In-Medium Vector Meson Spectral Functions and Polarization from FRG

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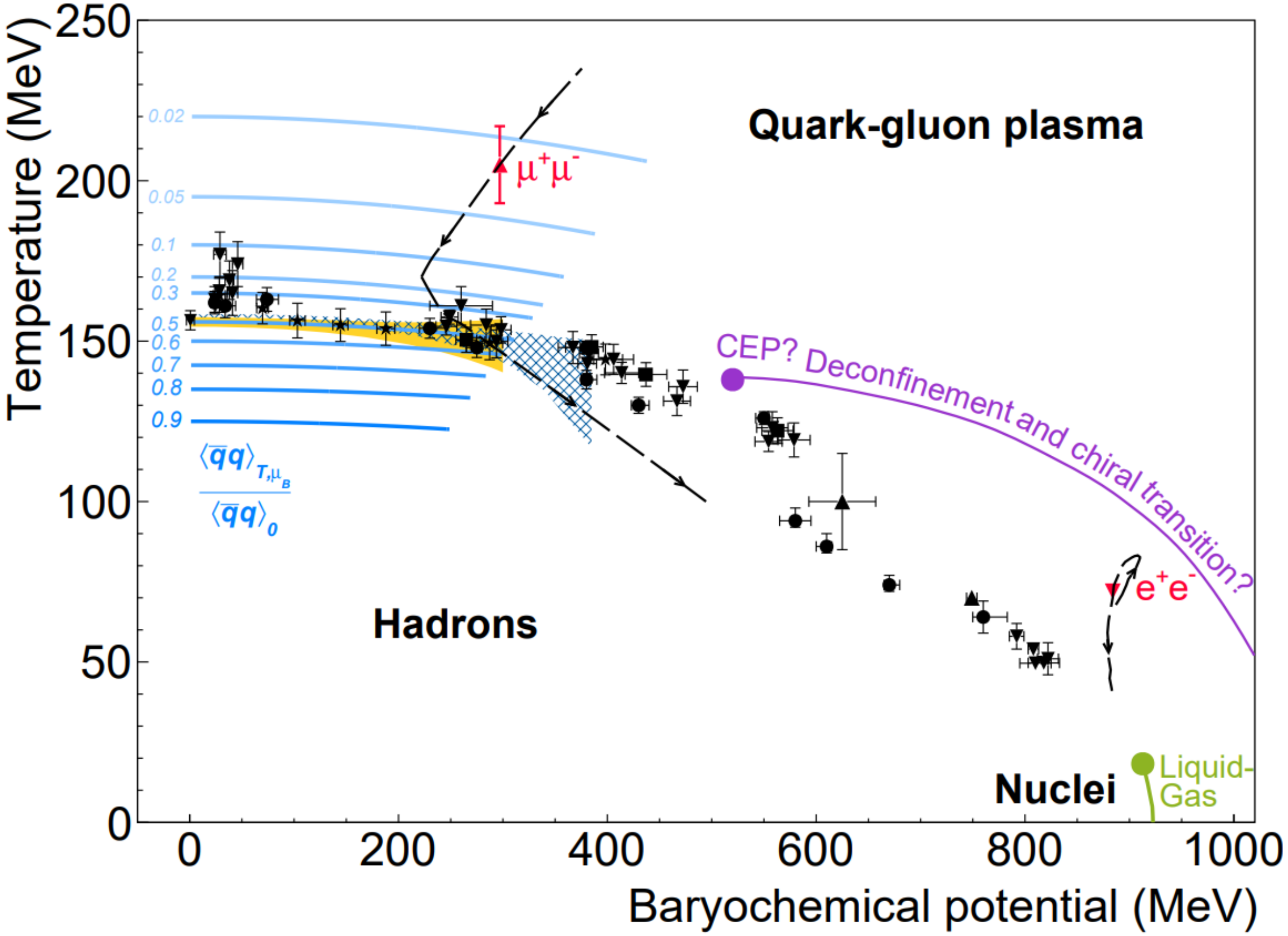
Dileptons from hot and dense QCD matter

CRC-TR 211

- Goal: Obtain chirally consistent EM spectral functions across the QCD phase diagram
- Identify the impact of possible phase transitions and CP on the EM rates

--> EM spectral function calculated from analytically continued FRG flow equations



Nature Phys. 15(2019) 10, 1040-1045
(update 2023 T. Galatyuk)

What is FRG?

- **F**unctional **R**enormalization **G**roup
- If you do **NOT** like equations:

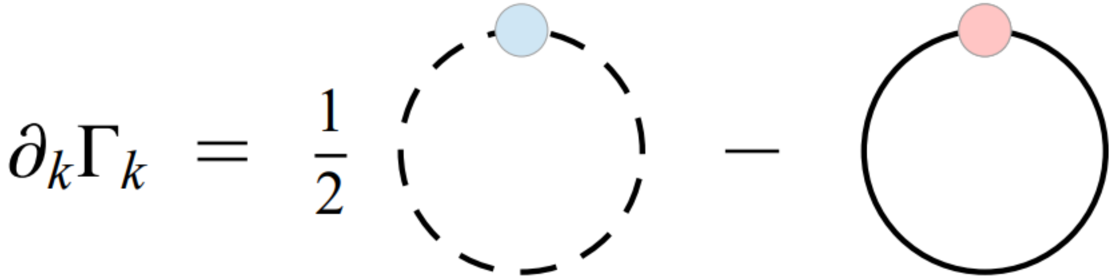
FRG is a machine which takes a Lagrange density and returns a phase diagram and Spectral Functions

- If you do like equations:

FRG approaches aim to solve the Wetterich equation for the effective average action and its functional derivatives to obtain an effective potential and n-point-Functions, treating thermal and quantum fluctuations consistently.

Wetterich equation

- Wetterich, Phys.Lett.B 301 (1993) 90-94
- Describes change of effective average action with scale k
- Simple one loop structure, but:
  - Full propagators in the loops
  - Exact equation!
  - Non-perturbative!
- Regulators cut off fluctuations below renormalization scale k (Litim-Regulators)



R. A. Tripolt, PhD thesis, 2015

Parity doublet model: Bare action Γ_k

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- Model includes $\pi, \sigma, N, \rho, a_1, N^*(1535)$
- Obeys chiral symmetry
- Has a mass term thanks to „Mirror Baryon Prescription“

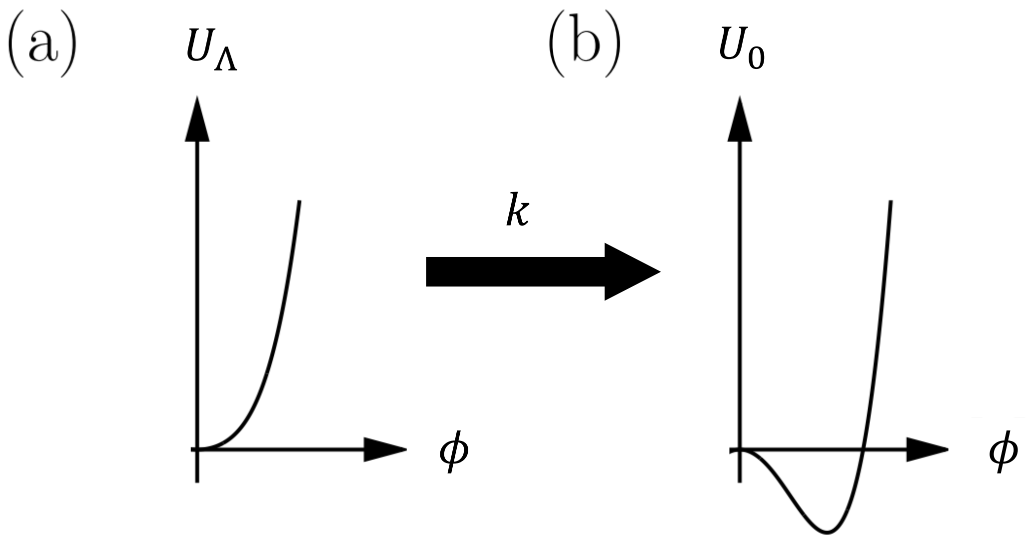
$$\Gamma_k = \int d^4x \Big\{ \bar{N}_1 \left(\not{\partial} - \mu_B\gamma_0 + h_{s,1}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,1}(\gamma_\mu\vec{\tau}\cdot\vec{\rho}_\mu + \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu}) \right) N_1$$
$$+ \bar{N}_2 \left(\not{\partial} - \mu_B\gamma_0 + h_{s,2}(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,2}(\gamma_\mu\vec{\tau}\cdot\vec{\rho}_\mu - \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu}) \right) N_2 + \boxed{m_{0,N}\left(\bar{N}_1\gamma^5 N_2 - \bar{N}_2\gamma^5 N_1\right)}$$
$$\boxed{+ U_k(\phi^2) - c\sigma} + \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{4}\,\mathrm{tr}\,\partial_\mu\rho_{\mu\nu}\partial_\sigma\rho_{\sigma\nu} + \frac{m_v^2}{8}\,\mathrm{tr}\,\rho_{\mu\nu}\rho_{\mu\nu} \Big\} + \Delta\Gamma_{\pi a_1}\,.$$

$m_{0,N}$ [MeV]	$h_{s,1} = h_{v,1}$	$h_{s,2} = h_{v,2}$	$f_\pi \equiv \sigma_0$ [MeV]	m_π [MeV]	m_σ [MeV]	m_{N_1} [MeV]	m_{N_2} [MeV]
800	6.94073	13.3493	92.8	137	474	938	1533

Tripolt et al., Phys.Rev.D 104 (2021) 5, 054005

Effective potential

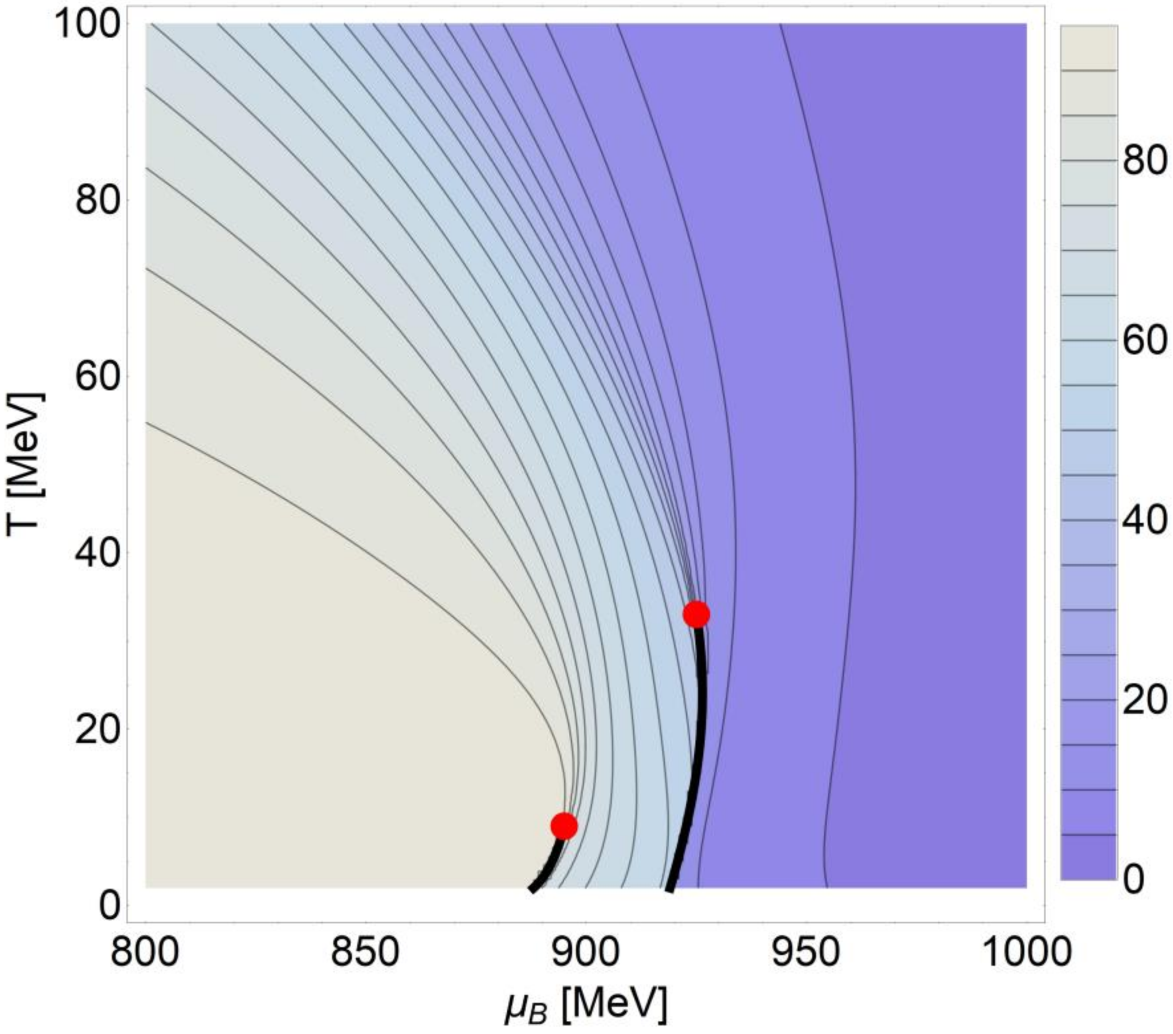
- Insert bare action Γ_Λ into Wetterich equation, get flow equation for U_k
- Potential has a minimum somewhere in field configuration at k=0.
- Position of minimum shows breaking of underlying symmetries
  - Order parameter!
  - Phase diagram!



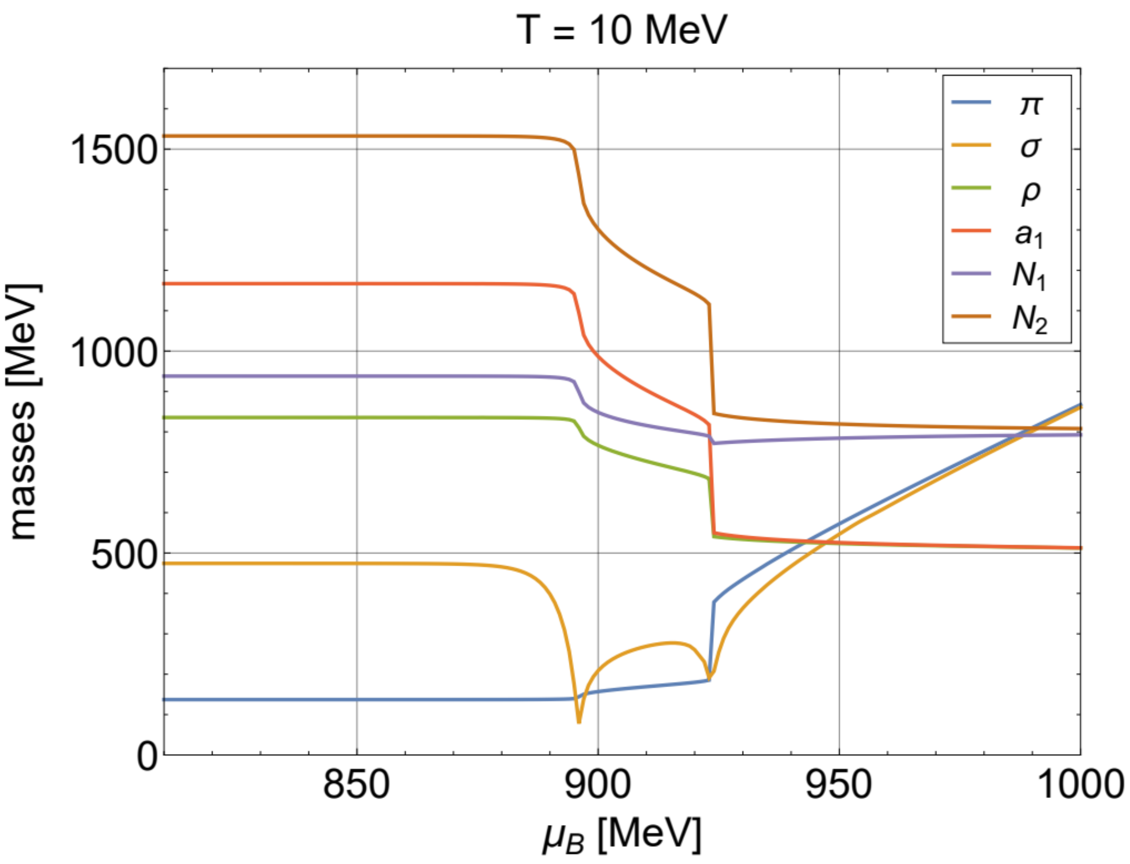
Lancaster, Blundell, QFT for gifted amateur

Phase diagram in parity doublet model

Phase diagram in parity doublet model



Mass degeneracy

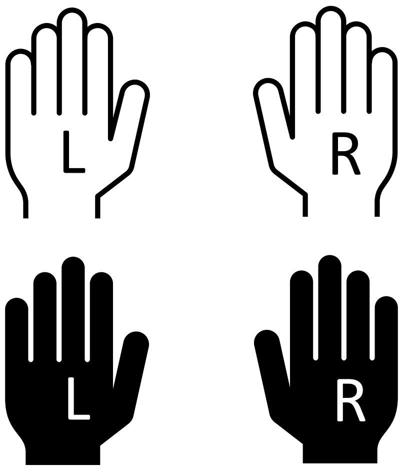


Tripolt et al., Phys.Rev.D 104 (2021) 5, 054005

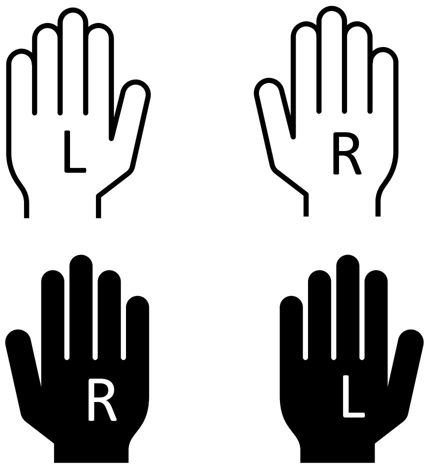
Parity in linear σ model

- Inclusion of parity partners in linear σ model:
  - 2 parity conserving ways:

„naive“ prescription



„mirror“ prescription



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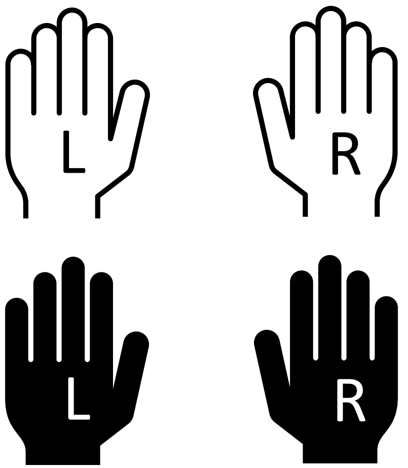
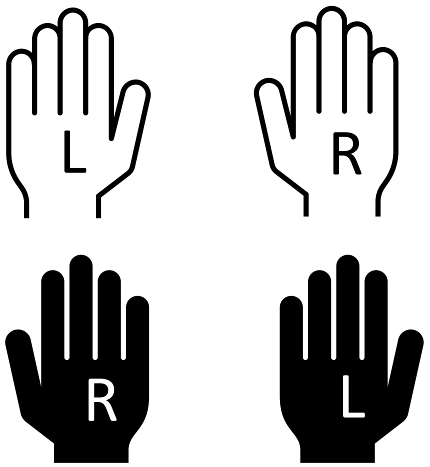


„mirror“ prescription



Consequences of prescriptions

- In both cases, mass terms for single baryon species are not allowed

$$\mathcal{L}_m = m\bar{\psi}_i\psi_i$$

- However: In mirror case, mixing allows for mass term

$$\mathcal{L}_m = m(\bar{\psi}_2\psi_1 + \bar{\psi}_1\psi_2)$$

- Chirally restored phase: Massless baryons vs. degenerate massive baryons

Weyrich et al., Phys.Rev.C 92 (2015) 1, 015214

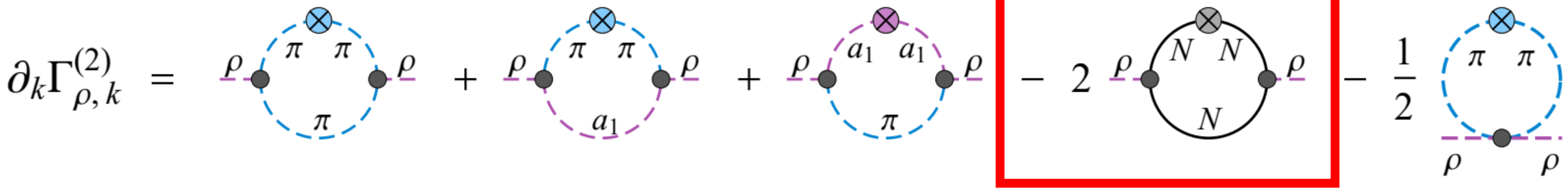
Consequences of mirror prescription

- Remember:

$$\boxed{m_{0,N}(\bar{N}_1\gamma^5 N_2 - \bar{N}_2\gamma^5 N_1)}$$

- In mirror prescription, chiral symmetry breaking explains part of baryon mass generation, but not full picture
- Instead, additional mass generated by different source
  - E.g. QCD scale anomaly, see e.g. Shifman et al., Phys.Lett.B 78 (1978) 443-446
- Possible experimental signals for mirror prescription?
  - Hadronic signals? Proposed in GiBUU by Larionov, v. Smekal, Phys.Rev.C 105 (2022)
  - η-meson enhancement? N^{*1535} enhancement?
- Other signals?

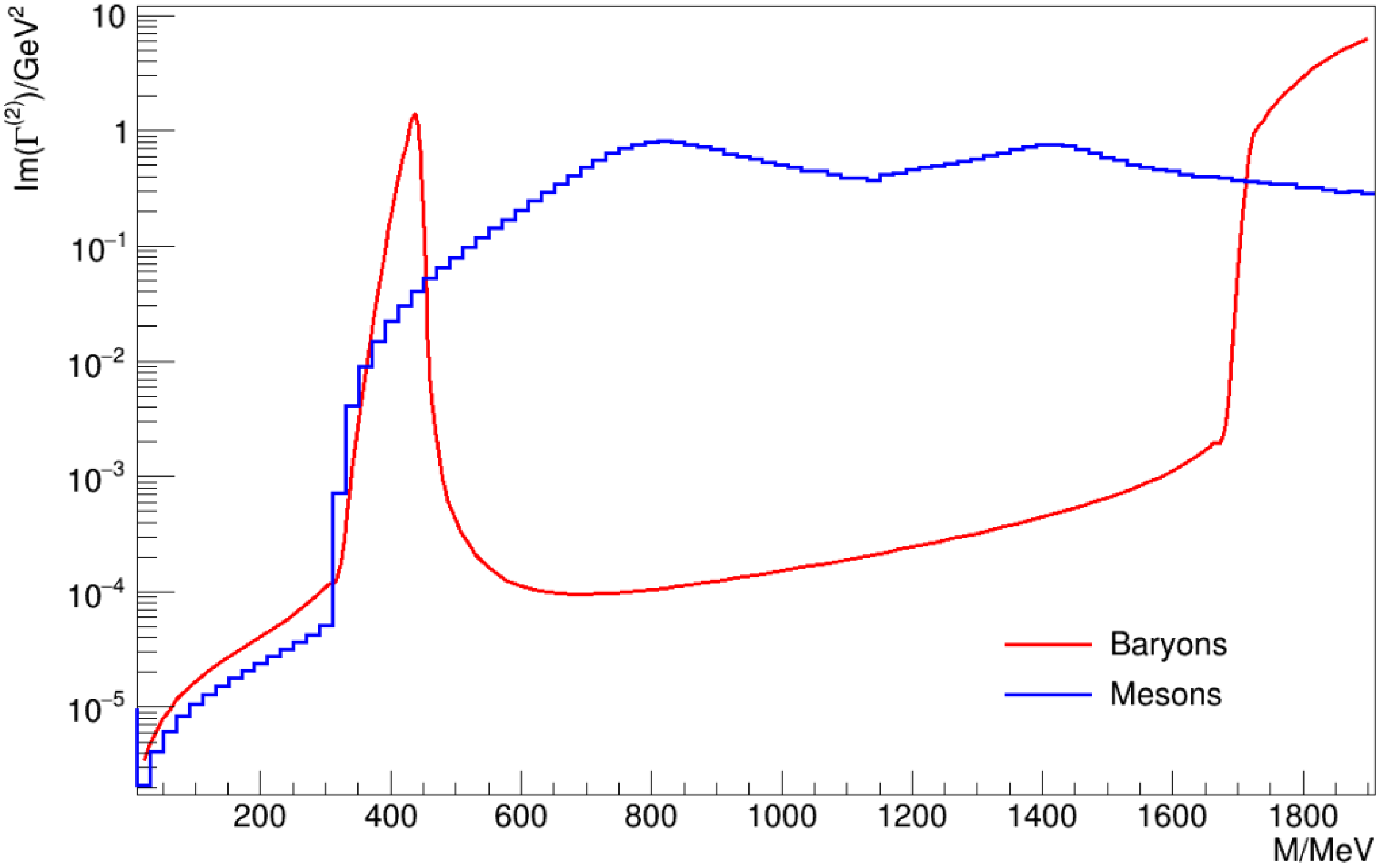
Prescription shows up in ρ spectral function!



- Functional derivatives of Γ_k w.r.t. fields give n-point functions
- Mirror prescription gives rise to low energy peak in ρ $\Gamma^{(2)}$ function due to $\rho^* + N_1 \rightarrow N_2$
  - Unique to parity doublet model with mirror prescription!

Tripolt et al., Phys.Rev.D 104 (2021) 5, 054005

Example: T=40 & μ_B=890, p=0 MeV, Im Γ_ρ^2



EM vs. Vector spectral functions?

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- Model includes $\pi, \sigma, N, \rho, a_1, N^*(1535)$
- No photons!

$$\Gamma_k = \int d^4x \Big\{ \bar{N}_1 \left(\partial\!\!\!/ - \mu_B \gamma_0 + h_{s,1}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,1}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu + \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu})\right) N_1$$
$$+ \bar{N}_2 \left(\partial\!\!\!/ - \mu_B \gamma_0 + h_{s,2}(\sigma - i\vec{\tau}\cdot\vec{\pi}\gamma^5) + h_{v,2}(\gamma_\mu \vec{\tau}\cdot\vec{\rho}_\mu - \gamma_\mu\gamma^5\vec{\tau}\cdot\vec{a}_{1,\mu})\right) N_2 + \boxed{m_{0,N}\left(\bar{N}_1\gamma^5 N_2 - \bar{N}_2\gamma^5 N_1\right)}$$
$$\boxed{+ U_k(\phi^2) - c\sigma} + \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{4}\,\mathrm{tr}\,\partial_\mu\rho_{\mu\nu}\partial_\sigma\rho_{\sigma\nu} + \frac{m_v^2}{8}\,\mathrm{tr}\,\rho_{\mu\nu}\rho_{\mu\nu} \Big\} + \Delta\Gamma_{\pi a_1}.$$

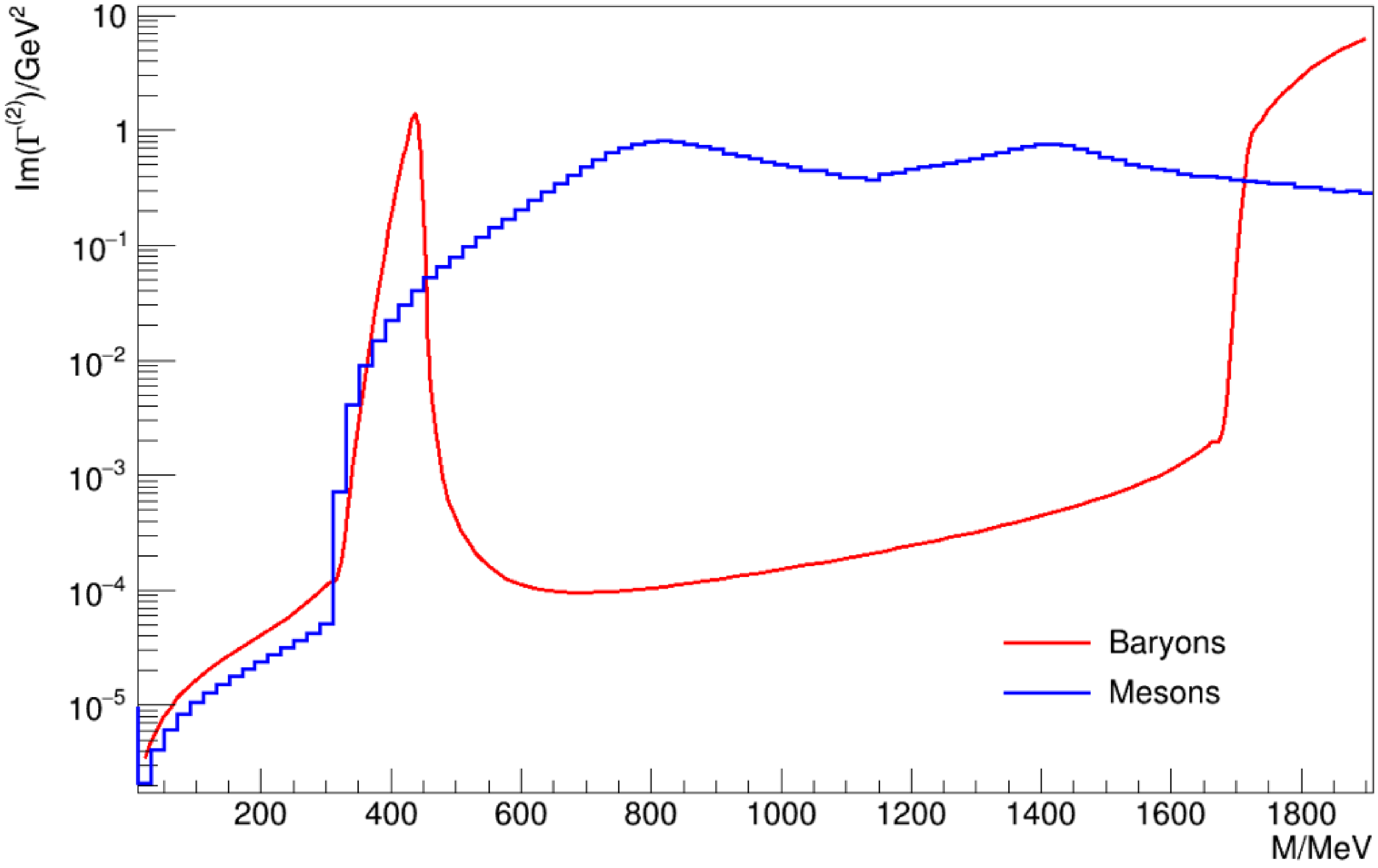
- Solution: Vector dominance model

$$\mathrm{Im} D_{\mathrm{EM}} = \sum_{\nu=\rho,\omega,\phi} \left(\frac{m_\nu^2}{g_\nu}\right)^2 \mathrm{Im} D_\nu$$

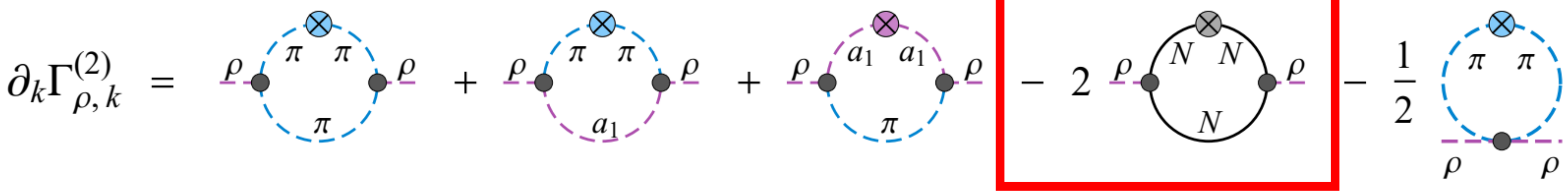
Why just ρ meson?

