Impact of Diquarks on the QCD Phase Structure and Spectral Functions

Ugo Mire

based on Ugo Mire and Bernd-Jochen Schaefer [in preparation]

Lunch Club Seminar

Gießen - January 15, 2024

CRC-TR 211
Strong-interaction matter under extreme conditions

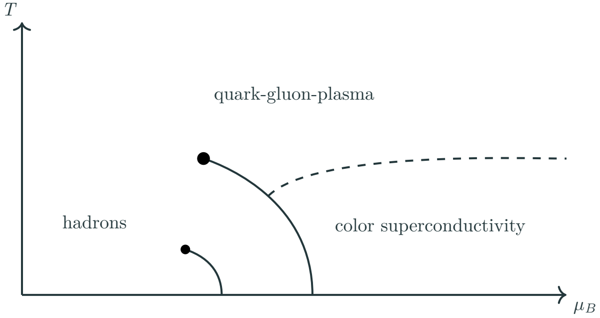
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Outline

Color Superconductivity and Quark-meson-diquark Model

QCD Phase Diagram



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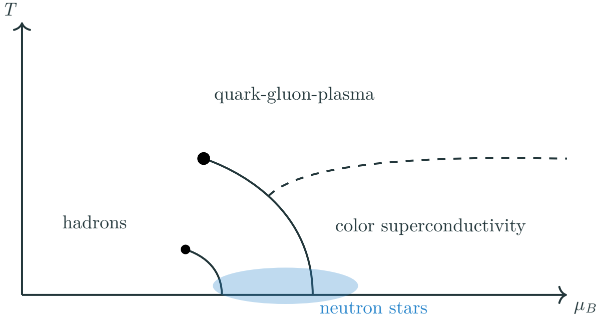

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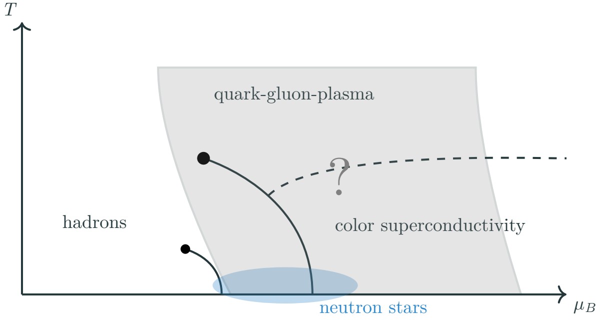

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- Fermi surface is unstable in presence of an attractive interaction!

Why Color Superconductivity?

2. Directly from QCD

- QCD action: $S = \int_x \left\{ -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{q} \not{D} q \right\}$

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$$S_{\text{eff}} = \int_x \left\{ \bar{q} \not{\partial} q + \sum_A (\bar{q} \Gamma_A q)^2 \right\}$$

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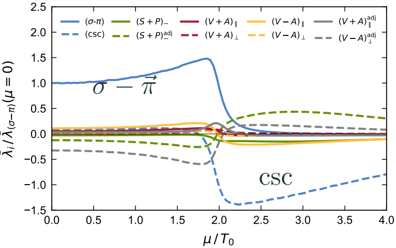
- Chiral symmetry breaking at low μ, color superconductivity at high μ



QMD Four Quark Interactions

Effective NJL model with the most important interactions

$$
\begin{aligned}
S_{4\mathrm{q}} = \int_x \Big\{ & \bar{q}\left(\gamma_\mu \partial_\mu - \mu\gamma_0 + m_0\right) q \\
& - \lambda_{\mathrm{sp}} \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 \vec{\tau} q)^2 \right] \\
& + \frac{1}{2}\lambda_{\mathrm{csc}} (\bar{q}^C \gamma_5 \tau_2 i\epsilon_a q)(\bar{q}\gamma_5 \tau_2 i\epsilon_a q^C) \Big\} \, .
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Bosonization

$$
e^{(\bar{q}q)^2} \sim \int \mathcal{D}\sigma \, e^{g_\sigma \sigma \bar{q}q + \frac{1}{2} m_\sigma^2 \sigma^2}
$$

after bosonization and kinetic term $\rightarrow$ quark-meson-diquark model

quark-meson
scalar-pseudoscalar
channel

$$
\begin{aligned}
S_{\mathrm{QMD}} = \int_x \Big\{ & \bar{q}\left(\not{\partial} - \mu\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}\right)\right) q \\
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& + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 \\
& + \left(\left(\partial_\nu + \delta_{\nu 0}2\mu\right)\Delta_a^*\right)\left(\partial_\nu - \delta_{\nu 0}2\mu\right)\Delta_a \\
& + U(\sigma^2 + \vec{\pi}^2, \Delta_a\Delta_a^*) - c\sigma \Big\}
\end{aligned}
$$

quark-diquark
interaction

diquark kinetic
term, couples to
μ

meson and diquark interactions,
arbitrary potential constrained by
symmetries

explicit chiral
symmetry breaking
term

Symmetries of the Model I

Color symmetry $SU(3)_c$

- Diquark field is composite

$$\Delta_a \sim \bar{q}_b \gamma_5 \tau_2 i\epsilon_{abc} C\bar{q}_c^T$$

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$$U_{ad}U_{be}U_{cf}\epsilon_{def} = \epsilon_{abc}$$

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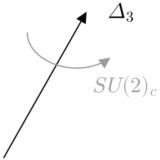
- Assume $\Delta_a = \delta_{a3} \Delta$ with $\Delta \neq 0 \to$ break color symmetry $SU(3)_c \to SU(2)_c$



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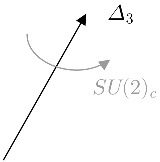
- 1 massive mode and 5 Goldstone modes



Chiral symmetry $SU(2)_A \times SU(2)_V$

- $\langle\sigma\rangle \neq 0$ break chiral symmetry $SU(2)_V \times SU(2)_A \to SU(2)_V$
- 1 massive mode σ and 3 (pseudo-)Goldstone modes $\vec{\pi}$

Symmetries of the Model II

Chiral symmetry $SU(2)_A \times SU(2)_V$

- $\langle\sigma\rangle \neq 0$ break chiral symmetry $SU(2)_V \times SU(2)_A \to SU(2)_V$
- 1 massive mode σ and 3 (pseudo-)Goldstone modes $\vec{\pi}$

Baryon number conservation $U(1)_B$

- Broken by $\langle\Delta_3\rangle \neq 0$
- Combination of $SU(3)_c$ and $U(1)_B$ is left unbroken $\to$ no Goldstone mode associated

Quark Spectrum I

What happens to the quarks?

