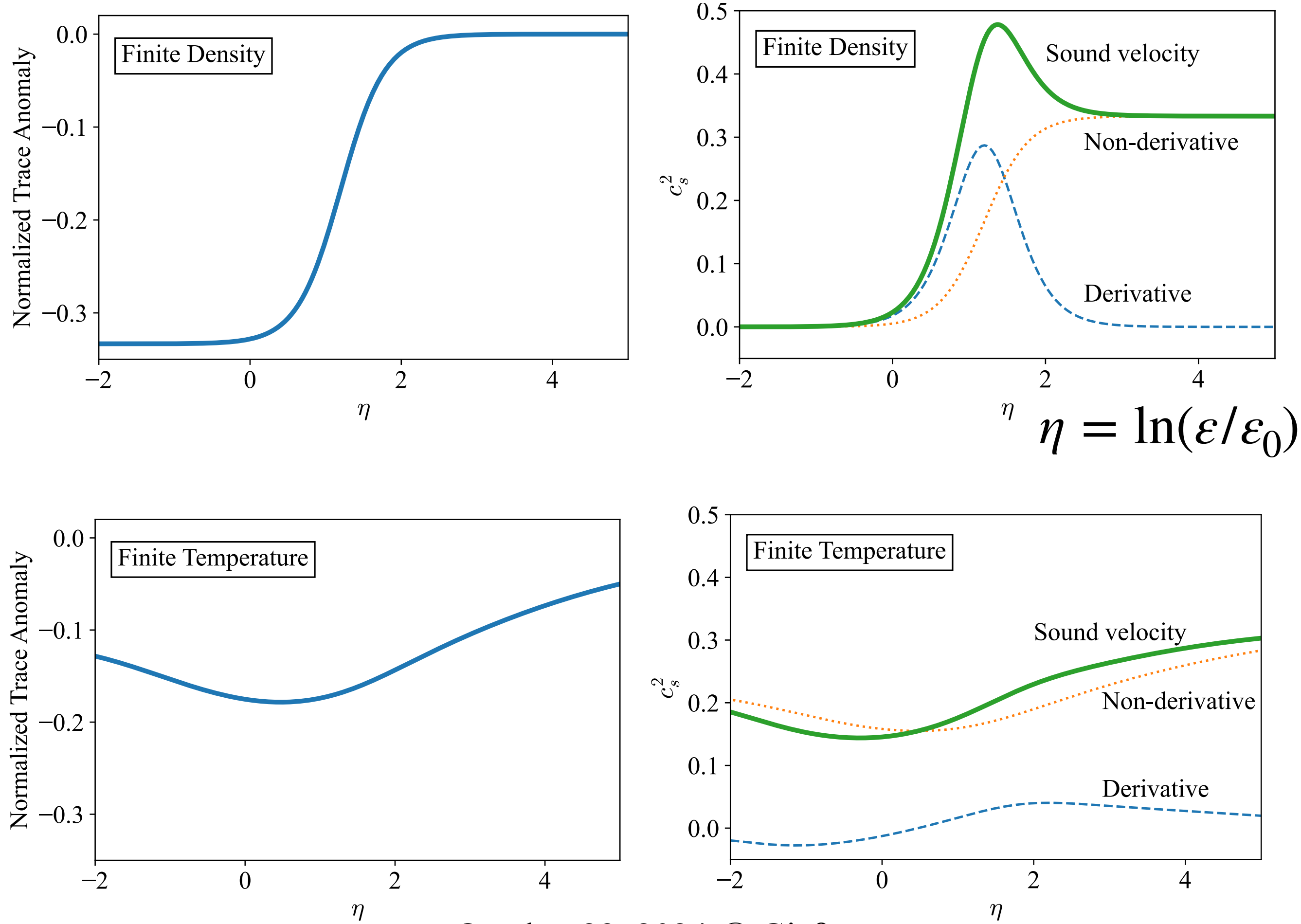


Newer analysis suggests that the trace anomaly goes negative!

Interpretation ?

Derivative contribution makes a peak structure!