- $j^{\mu}_{EM} = \sum_{q=u,d,s} \bar{q}\gamma^{\mu} q e_q = \frac{1}{\sqrt{2}} j^{\mu}_{\rho} + \frac{1}{3\sqrt{2}} j^{\mu}_{\omega} + \frac{1}{3} j^{\mu}_{\phi}$
- Check prefactors:
- ω weaker by factor 9, ϕ weaker by factor 5
- More importantly: ρ has by far the shortest lifetime
- ω, ϕ experience final state interactions

Example: T=40 & μ_B=890, p=0 MeV, Im Γ_ρ^2



Prescription shows up in ρ spectral function!

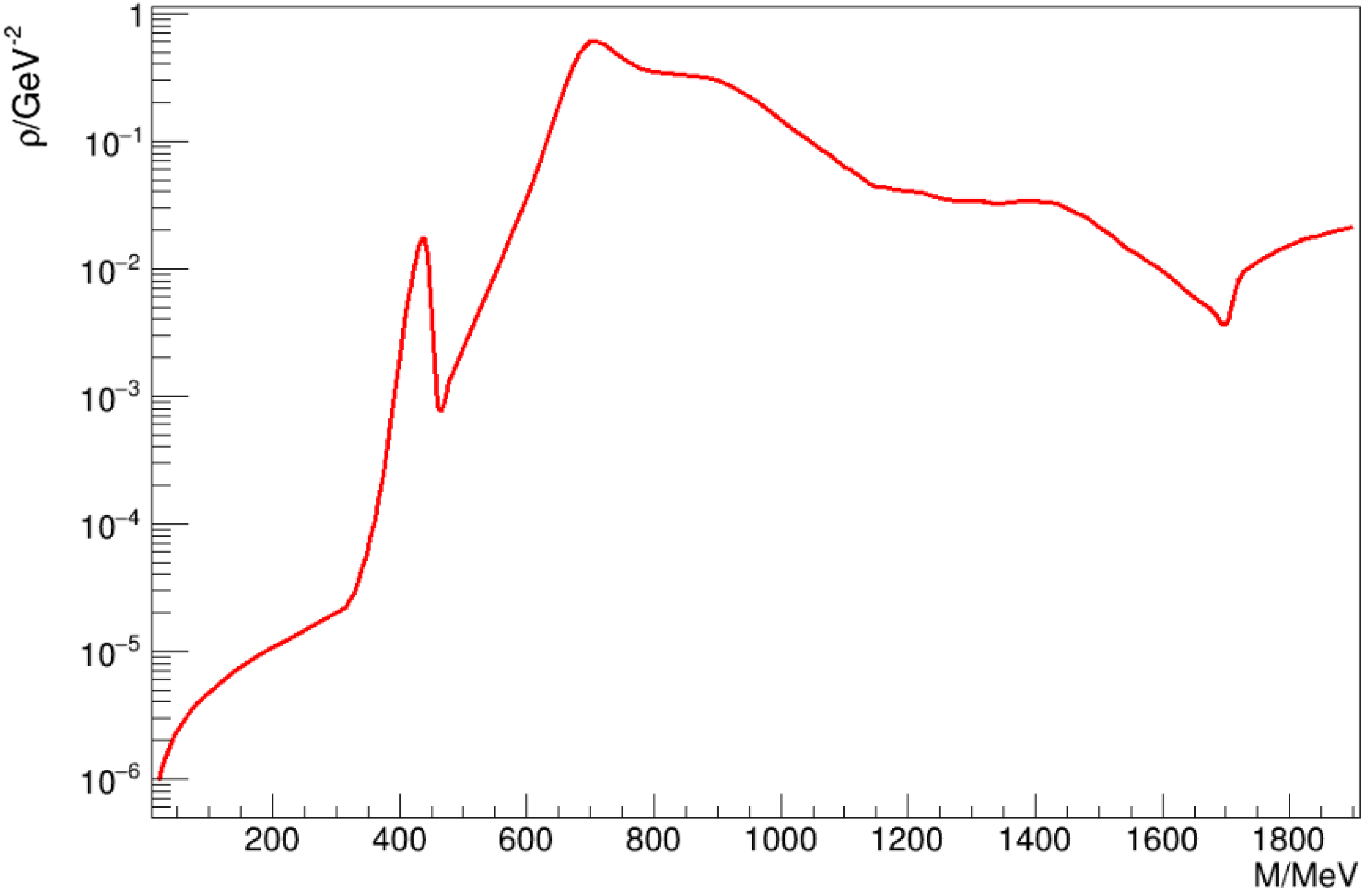


- Functional derivatives of Γ_k w.r.t. fields give n-point functions
- Mirror prescription gives rise to low energy peak in ρ $\Gamma^{(2)}$ function due to $\rho^* + N_1 \rightarrow N_2$
  - Unique to parity doublet model with mirror prescription!
- Also manifests in spectral function!

$$\Pi_\rho \propto \frac{Im\Gamma^{(2)}_\rho}{\left(Re\, \Gamma^{(2)}_\rho\right)^2 + \left(Im\, \Gamma^{(2)}_\rho\right)^2}$$

Tripolt et al., Phys.Rev.D 104 (2021) 5, 054005

Example: T=40 & μ_B=890, p=0 MeV



From spectral function to dilepton spectra

CRC-TR 211

- Important equation for extraction of thermal dilepton spectra

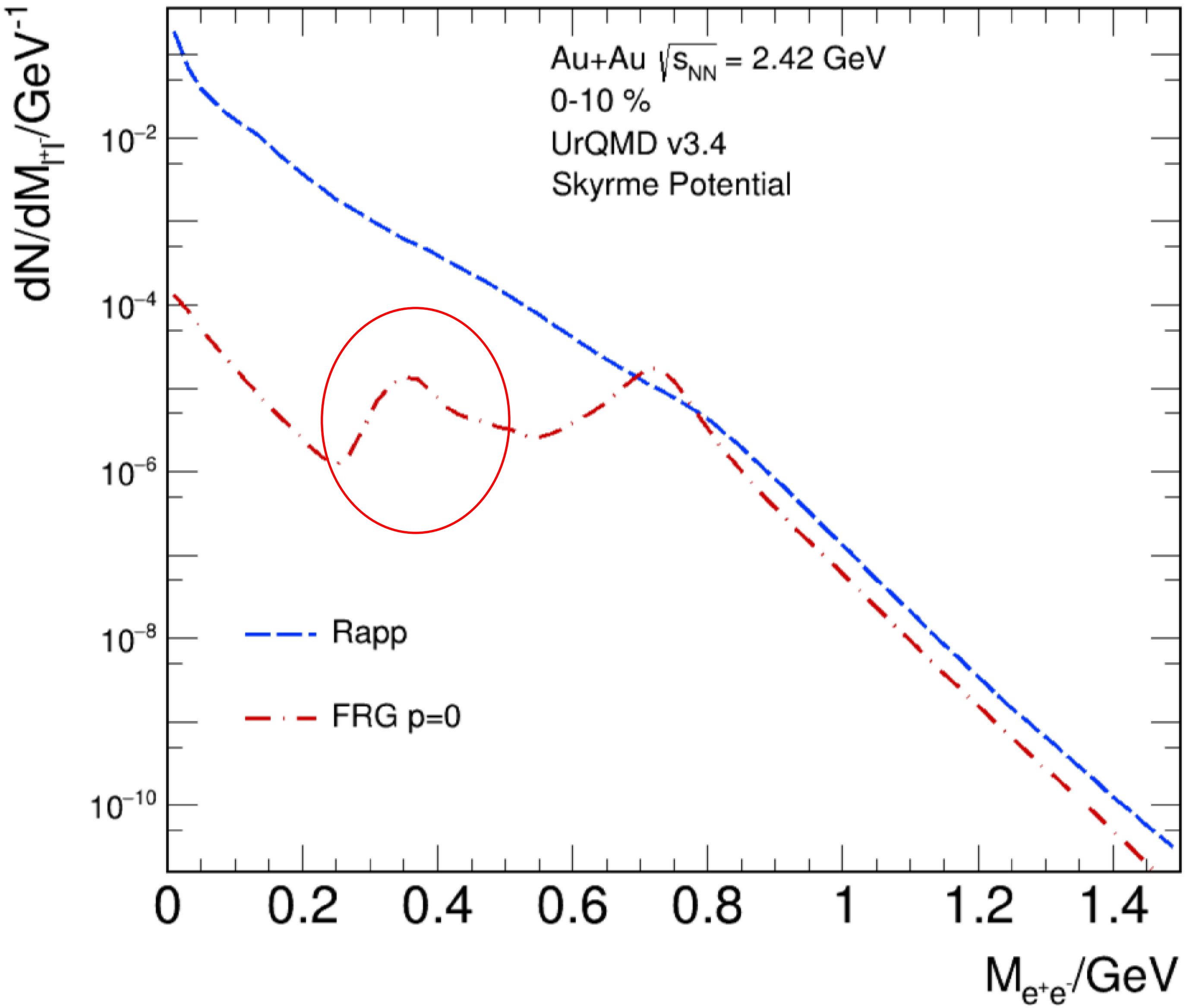
$$\frac{dN_{ll}}{d^4x d^4q} = -\frac{\alpha_{EM}^2}{\pi^3 M^2} L(M^2) f^{BE}(q_0, T) \mathrm{Im}\Pi_{EM}(M, q, \mu_B, T).$$

- VDM: vector spectral function proportional to electromagnetic SF
- Peak translates from spectral function to dilepton spectra!

Takeaway: Dilepton yield depends on T, μ_B (ρ_B), is obtained by integrating over space-time and 4-momentum

McLerran, Toimela, Phys. Rev. D 31 (1985), p. 545

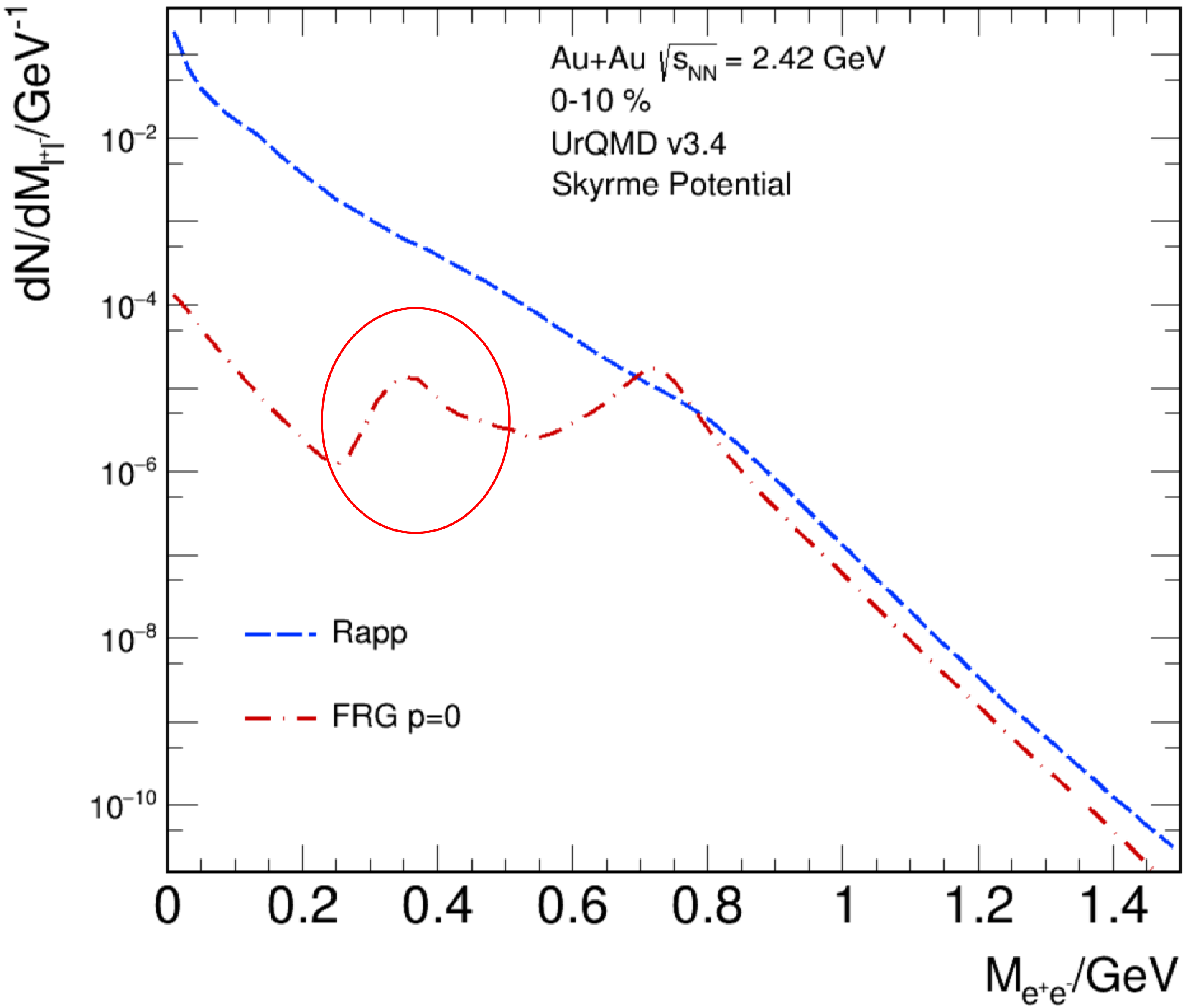
Mirror prescription in dilepton yield



- Take ρ, T distributions from UrQMD via Coarse Graining procedure
- For $\vec{p}$=0 MeV spectral function, peak is seen clearly in FRG SF
- Comparison with Rapp-Wambach spectral function
  - Describes spectra over wide range of energies

Rapp, Physics Letters B,Volume 731, 2014

Mirror prescription in dilepton yield

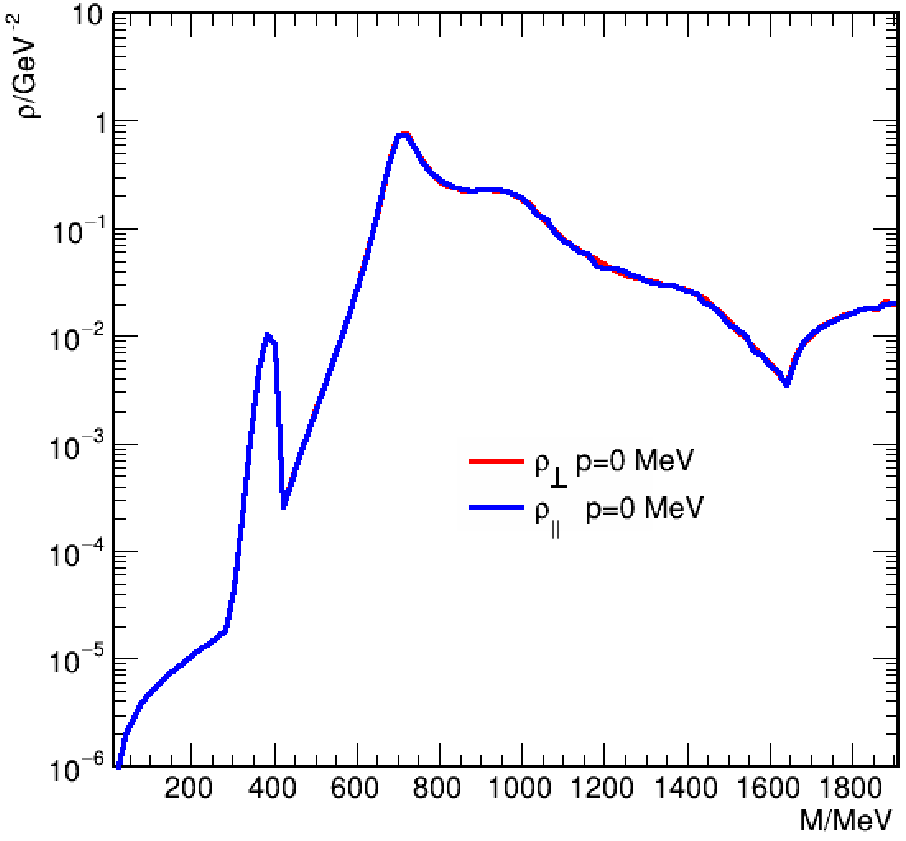


- Low energies?
  - 1st step: Full momentum dependence!
  - Does the peak survive for finite momentum?

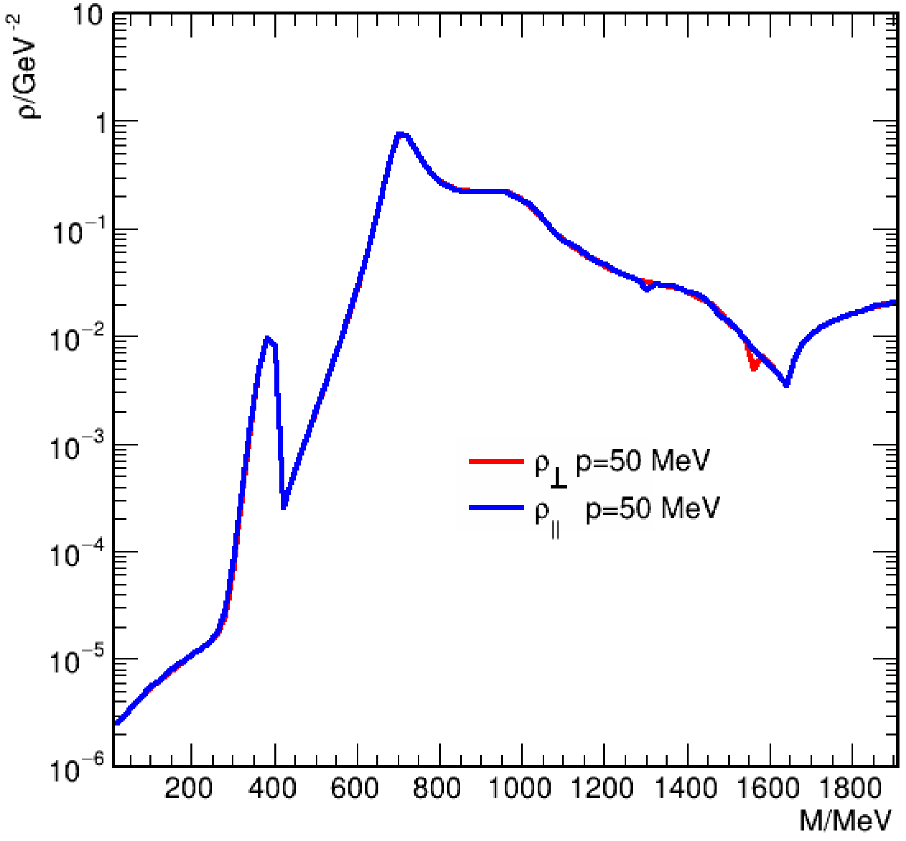
$$\rho = \frac{1}{3}(2\,\rho_{\perp} + \rho_{||})$$