- Symmetry breaking $SU(3)_c \to SU(2)_c \to$ quark color space splits in two parts $\{q_r, q_g\}$ and q_b

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- Symmetry breaking $SU(3)_c \to SU(2)_c \to$ quark color space splits in two parts $\{q_r, q_g\}$ and q_b
- Off-diagonal quark propagator

$$G_q^{-1} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad \leftarrow \quad \begin{array}{c} \text{in the basis} \\ \Psi^\top = (q, \bar{q}^\top) \end{array}$$

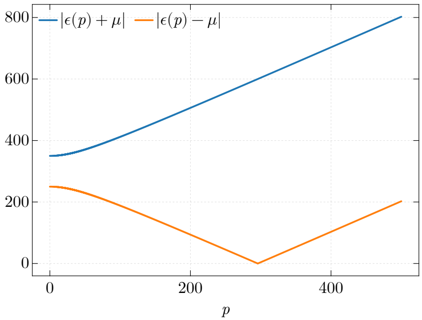
$\to$ find quark spectrum after diagonalisation

Quark Spectrum II

1. **Blue quarks**

$$\epsilon_q^{\pm} = \sqrt{p^2 + m_q^2} \pm \mu$$

with $m_q = g_\phi \sigma$.



Quark Spectrum II

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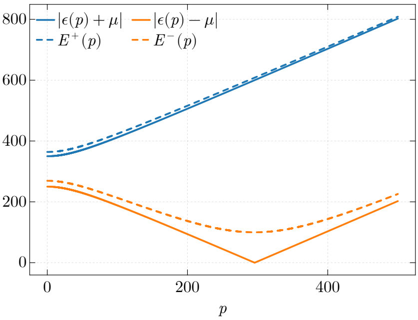
with $m_q = g_\phi \sigma$.

2. **Red and Green quarks**

$$E_q^{\pm} = \sqrt{(\epsilon_q^{\pm})^2 + \Delta_{\text{gap}}^2}$$

with $\Delta_{\text{gap}} = g_\Delta \Delta$.

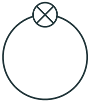


FRG and Phase Structure

Functional Renormalization Group

- Effective average action Γ_k: $\Gamma_{k=\Lambda} = S_{\text{classical}}$ and $\Gamma_{k=0} = \Gamma$ by means of regulator R_k.
- Γ_k follows a differential equation:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k \right] = \frac{1}{2}$$



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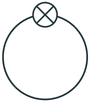

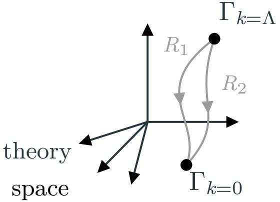

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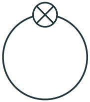

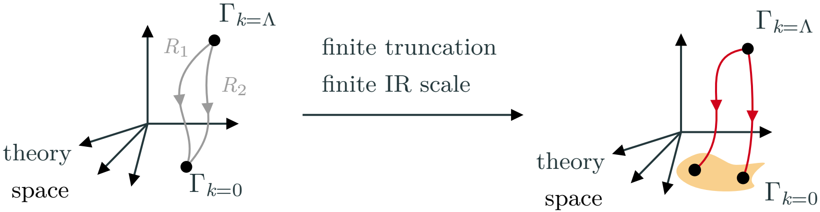

Truncation & Parameter Choice

Truncation Local Potential Approximation (LPA) for mesons, mean-field for diquarks

$$\begin{aligned}\Gamma_k = \int_x \Big\{ & \bar{q}\left[\partial\!\!\!/ - \mu\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec{\pi}\,\vec{\tau}\right)\right] q \\ & + \frac{1}{2} g_\Delta \left(\Delta_a^*\,\bar{q}_b\gamma_5\tau_2\, i\epsilon_{abc} C\bar{q}_c^T - \Delta_a\, q_b^T C\gamma_5\tau_2\, i\epsilon_{abc} q_c\right) \\ & + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U_k(\sigma^2 + \vec{\pi}^2, |\Delta|^2) - c\sigma \Big\}\end{aligned}$$

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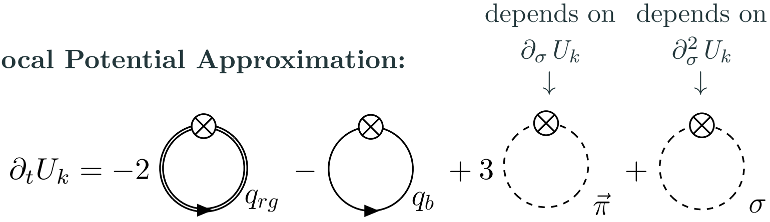
Parameter choice $\Lambda' = 600\,\mathrm{MeV}$

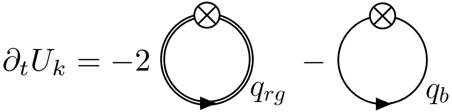
$$U_{\Lambda'}(\sigma, \Delta) = a_1 \sigma^2 + a_2 \sigma^4 + m_\Delta^2 \Delta^2$$

- Fix quark mass, pion decay constant, pion mass, sigma mass, and diquark curvature mass to $600\,\mathrm{MeV}$ in vacuum.
- g_Δ remains a free parameter.