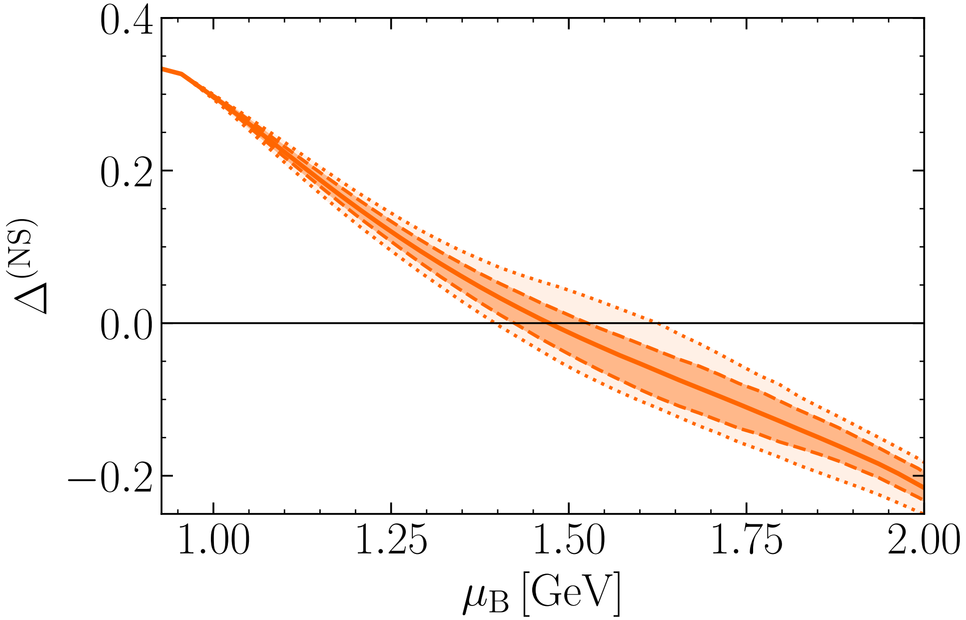
Sign Flipped $-\Delta$



Speed of Sound

Interpretation ??

Interesting question… $\Delta < 0$???



$$\Delta \propto \varepsilon - 3p$$

$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

Thermodynamic degrees of freedom

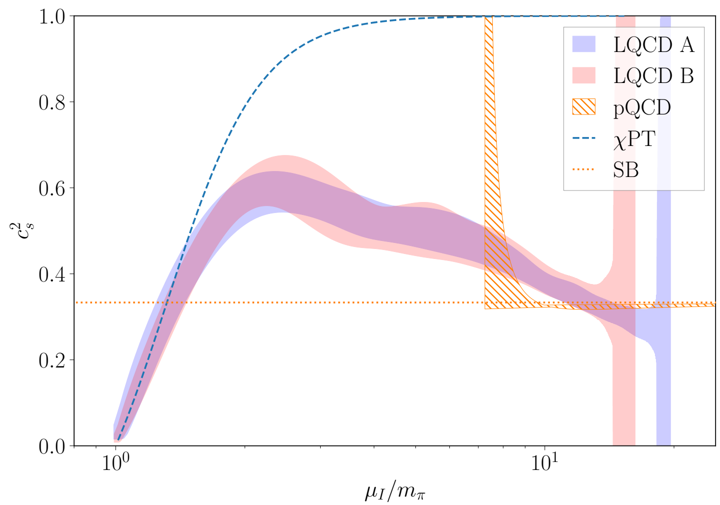
Negative trace anomaly implies the presence of "condensates"!?

Interpretation ??

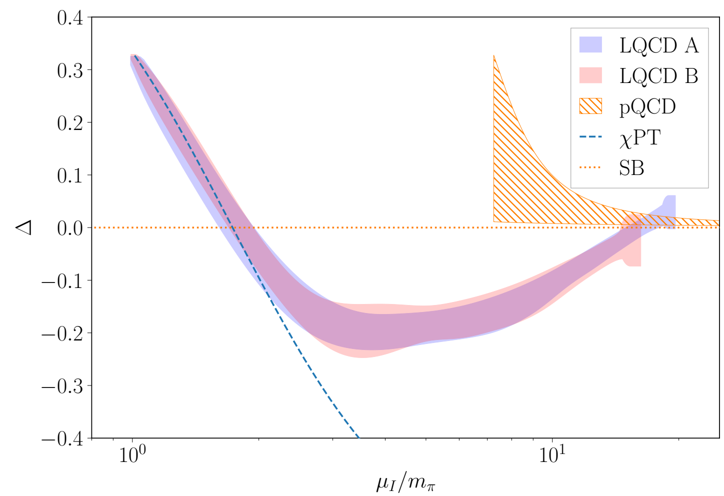
Lesson from high-isospin matter

Abbott+ (2023)

[Speed of sound peak]



[Negative trace anomaly]



$$p_{\chi\mathrm{PT}} = \frac{f_\pi^2 \mu_I^2}{2}\left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2$$

Pressure from the condensates

Interpretation ??

Quick derivation

Son-Stephanov (2001)

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}{\rm Tr}\nabla_\nu\Sigma\nabla_\nu\Sigma^\dagger - \frac{m_\pi^2 f_\pi^2}{2}{\rm ReTr}\Sigma$$

$$\nabla_0\Sigma = \partial_0\Sigma - \frac{\mu_I}{2}(\tau_3\Sigma - \Sigma\tau_3)$$

$\pi^0 = 0, \quad \pi^\pm \neq 0 \longrightarrow \overline{\Sigma} = \cos\alpha + i(\tau_1\cos\phi + \tau_2\sin\phi)\sin\alpha$