R. Rapp, J. Wambach, Eur. Phys. J. A 6, 415 (1999)

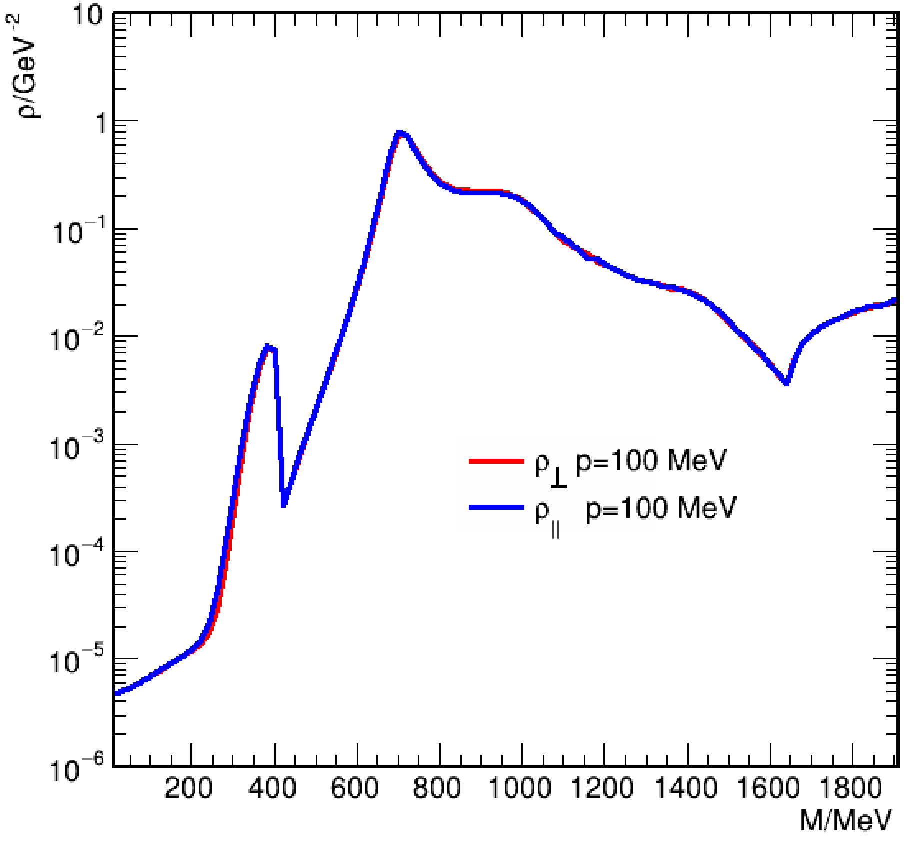
Example: T=40 & μ_B=890, p=0 MeV



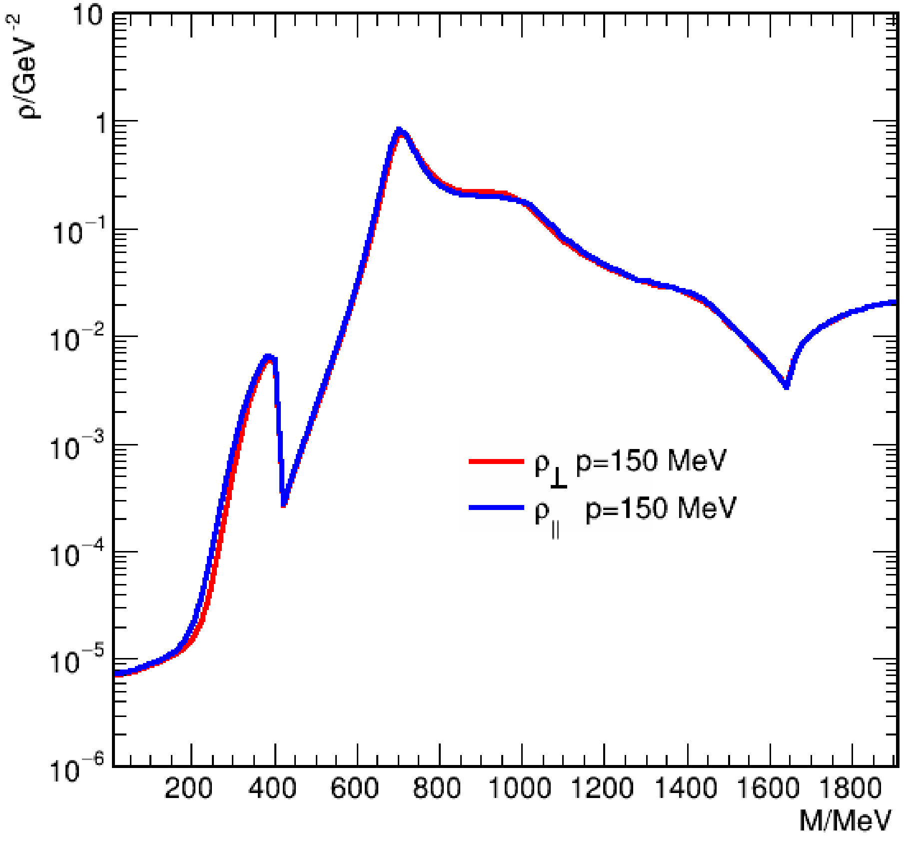
Example: T=40 & μ_B=890, p=50 MeV



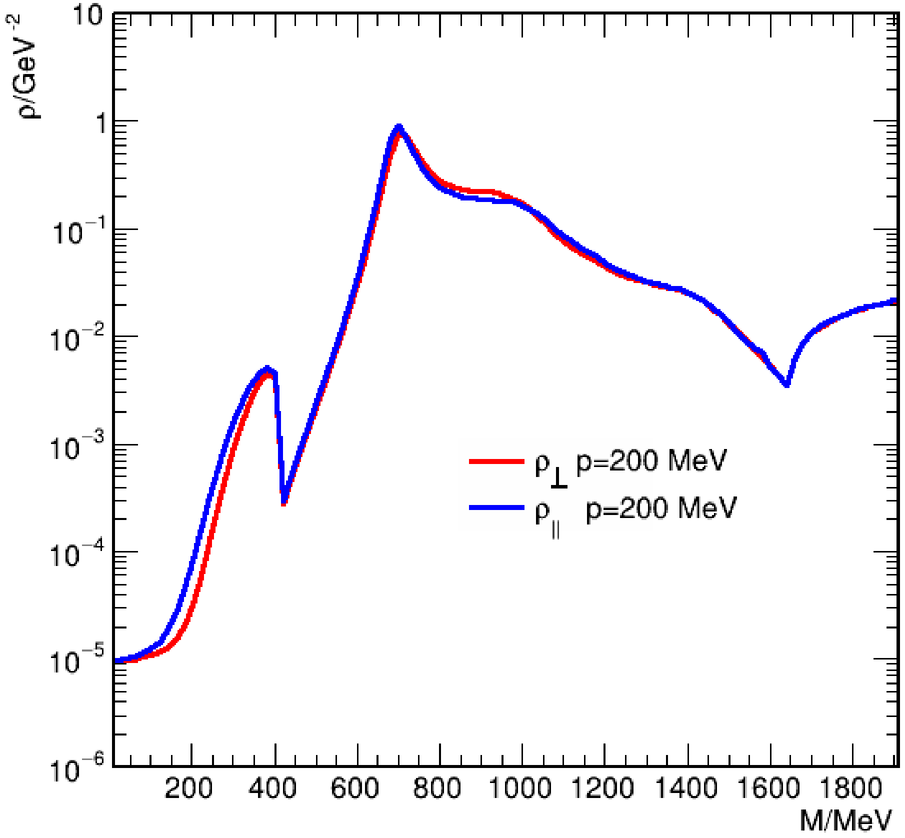
Example: T=40 & μ_B=890, p=100 MeV



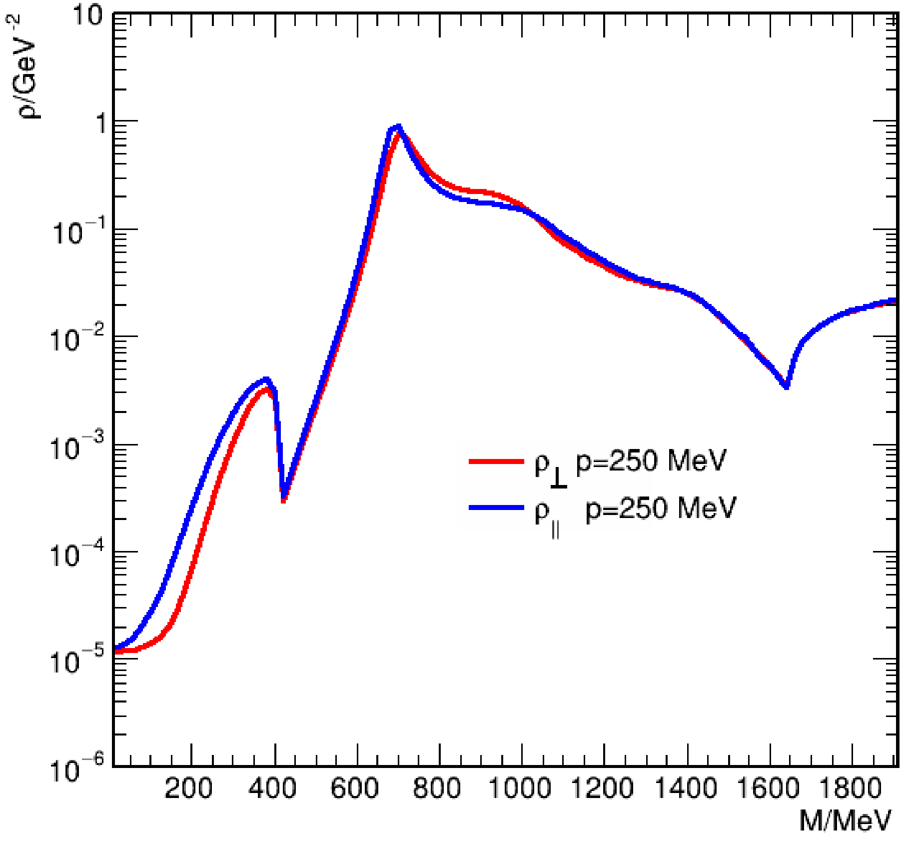
Example: T=40 & μ_B=890, p=150 MeV



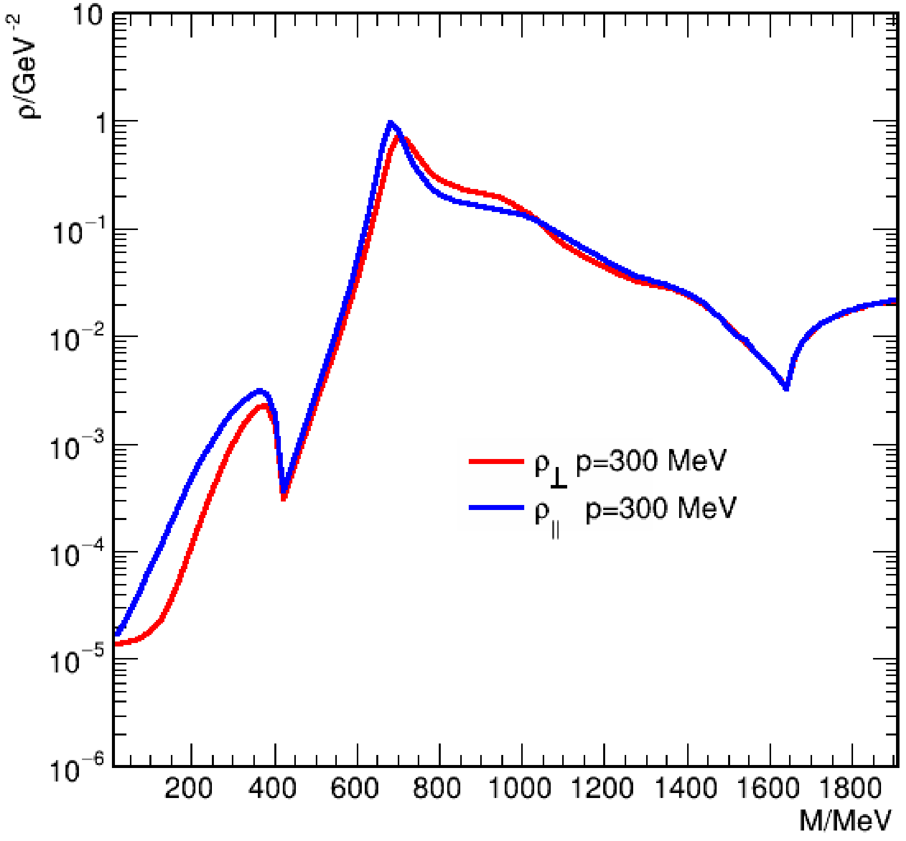
Example: T=40 & μ_B=890, p=200 MeV



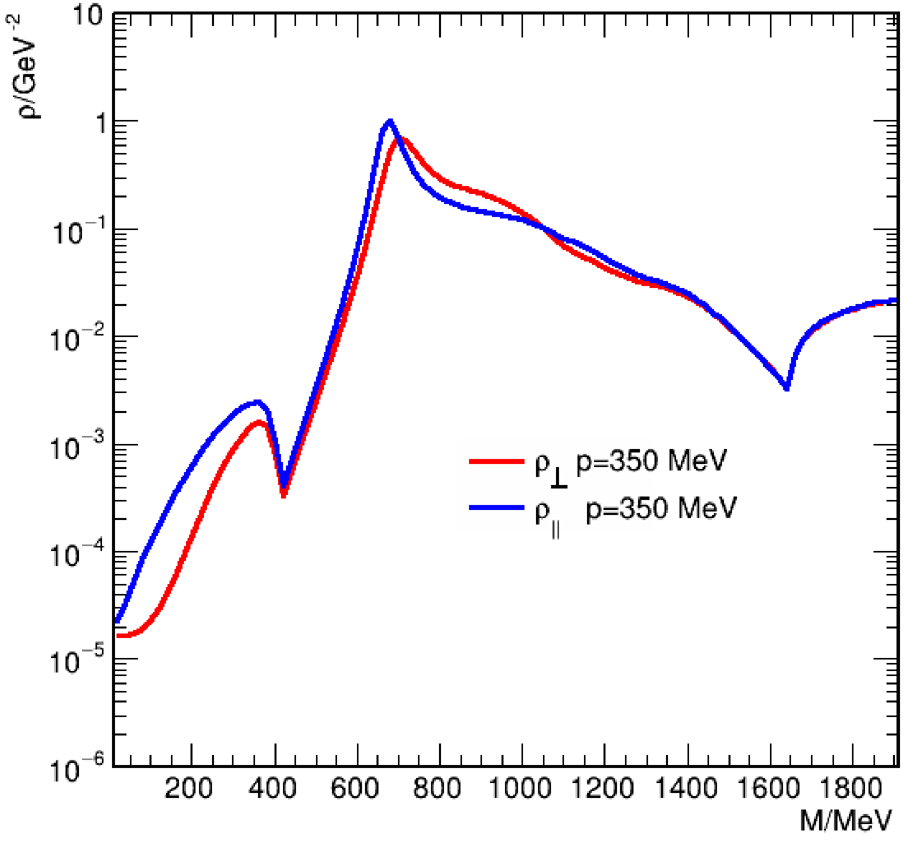
Example: T=40 & μ_B=890, p=250 MeV



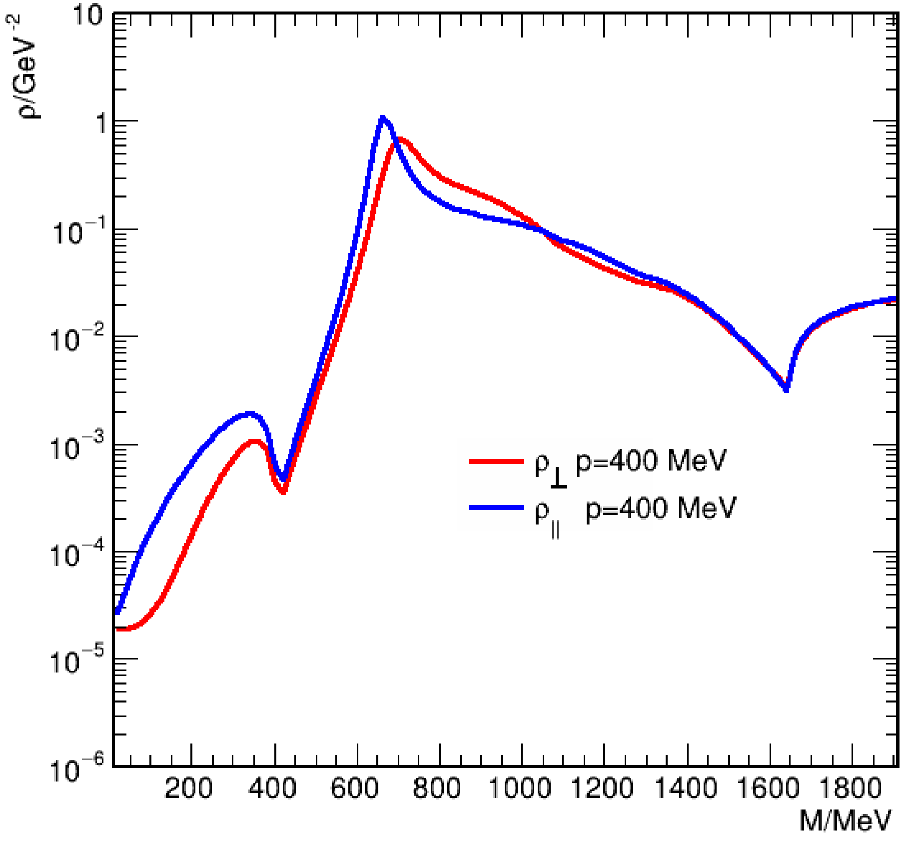
Example: T=40 & μ_B=890, p=300 MeV



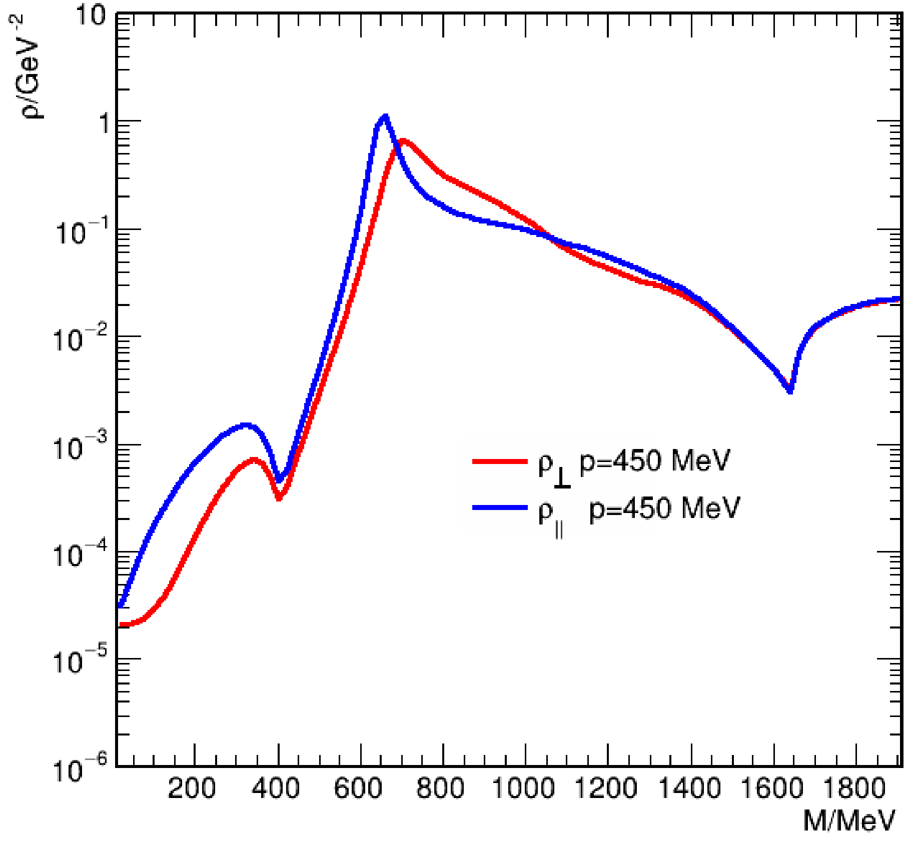
Example: T=40 & μ_B=890, p=350 MeV



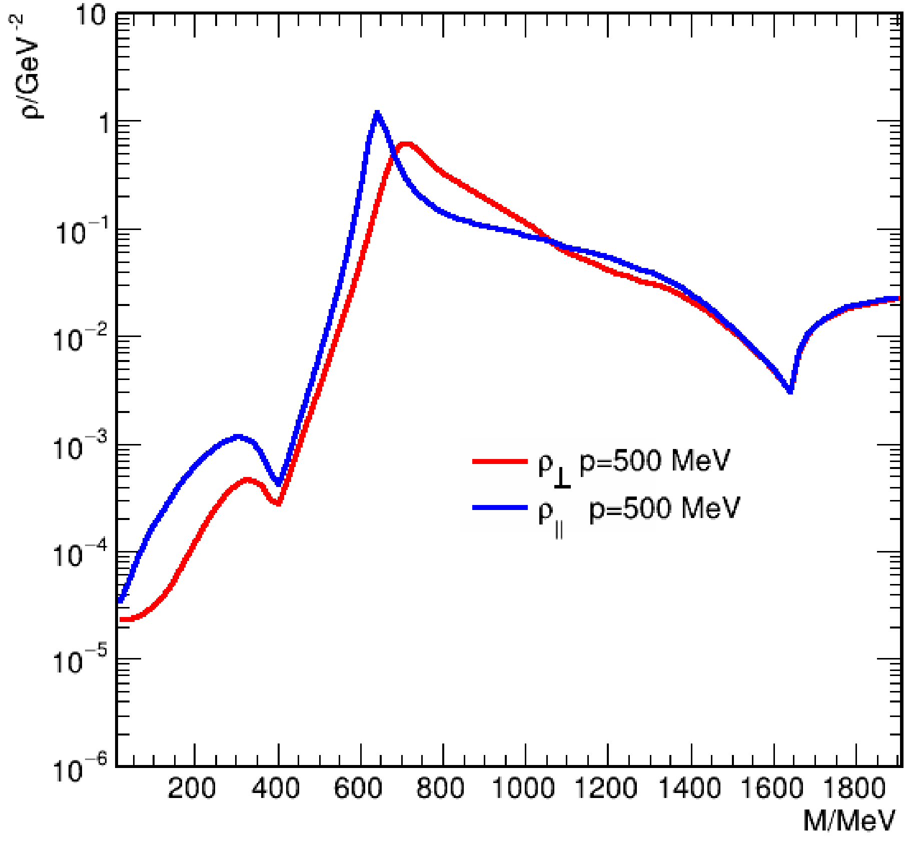
Example: T=40 & μ_B=890, p=400 MeV



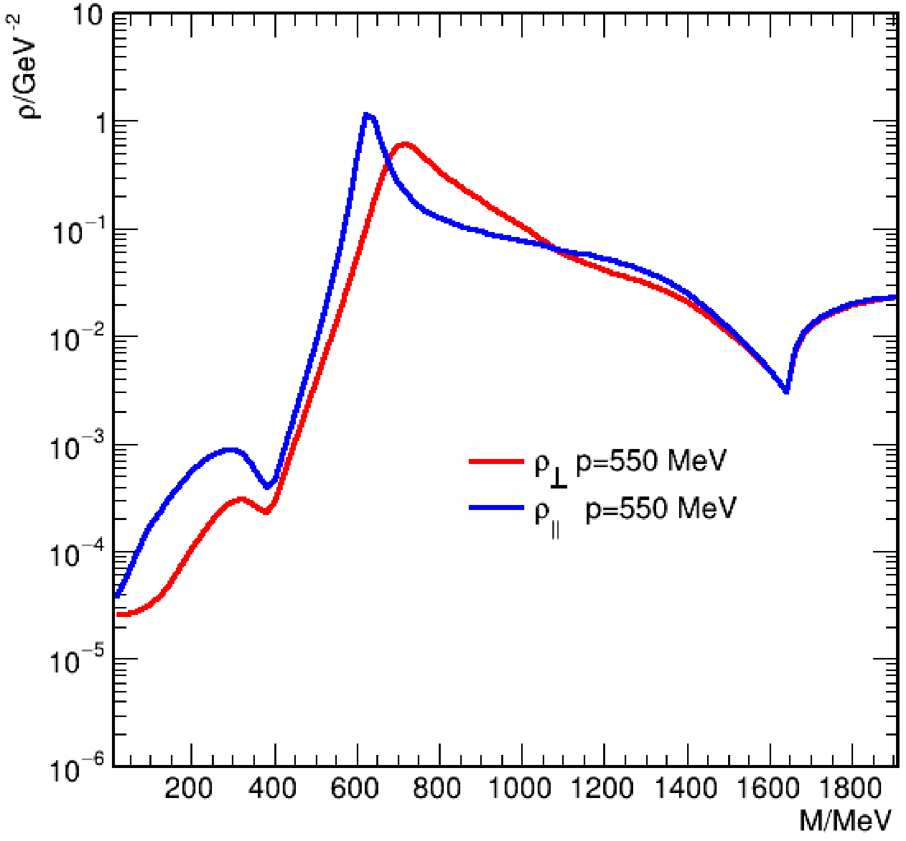
Example: T=40 & μ_B=890, p=450 MeV



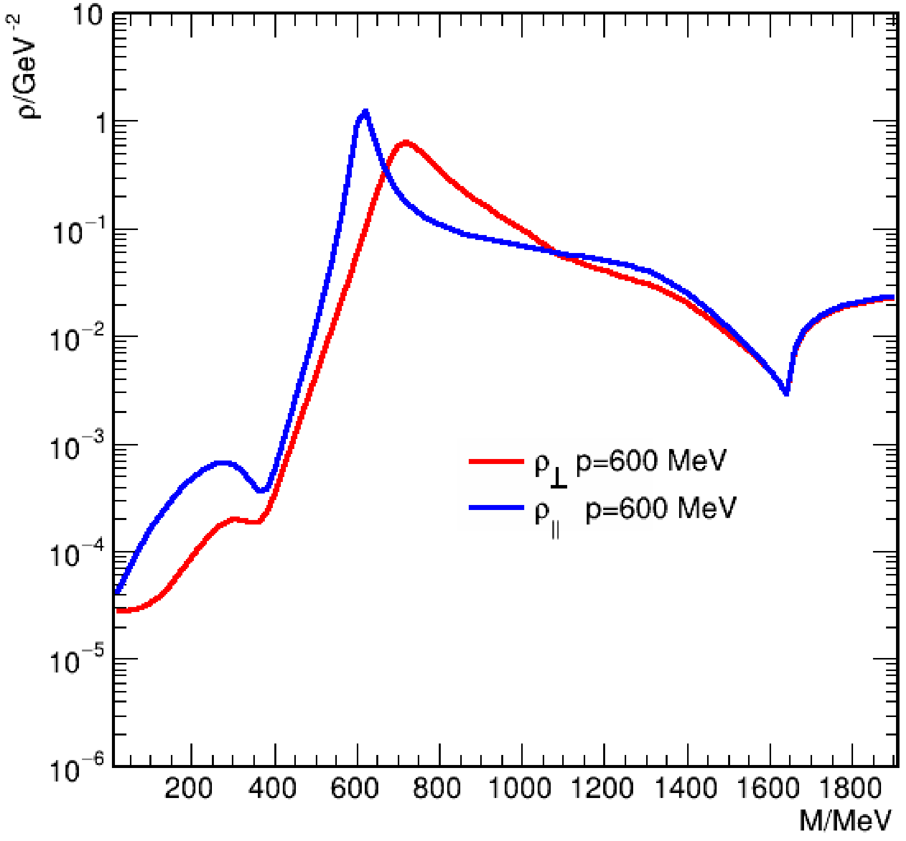
Example: T=40 & μ_B=890, p=500 MeV



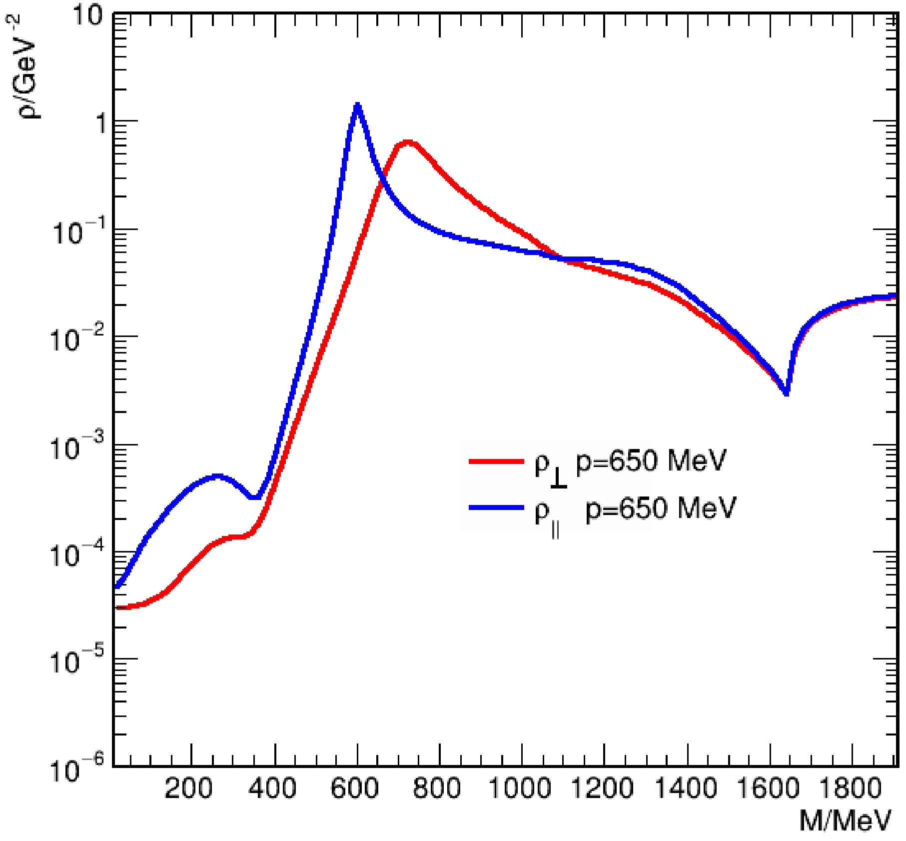
Example: T=40 & μ_B=890, p=550 MeV



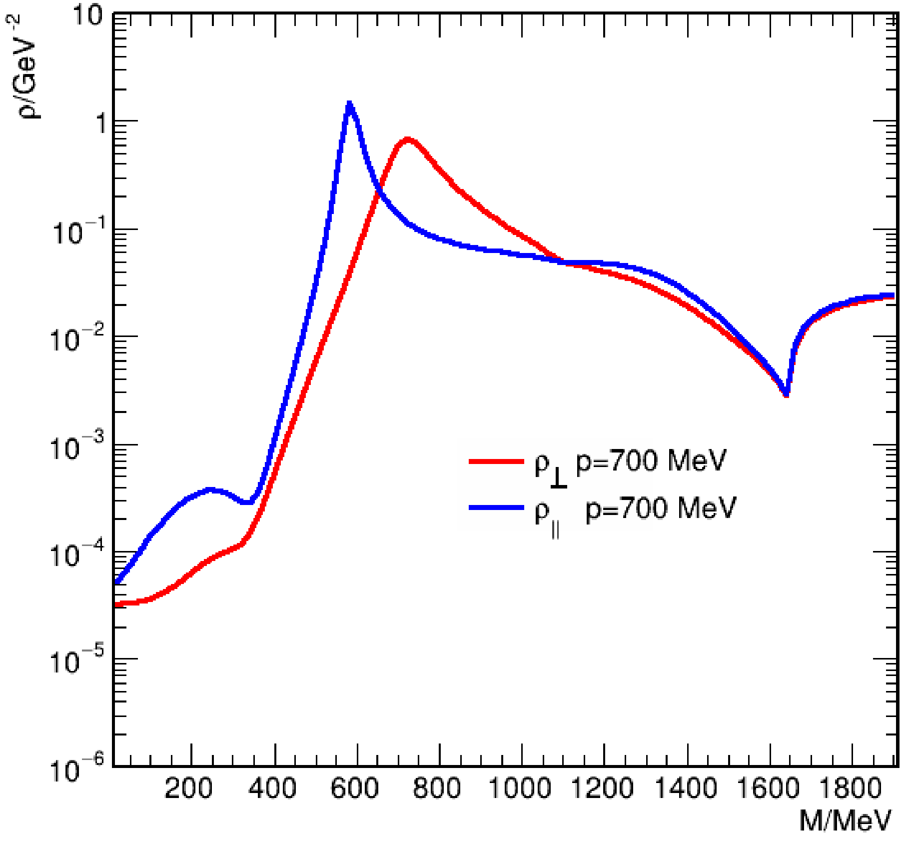
Example: T=40 & μ_B=890, p=600 MeV



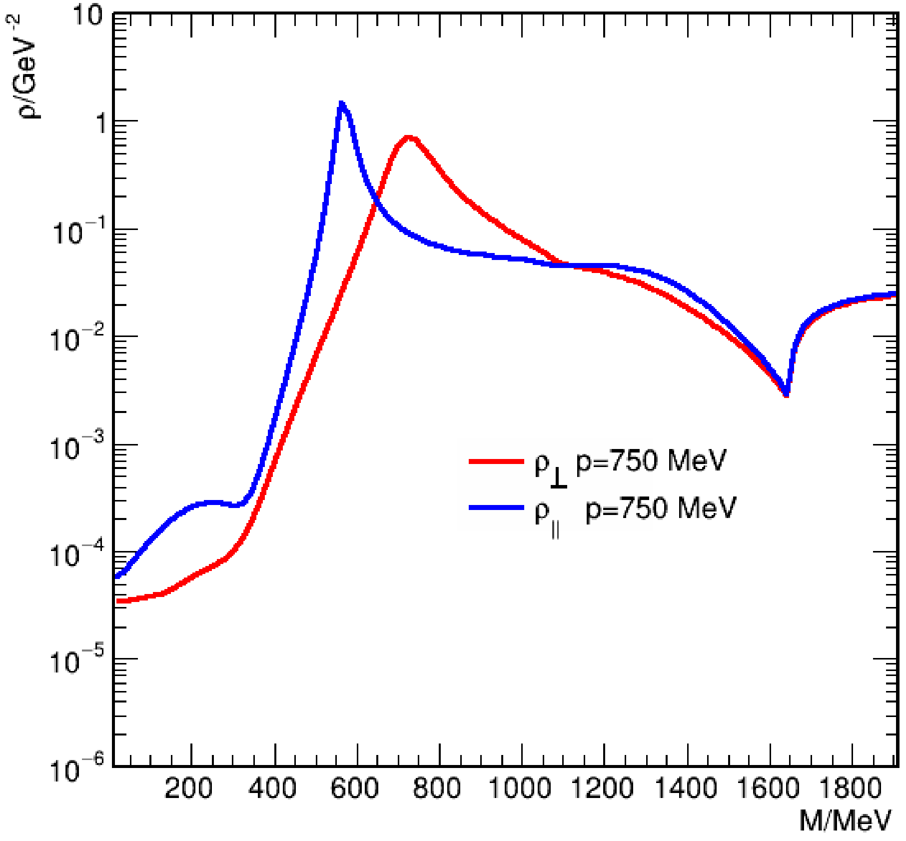
Example: T=40 & μ_B=890, p=650 MeV



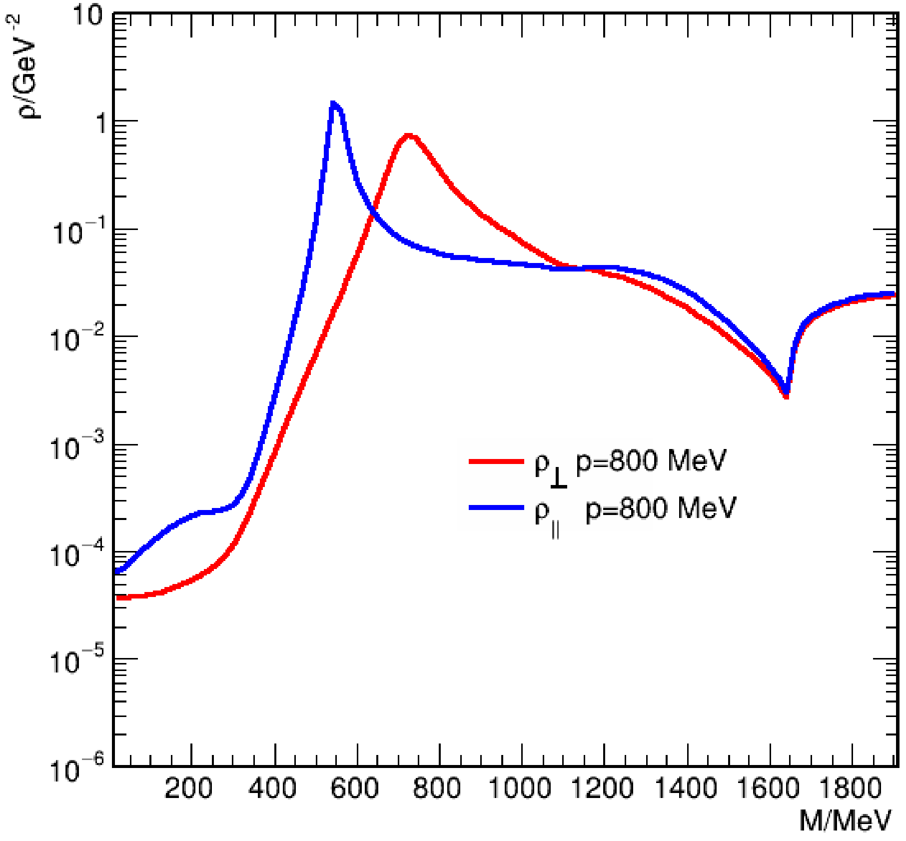
Example: T=40 & μ_B=890, p=700 MeV



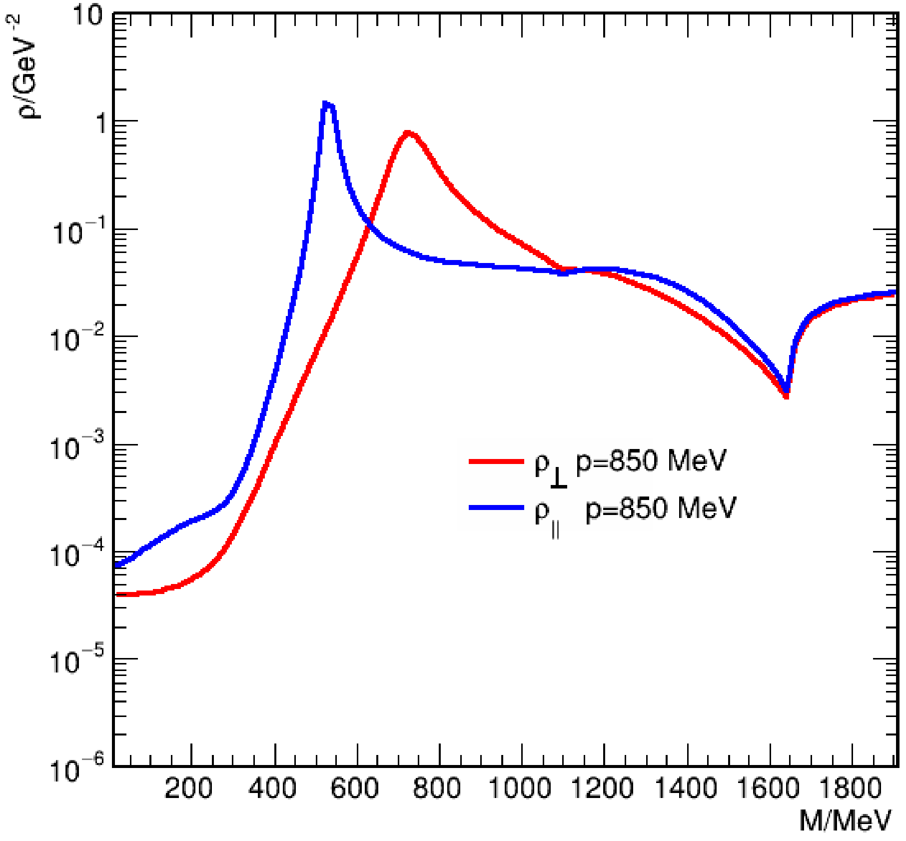
Example: T=40 & μ_B=890, p=750 MeV



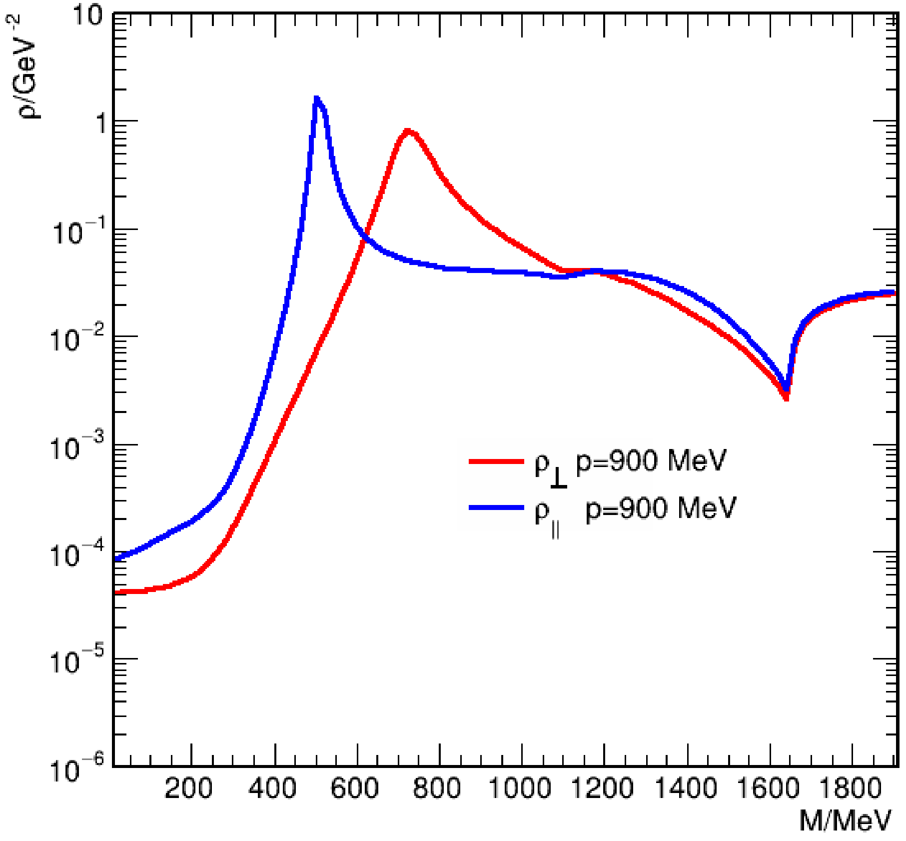
Example: T=40 & μ_B=890, p=800 MeV



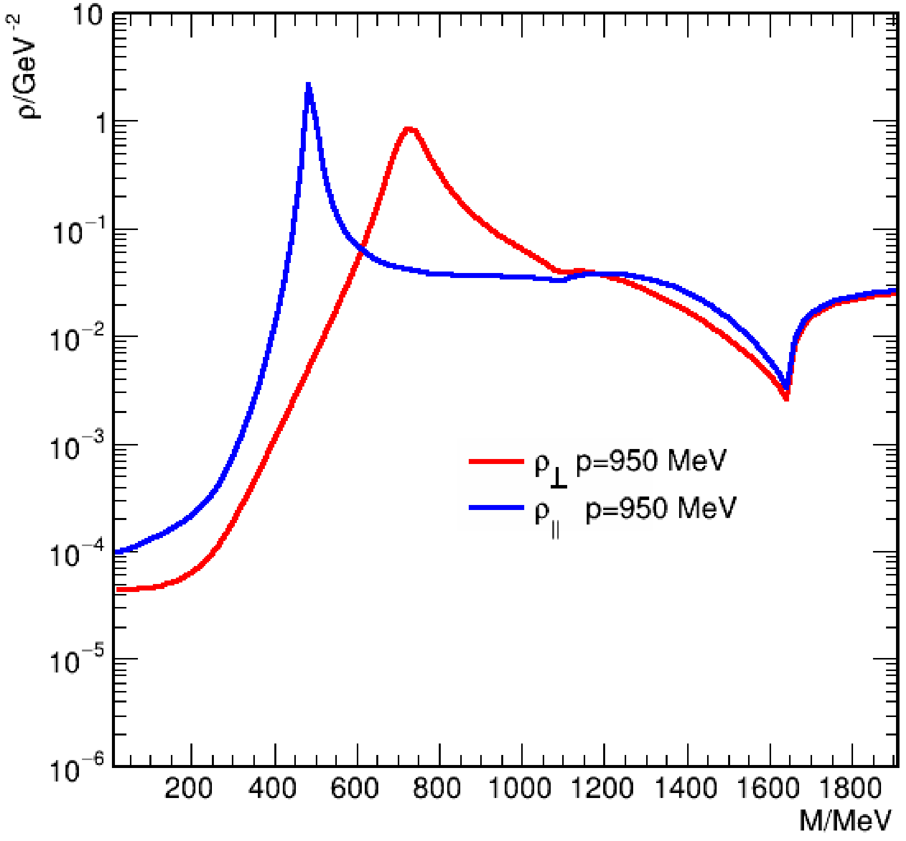
Example: T=40 & μ_B=890, p=850 MeV



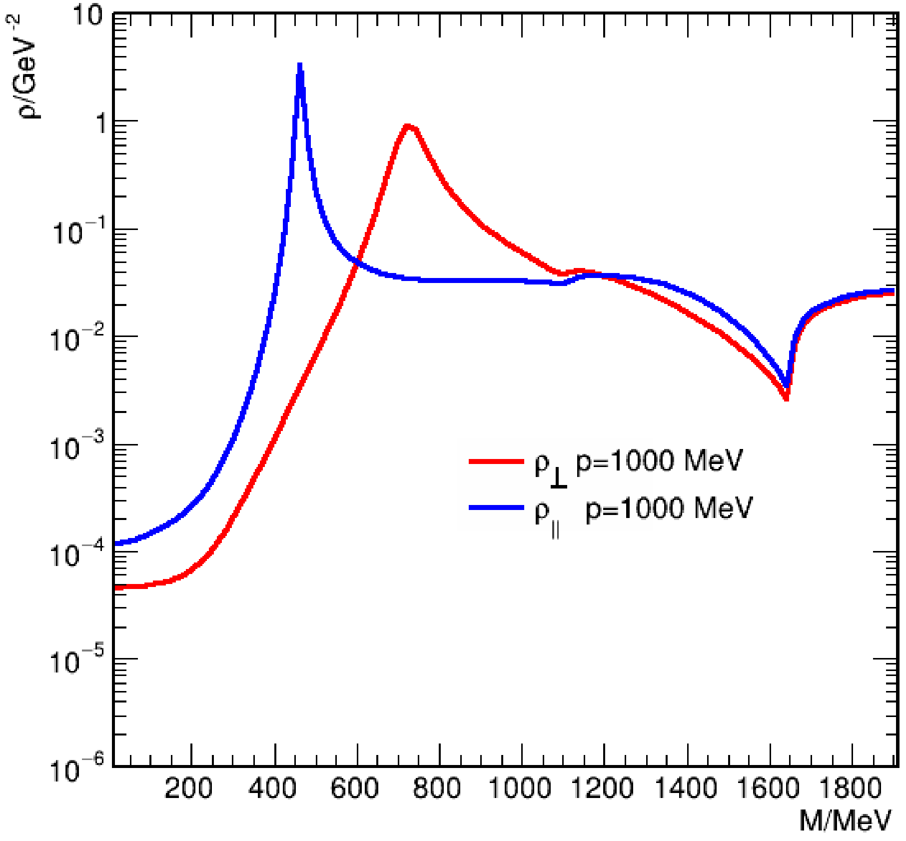
Example: T=40 & μ_B=890, p=900 MeV



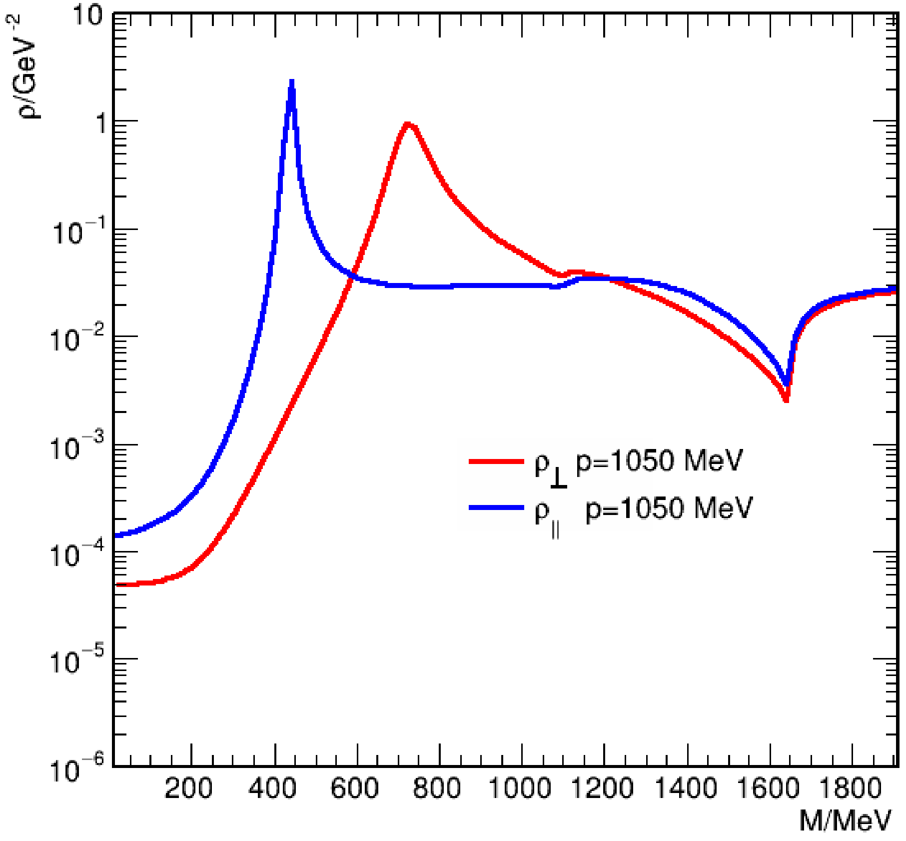
Example: T=40 & μ_B=890, p=950 MeV



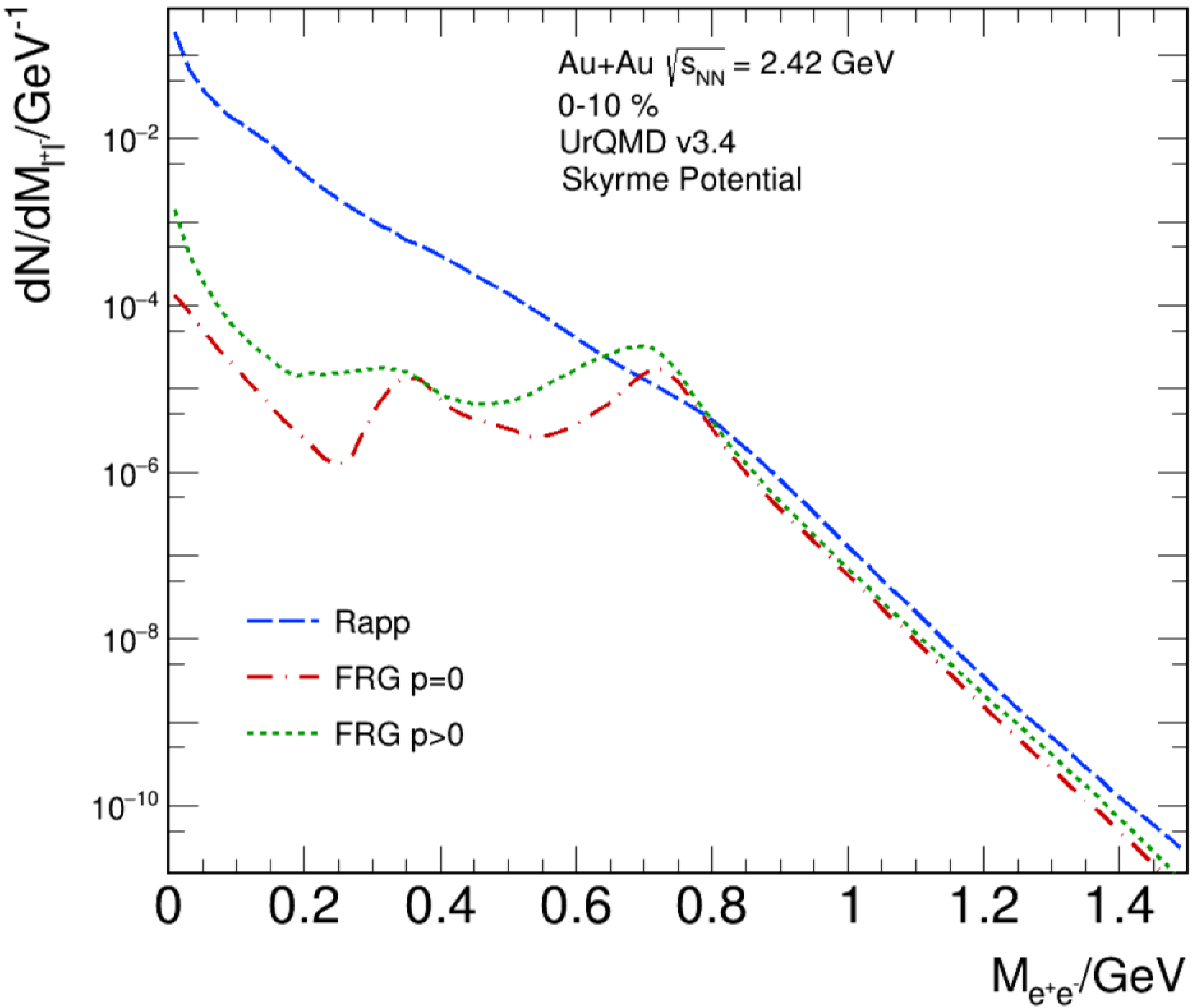
Example: T=40 & μ_B=890, p=1000 MeV



Example: T=40 & μ_B=890, p=1050 MeV

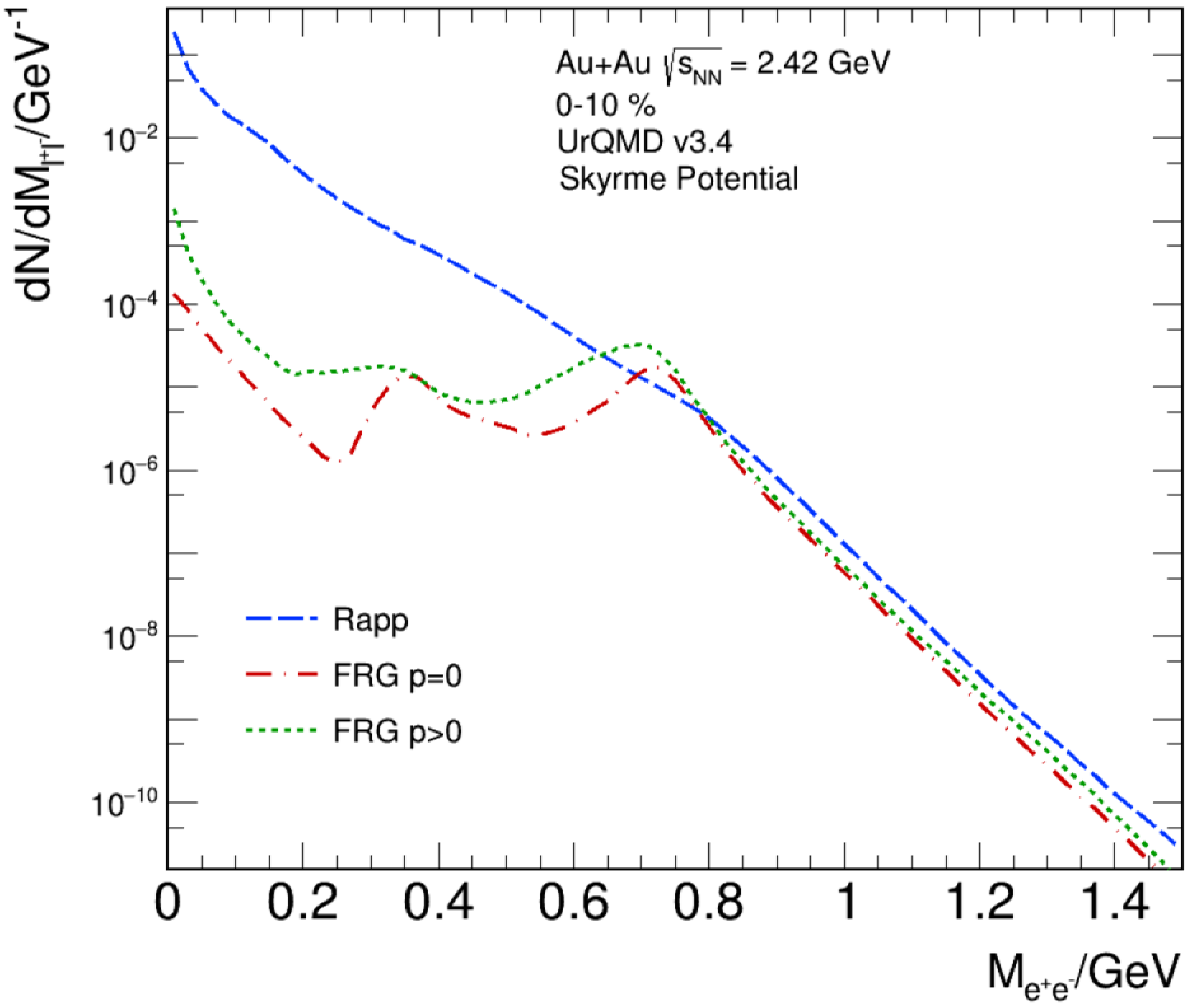


Dilepton invariant mass spectrum for p>0



- Integrated yield rises by a factor of ~2
- Peak is smeared out considerably

Dilepton invariant mass spectrum for p>0



- Integrated yield rises by a factor of ~2
- Peak is smeared out considerably