Local Potential Approximation:



Mean-field:

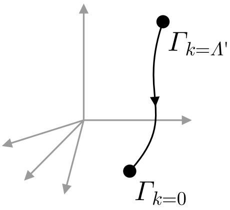


RG-consistency I

Main idea: remove cutoff artifacts at finite T and μ by "flowing upward"

- RG-consistency $\sim$ cutoff independence

$$\Lambda \frac{d\Gamma}{d\Lambda} = 0$$



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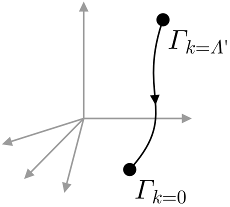

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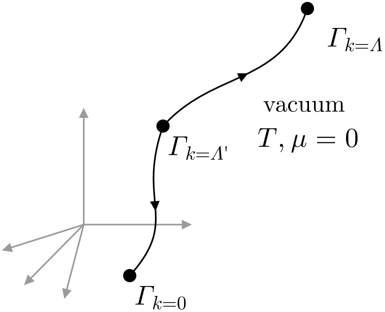

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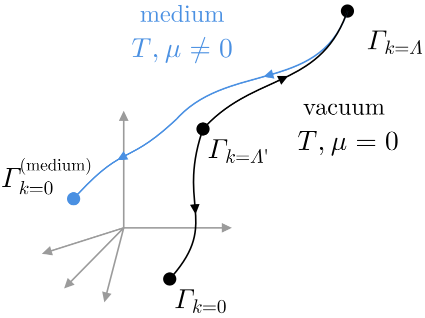

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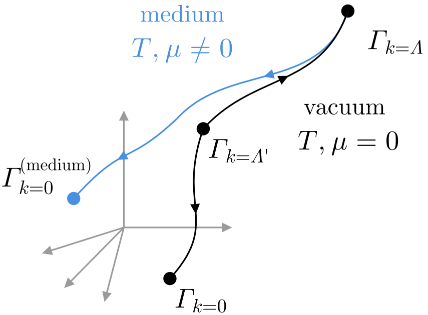

- Effective potential flow $\partial_k U_k(\sigma, \Delta) = \mathcal{F}_k(\sigma, \Delta) \rightarrow$ RG-consistent initial condition:

$$U_\Lambda(\sigma, \Delta) = U_{\Lambda'}(\sigma, \Delta) + \left(\int_{\Lambda'}^{\Lambda} dk \, \mathcal{F}_k(\sigma, \Delta) \right) \Bigg|_{T=\mu=0}$$

RG-consistency II

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- ... because of medium divergences

$$\Omega(\sigma, \Delta) \underset{\Lambda' \to \infty}{\simeq} -\mu^2 \Delta^2 \ln \Lambda' + \cdots$$

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$$\Omega(\sigma,\Delta) \underset{\Lambda'\to\infty}{\simeq} -\mu^2\Delta^2\ln\Lambda' + \cdots$$

- Capture medium divergences by Taylor expanding in $\mu \to$ modified initial condition becomes

$$U_\Lambda(\sigma,\Delta;\mu) = U_{\Lambda'}(\sigma,\Delta) + \left(\int_{\Lambda'}^{\Lambda} dk\,\mathcal{F}_k(\sigma,\Delta)\right)\bigg|_{T=\mu=0} + \frac{\mu^2}{2}\left(\partial_\mu^2\int_{\Lambda'}^{\Lambda} dk\,\mathcal{F}_k(\sigma,\Delta)\right)\bigg|_{T=\mu=0}$$

RG-consistency III

- LPA flow formulated in terms of $u_k(\sigma, \Delta) = \partial_\rho U_k(\sigma, \Delta)$ with $\rho = \sigma^2/2$

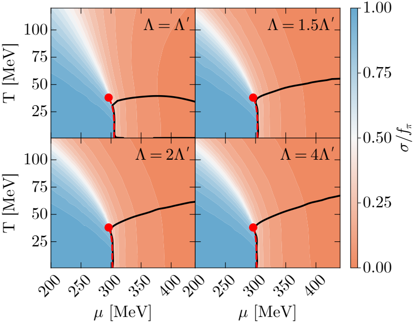
$$\partial_t u_k + \partial_\sigma F_k(\rho, u_k) = \partial_\sigma Q_k(\sigma, u_k, \partial_\rho u_k) + S_k(\rho, \Delta)$$

  $\rightarrow$ form of an advection-diffusion PDE.
- Advection is irreversible $\rightarrow$ flowing upward will not yield the initial condition!

But: Bosonic Fluctuations are supressed in the UV.

Solution: Flow upward using only the fermionic contribution (source term), add again the bosons at $k = \Lambda'$.

Phase Structure - Effect of RG-consistency



Mean-field Approximation

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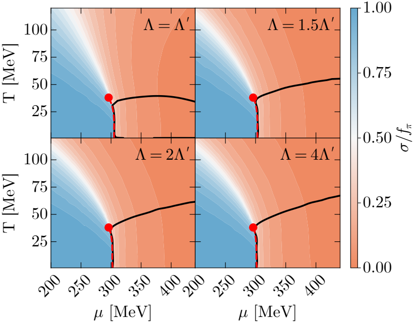

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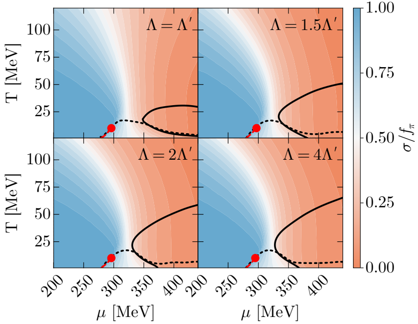
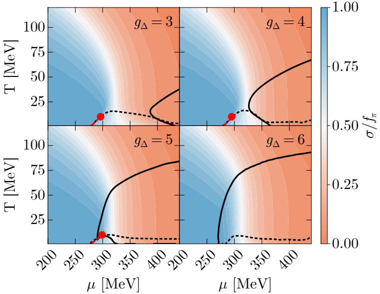


Local Potential Approximation

Phase Structure - Diquark and Back-bending

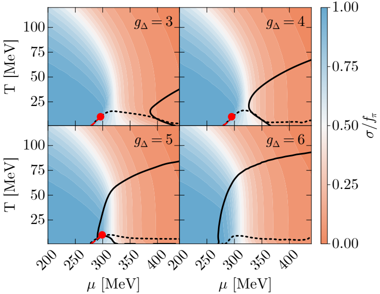
LPA phase structure



LPA

- Back-bending is reduced for high g_Δ but ...