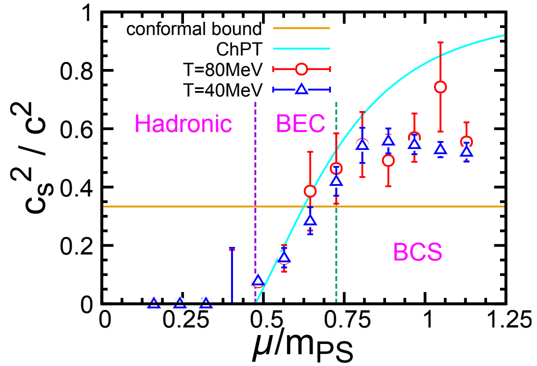
$F = -\frac{f_\pi^2}{2}\mu_I^2\sin^2\alpha - m_\pi^2 f_\pi^2\cos\alpha \longrightarrow \cos\alpha = m_\pi^2/\mu_I^2$

$$\longrightarrow F = -\frac{f_\pi^2}{2}\mu_I^2\left(1 + \frac{m_\pi^4}{\mu_I^4}\right)$$

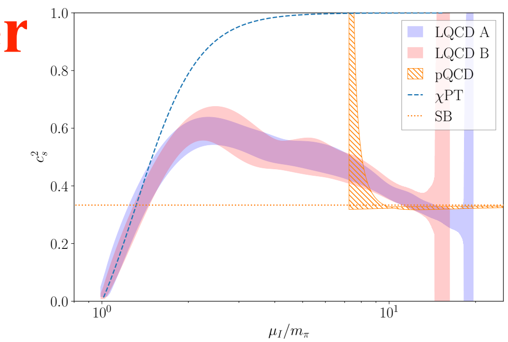
Perturbative Approaches

We should conduct extensive studies of the condensation effects on the speed of sound *perturbatively* for…

* **Diquark superfluid in QC_2D**
  To be compared with
  Lattice: Itou+ (2023-2024)



* **Pion-condensed high-isospin matter**
  To be compared with
  Lattice: Abbott+ (2023)

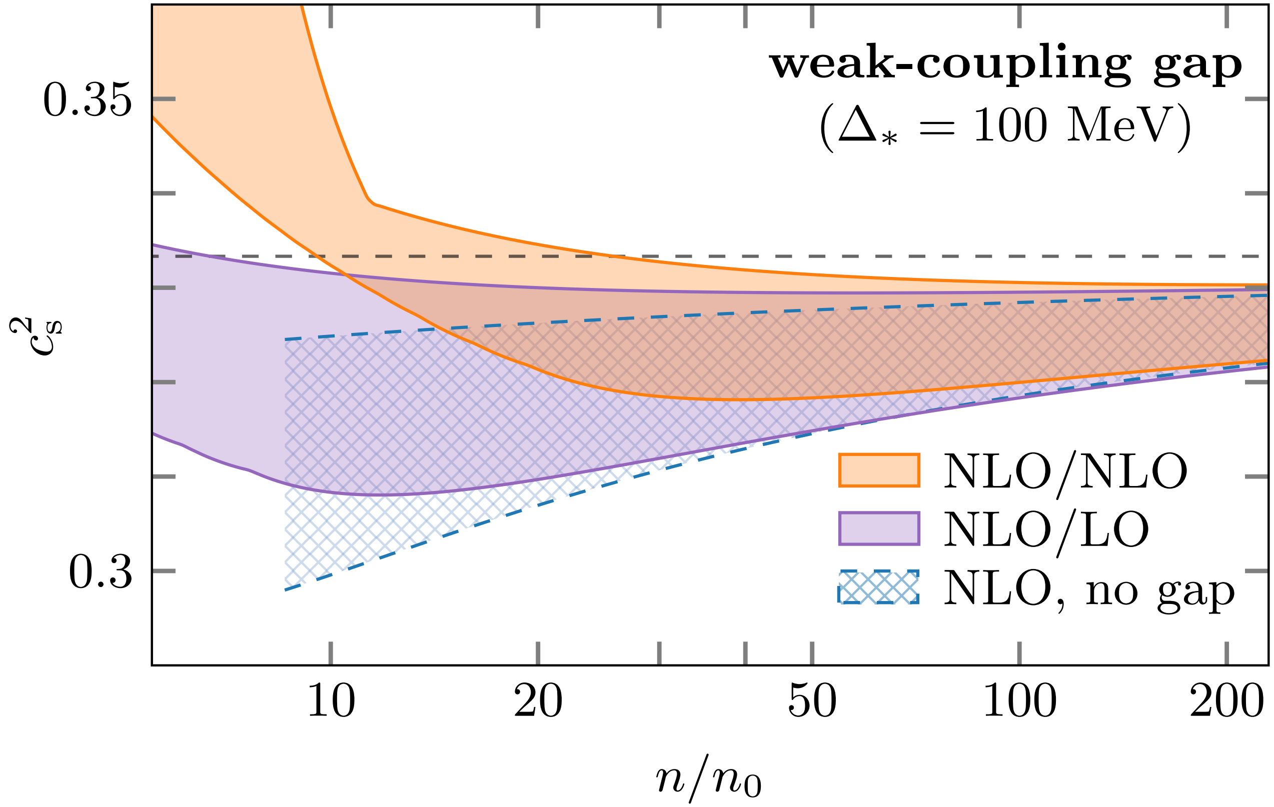


* **Two-flavor color-superconducting (2SC) matter**

Perturbative Approaches

Geissel-Gorda-Braun (2024)