- Still many things missing for “realistic” spectral function
  - More baryon species
  - Pion modifications

Vector meson polarization

- „Transverse“ and „longitudinal“ spectral functions?
- Vector mesons move through medium (at rest)
- Direction gives a quantization axis for spin projections
- $m_z = \pm 1, 0$
- Think: Real photons: ± 1, no 0 projection→Fully transversely polarized
- Very sensitive to production mechanism!

Vector Meson Polarization as Observable

- General angular distribution of dileptons in vector meson decay (ignoring weak interactions)

$$
\begin{aligned}
\frac{d\Gamma}{d^4q d\Omega_\ell} = \mathcal{N}\Big(1 &+ \lambda_\theta \cos^2\theta_\ell \\
&+ \lambda_\phi \sin^2\theta_\ell \cos 2\phi_\ell + \lambda_{\theta\phi} \sin 2\theta_\ell \cos\phi_\ell \\
&+ \lambda_\phi^\perp \sin^2\theta_\ell \cos 2\phi_\ell + \lambda_{\theta\phi}^\perp \sin 2\theta_\ell \cos\phi_\ell\Big)
\end{aligned}
$$

- Faccioli: Every production process has a „natural“ frame, in which only λ_θ contributes.

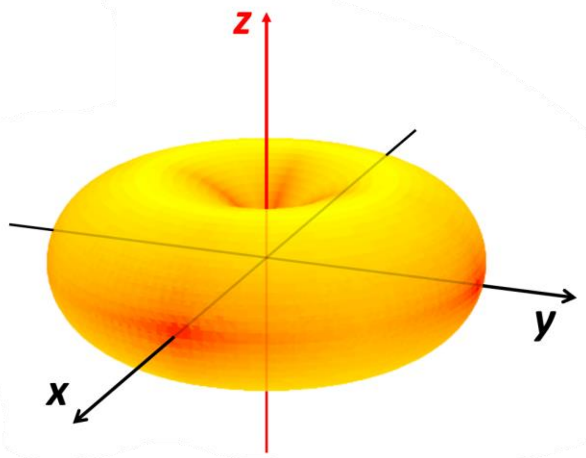
Faccioli, Lourenco, Lect.Notes Phys. 1002 (2022)

Special cases (Peanuts vs. Donuts)

$\lambda_\theta = -1$

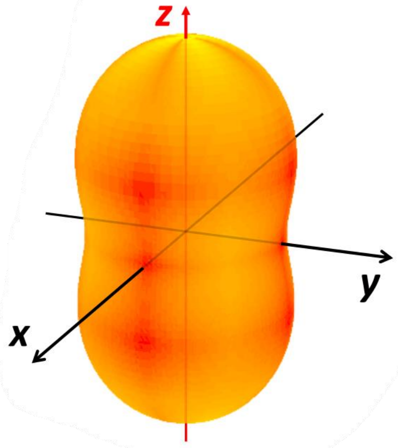
Caution!

$\lambda_\theta = 1$



„longitudinal"

„transverse"

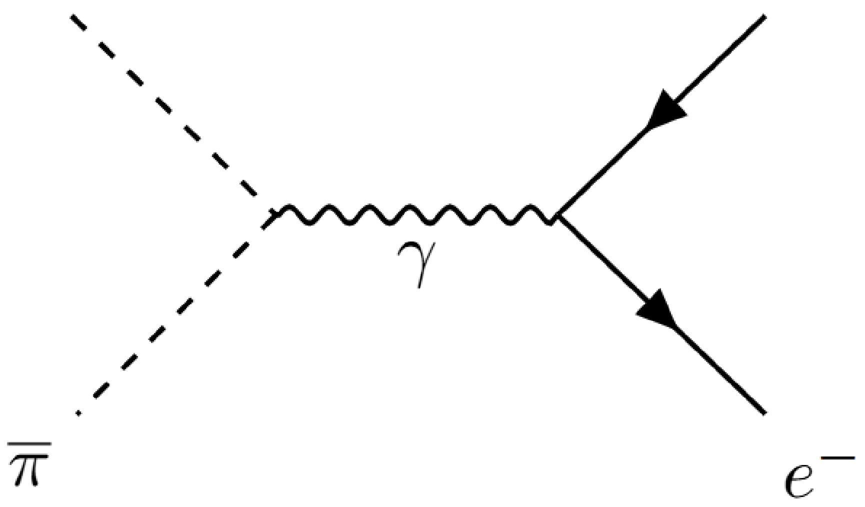


$$\frac{d\Gamma}{d^4q\,d\Omega_\ell} = \mathcal{N}\left(1 + \lambda_\theta \cos^2\theta_\ell\right)$$

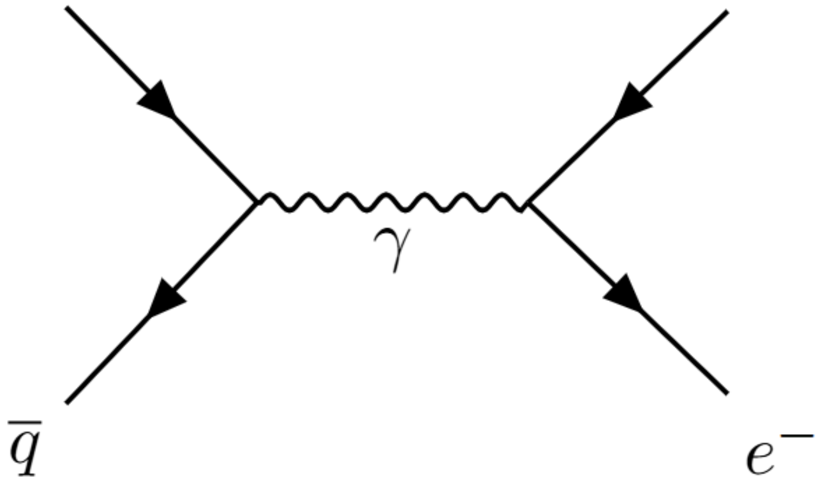
Faccioli, Lourenco, Lect.Notes Phys. 1002 (2022)

Example: $\pi\pi$ vs Drell-Yan

$\lambda_\theta = -1$



$\lambda_\theta = 1$

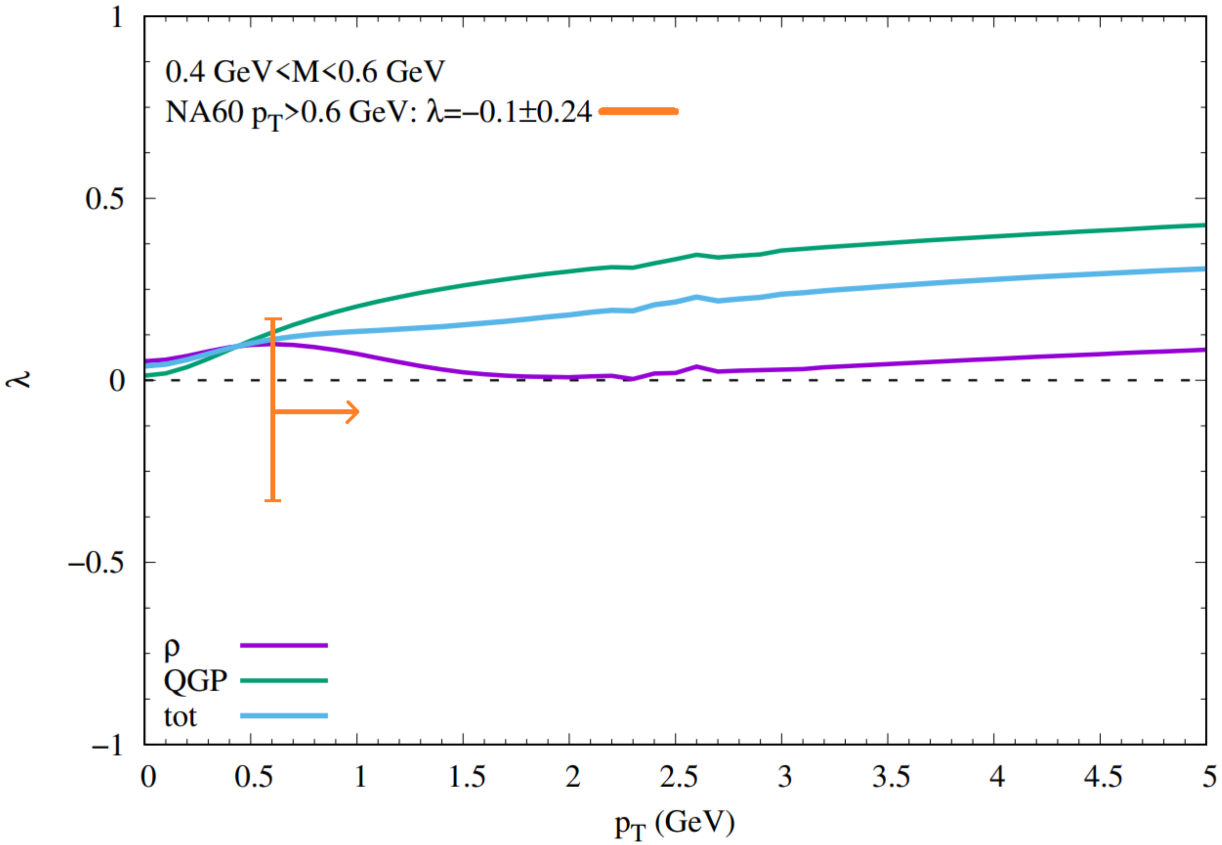


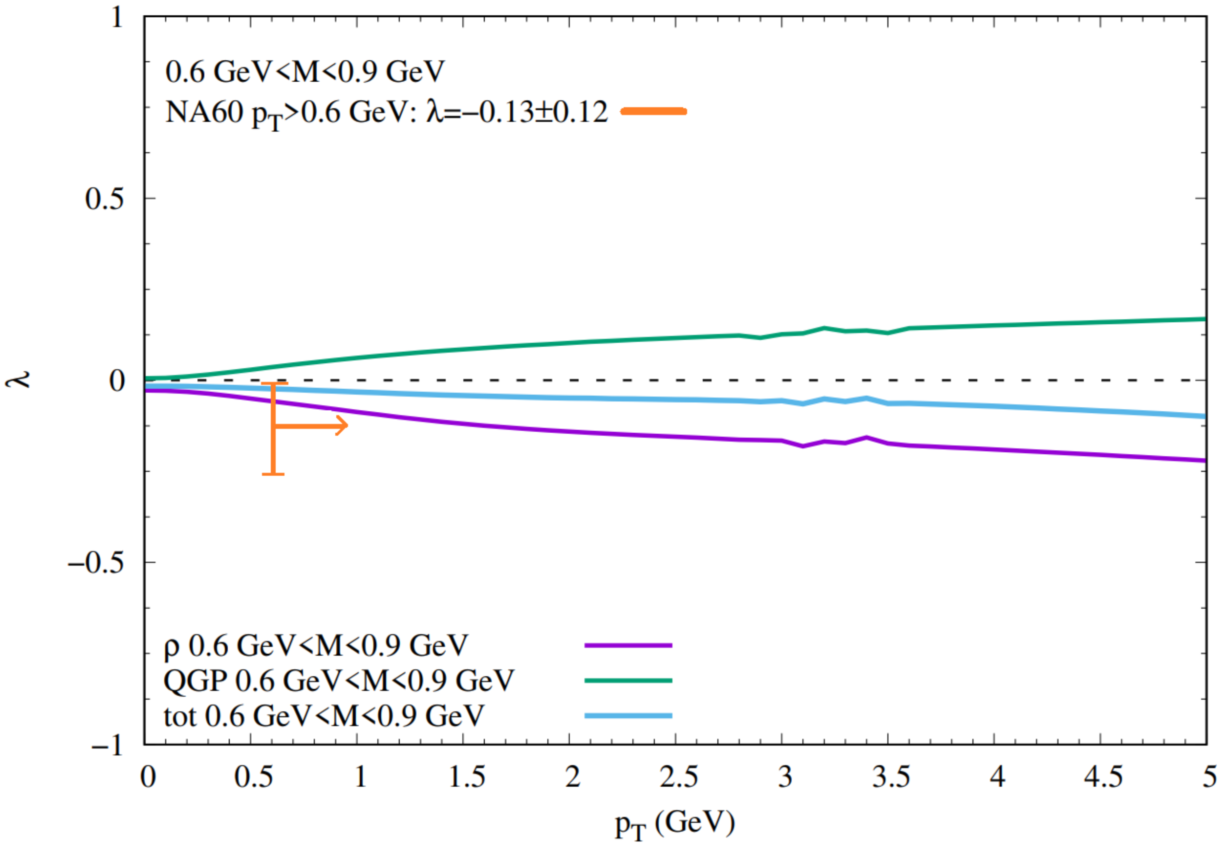
Pions have no spin, so spin contribution to polarization is 0 (though orbital contributions persist, as seen later)

Chiral limit: helicities of q and $\bar{q}$ are conserved and opposite, must couple to spin projection 1.

Bratkovskaya, Phys. Lett. B 348 (1995)

Polarization signatures of phase transition





v. Hees, presentation at ECT Workshop 2021

Polarisation of thermal dileptons?