LPA phase structure



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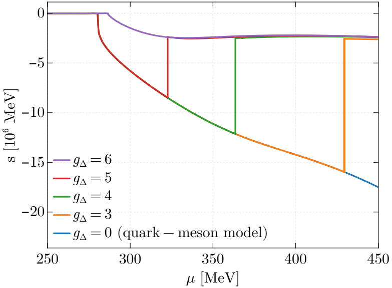
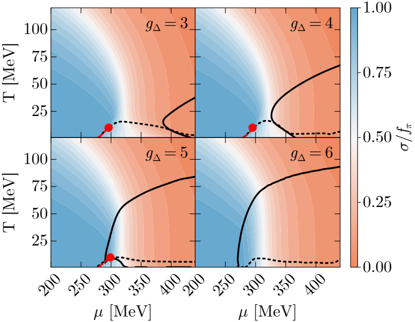


entropy density

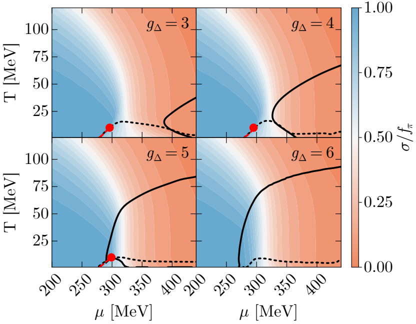
- Back-bending is reduced for high g_Δ but ...
- ... is still present through negative entropy density

Phase Structure - Silver Blaze

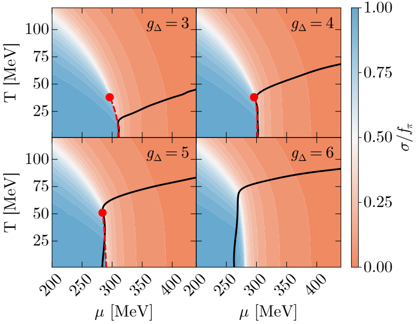


LPA

- Is Silver-Blaze violated?



LPA



MFA

- Is Silver-Blaze violated?

Reminder on Silver-Blaze

- Define correlations functions

$$\Gamma^{(n)}_{i_1 \ldots i_n}(p_1, \ldots, p_n; \mu) = \frac{\delta^n \Gamma[\Phi]}{\delta \Phi_{i_1}(p_1) \ldots \delta \Phi_{i_n}(p_n)}$$

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- Then Silver-Blaze implies for $T = 0$ and $\mu < \mu_c$

$$\Gamma^{(n)}_{i_1 \ldots i_n}(p_1, \ldots, p_n; \mu) = \Gamma^{(n)}_{i_1 \ldots i_n}(\tilde{p}_1, \ldots, \tilde{p}_n; 0)$$

with the shifted momenta

$$\tilde{p}_j = (p_0 + i c_j \mu, \vec{p})$$

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  with the shifted momenta

$$\tilde{p}_j = (p_0 + ic_j\mu, \vec{p})$$

- μ_c defined by lowest pole mass (of charged fields). For example

$$\Gamma^{(2)}_{\Delta_a\Delta^*_a}(p = (-im_{\Delta,\text{pole}}, \vec{0})) = 0$$

Spectral Functions

Two-point Flow

Definition of two-point functions:

$$\left.\frac{\delta^2\Gamma_{k=0}[\Phi]}{\delta\varphi_i^*(p)\delta\varphi_j(p)}\right|_{\varphi=\varphi_0} = \Gamma^{(2)}_{\varphi_i^*\varphi_j}(p) \times (2\pi)^d\delta^{(d)}(0)$$

for any bosonic field

$$\varphi^\top(p) = \big(\vec{\pi}^\top(p), \Delta^\top(p), \Delta^{*\top}(-p), \sigma(p)\big)$$

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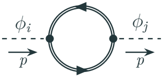
Mean-field flow for two-point functions

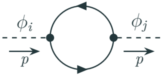
$$\partial_t\Gamma^{(2)}_{\varphi_i^*\varphi_j,k}(p) = \frac{1}{2}\partial_t\Pi_{\varphi_i^*\varphi_j,k}(p)$$

with the polarization loop

$$\Pi_{\varphi_i^*\varphi_j,k}(p) = \mathrm{Tr}\left(\Gamma^{(3)}_{\varphi_i^*\Psi\Psi}G_{q,k}(q)\Gamma^{(3)}_{\varphi_j\Psi\Psi}G_{q,k}(p+q)\right)$$

Diagrams for Two-point Flows

$\partial_t \Gamma^{(2)}_{\phi_i \phi_j}(p) = 2\, \tilde{\partial}_t$ 

$+\, \tilde{\partial}_t$ 

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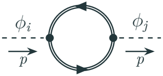
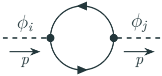
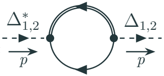
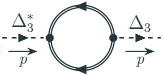
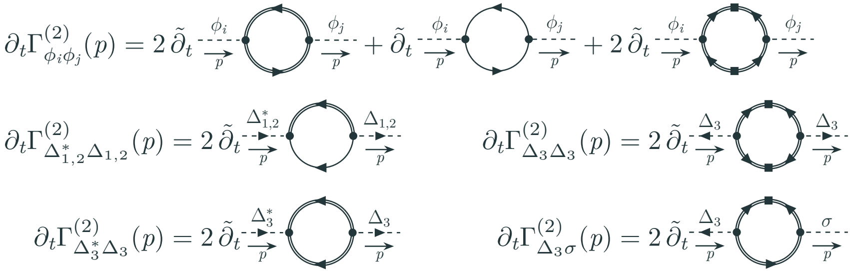
$\partial_t \Gamma^{(2)}_{\phi_i \phi_j}(p) = 2\, \tilde{\partial}_t$ 