* **Two-flavor color-superconducting (2SC) matter**



Perturbative Approaches

Fukushima-Minato (2024)

Perturbative master formula:

$$c_s^2 - \frac{1}{3} = -\frac{5}{36}\mu\frac{\partial\gamma_0(g)}{\partial\mu} + \frac{\gamma_1(g)}{18} \times \left\{ 2\frac{\Delta^2}{\mu^2} - \frac{\Delta}{\mu}\frac{\partial\Delta}{\partial\mu} - \left(\frac{\partial\Delta}{\partial\mu}\right)^2 - \Delta\frac{\partial^2\Delta}{\partial\mu^2} \right\}$$

$$\gamma_0(g) = 1 - \frac{g^2}{2\pi^2}$$

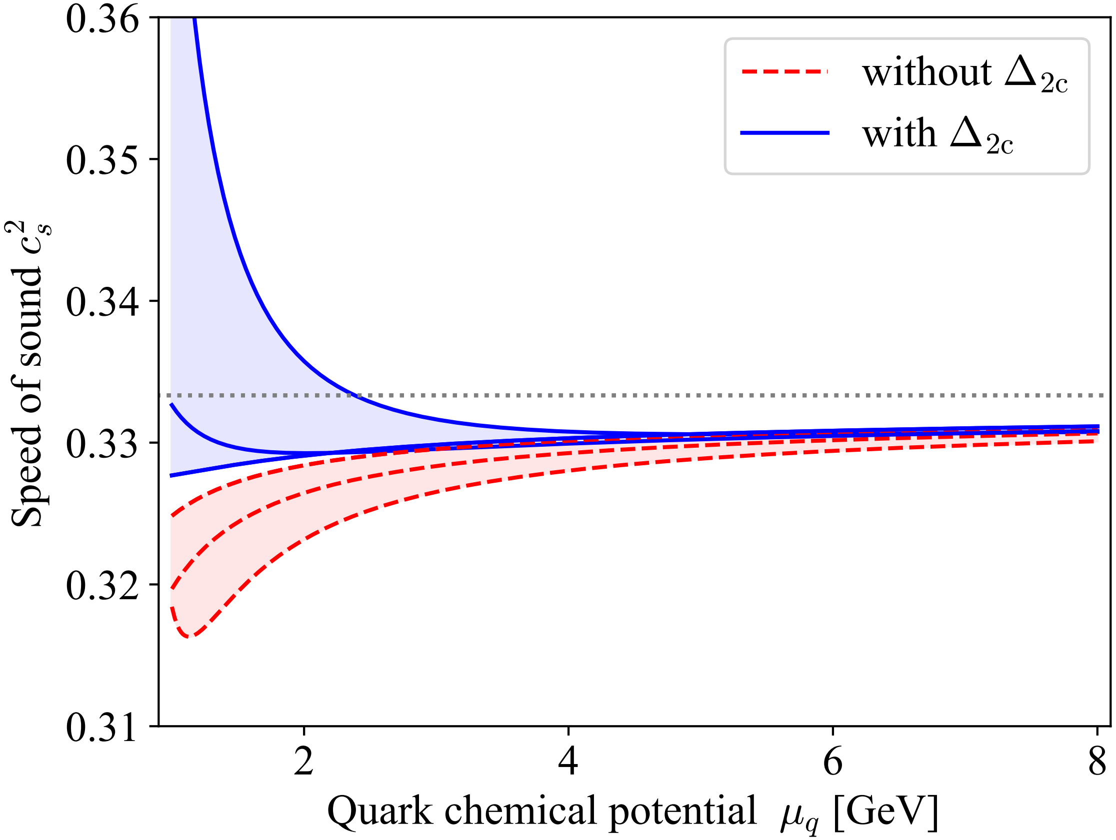
→ **NLO correction without condensate is negative!**

Perturbative Approaches

Fukushima-Minato (2024)

Looks pretty good!