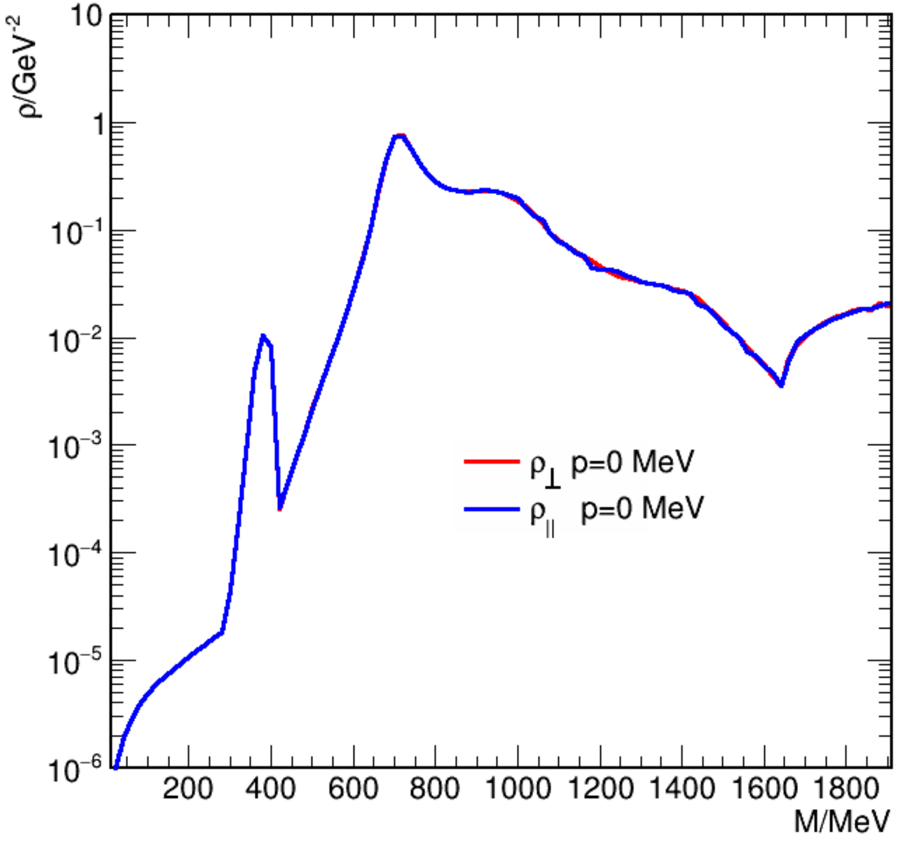
- Anisotropy of thermal virtual photons (in helicity frame) given by

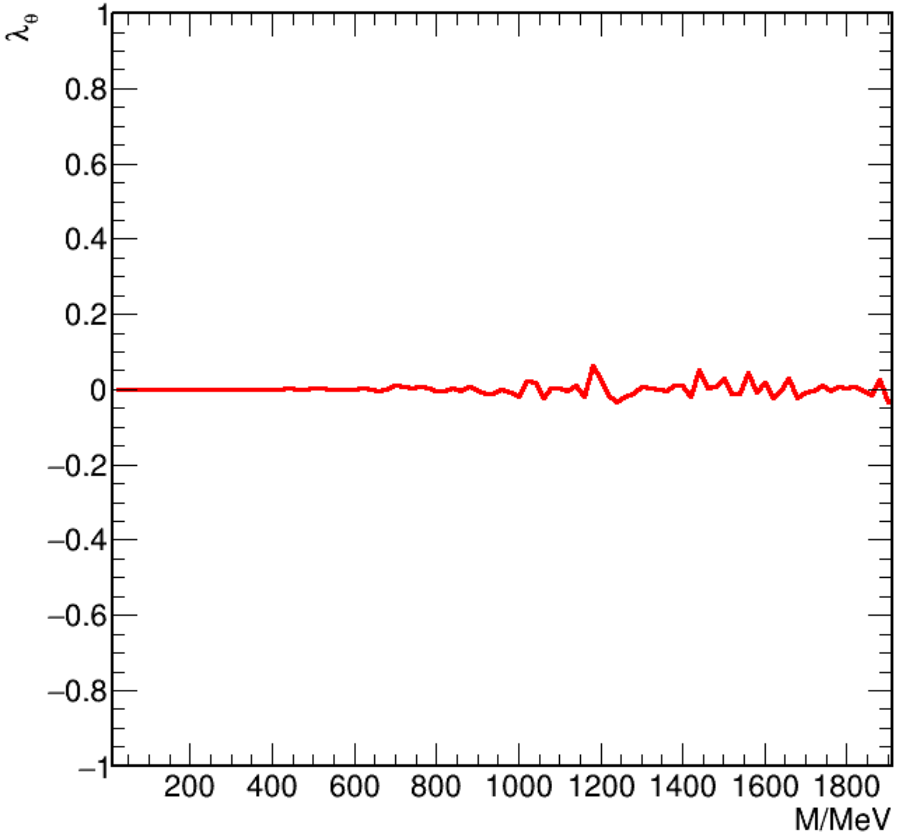
$$\lambda_\theta = \frac{\rho_\perp - \rho_{||}}{\rho_\perp + \rho_{||}}$$

- Gives a weighted convolution of all contributions to the spectral function

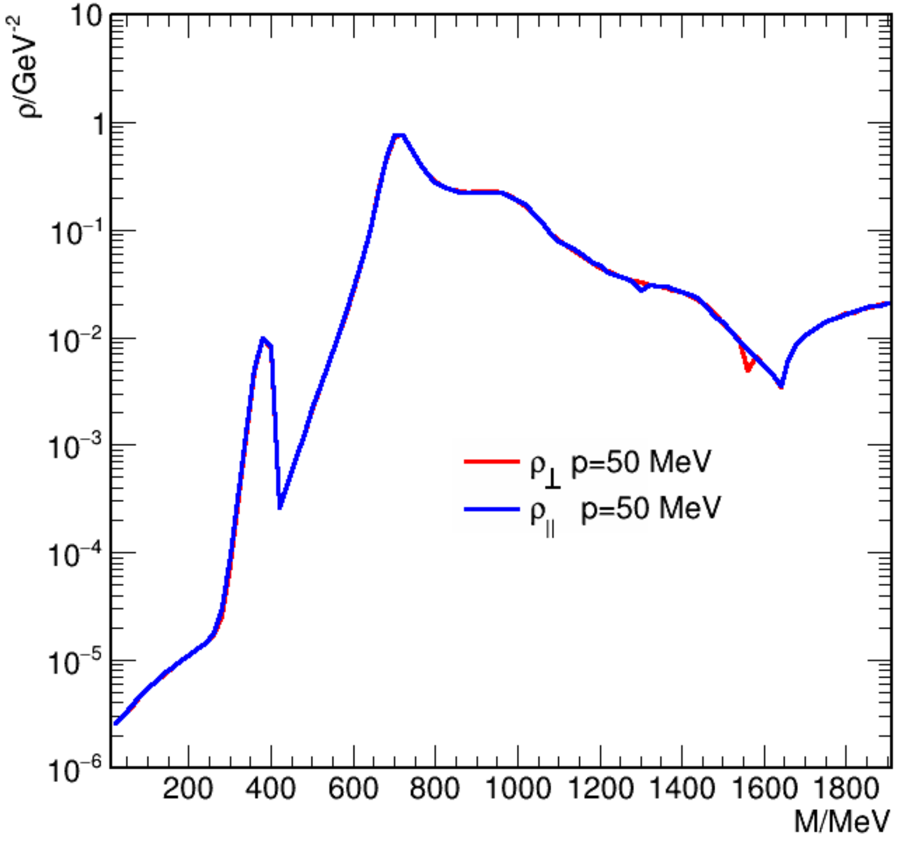
Speranza, PhD Thesis, TU Darmstadt, 2017