$+ \tilde{\partial}_t$ 

$+ 2\, \tilde{\partial}_t$ 

$\partial_t \Gamma^{(2)}_{\Delta^*_{1,2} \Delta_{1,2}}(p) = 2\, \tilde{\partial}_t$ 

$\partial_t \Gamma^{(2)}_{\Delta^*_3 \Delta_3}(p) = 2\, \tilde{\partial}_t$ 



- Non-vanishing off diagonal elements $\rightarrow$ **in-medium mixing** for $\sigma, \Delta_3, \Delta_3^*$ $\rightarrow$ diagonalize to find physical modes.

RG-consistency for Two-point Functions I

- Two-point functions are also affected by medium divergences.

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RG-consistency for Two-point Functions I

- Two-point functions are also affected by medium divergences.
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- Example

$$\Gamma^{(2)}_{\Delta_1^* \Delta_1}(p_0;\mu) \underset{\Lambda' \to \infty}{\simeq} (p_0 + i2\mu)^2 \ln \Lambda' + \cdots$$

RG-consistency for Two-point Functions II

- Taylor expand upward flow in μ **and** p_0

$$\Gamma^{(2)}_{\varphi^\dagger\varphi,\Lambda}(p_0;\mu) = \Gamma^{(2)}_{\varphi^\dagger\varphi,\Lambda'}(p_0;\mu) + \left(\int_{\Lambda'}^{\Lambda} dk\, \mathcal{F}_{\varphi^\dagger\varphi,k}(p_0;\mu)\right)\Bigg|_{T=\mu=p_0=0}$$

$$+ \frac{\mu^2}{2}\left(\partial_\mu^2 \int_{\Lambda'}^{\Lambda} dk\, \mathcal{F}_{\varphi^\dagger\varphi,k}(p_0;\mu)\right)\Bigg|_{T=\mu=p_0=0}$$

$$+ \frac{p_0^2}{2}\left(\partial_{p_0}^2 \int_{\Lambda'}^{\Lambda} dk\, \mathcal{F}_{\varphi^\dagger\varphi,k}(p_0;\mu)\right)\Bigg|_{T=\mu=p_0=0}$$

$$+ p_0\mu\left(\partial_\mu\partial_{p_0} \int_{\Lambda'}^{\Lambda} dk\, \mathcal{F}_{\varphi^\dagger\varphi,k}(p_0;\mu)\right)\Bigg|_{T=\mu=p_0=0}$$

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$$+\,p_0\mu\left(\partial_\mu\partial_{p_0}\int_{\Lambda'}^{\Lambda} dk\,\mathcal{F}_{\varphi^\dagger\varphi,k}(p_0;\mu)\right)\Bigg|_{T=\mu=p_0=0}$$

- RG-consistency can be extended to artifacts at finite p_0!

Analytical Continuation

Problem: $\Gamma^{(2)}_{\varphi_i^*\varphi_j}(p_0,\vec{0})$ for $p_0 \in \mathbb{C}$ is not well defined $\rightarrow$ need **analytical continuation**

- Make replacement

$$n_F(\epsilon + ip_0) \rightarrow n_F(\epsilon)$$

- Retarded two-point functions

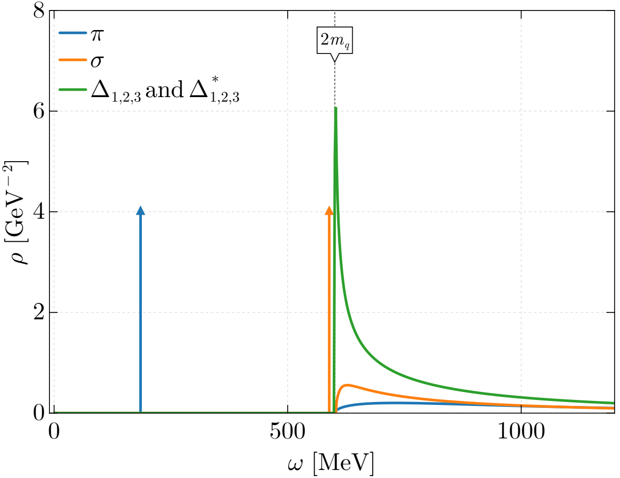
$$\Gamma^{(2),R}_{\varphi_i^*\varphi_j}(\omega) = -\lim_{\epsilon\rightarrow 0}\Gamma^{(2),E}_{\varphi_i^*\varphi_j}(p_0 = -i(\omega + i\epsilon),\vec{0})$$

- Spectral Function

$$\rho(\omega) = \frac{1}{\pi}\frac{\operatorname{Im}\Gamma^{(2),R}(\omega)}{\left(\operatorname{Re}\Gamma^{(2),R}(\omega)\right)^2 + \left(\operatorname{Im}\Gamma^{(2),R}(\omega)\right)^2}$$