*** Diquark superfluid in QC_2D**

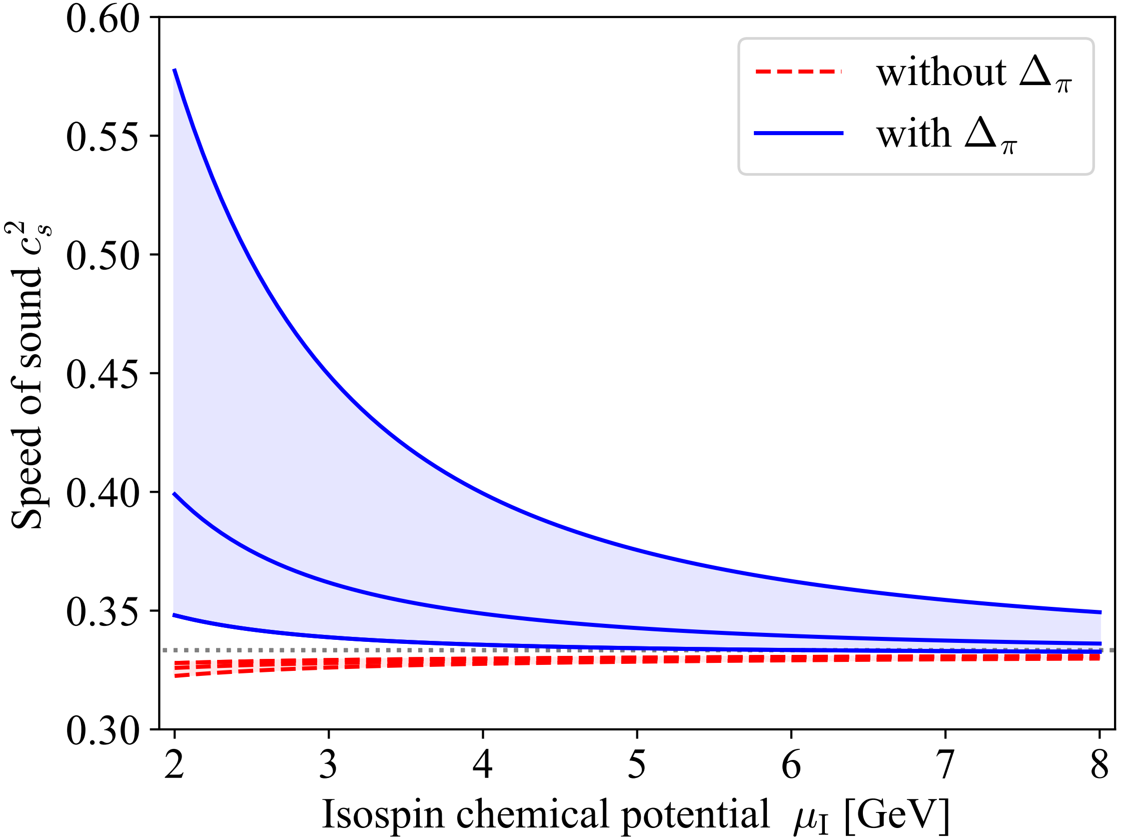


Perturbative Approaches

Fukushima-Minato (2024)

Looks not bad!

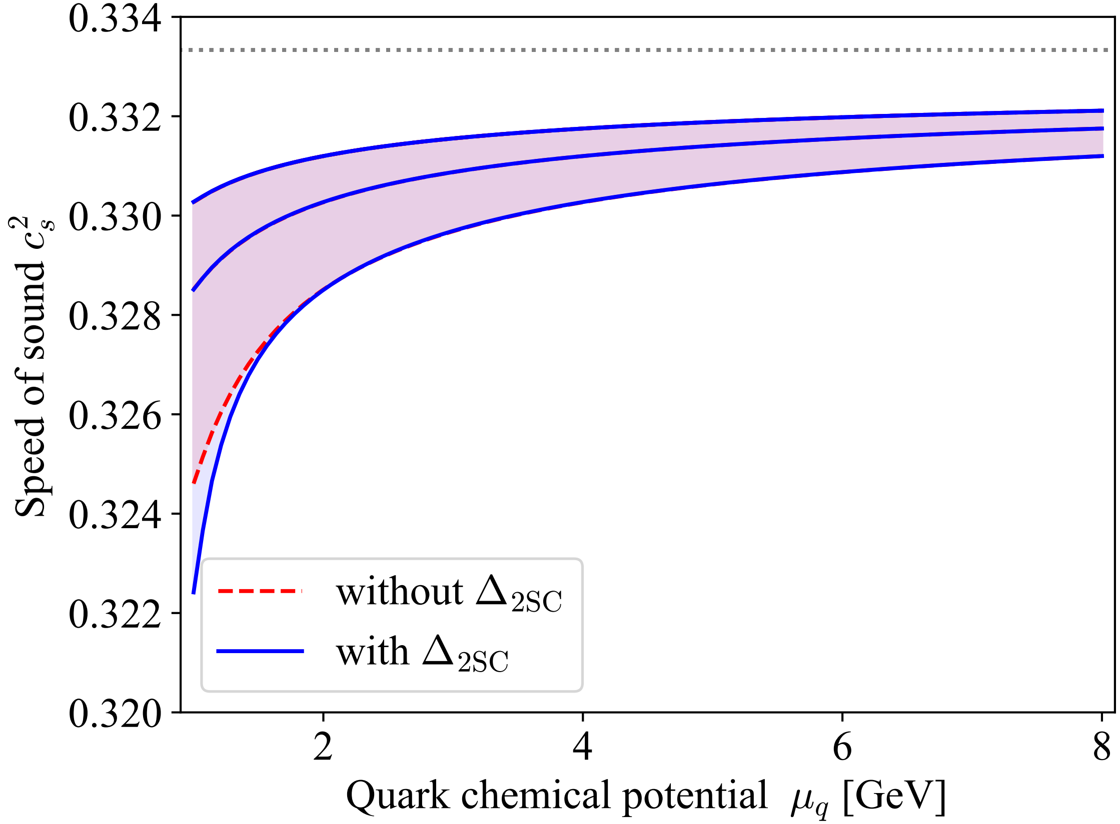
*** Pion-condensed high-isospin matter**



Perturbative Approaches

Fukushima-Minato (2024)

*** 2SC quark matter**



?