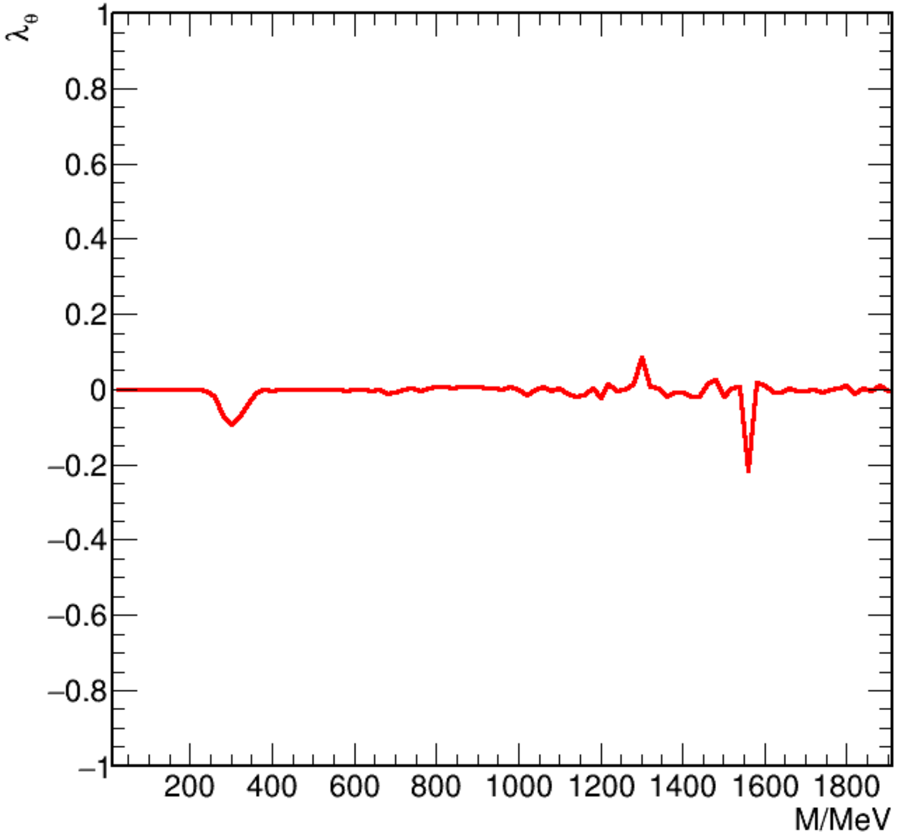
Example: T=40 & μ_B=890, p=0 MeV



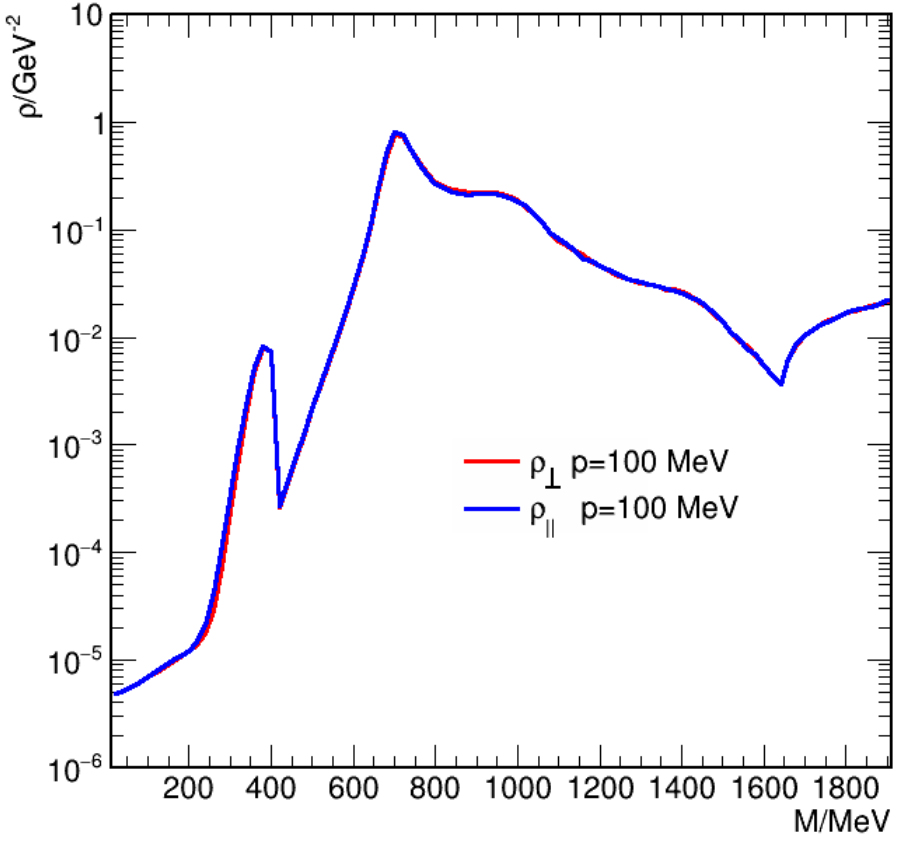


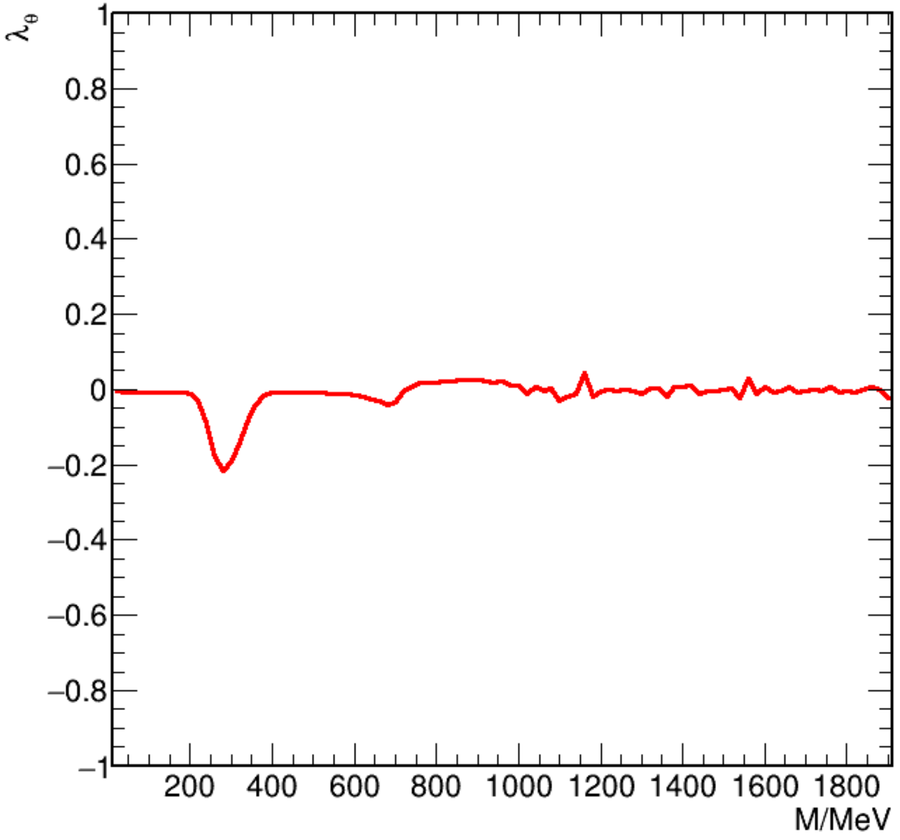
Example: T=40 & μ_B=890, p=50 MeV



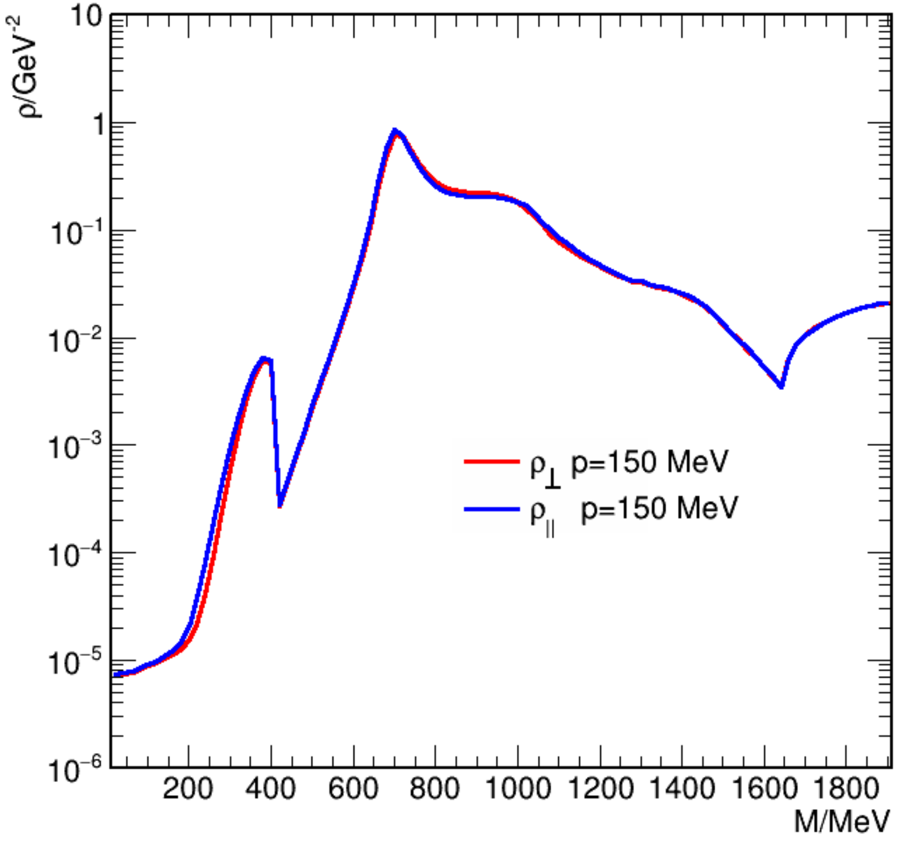


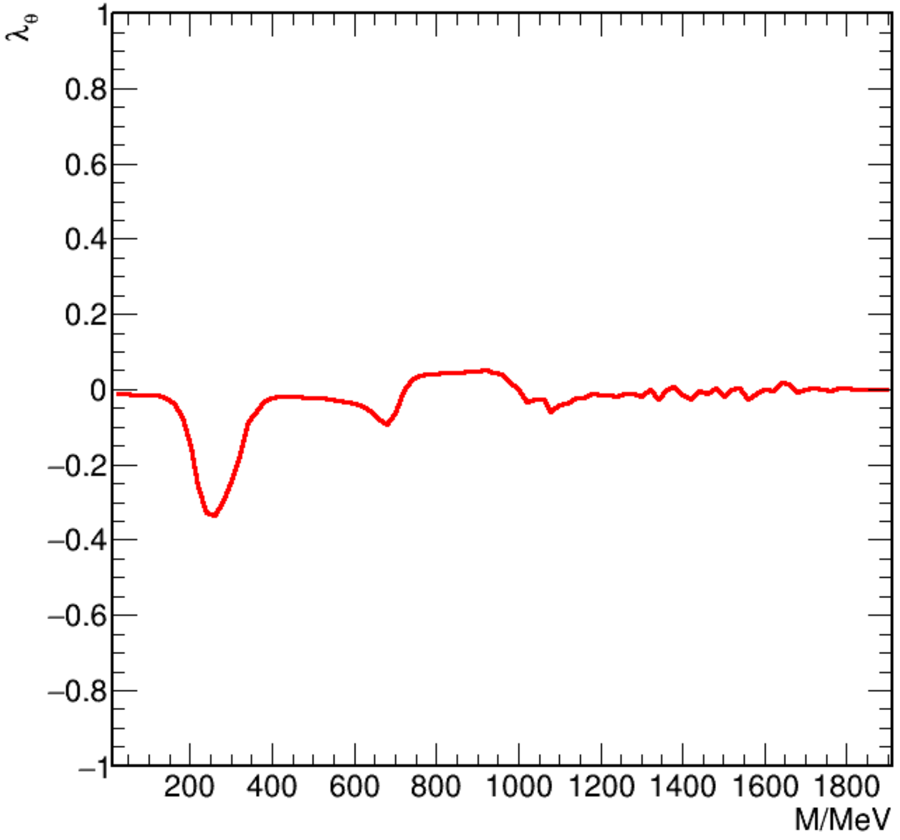
Example: T=40 & μ_B=890, p=100 MeV



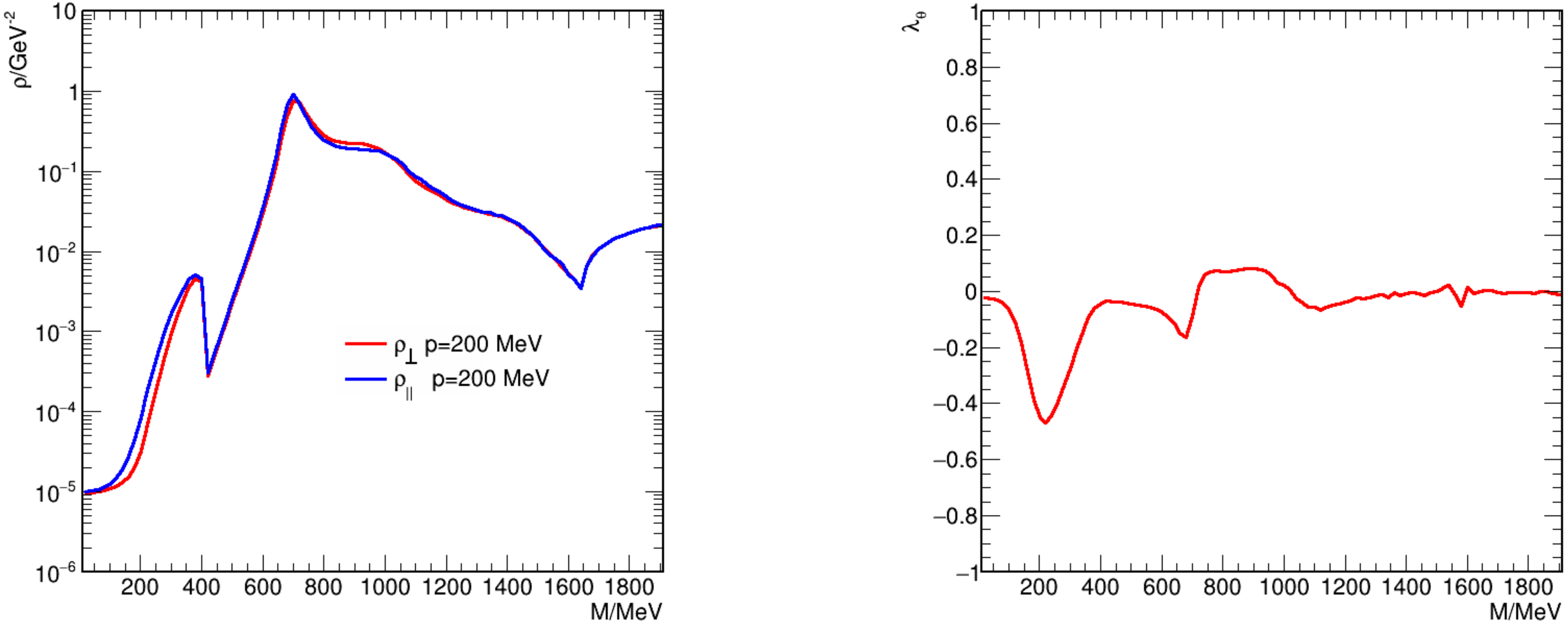


Example: T=40 & μ_B=890, p=150 MeV

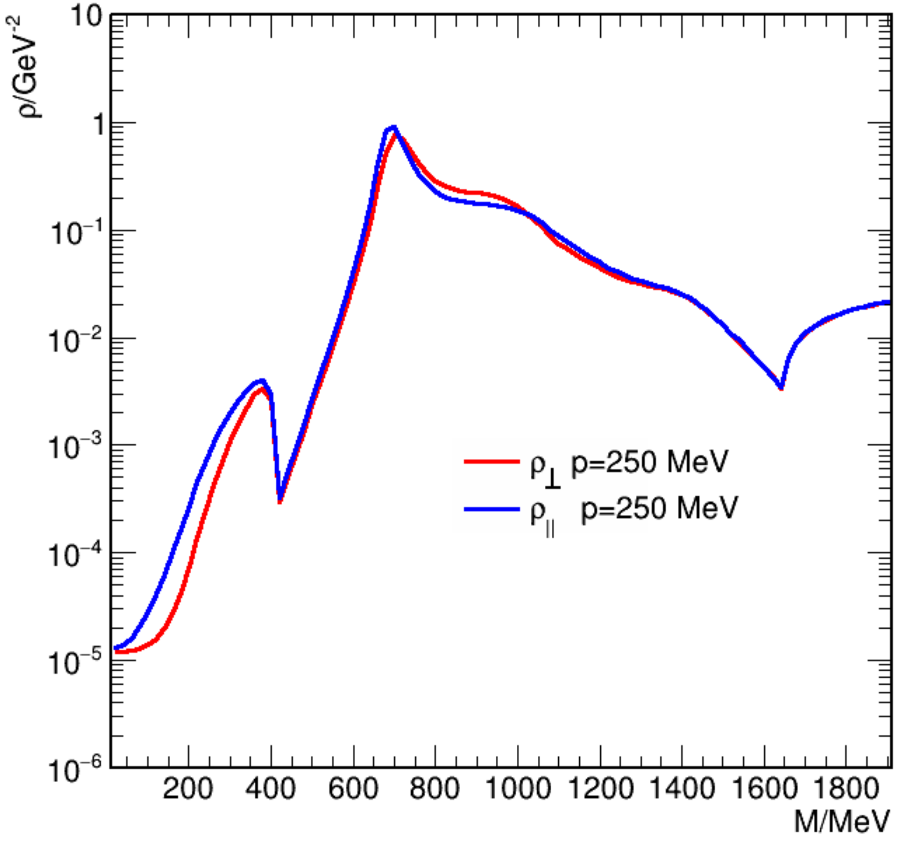


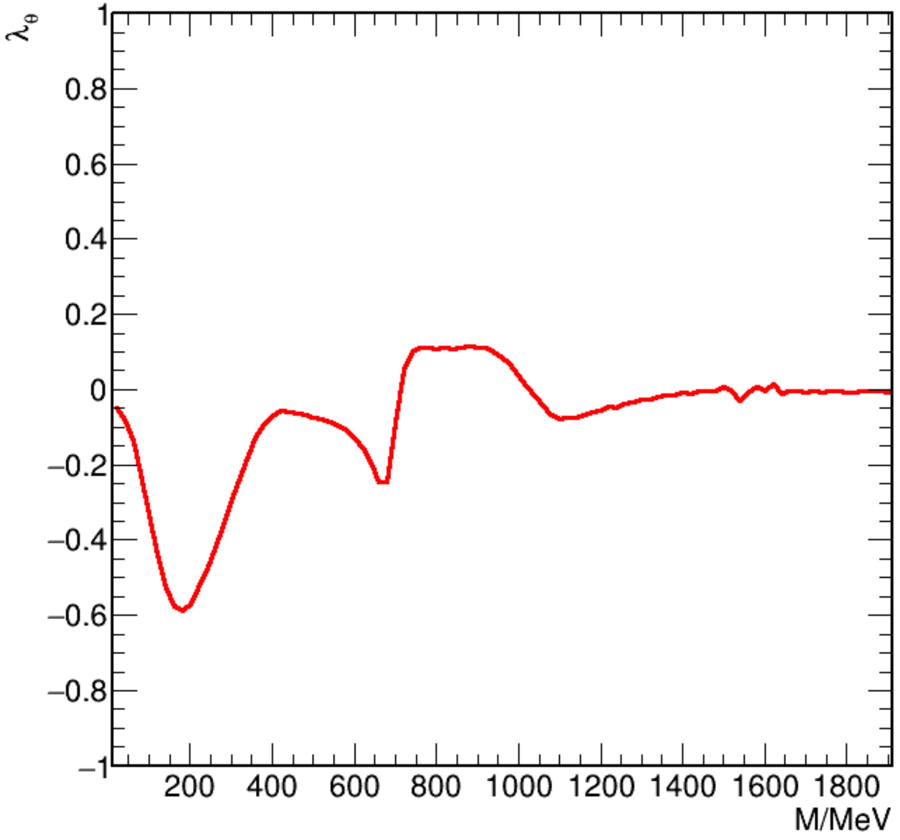


Example: T=40 & μ_B=890, p=200 MeV



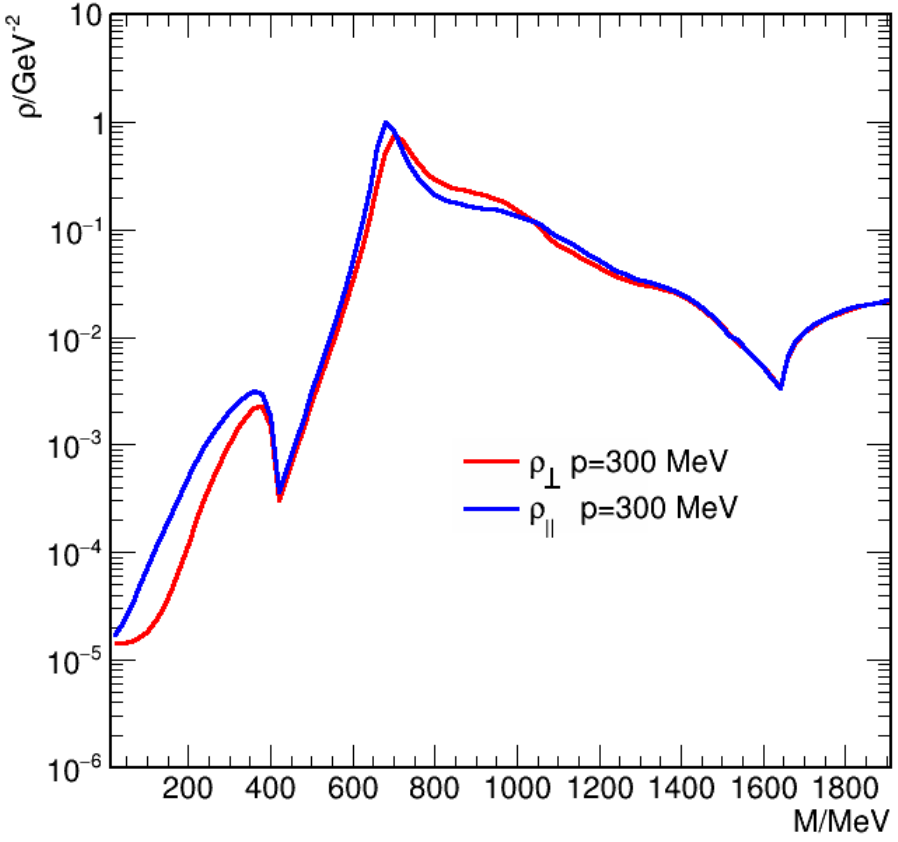
Example: T=40 & μ_B=890, p=250 MeV

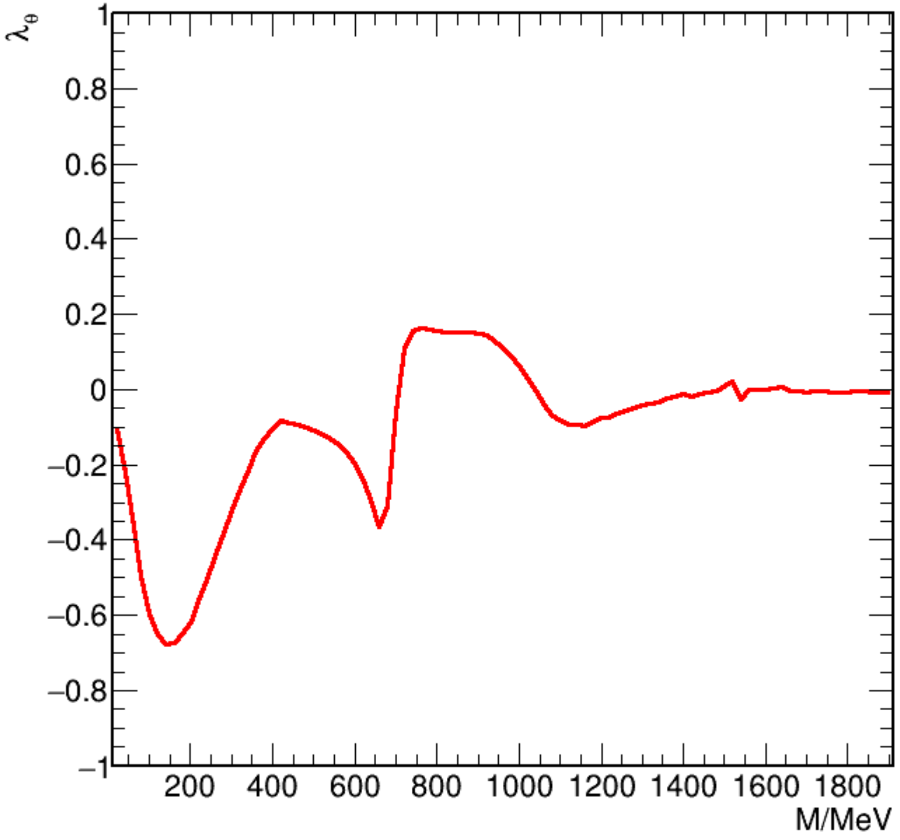




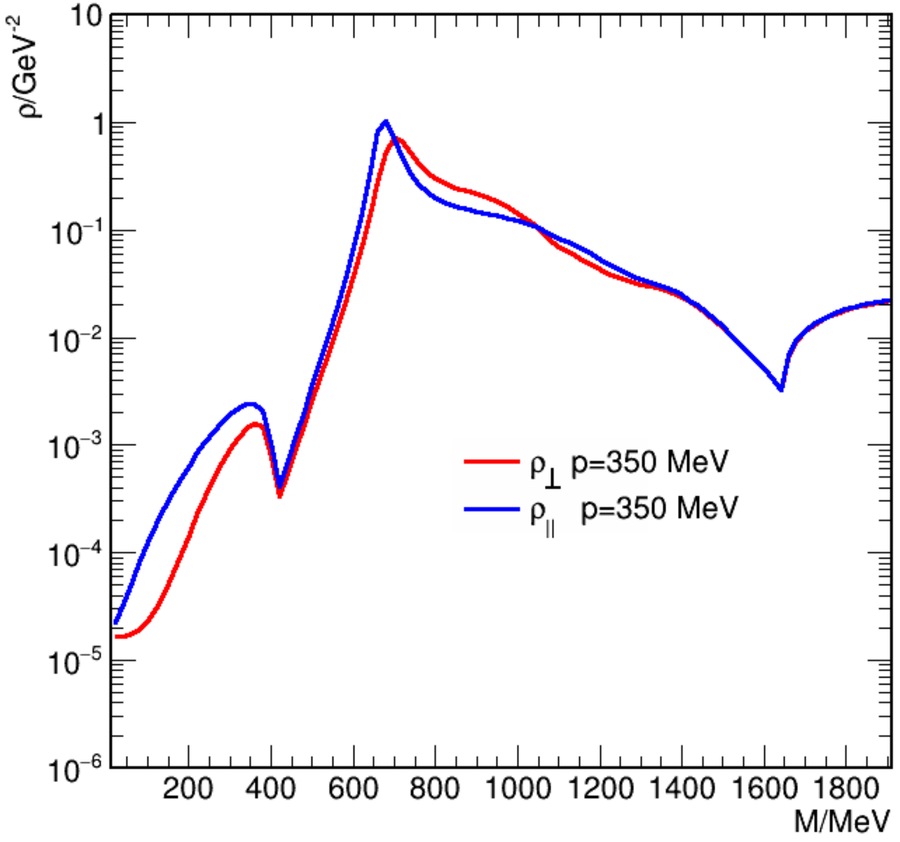
Example: T=40 & μ_B=890, p=300 MeV

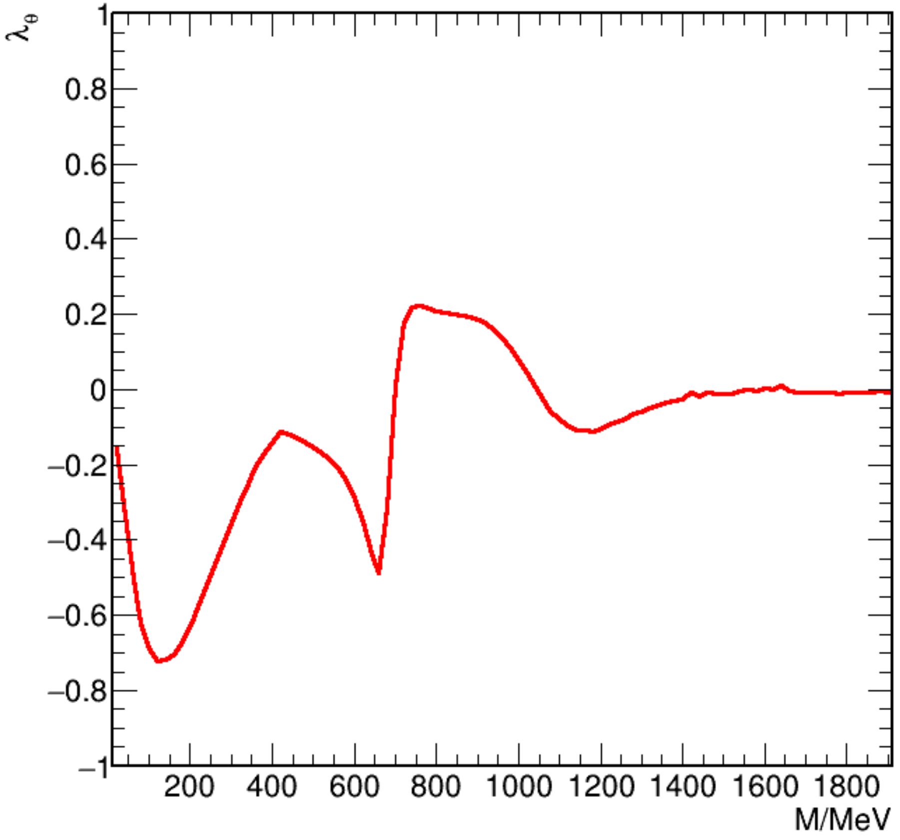
CRC-TR 211



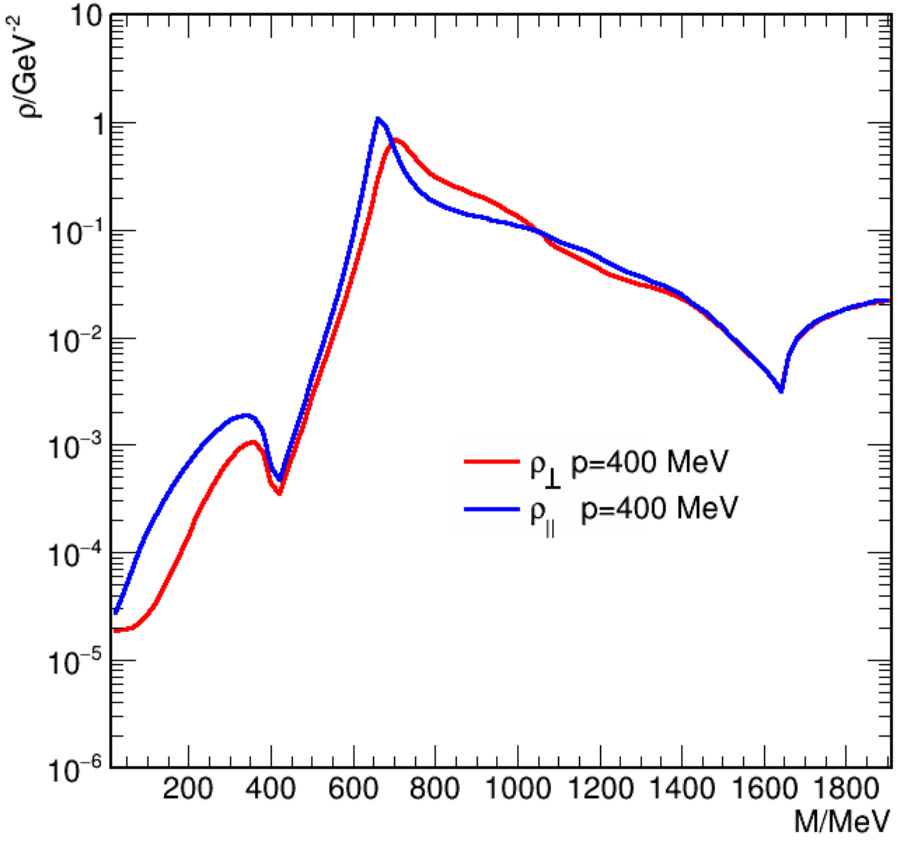


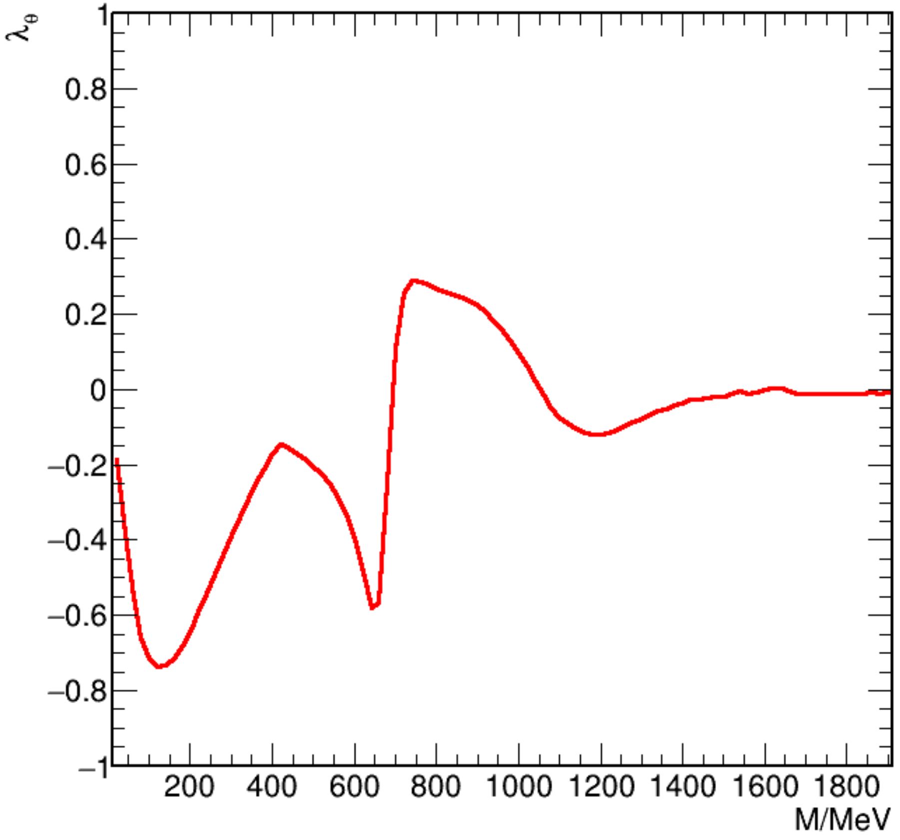
Example: T=40 & μ_B=890, p=350 MeV



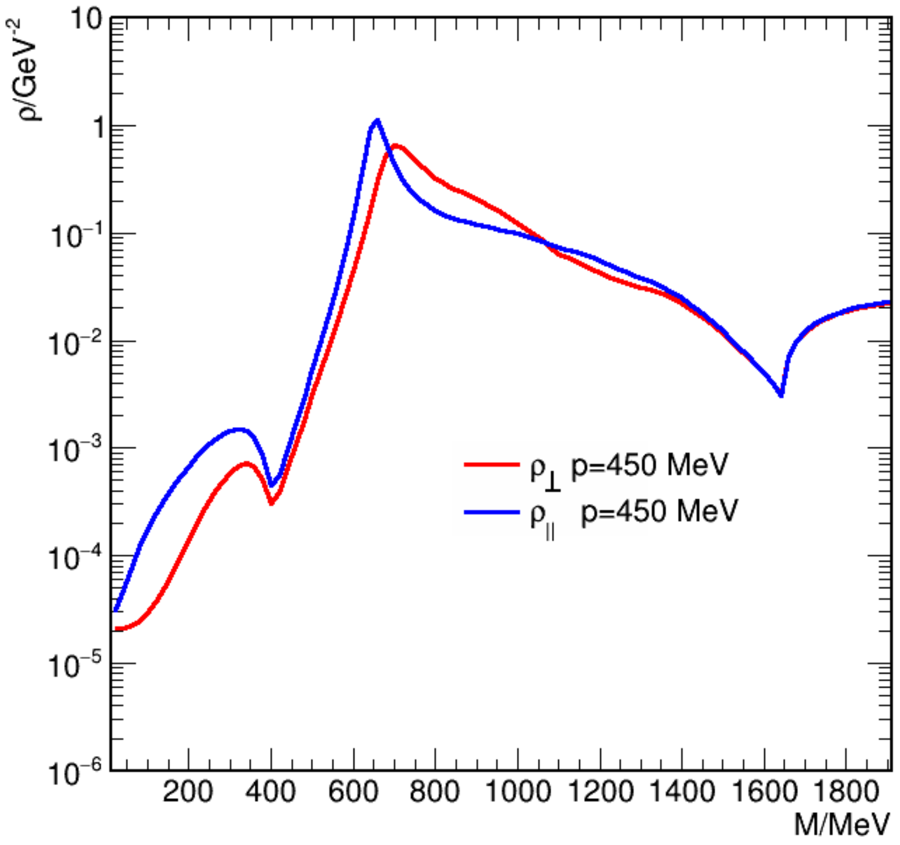


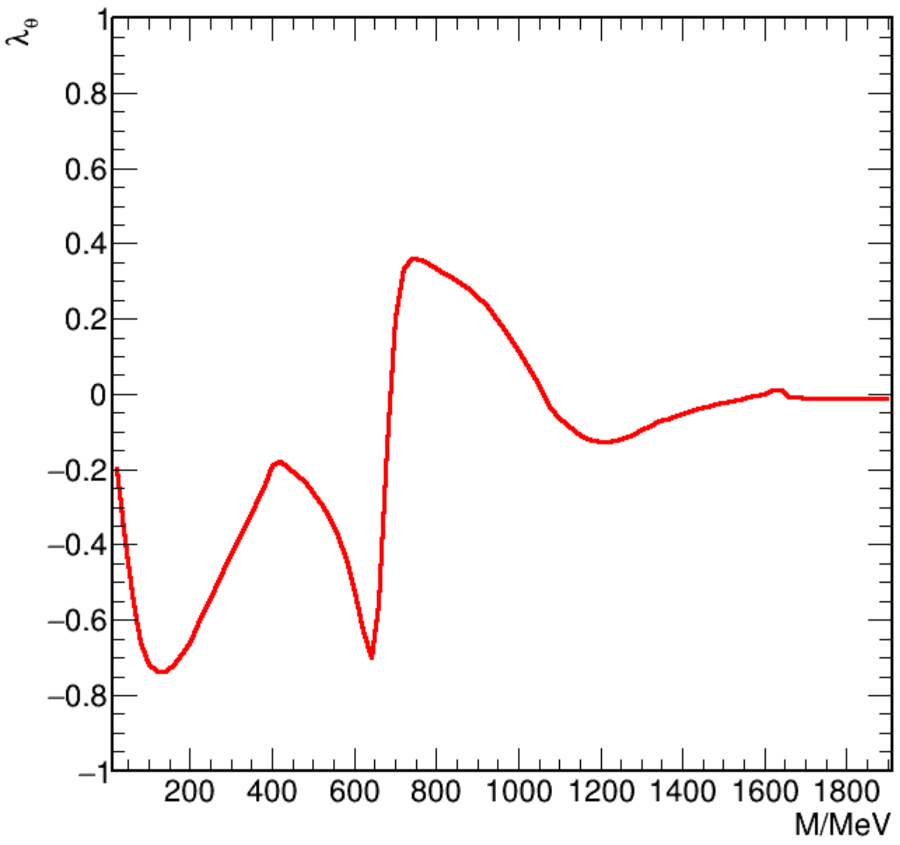
Example: T=40 & μ_B=890, p=400 MeV



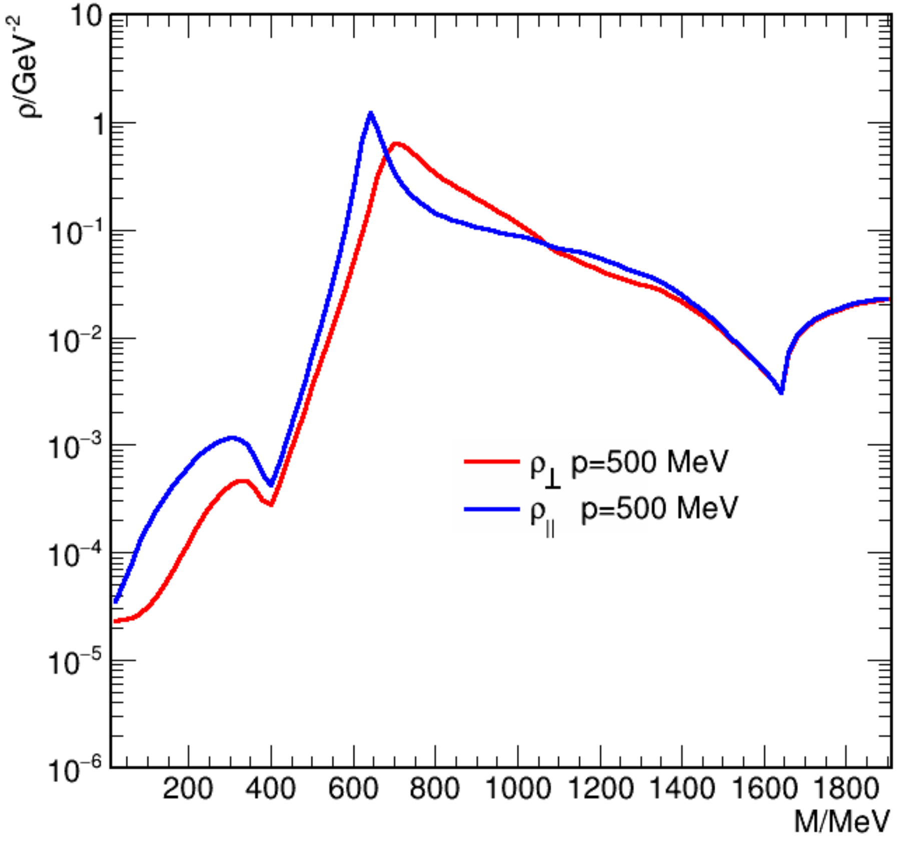


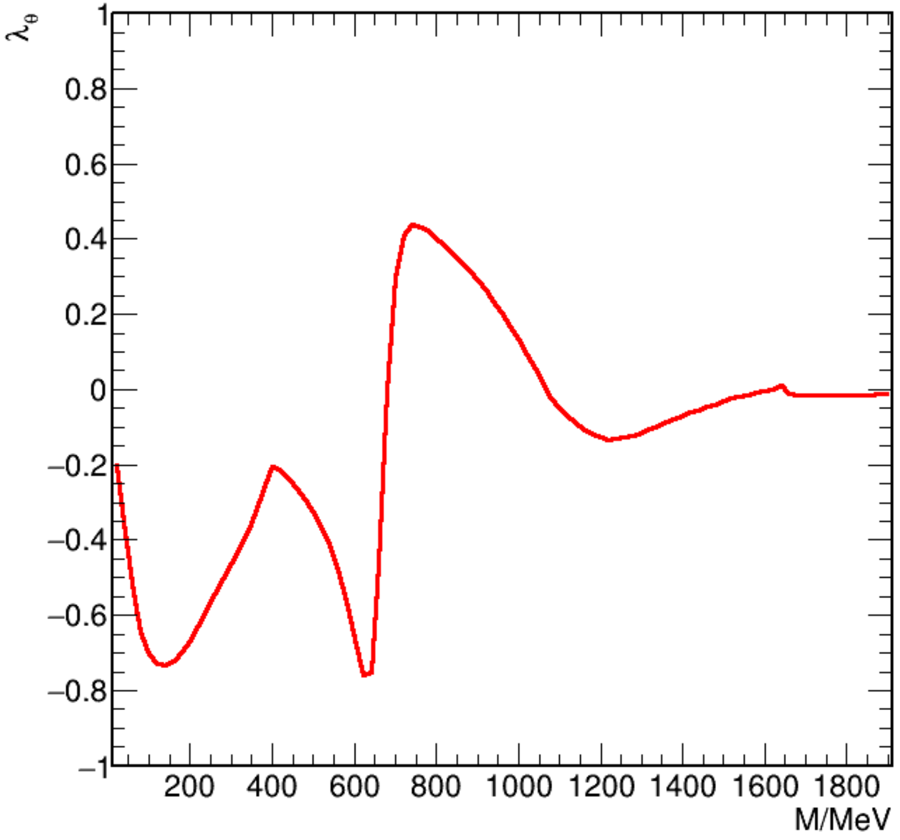
Example: T=40 & μ_B=890, p=450 MeV





Example: T=40 & μ_B=890, p=500 MeV





Example: T=40 & μ_B=890, p=550 MeV

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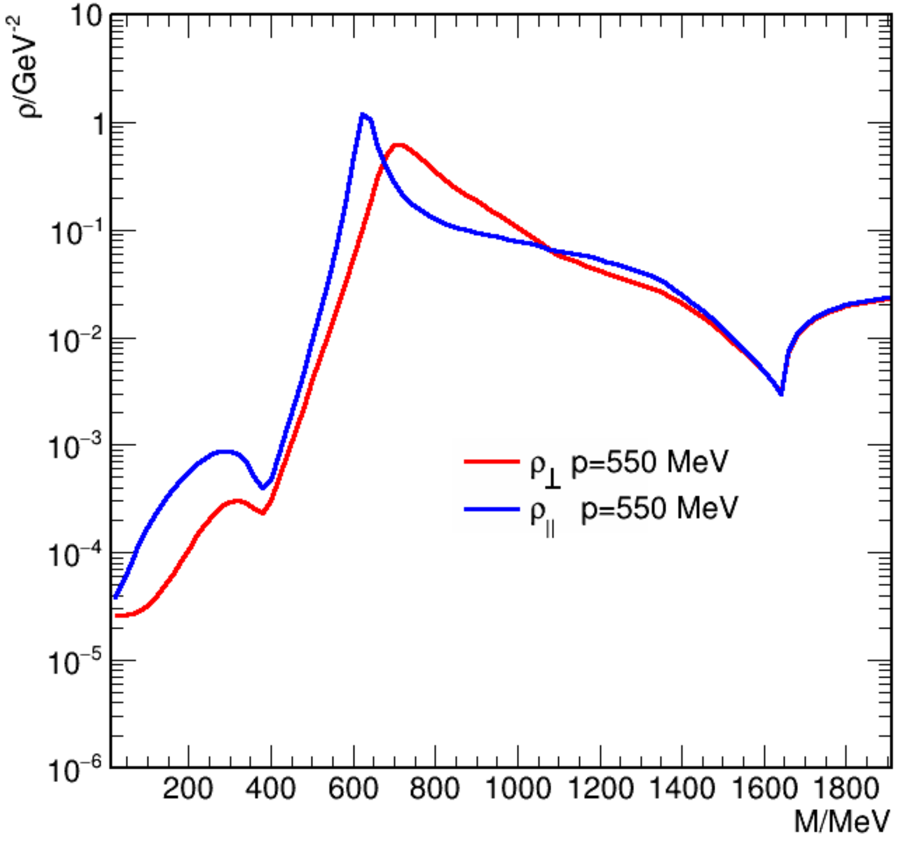
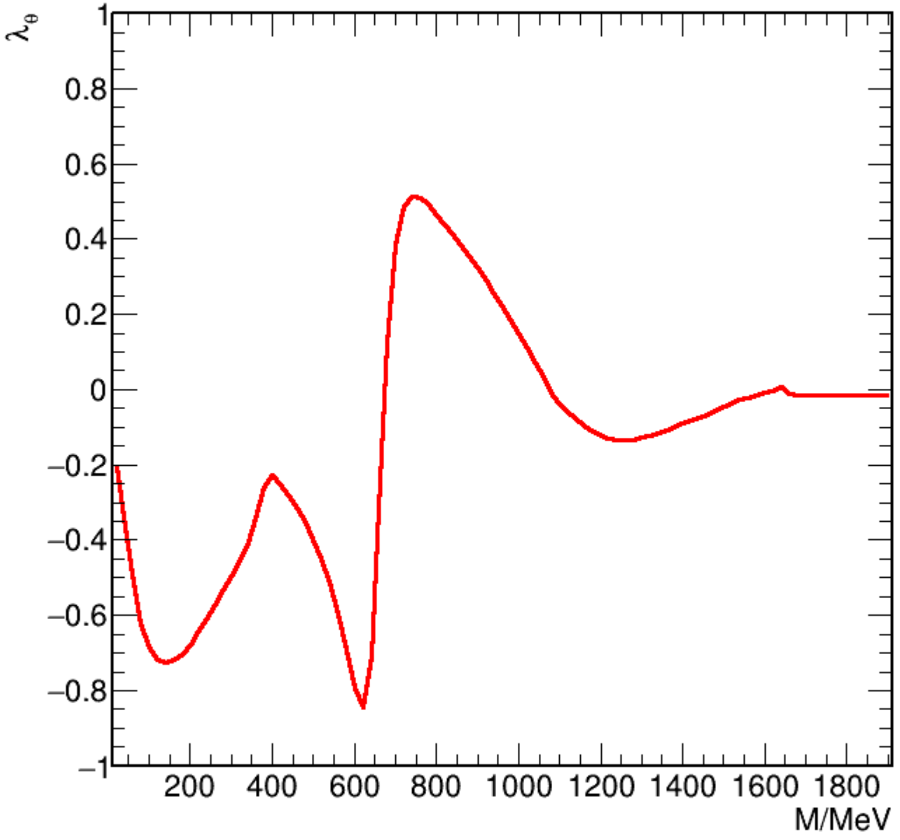
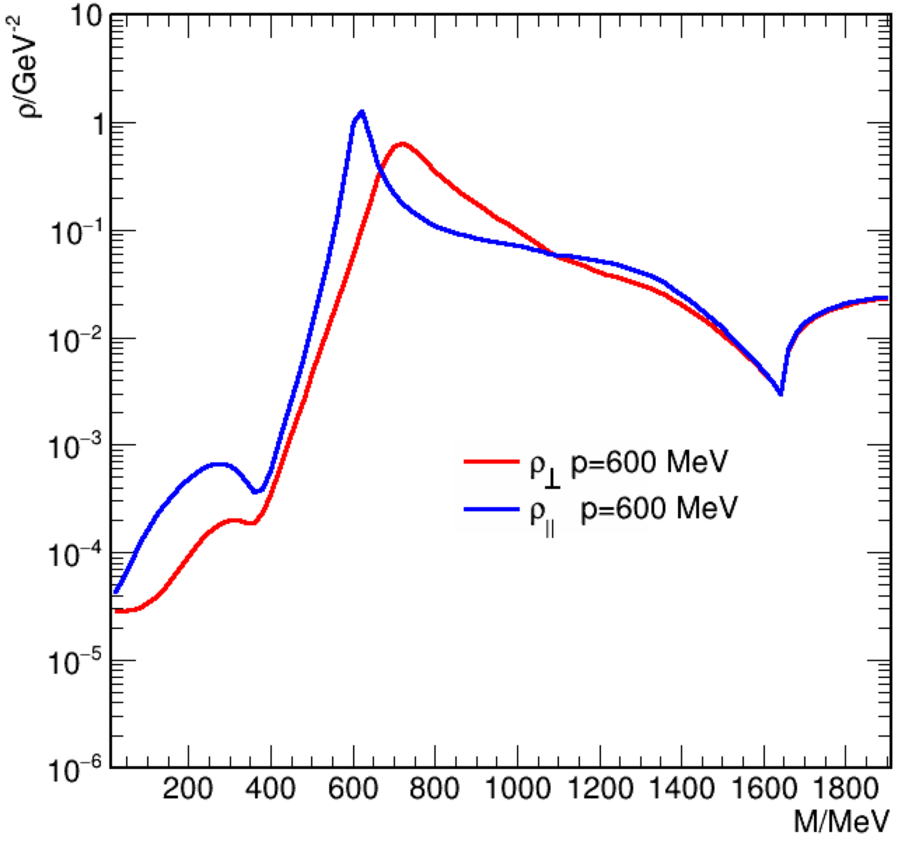
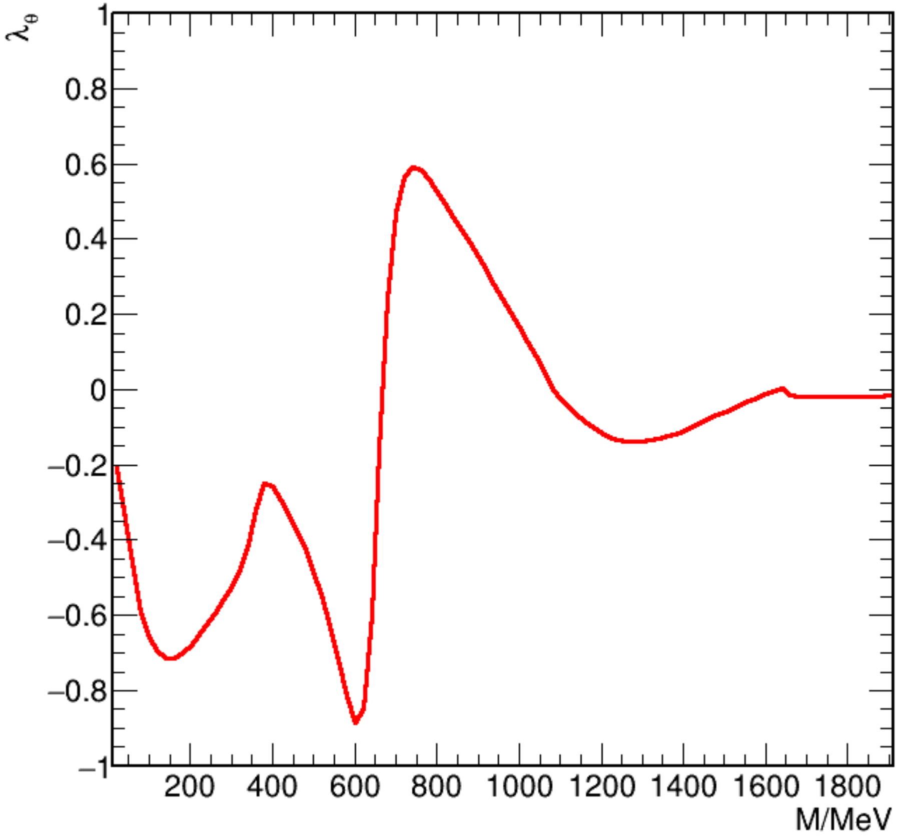