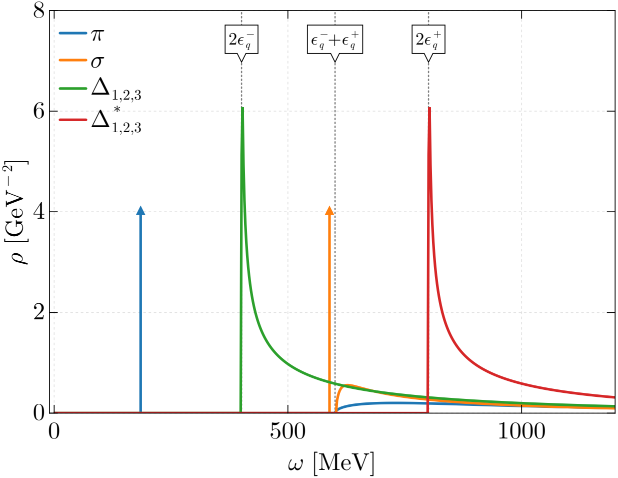
Spectral Functions ($T = 1\,\mathrm{MeV}$, $\mu = 0\,\mathrm{MeV}$)

Phase: $\sigma \neq 0$ and $\Delta = 0$



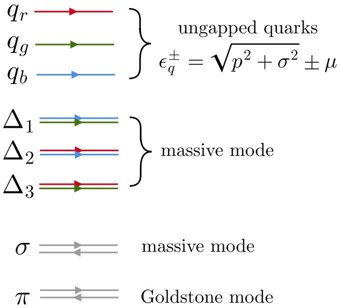
Spectral Functions ($T = 1\,\mathrm{MeV}$, $\mu = 100\,\mathrm{MeV}$)

Phase: $\sigma \neq 0$ and $\Delta = 0$



Chirally Broken & Non-superconducting

$$\sigma \neq 0 \ \& \ \Delta = 0$$



Chirally Broken & Non-superconducting

$$\sigma \neq 0 \;\&\; \Delta = 0$$

q_r, q_g, q_b: ungapped quarks $\epsilon_q^\pm = \sqrt{p^2+\sigma^2} \pm \mu$

Δ_1, Δ_2, Δ_3: massive mode

σ: massive mode

π: Goldstone mode

Chirally Restored & Superconducting

$$\sigma \approx 0 \;\&\; \Delta \neq 0$$

q_r, q_g: gapped quarks $E_q^\pm = \sqrt{(\epsilon_q \pm \mu)^2 + \Delta^2}$

q_b: ungapped quarks $\epsilon_q^\pm = \sqrt{p^2+\sigma^2} \pm \mu$

Δ_1, Δ_2: Goldstone mode

Δ_3, σ: 2 massive mode, 1 Goldstone mode

π: massive mode

Particle Summary

Chirally Broken & Non-superconducting

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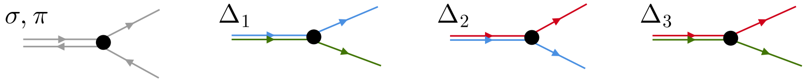
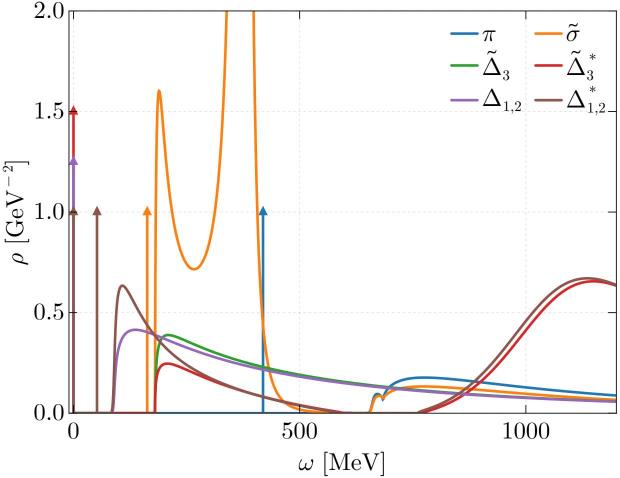
Interactions



Spectral Functions ($T = 1\,\mathrm{MeV}$, $\mu = 330\,\mathrm{MeV}$)

Phase: $\sigma \approx 0$ and $\Delta \neq 0$



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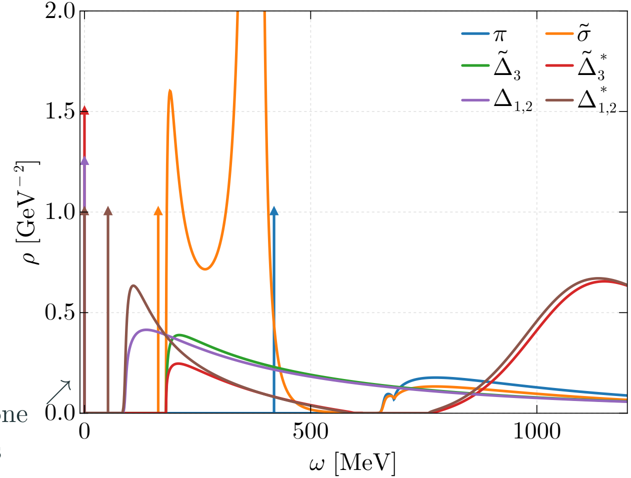

Goldstone modes

Spectral Functions ($T = 1\,\mathrm{MeV}$, $\mu = 330\,\mathrm{MeV}$)