Perturbative Approaches

Fukushima-Minato (2024)

Framework is common — gap equation

$$\tilde{\Delta}_\alpha(\nu) = \frac{1}{\pi^2}\int_0^\infty d\nu' \left[h_{1,\alpha} g^2 \ln\left(\frac{b}{\sqrt{|\nu^2 - \nu'^2|}}\right) + \frac{h_{2,\alpha}}{(2\pi)^2}\frac{\tilde{G}}{8(N_c^2-1)}\right] Z(\nu')\frac{\tilde{\Delta}_\alpha(\nu')}{\sqrt{\nu'^2 + \tilde{\Delta}_\alpha(\nu')^2}},$$

$$\Delta_\alpha \sim \mu e^{-\pi^2/2\sqrt{h_{1,\alpha}}g}$$

$$Z(\nu)^{-1} = 1 + \frac{g^2}{24\pi^2} C_F \ln\left(\frac{\tilde{m}_D^2}{4\nu^2}\right)$$

α	$c^F_{1,\alpha}$	$c^F_{g,\alpha}$	$h_{1,\alpha}$	$h_{2,\alpha}$
2SC	2	$\sqrt{2}/6$	$1/18$	$4N_f(N_c+1)/N_c$
2c	3	$\sqrt{2}/6$	$1/16$	$2N_f(N_c+1)$
π	3	$1/6$	$1/9$	$16(N_c^2-1)/N_c$

Quantitatively important (sometimes dropped)

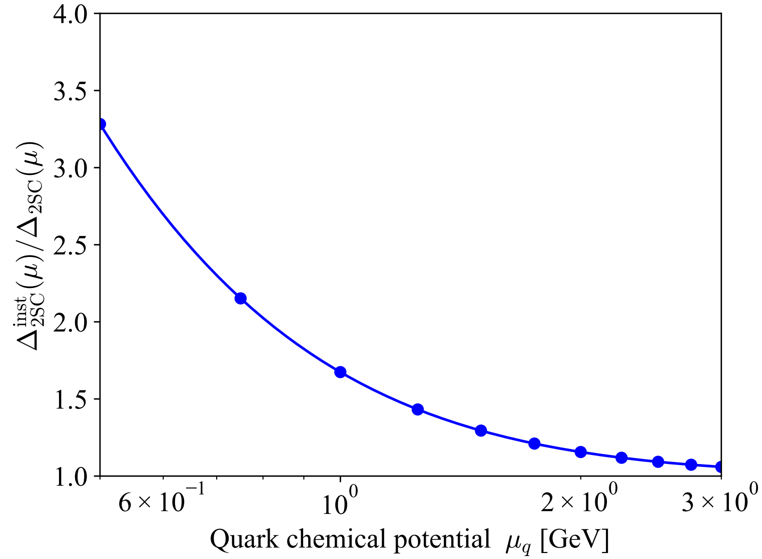
Perturbative Approaches

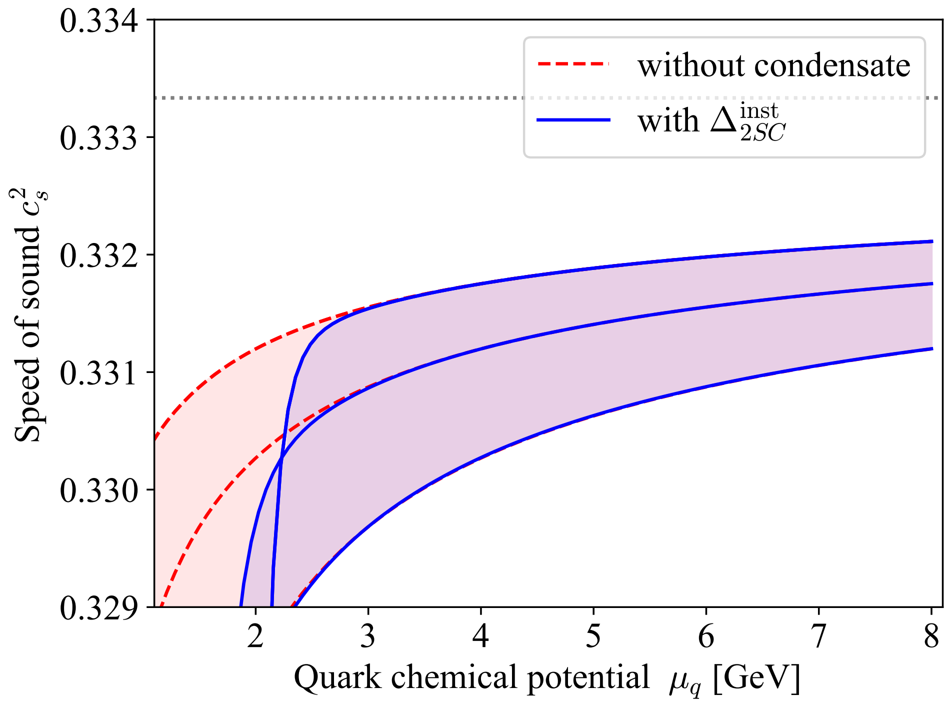
Fukushima-Minato (2024)

Instanton-induced interaction?