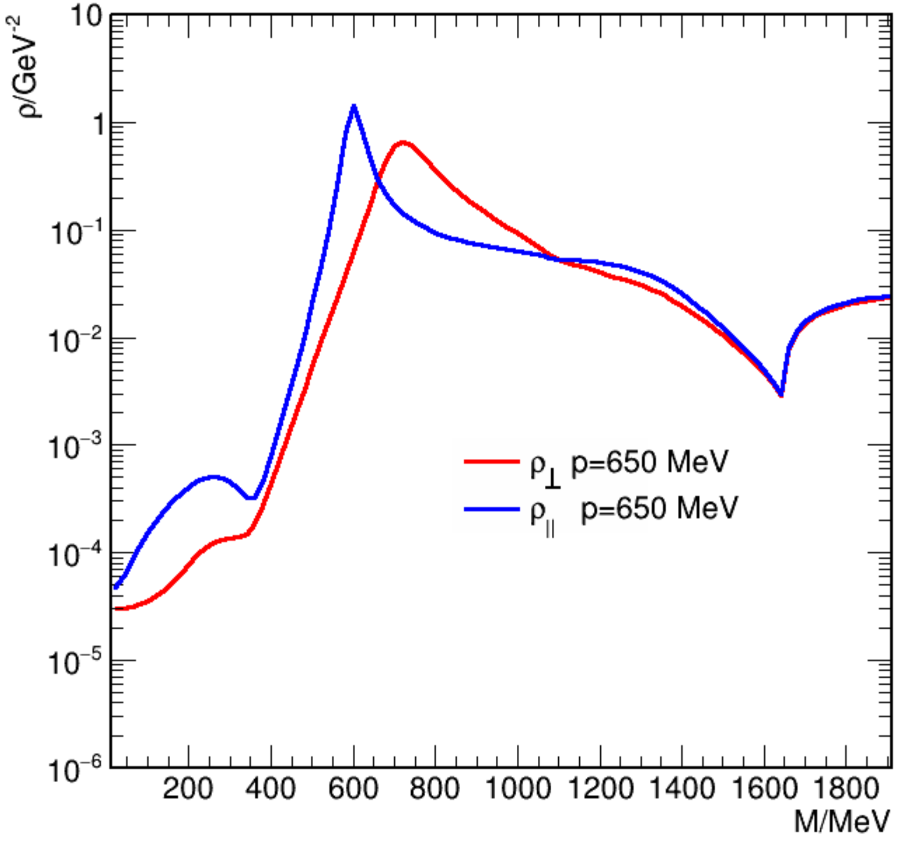


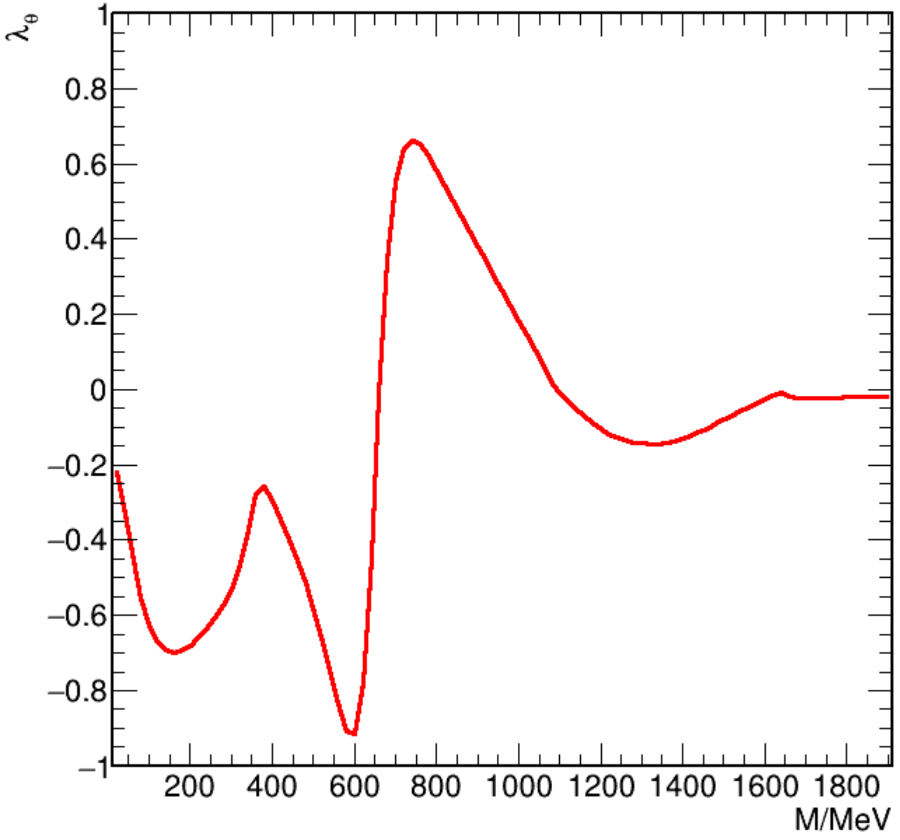
Example: T=40 & μ_B=890, p=600 MeV





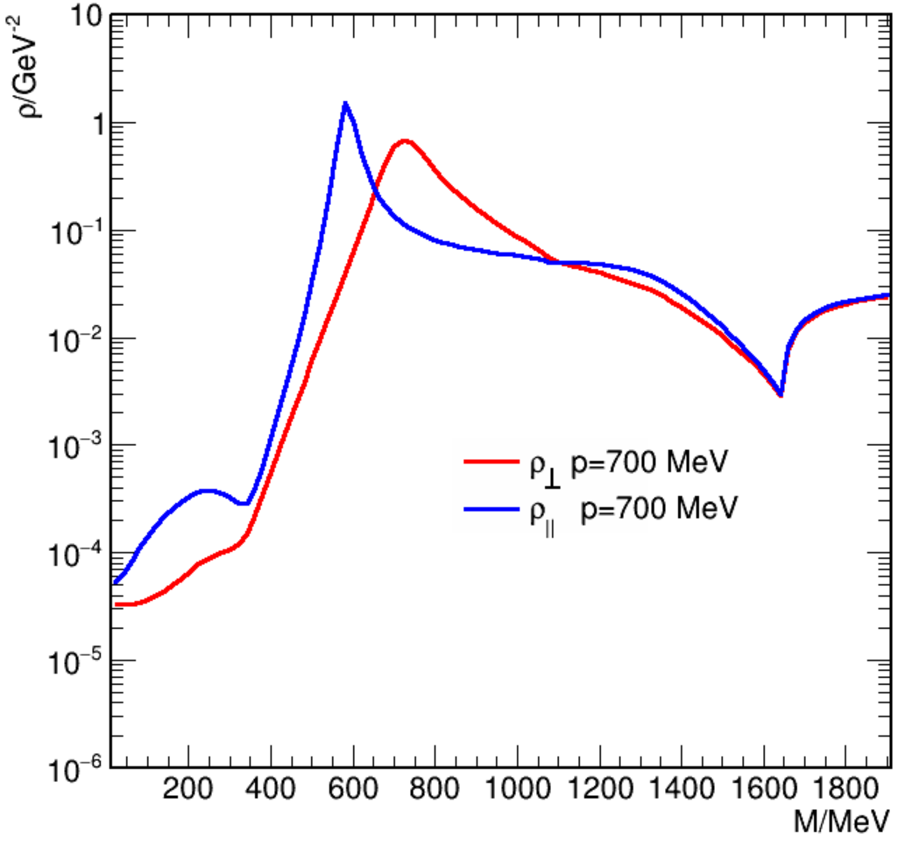
Example: T=40 & μ_B=890, p=650 MeV

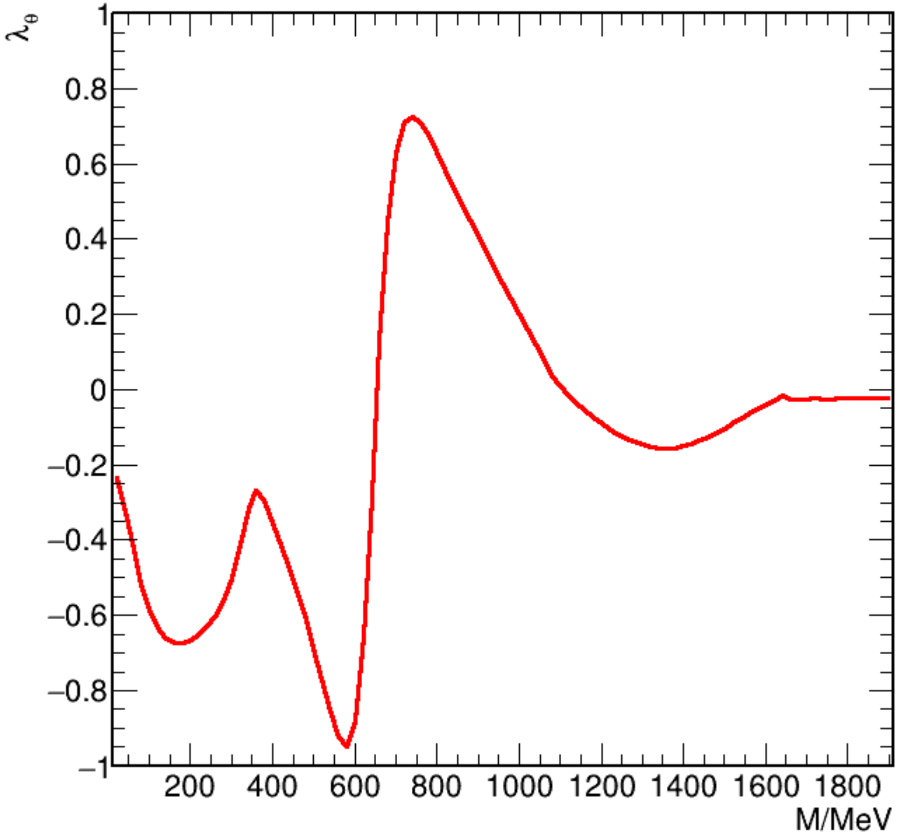




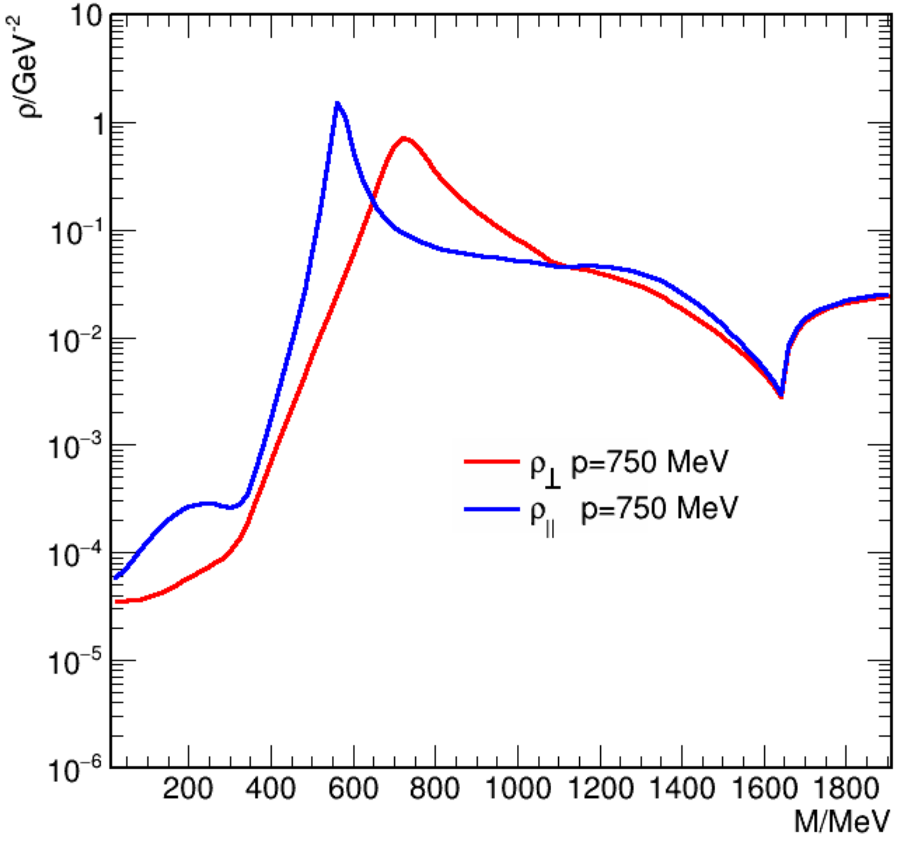
Example: T=40 & μ_B=890, p=700 MeV

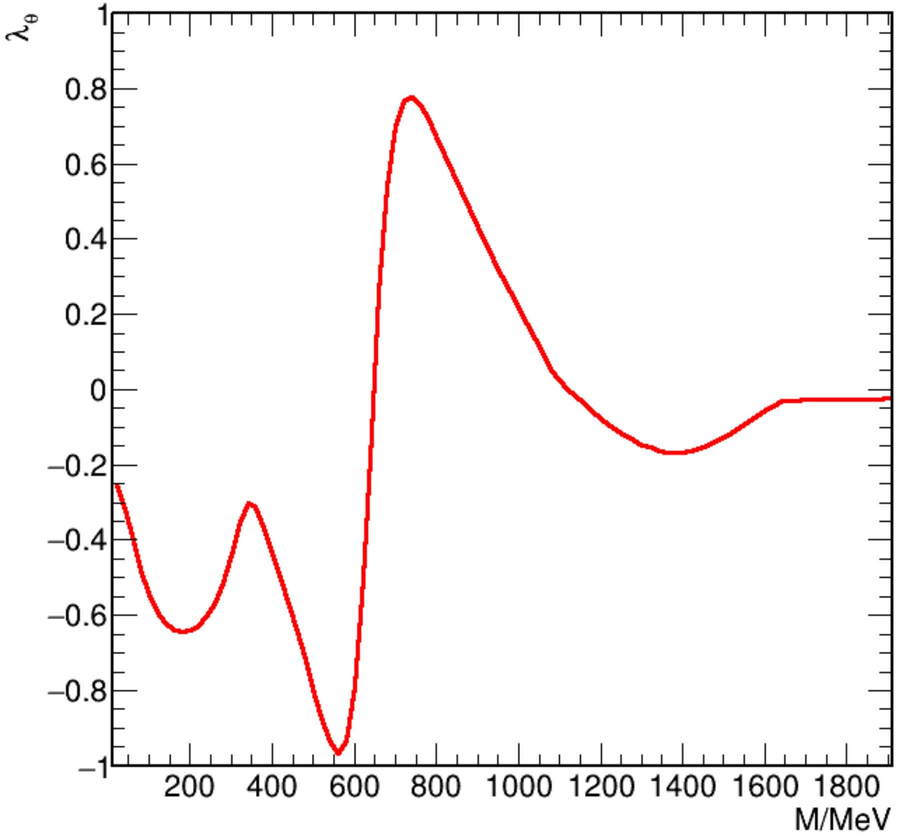
CRC-TR 211



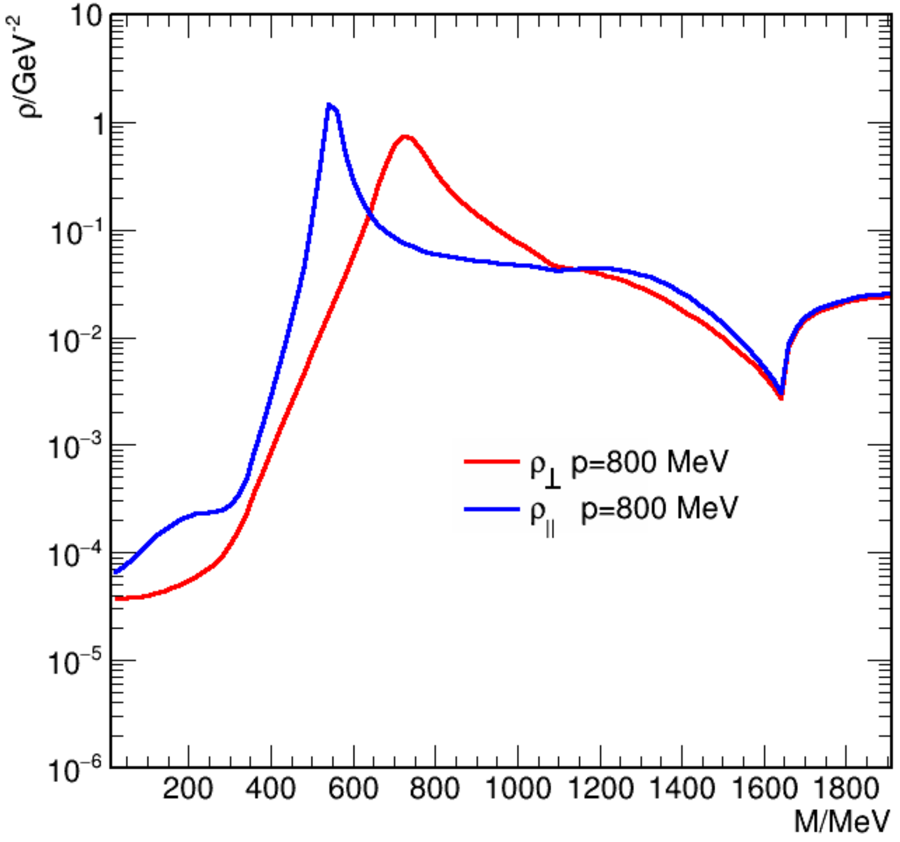


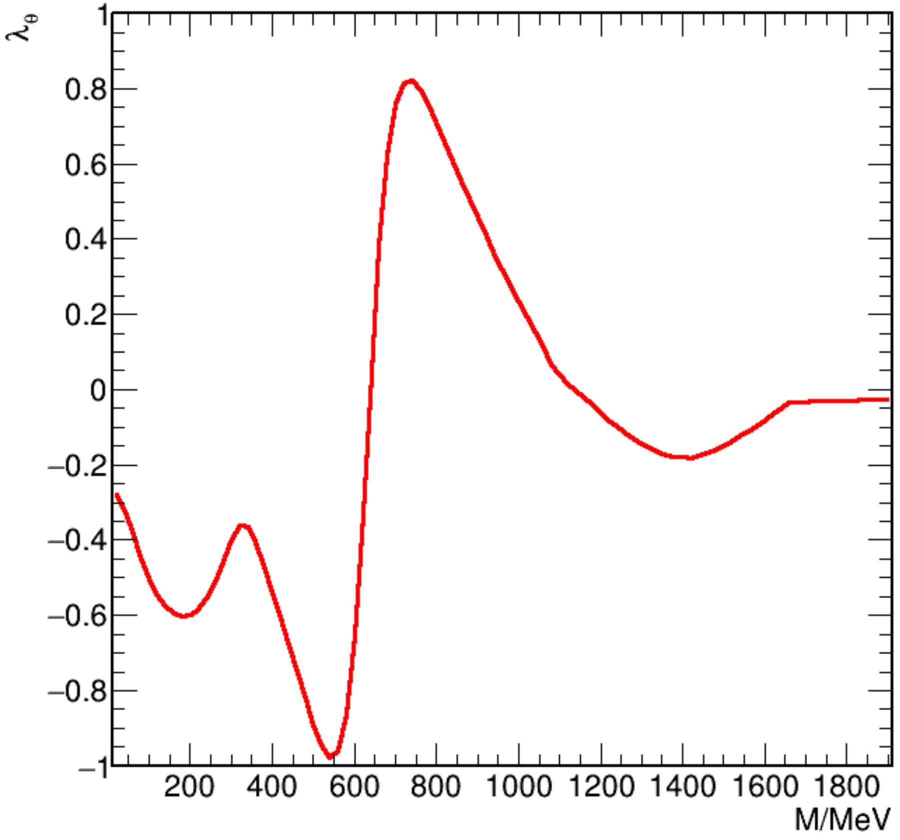
Example: T=40 & μ_B=890, p=750 MeV



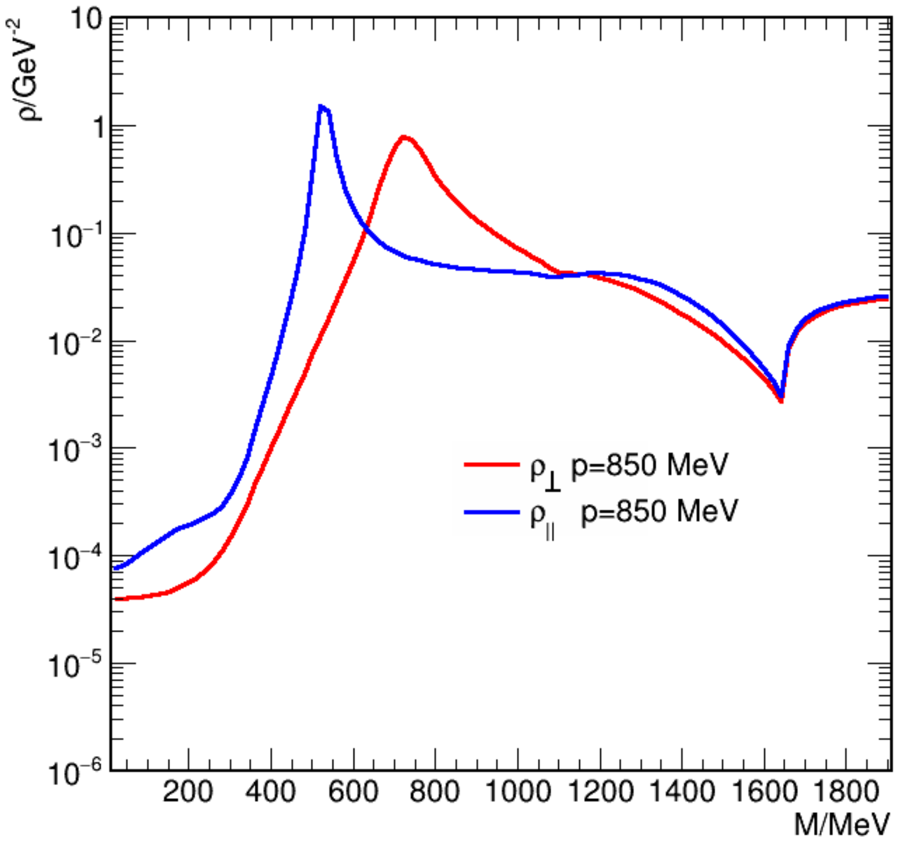


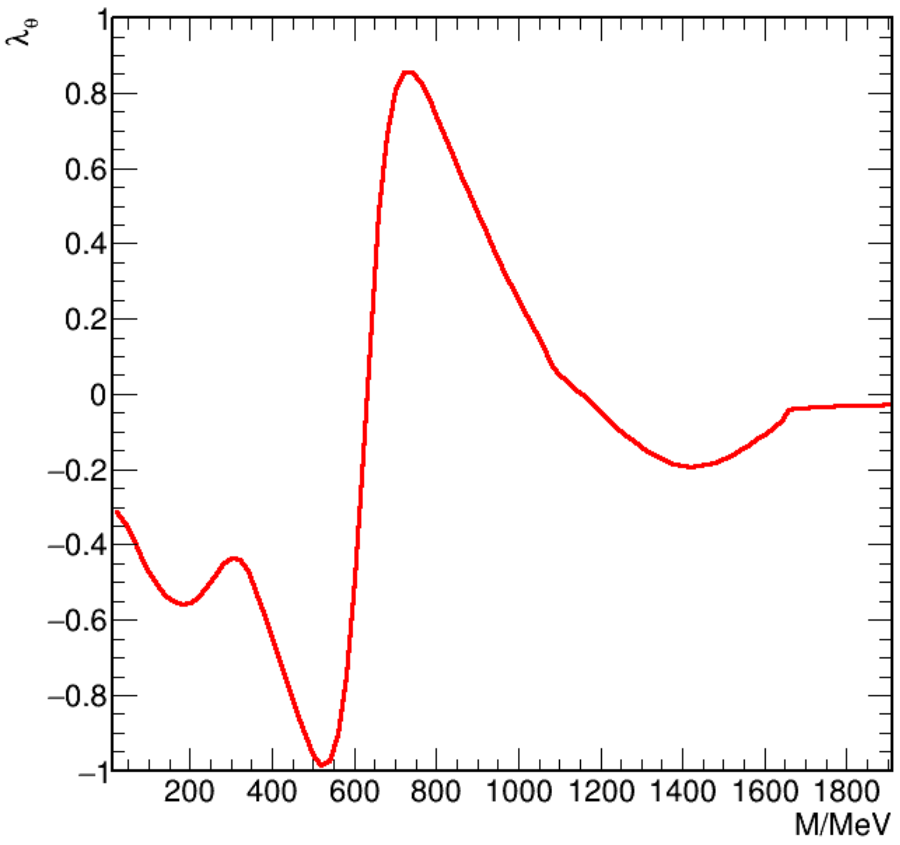
Example: T=40 & μ_B=890, p=800 MeV



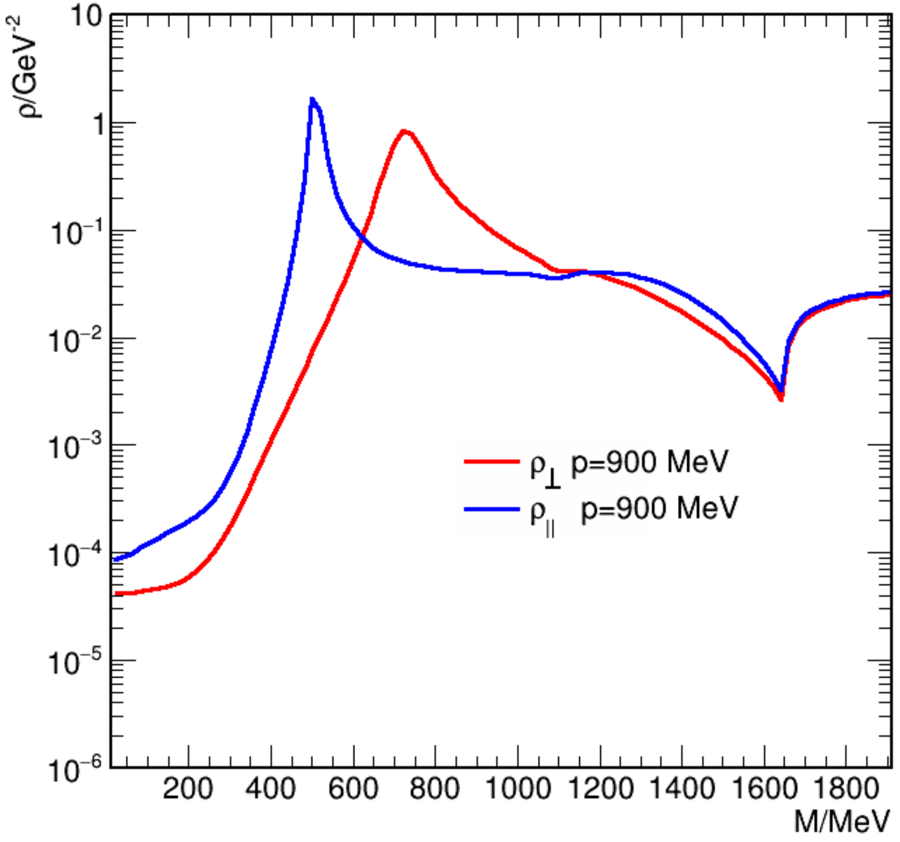


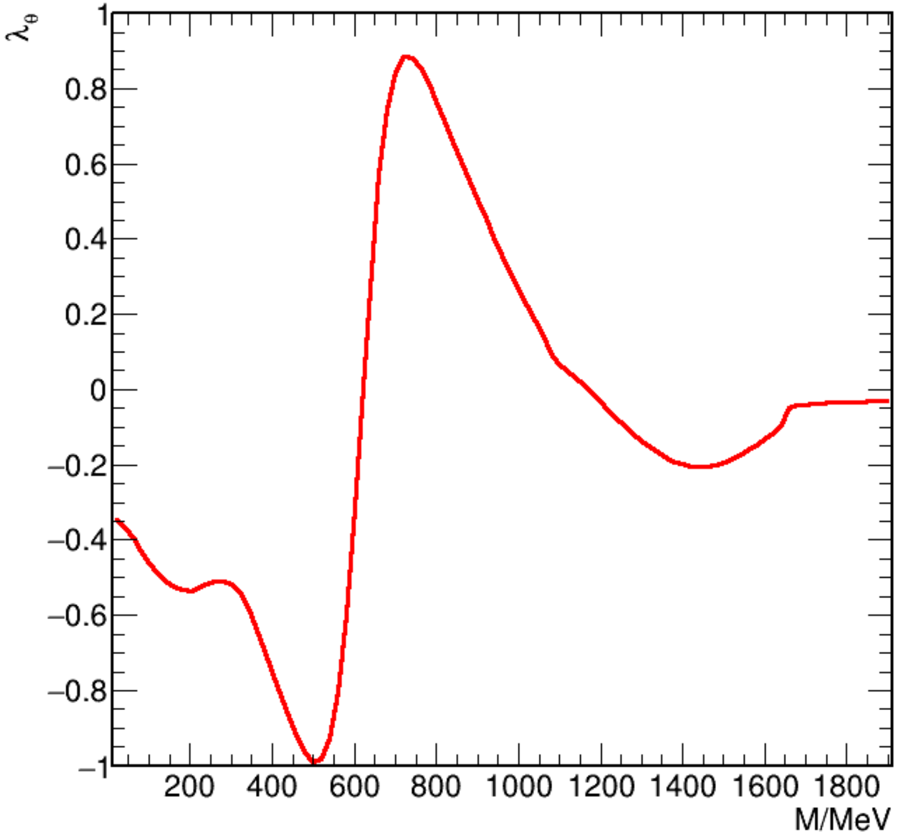
Example: T=40 & μ_B=890, p=850 MeV



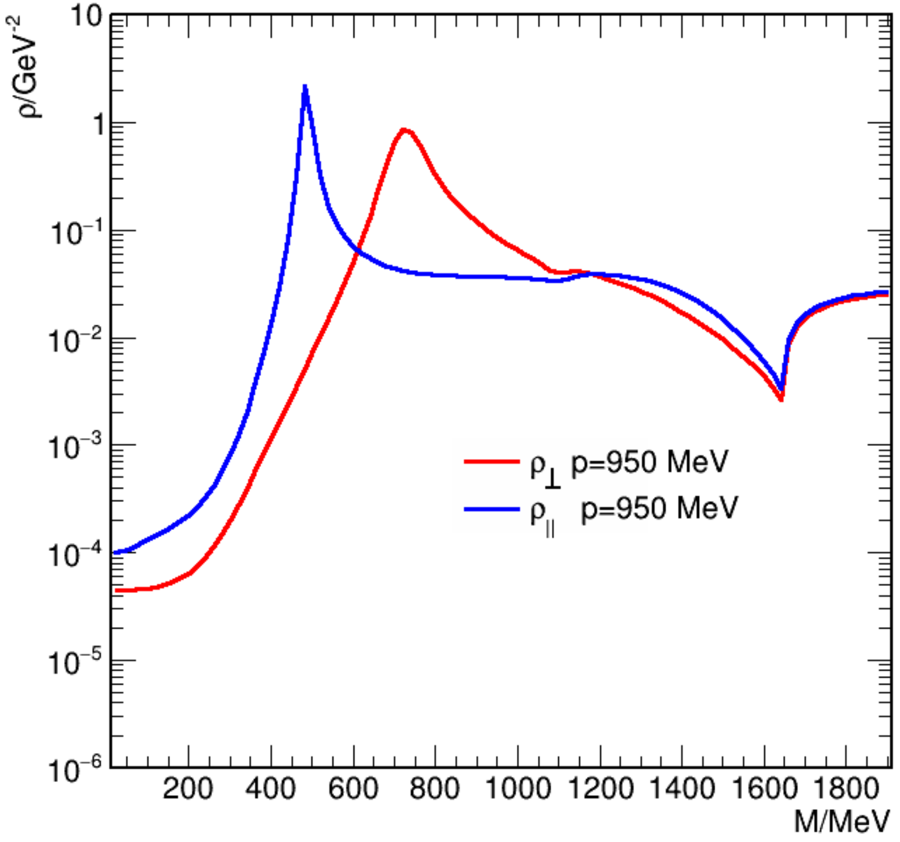


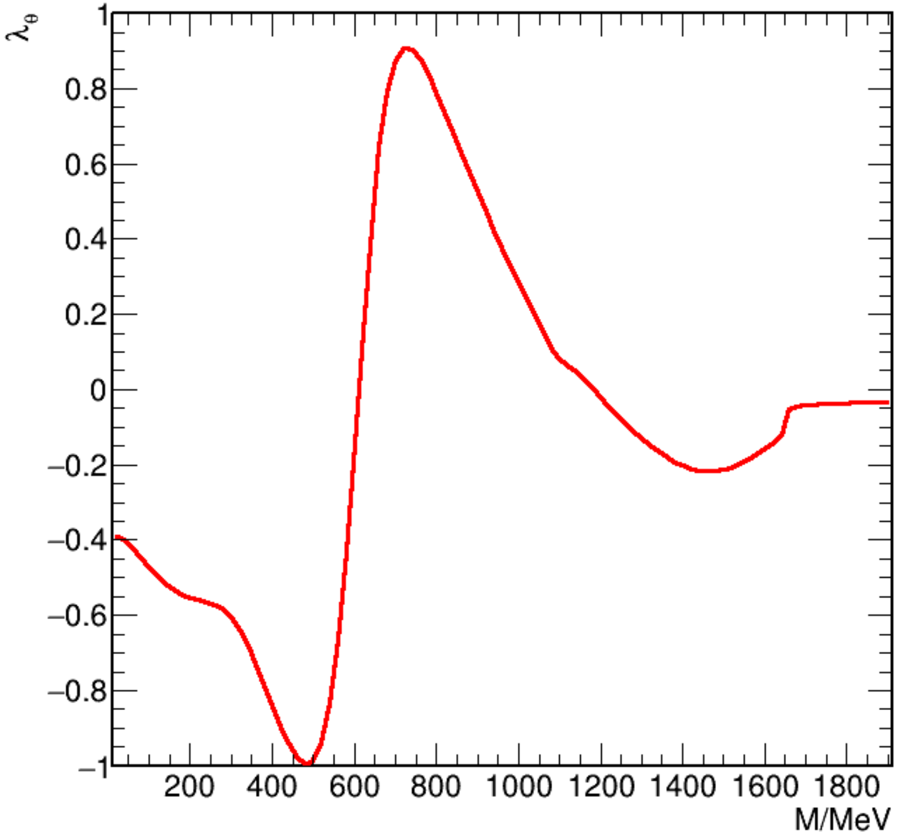
Example: T=40 & μ_B=890, p=900 MeV



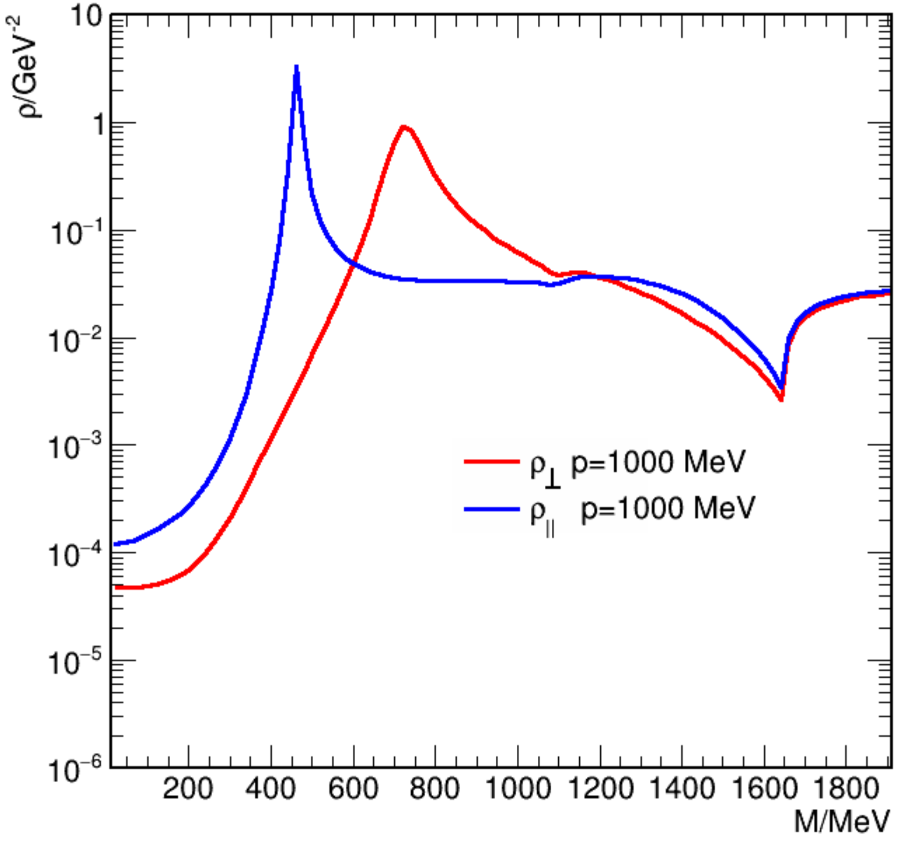


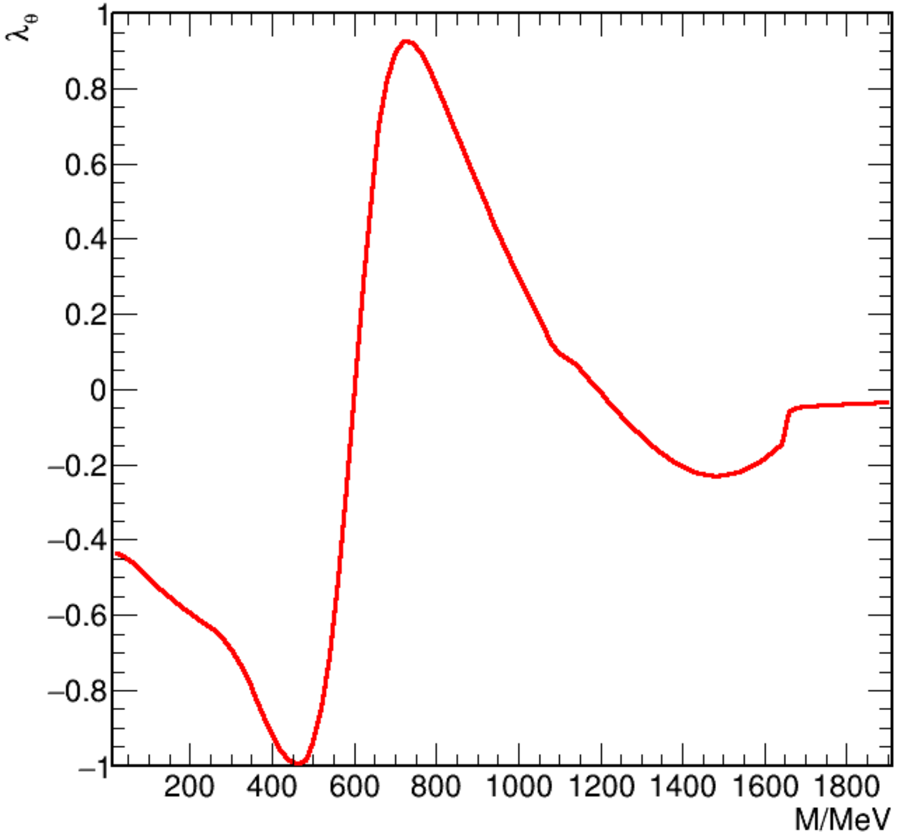
Example: T=40 & μ_B=890, p=950 MeV



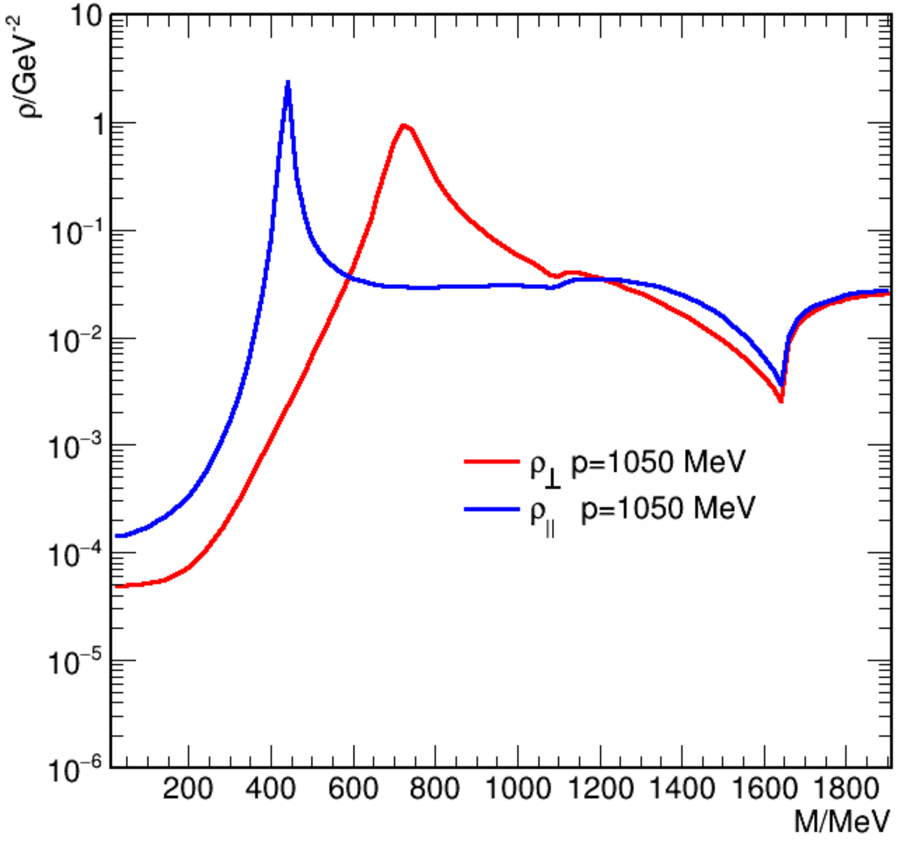


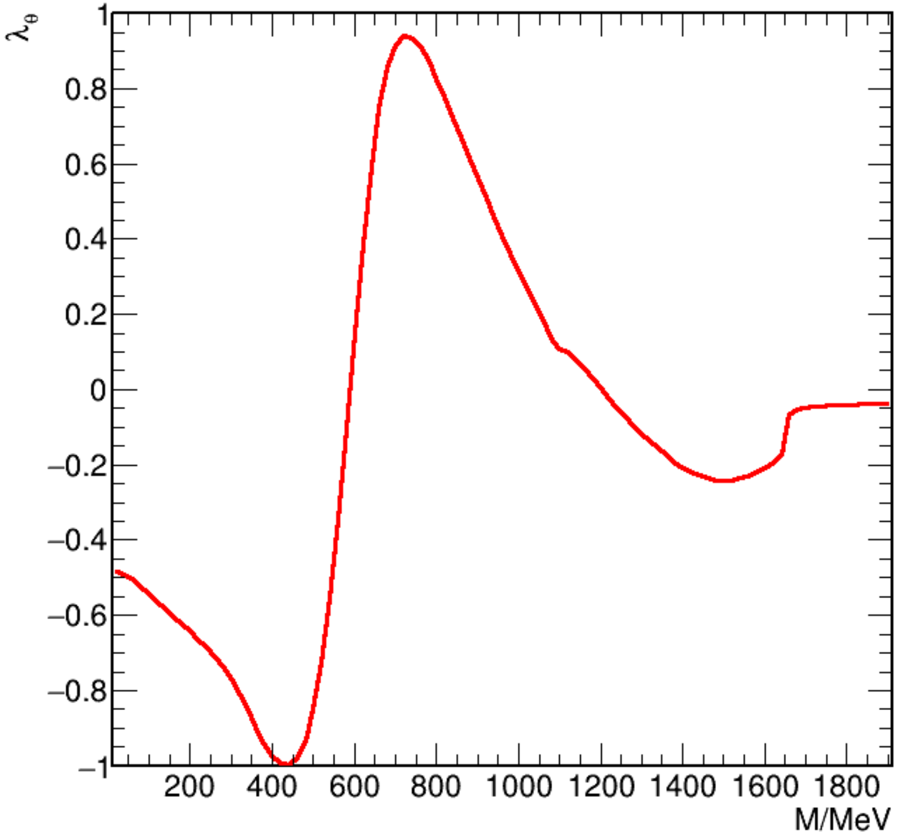
Example: T=40 & μ_B=890, p=1000 MeV



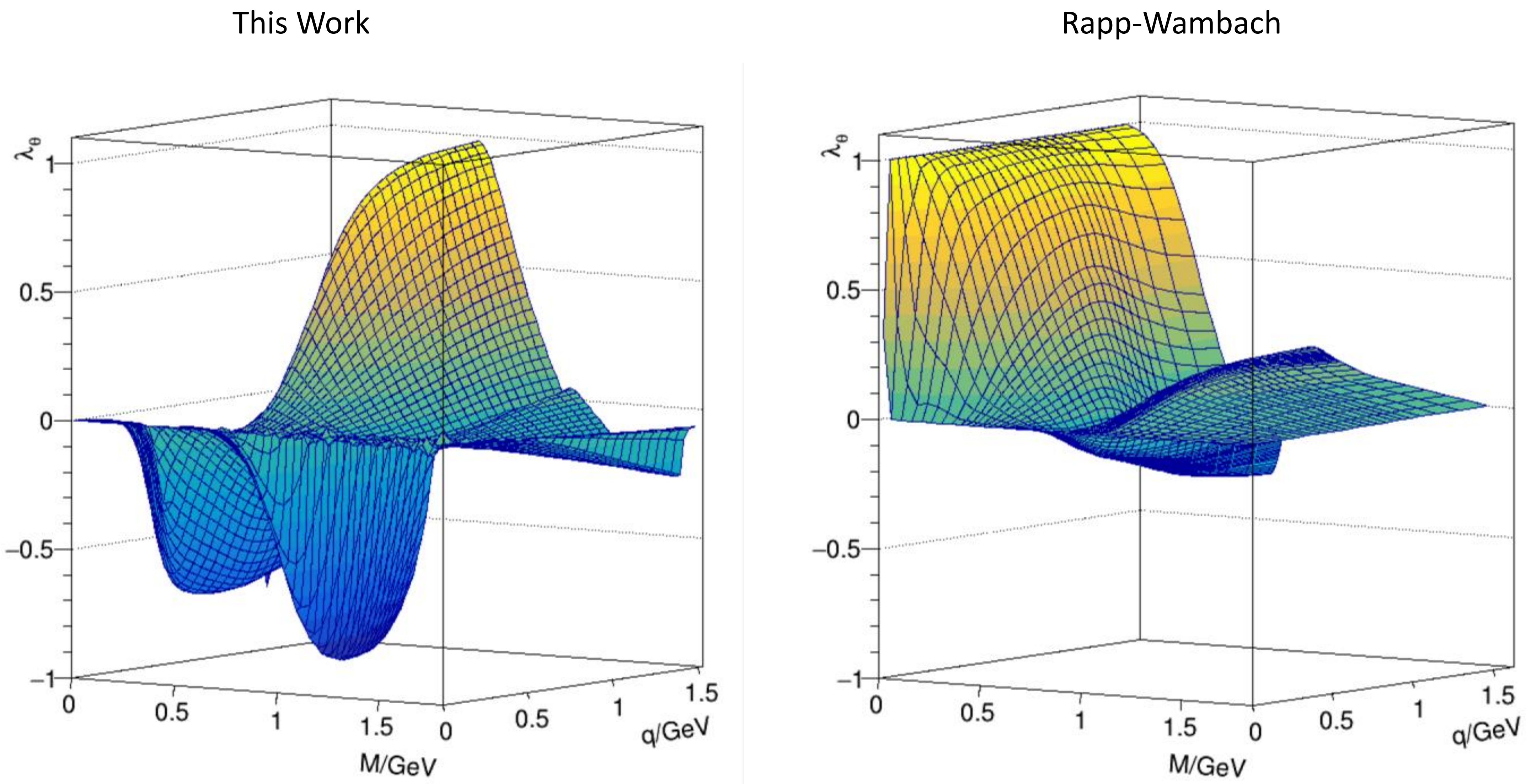


Example: T=40 & μ_B=890, p=1050 MeV

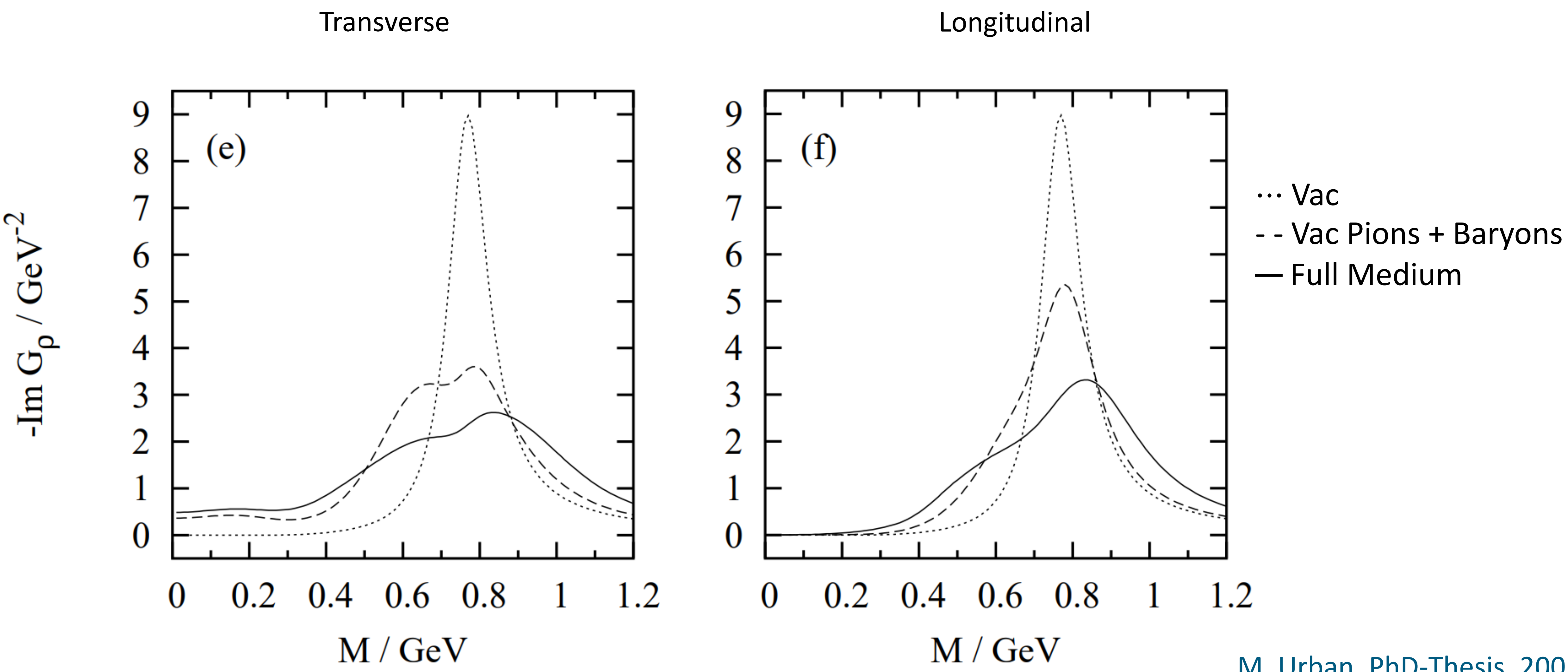




Polarisation: Comparison to Rapp-Wambach



Medium Effects: Transverse vs. Longitudinal

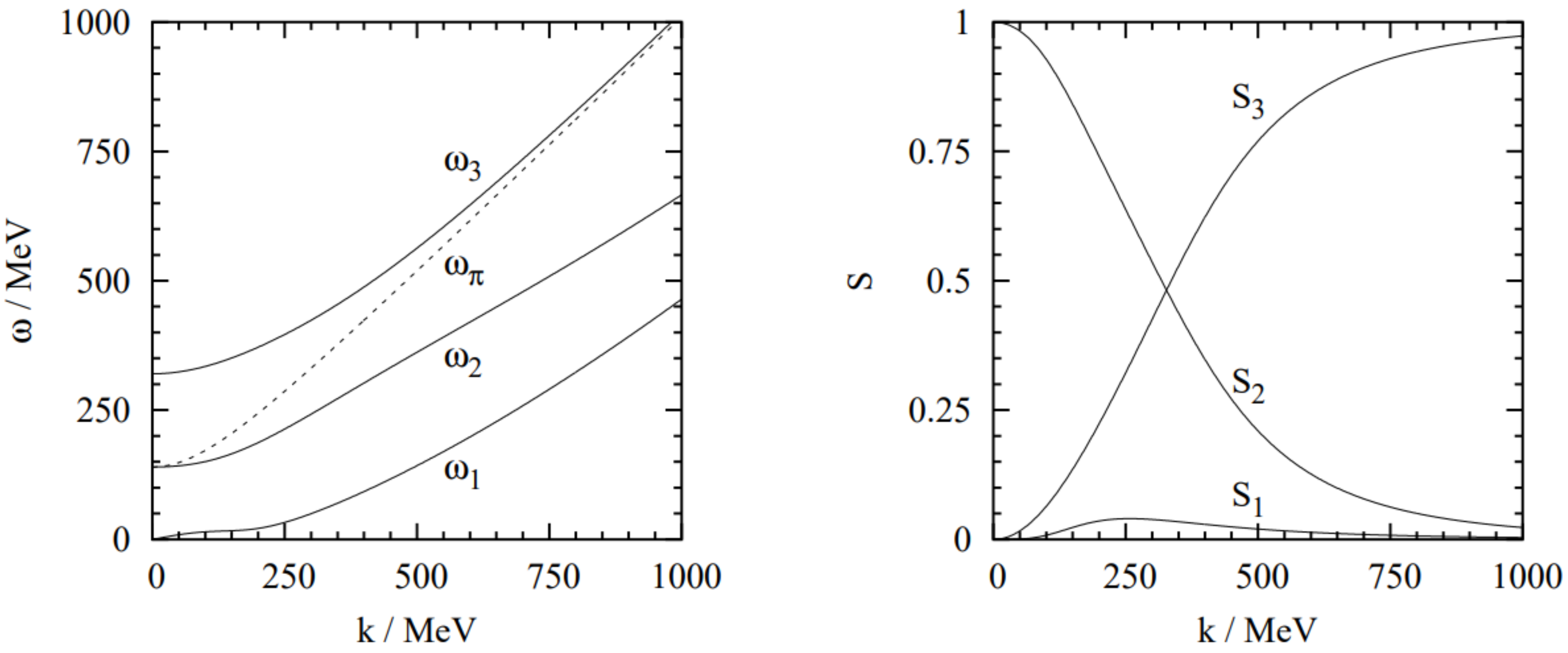


M. Urban, PhD-Thesis, 2001

Migdal parameters! Medium oszillations?

Introduce migdal parameters: $$\Pi = \frac{\Pi_{Nh} + \Pi_{\Delta h} - (g'_{11} - 2g'_{12} + g'_{22})\Pi_{Nh}\Pi_{\Delta h}}{1 - g'_{11}\Pi_{Nh} - g'_{22}\Pi_{\Delta h} + (g'_{11}g'_{22} - g'^{2}_{12})\Pi_{Nh}\Pi_{\Delta h}}$$

For T=0, μ_B small(ish): Approximate propagator with Three-level-system: $$G_\pi(k) = \frac{S_1(\vec{k})}{k_0^2 - \omega_1^2(\vec{k})} + \frac{S_2(\vec{k})}{k_0^2 - \omega_2^2(\vec{k})} + \frac{S_3(\vec{k})}{k_0^2 - \omega_3^2(\vec{k})}$$



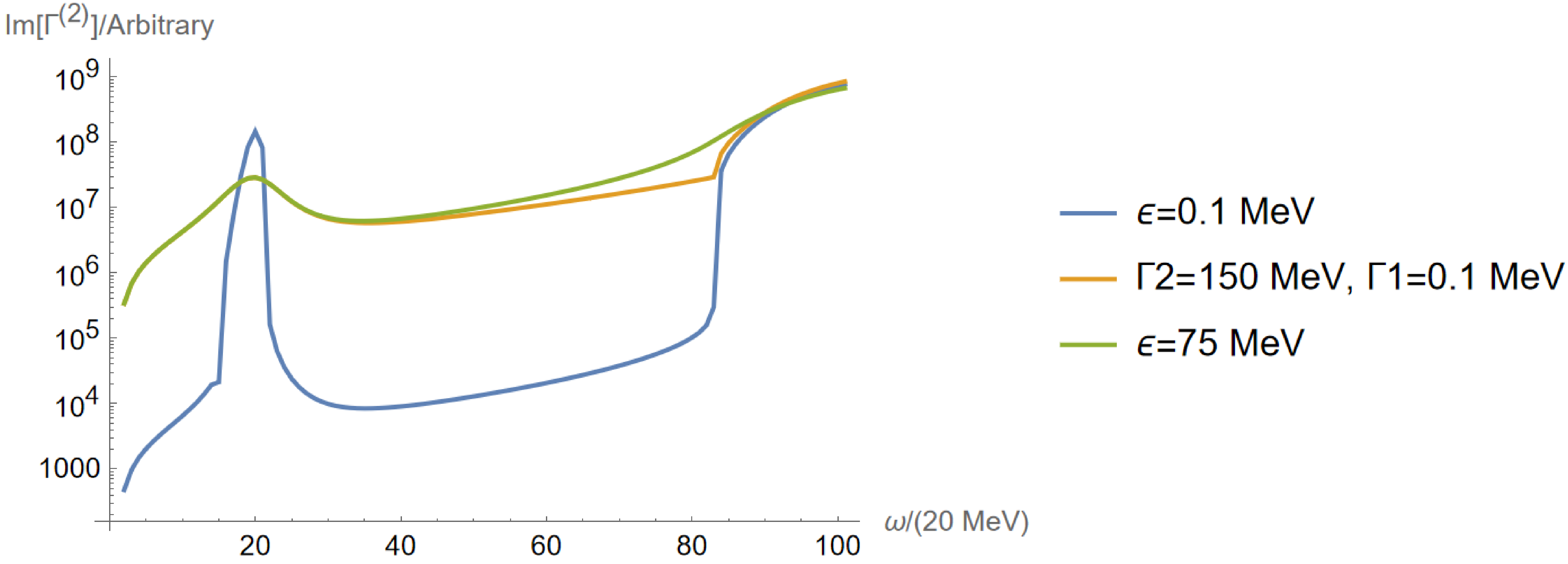
M. Urban, PhD-Thesis, 2001

Inclusion of more baryon species?

- Parity doubling with additional resonances?
- Spin 3/2 resonances?
- Widths?
  - Would increase yield dramatically
- Could e.g. follow Urban: Analytical continuation, with iΓ instead of $i\epsilon$
- $p0 \rightarrow -I(\omega + I\ \epsilon)$
- $p0 \rightarrow -I\left(\omega + I\,\frac{\Gamma_\mathrm{i}+\Gamma_j}{2}\right)$

Baryonic contribution at 0 momentum, widths

(T=50 MeV, μ=890 MeV)



Summary and Outlook