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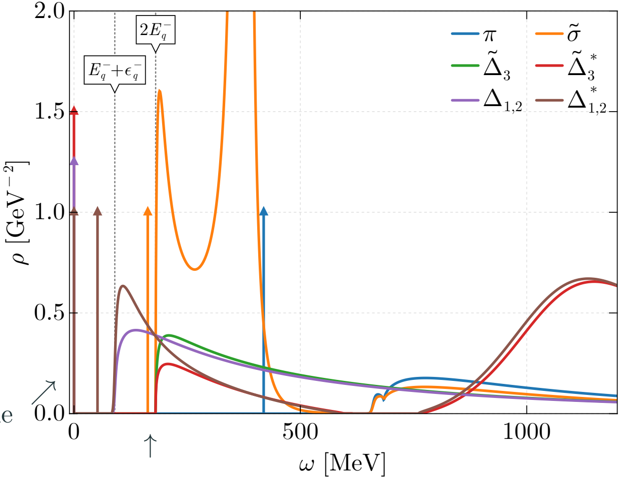
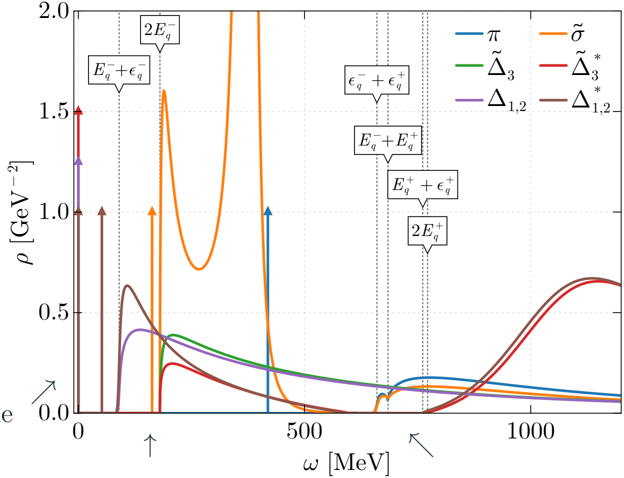


Goldstone modes

decay at the Fermi-surface
(produce particles with momenta p_F)

Phase: $\sigma \approx 0$ and $\Delta \neq 0$



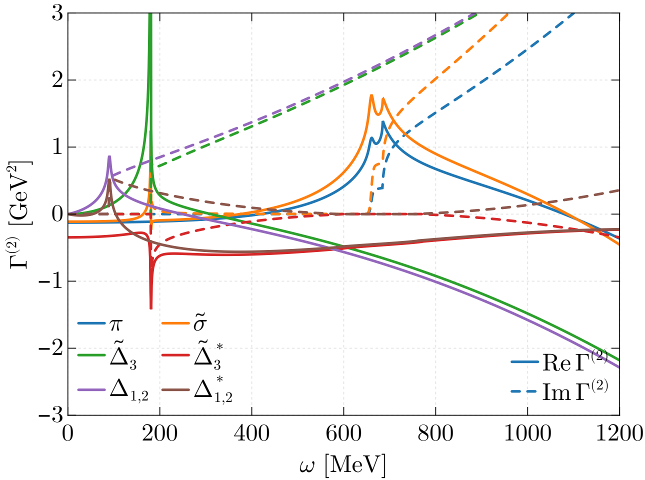
Goldstone modes

decay at the Fermi-surface (produce particles with momenta p_F)

normal decay

Two-point Functions ($T = 1\,\mathrm{MeV}$, $\mu = 330\,\mathrm{MeV}$)

Phase: $\sigma \approx 0$ and $\Delta \neq 0$



- $\mathrm{Im}\Gamma^{(2)}$ diverge around $\omega \sim 200\,\mathrm{MeV}$

Van-Hove Singularity

- Take analytic limit $\epsilon \to 0^+$

$$\lim_{\epsilon \to 0^+} \frac{1}{x \pm i\epsilon} = \mp i\pi\delta(x) + \mathcal{P}\left(\frac{1}{x}\right)$$

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- Decay to 2 gapped quarks produce

$$\mathrm{Im}\Gamma^{(2)} \propto \int_p \delta(4E_q^-(p)^2 - \omega^2)$$
$$= \int_p \sum_{p_0} \frac{\delta(p - p_0)}{|4\partial_p E_q^{-2}(p_0)|}$$

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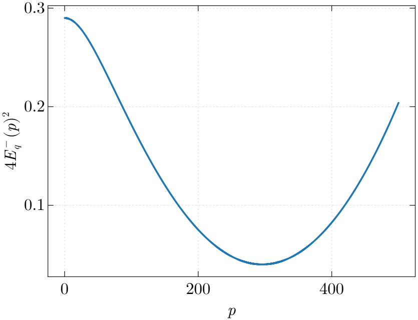
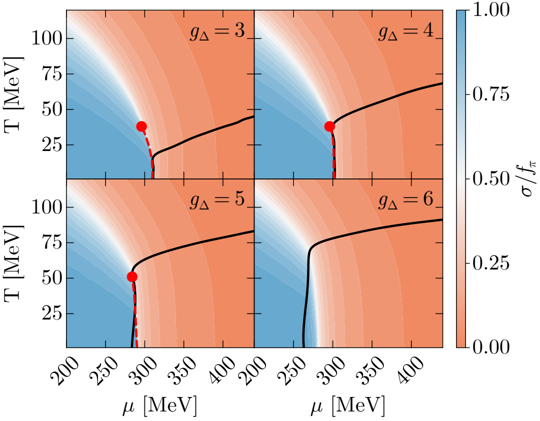


- $\mathrm{Im}\Gamma^{(2)}$ diverge at thresholds $\to$ could be relevant for phenomenology?

Pole Masses

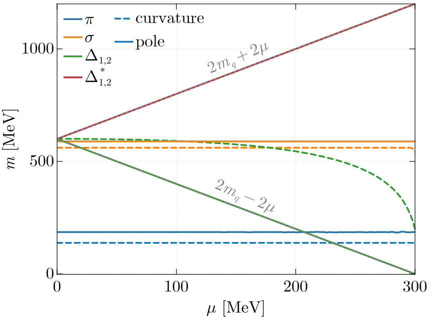
Diquark Pole Mass

Original question: is Silver-Blaze violated for high g_Δ?



- Compare diquark pole mass to curvature mass

Diquark Pole Mass $g_\Delta = 4$



MFA

- Very different behavior of diquark pole and curvature masses.