Even worse!?

Factor larger condensates





Perturbative Approaches

Fukushima-Minato (2024)

Subtle balance seem from

$$c_s^2 - \frac{1}{3} = -\frac{5}{36}\mu\frac{\partial\gamma_0(g)}{\partial\mu} + \frac{\gamma_1(g)}{18}$$

$$\times\left\{2\frac{\Delta^2}{\mu^2} - \frac{\Delta}{\mu}\frac{\partial\Delta}{\partial\mu} - \left(\frac{\partial\Delta}{\partial\mu}\right)^2 - \Delta\frac{\partial^2\Delta}{\partial\mu^2}\right\}$$

Even if the condensate develops large, almost no correction for $\Delta/\mu \sim \partial\Delta/\partial\mu$, or the overall sign could be flipped.

Perturbative Approaches

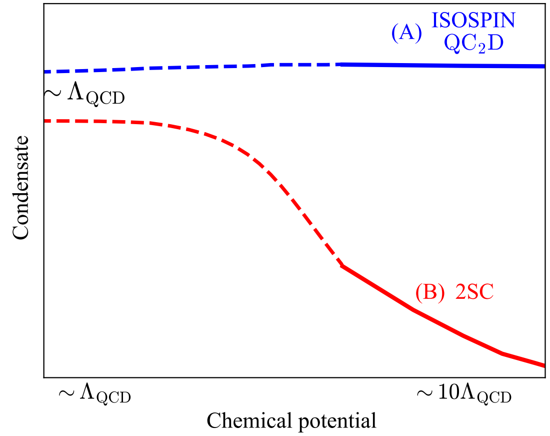
Fukushima-Minato (2024)

Cancellation NOT seen in the trace anomaly:

$$\mathfrak{T} = \frac{1}{3} - \frac{p}{\epsilon}$$

$$\simeq \frac{\mu}{9}\frac{\partial\gamma_0(g)}{\partial\mu} - \frac{2}{9}\gamma_1(g)\frac{\Delta}{\mu}\left(\frac{\Delta}{\mu} - \frac{\partial\Delta}{\partial\mu}\right)$$