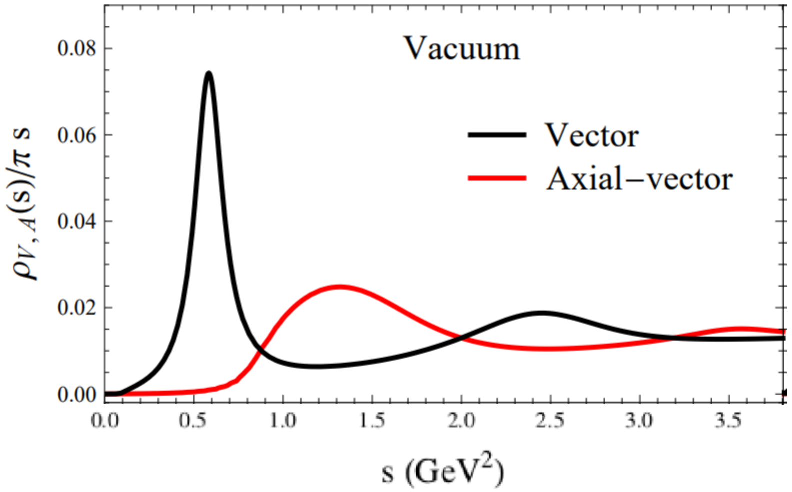
CRC-TR 211

- Dilepton invariant mass spectra could, at low momenta, provide information on mirror baryon scenario
- Finite momentum modifies low energy limit strongly, increases dilepton production
  - Peak structure is washed out integrating over momenta
- Dilepton polarization provides additional insights
  - Provided the rich structure obtained in the PDM survives the inclusion of additional effects
- More baryons, medium modifications of pions to be included!

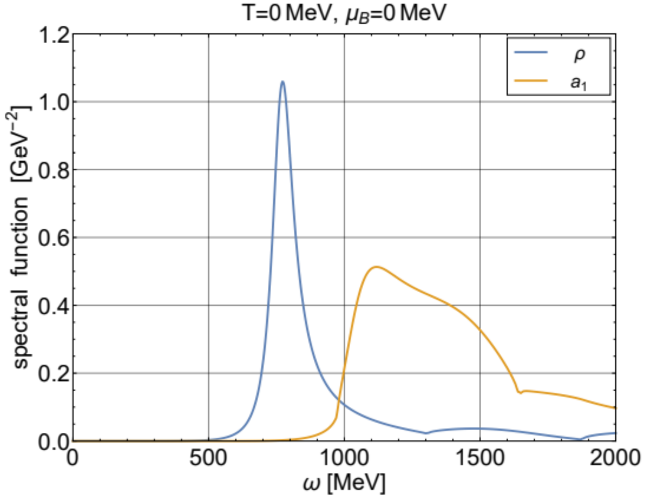
Appendix

Spectral function

- More precisely: An electromagnetic spectral function
  - Even more precisely: A vector-meson spectral function
  - Which is equivalent up to a constant in the Vector Dominance Model
- Mainly two:
  - Hadronic many body approach by Rapp
  - FRG approach by Tripolt



Rapp, Physics Letters B,Volume 731, 2014, Pages 103-109



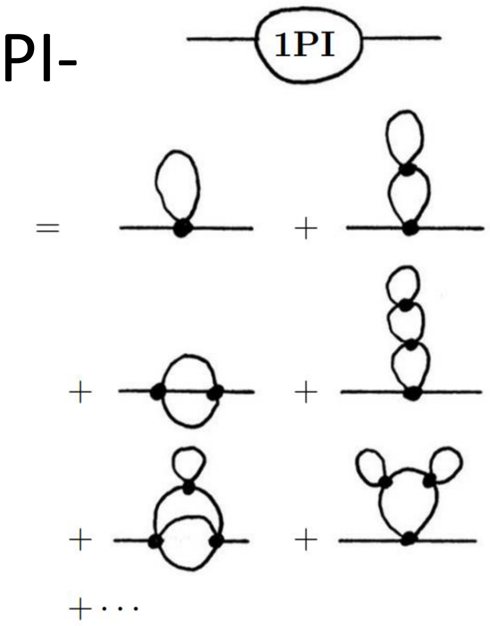
Tripolt, Phys.Rev.D 104 (2021)

Guide to deriving the Wetterich Equation

- Legendre-Transform of $\log Z[J]$ gives generating functional of 1-PI-Diagrams
  - effective action Γ
  - 1PI: Diagrams, which cannot be cut in 2 parts by cutting 1 line
- Add a fictional mass term $\propto m_k^2 \phi^2$ to Lagrangian

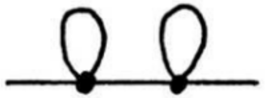
$$Z[J] = \int D\phi \exp(-\int d^4x\, \mathcal{L}[\phi(x)] + J(x)\phi(x))$$
$$Z_k[J] = \int D\phi \exp\left(-\int d^4x\, \mathcal{L}[\phi(x)] + J(x)\phi(x) + m_k^2 \phi(x)^2\right)$$

- Mass term m_k^2 often called „Regulator“ and written R_k



Lancaster, Blundell, QFT for gifted amateur

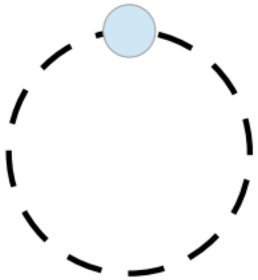
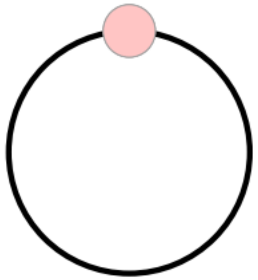
NOT 1-PI:



Guide to deriving the Wetterich Equation

- Mass term cuts away fluctuations with momentum scale $< k$
- Define Γ_k
  - Same as before, but averages action over Volume 1/k^3
  - Call Γ_k effective AVERAGE action
- Find change $\partial_k \Gamma_k$ with k
- Described by Wetterich equation
  - follows from very little assumptions of form of R_k
  - Uses in (but not limited to) QCD, magnets, condensed matter, statistical physics, critical phenomena

Quick Rundown:

$$\partial_k \Gamma_k = \frac{1}{2}$$  $$-$$ 

- Lines: (Euclidean) propagators
- Circles: Regulators $\propto \theta(k^2 - q^2)$
- Trace to be taken over all internal degrees of freedom: Isospin space, parity indices, fermion indices, Lorenz indices, internal momenta, etc.