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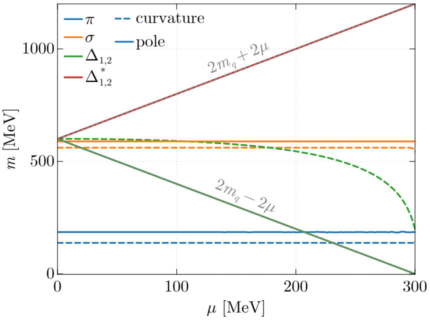
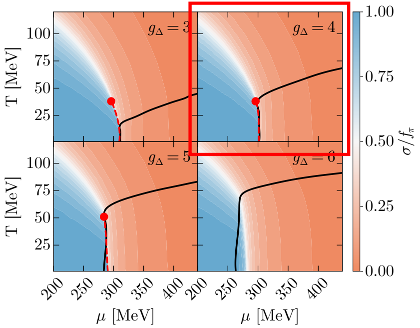


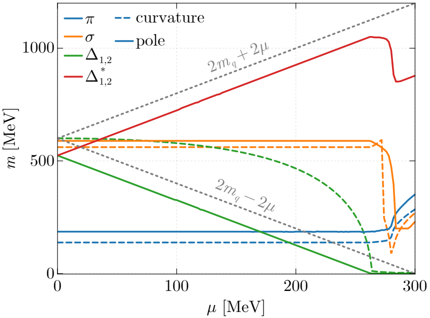
MFA



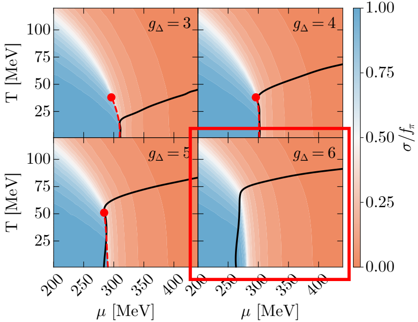
FRG

- Very different behavior of diquark pole and curvature masses.

Diquark Pole Mass $g_\Delta = 6$



MFA



FRG

- Finite vacuum diquark pole mass $< 600\,\mathrm{MeV} \rightarrow$ Silver-Blaze is not violated!

Conclusion

- Diquark at mean-field level reduce back-bending for high g_Δ.
- For high g_Δ, diquark pole mass becomes $< 600\,\mathrm{MeV}$ and diquarks dominate phase structure.
- Bosonic spectral functions already rich at the mean-field level.
- RG-consistency can be used to suppress cutoff artifacts in the thermodynamic potential and the two-point functions.
- Diquark curvature mass is very different from the diquark pole mass.

Backup Slides

Flow Equation

$$
\begin{aligned}
\partial_t U_k(\sigma,\Delta) =& \frac{k^5}{12\pi^2}\Bigg\{\frac{3}{\epsilon_\pi}\coth\frac{\epsilon_\pi}{2T}+\frac{1}{\epsilon_\sigma}\coth\frac{\epsilon_\sigma}{2T}\\
&-\frac{2N_f}{\epsilon_q}\Bigg[\tanh\frac{\epsilon_q^+}{2T}+\tanh\frac{\epsilon_q^-}{2T}+\\
&2\frac{\epsilon_q^+}{E_q^+}\tanh\frac{E_q^+}{2T}+2\frac{\epsilon_q^-}{E_q^-}\tanh\frac{E_q^-}{2T}\Bigg]\Bigg\}
\end{aligned}
$$

Mean-field Potential

$$\partial_t U_k = \partial_t f_k$$

$$f_k(\sigma, \Delta) = -2N_f \int_{\vec{p}} \Big\{ \epsilon_q + E_q^+ + E_q^- \\ + T \ln\left(1 + e^{-\beta\epsilon_q^+}\right) + T \ln\left(1 + e^{-\beta\epsilon_q^-}\right) \\ + 2T \ln\left(1 + e^{-\beta E_q^+}\right) + 2T \ln\left(1 + e^{-\beta E_q^-}\right) \Big\} \,,$$

$$\Omega(\sigma, \Delta) = U_{k=\Lambda'}(\sigma, \Delta) - c\sigma \quad + f_{k=0}(\sigma, \Delta) - f_{k=\Lambda'}(\sigma, \Delta) \,.$$

Mixing Matrix & Diagonalisation

$$\Gamma^{(2)}_{\varphi^\dagger\varphi}(p) \times (2\pi)^4\delta^{(4)}(0) = \begin{pmatrix} \frac{\delta^2\Gamma}{\delta\Delta_3^*(p)\delta\Delta_3(p)} & \frac{\delta^2\Gamma}{\delta\Delta_3^*(p)\delta\Delta_3^*(-p)} & \frac{\delta^2\Gamma}{\delta\Delta_3^*(p)\delta\sigma(p)} \\ \frac{\delta^2\Gamma}{\delta\Delta_3(-p)\delta\Delta_3(p)} & \frac{\delta^2\Gamma}{\delta\Delta_3(-p)\delta\Delta_3^*(-p)} & \frac{\delta^2\Gamma}{\delta\Delta_3(-p)\delta\sigma(p)} \\ \frac{\delta^2\Gamma}{\delta\sigma(-p)\delta\Delta_3(p)} & \frac{\delta^2\Gamma}{\delta\sigma(-p)\delta\Delta_3^*(-p)} & \frac{\delta^2\Gamma}{\delta\sigma(-p)\delta\sigma(p)} \end{pmatrix}$$

Can be diagonalized through

$$\varphi_i(p) \to U_{ij}(p)\varphi_j(p) \ ,$$

which implies

$$\Gamma^{(2)}_{\varphi^\dagger\varphi}(p) \to U^\dagger(p)\Gamma^{(2)}_{\varphi^\dagger\varphi}(p)\, U(p) \ .$$

Diagonalisation is possible because $\Gamma^{(2)}_{\varphi^\dagger\varphi}(p)$ is Hermitian.

RG-consistency for Spectral Functions

Phase: $\sigma \approx 0$ and $\Delta \neq 0$ ($T = 1\,\mathrm{MeV}$ and $\mu = 330\,\mathrm{MeV}$)