Positive

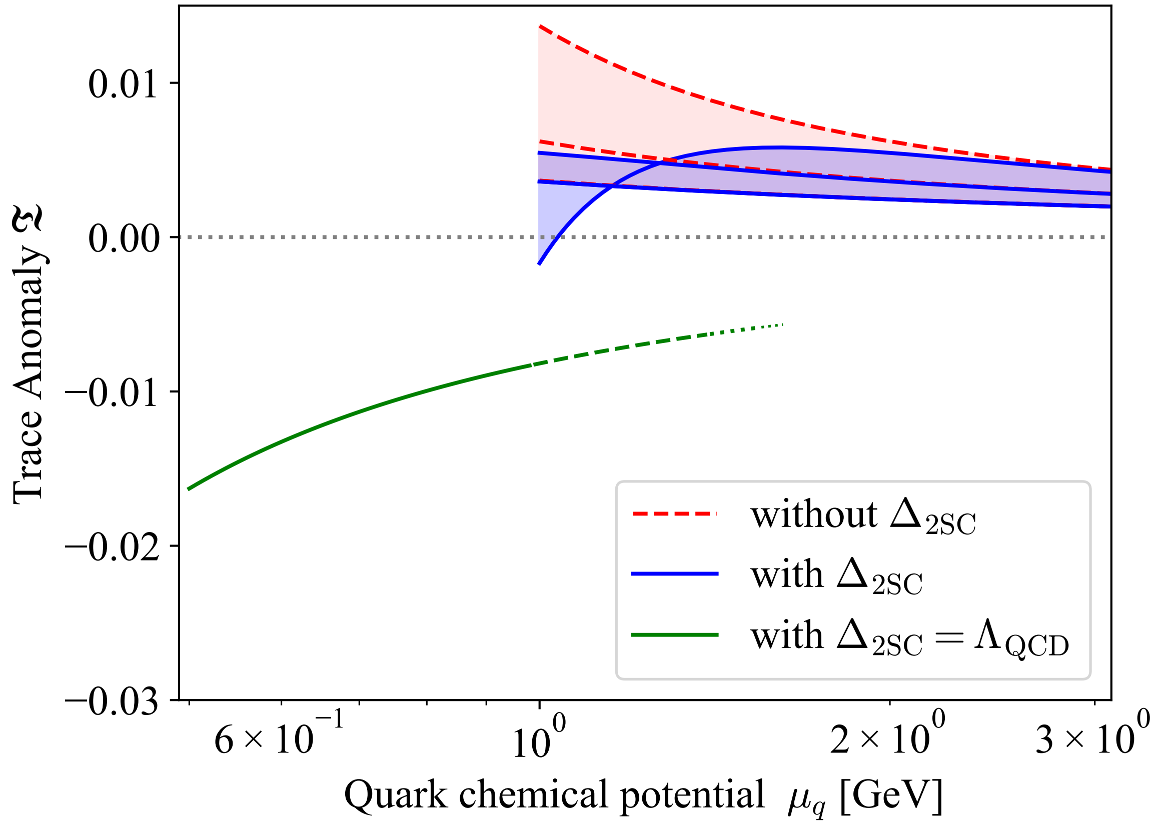


No cancellation if Δ is non-perturbatively large at lower density and goes smaller at higher density.

Perturbative Approaches

Fukushima-Minato (2024)

Cancellation NOT seen in the trace anomaly:



Perturbative Approaches

Fukushima-Minato (2024)

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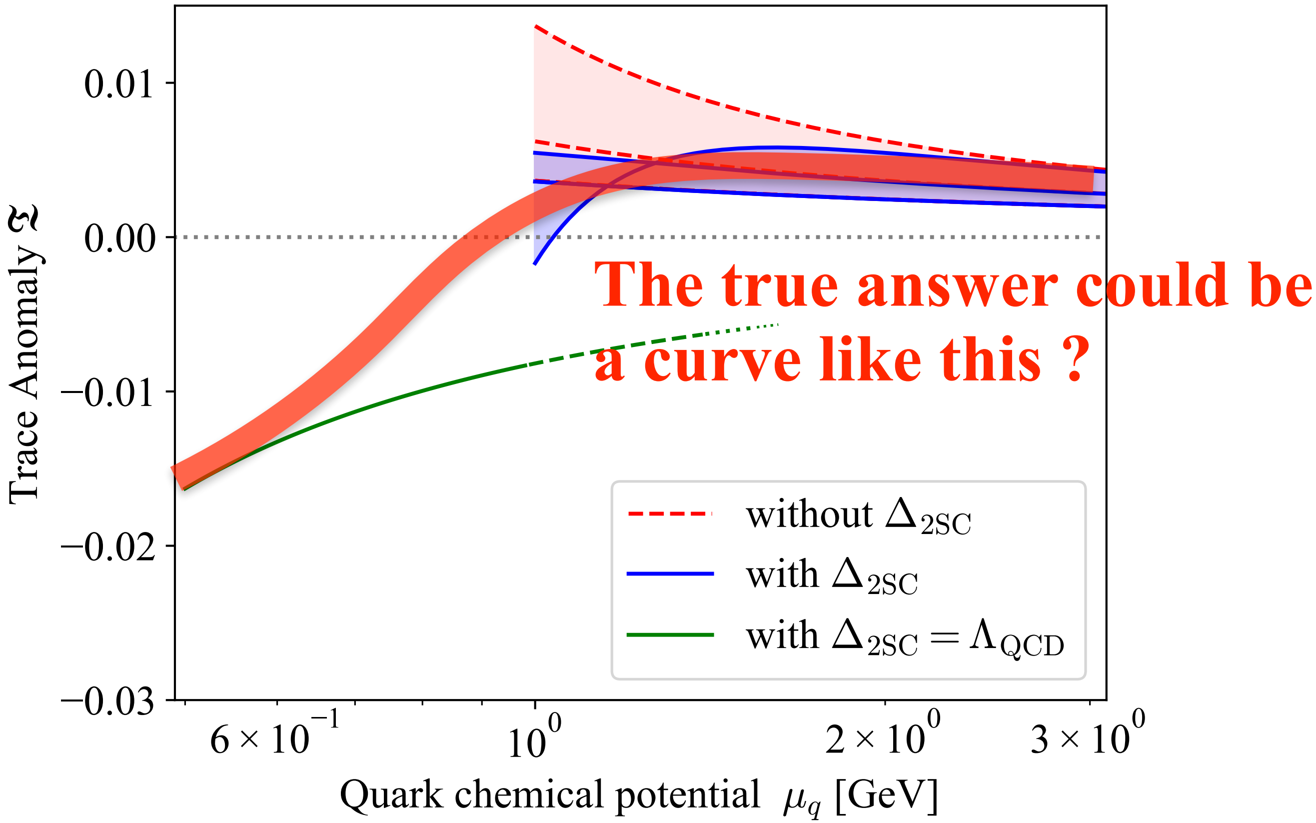

Conclusions

- **Speed of sound at high density may increase above the conformal value. This could be confirmed by the heavy-ion collision.**
- **Trace anomaly is going negative and it implies the presence of some condensates. Color-super?**
- **Perturbative calculations lead to too small gaps to account for phenomenological data.**
- **Trace anomaly is rather straightforward, but the speed of sound is more subtle…**